Exercices

1. Show that the equation $x^2 + ax + b = 0$ can be written as
$$(x - \alpha)^2 - \beta^2 = 0 \quad , \text{ or}$$
$$(x - \alpha)^2 = 0 \quad\quad , \text{ or}$$
$$(x - \alpha)^2 + \beta^2 = 0.$$

2. Prove that, if $x \geq -1$, then
$$(1 + x)^n \geq 1 + nx$$
for all positive integers n.

3. Prove by induction that $a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + \ldots + ab^{n-2} + b^{n-1})$.

4. Show that $|a + b| = |a| + |b|$ iff $ab \geq 0$.

5. A tank in the shape of an inverted cone is being filled with water. (See the figure. Express the volume of water in the tank as a function of the depth h.