Review of Quantum Mechanics

- *Concept of wave function*
- *Time dependent Schrödinger's equation*
- *Steady states Schrödinger equation.*

$$\boxed{E = h\nu}$$

Planck

$$\boxed{\lambda = \frac{h}{p}}$$

De Broglie

$$\Psi = \Psi_o e^{-i(2\pi \nu t - kx)}$$

$\Psi =$ *wave function*

$/\Psi_o/^2$: *density of probability*

$$\Psi = \Psi_o e^{-i2\pi(\frac{E}{h}t - \frac{P}{h}x)} = \Psi_o e^{-i\hbar^{-1}(Et - Px)}$$

$$\frac{\partial\Psi}{\partial x}=\frac{i}{\hbar}p\cdot\Psi_o e^{-\frac{i}{\hbar}(Et-px)}=\frac{i}{\hbar}p\cdot\Psi$$

$$\frac{\partial^2\Psi}{\partial x^2}=\left(\frac{i}{\hbar}p\right)^2\cdot\Psi=-\frac{p^2}{\hbar^2}\cdot\Psi$$

$$\frac{\partial^2\Psi}{\partial x^2}=-\frac{2mT}{\hbar^2}\cdot\Psi\Rightarrow\frac{\hbar}{2m}\frac{\partial^2\Psi}{\partial x^2}=-T\Psi$$

$$\frac{\partial \Psi}{\partial t} = -\frac{i}{\hbar} E \cdot \Psi \Rightarrow -\frac{\hbar}{i}\frac{\partial \Psi}{\partial t} = E\Psi$$

E = T + U ▷ E - T = U

$$\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} - \frac{\hbar}{i}\frac{\partial \Psi}{\partial t} = U \cdot \Psi$$

1 - D →
$$\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} - U \cdot \Psi = \frac{\hbar}{i}\frac{\partial \Psi}{\partial t}$$

3 - D →
$$\frac{\hbar^2}{2m}\Delta \Psi - U \cdot \Psi = \frac{\hbar}{i}\frac{\partial \Psi}{\partial t}$$

Time dependent SCHRÖDINGER EQUATION

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Steady States Schrödinger's equation

If U (x, $\cancel{t}$):

$$\Psi(x,t) = \Psi(x)\cdot\Psi(t)$$

$$\left.\begin{aligned} \frac{\partial^2\Psi}{\partial x^2} &= \Psi(t)\cdot\frac{\partial^2\Psi(x)}{\partial x^2} \\ \frac{\partial\Psi}{\partial t} &= \Psi(x)\cdot\frac{\partial\Psi(t)}{\partial t} \end{aligned}\right\}$$

Ψ(x) . Ψ(t)

$$\underbrace{\frac{\hbar^2}{2m}\frac{1}{\Psi(x)}\frac{\partial^2\Psi(x)}{\partial x^2} - U(x)}_{f(x)} = \underbrace{\frac{\hbar}{i}\frac{1}{\Psi}\frac{\partial\Psi}{\partial t}}_{g(t)};\ \forall x,t$$

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$$\frac{\hbar^2}{2m}\frac{1}{\Psi(x)}\frac{\partial^2\Psi(x)}{\partial x^2}-U(x)=\alpha \qquad \textbf{(1)}$$

$$\frac{\hbar^2}{i}\frac{1}{\Psi(x)}\frac{\partial\Psi(x)}{\partial x}=\alpha \qquad \textbf{(2)}$$

$$\ln\Psi(t)=\frac{i\alpha}{\hbar}t+cte \Rightarrow \Psi(t)=Me^{i\frac{\alpha}{\hbar}t}$$

$$\ln\Psi(t)=\Psi_o e^{-\frac{i}{\hbar}(Et-px)}=\Psi_o e^{i\frac{p}{\hbar}x}\cdot e^{-\frac{i}{\hbar}Et}$$

$$\Downarrow$$

$$\alpha=-E$$

Consequently

$$\boxed{\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x)}{\partial x^2}-(E-U)\Psi(x)=0}$$

SCHRÖDINGER' EQUATION

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Review of Quantum Mechanics

PHYSICS OF THE QUANTUM WELL:

- *1D confinement of electrons, holes and excitons*
- *Discret levels of energy (equidistants)*
- *Very good transistor structure: High Electron Mobility Transistor, (HEMT)*
- **Why?**

Quantum Mechanics has the answer!!!

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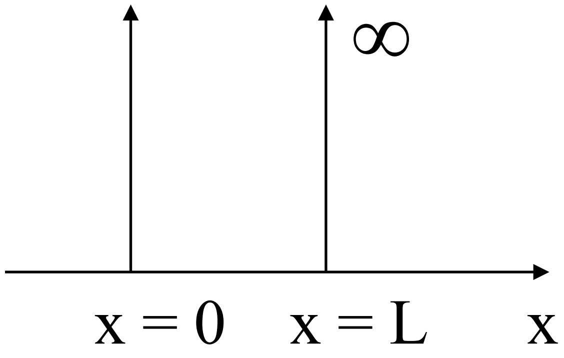
1-D POTENCIAL BOX



$U = 0$ en $0 < x < L$

$$U = \infty \text{ en } \begin{cases} x = 0 \\ x = L \end{cases}$$

Schrödinger's equation

$$\frac{\partial^2 \Psi(x)}{\partial x^2} + \frac{2m}{\hbar^2} E\Psi = 0 \quad \Rightarrow \quad \frac{\partial^2 \Psi(x)}{\partial x^2} + k^2 \Psi = 0$$

Solucion:

$$\Psi(x) = Ae^{ikx} + Be^{-ikx}$$

Boundary conditions

$$\begin{cases} \Psi(0) = 0 \\ \Psi(L) = 0 \end{cases}$$

De Ψ (0) = 0 $\Rightarrow$ 0 = A + B $\Rightarrow$ A = B

,

$$\Psi(x) = A\left(e^{ikx} - e^{-ikx}\right) = 2iA\sin kx$$

$$\Psi(x) = C\sin kx$$

Ψ (L) = 0

0 = sin k L $\Rightarrow$ k L = n π, amb n = 1, 2, 3

$$k = \frac{n\pi}{L} \Rightarrow k^2 = \frac{n^2\pi^2}{L^2}$$

:

$k = \sqrt{\dfrac{2mE}{\hbar^2}}$,consequently

Energy levels

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2} \Rightarrow \boxed{E = \frac{n^2\pi^2\hbar^2}{2mL^2}}$$

U = 0 $\Rightarrow$ $$E = T = \frac{p^2}{2m}$$

$$\boxed{p = n\pi \frac{\hbar}{L}}$$

quantització del moment

Wave functions

$$\Psi_n(x) = C sin\left(\frac{n\pi}{L}x\right)$$

C?

$$\int_{-\infty}^{\infty} |\Psi_n|^2 dx = C^2 \int_0^L sin^2\left(\frac{n\pi}{L}x\right) dx = 1$$

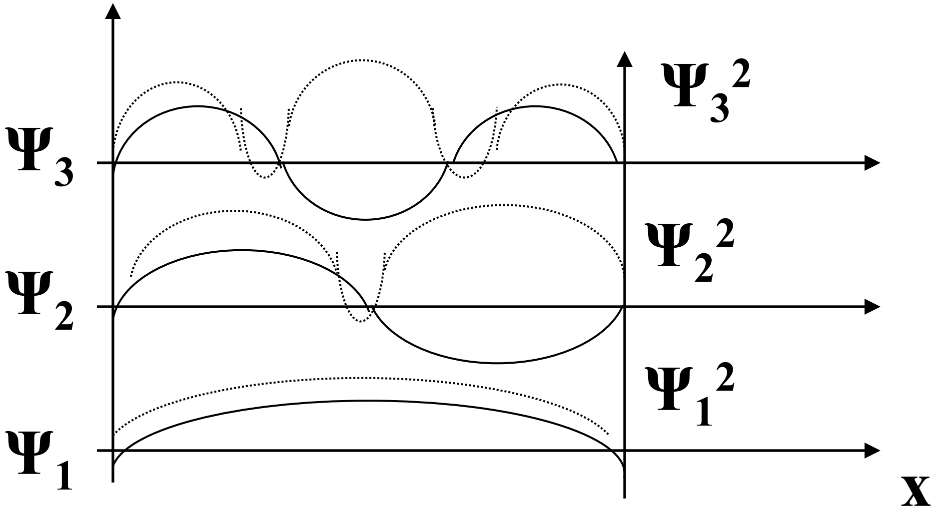
$$C^2\int_0^L sin^2\left(\frac{n\pi}{L}x\right)dx = C^2\left[\frac{1-\cos 2\alpha}{2}\right]_0^L = C^2\left[sin^2\alpha\right]_0^L =$$

$$= C^2\frac{L}{2} = 1 \Rightarrow \boxed{C = \sqrt{2/L}}$$

Consequently

$$\boxed{\Psi_n(x) = \sqrt{\frac{2}{L}}\cdot sin\left(\frac{n\pi}{L}x\right)}$$

$$\boxed{\Psi_n(x,t)=\sqrt{\frac{2}{L}}\cdot\sin\left(\frac{n\pi}{L}x\right)\cdot e^{-\frac{i}{\hbar}Et}}$$



Review of Quantum Mechanics

PHYSICS OF THE NANOWIRE/NANOROD

- 2D confinement of electrons.
- Discret levels of energy
- Very good unidimensional conductor.
- **Why?**

Quantum Mechanics has the answer!!!

2 Dimensions of confinement

Paticle in an infinite circular box

The Schrodinger equation here is

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = \varepsilon\Psi}$$

The potential is

$$V(x) = \begin{cases} 0 & \text{if} r < a \\ \infty & \text{if} r >= a \end{cases}$$

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Review of Quantum Mechanics

PHYSICS OF THE QUANTUM DOT

- 3D confinement of electrons.
- Discret levels of energy
- Very good light emitter.
- **Why?**

Quantum Mechanics has the answer!!!

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3 Dimensions of confinement

Particle in an infinite spherical box

This is a more complicated problem. Two approaches to a solution are illustrated with one leading to what are know as spherical Bessel function and the other to a solution involving regular Bessel functions of half integer order. The Schrodinger equation is

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = \varepsilon\Psi}$$

The potential is

$$V(x) = \begin{cases} 0 & \text{if} r < a \\ \infty & \text{if} r >= a \end{cases}$$

In the region inside the sphere where $V = 0$, this reduces to

$$-\frac{\hbar^2}{2m}\nabla^2\Psi = \varepsilon\Psi \qquad (6.24)$$

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THANKS A LOT FOR YOUR

FRIENDLY ATTENTION !!!

Tarragona, January 2014