Exercicis

54. The law of cosines for a triangle can be written

$$a^2 = b^2 + c^2 - 2bc\cos\theta.$$

At time t_0, $b_0 = 10$ inches, $c_0 = 15$ inches, $\theta_0 = \frac{1}{3}\pi$ radians.

(a) Find a_0 (the length a at time t_0).

(b) Find the rate of change of a with respect to b at time t_0 given that c and θ remain constant.

(c) Use the rate you found in part (b) to estimate by a differential the change in a that results from a 1-inch decrease in b.

(d) Find the rate of change of a with respect to θ at time t_0 given that b and c remain constant.

(e) Find the rate of change of c with respect to θ at time t_0 given that a and b remain constant.

(a) $a_0 = \left(b_0^2 + c_0^2 - 2b_0c_0\cos\theta_0\right)^{1/2} = 5\sqrt{7}$

(b) $\dfrac{\partial a}{\partial b} = (2b - 2c\cos\theta)\left(\dfrac{1}{2}\right)(b^2 + c^2 - 2bc\cos\theta)^{-1/2} = \dfrac{\sqrt{7}}{14}$

(c) $a \cong a_0 + \dfrac{\sqrt{7}}{14}(b - b_0) = 5\sqrt{7} + \dfrac{\sqrt{7}}{14}\cdot(-1)$ decreases by about $\dfrac{\sqrt{7}}{14}$ inches.

(d) $\dfrac{\partial a}{\partial \theta} = 2bc\sin\theta(\dfrac{1}{2})(b^2 + c^2 - 2bc\cos\theta)^{-1/2} = \dfrac{15}{7}\sqrt{21}$

(e) Differentiate implicitly: $\quad 0 = 2c\dfrac{\partial c}{\partial\theta} - 2b\,\dfrac{\partial c}{\partial\theta}\cos\theta + 2bc\sin\theta$

$$\frac{\partial c}{\partial\theta} = \frac{bc\sin\theta}{b\cos\theta - c} = -\frac{15}{2}\sqrt{3}$$