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4/22/2021

**1) Case 1: $\boldsymbol{\eta, U_\eta}$**

$$\eta = \frac{\dot{Q}_{user}}{\dot{Q}_{source} + \dot{W}_{\text{pump}}} = \frac{\eta_{user,hex}\dot{m}c_p(T_2 - T_3)}{\frac{\dot{m}c_p(T_1 - T_5)}{\eta_{source,hex}} + \frac{\dot{m}(P_4 - P_5)}{\rho\,\eta_{pump}}} = \frac{\eta_{user,hex}c_p(T_2 - T_3)}{\frac{c_p(T_1 - T_5)}{\eta_{source,hex}} + \frac{(P_4 - P_5)}{\rho\,\eta_{pump}}}$$

$$= \frac{0.8*45.86*4186(366.48 - 336.43)}{\frac{4186(368.54 - 335.15)}{0.8} + \frac{(335308.9 - 203655.79)}{0.8*980}} = 0.5752$$

| | Partials | Partial Formulas | Bias | Precision |
|---|---|---|---|---|
| 1 | $\frac{\partial\eta}{\partial\dot{m}}$ | $\dot{m}$ cancels so derivative is 0 | 0.1156 | 0.01 kg/s |
| 2 | $\frac{\partial\eta}{\partial T_2}$ | $\frac{\eta_{user,hex}\dot{m}c_p}{\frac{\dot{m}c_p(T_1 - T_5)}{\eta_{source,hex}} + \frac{\dot{m}(P_5 - P_4)}{\rho\,\eta_{pump}}} = 0.0191$ | 0.3665 | 0.1 K |
| 3 | $\frac{\partial\eta}{\partial T_3}$ | $-\frac{\partial\eta}{\partial T_2} = -0.0191$ | 0.3364 | 0.1 K |
| 4 | $\frac{\partial\eta}{\partial T_1}$ | $-\frac{\partial\eta}{\partial T_5} = -\,0.0172$ | 0.3685 | 0.1 K |
| 5 | $\frac{\partial\eta}{\partial T_5}$ | $\frac{\rho^2\, c_p{}^2\, \eta_{pump}{}^2 \eta_{source,hex}\eta_{user,hex}(T_2 - T_3)}{\left(\rho\, c_p\eta_{pump}(T_1 - T_5) + \eta_{source,hex}(P_5 - P_4)\right)^2} = 0.0172$ | 0.3351 | 0.1 K |
| 6 | $\frac{\partial\eta}{\partial P_5}$ | $-\frac{\partial\eta}{\partial P_4} = -\,0.4194$ | 83.8272 | 90 Pa |
| 7 | $\frac{\partial\eta}{\partial P_4}$ | $\frac{\eta_{source,hex}{}^2\rho\, c_p\eta_{pump}\eta_{user,hex}(T_2 - T_3)}{\left(\rho\, c_p\eta_{pump}(T_1 - T_5) + \eta_{source,hex}(P_5 - P_4)\right)^2} = 0.4194 * 10^{(-8)}$ | 50.9139 | 90 Pa |

$$B_\eta = \sqrt{\left(\frac{\partial\eta}{\partial\dot{m}}B_{\dot{m}}\right)^2 + \left(\frac{\partial\eta}{\partial T_2}B_{T_2}\right)^2 + \left(\frac{\partial\eta}{\partial T_3}B_{T_3}\right)^2 + \left(\frac{\partial\eta}{\partial T_1}B_{T_1}\right)^2 + \left(\frac{\partial\eta}{\partial T_5}B_{T_5}\right)^2 + \left(\frac{\partial\eta}{\partial P_5}B_{P_5}\right)^2 + \left(\frac{\partial\eta}{\partial P_4}B_{P_4}\right)^2}$$
$$= 0.0128$$

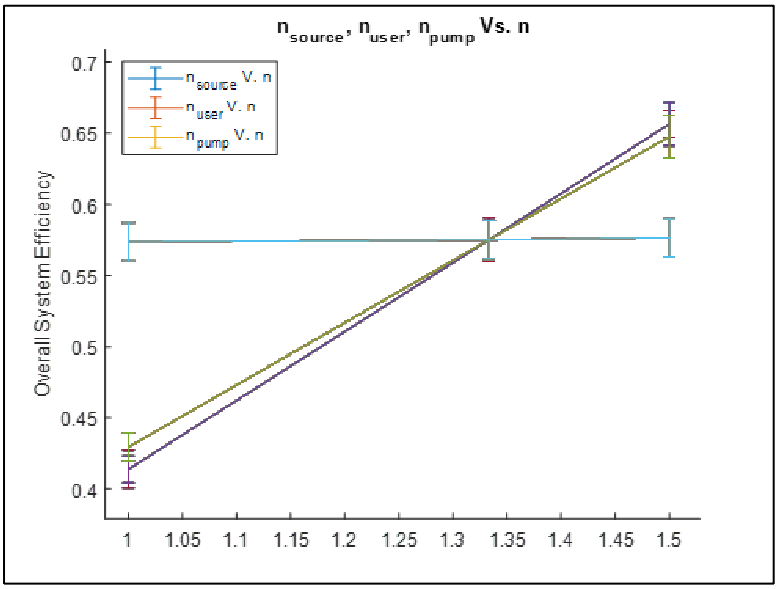
$$P_\eta = \sqrt{\left(\frac{\partial\eta}{\partial\dot{m}}P_{\dot{m}}\right)^2 + \left(\frac{\partial\eta}{\partial T_2}P_{T_2}\right)^2 + \left(\frac{\partial\eta}{\partial T_3}P_{T_3}\right)^2 + \left(\frac{\partial\eta}{\partial T_1}P_{T_1}\right)^2 + \left(\frac{\partial\eta}{\partial T_5}P_{T_5}\right)^2 + \left(\frac{\partial\eta}{\partial P_5}P_{P_5}\right)^2 + \left(\frac{\partial\eta}{\partial P_4}P_{P_4}\right)^2}$$
$$= 0.0036$$

$$U_\eta = \sqrt{B_\eta{}^2 + P_\eta{}^2} = \pm\,0.0133$$

## 2) $\eta, U_\eta$ for 22 Experimental Cases

| Case # | $\eta$ | $U_\eta$ | Case # | $\eta$ | $U_\eta$ |
|---|---|---|---|---|---|
| 1 | 0.5752 | $\pm$ 0.0133 | 12 | 0.5916 | $\pm$ 0.0139 |
| 2 | 0.6565 | $\pm$ 0.0153 | 13 | 0.6039 | $\pm$ 0.0144 |
| 3 | 0.4139 | $\pm$ 0.0094 | 14 | 0.5733 | $\pm$ 0.0133 |
| 4 | 0.6476 | $\pm$ 0.0150 | 15 | 0.5825 | $\pm$ 0.0136 |
| 5 | 0.4295 | $\pm$ 0.0099 | 16 | 0.5729 | $\pm$ 0.0132 |
| 6 | 0.5765 | $\pm$ 0.0133 | 17 | 0.5866 | $\pm$ 0.0137 |
| 7 | 0.5737 | $\pm$ 0.0133 | 18 | 0.5943 | $\pm$ 0.0140 |
| 8 | 0.5908 | $\pm$ 0.0139 | 19 | 0.5737 | $\pm$ 0.0133 |
| 9 | 0.5651 | $\pm$ 0.0130 | 20 | 0.5835 | $\pm$ 0.0137 |
| 10 | 0.5992 | $\pm$ 0.0142 | 21 | 0.4615 | $\pm$ 0.0097 |
| 11 | 0.3935 | $\pm$ 0.0077 | 22 | 0.5804 | $\pm$ 0.0135 |

## 3) Analyze Effect of Component Efficiencies on the Overall System Efficiency for Cases 1-7

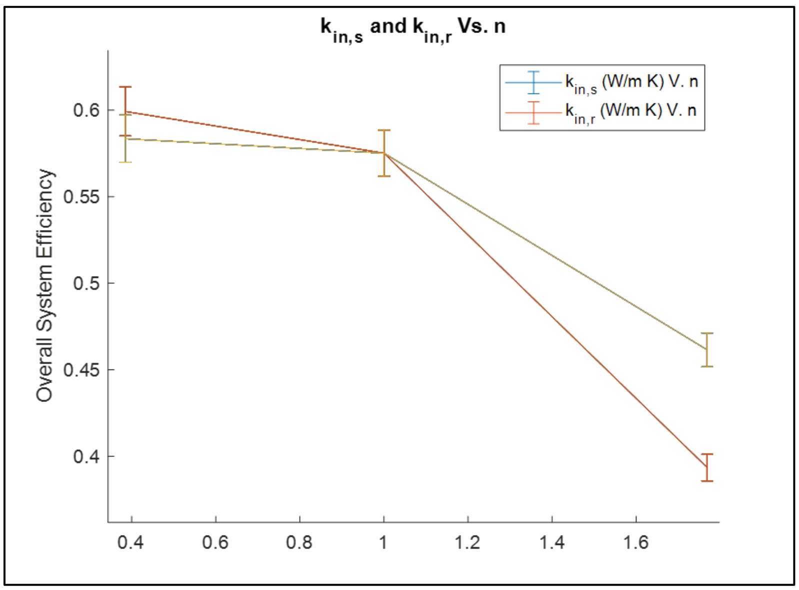



| Case | $\dot{Q}_{source}/\eta_{source,hex}$(W) | $\eta_{user,hex}\dot{Q}_{user}$(W) | $\dot{W}_{pump}/\eta_{pump}$(W) |
|---|---|---|---|
| 1 | 8013993.69 | 4613707.53 | -7700.32 |
| 2 | 8359529.05 | 5495819.19 | -11840.86 |
| 3 | 8016551.46 | 3319319.37 | -2940.0 |
| 4 | 8013317.28 | 5194489.14 | -7704.63 |
| 5 | 8041241.16 | 3457030.19 | -7698.01 |
| 6 | 8023678.39 | 4629997.27 | -6855.02 |
| 7 | 8008170.11 | 4600105.79 | -10271.32 |

*Note all these energy equations are written in first $\eta$ equation in this report.

Based on the plots and analysis, the overall system efficiency does not change at all with respect to changing source heat exchanger efficiency. But it increases with increasing pump and user heat exchanger efficiencies. This makes sense because we are running the pump to make this whole multi-heat exchanger system to work. If the pump cannot move the water at the necessary rate, then heat will dissipate faster in the surrounding environment. Also, the heat exchanger on the user side being multiplied directly by its efficiency while the one on the source side is being divided by its efficiency. The source would technically be more insulated while the use heat exchanger is ready to dissipate heat via air conditioning and other applications.

## 4) Supply and Return Line Insulation Materials Vs. Overall Efficiency for Cases 1,10,11,20,21

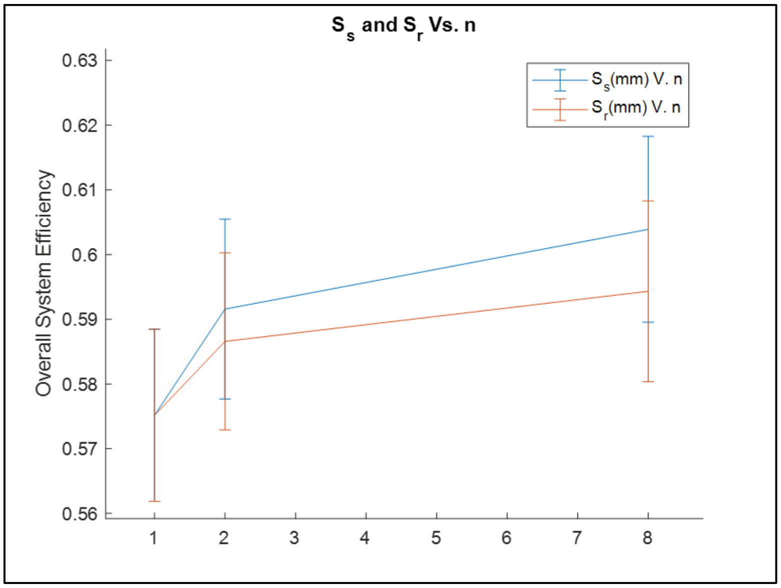



$$\dot{Q}_{loss,supply} = \dot{m}c_p(T_2 - T_1), \qquad \dot{Q}_{loss,supply} = \dot{m}c_p(T_3 - T_4)$$

| Case | $\dot{Q}_{loss,supply}$ (W) | $\dot{Q}_{loss,return}$ (W) |
|---|---|---|
| 1 | 396620.15 | 236083.08 |
| 10 | 150586.31 | 250893.11 |
| 11 | 2300656.31 | 173477.97 |
| 20 | 457336.61 | 91193.88 |
| 21 | 424621.08 | 1378771.28 |

According to the graph and the heat losses, the overall system efficiency decreases with increasing thermal conductivity. This makes sense because if the ability of a material to conduct heat is greater, then heat would leave the system faster. And over a certain range of time, more heat would leave the system. Making it less efficient overall. Increasing thermal conductivity seems to drastically decrease the efficiency.

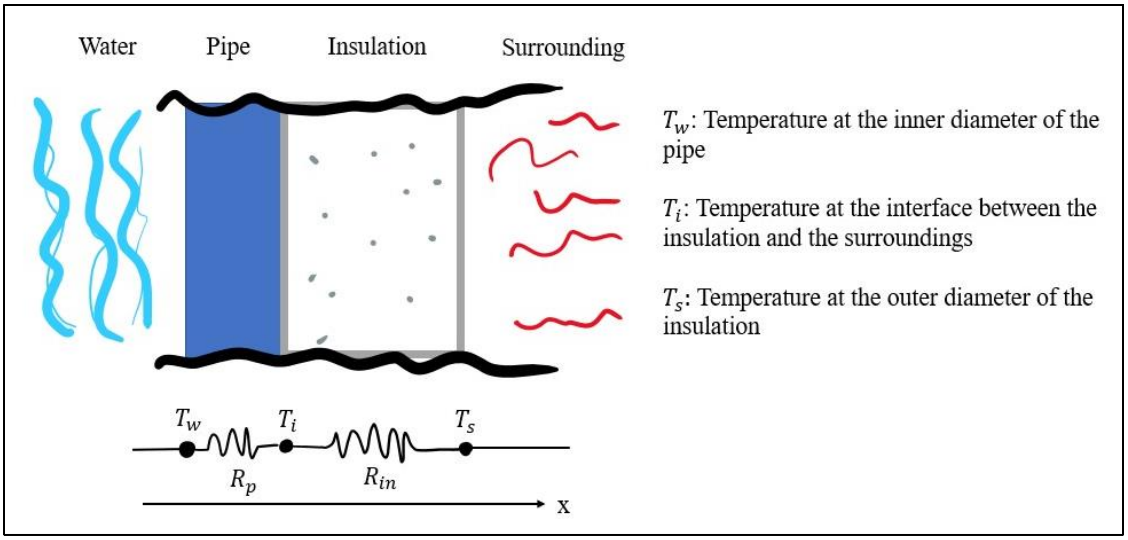
## 5) Supply and Return Line Insulation Thickness Vs. Overall Efficiency for Cases 1,12,13,17,18




| Case | $\dot{Q}_{loss,supply}$ (W) | $\dot{Q}_{loss,return}$ (W) |
|---|---|---|
| 1 | 396620.15 | 236083.08 |
| 12 | 233843.15 | 262384.56 |
| 13 | 94397.36 | 265515.52 |
| 17 | 406724.69 | 123024.55 |
| 18 | 510823.84 | 50042.64 |

Based on the graph and heat losses, thicker insulation improves the overall efficiency of the system. But, after a certain point, the efficiency tapers down. A reason for this phenomenon is that after a certain thickness threshold is reached, no matter how material there is, heat will still find a way to get out of the system and it is counter-productive to spend more money trying to stop heat from leaving by 100%. This makes the system a perpetual motion machine and according to the first law of thermodynamics they cannot exist, and heat has to be dissipated into a sink. But within the acceptable range, increasing insulation will increase the overall system efficiency.

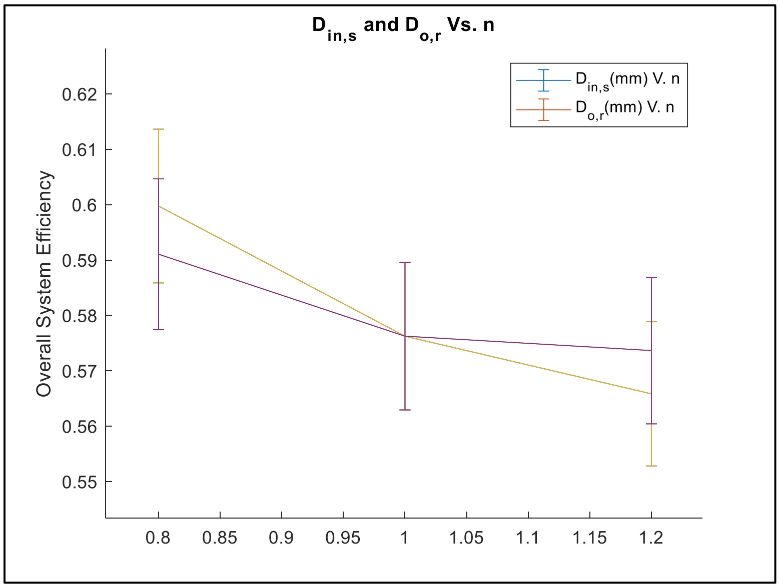
**6) Pipe Material Vs. Insulation Material with Respect to Thermal Effects for Cases 1 and 22**




| Case | Pipe Thermal Resistance (K m/W) | Insulation Thermal Resistance (K m/W) |
|---|---|---|
| 1 | $0.1459 \times 10^{-6}$ | 0.5611 |
| 22 | $0.0182 \times 10^{-6}$ | 0.5611 |

The larger the thermal resistance, the slower the heat can travel through a material. Which is why insulation is important. The thermal resistance of the steel stubes is significantly lower than that of the insulation. Which makes sense because metal has a tighter crystalline structure allowing for the motion of heat compared to a porous insulator. But steel pipes are smooth and are structurally sound for this application, so the rubber insulator takes care of making sure a significant amount of heat does not leave the system. Having more thermal resistance in general is good for the overall system efficacy as less heat would be leaving the system.

**7) Inner Diameters of the Supply and Return Pipes Vs. Overall Efficiency for Cases 1,8,9,15,16**



Looking at the graph above, the overall system efficiency decreases as the pipe diameter increases. But it is not drastic as indicated by the overlapping uncertainty points. This makes sense because the mass flow rate of the pipe should not affect the efficiency as it cancels out as indicated in the first equation in Section 1 of this report. But having a greater mass flow rate means having more surface area along the surface of the pipe for heat to leave the pipe. Additionally, the pump would have to exert more pressure to move the water. Changing pipe diameter can increase the insulation inner and outer diameters as well In essence, this parameter has little effect as it stands but drastically increasing it could decrease the system efficiency.