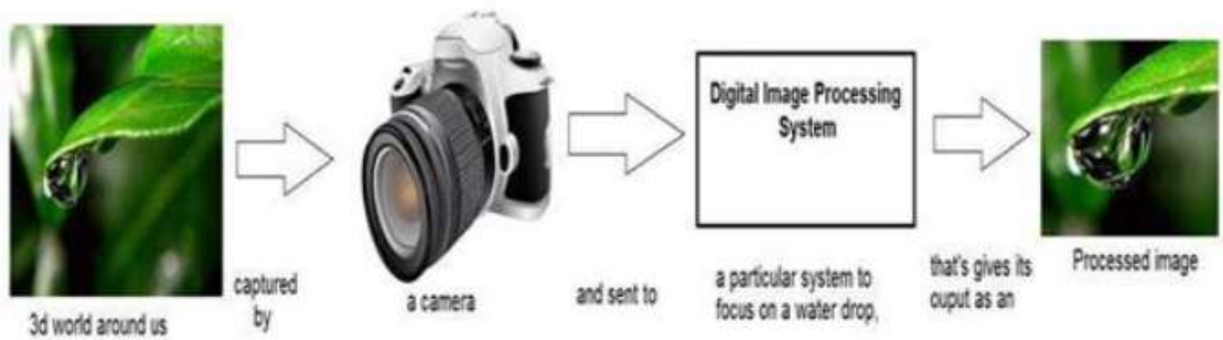
Module-1

Digital Image Fundamentals

The field of digital image processing refers to processing digital images by means of digital computer. Digital image is composed of a finite number of elements, each of which has a particular location and value. These elements are called picture elements, image elements, pels and pixels. Pixel is the term used most widely to denote the elements of digital image.

An image is a two-dimensional function that represents a measure of some characteristic such as brightness or color of a viewed scene. An image is a projection of a 3- D scene into a 2D projection plane.

An image may be defined as a two-dimensional function ***f(x,y),*** where x and y are spatial (plane) coordinates, and the amplitude of ***f*** at any pair of coordinates **(x,y)** is called the intensity of the image at that point.



The term gray level is used often to refer to the intensity of monochrome images. Color images are formed by a combination of individual 2-D images.

For example: The RGB color system, a color image consists of three (red, green and blue) individual component images. For this reason many of the techniques developed for monochrome images can be extended to color images by processing the three component images individually.

An image may be continuous with respect to the x- and y- coordinates and also in amplitude. Converting such an image to digital form requires that the coordinates, as well as the amplitude, be digitized.

APPLICATIONS OF DIGITAL IMAGE PROCESSING

Since digital image processing has very wide applications and almost all of the technical fields are impacted by DIP, we will just discuss some of the major applications of DIP.

Digital image processing has a broad spectrum of applications, such as

- Remote sensing via satellites and other space crafts
- Image transmission and storage for business applications
- Medical processing,
- RADAR (Radio Detection and Ranging)
- SONAR(Sound Navigation and Ranging) and
- Acoustic image processing (The study of underwater sound is known as underwater acoustics or hydro acoustics.)
- Robotics and automated inspection of industrial parts.

Images acquired by satellites are useful in tracking of

- Earth resources;
- Geographical mapping;
- Prediction of agricultural crops,
- Urban growth and weather monitoring
- Flood and fire control and many other environmental applications.

Space image applications include:

- Recognition and analysis of objects contained in images obtained from deep space-probe missions.
- Image transmission and storage applications occur in broadcast television
- Teleconferencing
- Transmission of facsimile images (Printed documents and graphics) for office automation

Communication over computer networks

- Closed-circuit television based security monitoring systems and
- In military communications.

Medical applications:

- Processing of chest X- rays

- Cineangiograms
- Projection images of transaxial tomography and
- Medical images that occur in radiology nuclear magnetic resonance (NMR)
- Ultrasonic scanning

IMAGE PROCESSING TOOLBOX (IPT) is a collection of functions that extend the capability of the MATLAB numeric computing environment. These functions, and the expressiveness of the MATLAB language, make many image-processing operations easy to write in a compact, clear manner, thus providing a ideal software prototyping environment for the solution of image processing problem.

There are **three types of computerized processes in the processing of image**

1) Low level process -these involve primitive operations such as image processing to reduce noise, contrast enhancement and image sharpening. These kind of processes are characterized by fact the both inputs and output are images.

2) Mid level image processing - it involves tasks like segmentation, description of those objects to reduce them to a form suitable for computer processing, and classification of individual objects. The inputs to the process are generally images but outputs are attributes extracted from images.

3) High level processing – It involves “making sense” of an ensemble of recognized objects, as in image analysis, and performing the cognitive functions normally associated with vision.

Components of Image processing System:

Image Sensors: With reference to sensing, two elements are required to acquire digital image. The first is a physical device that is sensitive to the energy radiated by the object we wish to image and second is specialized image processing hardware.

Specialize image processing hardware: It consists of the digitizer just mentioned, plus hardware that performs other primitive operations such as an arithmetic logic unit, which performs arithmetic such addition and subtraction and logical operations in parallel on images.

Computer: It is a general purpose computer and can range from a PC to a supercomputer depending on the application. In dedicated applications, sometimes specially designed computer are used to achieve a required level of performance

Software: It consists of specialized modules that perform specific tasks a well designed package also includes capability for the user to write code, as a minimum, utilizes the specialized module. More sophisticated software packages allow the integration of these modules.

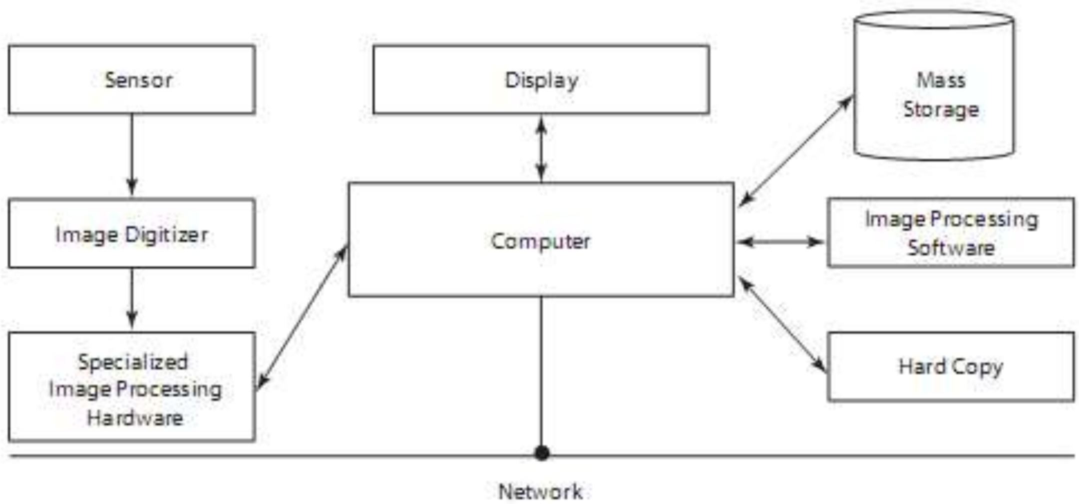


FIGURE 1.4: Components of an Image Processing System

Mass storage: This capability is a must in image processing applications. An image of size 1024 x1024 pixels, in which the intensity of each pixel is an 8- bit quantity requires one Megabytes of storage space if the image is not compressed .Image processing applications falls into three principal categories of storage

i) Short term storage for use during processing

ii) On line storage for relatively fast retrieval

iii) Archival storage such as magnetic tapes and disks

Image display: Image displays in use today are mainly color TV monitors. These monitors are driven by the outputs of image and graphics displays cards that are an integral part of computer system.

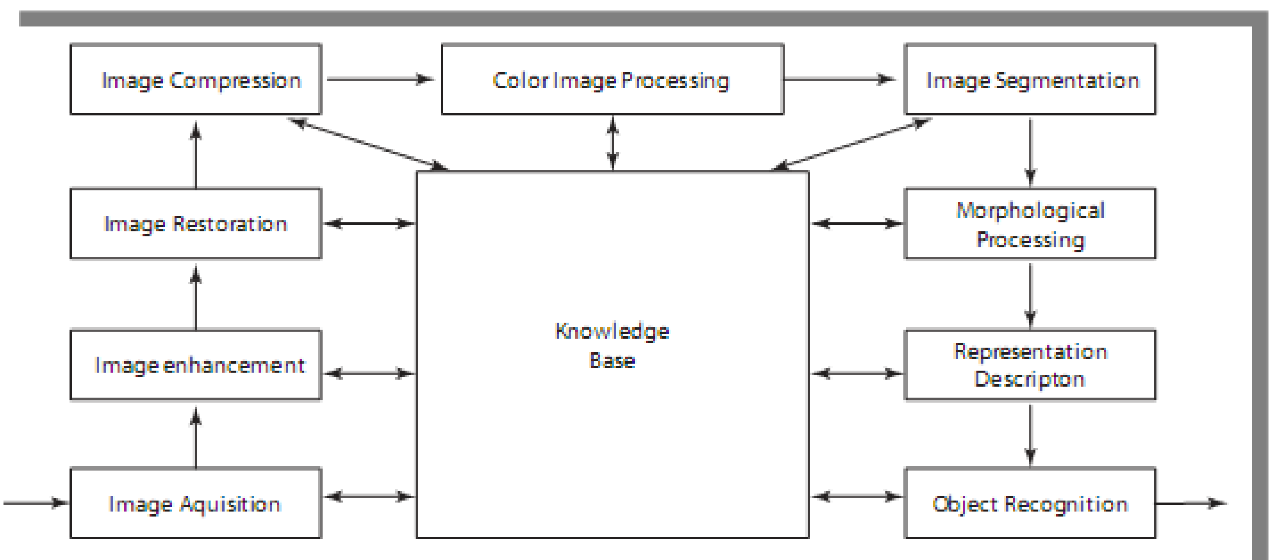
Hardcopy devices: The devices for recording image includes laser printers, film cameras, heat sensitive devices inkjet units and digital units such as optical and CD ROM disk. Films provide the highest possible resolution, but paper is the obvious medium of choice for written applications.

Networking: It is almost a default function in any computer system in use today because of the large amount of data inherent in image processing applications. The key consideration in image transmission bandwidth.

Fundamental Steps in Digital Image Processing:

There are two categories of the steps involved in the image processing –

1. Methods whose outputs are input are images.
2. Methods whose outputs are attributes extracted from those images.



IGURE 1.3: Fundamental steps in digital image processing

Image acquisition: It could be as simple as being given an image that is already in digital form. Generally the image acquisition stage involves processing such scaling.

Image Enhancement: It is among the simplest and most appealing areas of digital image processing. The idea behind this is to bring out details that are obscured or simply to highlight certain features of interest in image. Image enhancement is a very subjective area of image processing.

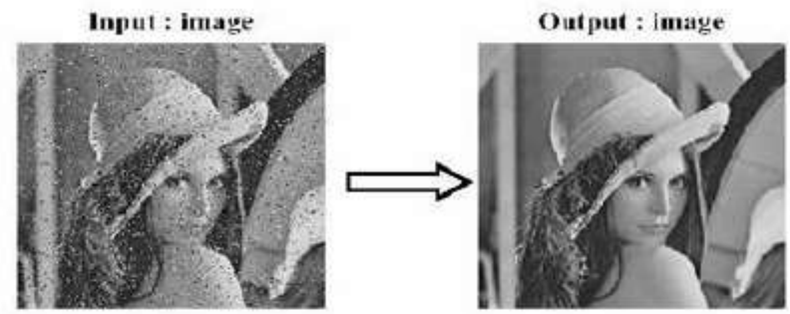
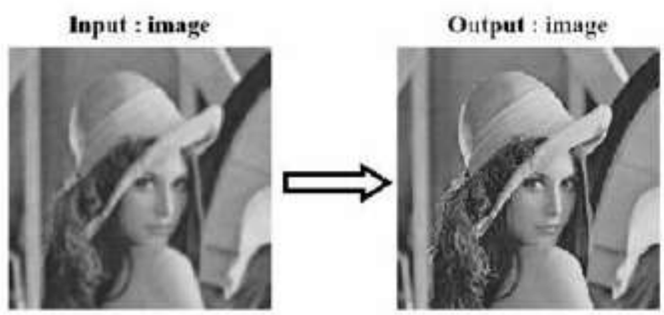


Image Restoration: It deals with improving the appearance of an image. It is an objective approach, in the sense that restoration techniques tend to be based on mathematical or probabilistic models of image processing. Enhancement, on the other hand is based on human subjective preferences regarding what constitutes a “good” enhancement result.



Color image processing: It is an area that is been gaining importance because of the use of digital images over the internet. Color image processing deals with basically color models and their implementation in image processing applications.

Wavelets and Multiresolution Processing: These are the foundation for representing image in various degrees of resolution.

Compression: It deals with techniques reducing the storage required to save an image, or the bandwidth required to transmit it over the network. It has to major approaches a) Lossless Compression b) Lossy Compression

Morphological processing: It deals with tools for extracting image components that are useful in the representation and description of shape and boundary of objects. It is majorly used in automated inspection applications.

Representation and Description: It always follows the output of segmentation step that is, raw pixel data, constituting either the boundary of an image or points in the region itself. In either case converting the data to a form suitable for computer processing is necessary.

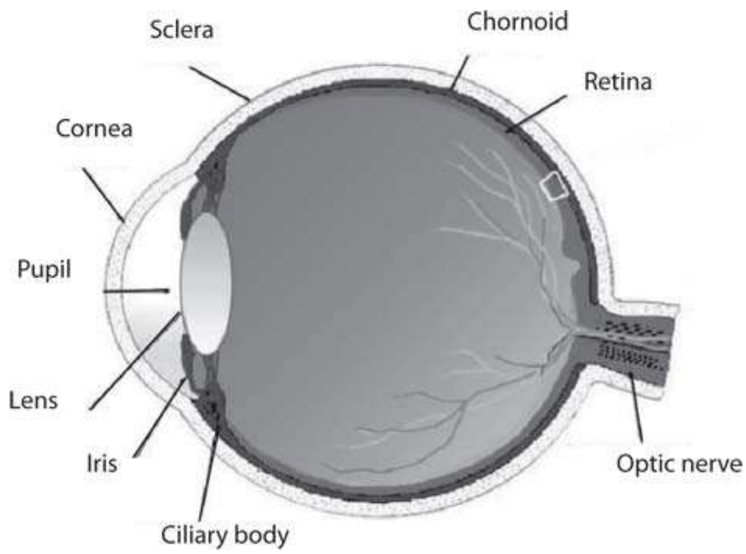
Recognition: It is the process that assigns label to an object based on its descriptors. It is the last step of image processing which use artificial intelligence of software.

Knowledge base:

Knowledge about a problem domain is coded into an image processing system in the form of a knowledge base. This knowledge may be as simple as detailing regions of an image where the information of the interest in known to be located. Thus limiting search that has to be conducted in seeking the information. The knowledge base also can be quite complex such interrelated list of all major possible defects in a materials inspection problems or an image database containing high resolution satellite images of a region in connection with change detection application.

The Human Eye

Eye functions as the biological equivalent of a camera. Fig shows a cross section of human eye. Eye perceives a scene by light rays that are either emitted or reflected from the object/scene. When light rays are within range of electromagnetic spectrum (300 to 700 nm), eye sends electrical signals to brain through optic nerve and brain interprets the object/scene. When light rays hit the eye, it first passes through the cornea, then iris, the lens, the vitreous humour and finally reaches retina.



Cornea

Cornea is a tough transparent tissue over pupil which acts as a lens and reflects the light. Cornea along with crystalline lens helps in the production of a sharp image at the retina.

Iris

Iris forms a round aperture that can vary in size and determines the amount of light entering the eye. During night or under dark situation, iris is wide open and lets as much light as possible into eye. During daylight, iris shrinks to a small hole to control light. Front part of iris contains the visible colour of eye and back part contains black pigment.

Lens

Lens can vary its shape to focus the object/scene on to the retina. It is made up of concentric layers of fibrous cells. Lens is coloured by yellow pigmentation.

Retina

Retina is the inner most membrane of the eye. In the retina, light rays are detected and converted to electrical signal by photo receptors. Eye has two types of photo - receptors: rods and cones.

Rods

Rods are abundant, about 100 million in a human eye, spread evenly about retina except at the **fovea.** Rods are sensitive to light. The fovea is the area of retina where there are no rods, vision is sharpest in fovea.

Cones

There are 6-7 million (much lesser as compared to rods) in fovea. Cones are responsible for colour sensitivity.

Rods are more sensitive to light and cones contribute to the colour information.

No photo receptors (cone & rods) are found at a point where optic nerve attaches to the eye, called as **Blind spot**. We cannot perceive anything at blind spot. Depending on the amount of light reaching the eye, three types of vision can be described:

1. Scotopic or Night Vision

At night or under dark conditions (when light source is absent), only rods are active.

Thus, we can perceive only shades of grey, as rods are not sensitive to colour information under darkness.

2. Photopic or Day Vision

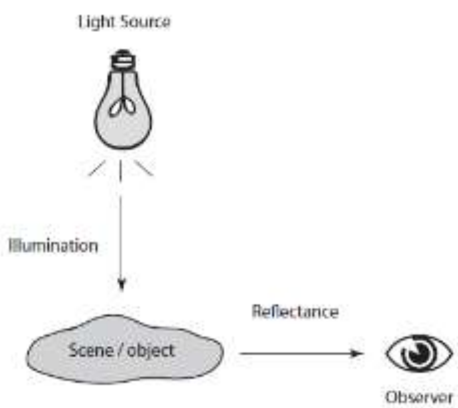
Under bright sunlight, cones are most active, we can see all colours and brightness of the scene.

3. Mesophic Vision

In dim light conditions, both rods and cones are active.

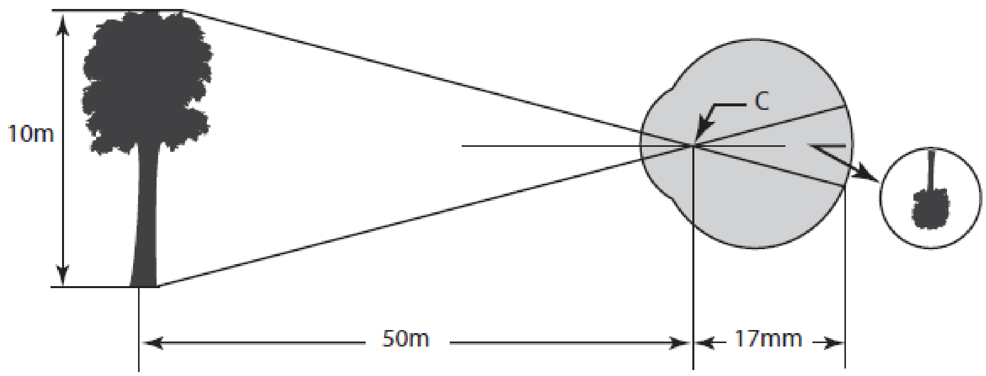
Image Formation

Energy from an illumination source is reflected from the scene/object and captured by the eye or the camera



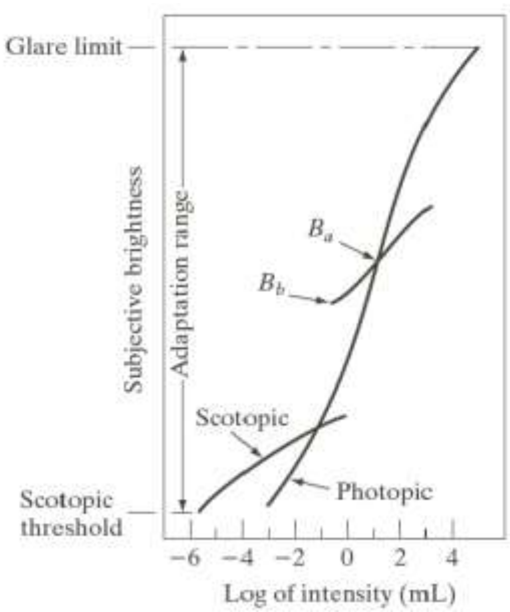
The lens of human eye is very flexible. The focal length as well as the shape of the lens can be changed very easily and to a large extent. Lens become relatively flat to focus on distant objects. The focal length of the eye is approximately 14 mm to 17 mm (distance between lens and retina). For far objects (distance > 3m), lens uses its lowest refractive power. For near objects, lens is strongly refractive. To find the height (h) of the object in an image formed in retina which is placed 50m from eye and actual size of object being 10m simple calculation can be used.

$$\frac{10}{50} = \frac{h}{17} \quad \Rightarrow \quad h = 3.4\text{mm}$$



Brightness Adaption and Discrimination

Digital image are displayed as a discrete set of intensities. The range of light intensity levels to which the human visual system can adopt is enormous- on the order of 10^{10} from scotopic threshold to the glare limit. Experimental evidences indicate that **subjective brightness is a logarithmic function of the light intensity incident on the eye.**



The curve represents the range of intensities to which the visual system can adopt. But the visual system cannot operate over such a dynamic range *simultaneously*. Rather, it is accomplished by change in its overcall sensitivity called **brightness adaptation.**

For any given set of conditions, the current sensitivity level to which of the visual system is called brightness adoption level , Ba in the curve. The small intersecting curve represents the range of subjective brightness that the eye can perceive when adapted to this level. It is restricted at level Bb , at and below which all stimuli are perceived as indistinguishable blacks. The upper portion of the curve is not actually restricted. Whole simply raise the adaptation level higher than Ba.

The ability of the eye to discriminate between change in light intensity at any specific adaptation level is also of considerable interest.

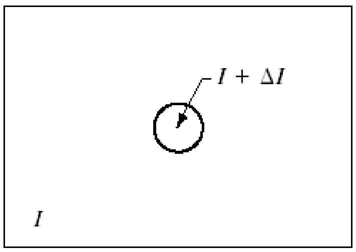


FIGURE 2.5 Basic experimental setup used to characterize brightness discrimination.

Take a flat, uniformly illuminated area large enough to occupy the entire field of view of the subject. It may be a diffuser such as an opaque glass, that is illuminated from behind by a light source whose intensity, I can be varied. To this field is added an increment of illumination ΔI in the form of a short duration flash that appears as circle in the center of the uniformly illuminated field. If ΔI is not bright enough, the subject cannot see any perceivable changes.

As ΔI gets stronger the subject may indicate of a perceived change. ΔIc is the increment of illumination discernible 50% of the time with background illumination I. Now,

ΔIc /I is called the Weber ratio.

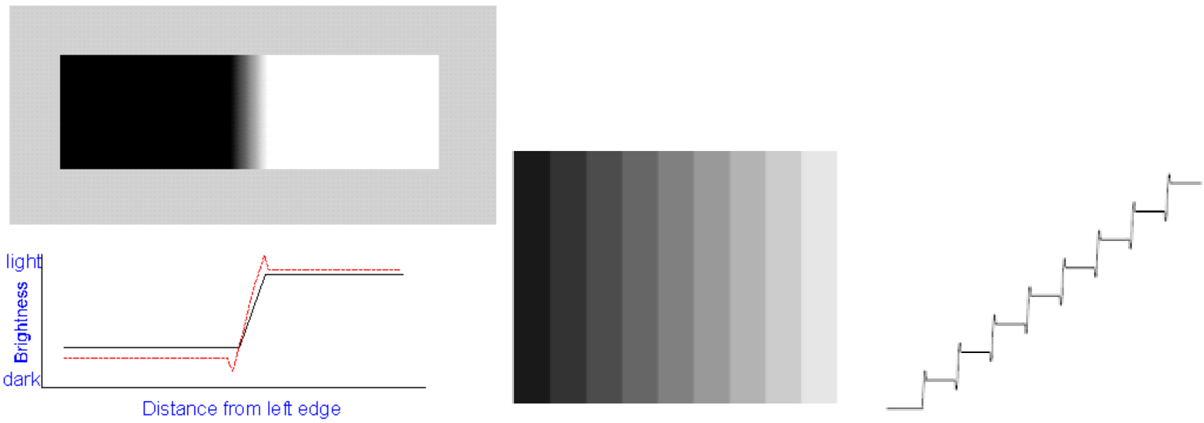
Small value means that small percentage change in intensity is discernible representing “good” brightness discrimination.

Large value of Weber ratio means large percentage change in intensity is required representing “poor brightness discrimination”.

Mach band effect:

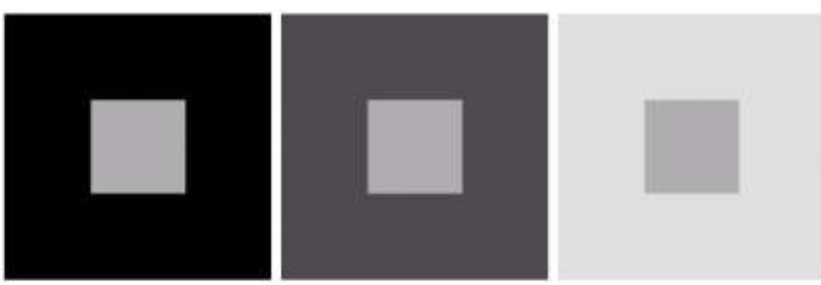
Mach band effect is shown in fig. Each bar is uniformly grey, but HVS perceives luminance change differently. Overshoots and undershoots appear in the brightness graph. Bright side of contrast jump appears extra light and dark side appears extra dark. Because of mach band effect, HVS sharpens the edges of the objects.

HVS tends to undershoot or overshoot around the boundary of regions of different intensities. Although the intensity of strips is constant, we perceive a brightness pattern that is strongly scalloped near the boundaries. **These scalloped bands are called Mach Bands.**



Simultaneous Contrast Effect

The perceived brightness doesn't depend on the brightness of the object alone, but also largely depends on the local background. Fig shows inner four squares with equal luminance, but they appear having different brightness because of the different background.



a b c

FIGURE 2.8 Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.

Optical illusion

In this the eye fills the non existing information or wrongly pervious geometrical properties of objects.

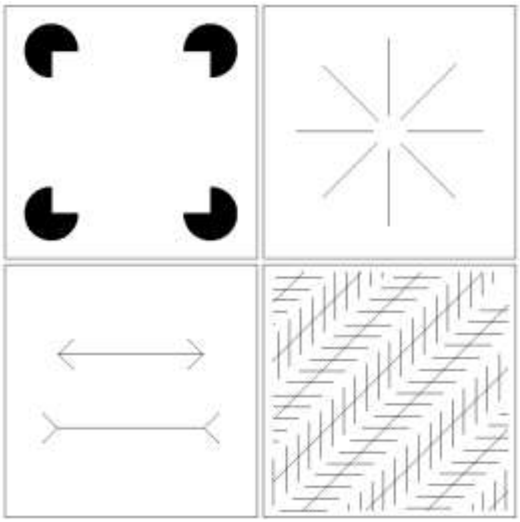
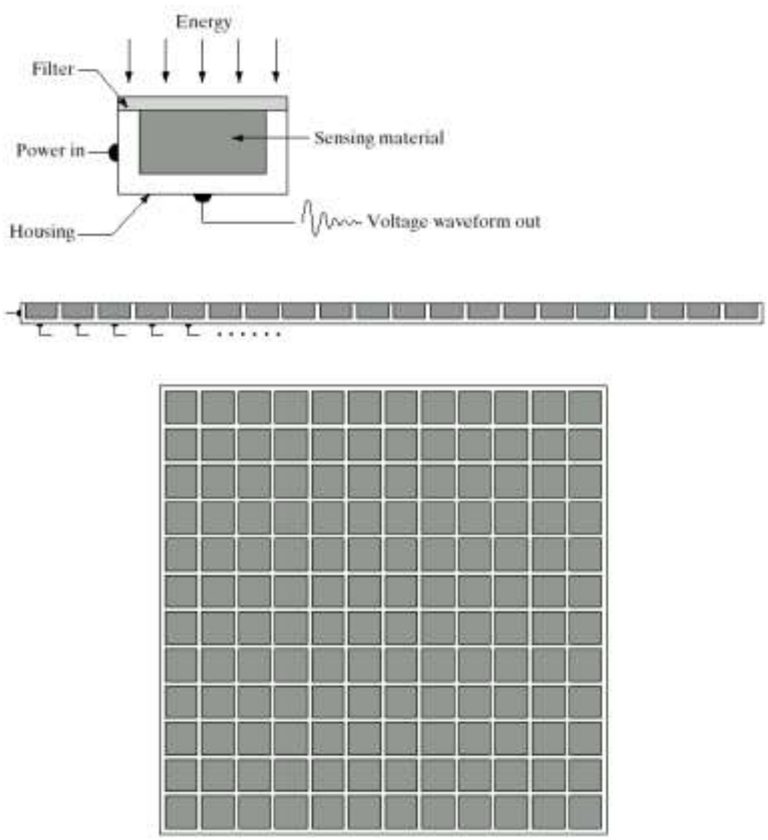


Image sensing and Acquisition:

The types of images in which we are interested are generated by the combination of an "illumination" source and the reflection or absorption of energy from that source by the elements of the "scene" being imaged. We enclose *illumination* and *scene* in quotes to emphasize the fact that they are considerably more general than the familiar situation in which a visible light source illuminates a common everyday 3-D (three-dimensional) scene. For example, the illumination may originate from a source of electromagnetic energy such as radar, infrared, or X-ray energy. But, as noted earlier, it could originate from less traditional sources, such as ultrasound or even a computer-generated illumination pattern. Similarly, the scene elements could be familiar objects, but they can just as easily be molecules, buried rock formations, or a human brain. We could even image a source, such as acquiring images of the sun. Depending on the nature of the source, illumination energy is reflected from, or transmitted through, objects. An example in the first category is light reflected from a planar surface. An example in the second category is when X-rays pass through a patient's body for the purpose of generating a diagnostic X-ray film. In some applications, the reflected or transmitted energy is focused onto a photo converter (e.g., a phosphor screen), which converts the energy into visible light. Electron microscopy and some applications of gamma imaging use this approach. The idea is simple: Incoming energy is transformed into a voltage by the combination of input electrical power and sensor material that is responsive to the particular type of energy being detected. The output voltage waveform is the response of the sensor(s), and a digital quantity is obtained from each sensor by digitizing its response. In this section, we look at the principal modalities for image sensing and generation.

Image Acquisition using a Single sensor:

The components of a single sensor. Perhaps the most familiar sensor of this type is the photodiode, which is constructed of silicon materials and whose output voltage waveform is proportional to light. The use of a filter in front of a sensor improves selectivity. For example, a green (pass) filter in front of a light sensor favors light in the green band of the color spectrum. As a consequence, the sensor output will be stronger for green light than for other components in the visible spectrum. In order to generate a 2-D image using a single sensor, there has to be relative displacements in both the x- and y-directions between the sensor and the area to be imaged. Figure shows an arrangement used in high-precision scanning, where a film negative is mounted onto a drum whose mechanical rotation provides displacement in one dimension. The single sensor is mounted on a lead screw that provides motion in the perpendicular direction. Since mechanical motion can be controlled with high precision, this method is an inexpensive (but slow) way to obtain high-resolution images. Other similar mechanical arrangements use a flat bed, with the sensor moving in two linear directions. These types of mechanical digitizers sometimes are referred to as **microdensitometers.**



a
b
c

FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Image Acquisition using a Sensor strips:

A geometry that is used much more frequently than single sensors consists of an in-line arrangement of sensors in the form of a sensor strip, shows. The strip provides imaging elements in one direction. Motion perpendicular to the strip provides imaging in the other direction. This is the type of arrangement used in most flat-bed scanners. Sensing devices with 4000 or more in-line sensors are possible. In-line sensors are used routinely in airborne imaging applications, in which the imaging system is mounted on an aircraft that flies at a constant altitude and speed over the geographical area to be imaged. One dimensional imaging sensor strips that respond to various bands of the electromagnetic spectrum are mounted perpendicular to the direction of flight. The imaging strip gives one line of an image at a time, and the motion of the strip completes the other dimension of a two-dimensional image. Lenses or other focusing schemes are used to project area to be scanned onto the sensors. Sensor strips mounted in a ring configuration are used in medical and industrial imaging to obtain cross-sectional (“slice”) images of 3-D objects.

Image Acquisition using a Sensor Arrays:

The individual sensors arranged in the form of a 2-D array. Numerous electromagnetic and some ultrasonic sensing devices frequently are arranged in an array format. This is also the predominant arrangement found in digital cameras. A typical sensor for these cameras is a CCD array, which can be manufactured with a broad range of sensing properties and can be packaged in rugged arrays of elements or more. CCD sensors are used widely in digital cameras and other light sensing instruments. The response of each sensor is proportional to the integral of the light energy projected onto the surface of the sensor, a property that is used in astronomical and other applications requiring low noise images. Noise reduction is achieved by letting the sensor integrate the input light signal over minutes or even hours. The two dimensional, its key advantage is that a complete image can be obtained by focusing the energy pattern onto the surface of the array. Motion obviously is not necessary, as is the case with the sensor arrangements. The figure shows the energy from an illumination source being reflected from a scene element, but, as mentioned at the beginning of this section, the energy also could be transmitted through the scene elements. The first function performed by the imaging system is to collect the incoming energy and focus it onto an image plane. If the illumination is light, the front end of the imaging system is a lens, which projects the viewed scene onto the lens focal plane. The sensor array, which is coincident with the focal plane, produces outputs proportional to the integral of the light received at each sensor. Digital and analog circuitry sweep these outputs and convert them to a video signal, which is then digitized by another section of the imaging system.

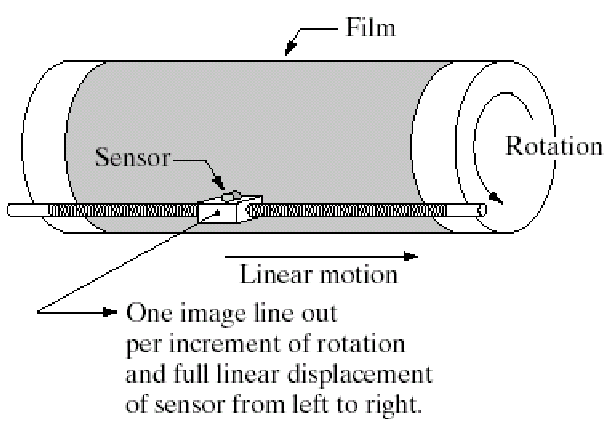


FIGURE 2.13 Combining a single sensor with motion to generate a 2-D image.

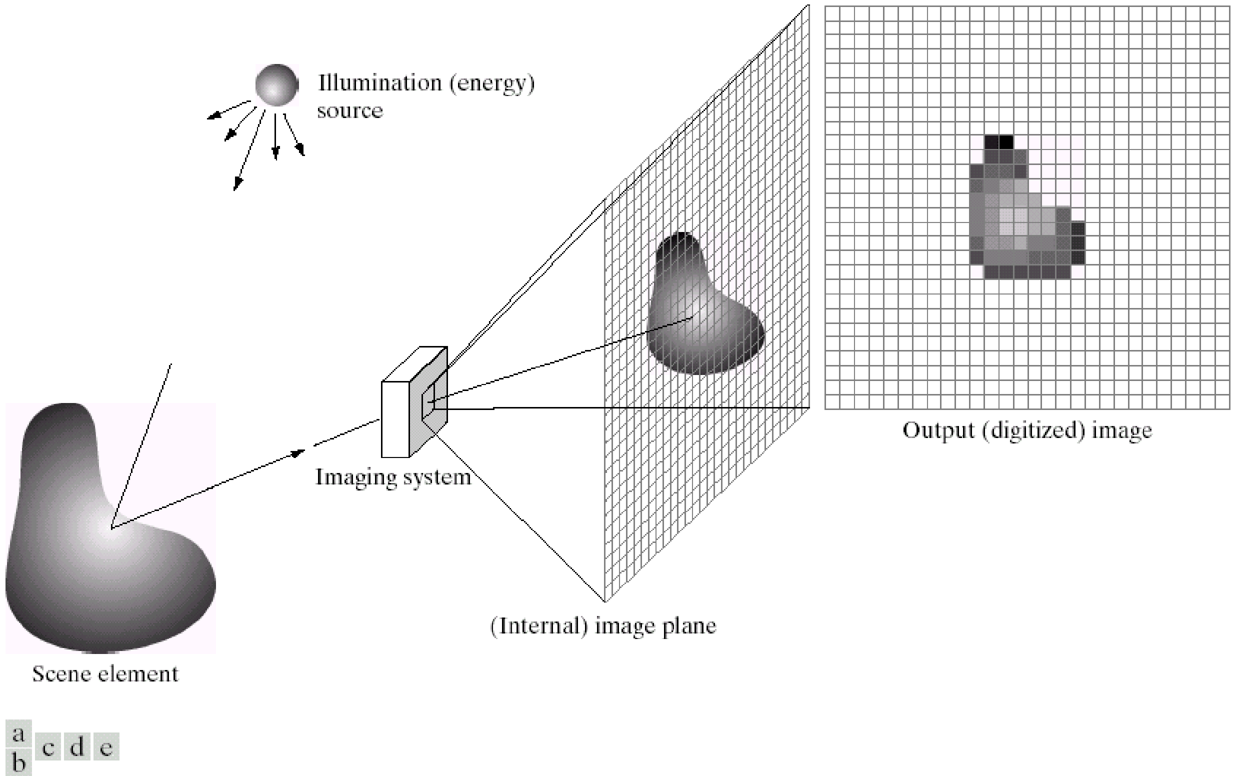


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

A Simple Image Model:

An image is denoted by a two dimensional function of the form f{x, y}. The value or amplitude of at spatial coordinates {x,y} is a positive scalar quantity whose physical meaning is determined by the source of the image. When an image is generated by a physical process, its values are proportional to energy radiated by a physical source. As a consequence, f(x,y) must be nonzero and finite; that is

$$O < f(x,y) < \infty$$

The function f(x,y) may be characterized by two components-

(a) The amount of the source illumination incident on the scene being viewed.

(b) The amount of the source illumination reflected back by the objects in the scene

These are called illumination and reflectance components and are denoted by **i(x,y) and r(x,y)** respectively.

The functions combine as a product to form f(x,y).

$$\mathbf{f(x,y)=i(x,y)r(x,y)}$$

$$\text{where } o < i(x,y) < \infty \text{ and } o < r(x,y) < 1$$

We call the intensity of a monochrome image at any coordinates (x,y) the gray level (l) of the image at that point l= f (x, y.)

$L\ min \leq l \leq Lmax$

Lmin is to be positive and Lmax must be finite

$Lmin = i_{min}\ r_{min}$

$Lmax = i_{max}\ r_{max}$

The interval [Lmin, Lmax] is called gray scale. Common practice is to shift this interval numerically to the interval [0, L-l] where l=0 is considered black and l= L-1 is considered white on the gray scale. All intermediate values are shades of gray of gray varying from black to white.

SAMPLING AND QUANTIZATION:

To create a digital image, we need to convert the continuous sensed data into digital from. This involves two processes – sampling and quantization. An image may be continuous with respect to the x and y coordinates and also in amplitude. To convert it into digital form we have to sample the function in both coordinates and in amplitudes.

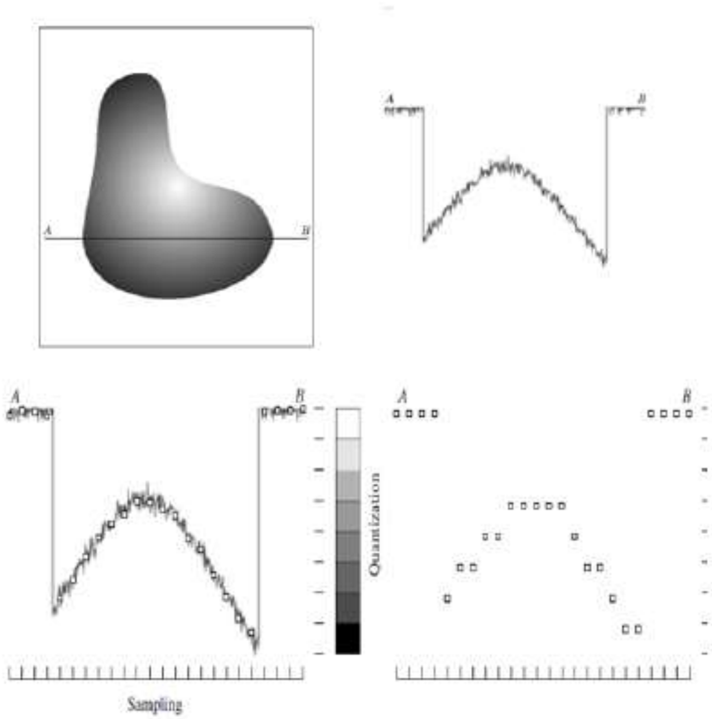
Digitalizing the coordinate values is called sampling. Digitalizing the amplitude values is called quantization. There is a continuous the image along the line segment AB. To simple this function, we take equally spaced samples along line AB. The location of each samples is given by a vertical tick back (mark) in the bottom part. The samples are shown as block squares superimposed on function the set of these discrete locations gives the sampled function.

In order to form a digital, the gray level values must also be converted (quantized) into discrete quantities. So we divide the gray level scale into eight discrete levels ranging from eight level values. The continuous gray levels are quantized simply by assigning one of the eight discrete gray levels to each sample. The assignment it made depending on the vertical proximity of a simple to a vertical tick mark.

Starting at the top of the image and covering out this procedure line by line produces a two dimensional digital image.

Image sampling and Quantization:

To create a digital image, we need to convert the continuous sensed data into digital form.
This involves two processes: *sampling* and *quantization*. A continuous image, f(x, y), that we want to convert to digital form. An image may be continuous with respect to the x- and y coordinates, and also in amplitude. To convert it to digital form, we have to sample the function in both coordinates and in amplitude. Digitizing the coordinate values is called *sampling*. Digitizing the amplitude values is called *quantization*.



Digital Image definition:

A digital image f(m,n) described in a 2D discrete space is derived from an analog image f(x,y) in a 2D continuous space through a sampling process that is frequently referred to as digitization we will look at some basic definitions associated with the digital image. The effect of digitization is shown in figure.

The 2D continuous image f(x,y) is divided into N rows and M columns. The intersection of a row and a column is termed a pixel. The value assigned to the integer coordinates (m,n) with m=0,1,2..N-1 and n=0,1,2…N-1 is f(m,n). In fact, in most cases, is actually a function of many variables including depth, color and time (t).

Representing Digital Images:

The result of sampling and quantization is matrix of real numbers. Assume that an image f(x,y) is sampled so that the resulting digital image has M rows and N Columns. The values of the coordinates (x,y) now become discrete quantities thus the value of the coordinates at origin become (X,Y) =(o,o) The next Coordinates value along the first signify the image along the first row. it does not mean that these are the actual values of physical coordinates when the image was sampled.

$$f(x,y)=\begin{bmatrix} f(1,1) & f(1,2) & \dots & f(1,N) \\ f(2,1) & f(2,2) & \dots & f(2,N) \\ \dots & \dots & \dots & \dots \\ f(M,1) & f(M,2) & \dots & f(M,N) \end{bmatrix}$$

Thus the right side of the matrix represents a digital element, pixel or pel. The matrix can be represented in the following form as well. The sampling process may be viewed as partitioning the xy plane into a grid with the coordinates of the center of each grid being a pair of elements from the Cartesian products Z2 which is the set of all ordered pair of elements (Zi, Zj) with Zi and Zj being integers from Z. Hence f(x,y) is a digital image if gray level (that is, a real number from the set of real number R) to each distinct pair of coordinates (x,y). This functional assignment is the quantization process. If the gray levels are also integers, Z replaces R, and a digital image become a 2D function whose coordinates and the amplitude value are integers. Due to processing storage and hardware consideration, the number gray levels typical lly is an integer power of 2.

$L=2^k$

Then, the number, b, of bites required to store a digital image is

$b=M*N*k$ When M=N, the equation become

$b=N^2*k$

When an image can have 2k gray levels, it is referred to as "k- bit". An image with 256 possible gray levels is called an "8- bit image" ($256=2^8$).

Spatial and Gray level resolution:

Spatial resolution is the smallest discernible details are an image. Suppose a chart can be constructed with vertical lines of width w with the space between the also having width W, so a line pair consists of one such line and its adjacent space thus. The width of the line pair is 2w and there is 1/2w line pair per unit distance resolution is simply the smallest number of discernible line pair unit distance. **Images of very low spatial resolution produce a checkerboard effect.**

Gray levels resolution refers to smallest discernible change in gray levels. Measuring discernible change in gray levels is a highly subjective process reducing the number of bits R while repairing the spatial resolution constant creates the problem of false contouring. It is caused by the use of an insufficient number of gray levels on the smooth areas of the digital image. It is called so because the rides resemble top graphics contours in a map. It is generally quite visible in image displayed using 16 or less uniformly spaced gray levels. **The use of insufficient number of gray levels in smooth areas of a digital image results in false contouring**

For enough human perception the number of quantization levels should be high. Image quantized **insufficient brightness levels cause the false contour which is the main problem in the image. False contour occurs when the brightness level is lower than which human can easily distinguish.**

Digital Image representation:

Digital image is a finite collection of discrete samples (*pixels*) of any observable object. The pixels represent a two- or higher dimensional "view" of the object, each pixel having its own discrete value in a finite range. The pixel values may represent the amount of visible light, infra-red light, absorption of x-rays, electrons, or any other measurable value such as ultrasound wave impulses.

The image does not need to have any visual sense; it is sufficient that the samples form a two-dimensional spatial structure that may be illustrated as an image.

The images may be obtained by a digital camera, scanner, electron microscope, ultrasound stethoscope, or any other optical or non-optical sensor. Examples of digital image are:

- digital photographs
- satellite images
- radiological images (x-rays, mammograms)
- binary images, fax images, engineering drawings

Computer graphics, CAD drawings, and vector graphics in general are not considered in this course even though their reproduction is a possible source of an image. In fact, one goal of intermediate level image processing may be to reconstruct a model (e.g. vector representation) for a given digital image.

RELATIONSHIP BETWEEN PIXELS:

We consider several important relationships between pixels in a digital image.

NEIGHBORS OF A PIXEL

A pixel *p* at coordinates (*x*,*y*) has four *horizontal* and *vertical* neighbors whose coordinates are given by: (x+1,y), (x-1, y), (x, y+1), (x,y-1)

	(x, y-1)	
(x-1, y)	*P* (*x*,*y*)	(x+1, y)
	(x, y+1)	

This set of pixels, called the 4-*neighbors* or *p*, is denoted by *N*4(*p*). Each pixel is one unit distance from (x,y) and some of the neighbors of p lie outside the digital image if (x,y) is on the border of the image. The four *diagonal* neighbors of *p* have coordinates and are denoted by *ND* (*p*).

(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)

(x-1, y+1)		(x+1, y-1)
	P (*x*,*y*)	
(x-1, y-1)		(x+1, y+1)

These points, together with the 4-neighbors, are called the 8-neighbors of p, denoted by N_8 (*p*).

(x-1, y+1)	(x, y-1)	(x+1, y-1)
(x-1, y)	*P (x,y)*	(x+1, y)
(x-1, y-1)	(x, y+1)	(x+1, y+1)

As before, some of the points in *ND* (*p*) and *N8* (*p*) fall outside the image if (x,y) is on the border of the image.

ADJACENCY AND CONNECTIVITY

Let v be the set of gray –level values used to define adjacency, in a binary image, v={1}.

In a gray-scale image, the idea is the same, but *V* typically contains more elements, for example, *V* = {180, 181, 182, …, 200}.

If the possible intensity values 0 – 255, *V* set can be any subset of these 256 values. if we are reference to adjacency of pixel with value.

Three types of adjacency

- 4- Adjacency – two pixel P and Q with value from V are 4 –adjacency if A is in the set N_4(P)
- 8- Adjacency – two pixel P and Q with value from V are 8 –adjacency if A is in the set N_8(P)
- M-adjacency –two pixel P and Q with value from V are m – adjacency if
    - (i) Q is in N4(p) or
    - (ii) Q is in ND(q) and the set N4(p) ∩ N4(q) has no pixel whose values are from V.

• Mixed adjacency is a modification of 8-adjacency. It is introduced to eliminate the ambiguities that often arise when 8-adjacency is used.

• For example:

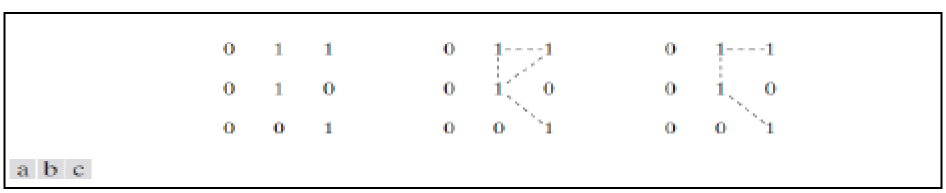


Fig:1.8(a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

Types of Adjacency:

• In this example, we can note that to connect between two pixels (finding a path between two pixels):

– In 8-adjacency way, you can find multiple paths between two pixels

– While, in m-adjacency, you can find only one path between two pixels

• So, m-adjacency has eliminated the multiple path connection that has been generated by the 8-adjacency.

• Two subsets *S1* and *S2* are adjacent, if some pixel in *S1* is adjacent to some pixel in *S2*. Adjacent means, either 4-, 8- or m-adjacency.

A Digital Path:

• A digital path (or curve) from pixel *p* with coordinate (x,y) to pixel *q* with coordinate (s,t) is a sequence of distinct pixels with coordinates (x_0,y_0), (x_1,y_1), …, (x_n, y_n) where $(x_0,y_0) = (x,y)$ and $(x_n, y_n) = (s,t)$ and pixels (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$

• n is the length of the path

• If $(x_0,y_0) = (x_n, y_n)$, the path is closed.

We can specify 4-, 8- or m-paths depending on the type of adjacency specified.

• Return to the previous example:

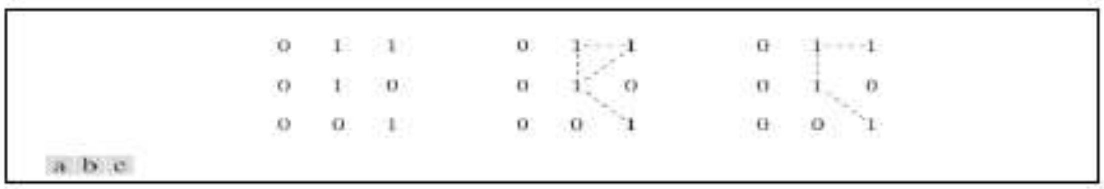


Fig:1.8 (a) Arrangement of pixels; (b) pixels that are 8-adjacent(shown dashed) to the center pixel; (c) m-adjacency.

In figure (b) the paths between the top right and bottom right pixels are 8-paths. And the path between the same 2 pixels in figure (c) is m-path

Connectivity:

• Let *S* represent a subset of pixels in an image, two pixels *p* and *q* are said to be connected in *S* if there exists a path between them consisting entirely of pixels in *S*.

For any pixel *p* in *S*, the set of pixels that are connected to it in *S* is called a *connected component* of *S*. If it only has one connected component, then set *S* is called a *connected set*.

Region and Boundary:

• REGION: Let *R* be a subset of pixels in an image, we call *R* a region of the image if *R* is a connected set.

• BOUNDARY: The *boundary* (also called *border* or *contour*) of a region *R* is the set of pixels in the region that have one or more neighbors that are not in *R*.

If *R* happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns in the image. This extra definition is required because an image has no neighbors beyond its borders. Normally, when we refer to a region, we are referring to subset of an image, and any pixels in the boundary of the region that happen to coincide with the border of the image are included implicitly as part of the region boundary.

DISTANCE MEASURES:

For pixel p,q and z with coordinate (x.y) ,(s,t) and (v,w) respectively D is a distance function or metric if

D [p.q] ≥ 0 {D[p.q] = O iff p=q}

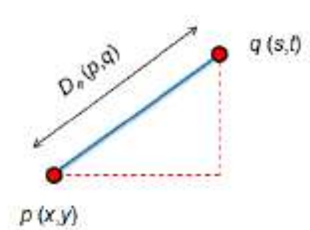
D [p.q] = D [p.q] and

D [p.q] ≥ {D[p.q]+D(q,z)

• The ***Euclidean Distance*** between *p* and *q* is defined as:

$$D_e (p,q) = [(x - s)^2 + (y - t)^2]^{1/2}$$

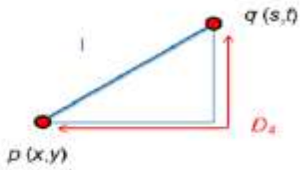
Pixels having a distance less than or equal to some value r from (x,y) are the points contained in a disk of radius „ r „centered at (x,y)



The D_4 *distance* (also called ***city-block distance***) between *p* and *q* is defined as:

$D4\ (p,q) = | x – s | + | y – t |$

Pixels having a D_4 distance from (x,y), less than or equal to some value r form a Diamond centered at (x,y).



Example:

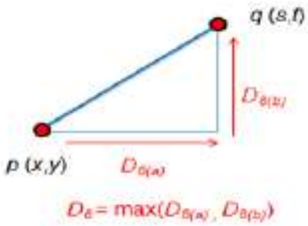
The pixels with distance $D4 \leq 2$ from (x,y) form the following contours of constant distance.

The pixels with $D4 = 1$ are the 4-neighbors of (x,y)

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

The *D8 distance* (also called ***chessboard distance***) between p and q is defined as:

$D8\,(p,q) = \max(|\,x - s\,|, |\,y - t\,|)$



Pixels having a *D8* distance from (x,y), less than or equal to some value r form a square Centered at (x,y).

Example:

D_8 distance ≤ 2 from (x,y) form the following contours of constant distance.

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

• **D_m distance:**

It is defined as the shortest m-path between the points.

In this case, the distance between two pixels will depend on the values of the pixels along the path, as well as the values of their neighbors.

• Example:

Consider the following arrangement of pixels and assume that *p*, *p2*, and *p4* have value 1 and that *p1* and *p3* can have can have a value of 0 or 1. Suppose that we consider the adjacency of pixels values 1 (i.e. *V* = {1})

	p_3	p_4
p_1	p_2	
p		

Now, to compute the *Dm* between points *p* and *p4*

Here we have 4 cases:

Case1: If *p1* =0 and *p3* = 0

	0	1
0	1	
1		

The length of the shortest m-path (the *Dm distance) is 2 (p, p2, p4)*

Case2: If *p1* =1 and *p3* = 0

	0	1
1	1	
1		

now, *p1 and p will no longer be adjacent (see m-adjacency definition)* then, the length of the shortest path will be 3 (*p*, *p1*, *p2*, *p4*)

Case3: If p_1 =0 and p_3 = 1

	1	1
0	1	
1		

The same applies here, and the shortest –m-path will be 3 (p, p_2, p_3, p_4)

Case4: If p_1 =1 and p_3 = 1

	1	1
1	1	
1		

The length of the shortest m-path will be 4 (p, p_1 , p_2, p_3, p_4)