Exercises

1. a) Find the domain of the scalar function:

$$f(x,y) = \sum_{k=0}^{\infty} (xy)^k$$

   b) Find and classify the critical points of $f(x,y)$

2. Assume that y is a differentiable function of x which satisfies the equation. Obtain dy/dx by implicit differentiation.

   $x^2 - 2xy + y^4 = 4.$
   $x e^y + y e^x - 2x^2 y = 0.$
   $x^{2/3} + y^{2/3} = a^{2/3}.$ (a *is* a constant)
   $x \cos xy + y \cos x = 2.$

3. Let $u = u(x,y)$, where $x = x(s)$ and $y = y(s)$, and assume that these functions have continuous second derivatives. Find the expression for $\frac{d^2u}{ds^2}$.

4. a) Show that the expression

$$x^6 y + y^2 \int_0^x \frac{1}{1+\sin^6 t}\,dt + y^5 - 1 = 0$$

   defines y as an implicit differentiable function $y = f(x)$ around the point (0,1).

   b) Write the equation of the tangent line to $y = f(x)$ at the point (0,1).

5. Assume that $u = u(x,y)$ is differentiable.

   a) Show that the change of variables to polar coordinates $x = r\cos\theta$, $y = r\sin\theta$ gives

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\cos\theta + \frac{\partial u}{\partial y}\sin\theta$$
$$\frac{\partial u}{\partial \theta} = -\frac{\partial u}{\partial x}\mathrm{r}\sin\theta + \frac{\partial u}{\partial y} r\cos\theta$$

   b) Express in terms of $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ the following expression:

$$\left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r}\left(\frac{\partial u}{\partial \theta}\right)^2$$