Exercicis

37. The magnitude of the gravitational force exerted by a body of mass M situated at the origin on a body of mass m located at the point (x, y, z) is given by the function

$$F(x, y, z) = \frac{GmM}{x^2 + y^2 + z^2}$$

where G is the universal gravitational constant. What are the level surfaces? What is the physical significance of these surfaces?

Les superfícies de força gravitatòria constant són superfícies esfèriques concèntriques, amb centre a l'origen:

$$\frac{GmM}{x^2 + y^2 + z^2} = c \quad \Longrightarrow \quad x^2 + y^2 + z^2 = \frac{GmM}{c}$$

38. The strength E of an electric field at a point (x, y, z) due to an infinitely long charged wire lying along the y-axis is given by the function

$$E(x, y, z) = \frac{k}{\sqrt{x^2 + z^2}}$$

where k is a positive constant. Describe the level surfaces of E.

Són superfícies cilíndriques circulars al voltant de l'eix y:

$$x^2 + z^2 = \frac{k^2}{c^2}$$

39. A metal solid occupies a region in three-space. The temperature T (in °C) at the point (x, y, z) in the solid is inversely proportional to its distance from the origin.

(a) Express T as a function of x, y, z.

(b) Describe the level surfaces and sketch a few of them. NOTE: The level surfaces of T are known as *isothermals*; at all points of an isothermal the temperature is the same.

(c) Suppose the temperature at the point (1, 2, 1) is 50°. What is the temperature at the point (4, 0, 3)?

(a) $T(x,y,z) = \dfrac{k}{\sqrt{x^2+y^2+z^2}}$, where k is a constant.

(b) $\dfrac{k}{\sqrt{x^2+y^2+z^2}} = c \implies x^2+y^2+z^2 = \dfrac{k^2}{c^2}$; the level surfaces are concentric spheres.

(c) $T(1,2,1) = \dfrac{k}{\sqrt{1^2+2^2+1^2}} = 50 \implies k = 50\sqrt{6} \implies T(x,y,z) = \dfrac{50\sqrt{6}}{\sqrt{x^2+y^2+z^2}}$

Now, $T(4,0,3) = \dfrac{50\sqrt{6}}{\sqrt{3^2+4^2}} = 10\sqrt{6}$ degrees

46. The surface $z = \sqrt{4-x^2-y^2}$ is a hemisphere of radius 2 centered at the origin.

(a) The line l_1 is tangent at the point $(1, 1, \sqrt{2})$ to the curve in which the hemisphere intersects the plane $x = 1$. Find equations that specify l_1.

(b) The line l_2 is tangent at the point $(1, 1, \sqrt{2})$ to the curve in which the hemisphere intersects the plane $y = 1$. Find equations that specify l_2.

(c) The tangent lines l_1 and l_2 determine a plane. Find an equation for this plane. [This plane can be viewed as tangent to the surface at the point $(1, 1, \sqrt{2})$.]

$f(x,y) = (4-x^2-y^2)^{1/2}$, $f_x(x,y) = -x(4-x^2-y^2)^{-1/2}$, $f_y(x,y) = -y(4-x^2-y^2)^{-1/2}$

(a) $f_y(1,1) = -\dfrac{\sqrt{2}}{2} \implies x = 1,\ z - \sqrt{2} = -\dfrac{\sqrt{2}}{2}(y-1)$

(b) $f_x(1,1) = -\dfrac{\sqrt{2}}{2} \implies y = 1,\ z - \sqrt{2} = -\dfrac{\sqrt{2}}{2}(x-1)$

(c) l_1 and l_2 have direction vectors $\mathbf{j} - \frac{\sqrt{2}}{2}\mathbf{k}, \mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{k}$ respectively. The normal to the plane is

$$\left(\mathbf{j} - \frac{\sqrt{2}}{2}\mathbf{k}\right) \times \left(\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{k}\right) = -\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j} - \mathbf{k},$$ so the tangent plane is

$$-\frac{\sqrt{2}}{2}(x-1) - \frac{\sqrt{2}}{2}(y-1) - (z-\sqrt{2}) = 0, \quad \text{or} \quad (x-1)+(y-1)+\sqrt{2}(z-\sqrt{2}) = 0$$