

### Exercicis

5. Consider the circle  $C$  of radius 2, centered at the origin.
- (a) Find a parametrization for  $C$  inducing a counterclockwise orientation and starting at  $(2, 0)$ .
  - (b) Find a parametrization for  $C$  inducing a clockwise orientation and starting at  $(0, 2)$ .
  - (c) Find a parametrization for  $C$  if it is now centered at the point  $(4, 7)$ .
6. Give a parametrization for each of the following curves:
- (a) The line passing through  $(1, 2, 3)$  and  $(-2, 0, 7)$
  - (b) The graph of  $f(x) = x^2$
  - (c) The square with vertices  $(0, 0)$ ,  $(0, 1)$ ,  $(1, 1)$ , and  $(1, 0)$  (Break it up into line segments.)
  - (d) The ellipse given by  $\frac{x^2}{9} + \frac{y^2}{25} = 1$

In **11** to **14**, compute the tangent vectors to the given paths:

**11.**  $\mathbf{c}(t) = (e^t, \cos t)$

**12.**  $\mathbf{c}(t) = (3t^2, t^3)$

**13.**  $\mathbf{c}(t) = (t \sin t, 4t)$

**14.**  $\mathbf{c}(t) = (t^2, e^2)$

- 15.** When is the velocity vector of a point on the rim of a rolling wheel *horizontal*? What is the speed at this point?

- 16.** If the position of a particle in space is  $(6t, 3t^2, t^3)$  at time  $t$ , what is its velocity vector at  $t = 0$ ?

In **17** and **18**, determine the equation of the tangent line of the curve at the specified point:

**17.**  $(\sin 3t, \cos 3t, 2t^{5/2}); t = 1$

**18.**  $(\cos^2 t, 3t - t^3, t); t = 0$

24. Consider the spiral given by  $\mathbf{c}(t) = (e^t \cos(t), e^t \sin(t))$ . Show that the angle between  $\mathbf{c}$  and  $\mathbf{c}'$  is constant.
25. Let  $\mathbf{c}(t) = (t^3, t^2, 2t)$  and  $f(x, y, z) = (x^2 - y^2, 2xy, z^2)$ .
- Find  $(f \circ \mathbf{c})(t)$ .
  - Find a parametrization for the tangent line to the curve  $f \circ \mathbf{c}$  at  $t = 1$ .

### Solucions

5. (a)  $\mathbf{c}(t) = (2 \cos(t), 2 \sin(t)), t \in [0, 2\pi]$   
 (b)  $\mathbf{c}(t) = (2 \sin(t), 2 \cos(t)), t \in [0, 2\pi]$   
 (c)  $\mathbf{c}(t) = (2 \cos(t) + 4, 2 \sin(t) + 7), t \in [0, 2\pi]$
11.  $\mathbf{c}'(t) = (e^t, -\sin t)$
13.  $\mathbf{c}'(t) = (t \cos t + \sin t, 4)$
15. Horizontal when  $t = (R/v)n\pi, n$  an integer; speed is zero if  $n$  is even; speed is  $2v$  if  $n$  is odd.
17.  $(\sin 3, \cos 3, 2) + (3 \cos 3, -3 \sin 3, 5)(t - 1)$
25. (a)  $(f \circ \mathbf{c})(t) = (t^6 - t^4, 2t^5, 4t^2)$   
 (b)  $\mathbf{l}(t) = (0, 2, 4) + t(2, 10, 8)$