Exercices

(1) Show that $x^2+ax+b=0$ can be written as

(i) $(x-\alpha)^2-\beta^2=0$

$x^2-2\alpha x+\alpha^2-\beta^2=\emptyset$

$x^2+a x+b=0$

$a=-2\alpha \longrightarrow \boxed{\alpha=-a/2}$

$b=\alpha^2-\beta^2 \longrightarrow b=\left(-\frac{a}{2}\right)^2-\beta^2 \rightarrow b=\frac{a^2}{4}-\beta^2$

$\longrightarrow \boxed{\beta=\sqrt{-(b-a^2/4)}=\sqrt{\frac{a^2}{4}-b}}$ $\quad \boxed{a^2/4\geq b}$

when

(ii) $(x-\alpha)^2=\emptyset$

$x^2-2\alpha x+\alpha^2=0$

$x^2+a x+b=0$

$a=-2\alpha \rightarrow \alpha=-a/2$

$b=\alpha^2 \rightarrow \alpha=\sqrt{b}$

$\Big\} \Rightarrow \sqrt{b}=-a/2$

$\boxed{b=a^2/4}$

when

(iii) $(x-\alpha)^2+\beta^2=\emptyset$

$x^2-2\alpha x+\alpha^2+\beta^2=0$

$x^2+a x+b=0$

$a=-2\alpha \longrightarrow \alpha=-a/2$

$b=\alpha^2+\beta^2 \longrightarrow b=\frac{a^2}{4}+\beta^2$

when

$\beta=\sqrt{b-a^2/4} \Rightarrow \boxed{b\geq -a^2/4}$

② if $x \geq -1$

Desigualtat Bernoulli

$$(1+x)^n \geq 1+nx \qquad \forall n \in \mathbb{N}$$

Inducció :

$n = 1$

$$(1+x)^1 = 1 + 1 \cdot x = 1+x \quad ✓$$

Hip. Inducció :

$$(1+x)^n \geq 1+nx$$

$$(1+x)^{n+1} \overset{?}{\geq} 1+(n+1)x$$

$$\underbrace{(1+x)^n(1+x)}_{\text{prod. potències}} \geq 1+x+nx$$

Sabem

$$(1+x)^n \geq 1+nx$$

$$(1+x)^n \cdot \underbrace{(1+x)}_{\substack{\geq 0 \text{ donat que} \\ x \geq -1}} \geq (1+nx)\cdot(1+x) = 1+nx+x+nx^2$$

$$\cancel{1}+\cancel{nx}+\cancel{x}+nx^2 \geq \cancel{1}+\cancel{x}+\cancel{nx}$$

$$\underset{\geq 0}{n}\,\underset{\geq 0}{x^2} \geq 0 \quad ✓$$

③ $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}\cdot b + \dots + ab^{n-2} + b^{n-1})$ $\boxed{n \geq 2}$

Inducció:

cas $n=2$

$$a^2 - b^2 = (a-b)(a+b) = a^2 - \cancel{ab} + \cancel{ab} - b^2 \checkmark$$

Hip. inducció:

$$a^n - b^n = (a-b)\underbrace{(a^{n-1} + a^{n-2}\cdot b + \dots + ab^{n-2} + b^{n-1})}_{*}$$

vole demostrar

$$a^{n+1} - b^{n+1} = (a-b)(a^n + a^{n-1}b + \dots + ab^{n-1} + b^n)$$

separo en:

$$a^n + b\underbrace{(a^{n-1} + a^{n-2}b + \dots + ab^{n-2} + b^{n-1})}_{*}$$

$$* = \frac{(a^n - b^n)}{a-b}$$

$$\Rightarrow a^{n+1} - b^{n+1} = (a-b)\left[a^n + b\cdot\frac{(a^n - b^n)}{a-b}\right] =$$

$$\cancel{(a-b)}\cdot\frac{a^n(a-b) + b(a^n - b^n)}{\cancel{(a-b)}} =$$

$$a^{n+1} \cancel{- a^n b} + \cancel{ba^n} - b^{n+1} = a^{n+1} - b^{n+1} \checkmark$$

(4) $$|a+b| = |a| + |b| \text{ iff } ab \geq \emptyset$$

per definició

$$|a+b| = \begin{cases} (a+b) & \text{iff } (a+b) \geq \emptyset \\ -(a+b) & \text{iff } (a+b) < \emptyset \end{cases}$$

$$|a| = \begin{cases} a & \text{iff } a \geq \emptyset \\ -a & \text{iff } a < \emptyset \end{cases}$$

$$|b| = \begin{cases} b & \text{iff } b \geq \emptyset \\ -b & \text{iff } b < \emptyset \end{cases}$$

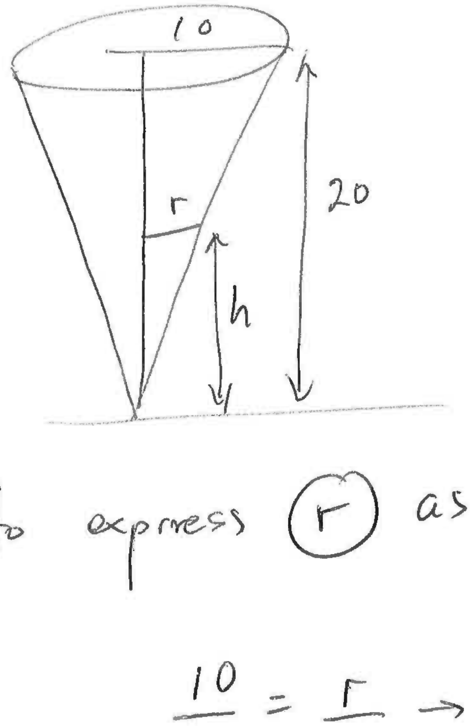
$$a \cdot b \geq 0$$

$$\Downarrow$$

$$(a \geq \emptyset \wedge b \geq \emptyset) \vee (a < \emptyset \wedge b < \emptyset)$$

$$|a| + |b| = \left.\begin{cases} a+b & \text{iff } (a+b) \geq \emptyset \\ -(a+b) & \text{iff } (a+b) < \emptyset \end{cases}\right\} = |a+b|$$

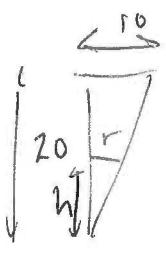
(5)



$V(h)$

$$V_{cone} = \frac{1}{3} \pi r^2 \cdot h$$

How to express r as a function of h



$$\frac{10}{20} = \frac{r}{h} \rightarrow r = \frac{h}{2}$$

$$\boxed{V(h) = \frac{1}{3} \pi \frac{h^2}{4} \cdot h = \frac{\pi}{12} h^3}$$