Solutions to the exercises Quantum Physics, GEMF

Coen de Graaf, URV, Departament de Química Física i Inorgànica

May 6, 2023

1 Introduction

1. $\frac{1}{3}$
2. $a = \frac{1}{2}\sqrt{3}, \qquad b = \frac{1}{2}$
3. 0.442, -0.147, 0.885 ; 0.783.
4. –
5. $\frac{1}{2}$, $\frac{1}{2}$; ≈ 0.854, ≈ 0.146
6. first splitter 1:2, second splitter 1:1
7. All probabilities equal $\frac{1}{2}$
8. $-\frac{\hbar}{2}$, 0, 0, 0, 0,$-\frac{\hbar}{2}$
9. $i\hbar\hat{S}_z$
10. $\frac{\hbar}{2}$
11. $\Delta S_z = \frac{\sqrt{3}}{4}\hbar^2$

2 The origins of Quantum Physics

1. $7.6 \cdot 10^{29}$
2. 1.11 s^{-1} ; 6.125 J; $7.35 \cdot 10^{-34}$ J; $8.33 \cdot 10^{33}$
3. $5.0208 \cdot 10^{14}$ Hz. $6.6055 \cdot 10^{-34}$ J/Hz.
4. (i) $1.67 \cdot 10^{-10}$ m. (ii) $3.42 \cdot 10^{-15}$ m (or $3.36 \cdot 10^{-15}$ m with relativistic corrections) (iii) $5.52 \cdot 10^{-36}$ m.
5. 1.45 Å, yes
6. 3.39 pm.
7. $r_1 = 5.31734 \cdot 10^{-11}$ m; $E_1 = 13.540$ eV
8. (i) 1.95 mm; (ii) 20 cm; (iii) $1.02 \cdot 10^{-5}$ m; (iv) $\Delta x_0\sqrt{1 + 1.1 \cdot 10^{-62}}$.
9. (i) $t = 0.17$ s and (iV) $t = 1.89 \cdot 10^{28}$ s.

3 Mathematical concepts

1. –

2. –

3. –

4. 1, -1 ,1, -1, 1, -1

5. –

6. (a) $\frac{1}{\sqrt{2}}\begin{pmatrix} -9 & 9i \\ 2i & 2 \end{pmatrix}$ and $\frac{1}{\sqrt{2}}\begin{pmatrix} -9 & -2i \\ -9i & 2 \end{pmatrix}$
   (b) –
   (c) $\begin{pmatrix} 81 & 18i \\ -18i & 4 \end{pmatrix}$ and $\frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$
   (d) $81 + 4 \neq 1$ and $\frac{1}{2}(1+1) = 1$
   (e) $49/2 \leq 85$

7. $\left(\frac{d}{dx}\right)^\dagger = -\frac{d}{dx}$ and $\left(i\frac{d}{dx}\right)^\dagger = i\frac{d}{dx}$

8. –

9. $-\frac{d}{dx}\hat{X}$

10. $\hat{L}_x$

11. $-2\frac{\partial}{\partial x}$

12. (a) $\sqrt{2/\sqrt{\pi}}$
    (b) $4\mathcal{E}$
    (c) $\frac{1}{2}$
    (d) $12\mathcal{E}$

4 The postulates of Quantum Mechanics

1. Yes, No

2. –

3. $\frac{1}{\sqrt{2\alpha}}$

4. (a) $E = 0$, $2\mathcal{E}$ and $-\mathcal{E}$

   (b) $a_1 = 0$ and $a_{2,3} = \pm\sqrt{17}a$. $P_1 = \frac{16}{17}$, $P_{2,3} = \frac{1}{34}$

   (c) $\langle \hat{A} \rangle = 0$

   (d) $\Delta A = a$

5. (a) $\frac{1}{3}$

   (b) $\frac{1}{3}$

   (c) $\frac{1}{6}$

6. $E_{1,2} = \epsilon \mp \gamma_{ab}$

7. –

8. $P_a(t) = \sin^2(\gamma_{ab}t/\hbar)$ and $P_b(t) = \cos^2(\gamma_{ab}t/\hbar)$

5 Model systems

1. $A = \sqrt{\frac{1}{L}}$, same for B.

2. $\lambda = 694$ nm

3. $R = 1$
   $E = 0.25: \quad P(1) = 8.63 \cdot 10^{-2}|A|^2$
   relative probabilities:
   $E = 0.25: \quad P(1) = 1 \quad P(2) = 0.086$
   $E = 0.1: \quad P(1) = 0.315 \quad P(2) = 0.022$
   $E = 0.01: \quad P(1) = 0.028 \quad P(2) = 0.002$

4. –

5. 0.1573

6. (a) $\begin{pmatrix} 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ 0 & 0 & 2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

(b) $\begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & \dots \\ 0 & 0 & \sqrt{2} & 0 & \dots \\ 0 & 0 & 0 & \sqrt{3} & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ \sqrt{1} & 0 & 0 & 0 & \dots \\ 0 & \sqrt{2} & 0 & 0 & \dots \\ 0 & 0 & \sqrt{3} & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

(c) $\sqrt{\frac{\hbar}{2m\omega}} \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & \dots \\ \sqrt{1} & 0 & \sqrt{2} & 0 & \dots \\ 0 & \sqrt{2} & 0 & \sqrt{3} & \dots \\ 0 & 0 & \sqrt{3} & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$ and $\sqrt{\frac{m\omega\hbar}{2}} \begin{pmatrix} 0 & -i\sqrt{1} & 0 & 0 & \dots \\ i\sqrt{1} & 0 & -i\sqrt{2} & 0 & \dots \\ 0 & i\sqrt{2} & 0 & -i\sqrt{3} & \dots \\ 0 & 0 & i\sqrt{3} & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

(d) –

(e) $\frac{1}{2}\hbar\omega \begin{pmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 3 & 0 & 0 & \dots \\ 0 & 0 & 5 & 0 & \dots \\ 0 & 0 & 0 & 7 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

(f) yes

6 Towards real-world problems

1. $\lambda_1 = 0$; $\lambda_{2,3} = \pm\sqrt{2}$

   $|1,0\rangle_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

   $|1,1\rangle_x = \frac{1}{2} \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$

   $|1,-1\rangle_x = \frac{1}{2} \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$

2. $E_1^{(1)} = 0$, $E_2^{(1)} = 0$, $E_3^{(1)} = 1$, $E_1^{(2)} = \frac{21}{20}$, $E_2^{(2)} = -\frac{1}{4}$, $E_3^{(2)} = -\frac{4}{5}$
   Exact energies: $E_1 = 7$, $E_2 = 2$, $E_3 = 1$

3. (a) –

   (b) –

   (c) –

(d) –

(e) $-\frac{\hbar\lambda}{4m\omega_0}$

(f) $\frac{3\beta\hbar^2}{4m^2\omega^2}$

4. $L_{opt} \approx 2.084$ and $E_{min} \approx 0.567$
   First excited state: $L_{opt} \approx 2.554$ and $E_{opt} = 1.658$
   Second excited state: $L_{opt} \approx 2.869$ and $E_{opt} = 2.698$
   Overlap: ground with first and first with second = 0; ground with second = not equal to zero

5.

6. ground state: (i) –; (ii) – ; (iii) $3h^2/4mL^2$, excited state: (i) –, (ii) –, (iii) $9h^2/8mL^2$

7. 54.4, 122.4, 217.6 eV ; 23.1, 91.1, 186.4 eV