Exercicis

1. Let $f(x, z) = e^{x+y}$.

(a) Find the first-order Taylor formula for f at $(0, 0)$.

(b) Find the second-order Taylor formula for f at $(0, 0)$.

In exercises 3 to 7, calculate the second-order Taylor formula for:

3. $f(x, y) = (x + y)^2$, where $x_0 = 0$, $y_0 = 0$

4. $f(x, y) = 1/(x^2 + y^2 + 1)$, where $x_0 = 0$, $y_0 = 0$

5. $f(x, y) = e^{x+y}$, where $x_0 = 0$, $y_0 = 0$

6. $f(x, y) = e^{-x^2-y^2} \cos(xy)$, where $x_0 = 0$, $y_0 = 0$

7. $f(x, y) = \sin(xy) + \cos(xy)$, where $x_0 = 0$, $y_0 = 0$

9. Calculate the second-order Taylor approximation to $f(x, y) = \cos x \sin y$ at the point $(\pi, \pi/2)$.

10. Let $f(x, y) = x \cos(\pi y) - y \sin(\pi x)$. Find the second-order Taylor approximation for f at the point $(1, 2)$.

11. Let $g(x, y) = \sin(xy) - 3x^2 \log y + 1$. Find the degree 2 polynomial which best approximates g near the point $(\pi/2, 1)$.

Solucions

1. (a) $f(h_1, h_2) = 1 + h_1 + h_2 + R_1(\mathbf{0}, \mathbf{h})$

(b) $f(h_1, h_2) = 1 + h_1 + h_2 + \frac{1}{2}h_1^2 + h_1 h_2 + \frac{1}{2}h_2^2 + R_2(\mathbf{0}, \mathbf{h})$

3. $f(h_1, h_2) = h_1^2 + 2h_1 h_2 + h_2^2 \quad [R_2(\mathbf{0}, \mathbf{h}) = 0$, in this case].

5. $f(h_1, h_2) = 1 + h_1 + h_2 + \dfrac{h_1^2}{2} + h_1 h_2 + \dfrac{h_2^2}{2} + R_2(\mathbf{0}, \mathbf{h})$

7. $f(h_1, h_2) = 1 + h_1 h_2 + R_2(\mathbf{0}, \mathbf{h})$

9. $g(x, y) = -1 + \dfrac{1}{2}(x - \pi)^2 + \dfrac{1}{2}\left(y - \dfrac{\pi}{2}\right)^2$

11. $$p(x, y) = 2 - \frac{3\pi^2}{4}(y - 1) - \frac{1}{2}\left(x - \frac{\pi}{2}\right)^2 + \frac{\pi^2}{4}(y - 1)^2 - \frac{7\pi}{2}\left(x - \frac{\pi}{2}\right)(y - 1)$$