EXERCISES - MICROCANONICAL ENSEMBLE

1. Compute the entropy and the equation of state for an ideal gas in two dimensions.

2. Excluded volume effects. Consider a 1-dimensional gas of non-interacting rods of length *R* in a volume of length *L.* Rods do not interact, but they cannot overlap or jump over one another. Compute the entropy and the equation of state.

3. Harmonic oscillators. Consider a collection of N non-interacting classical harmonic oscillators in 3 dimensions with frequency ω and total energy *E*. Compute the entropy *S* and the temperature *T.* Should particles be considered distinguishable or not? What is the expression for the equipartition theorem in this case?

4. Quantum oscillators. The energy values allows for a 1-dimensional quantum oscillator of frequency ω are $E_n = \hbar\omega\left(n + \frac{1}{2}\right) \qquad n = 0,1,2,3,\ldots$

   Consider a collection of N three-dimensional quantum oscillators of frequency ω and total energy *E,* compute the entropy and the temperature, and the dependency of energy with temperature.

5. Consider a model of a crystal which consists on a regular lattice with N sites. At a certain temperature T, $N - n$ sites are occupied by atoms and the remaining n sites are empty (Schottky vacancies). Each empty site (vacancy) increases the energy of the system by $\gamma > 0$.
   a. Compute the concentration of vacancies at equilibrium
   b. Compute the contribution of vacancies to the specific heat and study the behaviour of the system at low temperature.
   Hint: consider N, n large.

6. Consider N identical particles that can have two possible energy levels $\pm\epsilon$. Using the microcanonical ensemble, compute the entropy, the Helmholtz free-energy $F = E - TS$ and the occupation numbers (the expected number of particles in each level $n_{\pm\epsilon}$) as a function of T. What is the highest temperature at which the system can exist?

7. Consider a system of identical particles each o which has two energy levels $0, \epsilon > 0$. The upper energy level has a g-fold degeneracy (that is there are *g* different states with energy ϵ, while the 0 energy state is non-degenerate). The total energy of the system is E.
   A. Using the microcanonical ensemble, find the occupation numbers n_0, n_ϵ (that is the expected number of particles in each level) as a function of the temperature of the system.
   B. Consider the case $g = 2$. If the system has energy $E = 0.75N\epsilon$ and we put it in contact with a bath at constant temperature $T = 500\,K$, in what direction does the heat flow?