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Module 1

Satellite orbits and Trajectories.

Definition of an Orbit and a Trajectory:-

Orbit :- This is a trajectory that is periodically repeated.

Trajectory :- This is a path traced by the moving body.

Eg: The path followed by the motion of an artificial satellite around the earth is an orbit, whereas the path followed by a launch vehicle is called as launch trajectory / trajectory.

Orbiting satellites : Basic principles :-

The motion of natural and artificial satellites are governed by two forces:

(i) Centripetal force :- It is the force directed towards the centre of the earth due to gravitational force of attraction of earth.

(ii) Centrifugal force :- It is the force that acts outwards from the centre of the earth.

In case of satellite orbiting earth, the satellite exerts a centrifugal force, however the force that is causing the circular motion is the centripetal force.

Note:- In absence of the centripetal force, the satellite would have moved in straight line instead of circular motion. This centripetal force is directed at right angles to the satellite velocity towards centre of earth transforms straight line motion to circular motion.

Newtons Law of Gravitation:-

According to Newton's law of gravitation, every particle irrespective of its mass attracts every other particle with a gravitation force (F) whose magnitude is directly proportional to the product of the masses of two particles and inversely proportional to the square of the distance between them.

Mathematically, $F = \dfrac{G m_1 m_2}{r^2}$ — (1)

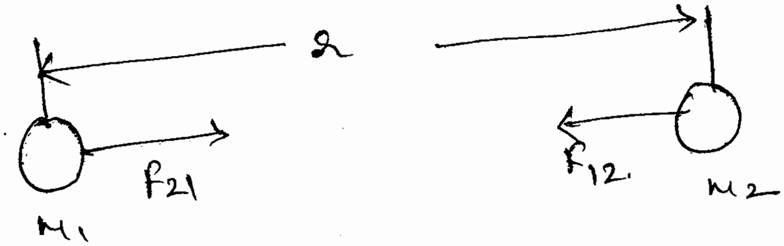
where $m_1, m_2 \rightarrow$ are masses of two particles

$r \rightarrow$ distance between the two particles

$G \rightarrow$ Gravitational constant $= 6.67 \times 10^{-11} m^3/kg s^2$

$$\left[G = \frac{N m^2}{kg^2} \right]$$

The force with which particle with mass m_1 attracts the particle with mass m_2 equals the force with which particle with mass m_2 attracts the particle with mass m_1. The forces are equal in magnitude but opposite in direction as shown in fig 1.



$F_{21} = F_{21}$

fig (i): Newton's law of Gravitation

Note :- * The acceleration, which is force per unit mass, $(a = F/M)$, experienced by two particles, depend upon their masses.

* A larger mass experiences lesser acceleration.
* The law is applicable for particles whose sizes are small compared to the distance between them.

* Mass of the Earth = 5.972×10^{24} kg.
* Approximate mass of satellite = 500 kg for small ~~sats~~ satellite.

Newton's Second law of motion :-

According to the Newton's second law of motion, the force equals the product of mass and acceleration.

In case of a satellite orbiting earth, if the orbiting velocity is v, then the acceleration called centripetal acceleration, experienced by the satellite at a distance r from the center of the earth would be v^2/r.

If the mass of the satellite is m, then it would experience a reaction force of mv^2/r. This reaction force is called as centrifugal force, directed outwards from the centre of the earth & for a satellite it is equal in magnitude to the gravitational force.

If the satellite travels with uniform velocity, then it travels in a circular orbit. Then equating the two forces mentioned, we get

$$\frac{GM_1 M_2}{r^2} = \frac{M_2 v^2}{r} \quad \leftarrow (2)$$

then $$\boxed{v = \sqrt{\frac{GM_1}{r}} = \sqrt{\frac{\mu}{r}}} \quad \leftarrow (3)$$

where $M_1 \rightarrow$ mass of the Earth.
$M_2 \rightarrow$ mass of the satellite

$\mu = GM_1 = 3.986013 \times 10^5 \, km^3/s^2 = 3.986013 \times 10^{14} \, Nm^2/kg$
$\hookrightarrow$ geocentric gravitational constant.

The orbital period (T) can be computed from,

$$\boxed{T = \frac{2\pi r^{3/2}}{\sqrt{\mu}}} \quad - (4)$$

For elliptical orbit, the forces governing the motion of the satellite are same. The velocity at any point on an elliptical orbit at a distance d from the centre of the earth is given by the formula,

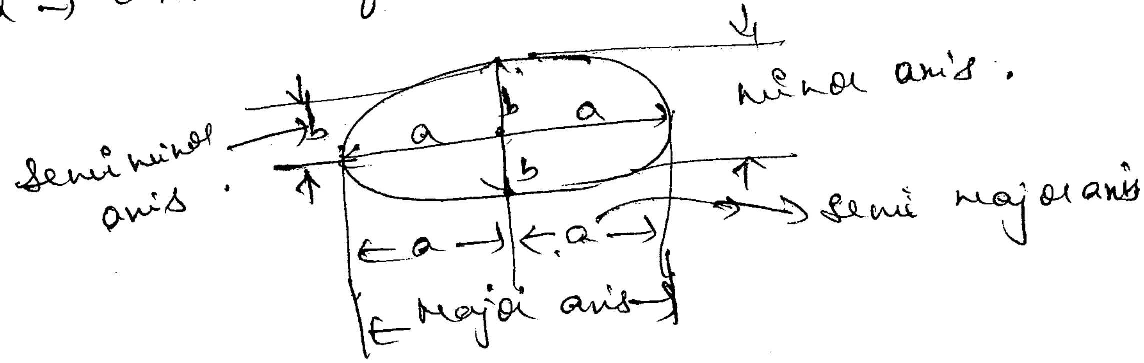
$$v = \sqrt{\mu\left(\frac{2}{d} - \frac{1}{a}\right)} \quad — (5)$$

where a → semi major axis of the elliptical orbit (refer below fig)

The orbital period in case of elliptical orbit is given by,

$$\boxed{T = \frac{2\pi a^{3/2}}{\sqrt{\mu}}} \quad — (6)$$

In eqn (5), d → distance of orbit from centre of the earth.



Kepler's law :-

The movement of a satellite in an orbit is governed by three kepler's law. These laws were given by Johannes kepler. He gave a set of three empirical expressions that explained planetary motion.

Kepler's first law :-

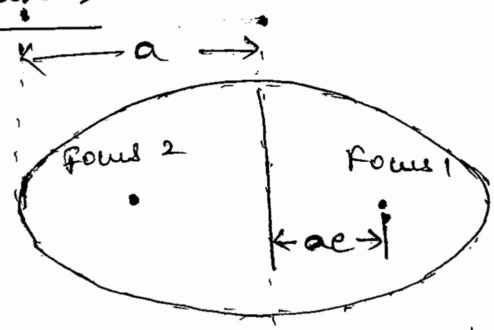


fig2: Kepler's first law

The law states that the orbit of a satellite around Earth is elliptical with the centre of the earth laying at one of the foci of the ellipse. The elliptical orbit is characterized by its semi major axis 'a' and eccentricity 'e'. Eccentricity is the ratio of distance between centre of ellipse and either of its foci (= ae) to the semimajor axis of the ellipse a.

i.e $e = \frac{\sqrt{a^2 - b^2}}{a}$

For a circular orbit $a = b \therefore e = 0$. i.e a circular orbit is a special case of elliptical orbit where foci merge together to a give a single central point and eccentricity becomes zero.

For any elliptical motion, the law of conservation of energy is valid at all points on the orbit. The law of conservation of energy states that energy can neither be created nor destroyed, it can only be transformed from one form to another. For satellites this means that sum of kinetic and the potential energies of a satellite always remain constant. The value of this constant is $-\frac{GM_1M_2}{2a}$

where M_1 is the mass of the earth
M_2 " " mass of the satellite
$a \rightarrow$ semi major axis of the orbit

The kinetic energy & potential energy of satellite is given by,

$$\text{kinetic energy} = \frac{1}{2}M_2v^2 \quad -(7)$$

$$\text{Potential energy} = -\frac{GM_1M_2}{r} \quad -(8) \quad [r \rightarrow \text{distance}]$$

from (7) & (8)

$$\frac{1}{2}M_2v^2 - \frac{GM_1M_2}{r} = -\frac{GM_1M_2}{2a}$$

$$\text{where } v^2 = GM_1\left[\frac{2}{r} - \frac{1}{a}\right]$$

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

Kepler's second law:

This property is used to increase the length of time a satellite can be seen from particular region of the earth.

The line joining the satellite and the centre of the earth sweeps out equal areas in the plane of the orbit in equal times (as shown in fig 3) that is, the rate (dA/dt) at which it sweeps area A is constant. The rate of change of swept out area is given by,

$$\frac{dA}{dt} = \frac{\text{Angular momentum of the satellite}}{2m}$$

where m is mass of the satellite.

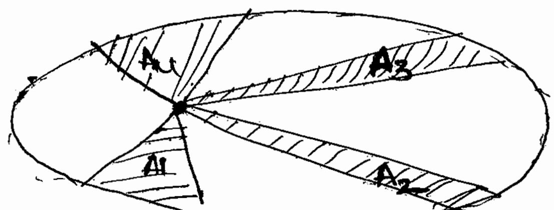


fig 3: kepler's second law.

Kepler's second law is also equivalent to the law of conservation of momentum, which implies that the angular momentum of the orbitting satellite is given by the product of the radius vector and component of linear momentum perpendicular to the radius vector is constant at all points on the orbit.

Angular momentum of satellite of mass 'm' is given by $\underline{mr^2\omega}$ or $\underline{mrv}$ $[\because v = \omega r]$

$\omega \rightarrow$ angular velocity of the satellite.

i.e $mr^2\omega = m\omega r(r) = mv'r$ remains constant.

$v' \rightarrow$ component of satellite velocity v in the direction perpendicular to the radius vector and expressed as $v\cos\gamma$

where $\gamma \rightarrow$ angle between the direction of motion of satellite and local horizontal, which is in the plane perpendicular to radius vector r (fig 4)

Hence the product $rv\cos\gamma$ is constant.

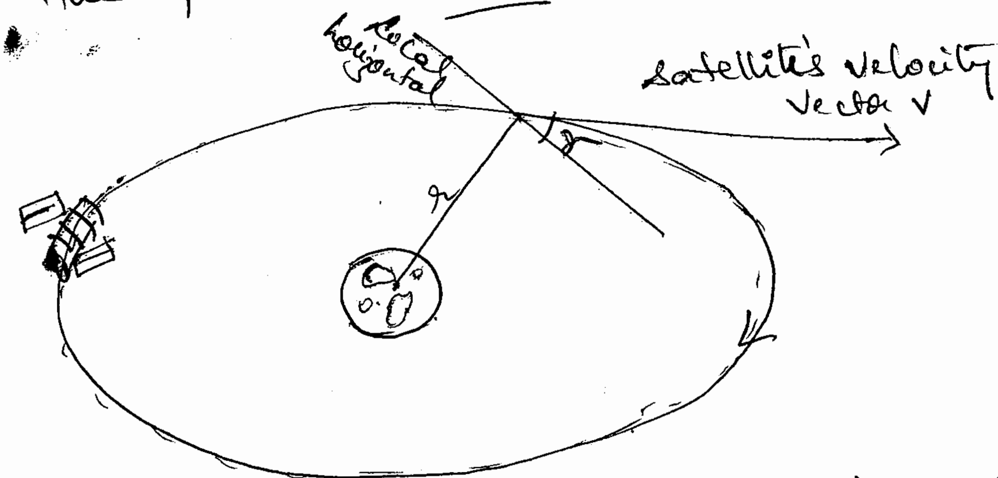


fig 4: Satellite position at any given time.

Note:- For any satellite in an elliptical orbit, the dot product of its velocity vector and radius vector at all point is constant. Hence $v_p r_p = v_a r_a = vr\cos\gamma$ — (7)

where $v_p \rightarrow$ velocity at the perigee point

$r_p \rightarrow$ perigee distance.

$v_a \rightarrow$ velocity at the apogee point.

$r_a \rightarrow$ apogee distance.

$v \rightarrow$ satellite velocity at any point in the orbit.

$r \rightarrow$ distant of the point.

$\gamma \rightarrow$ angle betn the direction of motion of the satellite

Kepler's third law :- This law is also known as law of periods. According to kepler's third law, the square of the time period of any satellite is proportional to the cube of the semi major axis of its elliptical orbit.

Expression for time period :-

A circular orbit with radius r is assumed. Equating the gravitational force with the centrifugal force, we get

$$\frac{GM_1M_2}{r^2} = \frac{M_2 v^2}{r} \quad \text{--- (9)}$$

replacing $v = \omega r$, in eq^n (9) we get,

$$\frac{GM_1M_2}{r^2} = \frac{M_2\omega^2 r^2}{r} = M_2\omega^2 r \quad \text{--- (10)}$$

$\therefore \omega^2 = GM_1/r^3$, Put $\omega = 2\pi/T$, gives,

$$T^2 = \left(\frac{4\pi^2}{GM_1}\right) r^3$$

or $$T = \left[\frac{2\pi}{\sqrt{\mu}}\right] r^{3/2} \quad \text{--- (11)}$$

In eq^n (11) if r is replaced by semi major axis 'a'

$$\boxed{T = \left(\frac{2\pi}{\sqrt{\mu}}\right) a^{3/2}} \quad \text{--- (12)}$$

Orbital parameters :-

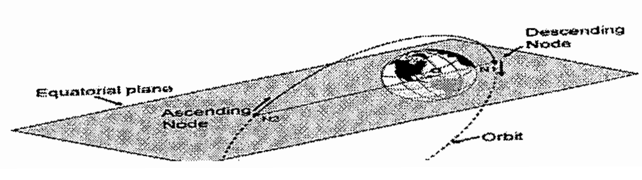
The satellite orbit is in generally elliptical & is characterised by number of parameters. Some of the orbital elements and parameters are:

1. Ascending and Descending nodes.
2. Equinoxes
3. Solstices
4. Apogee
5. Perigee
6. Eccentricity
7. Semi-Major axis.
8. Right Ascension of Ascending Node.
9. Inclination
10. Argument of Perigee
11. The Anomaly of satellite
12. Angles defining direction of satellite.

Each parameters are as explained below.

1) Ascending and descending nodes :-

The satellite orbit cuts the equatorial plane at two points. First called the descending node (N1), where the satellite passes from the northern hemisphere to southern hemisphere and the second, called ascending node (N2), where the satellite passes from the southern hemisphere to the northern hemisphere [see fig 5]



The nodes must be specified in order to define the orientation of elliptical orbit.

2. Equinoxes :- (means equal day and night) The inclination of the equatorial plane of Earth with respect to the direction of sun, defined by the angle formed by the line joining the centre of the Earth and the sun with the Earths equatorial plane follows a sinusoidal variation and completes one cycle of sinusoidal variation over a period of 365 days (see fig 6)

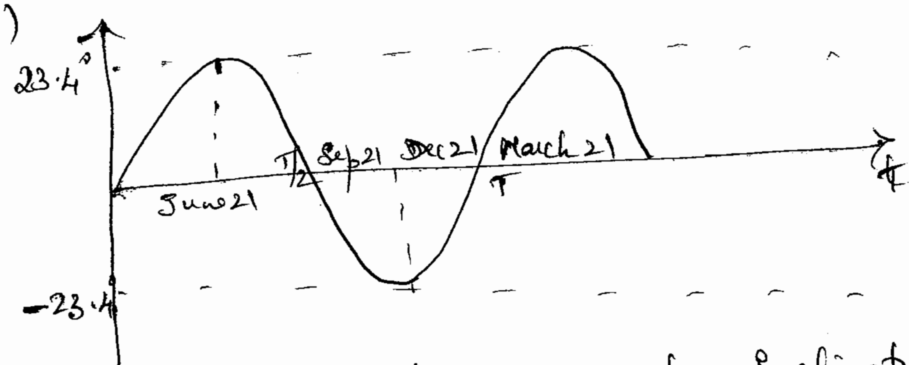


fig 6: yearly variation of angular inclination of Earth with Sun.

The sinusoidal variation of angle of inclination is defined by,

$$\text{Inclination angle (in degree)} = 23.4 \sin\left(\frac{2\pi t}{T}\right) \quad -(13)$$

where $T = 365$ days.

Expression (13) indicates that the inclination angle is zero for $t = T/2$ & T. This is observed to occur on 21 March, called spring equinox & on
[During equinox the sun is exactly above the

21 September, called autumn equinox. [12hrs of daylight & 12hrs of darkness]
The 2 equinoxes are spaced 6 months apart.
During equinoxes, it can be seen that the equatorial plane of Earth will be aligned with the direction of the Sun.

Also the line of intersection of Earth's equatorial plane & Earths orbital plane passes through centre of Earth is known as the line of Equinoxes. The direction of this line with respect to the direction of the Sun on 21 March determines a point at infinity called the vernal equinox (γ) [see fig 7)

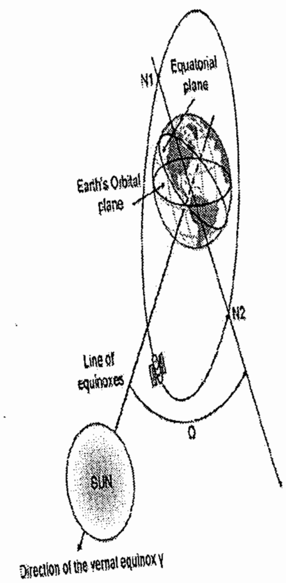


Fig 7

3. Solstices :- Solstices are the times ~~which~~ when the inclination angle is at its maximum (ie 23.4°). These occur twice during a year on 21 June, called summer solstice & 21 December called (days are longer)
winter solstice (days are shorter)
[Days are longer in summer solstice & shorter

4. Apogee : Apogee is the point on the satellite orbit that is at the farthest distance from the centre of the Earth (see fig 8). The Apogee distance A can be computed from the known values of orbit eccentricity e & the semi-major axis a as,

$$A = a[1+e] \quad \text{— (14).}$$

The apogee distance A can also be computed from the known values of perigee distance P and velocity at the perigee v_p as,

$$v_p = \sqrt{\frac{2\mu}{P} - \frac{2\mu}{A+P}} \quad \text{—(15)}$$

where $v_p = v\,d\cos\gamma / P$, where v being velocity of the satellite at a distance d from the centre of the Earth.

& P → Perigee distance. γ → inclination angle

A → Apogee distance.

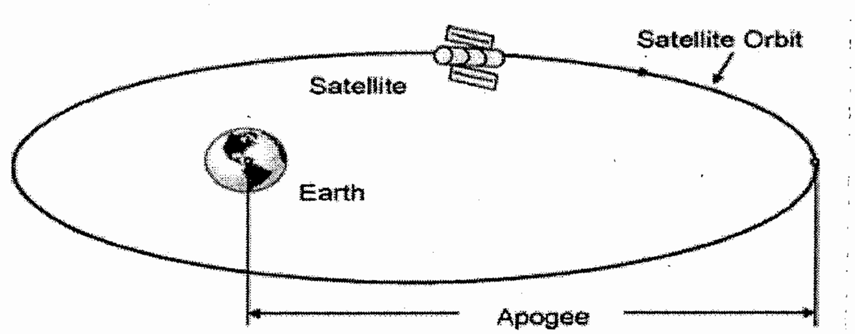


fig 8: Apogee.

5. <u>Perigee</u> :– Perigee is the point on the orbit that is nearest to the centre of the Earth (see fig 9). The perigee distance P can be computed from the known values of orbit eccentricity e and the semi-major axis a as,

$$P = a(1-e) \quad \text{— (16)}$$

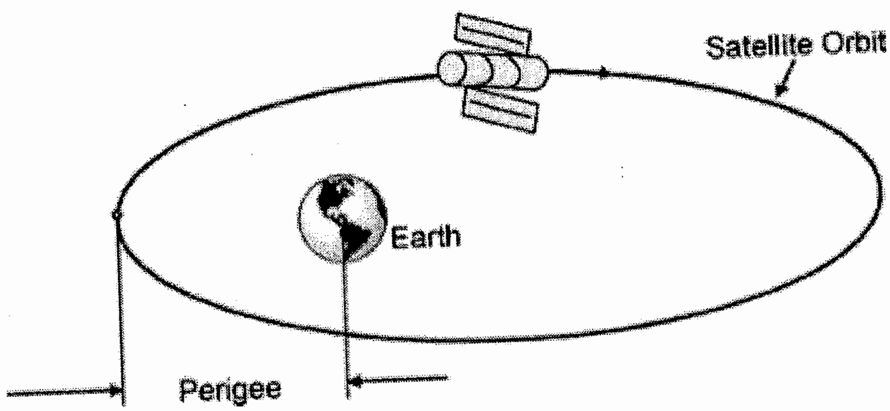


fig: Perigee

6. <u>Eccentricity</u> : The orbit eccentricity e is the ratio of the distance between the centre of the ellipse and the centre of the Earth to the semi major axis of the ellipse. It can be computed from any of the following eqns.

$$e = \frac{A-P}{A+P} \quad \text{— (17)}$$

$$e = \frac{A-P}{2a} \quad \text{— (18)}$$

$$e = \frac{\sqrt{a^2-b^2}}{a} \quad \text{— (19)}$$

where a & b are semi-major & semi minor axes respectively.

7. Semi-Major Axis :- It is a geometrical parameter of an elliptical orbit. It can however be computed from known values of apogee & perigee distances as,

$$a = \frac{A+P}{2} \quad \text{— (20).}$$

$$= \frac{\text{apogee} + \text{perigee}}{2}.$$

8. Right Ascension of Ascending Node :-

The right ascension of the ascending node tells about the orientation of the line of nodes, which is the angle made by the line joining the ascending and descending nodes, with respect to the direction of the vernal equinox. It is expressed as an angle 'Ω' measured from the vernal equinox towards the line of nodes in direction of rotation of Earth. The angle could be anywhere from $0°$ to $360°$ (see fig 10)

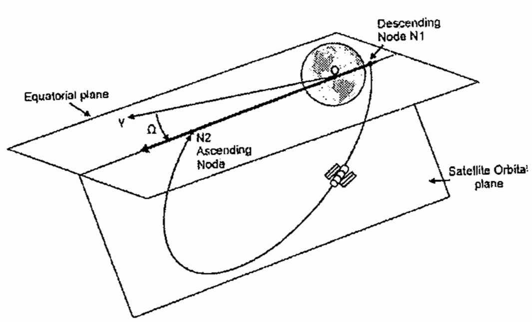


Fig: Right ascension of the ascending node

Acquisition of the correct angle of right ascension of ascending node Ω is important to ensure that the satellite orbit in the given plane. This can be acheived by choosing an appropriate injection time depending upon the longitude.

Calculation of Ω:-

Angle Ω can be computed as the difference between 2 angles. One is the angle α between direction of Vernal equinox & longitude of injection point and other angle is β. where β is the angle between line of nodes & longitude of injection point.

Angle β is computed from:

$$\sin\beta = \frac{\cos i \sin l}{\cos l \sin i} \quad - 21$$

where $\angle i$ is orbit inclination & l is the latitude at the injection point.

9. Inclination:- Inclination is the angle that the orbital plane of satellite makes with the Earth's equatorial plane.

Measurement of Inclination! -

The line of nodes divides both Earth's Equatorial plane as well as the satellite's orbital plane into two halves.

Inclination is measured as the angle between half of satellite's orbital ~~pole~~ plane containing the trajectory of the satellite from the descending ~~no~~ node to the ascending node to that half of the equatorial plane containing the trajectory of a point on the equator from n1 to n2 where n1 & n2 are respectively points vertically below the descending and ascending nodes.

The inclination angle can be determined from the latitude 'l' at the injection point & angle A_z between the projection of satellite's velocity vector on the local horizontal and north. It is given by,

$$\boxed{\cos i = \sin A_z \cos l} \quad \text{—(22)}$$

10. Argument of the perigee! - This parameter defines the location of the major axis of the satellite orbit. It is measured as angle ω between the line joining the perigee & the centre of the Earth and the line of nodes

from the ascending to the descending node in the same direction as that of satellite orbit.
(see fig 11)

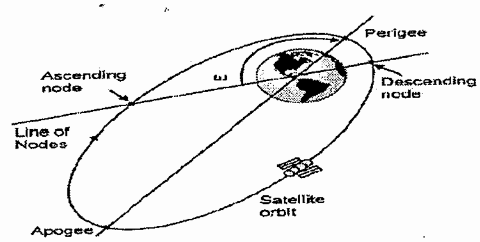


Fig: argument of perigee

fig 11: Argument of the perigee.

11. True Anomaly of satellite – This parameter is used to indicate the position of the satellite in its orbit. This is done by defining an angle θ, called true anomaly of the satellite, formed by the line joining the perigee and center of the Earth with the line joining the satellite and the center of the Earth. [see fig 11]

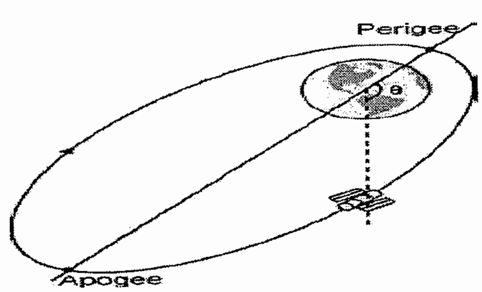


Fig: True anomaly of the satellite

fig 11: True anomaly of a satellite.

12. Angles defining direction of satellite! – The direction of the satellite is defined by two angles, the first by angle γ between the direction of the satellites velocity vector & its projection in the local horizontal & second by angle A_z between the north & the projection of the satellites velocity vector on the local horizontal. [see fig 12]

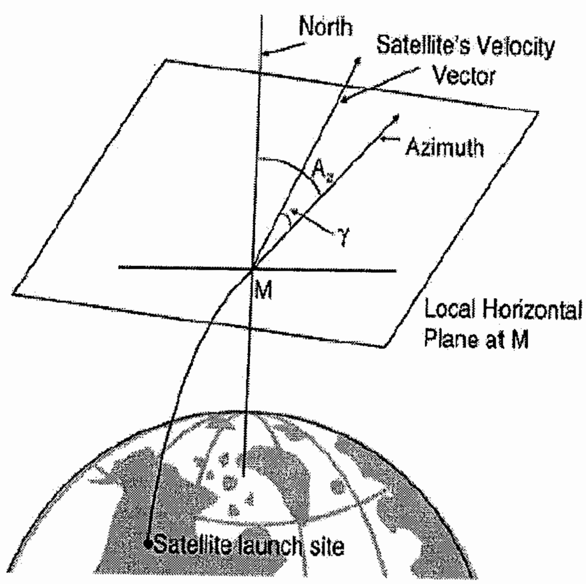


Fig : Angles defining the direction of the satellite

Prob 1: A satellite is orbiting Earth in a uniform circular orbit at a height of 630 km from the surface of Earth. Assume the radius of Earth & its mass to be 6370 km & 5.98×10^{24} kg respty. Determine the velocity of the satellite. (Take Gravitational Const $G = 6.67 \times 10^{-11}\ N m^2/kg^2$)

Solⁿ :- Orbit radius $r = \overset{\text{ht + radius of Earth}}{6370 + 630}$

$= 7000\ km = 7000 \times 10^3\ m$

$$\text{Const} = \mu = GM = 6.67 \times 10^{-11} \times 5.98 \times 10^{24}$$

$$= 39.8 \times 10^{13}\ N m^2/kg = 39.8 \times 10^{13}\ m^3/s^2$$

The velocity of satellite can be computed from,

$$v = \sqrt{\mu/R} = \sqrt{\frac{39.8 \times 10^{13}}{7000 \times 10^3}} = 7.54\ km/s.$$

Prob 2: The apogee & perigee distances of a satellite orbiting in a elliptical orbit are respectively 45000 km & 7000 km. Determine the foll

(a) Semi-major axis of the elliptical orbit.

(b) Orbit Eccentricity

(c) Distance between the center of Earth & the center of elliptical orbit.

Solⁿ (a) Semi major axis of elliptical orbit is $\Big\}$ $a = (\text{Apogee} + \text{Perigee})/2 = \frac{A+P}{2}$

$$= \frac{45000 + 7000}{2} = 26,000\ km$$

(b) Eccentricity $e = \frac{\text{Apogee} - \text{Perigee}}{2a} = \frac{45000 - 7000}{2 \times 26000} = \underline{0.73}$

(c) Distance betn centre of Earth and Centre of ellipse $\Big\} = ae = 26000 \times 0.73$
$= \underline{18980 \text{ km}}$

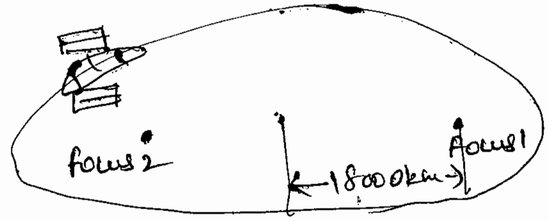
Prob3: A satellite is moving in an elliptical orbit with the major axis equal to 42000 km. If the perigee distance is 8000km, find the apogee & the orbit eccentricity.

Soln: Major axis = 42000 km
= Apogee + Perigee = 42000 km.

$\therefore$ Apogee = 42000 − Perigee
= 42000 − 8000 = 34000 km.

Eccentricity $e = \frac{\text{Apogee} - \text{Perigee}}{\text{Major axis}} = \frac{34000 - 8000}{42000} = \underline{0.62}$

Prob4: Refer to the satellite orbit shown in fig(a). Determine the apogee & perigee distances if the orbit eccentricity is 0.6.



fig(a)

Sol^n: If e is the orbit eccentricity and a the semi-major axis of the elliptical orbit, then the distance between the centre of the Earth and the centre of the ellipse is equal to ae.

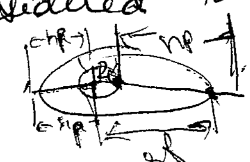
$$\therefore \quad ae = 18000\,km$$

$$a = \frac{18000\,km}{e} = \frac{18000 \times 10^3}{0.6} = 30{,}000\,km.$$

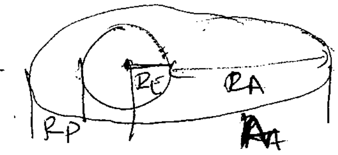
Apogee distance $= a \times (1+e) = 30000(1+0.6) = 48000\,km.$

Perigee " $= a \times (1-e) = 30000(1-0.6) = 12000\,km$

Prob 5: The difference between the farthest and the closest points in a satellite's elliptical orbit from the surface of the Earth is 30000 km & the sum of the distances is 50000 km. If the mean radius of the Earth is considered to be 6400 km, determine the orbit eccentricity



apogee $= R_a + h_a$
perigee $= R_p + h_p$



Sol^n: Apogee − Perigee $= 30000\,km$, as the radius of the earth cancel in this case.

Apogee + Perigee $= 50000 + 2(6400) = 62800\,km.$

$A = R_E + R_A$
$P = R_E + R_P$

orbit Eccentricity $= \dfrac{(\text{Apogee} - \text{Perigee})}{\text{Apogee} + \text{Perigee}} = \dfrac{30000}{62800} = 0.478$

Prob 6: Refer to fig (b). Satellite A is orbiting Earth in a circular orbit of radius 7000 km. Satellite B is orbiting Earth in an elliptical orbit with its

apogee and perigee distances of 47000km & 7000km resply. Determine the velocities of two satellites at point X.
(Take $\mu = 39.8 \times 10^{13} Nm^2/kg$)

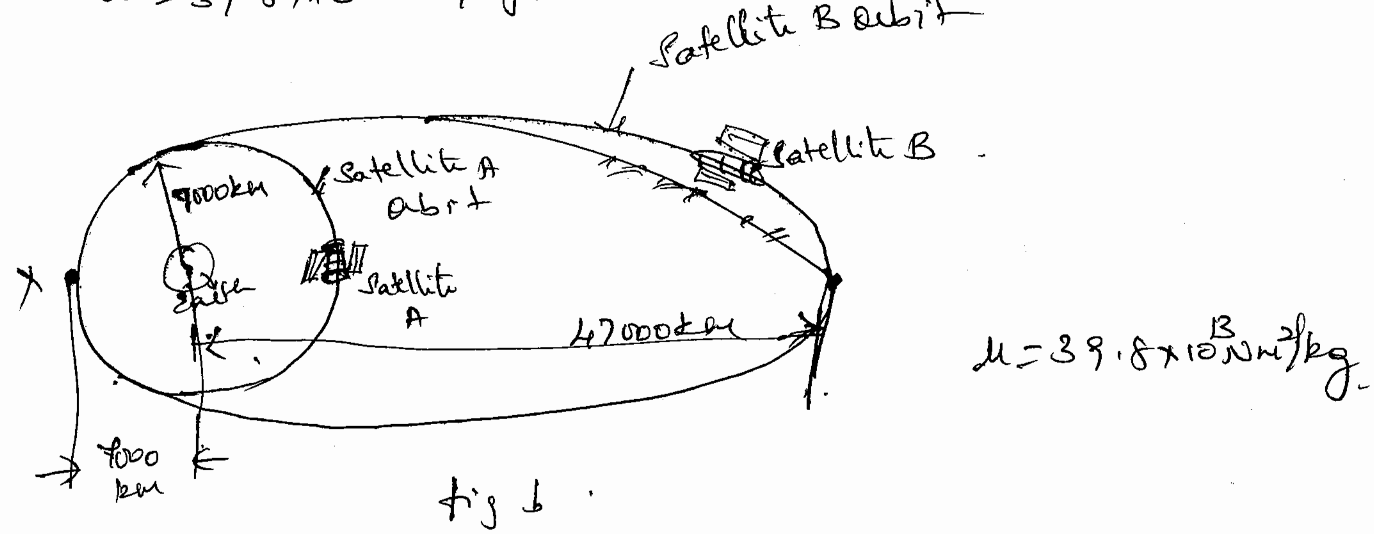


fig b.

Soln - Velocity of a satellite moving a circular orbit (i.e sat A) is constant throughout the orbit & is given by, $v = \sqrt{\frac{\mu}{R}}$.

∴ velocity of satellite A at point X is,

$$v = \sqrt{\frac{\mu}{R}} \Rightarrow \sqrt{\frac{39.8 \times 10^{13}}{7000 \times 10^{3}}} = 7.54 \text{ km/s}$$

Velocity of a satellite at any point in an elliptical orbit is given by, $v = \sqrt{\mu\left[\frac{2}{R} - \frac{1}{a}\right]}$

where $a \rightarrow$ Semi-major axis. $= (47000 + 7000)/2 = 27000$km.
$R \rightarrow$ distance from centre of earth $= 7000$km

$$\therefore v = \sqrt{39.8 \times 10^{13}\left[\frac{2}{7000 \times 10^{3}} - \frac{1}{27000 \times 10^{3}}\right]}$$

$= 9.946$ km/s

PROBLEMS & SOLUTIONS:

① A satellite is moving in an elliptical orbit with the major axis equal to 42000 km. If the perigee distance is 8000 km, find the apogee and the orbit eccentricity.

Ans:

$A \rightarrow$ Apogee
$P \rightarrow$ Perigee
$P = 8000$ km

$$\text{Major Axis} = 2a = 42000 \text{ km}$$

$$\text{Also} \quad 2a = A + P = 42000 \text{ km}$$

$$\therefore A = 42000 - P = 42000 - 8000$$

$$= 34000 \text{ km}$$

$$\therefore \text{Eccentricity} = e = \frac{A - P}{2a} = \frac{34000 - 8000}{42000}$$

$$= 0.62$$

② The difference between the farthest and closest points in a satellite's elliptical orbit from the surface of the Earth is 30000 km and the sum of the distances is 50000 km. If the mean radius of the Earth is 6400 km, determine the orbit eccentricity.

$$A - P = 30,000 \text{ km}$$

$$A + P = 50000 + 2R = 50000 + 2 \times 6400 = 62800 \text{ km}$$

$$e = \frac{A - P}{A + P} = \frac{30000}{62800} = 0.4777$$

done

③ Satellite A is orbiting Earth in a circular orbit of radius 7000 km, Satellite B is orbiting Earth in an elliptical orbit with ~~a~~ its apogee and perigee distances of 47000 km and 70,000 km respectively. Determine the velocities of the two satellites at the point X, the perigee point.

$$(\mu = 39.8 \times 10^{13} \text{ Nm}^2/\text{kg})$$

Velocity of the satellite A ~~in~~ moving in a circular orbit is

$$v_A = \sqrt{\frac{\mu}{R}}$$

$$= \sqrt{\frac{39.8 \times 10^{13}}{7000 \times 1000\,m}} = \sqrt{\frac{39.8 \times 10^{13}}{7 \times 10^6}}$$

$$= 7.54\ km/s. = \text{Velocity at } P.$$

Velocity of satellite B at point P is given by:

$$v_B = \sqrt{\mu\left[\frac{2}{P} - \frac{1}{a}\right]} \qquad a = \frac{A+P}{2} = 27000\ km$$

$$= \sqrt{39.8 \times 10^{13}\left[\frac{2}{7000{,}000} - \frac{1}{27000{,}000}\right]}$$

Note: Express km into meters.
Since 'N' is in ~~N~~ Nm^2/kg

$$= 10^3\sqrt{398 \times 0.2487} \simeq 9.946\ km/s.$$

(4) Calculate the orbital period of a satellite moving in an elliptical orbit with distance between ~~A & P~~ ~~being 50000 km~~ Perigee point & Apogee point being 50 000 km.

$\mu = 39.8 \times 10^{13}\ Nm^2/kg$

$$2a = 50000\ km \quad \therefore \quad a = 25000\ km \qquad 1\,km = 10^3\,m$$

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{(25 \times 10^3 \times 10^3)^3}{(39.8 \times 10^{13})}}$$

$$= 39368\,s$$

= 10h ~~55 minutes 48 seconds~~
56 min 8 sec

32

⑤ The semimajor axes of two satellites are 18000 km and 24000 km respectively. Determine the relationship between their orbital periods.

Let a_1 & T_1 → Semi-major axis & orbital period of satellite 1

a_2 & T_2 → " " " 2

$$T_1 = 2\pi\sqrt{\frac{a_1^3}{\mu}} \qquad T_2 = 2\pi\sqrt{\frac{a_2^3}{\mu}}$$

$$\frac{T_2}{T_1} = \left(\frac{a_2}{a_1}\right)^{3/2} = \left(\frac{24000}{18000}\right)^{3/2} = \underline{1.54}$$

not req'd

⑥ Satellite A is orbiting Earth in an equatorial orbit of radius 42000 km. Satellite B is orbiting Earth in an elliptical orbit with an apogee and perigee distances of 42000 km and 7000 km respectively. Determine the velocities of the two satellites at the perigee point X ($\mu = 39.8 \times 10^{13}\ Nm^2/kg$)

Also (~~R = 42000 km~~) OX = 42000 km Also $a = 24500$ km

SOLUTION:

Velocity of satellite A, at perigee point is given by :(

$$v = \sqrt{\mu/R} = \sqrt{39.8 \times 10^{13}/42000 \times 1000}$$

$$= 3.078 \text{ km/s}.$$

Velocity of satellite B at point X is given by

$$v = \sqrt{\mu\left[\frac{2}{R} - \frac{1}{a}\right]}$$

a = semi-major axis

= 24500 km

$$= \sqrt{39.8 \times 10^{13}\left[\frac{2}{42000{,}000} - \frac{1}{24500 \times 1000}\right]}$$

$$= 1.645 \text{ km/s}.$$

(7) The elliptical orbit of a satellite has its semi-major and semi-minor axes as 25000 km and 18330 km. respectively. Determine the apogee and perigee distances.

SOLUTION:

Let

A = Apogee distance

P = Perigee distance

a = Semi-major axis

b = Semi-minor axis

e = eccentricity,

Then

$$\cancel{a = \frac{A+P}{2}} \qquad a = 25000\ \text{km}$$

$$\cancel{b = \sqrt{A \times P}} \qquad b = 18330\ \text{km}$$

$$\therefore \quad e = \frac{\sqrt{a^2 - b^2}}{a}$$

$$= \frac{\sqrt{(25000)^2 - (18330)^2}}{25000}$$

$$= 0.68$$

$$P = a(1-e) = 25000\,(1-0.68)$$

$$= 8000\ \text{km}$$

$$A = a(1+e) = 25000\,(1+0.68)$$

$$= 42000\ \text{km}.$$

34 (8) The apogee and perigee distance of a certain elliptical satellite orbit are 42000 km & 8000 km respectively. If the velocity at perigee distance is 9.142 km/s what would be the velocity at the apogee point?

If P = perigee distance = 8000 km
A = apogee distance = 42000 km
v_{PA} = Velocity at apogee
v_{PP} = Velocity at perigee = 9.142 km/s.

$$v_P \times P = v_A \times A$$

$$\therefore \quad v_A = \frac{v_P \times P}{A} = \frac{8000 \times 9.142}{42000}$$

$$= 1.741 \text{ km/s.}$$

For Injection velocity

(9) A Satellite launched with an injection velocity v_1 from a point above the Surface of the Earth at a distance P from the center of the Earth attains an elliptical orbit with an apogee distance A_1. The same Satellite when launched with an injection velocity v_2 from the same perigee distance attains an elliptical orbit with an apogee distance of A_2. Derive the relationship between v_1 & v_2 in terms of P, A_1 & A_2

SOLUTION: Velocity at Perigee can be written as:

See last page

$$v_1 = \sqrt{2\mu\left[\frac{1}{P} - \frac{1}{A_1+P}\right]}$$

$$v_2 = \sqrt{2\mu\left[\frac{1}{P} - \frac{1}{A_2+P}\right]}$$

$$\frac{\frac{A_2+\cancel{P}-\cancel{P}}{P(A_2+P)}}{\frac{A_1+\cancel{P}-\cancel{P}}{P(A_1-P)}}$$

$$\left(\frac{v_2}{v_1}\right)^2 = \frac{\frac{1}{P} - \frac{1}{A_2+P}}{\frac{1}{P} - \frac{1}{A_1+P}} = \frac{1 + P/A_1}{1 + P/A_2}$$

$$\frac{A_2}{A_1} \times \frac{(A_1-P)}{(A_2+P)}$$

$$\frac{A_2+P-\cancel{P}}{} \times \frac{P(A_1+P)}{} = \frac{A_2 P(A_1+P)}{P(A_1+P)} = \frac{A_2 A_1(1+P/A_1)}{A_1 A_2(1+P/A_2)}$$

(10) A rocket injects a satellite at perigee with a horizontal velocity of 8 km/s from a height of 1620 km from the surface of the Earth. What will be the velocity of the satellite at a point distant 10000 km from the center of the Earth if the direction of satellite makes an angle of 30° with the local horizontal at that point? Assume radius of Earth to be 6380 km.

SOLUTION: We know

$$v_p = \sqrt{2\mu\left[\frac{1}{P} - \frac{1}{A+P}\right]}$$

$$= v \cdot d \cdot \cos\gamma / P$$

$v \to$ velocity of satellite

$v_p = 8$ km/s, $P = 1620 + 6380 = 8000$ km

If $d = 10{,}000$ km & $\gamma = 30°$,

$$v = \frac{v_p \times P}{d \cos\gamma} = \frac{(8000) \times 8}{10000 \times \cos 30°}$$

$\cos 30° \to 0.866$

$$= \frac{8 \times 8}{10 \times 0.866} = 7.39 \text{ km/s}.$$

(11) A typical Molniya orbit has a perigee and apogee ~~distance~~ heights above the surface of the Earth as 400 km & 40000 km respectively. Verify that the orbit has a 12 hour time period assuming radius of Earth to be 6380 km and $\mu = 39.8 \times 10^{13}$ Nm²/kg

(11) continued:

SOLUTION: The orbital period of a Satellite in elliptical orbit is given by

$$T = \frac{2\pi a^{3/2}}{\sqrt{\mu}} \qquad a \Rightarrow \text{Semi-major axis:}$$

$$a = \frac{40000 + 400 + 6380 + 6380}{2}$$

$$= 26580 \text{ km}$$

$$\therefore T = \frac{2 \times 3.14 \times (26580 \times 10^3)^{3/2}}{\sqrt{39.8 \times 10^{13}}}$$

Note
a is expressed in meters.
Since μ is in Nm^2/kg

$$= 43137 \text{ s}$$

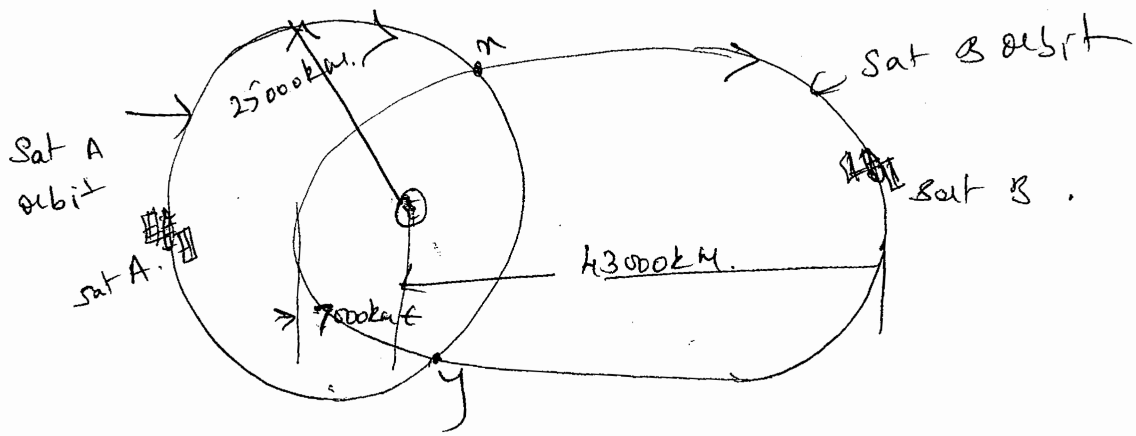
$$\simeq \underline{12 \text{ hours}}$$

Probl A satellite launched with an injection velocity v_1 from a point above the surface of the Earth at a distance P of 8000km from the centre of the Earth attains an elliptical orbit with an apogee distance of 12000km. The same satellite when launched with an injection velocity v_2 that is 20% higher than v_1 from the same perigee distance attains an elliptical orbit with an apogee distance A_2. Determine new apogee distance.

Soln:- We know $\left(\frac{v_2}{v_1}\right)^2 = \left[\frac{1+P/A_1}{1+P/A_2}\right]$ $\quad v_2 = 1.2\,v_1$

$\therefore 1.44 = \left[1 + \frac{8000}{12000}\right]$ $\quad \therefore A_2 = \underline{50826 \text{ km}}$

7. Satellite A is orbiting Earth in a circular orbit of radius 25000 km. Satellite B is orbiting Earth in an elliptical orbit with its apogee & perigee distances of 43000 km & 7000 km resply. Determine the velocities of the two satellites at the indicated points X & Y. (Take $\mu = 39.8 \times 10^3\ Nm^2/kg$)



Sol^n: Velocity of a satellite moving in a circular orbit is constant throughout the orbit & is given by

$$v = \sqrt{\frac{\mu}{R}}.$$

∴ velocity of satellite A at pts X & Y is given by,

$$v = \sqrt{\frac{39.8 \times 10^{13}}{25000 \times 10^3}} = \underline{3.989\ km/s}.$$

Velocity of a satellite at any point in an elliptical orbit is given by,

$$v = \sqrt{\mu\left(\frac{2}{R} - \frac{1}{a}\right)} \qquad a = \frac{(43000 + 7000)}{2}\ km$$

$$= 25000\ km$$

∴ velocity of satellite B at pts X & Y is

$$v = \sqrt{39.8 \times 10^{3} \times \left[\frac{2}{25000000} - \frac{1}{25000000}\right]}$$

$$= 3.989 \, km/s$$

(7) The elliptical orbit of a satellite has its semi-major & semi-minor axis as 25000km & 18330km resply. Determine the apogee & perigee distances.

Sol^n We know $a = \frac{A+P}{2}$ & $b = \sqrt{A \times P}$

Given $\frac{A+P}{2} = 25000 km$ ie $A + P = 50000 km$
$\therefore P = 50000 km - A$ —(1)

$b = \sqrt{A \times P} = 18300$ or $A \times P = 335988900$ —(2)

Substituting P from (1) in (2) we get

$$A \left[50000km - A\right] = 335988900$$

$$A^2 - 50000A \times 10^3 + 335988900 = 0$$

On solving Quadratic eq^n, we get
$A = 42000 km$ & $8000 km$.

If semi major axis is 25000 km then A cannot be 8000km.

∴ Apogee distance $A = 42000 km$.

∴ $P = 50000 - 42000 = 8000 km$.

$A = a(1+e) = 42000 km$

Injection Velocity and Resulting Satellite Trajectories:-

The satellite trajectory directly depends on the horizontal velocity with which a satellite is injected into space by a launch vehicle. This phenomenon is explained in terms of 3 cosmic velocities.

The general expression for the velocity of a satellite at a perigee point (v_p), assuming an elliptical orbit is given by,

$$v_p = \sqrt{\left(\frac{2\mu}{P}\right) - \left(\frac{2\mu}{A+P}\right)} \quad \text{—— (1)}$$

where $A \rightarrow$ apogee distance, $P \rightarrow$ Perigee distance

$\mu = GM = \text{Const}$ $\quad G = 6.67 \times 10^{-11} \, Nm^2/kg^2$

$M \rightarrow$ Mass of Earth $= 5.98 \times 10^{24}$ kg.

$\mu = 3.98 \times 10^{14} \, Nm^2/kg$.

① First cosmic velocity is the one at which apogee and perigee distances are equal. that is $A = P$ & the orbit is circular. Then exp^r ① reduces to,

$$v_1 = \sqrt{\frac{\mu}{P}}$$

Conclusion:- Irrespective of distance P of satellite from centre of earth, if the injection velocity is equal to first cosmic velocity also sometimes called the first orbital velocity. The satellite follows a circular orbit as shown in fig 13 and moves with a uniform velocity equal to $\sqrt{\mu/p}$

A simple calculations shows that [geostat sat] for a satellite 35,786 km above the Earth, the first cosmic velocity is 3.075 km/s & the orbital period is 23h 56m, which is equal to time period of one side real day, i.e the time taken by Earth to complete one full rotation around its axis with reference to distant stars.

$$v = \sqrt{\frac{\mu}{p}} = \sqrt{\frac{3.98 \times 10^{14}}{35786 \times 10^{3} + 6370 \times 10^{3}}}$$

↑ Radius of the earth

$= 3.075$ km/s

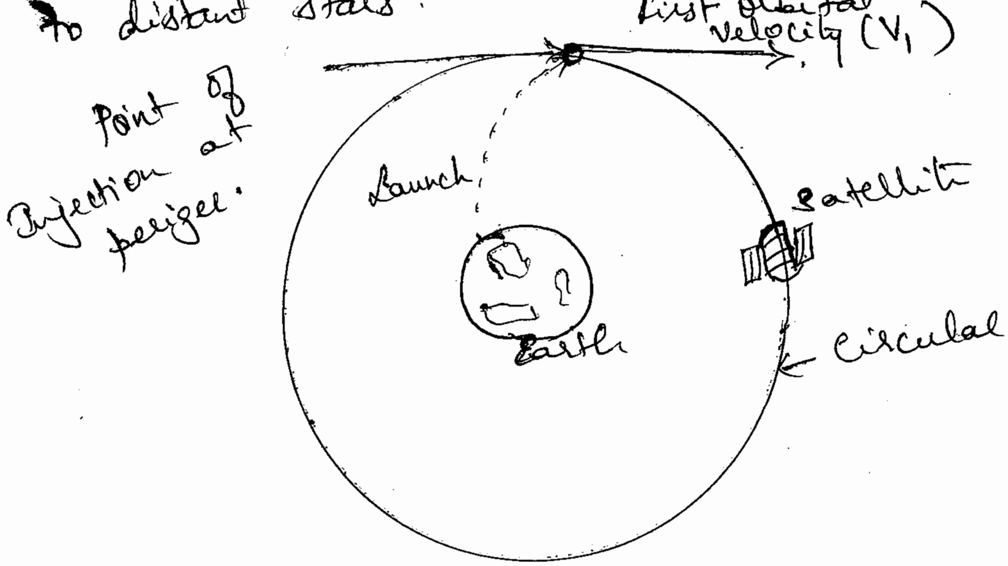


fig 13.

② If injection velocity is less than first cosmic velocity :-

* The satellite follows a ballistic trajectory and falls back to Earth.
* In this case orbit is elliptical & injection point is at the apogee and not the perigee.
* If perigee is in atmosphere then satellite accomplishes a ballistic flight and falls back to Earth. as shown in fig 14.

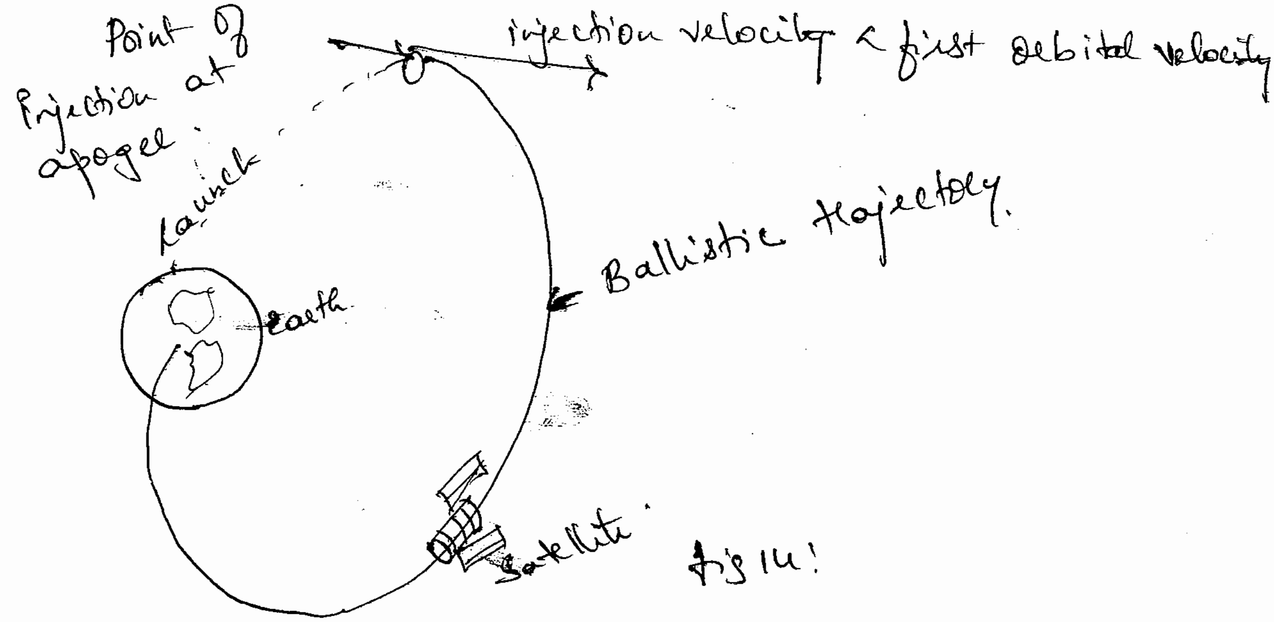


fig 14!

* For injection velocity greater than first cosmic velocity and less than second cosmic velocity ie $v > \sqrt{\mu/P}$ & $v < \sqrt{\frac{2\mu}{P}}$, the orbit is elliptical and eccentric. The orbit eccentricity is between 0 & 1.

$$\text{Consider } v_p = \sqrt{\frac{2\mu}{P} - \frac{2\mu}{A+P}} = \frac{v_d \cos\tau}{P}$$

when injection velocity $v_p = \sqrt{\frac{2\mu}{P}}$, then apogee distance A becomes infinite and the orbit takes the shape of a parabola and the eccentricity is 1. This is the second cosmic velocity v_2 or the second orbital velocity.

At this velocity v_2, the satellite escapes the Earth's gravitational pull.

* For injection velocity > second cosmic velocity, trajectory is hyperbolic & eccentricity is > 1

3) Third cosmic velocity (v_3) :-

If the injection velocity is greater than the above velocity (v_2), a stage is reached where the satellite succeeds in escaping from the solar system.

Mathematically, $v_3 = \sqrt{\frac{2\mu}{\rho} - v_t^2 (3-2\sqrt{2})}$ —(1).

where $v_t \rightarrow$ speed of Earth's revolution around the sun.

Note :- for a given Perigee distance P, it can be proved that injection velocities and corresponding apogee distances are related by,

$$\left(\frac{v_2}{v_1}\right)^2 = \frac{1+P/A_1}{1+P/A_2}$$ [Ref P15 for der^n in text book]

Types of Satellite Orbits :-

The satellite orbits can be classified on the basis of

1) Orientation of orbital plane.
2) Eccentricity
3) Distance from Earth.

Orientation of Orbital Plane :-

The orbital plane of the satellite can have various orientations with respect to equatorial plane of earth. we know angle between these two planes is the angle of inclinations.

Based on these inclination, the orbits can be classified as equatorial orbits, polar orbits and inclined orbits.

Equatorial orbit — Angle of inclination is zero. ie orbital plane of satellite coincides with the equatorial plane of the Earth. [Refer fig 15]. A satellite in the equatorial orbit has a latitude of 0°.

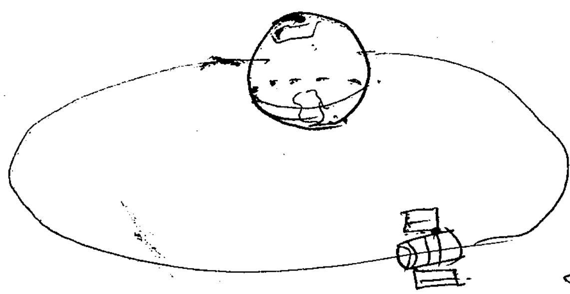


fig 15: Equatorial orbit

For an angle of inclination equal to 90°, the satellite is said to be in polar orbit [Refer fig 16]

For an angle of inclination betn 0° & 180°, the orbit is said to be inclined orbit.

For angle of inclination between 0° & 90°, the satellite travels in the same direction as the direction of rotation of earth. The orbit in this case is referred as direct or progeade orbit. For an angle of inclination between 90° & 180°, the satellite orbits in a direction opposite to the direction of rotation of Earth & orbit is called retrogeade orbit.

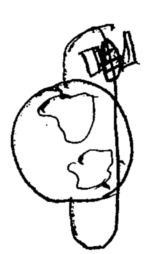


fig 16: Polar orbit.

Eccentricity of orbit :- On the basis of eccentricity we can classify orbits as circular and elliptical.

* when eccentricity is zero, the orbit is circular.
* when eccentricity lies betn 0 & 1, it is elliptical with centre of earth lying at one of the foci of the ellipse. All (practical) circular orbits are eccentric to some extent.

Eg: 1) Eccentricity of orbit of geostationary satellite INSAT-3B. is 0.0002526, which provide communi-cation & ~~meteo~~ meteorological services.

(2) Eccentricity figures for GOES-9 is 0.0004233, offers weather forecasting services.

Molniya orbit :-

It is widely used by Russia. It is also used by other countries of former soviet union to provide communication services. This is highly eccentric, inclined and elliptical orbit, such orbits give higher latitudes coverage, which cannot be given by geostationary orbit.

Typical eccentricity for Molniya orbit = 0.75.

orbit inclination is 65°.

The apogee distance is 40,000 km from surface of Earth.

Perigee distance is 400 km from surface of Earth.

Distance from the Earth :-

Satellite may be placed in different orbits based on the intended mission. The orbits are at various distances from the surface of the Earth. Depending on these distances, they are classified as, LEO's → Low earth orbits, height ranges from 160 to 500km.
MEO's → Medium Earth Orbits → ht 10,000 to 20,000 km
GEO → Geostationary earth orbit → needs a height of 36000km, 35786 km to be precise above surface of the Earth.

Applications :-
LEO's → These are closer to surface of earth, hence they have shorter orbital period as well as smaller propogation delays. They are highly suited for communication applications.
The power required is also less, they are small in sizewize & less expensive to build.
For Eg: under the LEO, ~~the~~ One important application of LEO satellites for Communication is project Iridium, which is a global communication system conceived by Motorola. A total of 66 satellites are arranged in a distributed architecture, with each satellite carrying (1/66) of total system capacity.

* This system gives telecommunication services at global level.
* This project is named Iridium since earlier, since the atomic number of Iridium is 77.
* other applications include surveillance, remote sensing, weather forecasting & scientific studies.

2) MEO satellites :-

* MEO's are orbit at a distance approximately 10000km to 20000km above surface of the Earth.
* They have an orbital period of 6 to 12 hours.
* These satellites stay in sight over a particular region of Earth for a longer time.
* The transmission distance and propagation delays are greater than those for LEO satellites.
* These satellites are mainly used for communication & navigation applications.

3) GEO satellites :-

* These satellites needs a height about 36000km (precisely 35786km) to have required orbital velocity.
* A GEO satellite must fulfill the following condition

1. It must have a constant latitude, which is possible only at 0° latitude.

2. The orbit inclination should be zero.
3. It should have a constant longitude & thus have a uniform ~~lon~~ angular velocity, which is possible when the orbit is circular.
4. The orbital period should be equal to 23 hrs 56 min, which implies that the satellite must orbit at a height of 35786 km above the surface of the Earth.

* Satellites in GEO's play a major role in relaying communication & TV broadcast signals around the globe. They also perform meteorological & military surveillance functions very effectively.

Orbital Perturbations :-

The satellite once placed in its orbit, experiences various perturbing torques that cause the orbital parameters of a satellite to vary with time. These torques are

* Gravitational forces from other bodies like solar and lunar attraction.
* Magnetic field interaction.
* Solar radiation pressure
* Assymetry of Earth's gravitational field.

~~Robot~~

Effects of Torques:-

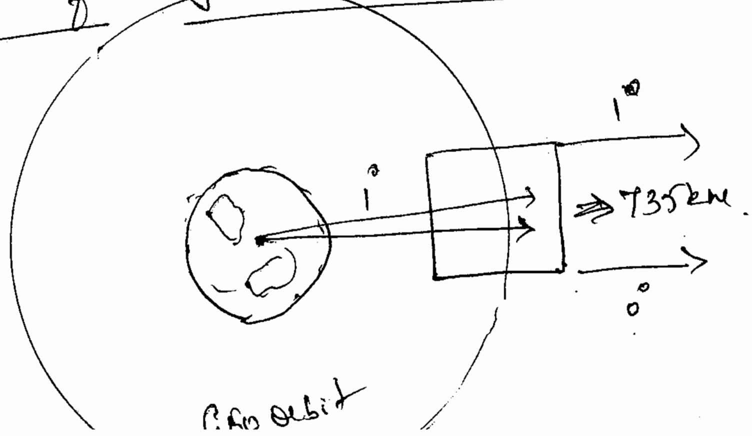
* Satellite orbit tends to drift.
* Orientation changes.
* True orbit will be defined other than that defined by kepler's laws.

Thus to overcome the effects of torques, the satellite position needs to be controlled both in East-west (EW) as north-South directions.

* The East west location needs to be (~~controlled~~ ~~both in East~~) maintained to prevent radio frequency (RF) interference from neighbouring satellites.
* The north-south orientation has to be maintained to have proper satellite inclination.

Note: In case of geostationary satellites, a 1° drift in east west direction is equivalent to a drift of about 735 km along the orbit.

Drift of a geostationary satellite:-



Reasons :-

(1) Non-uniform gravitational field around Earth.

This happens because,

* Earth is not perfect sphere and flattened at poles.
* Equatorial radius of Earth is not constant
* Average density of Earth is not uniform.

Total effect is that, there is a variation in gravitational force acting on satellite due to Earth. This effect is more on Geosatellite than for low earth orbit satellite.

As a result, these forces result in an acceleration or deacceleration component that varies with longitudinal location of satellite.

(2) Gravitational pulls of Sun & Moon :-

* Satellite apart from the gravitational field of Earth is also subjected to pulls of Sun & moon.
* The Earth's orbit around the Sun is an ellipse, whose plane is inclined at an angle of 7° with respect to equatorial plane of the Sun. The Earth is tilted around 23° away from the normal to ecliptic.

* The moon revolve around the Earth with an inclination of 5° to equatorial plane of Earth.

Hence the satellite in orbit is subjected to variety of out of plane forces, which change the inclination on the satellite's orbit.

[Note :- for LEO's effect of atmospheric drag is prominent, than pulls of sun & moon].

After perturbation the orbit is not an ellipse anymore and after one revolution, the satellite does not return to the same point in space. "The time elapsed between the successive perigee passages is referred to as anomalistic period." The anomalistic period (t_A) is given by,

$$t_A = \frac{2\pi}{\omega_{mod}} \quad -①.$$

where $$\omega_{mod} = \omega_0 \left[1 + \frac{k(1 - 1.5\sin^2 i)}{a^2(1-e^2)^{3/2}} \right] \quad -②.$$

where $\omega_0 \rightarrow$ angular velocity for spherical earth

$k = 66063.1704 \ km^2$

$a \rightarrow$ semi major axis ; $e \rightarrow$ eccentricity

$i = \cos^{-1} n_z$ (n_z is the x axis component of the orbit normal)

· Functions of Attitude and Orbit Control system! - 41

* The attitude and orbit control system maintains satellite position and
* keeps antenna pointed correctly in desired direction.

→ The orbital control is performed by firing thrusters in desired direction and also by releasing a jet of gas. This is called Station keeping.

* Thrusters and gas jets are used to correct the longitudinal drifts (in plane changes) and the inclination changes (out of plane).

* Longitudinal drifts are corrected by maneuvers called as North-South maneuvers. They require a larger velocity increment as compared to maneuvers required for correction inclination changes, which are called as East west maneuvers. A separate set of thrusters/Jets are required for E-W & N-S maneuvers.

Satellite Stabilization :-

Commonly employed techniques for satellite attitude control include:

1. Spin stabilization
2. Three axis of body stabilization.

Spin Stabilization :-

* In a spin-stabilized satellite, the satellite body is spun at a rate between 30 and 100 rpm about an axis perpendicular to the orbital plane as shown in fig below.

Such satellite consists of cylindrical drum covered with solar cells & rocket motor.

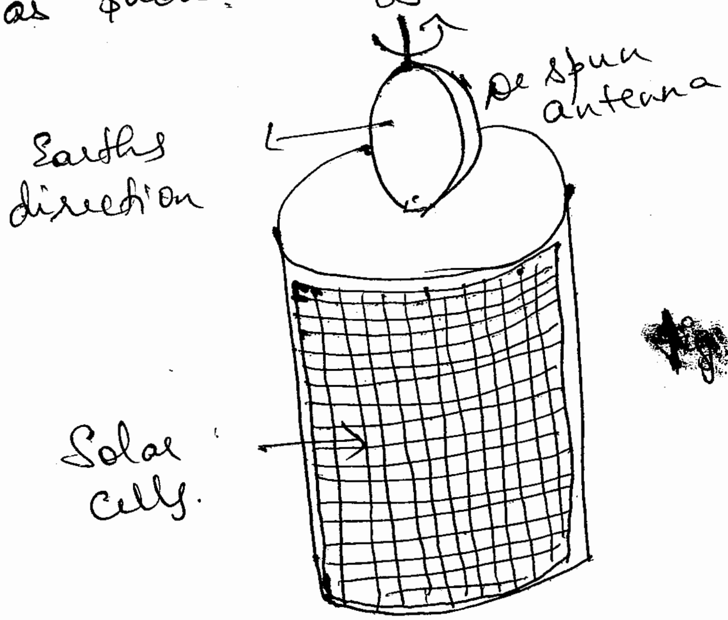


fig: Spin stabilized satellite.

are used for attitude control Stn that is necessary for spacecraft to maintain antenna in correct direction.

* Similar to a spinning top, the rotating body offers inertial stiffness, which prevents satellite from drifting from its desired orientation.
* Spin stabilized satellites are generally cylindrical in shape.
* For stability the satellite is to be spun about

about its major axis, having a maximum moment of Inertia.

Two types of spinning configurations employed in spin stabilized satellites.

1) Simple spinner configuration
2) Dual spinner configuration.

<u>Simple Spinner Configuration</u>:-

The satellite payload and other subsystems are placed in the spinning section, while the antenna & the feed are placed in ~~despun~~ de-spun platform. The de-spun platform is spun in a direction opposite to that of spinning satellite body.

<u>Dual Spinner Stabilization</u>:-

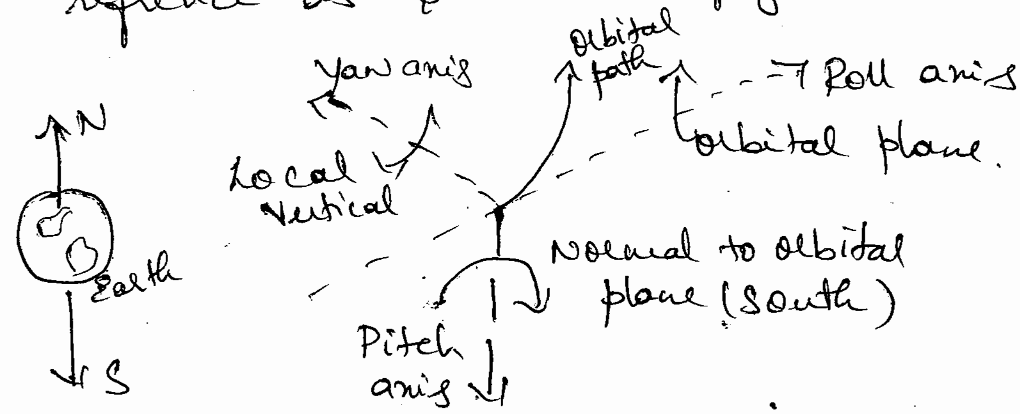
Both payload and antenna and feed are placed in the de-spun platform and other subsystems are located on the spinning body.

<u>Note</u>:- * Modern spin stabilized sats employ dual spinner configuration.

* Antenna mount on de-spun platform ensures that there is a constant direction pointing of antennae.

Three axis or body Stabilization! -

The Stabilization in this case is acheived by controlling the movement of satellite along the 3 axes - ie yaw, pitch and roll with respect to a refrence as shown in fig below.



Method of Stabilization! -

* The system uses reaction wheels or momentum wheels to correct orbit perturbations or disturbances.
* Stability is provided by active control system, which applies corrective forces on the wheels to correct the undesirable changes in the satellite orbit.
* Most 3-axis Stabilised satellites use momentum wheels.
* The basic technique used is to speed up or slow down of momentum wheel depending on the direction in which the satellite perturbed (~~disterb~~ disturbed).

For Eg: An increase in the speed of the wheel

- In clockwise direction, will make the satellite to rotate in a counterclockwise direction.

[Note] The momentum wheels rotate in one direction and can be twisted by a gimbal motor to provide a dynamic force on satellite.

* An alternative approach or method is to use reaction wheels. Three reaction wheels are used, one for each axis.
* These reaction wheels can be rotated in any direction depending upon the active correction force.
* The satellite body is generally box-shaped for satellites using 3 axis stabilization.
* Antennas are mounted on Earth ~~face~~ facing side and solar panels are mounted above the / below the body of satellite, in such a way that they always point towards the sun.

Eg: Satellites using 3-axis stabilization

Intel Sat-5, Intel Sat-7, INSAT series of satellites.

Comparison between Spin Stabilized and 3-axis Stabilized Satellites :-

Spin Stabilized	3-axis Stabilised
1. Less power generation capability and less area for complex antennae structures.	1. More power generation is possible with larger mounting area for complex antennae structures.
2. Simple in design and less expensive.	2. They are complex in design and costly.
3. Power is given to satellite in ~~or~~ transfer orbit also.	3. Power to satellite is not possible during transfer orbit phase since solar array, is unable to provide to provide power during this phase as it is stored inside.

Station keeping :-

* This is the process of maintainance of satellites orbit in right position against temporal drift. This drift may be due to natural forces like gravitational pulls of sun and moon, solar radiation pressure, earth being imperfect sphere etc.

* The adjustments can be done by release of jets of gas or by firing small rockets tied to body of satellite.

Orbital effects on Satellite performance :-

The Satellite is revolving constantly around the Earth, this motion has significant effects on its performance. They are

* Doppler Shift
* Effect due to variation in orbital distance
* Effect of Solar Eclipse
* Sun's transit Outrage.

1) Doppler's Shift :-

The geostationary satellites appear stationary with respect to Earth station terminal, but the low Earth orbit satellites have satellites in relative motion with respect to terminal (ES). However for case of geostationary satellites also, there are some variations between satellite & Earth station terminal. As the satellite is moving with respect to Earth Station terminal the frequency of the satellite transmitter also w.r.t receiver on the Earth Station terminal changes.

If the frequency transmitted by the satellite is 'f_T' then the received frequency f_R is given by equation,

$$\left(\frac{f_R - f_T}{f_T}\right) = \left(\frac{\Delta f}{f_T}\right) = \left(\frac{v_T}{v_p}\right)$$

where, v_T is component of satellite transmitter velocity directed towards the Earth Stn receiver.

v_p is the phase velocity of light in free space (3×10^8 m/s)

2) Variation in orbital distance :-

Variation in orbital distance results in variation in the range between satellite and Earth station terminal. This becomes important where TDMA scheme is employed, the timing of frames within TDMA bursts have to be worked out carefully, so that user terminals receive correct data at correct time. Range variation is more in LEO & MEO orbiting satellites.

3) Solar Eclipse :-

Satellites do not receive solar radiation for power during eclipse during this period, they work on board batteries. The design of the battery is such that it provides continuous power during the period of eclipse.

The discharging and charging of the batteries are controlled by the ground control stations.

Sudden temperature stress situations, which the satellite may experience during the eclipse, requires that design of satellite in a way to cope with these thermal stresses.

4) Sun transit outage :-

There are times when satellite passes directly between Sun and Earth.

In this case Earth station antenna will receive signals from the satellite as well as the microwave radiation emitted by Sun (Temp bet^n 6000K to 11,000K) depending upon the time of the 11 year sunspot cycle. This might cause temporary outage if the magnitude of solar radiation exceeds the fade margin of the receiver, in such cases traffic of satellite may be shifted to other satellites during these cases.

Eclipses :- With reference to satellites, eclipse is said to occur when the sunlight fails to reach the solar panel of the satellite, due to obstruction from a celestial or heavenly body. The major and most frequent source of eclipse is due to satellite coming in the shadow of Earth. This is called solar eclipse (fig a)

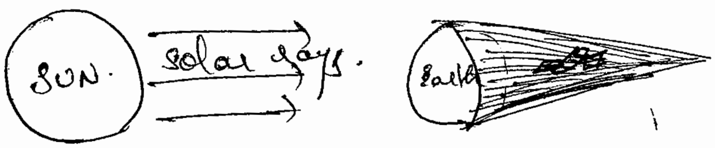
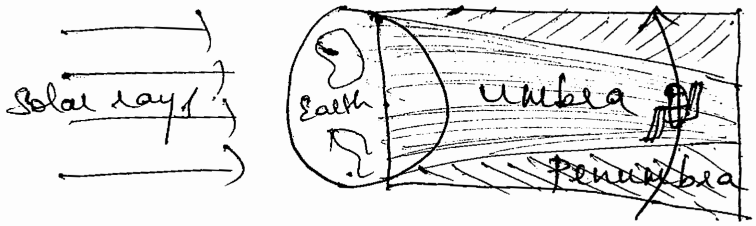


fig: Solar Eclipse

The eclipse is total; i.e the satellite fails to receive any light when it is in umbra region (dark central region of shadow during eclipse) and also in penumbra region (less dark region surrounding the umbra region). (fig 6)



* The eclipse occurs as the Earth's equatorial plane is inclined at a constant angle of about 23.5° to its ecliptic plane, which is the plane of Earth's orbit extended to infinity.

* Eclipse is seen on 42 nights during spring and equal number of nights during ~~antenna~~ autumn by the geostationary satellites.

* Effect of eclipse is worst during equinoxes and lasts for 72 minutes.

Design of Equinox — It is the point in time when Sun crosses the equator making the day and night equal in length.

Spring Equinox → 20-21 March.
Autumn Equinox → 22-23 September } Satellite is in total darkness for 72 minutes.

The duration of eclipse increases from zero to 72 minutes starting 21 days before equinox, then decreases from 72 minutes to zero during 21 days following equinoxes.

Lunar Eclipse:

This occurs when the moons shadow passes across the satellite. This is much less in common and occurs once in 29 years. In most of the cases, Solar eclipse is refered. Lunar Eclipse is as shown in fig below.

Effects of Solar Eclipse:

* It effects the battery recharging process.
* Satellite is depleted of its electrical power capacity
* High power satellites are most affected and they are shut down for all but essential services.

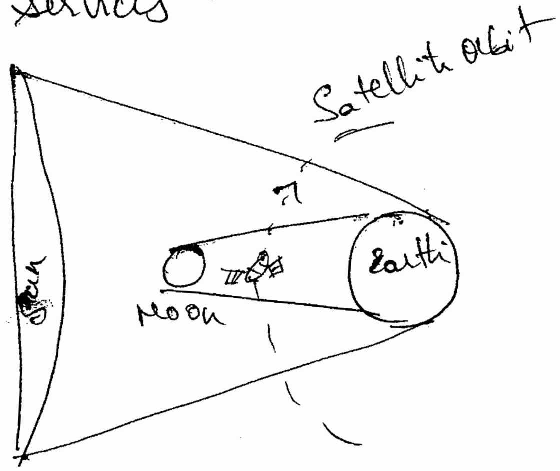


fig: Lunar Eclipse.

Look Angles of a Satellite:

By definition, the look angles of a satellite refer to the co-ordinates to which the Earth station must be pointed in order to communicate with the satellite and are expressed in terms of azimuth & elevation angles.

The process of pointing earth station antenna accurately towards the satellite can be done by knowing the elevation and azimuth angles of the Earth Station location.

Note :- Elevation angle affects the slant range also. Slant range is line of sight distance between Earth station and the satellite.

To determine look angles of a satellite :-

* Precise location of satellite has to be known

* The location is often determined by position of sub-satellite point. Sub-satellite is the location on the surface of the Earth that lies directly between the satellite and centre of the Earth. To any observer, the sub-satellite point will appear to be directly overhead.

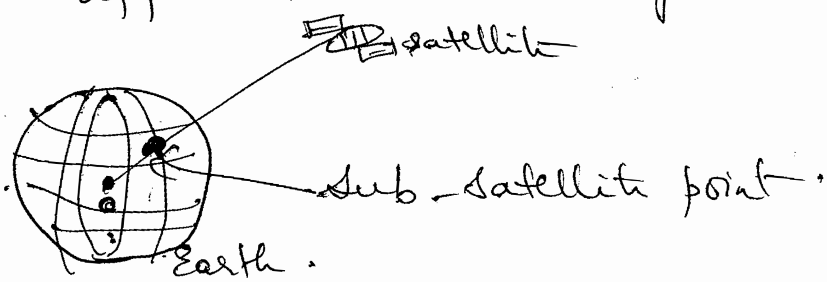


fig : sub-satellite

Azimuth Angle :- The azimuth angle 'A' of an earth station is defined as the angle produced by the line of intersection of local horizontal plane and the plane passing through Earth station, the satellite and centre of Earth with the true North. This is as shown in fig.

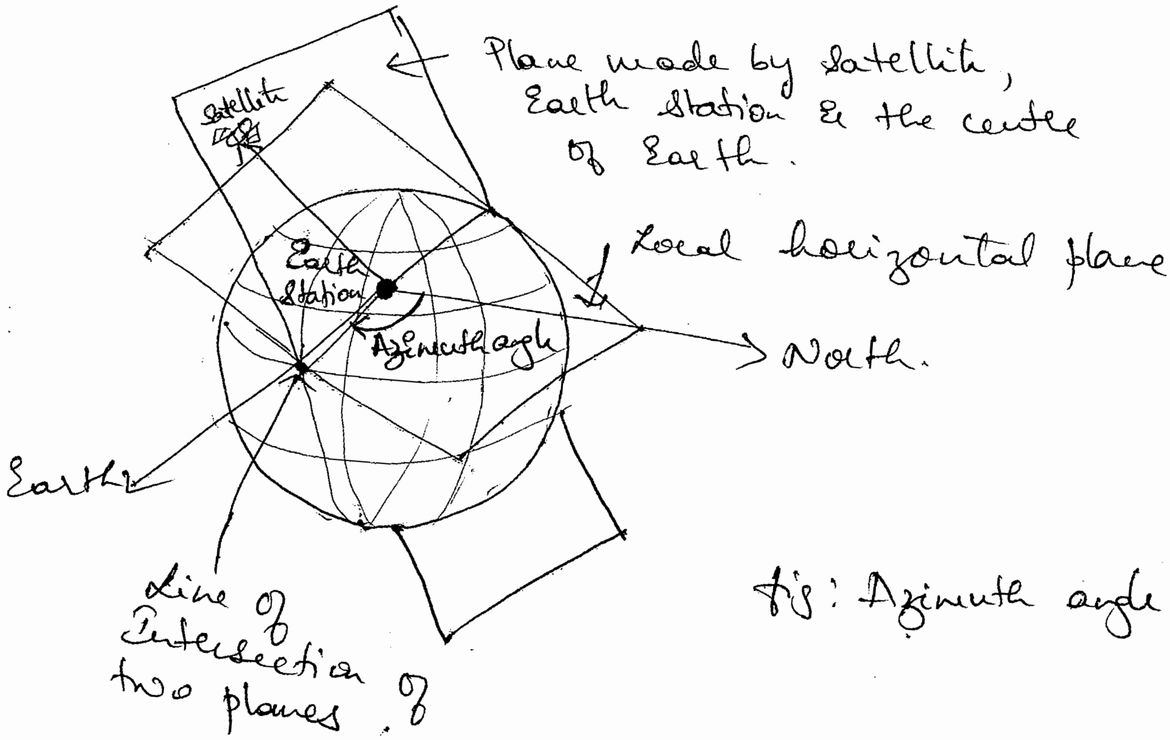


Fig: Azimuth angle.

Calculation of Azimuth angle

Depending upon the location of Earth station and the subsatellite point, the azimuth angle can be computed as follows:

Earth station in the northern hemisphere:

$A = 180^\circ - A'$, when Earth Stn is to the west of satellite $(\theta_S - \theta_L)$ is +ve

$A = 180^\circ + A'$, when the Earth Stn is to the East of satellite. $\theta_S - \theta_L \rightarrow$ +ve

Earth station in the Southern hemisphere:

$A = A'$ when the Earth Stn is to the west of satellite.

$A = 360^\circ - A'$ when the Earth Stn is to the East of Satellite

where A' can be computed from,

$$A' = \tan^{-1}\left[\frac{\tan|\theta_S - \theta_L|}{\sin\theta_l}\right]$$

here $\theta_S \rightarrow$ satellite longitude.

$\theta_L \rightarrow$ Earth Stn longitude; $\theta_l \rightarrow$ Earth Stn latitude.

Elevation Angle:

The earth station elevation angle ε is the angle between the line of intersection of the local horizontal plane and the plane passing through the Earth station, the satellite and centre of the Earth with the line joining the Earth station and the satellite.

It can be computed from,

$$\varepsilon = \tan^{-1}\left[\frac{r - R\cos\theta_l \cos|\theta_s - \theta_L|}{R\sin\left[\cos^{-1}(\cos\theta_l \cos|\theta_s - \theta_L|)\right]}\right] - \cos^{-1}(\cos\theta_l \cos|\theta_s - \theta_L|)$$

where $r \rightarrow$ is the orbital radius,

$R \rightarrow$ Earth's radius, $\theta_s \rightarrow$ satellite longitude

$\theta_L \rightarrow$ Earth stn longitude

$\theta_l \rightarrow$ Earth stn latitude.

Problems: [from text]

Computing the Slant range: The slant range of a satellite is defined as the range of the distance of the satellite from the Earth station. The elevation angle has a direct bearing on the slant range. Smaller the Earth station, larger is the elevelation angle of the slant range & coverage angle.

The slant range D can be computed from,

$$D = \sqrt{R^2 + (R+H)^2 - 2R(R+H)\sin\left[\varepsilon + \sin^{-1}\left\{\left(\frac{R}{R+H}\right)\cos\varepsilon\right\}\right]}$$

Coverage angle (α) is given by,

$$\alpha = \sin^{-1}\left[\left(\frac{R}{R+H}\right)\cos E\right]$$

where $R \rightarrow$ radius of Earth., $E \rightarrow$ angle of elevation
$H \rightarrow$ height of satellite above surface of earth.

Computing the line of sight distance between two satellites:-

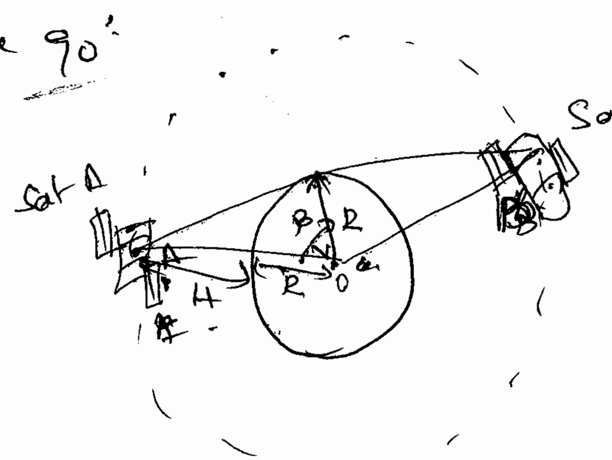
The line of sight distance between two satellites placed in the same circular orbit can be computed ~~from~~ by the points of loc^n of two satellites and centre of earth. The line of sight distance AB is given by,

$$AB = \sqrt{AC^2 + BC^2 - 2AC \times BC \times \cos\theta}.$$
$$\sqrt{D_1^2 + D_2^2 - 2D_1D_2\cos\theta}$$

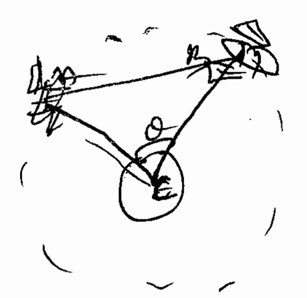
$\angle ABC$

$\theta \rightarrow$ is the angular separation of the longitudes of the two satellites. where $D_1 \rightarrow$ orbital radius of first satellite
$D_2 \rightarrow$ " " second satellite
$\theta \rightarrow$ angle formed by two radii

eg: If two satellites were located at $30°E$ & $60°E$, θ would be equal to $30°$ & if loc^n's were $30°W$ & $60°E$ θ would be $90°$



Max^m line of sight distance



Line of sight distance b/t^n two satellites

for b)

Max^m line of sight distance (AB) equals OA + OB (fig β)

Which equal 2OA = 2OB as OA = OB.

If R is the radius of the earth & H is the height of satellite above the surface of earth, then

$$OA = AC \sin\beta = (R+H)\sin\beta.$$

Now $\beta = \cos^{-1}\left(\frac{R}{R+H}\right)$

$$\therefore \quad OA = (R+H)\sin\left[\cos^{-1}\left(\frac{R}{R+H}\right)\right]$$

& Max^m line of sight distance $= 2(R+H)\sin\left[\cos^{-1}\left(\frac{R}{R+H}\right)\right]$

Computing line of sight distance bet^n two satellites :-

Eq^n Max^m line of sight distance $= 2(R+H)\sin\left[\cos^{-1}\left(\frac{R}{R+H}\right)\right]$

where R → Radius of the Earth, H → height of satellite above surface of earth

Line of sight distance can be also computed from,

$$\sqrt{D_1^2 + D_2^2 - 2 \times D_1 \times D_2 \times \cos\theta}$$

where D_1 → Orbital radius of first satellite

D_2 → " " " second "

θ → angle formed by two radii