Numerical Quadrature

1) Test the Composite Trapecium method to estimate the value of the integral:

$$I = \int_0^4 13(x - x^2)e^{-3x/2}dx = -1.5487883725278481333$$

2) Use the Composite Simpson method and the same number of divisions to obtain approximations to the integral

$$I = \int_0^4 13(x - x^2)e^{-3x/2}dx$$

3) Compare the different Newton-Cotes formulas to obtain estimations of the integral:

$$I = \int_0^{0.8} f(x)dx$$

where

$$f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5$$

4) Use the Composite Trapecium and the Composite Simpson to evaluate the erf function, defined as:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

for the value of $x = 0.5$. Compare your estimations with the tabulated value of $\mathrm{erf}(0.5) = 0.520500$.

5) Use the Trapecium rule to compute the integral of the following tabulated function:

x	0.0	0.12	0.22	0.32	0.36	0.40	0.44	0.54	0.64	0.70	0.80
$f(x)$	0.2000	1.3097	1.3052	1.7434	2.0749	2.4560	2.8430	3.5073	3.1819	2.3630	0.2320

Coming from the function

$$f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5$$

6) Wind forces exerted from the sails on the mast of a sailboat of total height $L = 30$, vary as a function of distance above the deck of the boat z. The total foce on the mast can be expressed as the integral of a continuous function:

$$F = \int_0^{30} 200\left(\frac{z}{5 + z}\right)e^{-2z/30}dz$$

This nonlinear integral is difficult to evaluate analytically. Use different methods to compute the value of the total force numerically and compare the results.

7) Use the Adaptive method to obtain approximations to the integral:

$$I = \int_0^4 13(x - x^2)e^{-3x/2}dx$$

8) The length of a cuve $y = g(x)$ is given by the integral

$$\int_a^b \sqrt{1 + (g'(x))^2}\, dx$$

Compute the length of the curve $y = \sin(3x)$ in the interval $[0, 2\pi]$

9) Use the Gaussian integration formulas up to order 5 to give estimations of the integrals:

$$I_1 = \int_{-1}^{1} x^4 - 3x^2 + 1\, dx$$

and

$$I_2 = \int_{2}^{3} x^4 - 3x^2 + 1\, dx$$

10) Use the Gaussian quadrature formula with $n = 4$ to compute approximations for the following integrals:

$$a)\ \int_0^1 \ln(1+x)dx = 2\ln 2 - 1$$

$$b)\ \int_0^1 \frac{1}{\sqrt{1+x^4}} dx = 0.92703733865069$$

$$c)\ \int_0^1 x(1-x^2)dx = \frac{1}{4}$$

$$d)\ \int_0^1 \frac{1}{1+x^3} = \frac{1}{3}\ln 2 + \frac{1}{9}\pi\sqrt{3}$$

$$e)\ \int_1^2 e^{-x^2} dx = 0.1352572580$$

11) Use the Gaussian quadrature formula with $n = 4$ to compute approximations for the following integrals:

$$a)\ \int_{-1}^1 \ln(1+x^2)dx = 2\ln 2 + \pi - 4$$

$$b)\ \int_{-1}^1 \sin^2 \pi x dx = 1$$

$$c)\ \int_{-1}^1 (x^8+1)dx = 20/9$$

$$d)\ \int_{-1}^1 e^{-x^2} dx = 1.493648266$$

$$e)\ \int_{-1}^1 \frac{1}{1+x^4} = 1.733945974$$

12) Use the MATLAB functions *integral2* and *integral3* to compute the multiple integrals

$$I_2 = \int_0^8 \int_0^6 2xy + 2x - x^2 - 2y^2 + 72\, dxdy$$

and

$$I_3 = \int_{-4}^4 \int_0^6 \int_1^3 x^3 - 2yz\, dxdydz$$