Probability Assignment

Basic Probability

1) 2 dies rolled at once

P(Sum of even and one face 6) $= \frac{\cancel{4}\,1}{\cancel{36}\,9}$

$= \frac{1}{9}$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

2) 2 dies rolled at once

P(Sum of no.'s < 7) $= \frac{15}{36}$

3) Flip of a coin 3 times

Total likely possible events = 8

HHH
HHT
HTT
THT
THH
TTH
HTH
TTT

$P(2H) = \frac{4}{8} = \frac{1}{2}$

P(2H/observed atleast 1H) $= \frac{P(2H \cap 1H)}{P(\text{atleast } 1H)}$

$= \frac{\frac{4}{8}}{\frac{7}{8}} = \frac{4}{\cancel{8}} \times \frac{\cancel{8}}{7}$

$= \frac{4}{7}$

4) A and B are a married couple with two kids. one of them is a girl.

GB, GG, BG

P(other kid also a girl) $= \frac{1}{3}$

Conditional, Joint and Marginal Probability

5)

rainy 1/3
- heavy traffic 1/2 — late 1/2
- no traffic 1/2 — late 0.25

not rainy $1 - \frac{1}{3} = \frac{2}{3}$
- heavy traffic 1/4 — late 0.25
  - not late 0.75
- no traffic 3/4 — late 1/8

i) P(Not rainy ∩ Heavy Traffic ∩ Not late) $= \frac{\cancel{2}}{\cancel{3}} \times \frac{1}{\cancel{4}} \times \frac{\cancel{3}}{4}$

$= \frac{1}{8}$

ii) P(late) = P(Rainy) × P(HT/R) × P(Late / Rainy ∩ Traffic) +
P(Rainy) × P(NT/R) × P(Late / Rainy ∩ NT) +
~~P(NT) × P(~~ P(Not Rainy) × P(HT/N Rainy) × P(Late / N Rainy ∩ Traffic) +
P(Not Rainy) × P(NT/N Rainy) × P(Late / N Rainy ∩ NT)

$= \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{4} + \frac{2}{3} \times \frac{1}{4} \times \frac{1}{4} + \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{1}{8}$

$= \frac{1}{12} + \frac{1}{24} + \frac{2}{48} + \frac{1}{16}$

$= \frac{4+2+2+3}{48} = \frac{11}{48}$

$P(\text{late}) = \frac{11}{48}$

$P(\text{Raining} / \text{Late}) = \frac{P(R \cap L)}{P(\text{Late})}$

$= \dfrac{\frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{4}}{\frac{11}{48}}$

$= \dfrac{\frac{1}{12} + \frac{1}{24}}{\frac{11}{48}}$

$= \frac{3}{24} \times \frac{48}{11} = \frac{6}{11}$

6) 3 Coins
2 Regular Coins
1 fake ~~two~~-headed Coin

normal 2/3 H $\frac{1}{2}$

1/3 H 1

2S

i) $P(H) = \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times 1$

$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

ii) $P(H) = \frac{2}{3}$

$P(2S / H) = \frac{P(2S \cap H)}{P(H)}$

$= \dfrac{\frac{1}{3} \times 1}{\frac{2}{3}} = \frac{1}{3} \times \frac{3}{2}$

$= \frac{1}{2}$

$\frac{5}{6} \times \frac{1}{9} + \frac{1}{6} \times \frac{1}{9}$

$\frac{5}{54} + \frac{1}{54} = \frac{6}{54} = \frac{1}{9}$

7) $P(\text{Coffee}) = 0.7$

$P(\text{Cake}) = 0.4$

$P(\text{Coffee} \cap \text{Cake}) = 0.2$

$P(\text{Coffee} / \text{Cake}) = \frac{P(\text{Coffee} \cap \text{Cake})}{P(\text{Cake})}$

$= \frac{0.2}{0.4} = \frac{1}{2}$

8) ~~truth~~ 5/6 black 8/9, white 1/9; lie 1/6 black, white

~~8)~~ black 8/9 — truth 5/6, lie 1/6; white 1/9 — truth 5/6, lie 1/6

~~P(b/At) = b(At)~~

$P(w/t) = \frac{P(t/w) \times P(w)}{P(t)}$

$= \dfrac{5/6 \times 1/9}{\frac{40}{54} + \frac{5}{54}} = \dfrac{\frac{5}{54}}{\frac{45}{54}}$

$\frac{8}{9} \times \frac{5}{6} + \frac{1}{9} \times \frac{5}{6}$

~~P(white ball) = P(lie / black) + P(truth / white)~~

$P(\text{White ball}) = P(\text{black ball}) \times P(\text{lie} / \text{black ball}) +$
$P(\text{White ball}) \times P(\text{truth} / \text{White ball})$

8/9 5/6 1/6

1/9 5/6 1/6

$= \frac{8}{9} \times \frac{1}{6} + \frac{1}{9} \times \frac{5}{6}$

$= \frac{8}{54} + \frac{5}{54} = \frac{13}{54}$

$P(w/t) = \frac{P(w \cap t)}{P(t)}$

$= \dfrac{1/9 \times \frac{5}{6}}{\frac{8}{9} \times \frac{5}{6} + \frac{1}{9} \times \frac{5}{6}}$

$\dfrac{\frac{5}{54}}{\frac{45}{54}} = \frac{9}{45}$

$= \dfrac{\frac{5}{54}}{\frac{40}{54} + \frac{5}{54}}$

8) $P(\text{white ball}) = \frac{1}{9}$

$P(\text{truth} / \text{white ball}) = \frac{5}{6}$

$P(\text{black ball}) = \frac{8}{9}$

$P(\text{truth} / \text{black ball}) = 1 - \frac{5}{6}$
$= \frac{1}{6}$

Using Baye's theorem

$$P(\text{white ball}/\text{truth}) = \frac{P(\text{truth}/\text{white ball}) \times P(\text{white ball})}{P(\text{truth})}$$

$$= \frac{\frac{5}{6} \times \frac{1}{9}}{\frac{1}{9} \times \frac{5}{6} + \frac{8}{9} \times \frac{1}{6}}$$

$$= \frac{\frac{5}{54}}{\frac{13}{54}} = \frac{5}{54} \times \frac{54}{13} = \frac{5}{13}$$

9) $P(\text{truth}) = \frac{4}{5}$

$P(\text{truth}/6) = \frac{4}{5}$

$P(6) = \frac{1}{6}$

$P(\text{Not}6) = \frac{5}{6}$

$$P(6/\text{truth}) = \frac{P(\text{truth}/6) \times P(6)}{P(\text{truth}/6) \times P(6) + P(\text{truth}/!6) \,/\, P(!6)}$$

$$= \frac{\frac{4}{5} \times \frac{1}{6}}{\frac{4}{5} \times \frac{1}{6} + \frac{1}{5} \times \frac{5}{6}} = \frac{\frac{4}{30}}{\frac{9}{30}} = \frac{4}{30} \times \frac{30}{9} = \frac{4}{9}$$

10) ~~P(Math) = 0.4~~

~~P(Science) = 0.6~~

~~P(Science / Math) =~~

~~P(Math ∩ Science) = 0.4~~

10) $P(\text{Math} \cap \text{Science}) = 0.4$

$P(\text{Math}) = 0.6$

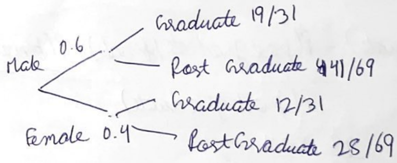
$$P(\text{Science}/\text{Math}) = \frac{P(\text{Science} \cap \text{Math})}{P(\text{Math})}$$

$$= \frac{0.4}{0.6} = \frac{2}{3}$$

11)

	Graduate	Post Graduate	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

a)



$P(\text{Male} \cap \text{Graduate}) = P(\text{Male}) \times P(\text{Graduate} / \text{Male})$
$= 0.6 \times \frac{19}{31} = \frac{11.4}{31}$

Its a Joint probability as we taking intersection of Males and Graduates.

b) P(Male) = 0.6

c) P(Graduate) = P(Male ∩ Graduate) + P(Female ∩ Graduate)

~~= P(Male) X P(Male~~

$= P(\text{Male}) \times P(\text{Graduate}/\text{Male}) + P(\text{Female}) \times P(\text{Graduate}/\text{Female})$

$= 0.6 \times \frac{19}{31} + 0.4 \times \frac{12}{31}$

$= \frac{11.4}{31} + \frac{4.8}{31}$

$= \frac{16.2}{31}$

It is Marginal or whole probability as we are ~~d)~~ calculating the probability of graduates whether Male or female.

~~d) P(Postgraduate~~

d) $P(\text{female}/\text{postgraduate}) = \frac{P(\text{postgraduate}/\text{female}) \times P(\text{female})}{P(\text{postgraduate})}$

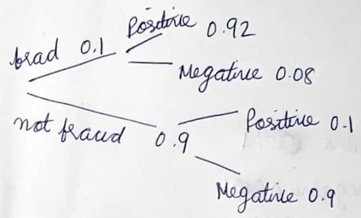
$= \frac{\frac{28}{69} \times 0.4}{1 - P(\text{graduate})}$

$= \frac{\frac{19.2}{69}}{1 - \frac{16.2}{31}} = \frac{\cancel{19.2}^{\cancel{96}\,\cancel{48}\,16}}{\cancel{69}_{23}} \times \frac{31}{\cancel{148}_{\cancel{74}\,37}}$

$= \frac{496}{851}$

Bayes Theorem

12)



P(Positive / fraud) = 0.92

P(fraud) = 0.1

P(Negative / fraud) = 0.08

P(Not fraud) = 0.9

P(Positive / Not fraud) = 0.1

P(Negative / Not fraud) = 0.9

~~P(Pos~~ $P(\text{fraud}/\text{Positive}) = \frac{P(\text{Positive}/\text{fraud}) \times P(\text{fraud})}{\cancel{P(\text{fraud})}\ P(\text{Positive})}$

$= \frac{0.92 \times 0.1}{0.1 \times 0.92 + 0.9 \times 0.1}$

$= \frac{0.9}{0.92 + 0.9}$

$= \frac{0.19 \times \cancel{100}}{\cancel{182}_{91} \times \cancel{10}} = \frac{45}{91}$

14) $P(\text{Swine flu}) = \frac{1}{10000}$

$= 0.0001$

Swine flu 0.0001 — positive 1
— negative 0

no Swine flu 0.9999 — Positive 0.01
— Negative 0.99

$$P(\text{Swine flu} / \text{positive}) = \frac{P(\text{positive}/\text{Swine flu}) \times P(\text{Swine flu})}{P(\text{Positive})}$$

$$= \frac{1 \times 0.0001}{0.0001 \times 1 + 0.999 \times 0.01}$$

$$= \frac{0.0001}{0.001 + 0.00999}$$

$$= \frac{0.0001}{0.01009}$$

$$= 0.0099$$

$$\simeq 0.01$$