54. The law of cosines for a triangle can be written

$$a^2 = b^2 + c^2 - 2bc \cos \theta.$$

At time t_0, $b_0 = 10$ inches, $c_0 = 15$ inches, $\theta_0 = \frac{1}{3}\pi$ radians.

(a) Find a_0 (the length a at time t_0).

(b) Find the rate of change of a with respect to b at time t_0 given that c and θ remain constant.

(c) Use the rate you found in part (b) to estimate by a differential the change in a that results from a 1-inch decrease in b.

(d) Find the rate of change of a with respect to θ at time t_0 given that b and c remain constant.

(e) Find the rate of change of c with respect to θ at time t_0 given that a and b remain constant.