DS 5240 Homework 1

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1. Let $\underline{u} = [2,7,1]^T$, $\underline{v} = [3\ 0\ 4]^T$, $\underline{w} = [1,5,8]^T$

Find: (a) $3\underline{u} - 4\underline{v}$

(b) $2\underline{u} + 4\underline{v} - 5\underline{w}$

a) $3\begin{bmatrix}2\\7\\1\end{bmatrix} - 4\begin{bmatrix}3\\0\\4\end{bmatrix} = \begin{bmatrix}6\\21\\3\end{bmatrix} - \begin{bmatrix}12\\0\\16\end{bmatrix} = \begin{bmatrix}-6\\21\\-13\end{bmatrix}$ ✓

b) $2\begin{bmatrix}2\\7\\1\end{bmatrix} + 4\begin{bmatrix}3\\0\\4\end{bmatrix} - 5\begin{bmatrix}1\\5\\8\end{bmatrix} = \begin{bmatrix}4\\14\\2\end{bmatrix} + \begin{bmatrix}12\\0\\16\end{bmatrix} - \begin{bmatrix}5\\25\\40\end{bmatrix} = \begin{bmatrix}11\\-11\\-22\end{bmatrix}$ ✓

2. Find the coefficients c_1, c_2, c_3 such that $\sum_{i=1}^{3} c_i \underline{u}_i = [0,3,5]^T$. Here, $\underline{u}_1 = [1,-1,1]^T$, $\underline{u}_2 = [1,3,2]^T$, and $\underline{u}_3 = [2,0,1]^T$.

$$c_1 u_1 + c_2 u_2 + c_3 u_3 = \begin{bmatrix}0\\3\\5\end{bmatrix}$$

$$c_1\begin{bmatrix}1\\-1\\1\end{bmatrix} + c_2\begin{bmatrix}1\\3\\2\end{bmatrix} + c_3\begin{bmatrix}2\\0\\1\end{bmatrix} = \begin{bmatrix}0\\3\\5\end{bmatrix}$$

$c_1 + c_2 + 2c_3 = 0$

$-c_1 + 3c_2 + 0 = 3$

$c_1 + 2c_2 + c_3 = 5$

$c_1 = -c_2 - 2c_3$

$3c_2 - c_1 = 3 \Rightarrow 3c_2 + c_2 + 2c_3 = 3$

$4c_2 + 2c_3 = 3$

$-c_2 - 2c_3 + 2c_2 + c_3 = 5$

$c_2 - c_3 = 5$

$c_2 = 5 - c_3$

$20 - 4c_3 + 2c_3 = 3$

$20 - 2c_3 = 3 \Rightarrow \boxed{c_3 = 17/6}$ -0.5

$\boxed{c_1 = 7/2}$ ✓

$\boxed{c_2 = 13/6}$ ✓

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3. Find $\underline{u}\cdot\underline{v} = \underline{u}^T\underline{v}$, where $\underline{u} = [2, 3, 5, 7]^T$ and $\underline{v} = [1, -1, 1, -1]^T$.

$$\begin{bmatrix} 2 \\ 3 \\ 5 \\ 7 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad u\cdot v \quad 2-3+5-7=-3 \qquad u^T v \quad \begin{bmatrix} 2 & 3 & 5 & 7 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = 2-3+5-7=-3$$

$$\boxed{u\cdot v = u^T v}$$

4. Let $\underline{u} = [5, 4, 1]^T$, $\underline{v} = [3, -4, 1]^T$, $\underline{w} = [1, -2, 3]^T$.

(a.) Compute the generalized cosine between all possible pairs.

(b.) Which pair of distinct vectors is most colinear? $u\cdot v = 0, 180$

(c.) Which pair are most orthogonal?

(d) Are $\underline{u}, \underline{v}$, and $\underline{w}$ linearly independent?

$$\cos\theta = \frac{u\cdot v}{\|u\|\|v\|}$$

a) u, v : $\dfrac{15-16+1}{\sqrt{25+16+1}+\sqrt{9+16+1}} = 0$

$u, w = \dfrac{5-2+3}{\|u\|\|w\|} = 0$

$w, v = \dfrac{3+2+3}{\sqrt{9+16+1}\sqrt{1+4+9}} = \dfrac{14}{\sqrt{26}\sqrt{14}} = 0.73$

b) for colinearity angle should be 0 or 180

∴ No vectors have angle 0, 180

(u, v) most colinear

c) (u, v) and (u, w) are orthogonal

d) $a\begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix} + b\begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} + c\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$

$5a+3b+c=0$ ①

$4a-4b-2c=0$

$a+b+3c=0$

$5a-3b+c=0$

$5a+c=3b$

$3b+3b=0$

$\boxed{b=0}$ $\boxed{a=0}$

$5a+c=0$

$5a+15c=0$

$-14c=0$

$\boxed{c=0}$

∴ $a=b=c=0$

∴ u, v, w are linearly independent

5. Let $\underline{u} = [1, 2, 1]^T$ and $\underline{v} = [3, 5, 8]^T$. Find

Find:

(a) the generalized cosine between $\underline{u}$ and $\underline{v}$,

(b) the projection of $\underline{u}$ onto $\underline{v}$,

(c) the L_2 distance between $\underline{u}$ & $\underline{v}$,

(d) the L_1 distance between $\underline{u}$ & $\underline{v}$.

a) $$\frac{3+10+8}{\sqrt{1+4+1}\,\sqrt{9+25+64}} = \frac{21}{\sqrt{6}\,\sqrt{98}} = \sqrt{3}/2$$

b) vectorProj u onto v $= \frac{v \cdot u}{|v|^2} \cdot v$

$$= \frac{3+10+8}{9+25+64} \cdot [3\ 5\ 8]^T = \frac{\cancel{21}^{\,3}}{\cancel{98}_{\,14}} [3\ 5\ 8]^T = \frac{3}{14}[3\ 5\ 8]^T$$

Scalar projection u onto v

$$\frac{u \cdot v}{|v|} = \frac{21}{\sqrt{98}} = \frac{21}{\sqrt{7 \cdot 7 \cdot 2}} = \frac{3}{\sqrt{2}}$$

c) L_2 distance $= \sqrt{(3-1)^2 + (5-2)^2 + (8-1)^2} = \sqrt{4+9+49} = \sqrt{62}$

d) L_1 distance $= |3-1| + |5-2| + |8-1| = |2| + |3| + |7| = 12$

6. A hyperplane can be described by two vectors $\underline{p}$ and $\underline{w}$, and is the set of vectors $\underline{x}$ such that $(\underline{x} - \underline{p})^T \underline{w} = \underline{0}$.

In $\mathbb{R}^2$, a hyperplane consists of all vectors $[x_1\ x_2]^T$ such that $x_2 = m x_1 + b$, for fixed m and b. Find $\underline{p}$ and $\underline{w}$ in terms of m and b. Let $\underline{p} = [0, ?]^T$ and $\underline{w} = [?\ 1]$ for unique answers.

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Given $x_2 = mx_1 + b$

Consider $P = [0, P_1]^T$

$w = [w_1, 1]^T$

Given $(x-P)^T w = 0$. we can define $[x_1, x_2]^T = x$

$x = \begin{bmatrix} x_1 \\ mx_1 + b \end{bmatrix}$ since $w \cdot (x-P)^T = 0$ and are orthogonal

$$\left(\begin{bmatrix} x_1 \\ mx_1 + b \end{bmatrix} - \begin{bmatrix} 0 \\ P_1 \end{bmatrix} \right)^T \begin{bmatrix} w_1 \\ 1 \end{bmatrix} = 0$$

$w_1 x_1 + mx_1 + b - P_1 = 0 \Rightarrow w_1 x_1 - P_1 = -(mx_1 + b)$

$w_1 x_1 - P_1 = -mx_1 - b$

comparing corresponding coefficients

$w_1 = -m$

$P = b$

$\therefore P = [0, b]^T$

$w = [-m, 1]^T$

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7. Write the vector $\underline{z} = [1\ 0\ 5]^T$ as a linear combination of $\underline{p}_1 = [1\ 0\ 1]^T$, $\underline{p}_2 = [1\ 1\ 3]^T$, and $\underline{p}_3 = [2\ 0\ -1]^T$.

$$\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$$c_1 + c_2 + 2c_3 = 1 \quad \Rightarrow \quad c_1 + 2c_3 = 1$$

$$c_2 = 0$$

$$c_1 + 3c_2 - c_3 = 5 \qquad c_1 - c_3 = 5$$

$$3c_3 = -4$$

$$c_3 = -4/3$$

$$c_1 = 11/3$$

$$c_2 = 0$$

$$\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \frac{11}{3} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + 0 + \frac{-4}{3} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 3 & -1 \end{bmatrix} \begin{bmatrix} 11/3 \\ 0 \\ -4/3 \end{bmatrix}$$

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8. Find all t, k so that $\underline{u} = [t \ k]^T$ and $\underline{v} = [3 \ 5]^T$ are orthogonal and $\underline{u}$ has unit length.

$u \cdot v = 0$

$3t + 5k = 0$

$t^2 + k^2 = 1$

$25k^2 + 9k^2 - 9 = 0$

$34k^2 = 9$

$k^2 = 9/34$

$k = \pm\sqrt{9/34}$ ✓

$t^2 = 1 - k^2$

$= 1 - \frac{9}{34}$

$= 25/34$

$t = \pm\sqrt{\frac{25}{34}}$

−0.5

9. Consider a hyperplane which is normal to $\underline{u}_1 = [1 \ -1 \ 1]^T$ and passes through $\underline{w} = [2 \ 2 \ 2]^T$. Prove or disprove that $\underline{x} = [2 \ 3 \ 3]$ is in the hyperplane.

If x is in hyperplane, it should be normal to u

i.e, $u \cdot x = 0$.

$2 - 3 + 3 \neq 0$

x is ~~not~~ in hyperplane.

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10 Let L be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$, with $L([0\ 1]^T) = [-1\ 0]^T$, and $L([1\ 0]^T) = [3\ 5]^T$.

(a) Find a matrix representation for L in the standard ordered basis.

$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$L(x) = A(x)$ and

$A = [L(e_1), L(e_2)]$

$A = \begin{bmatrix} 3 & -1 \\ 5 & 0 \end{bmatrix}$

(b) Find $L([2\ 2]^T)$ and $L([1\ -1]^T)$

$L([2\ 2]^T) = \begin{bmatrix} 3 & -1 \\ 5 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6-2 \\ 10 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$.

$L([1\ -1]^T) = \begin{bmatrix} 3 & -1 \\ 5 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3+1 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

(c) Prove or disprove: $[2\ 3]^T$ is an eigenvector of L.

We say λ is an eigen vector if there exists a non-zero vector such that $L(v) = \lambda v$ for each eigen value λ of L.

$L\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 3 & -1 \\ 5 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6-3 \\ 10 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \end{bmatrix}$

$L\begin{bmatrix} 2 \\ 3 \end{bmatrix} \neq \lambda \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ not an eigen vector

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11. Prove or disprove: $v = [2\ 3]^T$ is an eigenvector of $M = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$

$$L(x) = Ax$$

$$L\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6-6 \\ -4+9 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

not an eigen vector

12. Consider the linear operator from $\mathbb{R}^2$ to $\mathbb{R}^2$ defined by matrix $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $V = \{[1\ 1]^T, [1\ -1]^T\}$, both in the standard ordered basis.

(a) Show that V is a basis for $\mathbb{R}^2$

To prove V is a basis of $\mathbb{R}^2$, v_1, v_2 must be independent i.e, if two vectors are linearly independent & live in a two dimensional space they must span.

To show $V = \{[1,1]^T, [1,-1]^T\}$ are linearly independent, find $\det(1\ 1)^T$ $\det(1\ -1)^T$ and if $\neq 0$ they are linearly independent.

$\text{Det}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \neq 0 \therefore$ given two vectors are linearly independent and V is a basis for $\mathbb{R}^2$

(b) Find matrix K to express the linear operator in the basis V.

$$K = V^{-1} B V$$

$$V^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ -2 & 0 \end{bmatrix}$$

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13.) Let $\underline{X}$ denote a Gaussian column vector with mean vector $\underline{m}_X = [2\ 3]^T$, and covariance matrix $C_X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The random vector $\underline{Y} = A\underline{X}$, where $A = \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix}$.

(a.) Determine $E[\underline{Y}]$.

$$E(Y) = E(AX)$$
$$= AE(X)$$
$$= \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$
$$\begin{bmatrix} -2-6 \\ 4+9 \end{bmatrix} = \begin{bmatrix} -8 \\ 13 \end{bmatrix}$$

(b.) Find and simplify the covariance matrix of $\underline{Y}$.

$$Cov_Y = E\left[(Y-M_Y)(Y-M_Y)^T\right]$$
$$= E\left[A(X-M_X)(A(X-M_X))^T\right]$$
$$= E\left[A(X-M_X)(X-M_X)^T A^T\right]$$
$$= E\left[A X^2 A^T\right]$$
$$= A E(X^2) A^T$$
$$= A\, Cov_X A^T$$
$$= \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$
$$\begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ -8 & 13 \end{bmatrix}$$

$$\boxed{Cov_Y = \begin{bmatrix} 5 & -8 \\ -8 & 13 \end{bmatrix}}$$

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(c.) Completely specify the probability density function of vector $\underline{Y}$.

$$F_Y(y) = \frac{1}{(2\pi)^{N/2}|C_y|^{1/2}} \exp\left[-\frac{1}{2}(Y-\mu_Y)^T C_y^{-1}(y-X_y)\right]$$

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14. Let X and Y be random variables with joint pdf

$$f_{XY}(x,y) = \begin{cases} \frac{k}{16}, & \text{if } -4 < x < 4 \text{ and } 2 < y < 4, \\ 0, & \text{otherwise} \end{cases}$$

(a) Find constant K.

$$\int_{-4}^{4} \int_{2} \frac{k}{16} \cdot dy \cdot dx = 1 \Rightarrow \frac{k}{16} \int_{-4}^{4} y \Big|_{2}^{4} dx = 1$$

$$\Rightarrow \frac{k}{16} \int_{-4}^{4} (4-2) dx = 1 \Rightarrow \frac{k}{16} \int_{-4}^{4} 2 \, dx = 1 \Rightarrow \frac{2k}{16} \int_{-4}^{4} dx = 1$$

$$\Rightarrow \frac{2k}{16} [4 - (-4)] = 1 \Rightarrow \frac{k}{8}(8) \Rightarrow k = 1$$

$$\boxed{\therefore k = 1}$$

(b.) Prove or disprove: X and Y are orthogonal.

$$E[XY] = 0$$

$$\Rightarrow \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy \, f_{xy}(x,y) \, dx \, dy = 0$$

$$\Rightarrow \int_{2}^{4} \int_{-4}^{4} xy \left(\frac{1}{16}\right) dx \, dy$$

$$\Rightarrow \int_{2}^{4} y \left. \frac{x^2}{2} \right|_{-4}^{4} dy \Rightarrow \int_{2}^{4} y \left(\frac{4^2}{2} - \frac{4^2}{2} \right) dy$$

$$\Rightarrow \int_{2}^{4} y(0) \cdot dy \Rightarrow 0$$

$\therefore$ x & y are orthogonal

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(c) Prove or disprove: X and Y are independent.

$f_{XY}(x,y) = f_X(x)\, f_Y(y)$ if x & y are independent.

we know $f_{XY}(x,y) = \frac{k}{16} = \frac{1}{16}$

$\because f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy = \int_2^4 \frac{1}{16}\cdot dy = \frac{1}{16}\int_2^4 dy$

$= \frac{1}{16}\, y\Big|_2^4 = \frac{1}{16}(4-2) = \frac{2}{16} = \frac{1}{8}$

$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$

$= \int_{-4}^{4} \frac{1}{16}\,dx = \frac{1}{16}\, x\Big|_{-4}^{4} = \frac{1}{16}(4-(-4)) = \frac{8}{16}$

$\therefore f_{X\cdot Y}(x,y) = \frac{1}{16} = f_X(x)\, f_Y(y)$

$= \frac{1}{8}\cdot\frac{1}{2} = \frac{1}{16}$

$\therefore$ x & y are independent

(d.) Find $P[Y \le 3 / X \ge 0]$

$f_{XY}(y/x) = \frac{f(x,y)}{f_X(x)} = \frac{1\times 8}{16\times 1} = 1/2$

$\Rightarrow f_{XY}(y \le 3 / x \ge 0) = 1/2$ $\quad \because x \ge 0$ and x, y are independent

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15. Coin C_1 is selected with probability $1/3$; otherwise coin C_2 is chosen for a single flip. C_1 produces heads with probability $1/7$ and C_2 produces heads with probability $7/8$.

(a) A head is produced. Find the chance that C_2 was selected.

$P(C_1) = 1/3 \quad P(C_2) = 2/3$

$P(H|C_1) = 1/7 \quad P(H|C_2) = 7/8)$

$P(C_2|H) = P(C_2, H) / P(H)$

$P(H|C_2) * P(C_2) / P(H)$

$P(H|C_2) * P(C_2) / [P(H|C_1) * P(C_1) + P(H|C_2) * P(C_2)]$

$= (7/8 * \frac{2}{3}) / [\frac{1}{7} * \frac{1}{3} + \frac{7}{8} * \frac{2}{3}]$

$= 7/12 [\frac{1}{21} + 7/12] = \frac{49}{53}$

(b) Prove or disprove: the chance of heads is $1/2$.

$P(H) = P(H|C_1) * P(C_1 + P(H|C_2) * P(C_2)$

$= 1/7 * 1/3 + 7/8 * 2/3$

$= 1/21 + 14/24$

$= 53/84$

chance of heads is not $1/2$

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16. Let X have probability density function $f_X(x) = \begin{cases} 2x, & 0 < x < \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$. Find the characteristic function of X.

We know

$$\phi_X(t) = E\left[e^{itx}\right]$$

$$= \int_R e^{itx}\, d F_X(x)$$

$$= \int_R e^{itx} f_X(x)\, dx, \quad \text{where } f_X(x) \text{ is pdf of } X$$

$$= \int_R e^{itx} f_X(x)\, dx$$

Given $f_X(x) = \begin{cases} 2x & 0 < x < 1/2 \\ 0 & 0, \text{ otherwise} \end{cases}$

$$\phi_X(t) = \int_0^{1/2} e^{itx}\, 2x\, dx$$

we know $\int e^{ax} = 1/a\, e^{ax}$

$$\Rightarrow \int_0^{1/2} e^{itx}\, 2x\, dx = \frac{1}{it} e^{itx} \Big]_0^{1/2}$$

$$= \frac{1}{2it}\left[e^{it\,2/2} - e^{it(0)}\right]$$

$$= \frac{1}{2it}\left(e^{it} - 1\right)$$

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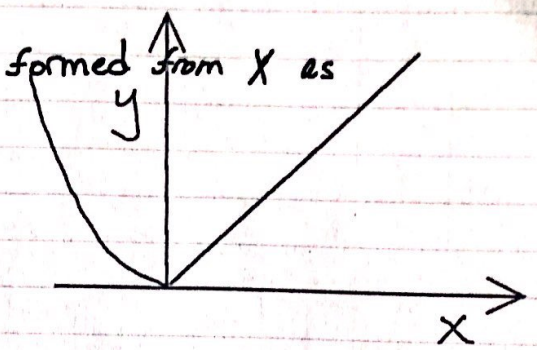
17. Let Y be a random variable formed from X as

$$Y = \begin{cases} X, & X > 0 \\ 2X^2, & X \leq 0 \end{cases}$$



(a.) Compute the cdf of Y, $F_Y(y)$, in terms of the cdf of X, $F_X(x)$.

$$F_Y(y) = P(Y \leq y) = P(X \leq y) + P(2X^2 \leq y)$$

$$F_Y(y) = P(X \leq y) + P(X \leq +\sqrt{y/2})$$

$$F_Y(y) = P(X \leq y) + P(-\sqrt{1/2}\,X \leq \sqrt{1/2})$$

$$F_Y(y) = F_X(y) + \left[F_X(\sqrt{y/2}) - F_X(-\sqrt{y/2})\right]$$

$$F_Y(y) = F_X(y) - F_X(-\sqrt{y/2})$$

$$\therefore F_Y(y) = \begin{cases} F_X(y) - F_X(-\sqrt{y/2}) & y \geq 0 \\ 0 & y < 0 \end{cases}$$

(b) Find the pdf of Y, $f_Y(y)$, when $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

$$PDF = \frac{d}{dy} F_Y(y) = \frac{d}{dy}\left(F_X(y) - F_X(-\sqrt{y/2})\right)$$

$$= F_X(y)\frac{dy}{dy} - F'_X(-\sqrt{y/2}) * \frac{d(-\sqrt{y/2})}{dy}$$

$$= f_X(y) + \frac{1}{2\sqrt{2}} f_X(-\sqrt{y/2}) + 1/\sqrt{y}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-y/2} + \frac{1}{2\sqrt{2}} * \frac{1}{\sqrt{2\pi}} e^{\frac{-(-\sqrt{y})^2}{2}} * \frac{1}{\sqrt{y}}$$

$$= \frac{1}{2\pi} e^{-y/2} + \frac{1}{4\sqrt{\pi}} e^{-y/2} * \frac{1}{\sqrt{y}} \quad ; y \geq 0$$

$0 \quad ; \quad y < 0$

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