Exercises

In Exercises 1 to 4, evaluate the following integrals if they exist

1. $\displaystyle\iint_D \frac{1}{\sqrt{xy}}\, dA$, where $D = [0, 1] \times [0, 1]$

2. $\displaystyle\iint_D \frac{1}{\sqrt{|x - y|}}\, dx\, dy$, where $D = \{(x, y) \mid 0 \le x \le 1, 0 \le y \le 1, y \le x\}$

3. $\displaystyle\iint_D (y/x)\, dx\, dy$, where D is bounded by $x = 1$, $x = y$, and $x = 2y$

4. $\displaystyle\int_0^1 \int_0^{e^y} \log x\, dx\, dy$

5. Let $D = [0, 1] \times [0, 1]$. Let $0 < \alpha < 1$ and $0 < \beta < 1$. Evaluate:

$$\iint_D \frac{dx\, dy}{x^\alpha y^\beta}.$$

7. (a) Evaluate

$$\iint_D \frac{dA}{(x^2 + y^2)^{2/3}},$$

where D is the unit disk in $\mathbb{R}^2$.

(b) Determine the real numbers λ for which the integral

$$\iint_D \frac{dA}{(x^2 + y^2)^\lambda}$$

is convergent, where again D is the unit disk.

11. Discuss whether the integral

$$\iint_D \frac{x+y}{x^2+2xy+y^2}\,dx\,dy$$

exists where $D = [0, 1] \times [0, 1]$. If it exists, compute its value.

13. Let W be the first octant of the ball $x^2 + y^2 + z^2 \leq a^2$, where $x \geq 0$, $y \geq 0$, $z \geq 0$. Evaluate the improper integral

$$\iiint_W \frac{(x^2+y^2+z^2)^{1/4}}{\sqrt{z+(x^2+y^2+z^2)^2}}\,dx\,dy\,dz$$

by changing variables.

Solutions

1. 4

3. 3/16

5. $\frac{1}{(1-\alpha)(1-\beta)}$

7. (a) 3π

(b) $\lambda < 1$

11. Integrate over $[\varepsilon, 1] \times [\varepsilon, 1]$ and let $\varepsilon \to 0$ to show the improper integral exists and equals $2 \log 2$.

13. $\frac{2\pi}{9}[(1 + a^3)^{3/2} - a^{9/2} - 1]$