Classical Interpolation

1) Use the Lagrange and Newton interpolation to interpolate the functon $y = \ln(x)$ in the interval $[0.1, 2]$

2) Plot the Lagrange basis polynomials for the interpolation on the set of nodes

$$x = (0.1, 0.2, 0.5, 0.9, 1.0, 1.1, 1.7, 1.75)$$

3) Use Lagrange polynomials of different degrees to intepolate the function $y = \sin(x)$ within the interval $[0, 2\pi]$

4) Use Newton polynomials of different degrees to interpolate the function $y = 1/x$ within the interval $[0.1, 3]$

5) Compute the interpolating polynomial computing the coefficients in the expression

$$P_n(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n$$

solving directly the systems of equations that results when using a table of $y = \ln(x)$ of five points and imposing that the interpolating polynomial goes through these points.

6) Interpolate the table of nodes

x	1	2	4	8
y	0.5	0.3	0.1	0.05

Imposing the condition $y(x)$ must be a function of the form:

$$y = a_0 + a_1 e^{-x} + a_2 e^{-2x} + a_3 e^{-3x}$$

7) Interpolate the polynomial

$$P_6(x) = \prod_{i=1}^{6} (x - i)$$

a) Using the nodes

$$x = (0, 1.5, 2.5, 3.5, 4.5, 5.5, 6.6)$$

b) Repeat the interpolation using the set of nodes

$$x = (0, 1, 2, 3, 4, 5)$$

c) Repeat the interpolation using the set of nodes

$$x = (0, 1.5, 2.5, 3.5, 4.5, 5.5, 6.6, 7.7)$$

8) Use the point interpolation polynomials to estimate the errors in the interpolation of the table

x	-1.5	0	1.0	1.6	2.1
y	0.2231	1.0000	2.7182	4.9530	8.1551

of the function $y = \exp(x)$ when interpolating the value of the function at the point $x = 1.2$. Repeat the analysis for the table

x	1	2	3	4	5
y	0.3679	0.1353	0.0498	0.0183	0.0067

of the function $y = \exp(-x)$ at the point $x = 3.2$. Finally use the table

x	6	7	8	9	10
y	0.0025	0.0009	0.0003	0.0001	0.0000

9) Fit a polynomial to the "perfidius polynomial"

$$pf(x) = \prod_{i=1}^{20}(x - i)$$

using the nodes

$$x = (0, 1, 2, \ldots, 19).$$

Repeat the analysis for the polynomial

$$p_{20}(x) = \prod_{i=1}^{20}(x - 2^{-i})$$

using the nodes

$$x = (1, 2^{-1}, 2^{-2}, \ldots, 2^{-19}).$$

10) Find the interpolating Polynomials of degree $4, 8, 18$ to the function

$$f(x) = \frac{1}{1 + x^2}$$

in the interval $[-5, 5]$ using equiespaced nodes. Repeat the analysis using the Chebysev nodes.

11) Use the following tables to compute the Hermite polynomial approximation of the following functions:

a)

$$f(x) = x\ln(x)$$

x	8.3	8.6
y	17.56492	18.50515
y'	3.116256	3.151762

b)

$$f(x) = x\cos(x) - 2x^2 + 3x - 1$$

x	0.1	0.2	0.3	0.4
y	−0.62049958	−0.28398668	0.00660095	0.24842440
y'	3.58502082	3.14033271	2.66668043	2.16529366

c)

$$f(x) = \ln(e^x + 2)$$

x	−1	−0.5	0.0	0.5
y	0.8619940	0.95802009	1.0986123	1.2943767
y'	0.15536240	0.23269654	0.3333333	0.45286776