

Exercicis

37. The magnitude of the gravitational force exerted by a body of mass M situated at the origin on a body of mass m located at the point (x, y, z) is given by the function

$$F(x, y, z) = \frac{GmM}{x^2 + y^2 + z^2}$$

where G is the universal gravitational constant. What are the level surfaces? What is the physical significance of these surfaces?

38. The strength E of an electric field at a point (x, y, z) due to an infinitely long charged wire lying along the y -axis is given by the function

$$E(x, y, z) = \frac{k}{\sqrt{x^2 + z^2}}$$

where k is a positive constant. Describe the level surfaces of E .

39. A metal solid occupies a region in three-space. The temperature T (in $^{\circ}\text{C}$) at the point (x, y, z) in the solid is inversely proportional to its distance from the origin.

- (a) Express T as a function of x, y, z .
- (b) Describe the level surfaces and sketch a few of them.
NOTE: The level surfaces of T are known as *isothermals*; at all points of an isothermal the temperature is the same.
- (c) Suppose the temperature at the point $(1, 2, 1)$ is 50° . What is the temperature at the point $(4, 0, 3)$?

46. The surface $z = \sqrt{4 - x^2 - y^2}$ is a hemisphere of radius 2 centered at the origin.

- (a) The line l_1 is tangent at the point $(1, 1, \sqrt{2})$ to the curve in which the hemisphere intersects the plane $x = 1$. Find equations that specify l_1 .
- (b) The line l_2 is tangent at the point $(1, 1, \sqrt{2})$ to the curve in which the hemisphere intersects the plane $y = 1$. Find equations that specify l_2 .
- (c) The tangent lines l_1 and l_2 determine a plane. Find an equation for this plane. [This plane can be viewed as tangent to the surface at the point $(1, 1, \sqrt{2})$.]