Page No.

TUTORIAL 4

Master's Theorem.

$T(n) = aT(n/b) + f(n)$

$a \geq 1$ & $b > 1$ & $f(n)$ is asymptotically +ve.

① if $f(n) \leq n^c$

$T(n) = \Theta(n^c)$ $\quad c = \log_b a$

② $f(n) = n^c$

$T(n) = \Theta(n^c \log n)$ $\quad$ compare n^c & $f(n)$

③ $f(n) > n^c$

$T(n) = \Theta(f(n))$

Solutions

① $T(n) = 3T(n/2) + n^2$

$a = 3, \; b = 2$

$c = \log_2 3 = 1.5$

$n^2 > n^{1.5}$

$\therefore T(n) = \Theta(n^2)$

② $T(n) = 4T(n/2) + n^2$

$a = 4, \; b = 2$

$c = \log_2 4$

$c = 2$

$f(n) = n^c, \; n^2 = n^2, \; T(n) = \Theta(n^2 \log n)$

Date.
Page No.

(3) $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$c = \log_2 1$

$c = 0, (n^c = 1)$

$2^n > 1$

$T(n) = O(2^n)$

(4) $T(n) = 2^n T(n/2) + 2^n$

$a = 2^n, b = 2$

$c = \log_2 2^n$

$c = n^2, n^c, n^n$

$f(n) = n^c \Rightarrow O(n^n)$

(5) $T(n) = 16T(n/4) + n$

$a = 16; b = 4$

$c = \log_4 4^2$

$c = 2$

compare $f(n)$ & n^c

$n < n^2$

$T(n) = O(n^2)$

(6) $T(n) = 2T(n/2) + n\log n$

$c = \log_2 2 = 1$

$n \log n > n$

$T(n) = O(n \log n)$

(7) $T(n) = 2T(n/2) + n/\log n$

$a = 2, b = 2$

$c = \log_2 2 \quad c = 1$

$\frac{n}{\log n} < n$

$f(n) < n^c$

$O(n)$

(8) $T(n) = 2T(n/4) + n^{0.5}$

$a = 2, b = 4$

$c = \log_4 2 \quad c = 1/2$

$f(n) = n^c$

$n^{0.5} = n^{0.5}$

$T(n) = O(n^{0.5} \log n)$

(9) $T(n) = 0.5\,T(n/2) + 1/n$

$c = \log \frac{1}{2} = -1$

$n^{-1} = n^{-1}$

$T(n) = \Theta(n^{-1} \log n)$

(10) $T(n) = 16\,T(n/4) + n!$

$a = 16, b = 4$

$c = \log_4 16, \quad c = 2$

$n! \quad n^2$

if $(n \geq 3)$ $n! > n^2$

$\Theta(n!)$

if $(n < 3)$ $n^2 > n!$

$\Theta(n^2)$

(11) $T(n) = 4T(n/2) + \log n$

$a = 4, \quad b = 2$

$c = \log_2 4, \quad c = 2$

$\log n < n^2$

$T(n) = \Theta(n^2)$

(12) $T(n) = \sqrt{n}\,T(n/2) + \log n$

$a = \sqrt{n}, \quad b = 2$

$c = \log_2 n^{1/2}$

(13) $T(n) = 8T(n/2) + n$

$a = 3, b = 2$

$c = \log_2 3 = 1.5$

$f(n) < n^c$

$n < n^{1.5}$

$T(n) = \Theta(n^{1.5})$

(14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3, \quad b = 3$

$c = \log_b a = 1$

$f(n) = n^{1/2}$

$n^c = n^1$

$f(n) = n^c$

$T(n) = \Theta(n)$

(15) $T(n) = 3T(n/3) + n/2$

$a = 3, b = 3$

$c = \log_b a = 1$

$f(n) = n/2$

$n^{c'} = n$

$f(n) < n^c$

$T(n) = n$

(17) $T(n) = 3T(n/4) + n\log n$

$a = 3, b = 4$

$c = \log_4 3$

$\therefore T(n) = n\log n$

$n^c = 0.1$

$f(n) > n^c$

$T(n) = n\log n$

(18) $T(n) = 6T(n/3) + n^2\log n$

$a = 6, b = 3$

$c = \log_3 6 = 1.6$

$T(n) > n^c$

$\therefore T(n) = \Theta(n^2\log n)$

(19) $T(n) = 4T(n/2) + n\log n$

$a = 4, b = 2$

$c = \log_2 4 = 2$

$f(n) = n\log n$

$n^2 = n^2$

$n\log n < n^2$

$TC = \Theta(n^2)$

(20) $T(n) = 64T(n/8) - n^2\log n$

$a = 64, b = 8$

$c = \log_8 64 = 2$

$f(n) > n^c$

$n^2\log n > n^2$

$TC = \Theta(n^2\log n)$

(21) $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3$

$c = \log_3 7 = 1.77$

$f(n) > n^0$

$n^2 > n^{1.77}$

$T(n) = \Theta(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, b = 2$

$\log_2 1 = 0$

$f(n) > n^c$

$n(2 - \cos n) > n^0$

$T(n) = \Theta(n(2 - \cos n))$