# Splines

1) Build and appoximation of natural splines for the following functions:

$$a)\, f(x) = 2x$$
$$b)\, f(x) = (x-1)^2 + 1$$
$$c)\, f(x) = \frac{1}{1+x^2}$$
$$d)\, f(x) = \arctan(x)$$

2) Use a clampled spline to build an approximation to the function

$$b)\, f(x) = (x-1)^2 + 1$$

in the interval $[0.5, 2]$

3) Consider the following table of the population of the United States from $1940$ to $1990$. Use free cubic spline interpolation to approximate the population in the years $1930, 1965$ and $2010$.

The population in $1930$ was approximately $123203000$.

| *Year* | 1940 | 1950 | 1960 | 1970 | 1980 | 1990 |
|---|---|---|---|---|---|---|
| *Population* $\times 10^3$ | 132165 | 151326 | 179323 | 203302 | 236542 | 249633 |

4) It is suspected that the high amounts of tannin in mature oak leaves inhibit the growth of the winter moth larvae that extensively damage these trees in certain years. The following table liste the average weight of two samples of larve at times in the first $28$ days after birth. The first sample was reared on young oak leavees, whereas the second sample was reared on mature leaves from the same tree.

a) Use a cubic spline to approximate the average weight curve for each sample.

b) Find an approximate maximum average weight for each sample by determining the maximum of the spline.

| Day | 0 | 6 | 10 | 13 | 17 | 20 | 28 |
|---|---|---|---|---|---|---|---|
| Sample 1 | 6.67 | 17.33 | 42.67 | 37.33 | 30.10 | 29.31 | 28.74 |
| Sample 2 | 6.67 | 16.11 | 18.89 | 15.00 | 10.56 | 9.44 | 8.89 |

5) The gamma functionc, denoted by $\Gamma(x)$, is an important special function in probability, combinatorics and other areas of applied mathematics. Because it can be shown that $\Gamma(n+1) = n!$, the gamma function is considered a generalization of the factorial function to non-integer arguments. The following table gives values of $\Gamma(x)$ in the $[1,2]$ interval.

| $x$ | $\Gamma(x)$ |
|---|---|
| $1.00$ | $1.0000000000$ |
| $1.10$ | $0.9513507699$ |
| $1.20$ | $0.9181687424$ |
| $1.30$ | $0.8974706963$ |
| $1.40$ | $0.8872638175$ |
| $1.50$ | $0.8862269255$ |
| $1.60$ | $0.8935153493$ |
| $1.70$ | $0.9086387329$ |
| $1.80$ | $0.9313837710$ |
| $1.90$ | $0.9617658319$ |
| $2.00$ | $1.0000000000$ |

Use an spline to approximate the values of the gamma function at the nodes $x = 1.1, 1.3, 1.5, 1.7$ and $1.9$ using the value of the nodes at $x = 1.0, 1.2, 1.4, 1.6, 1.8$ and $2.0$.

5) Given the table of value of the elliptic integral of the first kind $f(x,y)$

| $y\backslash x$ | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|
| 50 | $0.9401$ | $0.9647$ | $0.9876$ | $1.0044$ | $1.0107$ |
| 52 | $0.9835$ | $1.0118$ | $1.0387$ | $1.0587$ | $1.0662$ |
| 54 | $1.0277$ | $1.0602$ | $1.0915$ | $1.1152$ | $1.1242$ |
| 56 | $1.0725$ | $1.1097$ | $1.1462$ | $1.1743$ | $1.1851$ |
| 58 | $1.1180$ | $1.1605$ | $1.2030$ | $1.2362$ | $1.2492$ |

find an approximation to $f(65,53)$ by the following. (i) Interpolating horizontally to find $f(65,y)$ for $y = 50, 52, 54, 56, 58$ and then interpolating vertically. (ii) Interpolating vertically and then horizontally. (iii) Interpolating diagonally along the two diagonals. Compare the results with the correct value which is $f(65,53) = 1.0509$

5) Use the following tables to draw a clamped spline joining all the points:

| $x$ | 1 | 2 | 5 | 6 | 7 | 8 | 10 | 13 | 17 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 3.0 | 3.7 | 3.9 | 4.2 | 5.7 | 6.6 | 7.1 | 6.7 | 4.5 |
| $f'(x)$ | 1.0 | | | | | | | | −0.67 |

| $x$ | 17 | 20 | 23 | 24 | 25 | 27 | 27.7 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 4.5 | 7.0 | 6.1 | 5.6 | 5.8 | 5.2 | 4.1 |
| $f'(x)$ | 3.0 | | | | | | −4.0 |

| $x$ | 27.7 | 28 | 29 | 30 |
|---|---|---|---|---|
| $f(x)$ | 4.1 | 4.3 | 4.1 | 3.0 |
| $f'(x)$ | 0.33 | | | −1.5 |