HW 4 acm220004

1) $\sum_{k=1}^{n-1} P(k)P(n-k) = \Omega(2^{n-2})$ ← big O & little o

Guess: $\sum_{k=1}^{n-1} P(k)P(n-k) \geq 2^{n-2}$

$\sum_{k=1}^{n-1} P(k)P(n-k) \geq c \cdot 2^{n-2}$

Basis: $n=5 \rightarrow P(1)P(4) + P(2)P(3) + P(3)P(2) + P(4)P(1)$
$= 5+6+6+5 = 32$ & $2^3 = 8$
$32 \geq 8$ ✓

IH: $\sum_{k=1}^{x-1} P(k)P(x-k) \geq c \cdot 2^{n-2}$ $c=1$, $n \in \mathbb{R}$ & $1 \leq k < n$

↓ (plug in 2^{k-2})

IS: $\sum_{k=1}^{n-1} (2^{k-2})(2^{n-k-2}) = \sum_{k=1}^{n-1} 2^{k-2+n-k-2} = 2^{n-4}$

$= 2^{n-4} \sum_{k=1}^{n-1} 1 = 2^{n-4}(n-1)$ as long as $n \geq 5$, $n-4 \geq 1$

meaning 2^{n-4} is positive $\leftarrow \sum_{k=1}^{n} P(k)P(n-k) \geq P 2^{n-2}$

by using induction, we proved that $\sum_{k=1}^{n-1} P(k)P(n-k) = \Omega(2^{n-2})$

$2^{n-4}(n-1)$ so $2^{n-4}(4) \Rightarrow 2^{n-4} \cdot 2^2 = \underline{2^{n-2}}$

← at least 5

2) $\langle a, b, b, a, b, a, b, a \rangle$ & $\langle b, a, b, a, a, b, a, a, b \rangle$

2D

	0	b	a	b	a	a	b	a	a	b
0	0	0	0	0	0	0	0	0	0	0
a	0	1	1	1	1	1	1	1	1	1
b	0	1	1	2	2	2	2	2	2	2
b	0	1	1	2	2	2	3	3	3	3
a	0	1	2	2	3	3	3	4	4	4
b	0	1	2	3	3	3	4	4	4	5
a	0	1	2	3	4	4	4	5	5	5
b	0	1	2	3	4	4	5	5	5	6
a	0	1	2	3	4	5	5	6	6	6

max-subsequence

$$\left.\begin{array}{l} \checkmark \langle b, a, b, a, b, a \rangle \\ \checkmark \langle a, b, a, b, a, b \rangle \\ \checkmark \langle a, b, b, a, a, b \rangle \\ \checkmark \langle a, b, a, a, b, a \rangle \\ \checkmark \langle a, b, a, b, a, a \rangle \\ \langle b, b, a, a, b, a \rangle \\ \langle b, b, a, b, a, b \rangle \\ \langle b, b, a, b, a, a \rangle \end{array}\right\} 8 \text{ total}$$