# VIDYAPEETH

**BATCH CODE : 12AJ402 MA**

- Subject Name- Mathematics
- Chapter Name- Trigonometric Equation

Lecture No.- 01.

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# Today's Targets

1. Trigonometric Equation
2. 
3. 
4. 
5. 

# $\sin(A\pm B) = \sin A\cdot\cos B \pm \cos A\cdot\sin B$

$\cos(A\pm B) = \cos A\cdot\cos B \mp \sin A\sin B$

# $\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B$

$\cos(A+B)\cdot\cos(A-B) = \cos^2 A - \sin^2 B$

# $\sin\theta\cdot\sin(60-\theta)\cdot\sin(60+\theta) = \frac{1}{4}\sin 3\theta$

$\cos\theta\cdot\cos(60-\theta)\cdot\cos(60+\theta) = \frac{1}{4}\cos 3\theta$

$\tan\theta\cdot\tan(60-\theta)\cdot\tan(60+\theta) = \tan 3\theta$

$\boxed{60 \longleftrightarrow 120^\circ}$

***

# $2\sin A\cdot\cos B = \sin(A+B)+\sin(A-B)$

$2\cos A\cdot\sin B = \sin(A+B)-\sin(A-B)$

$2\cos A\cdot\cos B = \cos(A+B)+\cos(A-B)$

$2\sin A\cdot\sin B = \cos(A-B)-\cos(A+B)$

# $\sin(C)\pm\sin D = 2\sin\left(\frac{C\pm D}{2}\right)\cdot\cos\left(\frac{C\mp D}{2}\right)$

$\cos(C)+\cos D = 2\cos\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)$

$\cos(C)-\cos D = -2\sin\left(\frac{C+D}{2}\right)\cdot\sin\left(\frac{C-D}{2}\right)$

$\tan(A\pm B) = \dfrac{\tan A\pm\tan B}{1\mp\tan A\cdot\tan B}$

$\cot(A\pm B) = \dfrac{\cot A\cdot\cot B - 1}{\cot B\pm\cot A}$

$\tan(A_1+A_2\cdots A_n) = \dfrac{S_1-S_3\cdots}{1-S_2+S_4\cdots}$

$\sin(2A) = 2\sin A\cos A = \dfrac{2\tan A}{1+\tan^2 A}$

$\cos(2A) = \cos^2 A-\sin^2 A = 1-2\sin^2 A$

$= 2\cos^2 A - 1 = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

$\tan 2A = 2\tan A / 1-\tan^2 A$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

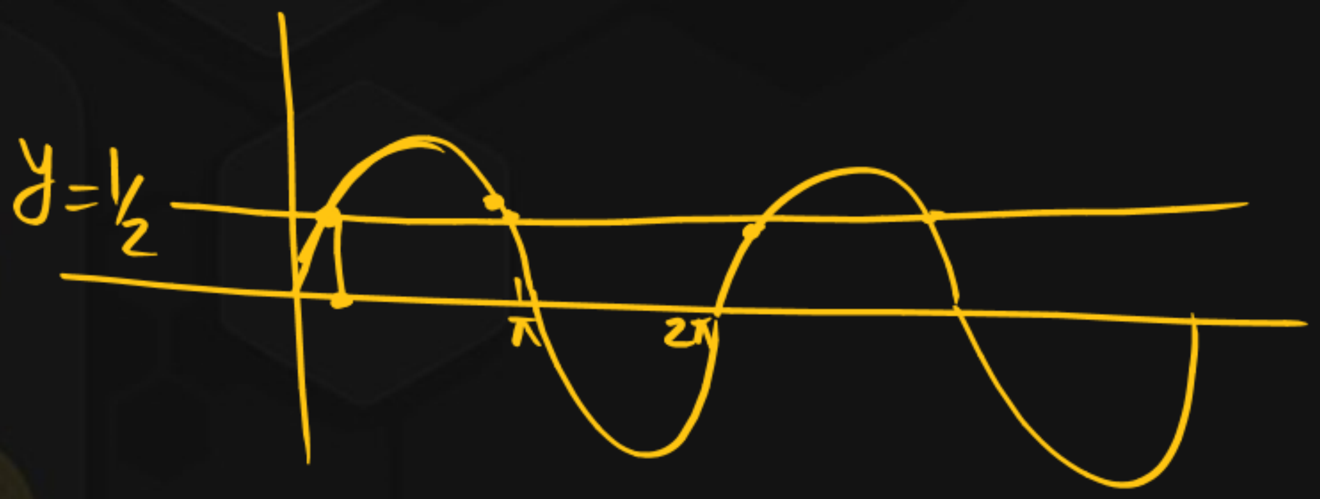
$\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

Angle AP →

$\dfrac{\sin\left(\frac{nd}{2}\right)}{\sin(d/2)} \begin{matrix}(\sin)\\(\cos)\end{matrix}\left[\dfrac{1^{st}+last}{2}\right]$

# $\dfrac{\sin(2^n\theta)}{2^n\sin\theta}$

$$\boxed{y_1 = y_2}$$




$\sin x = \frac{1}{2}$

$x = 30^\circ,$

$y_1 = \sin x \quad y_2 = \frac{1}{2}$

$x^2 + 3x + 2 = 0.$

$(x+2)(x+1) = 0$

$x+2=0 \;/\; x+1=0$

$x=-2 \;/\; x=-1$

# Trigonometric Equation

An equation involving one or more trigonometrical ratios of unknown angles.

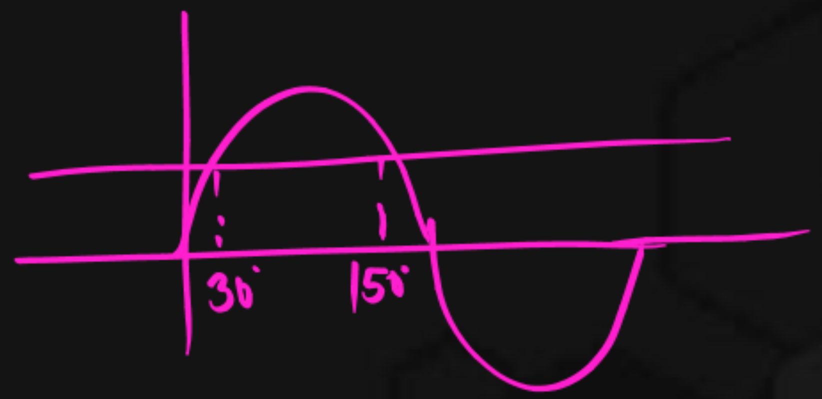
Eg.: $\sin\theta = \frac{1}{2}$ $\qquad$ $\sin\theta + \cos\theta = \sqrt{2}$

$\sin x = 1/2$

## ❖ Solution of Trigonometric Equation:

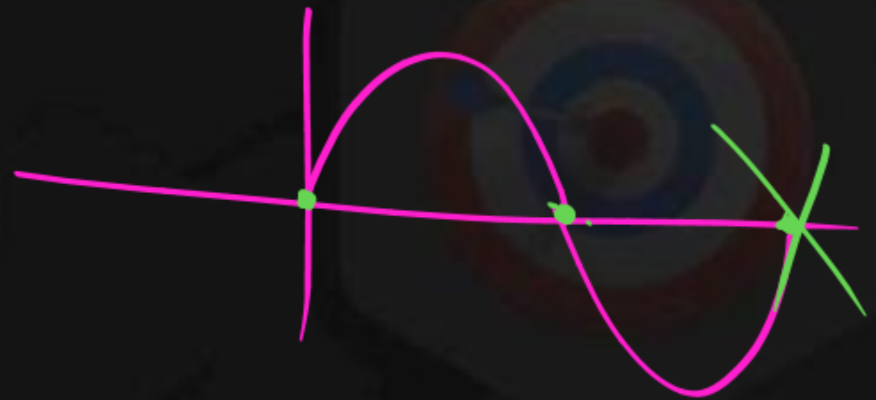
(a) Principal solution: $[0, 2\pi)$

$\sin x = \frac{1}{2}$




(b) General Solution: Compact form (all solution)
(Use of set.)

$\sin\theta = 0$

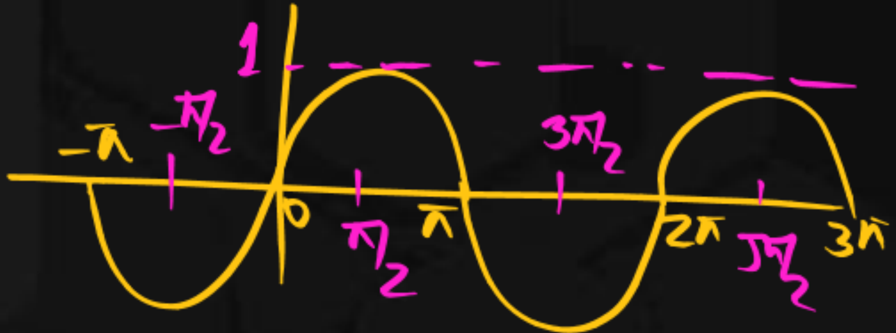


(c) Particular solution: $[a, b]$

## ❖ General Solution

If I got general solution, then automatically got the all principal & particular solution.

$\boxed{\sin\theta = 0}$




$$\boxed{\theta = n\pi \; ; n \in I}$$

$\sin\theta = 1$

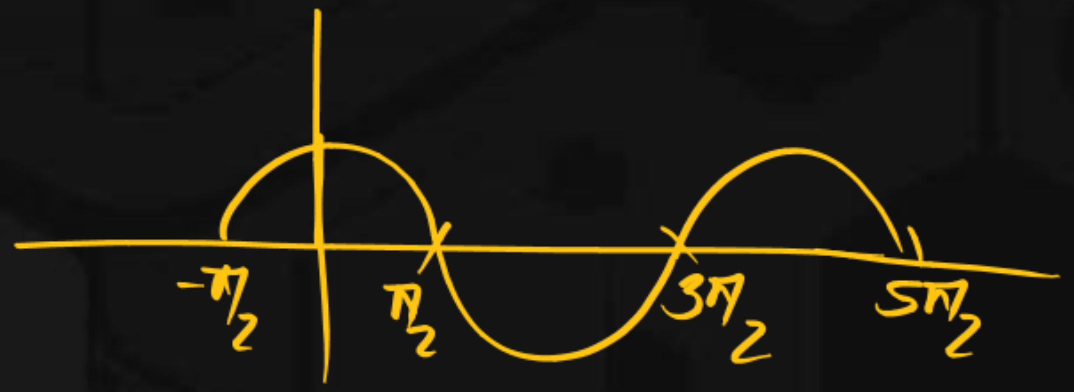
$$\boxed{\theta = (4n+1)\frac{\pi}{2}}$$

$n \in I$

$\boxed{\sin\theta = -1}$

$$\boxed{\theta = (4n-1)\frac{\pi}{2}, \; n \in I}$$

$$\boxed{\cos\theta = 0}$$




$$\theta = \pi/2,\ 3\pi/2,\ 5\pi/2$$

$$\theta = (2n+1)\pi/2 \ ;\ n \in I.$$

$\tan\theta = 0$

$\frac{\sin\theta}{\cos\theta} = 0$

$\sin\theta = 0.$

$\boxed{\theta = n\pi}$, $n \in I.$

# $\sin\theta = 0^\circ \longrightarrow \theta = n\pi$ $\quad n \in I.$

$\tan\theta = 0^\circ$

# $\cos\theta = 0^\circ$ $\quad \theta = (2n+1)\frac{\pi}{2}$, $n \in I.$

## ❖ NOTE

For cot, sec, and cosec, we convert easily in the form of sin, cos and tan and then make the solution.

**Question**

$n \in I$

Find general solution:

$\sin 3\theta = 0$

$3\theta = n\pi$

$\theta = \left(\frac{n\pi}{3}\right), n \in I$

$\tan\theta/3 = 0$

$\frac{\theta}{3} = n\pi$

$\theta = 3n\pi$

$\cos 4\theta = 0$

$4\theta = (2n+1)\pi/2$

$\theta = (2n+1)\pi/8$

$\text{Sin}(x+1) = 0$

$x+1 = n\pi$

$x = (n\pi - 1)$

$\cos(x+1) = 0$

$x+1 = (2n+1)\pi/2$

$x = (2n+1)\pi/2 - 1$

$\tan(x+1) = 0$

$x+1 = n\pi$

$x = n\pi - 1$

❖ General solution of $\boxed{\sin\theta = \sin\alpha}$


$$\sin\theta - \sin\alpha = 0$$

$$2\sin\left(\frac{\theta-\alpha}{2}\right)\cdot\cos\left(\frac{\theta+\alpha}{2}\right) = 0$$

$$\sin\left(\frac{\theta-\alpha}{2}\right) = 0 \qquad\qquad \cos\left(\frac{\theta+\alpha}{2}\right) = 0$$

$$\frac{\theta-\alpha}{2} = n\pi \qquad\qquad \frac{\theta+\alpha}{\cancel{2}} = (2n+1)\frac{\pi}{\cancel{2}}$$

$$\boxed{\theta = 2n\pi + \alpha} \qquad\qquad \boxed{\theta = (2n+1)\pi - \alpha}$$

↳ even $\pi + \alpha$ $\qquad$ (odd $\pi - \alpha$)

$$\boxed{\theta = n\pi + (-1)^n\alpha} \; ; \; \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

| | |
|---|---|
| $2\pi + \alpha$ | $\pi - \alpha$ |
| $4\pi + \alpha$ | $3\pi - \alpha$ |
| $6\pi + \alpha$ | $5\pi - \alpha$ |
| $8\pi + 1$ | $7\pi - \alpha$ |
| ⋮ | ⋮ |

Question

2sinx = 1. Find principal, general $ [0, 4π], solution.

$\sin x = \frac{1}{2}$

$\boxed{\sin x = \sin \pi/6}$

$\boxed{\frac{1}{2} = \sin \pi/6}$

$\boxed{\sin x = \sin \alpha}$

$\boxed{\alpha = \pi/6}$

$x = n\pi + (-1)^n \alpha \; ; \; n \in I$

Ans. $\boxed{x = n\pi + (-1)^n \pi/6 \; ; \; n \in I}$

# Principal solⁿ →

$x = \pi/6, \frac{5\pi}{6}, \cancel{2\pi + \pi/6}$

# Particular solⁿ ⇒ $[0, 4\pi]$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ Ans

**Question**

$2\cos^2\theta + 3\sin\theta = 0$

$2(1-\sin^2\theta) + 3\sin\theta = 0$

$2 - 2\sin^2\theta + 3\sin\theta = 0$

$2\sin^2\theta - 3\sin\theta - 2 = 0$

$2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$

$2\sin\theta(\sin\theta - 2) + 1(\sin\theta - 2) = 0$

$(2\sin\theta + 1)(\sin\theta - 2) = 0$

$\sin\theta = -\frac{1}{2}$

$\sin\theta = \sin\left(-\frac{\pi}{6}\right)$

$\alpha = -\pi/6$

$\theta = n\pi + (-1)^n\left(-\pi/6\right);\ n \in I$

Ans

$\cancel{\sin\theta = 2}$

❖ General solution of $\boxed{\cos\theta = \cos\alpha}$ ✓ $\alpha \in [0, \pi]$

$\cos\theta - \cos\alpha = 0$

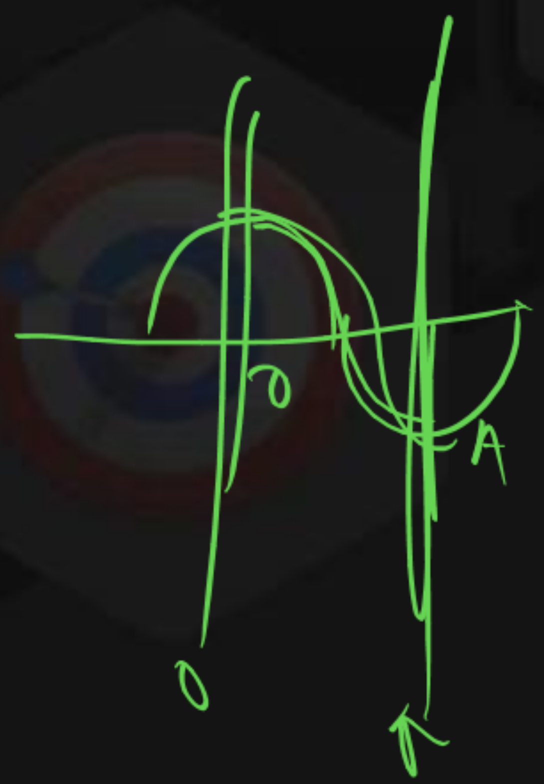
$-2\sin\left(\frac{\theta+\alpha}{2}\right) \cdot \sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\sin\left(\frac{\theta+\alpha}{2}\right) = 0$ or $\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\frac{\theta+\alpha}{2} = n\pi$ $\qquad$ $\frac{\theta-\alpha}{2} = n\pi$

$\theta = (2n\pi - \alpha)$ $\qquad$ $\theta = 2n\pi + \alpha$

$\boxed{\theta = 2n\pi \pm \alpha}$ ✓ $, n \in I, \alpha \in [0, \pi]$



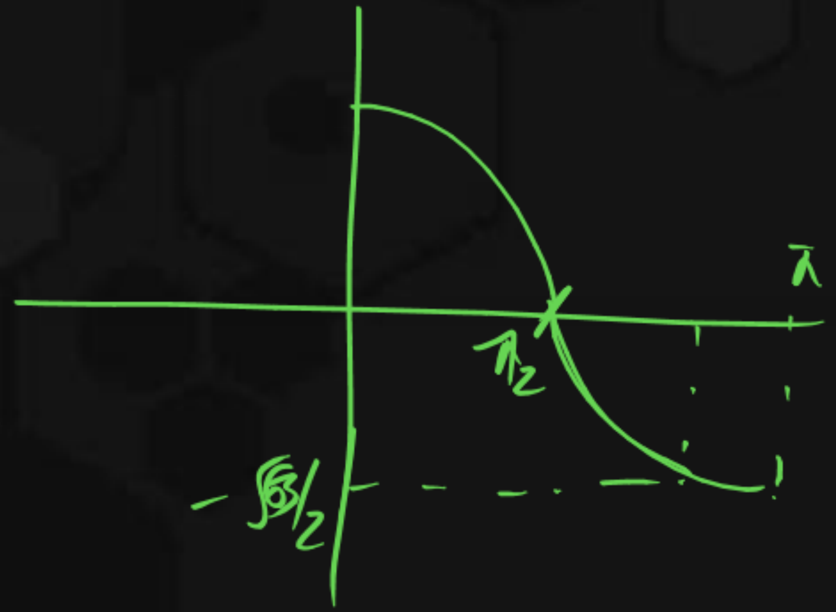
## Question

$2\cos x + \sqrt{3} = 0$

$\cos x = -\dfrac{\sqrt{3}}{2}$

$\boxed{\cos x = \cos \alpha}$

$\alpha = \left(\dfrac{5\pi}{6}\right)$




$x = 2n\pi \pm \alpha$

$\boxed{x = 2n\pi \pm 5\pi/6}, \; n \in I$

Ans.

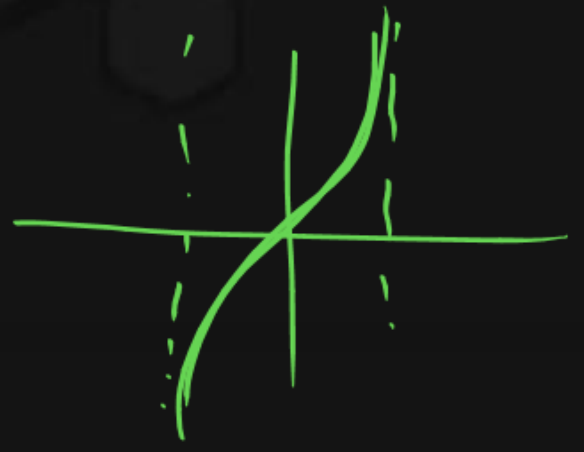
Question

$$\sec\theta = \frac{2}{\sqrt{3}}$$

H.W.

$\cos\theta = \sqrt{3}/2$

❖ General solution of $\boxed{\tan\theta = \tan\alpha}$



$\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} - \dfrac{\sin\alpha}{\cos\alpha} = 0.$

$\Rightarrow \dfrac{\sin\theta\cos\alpha - \sin\alpha\cdot\cos\theta}{\cos\theta\cdot\cos\alpha} = 0.$

$\Rightarrow \dfrac{\sin(\theta-\alpha)}{\cos\theta\cdot\cos\alpha} = 0$

$\Rightarrow \sin(\theta-\alpha) = 0.$

$\theta - \alpha = n\pi$

$\boxed{\theta = n\pi + \alpha}, \ n \in I.$

# (1) $\sin\theta = \sin\alpha$

$\downarrow$

$\theta = n\pi + (-1)^n \alpha$

$\downarrow$

$\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(2) $\cos\theta = \cos\alpha$

$\downarrow$

$\theta = (2n\pi \pm \alpha)$

$\downarrow$

$\alpha \in [0, \pi]$

(3) $\tan\theta = \tan\alpha$

$\downarrow$

$\theta = (n\pi + \alpha)$

$\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Question

$\tan 3x = 1$

$\tan\theta = \tan\alpha$

$\tan\alpha = 1$

$\boxed{\alpha = \pi/4}$

$\tan 3x = \tan \pi/4$

$3x = n\pi + \pi/4$

$x = \left(\frac{n\pi}{3} + \frac{\pi}{12}\right) \forall n,\ n \in I.$

Question

$$\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta \cdot \tan 4\theta \cdot \tan 7\theta$$

$\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C.$

$A + B + C = n\pi$

$\theta + 4\theta + 7\theta = n\pi$

$12\theta = n\pi$

$\theta = \dfrac{n\pi}{12}\ ;\ n \in I.$

Ans

## ❖ General solution of the Trigonometric equation.

$\Rightarrow \boxed{\sin^2\theta = \sin^2\alpha}$

$\Rightarrow \frac{\cancel{1}-\cos 2\theta}{\cancel{2}} = \frac{\cancel{1}-\cos 2\alpha}{\cancel{2}}$

$\cos(2\theta) = \cos(2\alpha)$.

$2\theta = 2n\pi \pm 2\alpha$

$\Rightarrow \boxed{\theta = n\pi \pm \alpha}$.

$\Rightarrow \boxed{\tan^2\theta = \tan^2\alpha}$

$\Rightarrow 1+\tan^2\theta = 1+\tan^2\alpha$

$\sec^2\theta = \sec^2\alpha$

$\cos^2\theta = \cos^2\alpha$

$\boxed{\cos^2\theta = \cos^2\alpha}$

$\cancel{1}-\sin^2\theta = \cancel{1}-\sin^2\alpha$.

$\sin^2\theta = \sin^2\alpha$.

$$\left.\begin{aligned}\sin^2\theta &= \sin^2\alpha \\ \cos^2\theta &= \cos^2\alpha \\ \tan^2\theta &= \tan^2\alpha\end{aligned}\right\} \Rightarrow \boxed{\theta = n\pi \pm \alpha}\ ,\ n \in I$$

$\sin\theta = \sin\alpha \rightarrow \theta = n\pi + (-1)^n\alpha$ ; $n \in I$. $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos\theta = \cos\alpha \rightarrow \theta = 2n\pi \pm \alpha$ ; $n \in I$. $\alpha \in [0, 2\pi]$

$\tan\theta = \tan\alpha \rightarrow \theta = n\pi + \alpha$ ; $n \in I$. $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

## ❖ NOTE

$$\sin\theta = \sin\alpha \rightarrow \theta = n\pi + (-1)^n\alpha$$

$$\cos\theta = \cos\alpha \rightarrow \theta = 2n\pi \pm \alpha$$

$$\tan\theta = \tan\alpha \rightarrow \theta = n\pi + \alpha$$

$$\left.\begin{matrix} \sin^2\theta = \sin^2\alpha \\ \cos^2\theta = \cos^2\alpha \\ \tan^2\theta = \tan^2\alpha \end{matrix}\right\} \Rightarrow \theta = n\pi \pm \alpha$$

**Question**

$\sin^2\theta = \frac{1}{4}$

$\sin^2\theta = \left(\frac{1}{2}\right)^2$

$\sin^2\theta = \left(\sin \pi/6\right)^2$

$\boxed{\sin^2\theta = \sin^2\alpha}$

$\theta = n\pi \pm \alpha.$

$\alpha = \pi/6$

$\theta = n\pi \pm \pi/6$ Ans

$n \in I.$

**Question**

$2\tan^2\theta = \sec^2\theta$ [redacted]

$2\tan^2\theta = 1 + \tan^2\theta$ $\quad\quad$ $\tan^2\theta = \tan^2\alpha.$

$\tan^2\theta = (1)^2$

$\tan^2\theta = \left(\tan \pi/4\right)^2$

$\alpha = \pi/4.$

$\boxed{\theta = n\pi \pm \pi/4}$ $, n \in I.$

**Question**

$4\cos^2\theta + 6\sin^2\theta = 5$

H.W.

**Question**

$$\frac{1-\cos 2\theta}{1+\cos 2\theta} = 3$$

$$\frac{\cancel{2}\sin^2\theta}{\cancel{2}\cos^2\theta} = 3$$

$$\tan^2\theta = 3$$

$$\tan^2\theta = (\sqrt{3})^2$$

$$\tan^2\theta = \left(\tan\frac{\pi}{3}\right)^2$$

$$\alpha = \frac{\pi}{3}$$

$$\theta = n\pi \pm \frac{\pi}{3} \quad n \in I$$

Ans

**Question**

$$4\tan^2\theta = 3\sec^2\theta$$

H.W.

## Question

Principal solution : $\sin^2 x + 2\tan^2 x + \frac{4}{\sqrt{3}}\tan x - \sin x + \frac{11}{12} = 0$

H.W.

$$\left(\sin^2 x - \sin x\right) + 2\left(\tan^2 x + \frac{2}{\sqrt{3}}\tan x\right) + \frac{11}{12} = 0.$$

**Question**

$$\frac{\tan 3x - \tan 2x}{1 + \tan 3x \cdot \tan 2x} = 1$$

H.W.

## Question

Find principal solution of $3\tan(\theta - 15) = \tan(\theta + 15)$.

H.W.

**Question**

Find number of solution of $8\tan^2\frac{x}{2} = 1 + \sec x$ in $[-2\pi, 3\pi]$.

H.W.

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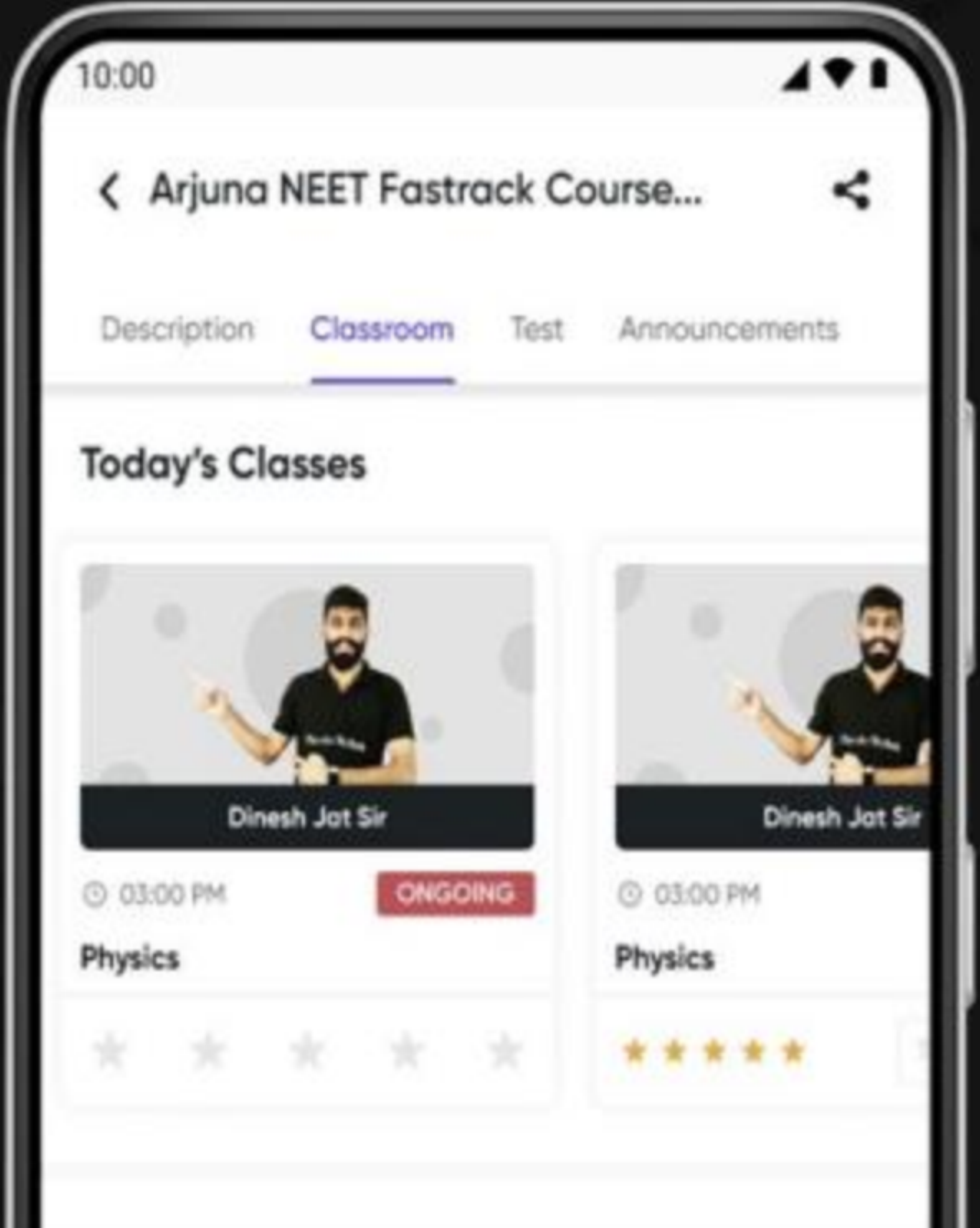


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