

Computer Problems. Approximation

Linear Squares Fit

1) Fit the following data

x	1	1.5	2	2.5	3	3.5	4
y	25	31	27	28	36	35	32

Give also the value of the error of the fit.

2) The following table lists the temperatures of a room recorded during the time interval $[1, 7]$. Find the best linear least squares that approximate the table.

x	1	2	3	4	5	6	7
y	13	15	20	14	15	13	10

Use your result to predict the temperature of the room at 8.

3) Suppose we consecutively apply forces of 1, 2, 3 and 4 Kg to a spring and find that its displacements are:

x	1	2	3	4
d	3.3	6.1	9.8	12.3

Hooke's law states that the force F and the displacement x are related by the linear function $F = -kx$, where $k > 0$ is a constant characteristic of the spring, called the **spring constant**. Determine k from the experimental data.

Polynomial Least Squares Fit

1) Fit polynomials up to degree $n = 5$ to the function $f(x) = \cos(2\pi x)$ in the interval $[0, 1]$.

2) Fit polynomials up to degree $n = 3$ to the following table:

x	1	2	3	4	5	6	7	8	9	10
y	1.3	3.5	4.2	5.0	7.0	8.8	10.1	12.5	13.0	15.6

Compute also the sum of the root of square differences between the data and the polynomial fit.

3) To determine the relationship between the number of fish and the number of species of fish in samples taken for a portion of the Great Barrier Reef, P. Sale and DR. Dybdahl fit a linear least squares polynomial to the following collection of data, which are collected in samples over a 2-year period. Let x be the number of fish in the sample and y be the number of species in the sample.

x	y	x	y	x	y
13	11	29	12	60	14
15	10	30	14	62	21
16	11	31	16	64	21
21	12	36	17	70	24
22	12	40	13	72	17
23	13	42	14	100	23
25	13	55	22	130	34

Compare the polynomial least square fits for these data.

4) Compare the polynomial fits of orders $n = 1$ to $n = 5$ for the nodes

$$x \quad 0.25 \quad 1.0 \quad 1.5 \quad 2.0 \quad 2.4 \quad 5.0$$

of the function

$$f(x) = \frac{1.44}{x^2} + 0.24x$$

Nonlinear Models

1) Find the least squares exponential of the form

$$f(x) = ae^{bx}$$

that best fits the following data

x	1	3	4	6	9	15
y	4.0	3.5	2.9	2.5	2.75	2.0

2) Find the least squares hyperbolic function of the form

$$f(x) = a + \frac{b}{x}$$

that fits the data set of the former problem.

x	1	3	4	6	9	15
y	4.0	3.5	2.9	2.5	2.75	2.0

3) Fit a exponential function of the form

$$f(x) = ae^{-2x} + be^{-3x}$$

to the following data set

x	0.1	0.2	0.3	0.4
y	0.76	0.58	0.44	0.35

Trigonometric Approximation

1) Use the trigonometric basis to make an approximation of the function

$$f(x) = |x|$$

in the interval $[-\pi, \pi]$.

2) Use the trigonometric basis to make an approximation of the function

$$f(x) = 2x^2 - 9$$

in the interval $[-\pi, \pi]$.

3) Use the trigonometric basis to make an approximation of the function

$$f(x) = x^4 - 3x^3 + 2x^2 - \tan(x(x - 2))$$

in the interval $[0, 2]$;