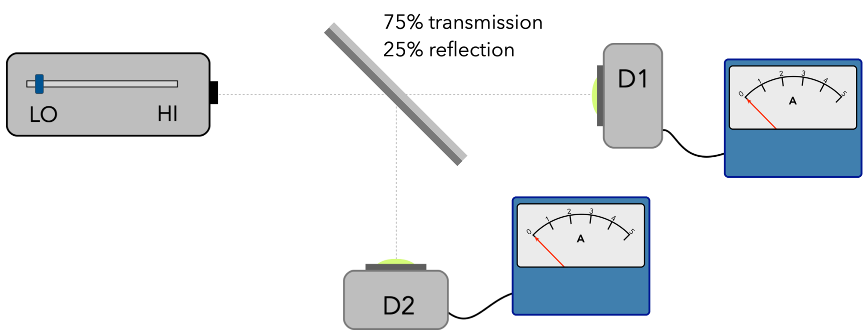
Exercises Quantum Physics (GEMF) – Introduction

Coen de Graaf — Departament de Química Física i Inorgànica, URV

1. Calculate the probability of finding a quantum particle in state $|\text{down}\rangle$ when the system is described with the state vector

$$|\psi\rangle = -\frac{1}{\sqrt{6}}|\text{top}\rangle + \frac{1}{\sqrt{2}}|\text{middle}\rangle - \frac{1}{\sqrt{3}}|\text{down}\rangle.$$

2. Write down the state vector of the photon emitted by the laser after passing through a beam splitter with 75% transmission and 25% reflection.



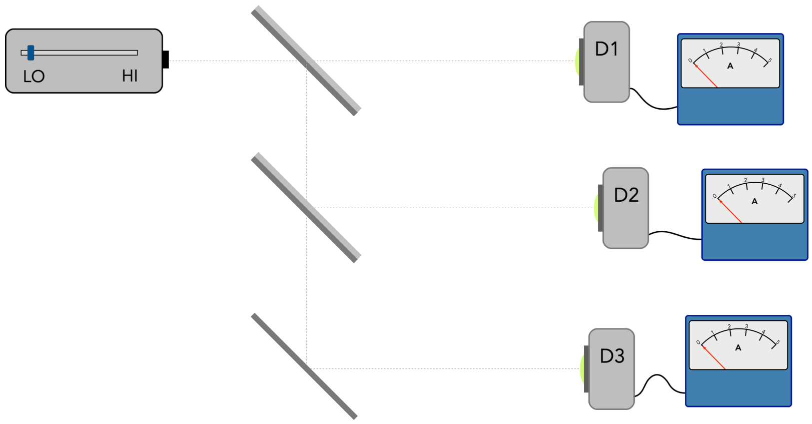
3. Modify the following state vector in such a way that it becomes a physically sound description of a quantum system and calculate the probability for measuring $|\text{green}\rangle$.

$$|\phi\rangle = 0.3|\text{red}\rangle + 0.1|\text{blue}\rangle + 0.6|\text{green}\rangle$$

4. Show that a photon emitted by the *top* laser only triggers the detector 2 when sent through the MZ interferometer as discussed in the theory sessions.

5. Calculate the probability of a photon send through a MZ interferometer for hitting detector 1 and for hitting detector 2 when the pathway for the reflected photon is longer than for the transmitted photon by $\frac{1}{4}\lambda$ (λ is the wavelength of the emitted light). Do the same for a difference in the length of $\frac{1}{8}\lambda$.

6. Write down the state vector of a photon emitted by the laser (i) at $t = t_1$ after passing the first beam splitter, and (ii) at $t = t_2$ after passing the second beam splitter. Both splitters have a reflection/transmission ratio of 1. What ratios should be used in the beam splitters to have equal changes to detect the photon in any of the detectors?



7. Show that an electron with a spin angular moment polarized along the y-direction has equal probability to be deflected by $+\Delta x$ as being deflected by $-\Delta x$ when sent through a magnetic field aligned with the x-axis.

8. Calculate the expectation value of the $\hat{S}_z$ operator for the state vectors $|\downarrow\rangle$, $|+\rangle$, and $|\circlearrowright\rangle$. Do the same for the $\hat{S}_x$ operator and the state vectors $|\uparrow\rangle$, $|-\rangle$ and $\circlearrowleft\rangle$.

9. Calculate the commutator of $\hat{S}_x$ and $\hat{S}_y$.

10. Show that $\Delta S_y = \hbar/2$ for the spin state $|\uparrow\rangle$.

11. Calculate the variance in S_z and S_x for the spin state $|\psi\rangle = \frac{1}{2}|\uparrow\rangle + \frac{1}{2}\sqrt{3}|\downarrow\rangle$.