Numerical Differentiation

1) Check the progressive and regressive formulas of first and second order to compute de dervatives of the function $y = \sin(x)$ in the interval $[0, 2\pi]$

2) Check the centered difference formulas of first and second order to compute de dervatives of the function $y = \exp(x)$ in the interval $[0, 6]$

3) Use the Richardson extrapolation to compute the derivatives of the function $y = \sin(x)$ in the interval $[0, 2\pi]$

4) Use the Richardson extrapolation to compute the derivatives of the function $y = \exp(x)$ in the interval $[0, 6]$

5) Add some random gaussian noise to the function $y = \sin(x)$ and compute the numerical derivatives that result using a centered derivation formula.

6) Check the presence of an optimal step in the computation of the numerical derivatives of the function $y = \sin(x)$ usig a first order progressive finite difference.

7) Compute the numerical derivative of order up to $n = 4$ of the function $f(x) = x^7$ in the point $x = 1$ using high order difference formulas.

8) Build a table of equiespaced points for the function

$$f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5$$

in the interval $[0, 0.8]$. Use this table and the *diff* and *gradient* functions of MATLAB to compute the numerical derivatives of the function. Compare with the exact results.

9) Consider the function

$$f(x) = \frac{e^x}{1 + x + 0.5x^2}$$

Evaluate the error in its first derivative $f'(0)$, computed using the central first derivative formula, for a step size ranging from $\triangle x = 0.1$ to $\triangle x = 10^{-7}$. Plot the error versus the step $\triangle x$ using a logarithmic scale. For zero values use *eps* instead.

10) Write a MATLAB function that calculates the first and second derivatives of a given function on a grid $a = x_0 < x_1 < \ldots < x_n = b$ with constant grid spacing $h = x_{j+1} - x_j$ for $j = 0, 1, \ldots, (n-1)$ to second order of accuracy in h for all grid points. Plot the calculated values against the exact derivatives of the function $f(x) = x\sin(x)$ for $a = 0$, $b = 2\pi$ and $n = 50$

11) Calculate the first derivative of the function

$$f(x) = \frac{e^x}{1 + 2x^2}$$

at the points $x = 0$ and $x = 1$ using Richardson extrapolation.

10) When a fluid flows over a surface, the shear stress τ (N/m^2) at the surface is given by the expression

$$\tau = \mu \frac{du}{dy}\bigg|_{surface}$$

where μ is the viscosity (Ns/m^2), u is the velocity parallel to the surface (m/s) and y is the distance normal to the surface (cm). Measurements of the velocity of an air stream flowing above a surfade are made with an LDV (Laser Doppler Velocimeter). The folowing values were obtained:

y	0.0	0.1	0.2	0.3
u	0.00	55.56	88.89	100.00

At the local temperature, $\mu = 0.00024\ Ns/m^2$. Calculate (a) du/dy at the surface based of first, second and third order polynomials. (b) The corresponding values of the shear stress at the surface. (b) The shear force acting on a plate $10\ cm$ long and $5\ cm$ wide.

11) When a fluid flows over a surface, the heat transfer rate $\dot{q}\ (J/s)$ to the surface is given by the expression

$$\dot{q} = -kA\frac{dT}{dy}\bigg|_{suface}$$

where k is the thermal conductivity (J/msK), T is the temperature (K) and y is the distance normal to the surface (cm). Measurements of the temperature of an air stream following above a surface are made with a themocouple. The following values were obtained:

y	0.0	0.1	0.2	0.3
T	1000.00	533.33	355.56	300.00

At the average temperature, $k = 0.030\ J/msK$. (a) dT/dy at the surface, based on first, second and third order polynomials. (b) The corresponding values of the heat flux $\dot{q}/A$ at the surface, and (c) The heat transfer to a flat plate $10\ cm$ long and $5\ cm$ wide.