Exercises - Central forces

6.1 Exponential spiral. Given the angular momentum L, find the central potential $V(r)$ that leads to a spiral path of the form $r(\theta) = r_0 \exp(a\theta)$. Choose $E = 0$. **Hint:** Obtain an expression for r' that does not contain θ, and then use conservation of energy.

6.2 Cross section. A particle moves in a potential, $V(r) = -C/(3r^3)$.

(a) Draw the effective potential V_{eff}.

(b) Find the maximum value of the effective potential for a given angular momentum L.

(c) Let the particle come in from infinity with speed v_0 and impact parameter b.[1] In terms of C, m, and v_0 , what is the largest value b_{max} of the impact parameter for which the particle is captured by the potential?

(d) What is the "cross section" for capture $\sigma = \pi b_{\text{max}}^2$, for this potential?

[1] The impact parameter b of a trajectory is the closest distance to the origin the particle would achieve if it moved in the straight line determined by its initial velocity far from the origin.

6.3 β/r^2 potential Consider a particle of mass m subject to a central potential $V(r) = \beta/r^2$.

(a) Draw the effective potential in the cases $\beta < -L^2/2m$, $\beta = -L^2/2m$, and $\beta > -L^2/2m$.

(b) Following the same general strategy that we used to solve the Kepler problem, obtain the differential equation descriving the orbit $r(\theta)$.

(c) Solve the orbits $r(\theta)$ in the same three ranges of β as in 6.3a. Use $a \equiv \beta + L^2/2m$ to write less.