Exercises Quantum Physics (GEMF) – Postulates

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1. **Eigenfunctions:** Are the following two functions eigenfunctions of the kinetic energy operator for a particle moving along the x-axis?

   (a) $e^{ikx} - 2\cos(kx)$

   (b) $\sin(kx) + 2\cos(2kx)$

2. **Linear momentum operator:** Calculate the partial derivative of the wave function of a one-dimensional free particle with respect to its position and use the Broglie relation to establish an expression of the linear momentum operator.

3. **Expectation values:** Calculate the expectation value of the position of a particle in the following quantum state

$$\psi(x) = \frac{1}{\sqrt{2}}\big[\psi_0(x) + \psi_1(x)\big]$$

$$\text{with } \psi_0(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2} \quad \text{and} \quad \psi_1(x) = \left(\frac{4\alpha^3}{\pi}\right)^{1/4} x e^{-\alpha x^2/2}$$

4. **Measurements 1:** Consider a system whose Hamiltonian is given by

$$\hat{H} = \mathcal{E}\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

   and an operator $\hat{A}$ defined as

$$\hat{A} = a\begin{pmatrix} 0 & 4 & 0 \\ 4 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

   (a) What are the possible outcomes of measuring the energy?

   (b) Immediately after measuring $-\mathcal{E}$ as energy of the system, the observable A represented by the operator $\hat{A}$ is measured. What values can be measured and with what probability?

   (c) What is the expectation value for observable A?

   (d) Calculate the uncertainty ΔA

5. **Measurements 2:** Consider the following two operators and a system described by $|\psi(t)\rangle$

$$\hat{A} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \hat{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad |\psi(t)\rangle = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

   (a) What is the probability to measure $A = -1$ at time t?

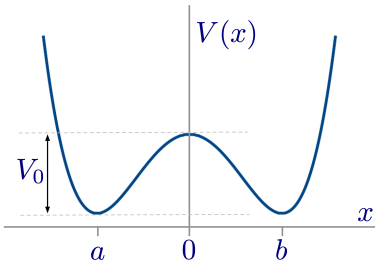
   (b) Immediately after measuring B, a measurement of A is performed. What is the probability of finding $B = 0$ and $A = 1$?

   (c) What is the probability of finding these values for A and B when the order of measurements is inverted?

6. **Time evolution:** Consider a two-dimensional Hilbert space whose basis vectors are $|\phi_a\rangle$ and $|\phi_b\rangle$, the left and right localized state vectors of a particle trapped in a double well potential. Being a symmetric potential, the energy of the two state vectors is the same, $\epsilon < V_0$. The Hamiltonian of the system is

$$\hat{H} = \begin{pmatrix} \epsilon & \gamma_{ab} \\ \gamma_{ab} & \epsilon \end{pmatrix}$$

   where γ_{ab} is a measure of the probability that the particle tunnels through the barrier, or in other words, the chance that the particle hops from a to b or *vice-versa*.



   (a) Find the eigenvalues and the corresponding eigenvectors of the system.

   (b) Are the left and right localized descriptions of the particle stationary? If not, what are the stationary states?

   (c) The system is prepared at $t = 0$ as $|\psi(x, t = 0)\rangle = |\phi_b\rangle$. Derive the time dependence of the wave function by (i) directly solving the time-dependent Schrödinger equation and (ii) applying the time-evolution operator.