

# Your Project Report Title Here

## CS7IS2 Project (2019/2020)

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**Abstract.** This paper demonstrates the creation, testing and performance evaluation of a game Connect-Four with the help of three state-of-the art AI algorithms namely Minimax algorithm, Expectimax Algorithm with alpha-beta pruning and Depth-Limited Search for improved performance and Monte Carlo Tree Search algorithm. Connect-Four is a familiar and well-known board game whose objective is to get four slots in a row.

**Keywords:** computational geometry, graph theory, Hamilton cycles//  
To be added

## 1 Introduction

Besides constructing intelligent systems, Artificial Intelligence (AI) is also the endeavour to understand them. It includes applications ranging from general purpose areas to particular tasks like proving mathematical theorems and diagnosing diseases. Game playing is one of the oldest fields of ventures in AI. General Game Playing (GGP) is the composition of AI programs' ability to play games successfully.

Connect-Four, a two-player game where the objective of the game is to win by making four or more discs a player's colour in a line horizontally, vertically or diagonally. The board is fashioned with six rows and seven columns where the discs are dropped vertically down occupying the last available slot within that respective column by taking turns. If the board is completely filled with discs but was unsuccessful in deciding a winner, then it ends with a tie. Although it is a solved game, it has  $10^{13}$  possible positions on the board making it absurd and illogical to store the moves in memory. As ardent players of board games and an inclination towards working on Artificial Intelligence, in this paper, we aim to depict and illustrate the different ways AI can be utilized to create gameplay agents for the Connect-Four board game. Agents in AI are entities which act and directs its movements towards accomplishing goals. However, goals alone are insufficient to develop high-quality behaviour. Utility based agents comprise of a utility function that maps a state onto a real number representing the associated degree of happiness (1). For the agent to achieve the goal, it needs to be equipped with the capability of knowing the states, possible actions, transition

model, goal test and path cost. The Connect-Four environment (3) comprises of the following environmental properties as follows:

- Accessible : It is completely observable since it consists of the board with constant dimensions and discs belonging to either the player or opponent.
- Deterministic : It is considered deterministic since no random elements are involved.
- Static : Since the present state cannot be altered be changed until the agent performs a move it is static.
- Discrete : Since the number of states is finite it is discrete.

There are many approaches that can be employed to solve the Connect-Four game based on the difficulty level set for the AI. As per (2) they are random, defensive and aggressive AI. Random AI as the name suggests randomly performs an action and is easiest to beat, Defensive AI prioritizes to oppose and block the opponents moves from winning and Aggressive AI gives precedence to only winning.

Connect-Four makes a good candidate for implementing and studying search algorithms to find the best solution from all feasible solutions. This paper demonstrates the creation, testing and performance evaluation of Connect-Four with the help of three state-of-the art AI algorithms namely Minimax algorithm, Expectimax Algorithm with alpha-beta pruning and Depth-Limited Search for improved performance and Monte Carlo Tree Search algorithm. In this paper, we aim to demonstrate a variation to the program with a mixture of ways to select the type of players to play and solve the game. The possible combinations of players are Human versus Human, Human versus Agent and Agent versus Agent. Our experiment involves five types of agent-players namely: *Agent1* – Random, *Agent2* – Forward checking, *Agent3* – Minimax algorithm with alpha-beta pruning and depth-limited search, *Agent4* – Expectimax algorithm with alpha-beta pruning and depth-limited search and *Agent5* – Monte Carlo Tree Search.

Search strategies are evaluated by measuring their effectiveness of performance in solving the problem. Ideally, effectiveness of a search is calculated using the search cost associated with the time and memory consumed to obtain the solution. For the Connect-Four game, we aim to calculate the time taken and total number of moves made by both the players.

The rest of the paper is organized as follows: Section 2 mentions the Related Work on different techniques and approaches, Section 3 comprises of the Problem Definition and Algorithms implemented, Section 4 describes the Experimental Results obtained by the utilised methodology and lastly, Section 5 includes the Conclusions and final discussions of the main results along with an acknowledgement of future work.

## 2 Related Work

Game playing is an important area of artificial intelligence since they are an interesting means in comparing interaction between user and machine's behaviour directly. Once the rules, permissible moves and conditions to succeed or lose the game are acknowledged search procedures can be implemented to see good moves to be generated and explored first. Game theory broadens decision theory to circumstances in which multiple agents interact. There has been extensive research performed in understanding different approaches to solving two-player games, strategical and turn-taking games and tactical adversarial games like Backgammon, Connect-Four, Othello and Chess. Typically, these games are modelled as decision trees using different AI algorithms. The general notion of characterizing and assessing the quality of a game is by playing the game with various controllers and algorithms and evaluating the performance through comparisons. As per (6), by investigating the relative performance of different general game-playing algorithms results suggest a significant positive relationship between intelligent agents, game design and success rate of the algorithm. In fact, these parameters differentiate between good and bad game playing algorithms on a well-designed game with the restriction to human-designed games.

In (5), provides literature based on current challenges faced in real-time strategy games. The paper describes the comparative analysis conducted on different AI based machine learning techniques like naïve Bayes classifier and support vector machines to evaluate the performance issues faced trying to achieve optimality. Comparison between the techniques was helpful and utilized in designing the characters of video games.

In (10), paper illustrates the usage of dynamic scripting to provide actions in a turn-based strategic game. The proposed algorithm incorporates rule ordering of dynamic scripting through reinforcement learning combined with minimax algorithm to obtain better performance. The performance is evaluated through a series of matches against a manually designed static AI.

In (8), demonstrates the use and evaluation of Minimax Tree Optimisation (MMTO) algorithms' performance in shogi to learn a heuristic evaluation function of a practical alpha-beta search program. This study involves a depth search where it repetitively searches and updates using partial derivatives. The paper concludes that MMTO outperformed existing methods and can adapt to forty million parameters. **(can be removed)**.

In (9), paper proposes MCTS-minimax hybrids that integrate shallow minimax searches into the MCTS framework. It investigates ways to combine the strategic toughness of MCTS with the tactical strength of minimax to exercise better results from universally useful hybrid search algorithms. The paper explores its effectiveness on games like Connect-Four with the objective to relate their performance to the strategies of the domains.

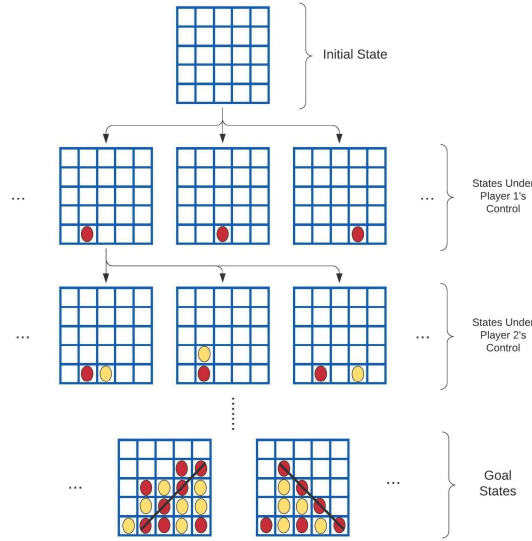
In (7), presents a study involving the Connect-Four game in a real-time environment with incorporation of time restraints. The paper uses an artificial intelligence based on influence mapping, minimax, minimax with alpha-beta pruning and A\* to evaluate its performance with time constraints.

#### Inferences from above papers

6 – using 3 different algorithms to analyse the performances and identify the best among them since there is a relationship between the results and agent incorporated with types of algorithms. 10 – The performance was evaluated through series of matches. 8 – uses MMTO with alpha-beta pruning on shogi. We aim to attempt the same on Connect-Four.

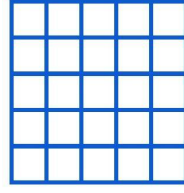
### 3 Problem Definition and Algorithm

Connect-Four being a two-player mathematical 6x7 board game, can be solved strategically using a mathematical solution. FigXXX depicts the factors that are to be defined to formulate the problem.

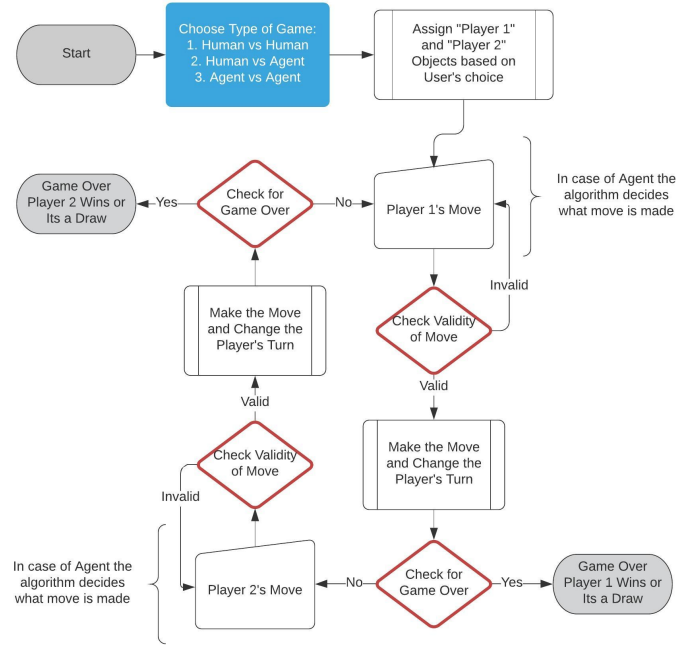


**Fig. 1.** Connect 4 - State Action

From a plethora of artificial intelligence algorithms available, a mixture of brute-force techniques and knowledge-based approaches can be utilised to solve the game. There are a few conclusions resolved from solving the game, for instance, the first player has a higher probability of winning if the game is started by placing the disc in the middle column. However, this highly depends on the agent's capacity to look ahead based on the algorithm imposed on it. The layout of the Connect-Four game and design flow of the game is shown in figVVV and figBBB respectively.



**Fig. 2.** Connect 4 - Empty Board



**Fig. 3.** Game Flow

### 3.1 Minimax Algorithm

Minimax, a decision-making algorithm is typically used in turn-based two-player games aiming to obtain the optimal next move. The general terms of the algorithm are maximiser, generally being the agent and minimizer, the agent's opponent. The algorithm is based on a zero-sum game theory concept where "the total utility score is divided among the players. An increase in one player's score results in decrease in another player's score" (11). This recursive algorithm prepares an optimal move for the agent when always assuming that the opponent plays optimally. To achieve this, the agents looks ahead by using depth-first search for all feasible moves and calculates the resultant score. However, it is not feasible to explore the entire game tree for Connect-Four and hence we opt to backtrack the score using depth-limited search in place of depth-first search which imposes a cut off on the maximum depth of a path. In this paper, we look ahead by 5-6 moves to determine an intermediate score in the "evaluation" function for which the pseudocode is depicted below. This function provides an estimate of the expected utility numeric value based on the weights at a given state for the player.

```
def evaluate_window(window):
    score = 0
    if window.count(agents_coin) == 4:
        score += 100
    elif window.count(agents_coin) == 3 and window.count(
        EMPTY) == 1:
        score += 5
    elif window.count(agents_coin) == 2 and window.count(
        EMPTY) == 2:
        score += 2
    if window.count(opponent_coin) == 3 and window.count(
        EMPTY) == 1:
        score -= 4
    return score

def evaluate(state):
    score = 0
    ## Score center column
    score += state.center_column.count(agents_coin) * 3

    ## Score Horizontal
    for window in state.all_possible_horizontal_windows:
        score += evaluate_window(window)

    ## Score Vertical
    for window in state.all_possible_vertical_windows:
        score += evaluate_window(window)
```

```

## Score positive sloped Diagonal
for window in state.all_possible_pos_sloped_windows:
    score += evaluate_window(window)

## Score negative sloped Diagonal
for window in state.all_possible_neg_sloped_windows:
    score += evaluate_window(window)

return score

```

Pruning is a technique implemented in search algorithms to reduce the length of the tree by eliminating the sections that are significantly negligent in terms of accuracy or efficiency. Therefore, we apply a pruning method to limit the number of nodes that the agent needs to examine when looking ahead. Alpha-beta pruning, a technique that can be applied to any depth of a tree is implemented on the standard minimax algorithm to obtain optimal moves. Below code depicts the pseudocode for the minimax algorithm.

```

def minimax(state, depth, alpha, beta, maximizingPlayer):
    valid_moves = state.get_valid_moves()
    is_terminal = state.is_terminal_node()

    if depth == 0 or is_terminal:
        if is_terminal:
            if state.check_win(agent):
                return (None, 100000)
            elif state.check_win(opponent):
                return (None, -100000)
            else: # Game is over, no more valid moves
                return (None, 0)
        else: # Depth is zero
            return (None, evaluate(state))

    if maximizingPlayer:
        utility = -math.inf
        best_move = random.choice(valid_moves)
        for move in valid_moves:
            state.agent_makes_move(move)
            new_utility = minimax(state, depth-1, alpha, beta,
                                False)[1]

            if new_utility > utility:
                utility = new_utility
                best_move = move

        alpha = max(alpha, utility)
        if alpha >= beta:
            break
    return best_move, utility

```

```

else: # Minimizing player
    utility = math.inf
    best_move = random.choice(valid_moves)
    for move in valid_moves:
        state.simulate_opponent_move(move)
        new_utility = minimax(state, depth-1, alpha, beta,
                               True)[1]

        if new_utility < utility:
            utility = new_utility
            best_move = move

    beta = min(beta, utility)
    if alpha >= beta:
        break
    return best_move, utility

```

### 3.2 Expectimax Algorithm

There are games that mirror the unpredictable external events by including a random element. Expectimax, a variant of minimax is an algorithm commonly used in zero-sum games where the outcome depends on the probability of the external element along with the player's skill. It generalises the games where transitions between states are made probabilistic. It uses a predefined opponent strategy to treat opponent decision nodes as chance nodes (1). In other words, the opponent generally being the minimizer in minimax is replaced with the role of a player whose moves depend on the probability element which is played by chance. These are no longer definite with a minimax value and calculated as an average or expected value. Below code depicts the pseudocode for the minimax algorithm.

```

utility = math.inf
best_move = random.choice(valid_moves)
for move in valid_moves:
    state.simulate_opponent_move(move)
    new_utility = expectimax(state, depth-1, alpha, beta,
                             True)[1]

    if new_utility <= utility:
        utility = new_utility
        best_move = move

    beta = utility/no_of_valid_moves
    if alpha >= beta:
        break
    return best_move, utility

```



Apart from the probabilistic element, the algorithm works similar to mini-max and hence we apply the same depth-limited search and alpha-beta pruning technique to improve the performance of the algorithm.

### 3.3 Monte Carlo Tree Search Algorithm

Monte Carlo Tree Search (MCTS) is a deeply utilized heuristic search algorithm known for incorporating an element of randomness into the search for variety of games. It was highly recommended as a search and planning structure for identifying optimal results in a given situation. This is demonstrated and confirmed because Monto Carlo rollouts of games gives it a strategic advantage over traditional depth-limited searches. Upper Confidence Bounds for Trees (UCT), a manifestation of MCTS was considered revolutionary for game-playing agents in artificial intelligence (4). The expression for UCT is depicted below from which the highest value calculated is utilised in decision making models.

$$\frac{w_i}{n_i} + c\sqrt{\frac{\ln N_i}{n_i}} \quad (1)$$

Where,

- $w_i$  : number of wins for the node after the  $i$ th move
- $n_i$  : number of simulations for the node after the  $i$ th move
- $N_i$  : total number of simulations after the  $i$ th move run by the parent node
- $c$  : the exploration parameter, chosen as 2 for the game

#### Pseudocode for UCT:

```
# returns child with largest UCT value
def selection():
    uct_value = lambda node: node.wins / node.visits + sqrt(2 *
                                                            log(parent_node.visits) / node
                                                            .visits)
    # applies the above uct_value lambda function to all the child
    # nodes (children)
    return sorted(children, key = uct_value)[-1]
```

The MCTS planning methodology incrementally constructs an asymmetric search tree guided in most assuring direction which is estimated iteratively and enhances its performance with an increase in the number of iterations ie., it samples moves rather than considering all legal moves from a given state (4). An MCTS iteration usually consists of four consecutive phases: (1)*Selection* where the tree choses the node with largest UCT value, in other words the node which possess the best chance of winning out of all, (2)*Expansion* into new child nodes of the tree after randomly choosing the node, (3)*Rollout* where simulations are conducted on the expanded nodes to review the status of the game - win, loss or tie, (4)*Backpropagation* updates the results obtained in the rollout step along

the chosen path like number of wins for the selected node, total number of simulations conducted on the selected node and parent node. Fig XXX depicts a working example of Connect-Four game by following the steps mentioned for the MCTS iteration (MIGHT REMOVE) which is followed by the pseudocode of the above mentioned MCTS steps.

**Fig XXX - ?????????????????????????**

**Pseudocode of MCTS:**

```
def montecarlo_tree_search(currentNode, state):
    rootnode = Node(state, PREV_PLAYER)

    if currentNode is not None:
        rootnode = currentNode

    for i in range(max_iterations):
        node = rootnode

        # selection
        while node.available_moves == [] and node.children != []:
            node = node.selection()
            player_makes_move(state, CURR_PLAYER, node.move)

        # expand
        if node.available_moves != []:
            move = random.choice(node.available_moves)
            player_makes_move(state, CURR_PLAYER, move)
            node = node.expand(move)

        # rollout
        while get_available_moves():
            move = random.choice(get_available_moves())
            player_makes_move(state, CURR_PLAYER, move)
            if check_player_win(PREV_PLAYER):
                break

        # backpropagate
        while node is not None:
            node = node.parent

        if timeout:
            break

    win_ratio = lambda node: node.wins/node.visits
    sorted_children = sorted(rootnode.children, key =
                             win_ratio[::-1])
```

```
return rootnode, sorted_children[0].move
```

## 4 Experimental Results

In this section, the experimental setup, evaluation criteria for the data and methodology used is described in detail followed by a discussion of findings and results.

### 4.1 Methodology

Apart from the three state-of-the-art artificial intelligence algorithms Minimax, Expectimax and Monte Carlo Tree Search, our experimental setup consists of a random and forward checking agent for naïve playing. We aim to formulate different combinations of games to be played against each other (10) and evaluate their performance through those series of five matches. Based on the time and number of moves made by each of the players, win percentages are calculated over 5 matches inclusive of games that ended in a tie.

### 4.2 Results

To begin the experiment, a variation of minimax and expectimax with depth as 3 and 4 were set to play among all combinations over a series of 5 matches. A detailed report of total number of moves made and total time taken by both players and the winners of each game were recorded. From this data, the win percentages of each agent against their opponents were recorded as depicted by Table bbb

% OF WINS BETWEEN PLAYER 1 (in red) AGAINST PLAYER 2 (in yellow)	Random	Forward Checking	Minimax (D=3)	Minimax (D=4)	Expectimax (D=3)	Expectimax (D=4)	MCTS (S = 5k, T = 2s)
Random	60 / 40	0 / 100	0 / 100	0 / 100	0 / 100	0 / 100	0 / 100
Forward Checking	80 / 20	20 / 80	0 / 100	0 / 100	20 / 80	20 / 40	0 / 100
Minimax (D=3)	100 / 0	100 / 0	100 / 0	100 / 0	100 / 0	100 / 0	60 / 40
Minimax (D=4)	100 / 0	100 / 0	0 / 100	100 / 0	100 / 0	100 / 0	40 / 60
Expectimax (D=3)	100 / 0	20 / 80	0 / 100	0 / 100	100 / 0	100 / 0	40 / 60
Expectimax (D=4)	100 / 0	60 / 40	0 / 100	0 / 100	100 / 0	100 / 0	60 / 40
MCTS (S = 5k, T = 2s)	100 / 0	100 / 0	0 / 100	20 / 80	40 / 60	20 / 80	100 / 0

Fig. 4. Win Ratio for Bots 2

On a brief look of the results, it is noticeable that the random agent has almost negligible wins against forward checking and the other agents. Based on manual analysis of these results thoroughly, it was apparent that the random and forward checking agent had insignificant and almost negligible win ratios against the agents equipped with Minimax, Expectimax and MCTS algorithms. This is a reflection of how good the agents equipped with algorithms have better win ratios i.e. they have a better chance of solving and winning the game against a random agent and naïve-play by the forward checking agent. Therefore, we resolved to exclude the random and forward checking agent from the next part of the experimental study.

The study was advanced by performing a series of another 5 matches with depth values as 5 and 6 for both minimax and expectimax and varying the number of iterations S as 5k and 10k with Time-out time T as 2s and 3s for the MCTS agent. A similar report of total number of moves made and total time taken by both players and the winners of each game were recorded from which the win percentages of each agents against their opponents was captured. Table hhh depicts the recordings.

% OF WINS BETWEEN PLAYER 1 (in red) AGAINST PLAYER 2 (in yellow)	Minimax (D=5)	Minimax (D=6)	Expectimax (D=5)	Expectimax (D=6)	MCTS (S = 10k, T = 3s)	MCTS (S = 5k, T = 3s)	MCTS (S = 10k, T = 2s)
Minimax (D=5)	0	100	0	0	40	20	20
Minimax (D=6)	0	100	0	0	20	20	0
Expectimax (D=5)	100	100	100	100	40	60	20
Expectimax (D=6)	100	100	100	100	60	60	20
MCTS (S = 10k, T = 3s)	80	80	40	20	60	40	40
MCTS (S = 10k, T = 2s)	80	80	40	20	60	60	40
MCTS (S = 5k, T = 3s)	100	100	40	60	40	60	20

Fig. 5. Win Ratio for Bots 1

(COMPARISON WITH BASELINE???)

### 4.3 Discussion

In order to determine the performance of each of the agents, they were made to contest with each other and calculate certain metrics for evaluation. This is conducted based on the fact that win rates suggest a relationship between intelligent agents based on the different types of algorithms that they are equipped

with. On analysis of the outcomes, it is observed that agents equipped with Expectimax and Monte Carlo Tree Search do substantially better than the Random and Forward-Checking agent. MCTS sampling is said to be robust and produce strong plays in contrast to minimax which is fragile towards noise in the evaluate function for the intermediate states. Even though MCTS has displayed reasonable success a primary weakness of MCTS, shared by most search heuristics, is that the dynamics of search are not yet fully understood, and the impact of decisions concerning parameter settings and enhancements to basic algorithms are hard to predict (12). Therefore, it appears that the traditional approach of Minimax algorithm at depth 6 with alpha-beta pruning and depth-limited search has the highest number of wins among all the agents overall. This is because minimax always provides the optimal move to the agent under the impression that the opponent also makes optimal moves. The ability of our program to look ahead by 5-6 moves and resolve an intermediate score is beneficial and can be witnessed in our results.

**(SOME SORT OF GRAPH SHOWING ALL AGENTS PERFORMANCE) and explanation for it**

## 5 Conclusions

Provide a final discussion of the main results and conclusions of the report. Comment on the lesson learnt and possible improvements.

**Future:** Could try hybrid of algorithms

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