Numerical Solution of Boundary Value Problems 1D

Shooting Methods

1) Solve the BVP :

$$y'' = \frac{2x}{1+x^2}y' - \frac{2}{1+x^2}y + 1$$

using the linear shooting method, where $x \in [0,4]$ and $x(0) = 1.25,\ x(4) = -0.95.$ Compare your results with the analithic solution:

$$y(x) = 1.25 + 0.4860896526x - 2.25x^2 + 2x\arctan(x) + \frac{1}{2}(x^2-1)\ln(1+x^2).$$

2) Solve the BVP :

$$y'' = -\frac{2}{x}y' + \frac{2}{x^2}y + \frac{\sin(\ln(x))}{x^2}$$

using the linear shooting method, where $x \in [1,2]$ and $x(1) = 1,\ x(2) = 2.$ Compare your results with the analithic solution:

$$y(x) = 1.1392070132x - 0.03920701320x^2 - \frac{3}{10}\sin(\ln(x^2)) - \frac{1}{10}\cos(\ln(x)).$$

3) Solve the BVP:

$$y'' = \frac{1}{8}(32 + 2x^3 - yy')$$

using the nonlinear shooting method, where $x \in [1,3]$ and $y(1) = 17,\ y(3) = 43/3.$ Compare your results with the exact solution

$$y(x) = x^2 + \frac{16}{x}$$

4) Solve the BVP:

$$y'' = -y'^2 - y + \ln(x)$$

using the nonlinear shooting method, where $x \in [1,2]$ and $y(1) = 0,$ $y(1) = \ln(2).$ Compare your results with the exact solution

$$y(x) = \ln(x)$$

5) Solve the BVP:

$$y'' = 2y^3$$

using the nonlinear shooting method, where $x \in [-1,0]$ and $y(-1) = 1/2,$ $y(0) = 1/3.$ Compare your results with the exact solution

$$y(x) = \frac{1}{x+3}$$

Finite Difference Methods

1) Solve the BVP :

$$y'' = \frac{2x}{1+x^20}y' - \frac{2}{1+x^2}y + 1$$

using the linear finite difference method, where $x \in [0,4]$ and $y(0) = 1.25,\ y(4) = -0.95.$ Compare your results with the analithic solution:

$$y(x) = 1.25 + 0.4860896526x - 2.25x^2 + 2x\arctan(x) + \frac{1}{2}(x^2-1)\ln(1+x^2).$$

2) Solve the BVP :

$$y'' = -\frac{2}{x}y' + \frac{2}{x^2}y + \frac{\sin(\ln(x))}{x^2}$$

using the linear finite difference method, where $x \in [1,2]$ and $y(1) = 1,\ y(2) = 2.$ Compare your results with the analithic solution:

$$y(x) = 1.1392070132x - 0.03920701320x^2 - \frac{3}{10}\sin(\ln(x^2)) - \frac{1}{10}\cos(\ln(x)).$$

3) Solve the BVP:

$$y'' = \frac{1}{8}(32 + 2x^3 - yy')$$

using the nonlinear finite difference method, where $x \in [1,3]$ and $y(1) = 17,$ $y(3) = 43/3.$ Compare your results with the exact solution

$$y(x) = x^2 + \frac{16}{x}$$

4) Solve the BVP:

$$y'' = -y'^2 - y + \ln(x)$$

using the nonlinear finite difference method, where $x \in [1,2]$ and $y(1) = 0,$ $y(1) = \ln(2).$ Compare your results with the exact solution

$$y(x) = \ln(x)$$

Finite Element Methods

1) Solve numerically the problem of the distribution of temperature in a rod:

$$\begin{aligned} -(ku')' &= f(x),\ x \in I = [2,8] \\ u(2) &= -1 \\ u'(8) &= 0. \end{aligned}$$

Using the Galerking Finite Element methods, where the conductivity is given by the function

$$k(x) = 0.1(5 - 0.6x)$$

while the heat source is given by the function

$$f(x) = 0.03(x-6)^4.$$

Use the midpoint or trapezoidal approximation for the numerical integrations.

2) Solve numerically the problem:

$$\begin{aligned} u'' + u &= -x^2,\ x \in I = [0,1] \\ u(0) &= 0 \\ u(1) &= 0 \end{aligned}$$

Using the Galerking Finite Element methods. Use the midpoint or trapezoidal approximation for the numerical integrations. Compare the numerical solution with the exact solution given by:

$$u(x) = \frac{\sin x + 2\sin(1-x)}{\sin 1} + x^2 - 2$$