AN INVESTIGATION OF LUCKY EDGE MEAN LABELING OF CORONA PRODUCT OF GRAPHS

Dissertation submitted in partial fulfillment of the

Requirements for the degree of

MASTER OF SCIENCE IN MATHEMATICS

Submitted by

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(Reg. No: 19MMA001)

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DEPARTMENT OF MATHEMATICS

SREE SARASWATHI THYAGARAJA COLLEGE

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THIPPAMPATTI, POLLACHI-642 107

MAY-2021

CERTIFICATE

This is to certify that the dissertation, entitled, **"AN INVESTIGATION OF LUCKY EDGE MEAN LABELING OF CORONA PRODUCT OF GRAPHS"**, submitted to **Sree Saraswathi Thyagaraja College(Autonomous), Pollachi** in partial fulfilment of the requirements for the award of the degree of **Master of Science in Mathematics** is a record of project work done by **Miss. A.AISHWARYA (Reg. No:19MMA001)** during the period **2019-2021** of his study in the Department of Mathematics at **Sree Saraswathi Thyagaraja College(Autonomous), Pollachi** under my supervision and guidance and the dissertation has not formed the basis for the award of any Degree / Diploma / Associateship / Fellowship or other similar title to any candidate of any University.

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DECLARATION

I, **Miss. A.AISHWARYA (Reg. No: 19MMA001)** hereby declare that the dissertation, entitled "**AN INVESTIGATION OF LUCKY EDGE MEAN LABELING OF CORONA PRODUCT OF GRAPHS**", submitted to **Sree Saraswathi Thyagaraja College(Autonomous), Pollachi** in partial fulfilment of the requirements for the award of the degree of **Master of Science in Mathematics** is a record of project work done by me during **2019-2021** under the supervision and guidance **Dr**.(**Mrs**). **P.SUGAPRIYA ,M.Sc., M.Phil., Ph.D.,**Assistant Professor, Department of Mathematics, **Sree Saraswathi Thyagaraja College(Autonomous),Pollachi** and it has not formed the basis for the award of any Degree / Diploma / Associateship / Fellowship or other similar title to any candidate in any University.

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Abstract

A function f:V $\rightarrow$ N is said to be lucky edge mean labeling if there exists a function f* : E $\rightarrow$ N such that f* (uv)= $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ and the edge set E(G)is a proper coloring of G, that is f* (uv) $\neq$ f* (vw) whenever uv and vw are adjacent edges. The least integer k for which a graph G has a lucky edge mean labeling from the set $\{$1,2,3, . . . ,k$\}$ is the lucky edge mean number of G denoted by $\eta_M^{'}$ (G) . A graph which admits lucky edge mean labeling is the lucky edge mean graph. In this project we have to prove that Corona product of path graphs with path graphs, star graphs, wheel graphs and complete graphs admits lucky edge mean labeling and also we obtain their lucky edge mean number of these graphs.

Contents

Chapter 1

Introduction

1.1 Graph Theory

Konigsberg in Prussia (now Kaliningrad, Russia), was set on both sides of the Pregel River, and included two large islands, which were connected to the rest of the city by seven bridges. The problem was to device a walk through the city that would cross each of those bridges once and only once. The residents of the city always challenged visitors and travellers to see whether they could solve the problem. However, no one except Leonard Euler could find a way. Euler proved that the problem had no solution. He was able to find a rule, using which, it was easy to determine whether it is possible to pass through all bridges, without passing twice on either of them. Its negative resolution laid the foundations of graph theory. Euler wrote a paper about "The Seven Bridges of Konigsberg" and published it in 1736. It was the first paper about graph theory in history.

1.2 Graph Labeling

The concept of graph labeling was first introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or edge labeling). In the interviewing years various labeling of graphs have been investigated in over 1100 papers.

Labeled graphs serve as useful models in a broad range of X-ray, crystallography, radar, astronomy, data base management and coding theory. Nellai Murugan.A and Maria Irudhaya Aspin Chitra.R introduced the concept of lucky edge labeling as a function f is an assignment of integers to vertices such that f(u)+f(v) is assigned to the edge uv and it is denoted as f^* (uv) , should satisfy the condition that f^* (uv) $\neq$ f^* (vw) whenever uv and vw are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set {1,2,3, . . . ,k} is the lucky edge number of G and is denoted by η (G). A graph which admits lucky edge labeling is the lucky edge labeled graph.

In 2014, they proved Pn ,Cn are lucky edge labeled graph.

In 2015, they proved planar grid graph are lucky edge labeling.

In 2016, they proved triangular graphs are lucky edge labeled graphs and some special graphs are also lucky edge labeled graphs.

In 2017, they proved subdivision of star and wheel graphs, H-super subdivision of graphs are lucky edge labeled graphs.

In 2018, they proved star related graphs are lucky edge labeled graph.

In 2020, Senthil Amutha.R , Sugapriya.P , Vaanmathi.S introdued lucky edge geometric mean labeling. In this project we have to show that corona product of path graphs with path graphs, star graphs, wheel graphs and complete graphs are lucky edge mean graphs.

Chapter 1 deals with introduction to lucky edge mean labeling.

Chapter 2 deals with basic definitions which are needed for subsequent chapter.

Chapter 3 deals with lucky edge mean labeling of path graphs with path graphs and star graphs.

Chapter 4 deals with lucky edge mean labeling of path graphs with wheel graphs and complete graphs.

Chapter 2

Preliminaries

Definition 2.0.1 *A walk of a graph G is an alternating sequence of vertices and edges* v_0 *,* x_1 *,* v_1 *,* x_2 *, . . . ,* v_{n-1} *,* x_n *, v beginning and ending with vertices such that each edge* x_i *is incident with* v_{i-1} *and* v_i *.*

Definition 2.0.2 *A walk is called a path[3] if all its points are distinct.*

Definition 2.0.3 *A bipartite graph is a graph whose vertex set V(G) can be partitioned into two subsets* V_1 *and* V_2 *such that every edge of G joins a vertex of* V_1 *with a vertex of* V_2 *. If every vertex of* V_1 *is adjacent with every vertex of* V_2 *, then G is a complete bipartite graph. If* $|V_1| = m$ *and* $|V_2| = n$*, then the complete bipartite graph is denoted by* $K_{m,n}$ *.*

Definition 2.0.4 $K_{1,n}$ *is called a Star[4]*

Definition 2.0.5 *A wheel graph[1]* W_n *is defined as an n-cycle with one additional* *vertex that is adjacent to each of the vertices in the cycle.*

Definition 2.0.6 *A complete graph[6] is a simple graph with exactly one edge between evry pair of distinct nodes and the complete graph with n vertices is denoted by* K_n .

Definition 2.0.7 *Let G and H be the two graphs of order m and n. The corona product[2]* $G \odot H$ *is defied as the graph obtained from G and H by taking one copy of G and m copies of H and joining by an edge each vertex from the i-th copy of H with the i-th vertex of G.*

Definition 2.0.8 *A function* $f{:}V(G) \rightarrow N$ *is said to be lucky edge mean labeling if there exists a function* $f^*{:}E(G) \rightarrow N$ *such that* $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ *and the edge set E(G)is a proper coloring of G, that is* $f^*(uv) \neq f^*(vw)$ *whenever uv and vw are adjacent edges. The least integer k for which a graph G has a lucky edge mean labeling from the set* $\{1,2,3, \ldots ,k\}$ *is the lucky edge mean number of G denoted by* $\eta'_M(G)$*. A graph which admits lucky edge mean labeling is the lucky edge mean graph.*

Chapter 3

Lucky Edge Mean Labeling of Corona Product of Path Graphs with Path Graphs and Star Graphs

In this chapter we investigate the Lucky Edge Mean Labeling of the Corona product of Path Graphs with Path Graphs and Star Graphs.

3.1 Corona Product of Path Graphs with Path Graphs ($P_m \odot P_n$):

Let P_m be the path graph of order m

V(P_m)={v_i;1 $\leq$ i $\leq$ m}

E(P_m)={v_i v_{i+1};1 $\leq$ i $\leq$ m-1}

Figure:3.1 Labeling of P_m

Let P_n be the path graph of order n

V(P_n)={ u_j ;1 $\leq$ j $\leq$ n}

E(P_n)={ u_j u_{j+1} ;1 $\leq$ j $\leq$ n-1}

Figure:3.2 Labeling of P_n

Corona product of Path Graphs P_m with Path Graphs P_n :

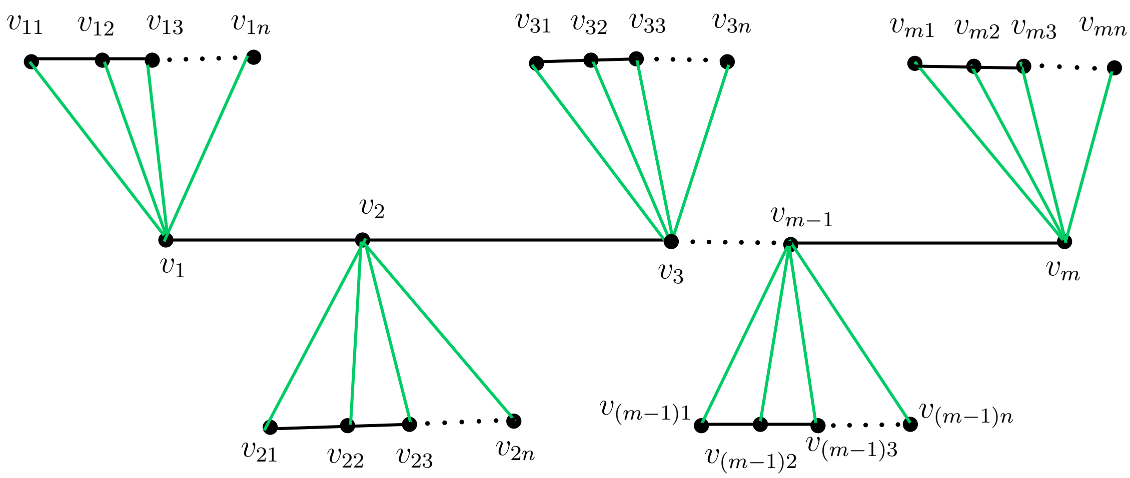

Figure:3.3 Labeling of $P_m \odot P_n$

Let P_m and P_n be the two graphs of order m and n. The product $P_m \odot P_n$ is defined as the graph obtained from P_m and P_n by taking one copy of P_m and m copies of P_n and joining by an edge each vertex from the i-th copy of P_n with the i-th vertex of P_m.

3.1.1 Characteristics of $P_m \odot P_n$:

i)V($P_m \odot P_n$)={ v_i ;1 $\leq$ i $\leq$ m} $\cup$ { v_{ij} ;1 $\leq$ i $\leq$ m , 1 $\leq$ j $\leq$ n}

ii)E($P_m \odot P_n$)={ v_i v_{i+1} ;1 $\leq$ i $\leq$ m-1}

$\cup$ { v_{ij} $v_{i(j+1)}$;1 $\leq$ i $\leq$ m,1 $\leq$ j $\leq$ n-1} $\cup$ { v_i v_{ij} ;1 $\leq$ i $\leq$ m,1 $\leq$ j $\leq$ n}

iii)| V($P_m \odot P_n$)| = m(n+1)

iv)| E($P_m \odot P_n$)| = 2mn-1

v)The maximum degree = n+2

vi)The minimum degree = 2

vii)The number of vertices having maximum degree = m-2

viii)The number of vertices having minimum degree = 2m

ix)The graph $P_m \odot P_n$ is isomorphic to rooted product at v of path P_m with fan graph $F_{1,n}$.

x)Diameter of the graph $P_m \odot P_n$ = m+n+1

xi)Number of C_3 = m(n-1)

xii)The number of $F_{1,n}$ = m

3.1.2 Theorem:

Corona product graph $P_m \odot P_n$ is a lucky edge mean graphs when m,n $\geq$ 3

Proof:

For m,n $\geq$ 3

Define f: V($P_m \odot P_n$) $\rightarrow$ N

The vertex labels are defines as,

f(v_1)=2

2 $\leq$ i $\leq$ m

f(v_i)= $\begin{cases} 2; i \equiv 0(mod4) \\ 3; i \equiv 1(mod4) \\ 1; i \equiv 2,3(mod4) \end{cases}$

For m $\equiv$ 0(mod4)

1 $\leq$ i $\leq$ m-1, 1 $\leq$ j $\leq$ n

i $\equiv$ 0(mod4)

f(v_{ij})= $\begin{cases} 2n+6-j;\ j\ is\ even \\ j+5;\ j\ is\ odd \end{cases}$

i $\equiv$ 1,2,3(mod4)

f(v_{ij})= $\begin{cases} 2n+4-j;\ j\ is\ even \\ j+3;\ j\ is\ odd \end{cases}$

For i=m , 1 $\leq$ j $\leq$ n

f(v_{mj})= $\begin{cases} 2n+4-j;\ j\ is\ even \\ j+3;\ j\ is\ odd \end{cases}$

For m $\equiv$ 1,2,3(mod4)

1 $\leq$ i $\leq$ m, 1 $\leq$ j $\leq$ n

i$\equiv$0(mod4)

f(v_{ij})= $\begin{cases} 2n+6-j; \ j \ is \ even \\ \\ j+5; \ j \ is \ odd \end{cases}$

i$\equiv$1,2,3(mod4)

f(v_{ij})= $\begin{cases} 2n+4-j; \ j \ is \ even \\ \\ j+3; \ j \ is \ odd \end{cases}$

Define the induced function :

f*:E(P_m $\odot$ P_n) $\rightarrow$ N

f* (uv)= $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$, for every uv$\in$E(P_m $\odot$ P_n)

For 1 $\leq$ i $\leq$ m - 1

f* (v_i v_{i+1})= $\begin{cases} 3; i \equiv 0(mod4) \\ \\ 1; i \equiv 2(mod4) \\ \\ 2; i \equiv 1,3(mod4) \end{cases}$

1 $\leq$ j $\leq$ n-1

m $\equiv$ 0(mod4)

1 $\leq$ i $\leq$ m-1, 1 $\leq$ j $\leq$ n-1

i $\equiv$ 0 (mod4)

f* (v_{ij} $v_{i(j+1)}$)= $\begin{cases} n+5; j \ is \ odd \\ \\ n+6; j \ is \ even \end{cases}$

i $\equiv$ 1,2,3(mod4)

f* (v_{ij} $v_{i(j+1)}$)= $\begin{cases} n+3; j \ is \ odd \\ \\ n+4; j \ is \ even \end{cases}$

For i=m

f* (v_{ij} $v_{i(j+1)}$)= $\begin{cases} n+3; j \ is \ odd \\ \\ n+4; j \ is \ even \end{cases}$

m $\equiv$ 1,2,3(mod4)

$1 \leq i \leq m, 1 \leq j \leq n-1$

$i \equiv 0 \pmod 4$

$$f^*(v_{ij}\ \ v_{i(j+1)}) = \begin{cases} n+5; j\ is\ odd \\ n+6; j\ is\ even \end{cases}$$

$i \equiv 1,2,3 \pmod 4$

$$f^*(v_{ij}\ \ v_{i(j+1)}) = \begin{cases} n+3; j\ is\ odd \\ n+4; j\ is\ even \end{cases}$$

For $1 \leq j \leq n$

$$f^*(v_1\ \ v_{1j}) = \begin{cases} \frac{j+5}{2}; j\ is\ odd \\ \frac{2n+6-j}{2};\ j\ is\ even \end{cases}$$

$m \equiv 0 \pmod 4$

For $2 \leq i \leq m-1, 1 \leq j \leq n$

$i \equiv 0,1 \pmod 4$

$$f^*(v_i\ \ v_{ij}) = \begin{cases} \frac{j+7}{2};\ j\ is\ odd; \\ \frac{2n+8-j}{2};\ j\ is\ even \end{cases}$$

$i \equiv 2,3 \pmod 4$

$$f^*(v_i\ \ v_{ij}) = \begin{cases} \frac{j+5}{2};\ j\ is\ odd; \\ \frac{2n+6-j}{2};\ j\ is\ even \end{cases}$$

For $i=m$

$$f^*(v_m\ \ v_{mj}) = \begin{cases} \frac{j+5}{2};\ j\ is\ odd; \\ \frac{2n+6-j}{2};\ j\ is\ even \end{cases}$$

$m \equiv 1,2,3 \pmod 4$

For $1 \leq i \leq m, 1 \leq j \leq n$

$i \equiv 0,1 \pmod 4$

$$f^*(v_i\ \ v_{ij}) = \begin{cases} \frac{j+7}{2};\ j\ is\ odd; \\ \frac{2n+8-j}{2};\ j\ is\ even \end{cases}$$

i $\equiv$ 2,3 (mod4)

f* (v_i v_{ij})= $\begin{cases} \frac{j+5}{2};\ j\ is\ odd; \\ \frac{2n+6-j}{2};\ j\ is\ even \end{cases}$

It is clear that lucky edge mean labeling of P_m $\odot$ P_n is ,

{1,2,3, . . . ,n+4} , 1 $\leq$ m $\leq$ 4

{1,2,3, . . . ,n+6} , m $\geq$ 5

Hence P_m $\odot$ P_n is a lucky edge mean graph.

Here η_M' (P_m $\odot$ P_n)= $\begin{cases} n+4;\ 1 \leq m \leq 4 \\ n+6;\ m \geq 5 \end{cases}$

3.1.3 Illustration:

Lucky edge mean labeling of P_4 $\odot$ P_3 is shown below

The Lucky edge mean number of P_4 $\odot$ P_3 is 7.

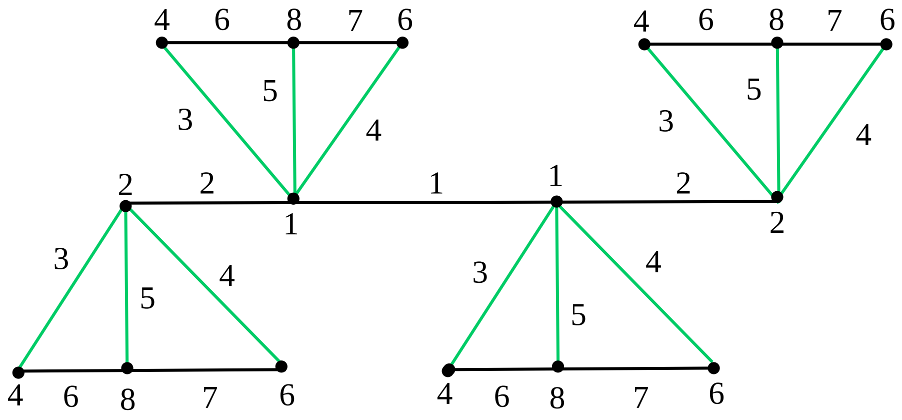

Figure:3.4 Lucky edge mean labeling of P_4 $\odot$ P_3

3.2 Corona Product of Path Graphs with Star Graphs ($P_m \odot K_{1,n}$):

Let P_m be the path graph of order m

V(P_m)={ v_i ;1 $\leq$ i $\leq$ m}

E(P_m)={ v_i v_{i+1} ;1 $\leq$ i $\leq$ m-1}

Figure:3.5 Labeling of P_m

Let $K_{1,n}$ be the Star graph of order n

V($K_{1,n}$)={ u }$\cup${ u_j ;1 $\leq$ j $\leq$ n}

E($K_{1,n}$)={ u u_j ;1 $\leq$ j $\leq$ n}

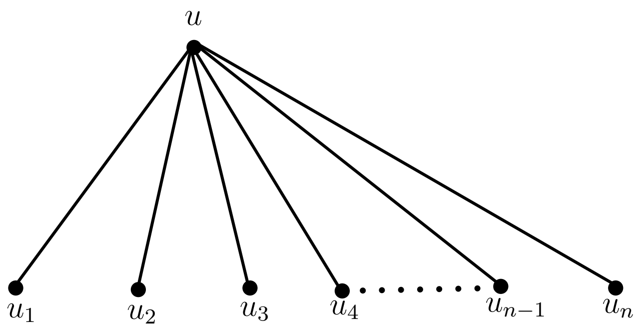

Figure:3.6 Labeling of $K_{1,n}$

Corona product of Path Graphs P_m with Star Graphs $K_{1,n}$:

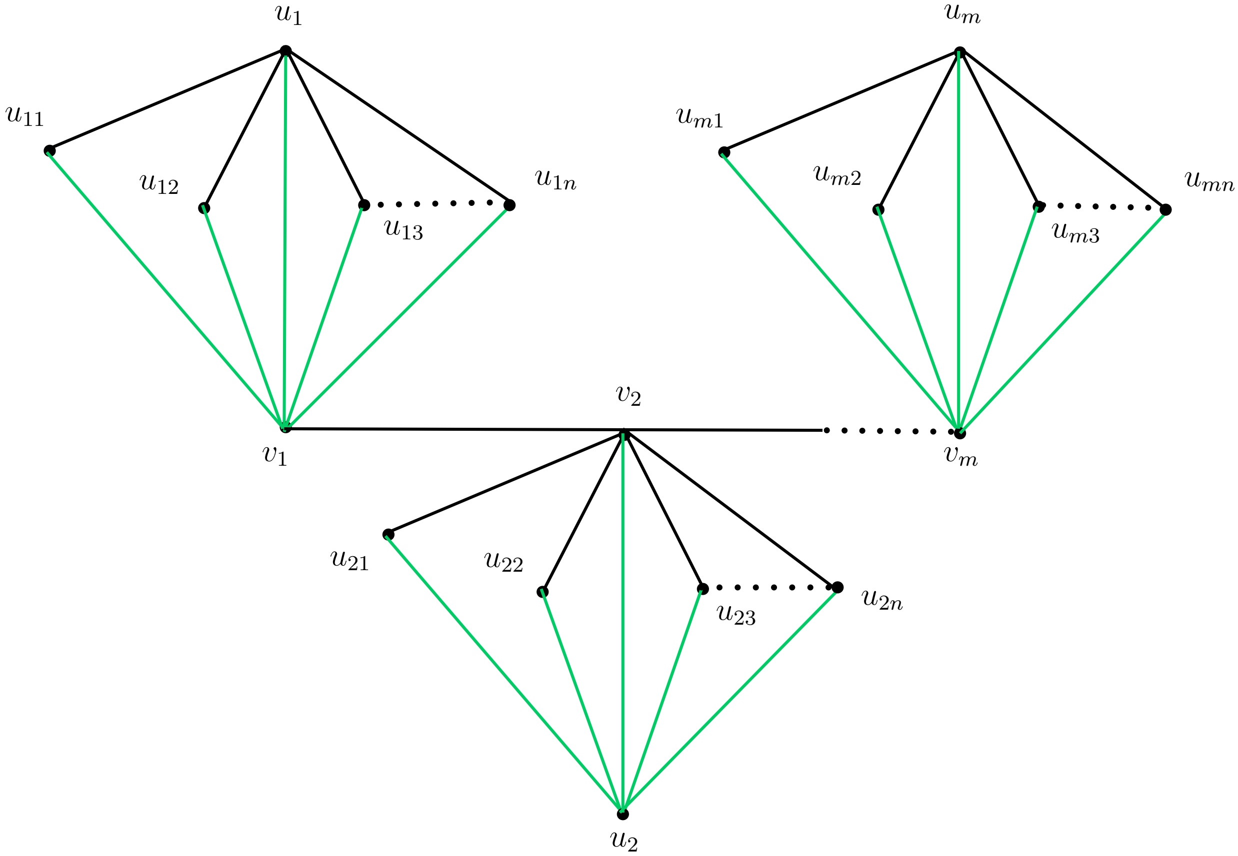

Figure:3.7 Labeling of $P_m \odot K_{1,n}$

Let P_m and $K_{1,n}$ be the two graphs of order m and n. The product $P_m \odot K_{1,n}$ is defined as the graph obtained from P_m and $K_{1,n}$ by taking one copy of P_m and m copies of $K_{1,n}$ and joining by an edge each vertex from the i-th copy of $K_{1,n}$ with the i-th vertex of P_m.

3.2.1 Characteristics of $P_m \odot K_{1,n}$:

i)V($P_m \odot K_{1,n}$)={ v_i ;1 $\leq$ i $\leq$ m}$\cup${ u_i ;1 $\leq$ i $\leq$ m}

$\cup$ { u_{ij} ;1 $\leq$ i $\leq$ m,1 $\leq$ j $\leq$ n}

ii)E($P_m \odot K_{1,n}$)={ v_i v_{i+1} ;1 $\leq$ i $\leq$ m-1}$\cup${ u_i u_{ij} 1 $\leq$ i $\leq$ m,1 $\leq$ j $\leq$ n}

$\cup${ v_i u_{ij} ;1 $\leq$ i $\leq$ m,1 $\leq$ j $\leq$ n}$\cup${ u_i v_i 1 $\leq$ i $\leq$ m }

iii)|V($P_m \odot K_{1,n}$)|=m(n+2)

iv)|E($P_m \odot K_{1,n}$)|=2m(n+1)-1

v)The maximum degree = n+3

vi)The minimum degree = 2

vii)The number of vertices having maximum degree = m-2

viii)The number of vertices having minimum degree = mn

3.2.2 Theorem:

Corona product graph $P_m \odot K_{1,n}$ is a lucky edge mean graphs when m,n $\geq$ 3

Proof:

For m,n$\geq$3

Define f: V($P_m \odot K_{1,n}$) $\rightarrow$ N

The vertex labeled are defined as,

$$f(v_i)=\begin{cases} 3; i \equiv 0(mod4) \\ 1; i \equiv 1,2(mod4) \\ 2; i \equiv 3(mod4) \end{cases}$$

f(u_1)=3 , f(u_2)=4

For m=3 , f(u_m)=4

For m $\geq$ 4

m$\equiv$0,1,3(mod 4)

3$\leq$ i $\leq$ m

f(u_i)= $\begin{cases} 5; i \equiv 0,3(mod4) \\ 4; i \equiv 1,2(mod4) \end{cases}$

m$\equiv$ 2(mod 4)

3 $\leq$ i $\leq$ m-1

f(u_i)= $\begin{cases} 5; i \equiv 03(mod4) \\ 4; i \equiv 1,2(mod4) \end{cases}$

For i=m, f(u_m)=3

f(u_{ij})=f(u_i)+2j ; 1 $\leq$ i $\leq$ m, 1 $\leq$ j $\leq$ n

Define the induced function:

f*:E(P_m $\odot$ $K_{1,n}$) $\rightarrow$ N

f*(uv)= $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$, for every uv$\in$E(P_m $\odot$ $K_{1,n}$)

For 1 $\leq$ i $\leq$ m-1,

f*(v_i v_{i+1})= $\begin{cases} 2; i \equiv 0,2(mod4) \\ 1; i \equiv 1(mod4) \\ 3; i \equiv 3(mod4) \end{cases}$

f*(u_1 v_1)=2

f*(u_2 v_2)=3

For m=3, f*(u_m v_m)=3

For m $\geq$ 4

m$\equiv$ 0,1,3(mod 4) , 3$\leq$ i$\leq$ m f* (u_i v_i)= $\begin{cases} 4; i \equiv 0,3(mod4) \\ 3; i \equiv 1,2(mod4) \end{cases}$

m$\equiv$ 2(mod 4) , 3$\leq$ i$\leq$ m-1

f* (u_i v_i)= $\begin{cases} 4; i \equiv 0,3(mod4) \\ 3; i \equiv 1,2(mod4) \end{cases}$

For i=m , f* (u_m v_m)=2

f* (v_i u_{ij})=f* (u_i v_i)+j

f* (u_i u_{ij})=f* (u_i v_i)+j+1

It is clear that lucky edge mean labeling of $P_m \odot K_{1,n}$ is,

{1,2,3, . . . ,m+n+1} , 3 $\leq$ m $\leq$ 4

{1,2,3, . . . ,5+n} , m $>$ 4

Hence $P_m \odot K_{1,n}$ is a lucky edge mean graph.

Here , $\eta^{'}_M$ ($P_m \odot K_{1,n}$)= $\begin{cases} m+n+1; 3 \leq m \leq 4 \\ 5+n; m > 4 \end{cases}$

3.2.3 Illustration:

Lucky edge mean labeling of $P_3 \odot K_{1,3}$ is shown.

The lucky edge number of $P_3 \odot K_{1,3}$ is 7

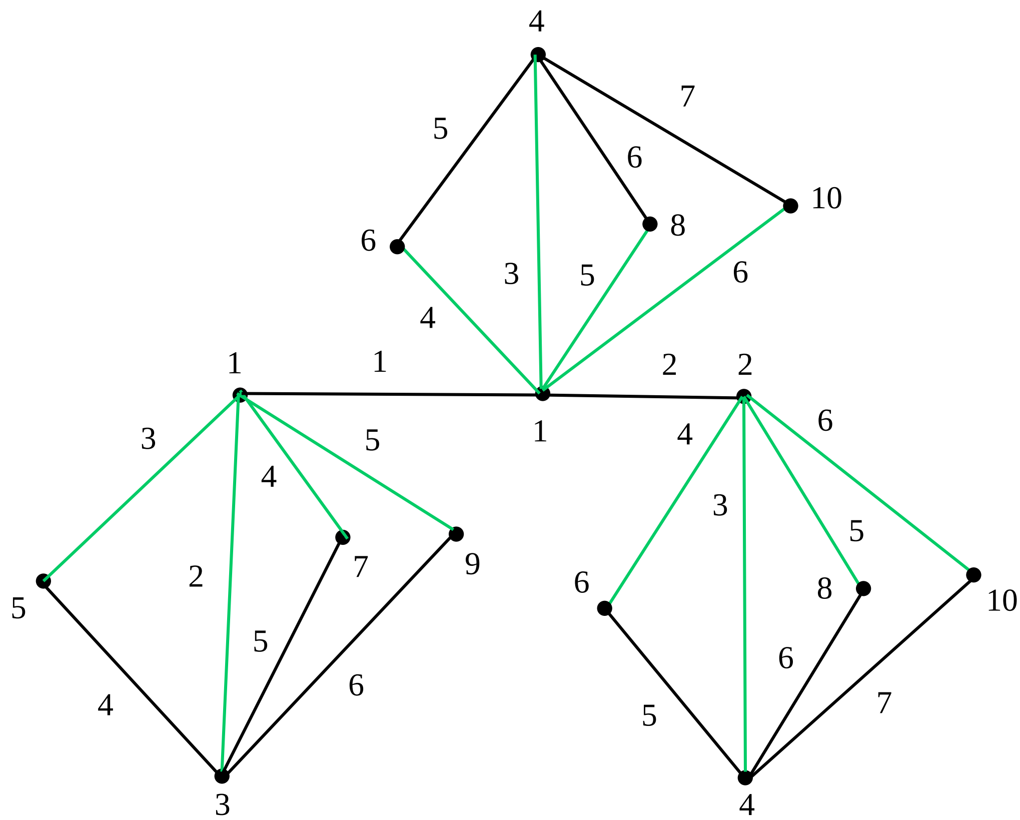

Figure:3.8 Lucky edge mean labeling of $P_3 \odot K_{1,3}$

Chapter 4

Lucky Edge Mean Labeling of Corona Product of Path Graphs with Wheel Graphs and Complete Graphs

In this chapter we investigate the Lucky Edge Mean Labeling of the Corona product of Path Graphs with Wheel Graphs and Complete Graphs.

4.1 Corona product of Path Graphs with Wheel Graphs ($P_m \odot W_n$):

Let P_m be the path graph of order m

V(P_m)={v_i;1 $\leq$ i $\leq$ m}

E(P_m)={ v_i v_{i+1} ;1 $\leq$ i $\leq$ m-1}

Figure:4.1 Labeling of P_m

Let W_n be the wheel graph of order n

V(W_n)={ u } $\cup$ { u_j ;1 $\leq$ j $\leq$ n}

E(W_n)={ u u_j ;1 $\leq$ j $\leq$ n} $\cup$ { u_j u_{j+1} ;1 $\leq$ j $\leq$ n-1} $\cup$ { u_n u_1 }

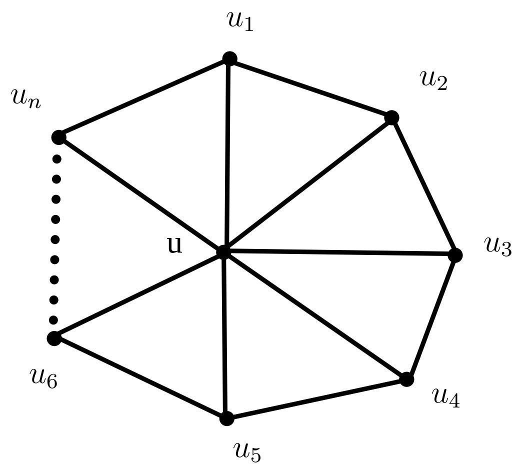

Figure:4.2 Labeling of W_n

Corona product of Path Graphs P_m with Wheel Graphs W_n :

Let P_m and W_n be the two graphs of order m and n. The product P_m $\odot$ W_n is defined as the graph obtained from P_m and W_n by taking one copy of P_m and m copies of W_n and joining by an edge each vertex from the i-th copy of W_n with the i-th vertex of P_m .

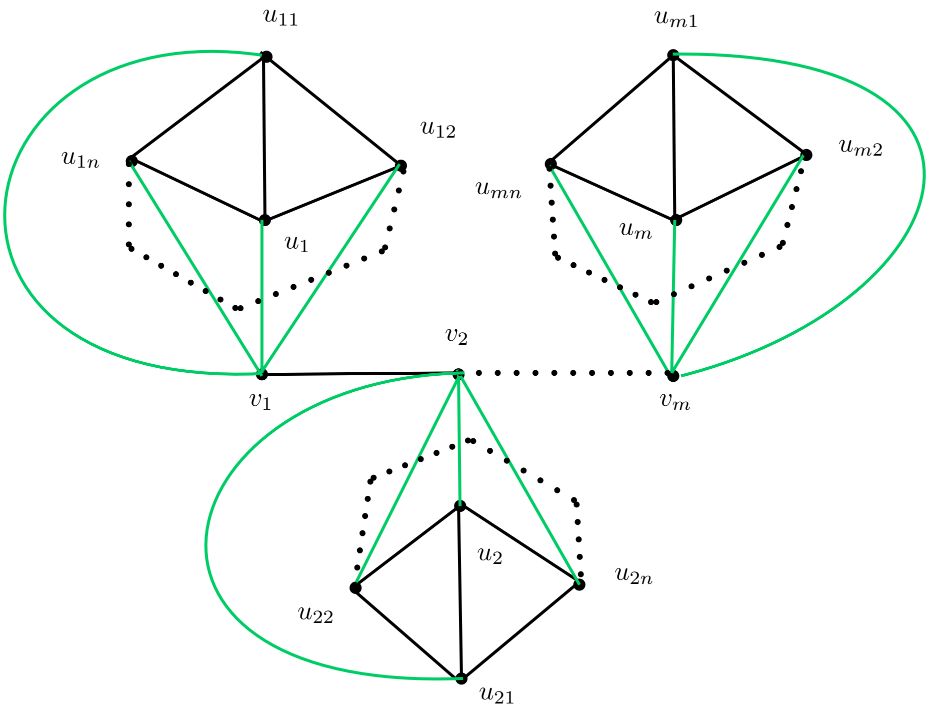

Figure:4.3 Labeling of $P_m \odot W_n$

4.1.1 Characteristics of $P_m \odot W_n$:

i)V($P_m \odot W_n$)={ v_i ;1 $\leq$ i $\leq$ m }$\cup$ { u_i ;1 $\leq$ i $\leq$ m}

$\cup$ { u_{ij} ;1 $\leq$ i $\leq$ m,1 $\leq$ j $\leq$ n}

ii)E($P_m \odot W_n$)={ v_i v_{i+1} ;1 $\leq$ i $\leq$ m-1 }$\cup$ { u_i u_{ij} ;1 $\leq$ i $\leq$ m,1 $\leq$ j $\leq$ n}

$\cup$ { u_{ij} $u_{i(j+1)}$;1 $\leq$ i $\leq$ m,1 $\leq$ j $\leq$ n} $\cup$ { u_{i1} u_{in} ;1 $\leq$ i $\leq$ m }

$\cup$ { u_i v_i ;1 $\leq$ i $\leq$ m }$\cup$ { v_i u_{ij} ;1 $\leq$i $\leq$ m,1 $\leq$ j $\leq$ n }

iii) | V($P_m \odot W_n$) | =m(2+n)

iv) | E($P_m \odot W_n$) | =m(3n+2)-1

v)The maximum degree = n+3

vi)The minimum degree = 4

vii)The number of vertices having maximum degree = m-2

viii)The number of vertices having minimum degree = mn

4.1.2 Theorem:

Corona product graph $P_m \odot W_n$ is a lucky edge mean graphs when m,n $\geq$ 3.

Proof:

For m,n $\geq$ 3,

Define f:V($P_m \odot W_n$) $\rightarrow$ N

The vertex labels are defined as,

$$f(v_i) = \begin{cases} 2; i \equiv 0(mod4) \\ 3; i \equiv 1(mod4) \\ 1; i \equiv 2,3(mod4) \end{cases}$$

f(u_1)=3 , f(u_2)=4

For m=3 , f(u_m)=4

For m $\geq$ 4

m $\equiv$ 0,1,3(mod4) , 3 $\leq$ i $\leq$ m

$$f(u_i) = \begin{cases} 5; i \equiv 0,3(mod4) \\ 4; i \equiv 1,2(mod4) \end{cases}$$

m $\equiv$ 2(mod4) , 3 $\leq$ i $\leq$ m-1

$$f(u_i) = \begin{cases} 5; i \equiv 0,3(mod4) \\ 4; i \equiv 1,2(mod4) \end{cases}$$

For i=m , f(u_m)=3

f(u_{ij})=f(u_i)+j+1; j is odd

f(u_{ij})=f(u_i)-j+12; j is even

Define the induced function :

f* :E(P_m $\odot$ W_n) $\rightarrow$ N

f* (uv)= $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$, for every uv $\in$ E(P_m $\odot$ W_n)

If n is odd ,

f* (u_i u_{ij})= $\begin{cases} f(u_i) + \frac{1+j}{2}; j \ is \ odd \\ f(u_i) + \frac{12-j}{2}; j \ is \ even \end{cases}$

If n is even ,

f* (u_i u_{ij})= $\begin{cases} f(u_i) + \frac{1+j}{2}; j \ is \ odd \\ f(u_i) + \frac{14-j}{2}; j \ is \ even \end{cases}$

If n is odd,

$1 \leq i \leq m, 1 \leq j \leq n-1$

f* (u_{ij} $u_{i(j+1)}$)= $\begin{cases} f(u_i) + 6; j \ is \ odd \\ f(u_i) + 7; j \ is \ even \end{cases}$

If n is even,

$1 \leq i \leq m, 1 \leq j \leq n-1$

f* (u_{ij} $u_{i(j+1)}$)= $\begin{cases} f(u_i) + 7; j \ is \ odd \\ f(u_i) + 8; j \ is \ even \end{cases}$

f* (u_{i1} u_{in})= $\begin{cases} f(u_i) + 4; j \ is \ odd \\ f(u_i) + 5; j \ is \ even \end{cases}$

f* (u_1 v_1)=2

f* (u_2 v_2)=3

For m=3 , f* (u_m v_m)=3

For m $\geq$ 4 , 3 $\leq$ i $\leq$ m

m$\equiv$0,1,3(mod4)

f* (u_i v_i)= $\begin{cases} 4; i \equiv 0,3(mod4) \\ 3; i \equiv 1,2(mod4) \end{cases}$

m$\equiv$ $2(mod4)$, $1 \leq i \leq m-1$

f* (u_i v_i)= $\begin{cases} 4; i \equiv 0,3(mod4) \\ 3; i \equiv 1,2(mod4) \end{cases}$

For i=m , f* (u_m v_m)=2

If n is odd,

1 $\leq i \leq m, 1 \leq j \leq$ n

f* (v_i u_{ij})= $\begin{cases} f(u_i) + \frac{j-1}{2}; j\ is\ odd \\ f(u_i) + \frac{10-j}{2}; j\ is\ even \end{cases}$

If n is even,

1 $\leq i \leq m, 1 \leq j \leq$ n

f* (v_i u_{ij})= $\begin{cases} f(u_i) + \frac{j-1}{2}; j\ is\ odd \\ f(u_i) + \frac{12-j}{2}; j\ is\ even \end{cases}$

It is clear that lucky edge mean labeling of P_m $\odot$ W_n is

{1,2,3, . . . ,m+n+3} , 3 $\leq$ m $\leq$ 4

{1,2,3, . . . ,7+n} , m $\geq$ 5

Hence P_m $\odot$ W_n is a lucky edge mean graph.

Here , $\eta_M^{'}$ (P_m $\odot$ W_n)= $\begin{cases} m+n+3; 3 \ \leq m \ \leq \ 4 \\ 7+n; m \ \geq \ 5 \end{cases}$

4.1.3 Illustration:

Lucky edge mean labeling of $P_3 \odot W_5$ is shown.

The lucky edge number of $P_3 \odot W_5$ is 11.

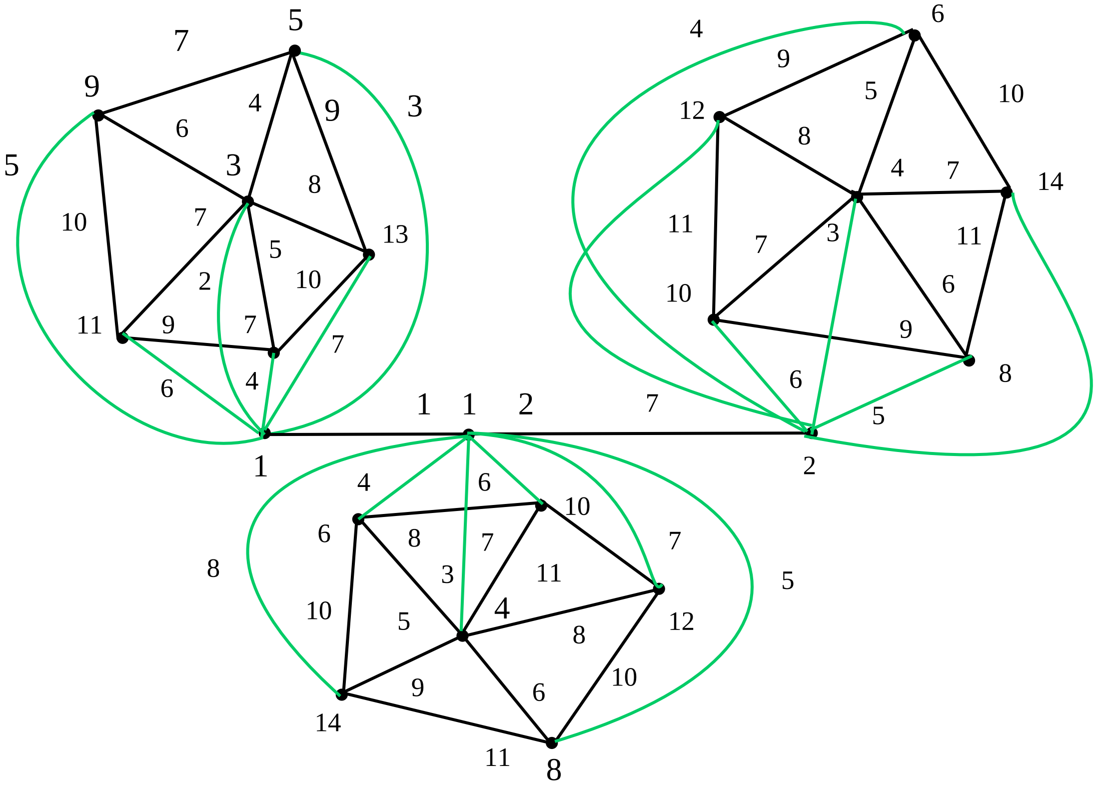

Figure:4.4 Lucky edge mean labeling of $P_3 \odot W_5$

4.2 Corona Product of Path Graphs with Complete Graphs ($P_m \odot K_n$)

Let P_m be the path graph of order m

V(P_m)={v_i;1 $\leq$ i $\leq$ m}

E(P_m)={ v_i v_{i+1} ;1 $\leq$ i $\leq$ m-1}

Figure:4.5 Labeling of P_m

Let K_n be the complete graph of order n

V(K_n)={ u_j ;1 $\leq$ j $\leq$ n}

E(K_n)={ u u_j ;1 $\leq$ j $\leq$ n} $\cup$ { u_j u_{j+1} ;1 $\leq$ j $\leq$ n-1} $\cup$ { u_n u_1 }

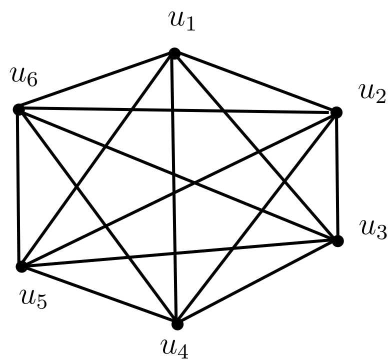

Figure:4.6 Labeling of K_6

Corona product of Path Graphs P_m with Complete Graphs K_n :

Let P_m and K_n be the two graphs of order m and n. The product $P_m \odot K_n$ is defined as the graph obtained from P_m and K_n by taking one copy of P_m and m copies of K_n and joining by an edge each vertex from the i-th copy of K_n with the i-th vertex of P_m .

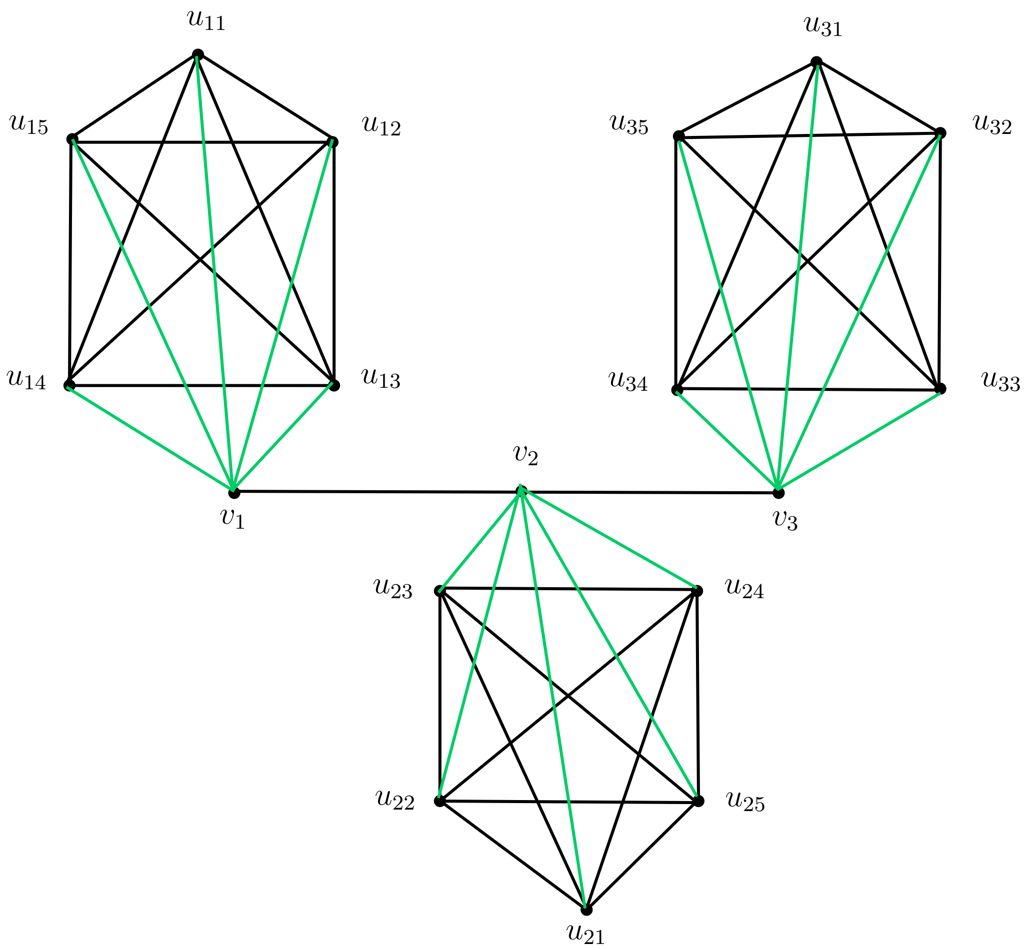

Figure:4.7 Labeling of $P_3 \odot K_5$

4.2.1 Characteristics of $P_m \odot K_n$:

i)V($P_m \odot K_n$)={ v_i ;1 $\leq$ i $\leq$ m}$\cup${ v_{ij} ;1 $\leq$ i $\leq$ m , 1 $\leq$ j $\leq$ n}

ii)E($P_m \odot K_n$)={ v_i v_{i+1} ;1 $\leq$ i $\leq$ m-1}$\cup${ v_i v_{ij} ;1 $\leq$ i $\leq$ m , 1 $\leq$ j $\leq$ n}

$\cup${ v_{ij} v_{ik} ;1 $\leq$ i $\leq$ m , 1 $\leq$ j $\leq$ n , j+1 $\leq$ k $\leq$ n}

iii)| V($P_m \odot K_n$) |=m(n+1)

iv)| E($P_m \odot K_n$) |= $\frac{m}{2}$ [2+ $n^2 + n$]-1 v)The maximum degree = n+2

vi)The minimum degree = n

vii)The number of vertices having maximum degree = m-2

viii)The number of vertices having minimum degree = mn

4.2.2 Theorem:

Corona product graph $P_m \odot K_n$ is a lucky edge mean graphs when m,n $\geq$ 3.

Proof:

For m,n $\geq$ 3

Define f:V($P_m \odot K_n$) $\rightarrow$ N

The vertex labels are defined as,

$$f(v_i)=\begin{cases} 3; i \equiv 0(mod4) \\ 1; i \equiv 1,2(mod4) \\ 2; i \equiv 3(mod4) \end{cases}$$

f(v_{1j})=2j+1, 1 $\leq$ j $\leq$ n

f(v_{2j})=2j+2, 1 $\leq$ j $\leq$ n

For m=3 , f(v_{3j})=2j+2, 1 $\leq$ j $\leq$ n

For m $\geq$ 4

m $\equiv$ 0,1,3(mod4) , 3 $\leq$ i $\leq m$

$$f(v_{ij})=\begin{cases} 2j+3;\ i \equiv 0,3(mod4) \\ 2j+2;\ i \equiv 1,2(mod4) \end{cases}$$

m $\equiv$ 2(mod4) , 3 $\leq$ i $\leq$ m-1 f(v_{ij})= $\begin{cases} 2j+3; \ i \equiv 0,3(mod4) \\ 2j+2; \ i \equiv 1,2(mod4) \end{cases}$

f(v_{mj})=2j+1 ; 1 $\leq$ j $\leq$ n

Define the induced function:

f*:E(P_m $\odot$ K_n) $\rightarrow$ N ,

f*(uv)= $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$, for every uv $\in$ E(P_m $\odot$ K_n)

For 1$\leq$i$\leq$m-1

f*(v_i v_{i+1})= $\begin{cases} 3; i \equiv 0(mod4) \\ 1; i \equiv 0(mod4) \\ 2; i \equiv 1,3(mod4) \end{cases}$

f*(v_1 v_j)=j+1; 1 $\leq$ j $\leq$ n

f*(v_2 v_j)=j+2; 1 $\leq$ j $\leq$ n

For m=3, f*(v_3 v_j)=j+2;1 $\leq$ j $\leq$ n

For m$\geq$4

m$\equiv$0,1,3(mod4),

3 $\leq$ i $\leq$ m , 1 $\leq$ j $\leq$ n

f*(v_i v_{ij})= $\begin{cases} j+3; i \equiv 0,3(mod4) \\ j+2; i \equiv 1,2(mod4) \end{cases}$

m$\equiv$2(mod4),

3 $\leq$ i $\leq$ m-1 , 1 $\leq$ j $\leq$ n

f*(v_i v_{ij})= $\begin{cases} j+3; i \equiv 0,3(mod4) \\ j+2; i \equiv 1,2(mod4) \end{cases}$

For i=m ,f*(v_m v_{mj})=j+1 , 1 $\leq$ j $\leq$ n

f*(v_{1j} v_{1k})=k+j+1 , 1 $\leq$ j $\leq$ n , j+1 $\leq$ k $\leq$ n f* (v_{2j} v_{2k})=k+j+2 , 1 $\leq$ j $\leq$ n , j+1 $\leq$ k $\leq$ n

For m=3, f* (v_{3j} v_{3k})=k+j+2 , 1 $\leq$ j $\leq$ n , j+1 $\leq$ k $\leq$ n

For m$\geq$4 , 3 $\leq$ i $\leq$ m

m$\equiv$0,1,3(mod4)

f* (v_{ij} v_{ik})= $\begin{cases} k+j+3, i \equiv 0,3(mod4) \\ k+j+2, i \equiv 1,2(mod4) \end{cases}$

m$\equiv$2(mod4), 3$\leq$i$\leq$m

f* (v_{ij} v_{ik})= $\begin{cases} k+j+3, i \equiv 0,3(mod4) \\ k+j+2, i \equiv 1,2(mod4) \end{cases}$

For i=m , f* (v_{mj} v_{mk})=k+j+1

It is clear that lucky edge mean labeling of $P_m \odot K_n$ is,

{1,2,3, . . . ,2n+1} , m,n=3

{1,2,3, . . . ,2n+2} , m,n$\geq$4

Hence $P_m \odot K_n$ is a lucky edge mean graph.

Hence , $\eta_M^{'}$ ($P_m \odot K_n$)= $\begin{cases} 2n+1; m,n=3 \\ 2n+2; m,n \geq 4 \end{cases}$

4.2.3 Illustration

Lucky edge mean labeling of $P_3 \odot K_4$ is shown.

The lucky edge mean number of $P_3 \odot K_4$ is 9.

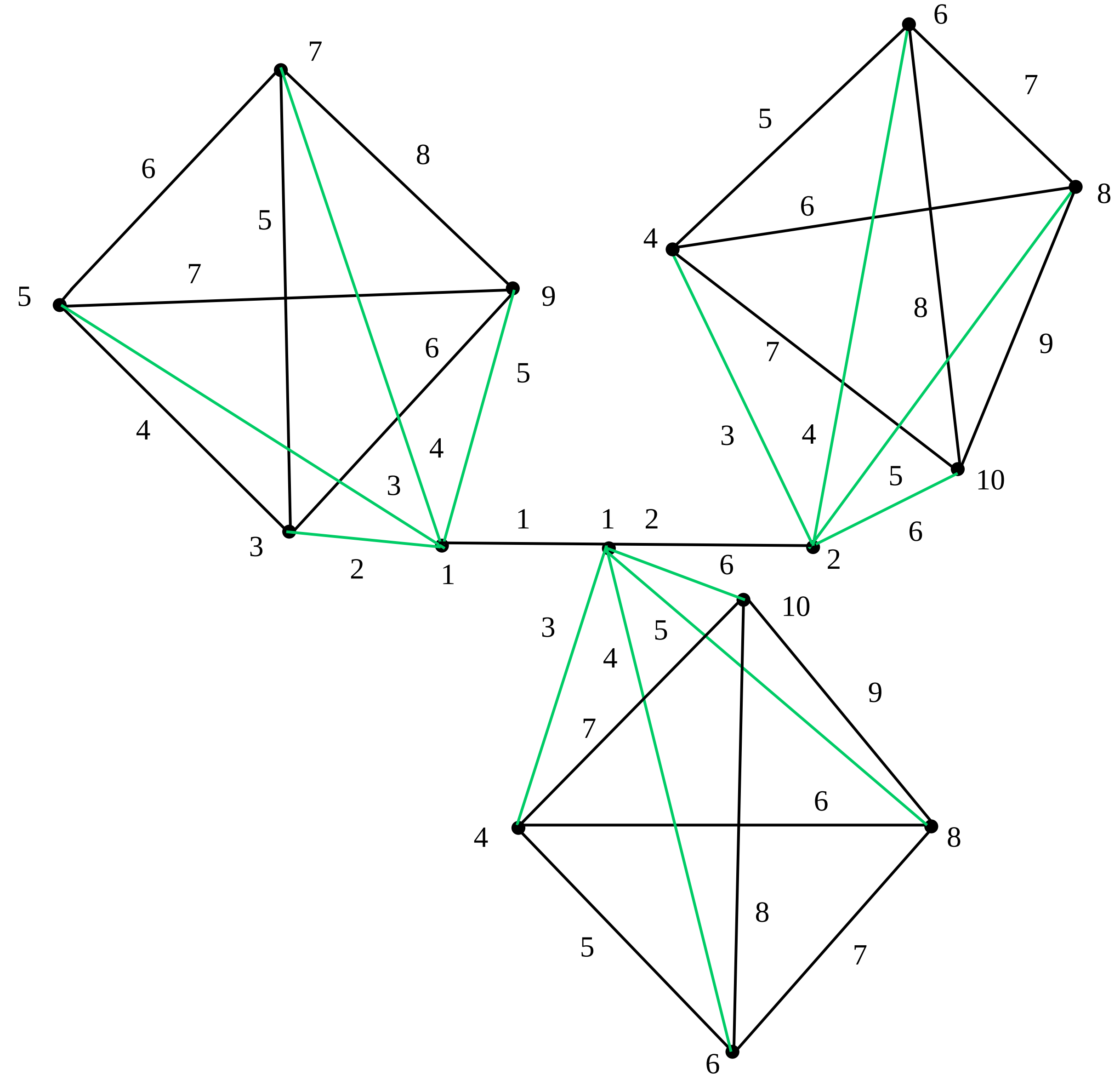

Figure:4.8 Lucky edge mean labeling of $P_3 \odot K_4$

Conclusion

In this project, we computed the lucky edge mean number of Corona product of path graphs with path graphs, star graphs, wheel graphs and complete graphs are lucky edge mean graphs. In future we can compute lucky edge geometric and harmonic mean number of various product graphs.

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