THE LAWS OF LOGIC:

1. Law of Double Negation: For any proposition p,

   $(\neg \neg p) \Leftrightarrow p$

2. Idempotent Law: For any proposition p,

   (a) $(p \vee p) \Leftrightarrow p$

   (b) $(p \wedge p) \Leftrightarrow p$

3. Identity Law: For any proposition p,

   (a) $(p \vee F_0) \Leftrightarrow p$

   (b) $(p \wedge F_0) \Leftrightarrow p$

4. Inverse Law: For any proposition p,

   (a) $(p \vee \neg p) \Leftrightarrow T_0$

   (b) $(p \wedge \neg p) \Leftrightarrow T_0$

5. Domination Law: For any proposition p,

   (a) $(p \vee T_0) \Leftrightarrow T_0$

   (b) $(p \wedge F_0) \Leftrightarrow F_0$

6. Commutative Law: For any Two propositions p and q,

(a) $(p \vee q) \Leftrightarrow (q \vee p)$

(b) $(p \wedge q) \Leftrightarrow (q \wedge p)$

7. Absorption Law: For any propositions p and q,

(a) $[\, p \vee (p \wedge q)] \Leftrightarrow p$

(b) $[\, p \wedge (p \vee q)] \Leftrightarrow p$

8. De-Morgan Law: For any propositions p and q,

(a) $\neg (p \vee q) \Leftrightarrow \neg p \wedge \neg q$

(b) $\neg (p \wedge q) \Leftrightarrow \neg p \vee \neg q$

9. Associative Law: For any Three propositions p, q, r,

(a) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$

(b) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

10. Distributive Law: For any Three propositions p, q, r,

(a) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

(b) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

Law for the Negation of a Conditional

Given a conditional $p \rightarrow q$, its negation is obtained by using the following law:

$$\neg (p \rightarrow q) \Leftrightarrow [p \wedge (\neg q)]$$

Proof: The following table gives the truth values of $\neg (p \rightarrow q)$ and $(p \wedge \neg q)$ for all possible truth values of p and q.

p	q	$p \rightarrow q$	$\neg (p \rightarrow q)$	$\neg q$	$p \wedge \neg q$
0	0	1	0	1	0
0	1	1	0	0	0
1	0	0	1	1	1
1	1	1	0	0	0

1. Let x be a specified number. Write down the negation of the following conditional:

"If x is an integer, then x is a rational number."

2. Let x be a specified number. Write down the negation of the following proposition:

"If x is not a real number, then it is not a rational number and not an irrational number."

3. Prove the following logical equivalences without using truth tables:

(i) $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$

(ii) $[p \vee q \vee (\neg p \wedge \neg q \wedge r)] \Leftrightarrow (p \vee q \vee r)$

(iii) $[(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$

4.Simplify the following compound propositions using the laws of logic:

(i) $(p \vee q) \wedge [\neg\{(\neg p) \wedge q\}]$

(ii) $(p \vee q) \wedge [\neg\{(\neg p) \vee q\}]$

(iii) $\neg [\neg \{(p \vee q) \wedge r\} \vee \neg q]$

5. Prove the following:

(i) $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$

(ii) $[\neg p \wedge (\neg q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

6.Prove the following result:

$$\neg [\{(p \vee q) \wedge r\} \rightarrow \neg q] \Leftrightarrow \neg [\neg [(p \vee q) \wedge r] \vee \neg q] \Leftrightarrow q \wedge r$$