Exercicis

Part A

3. Evaluate the following iterated integrals:

(a) $\int_{-1}^{1}\int_{0}^{1}(x^4y+y^2)\,dy\,dx$

(b) $\int_{0}^{\pi/2}\int_{0}^{1}(y\cos x+2)\,dy\,dx$

(c) $\int_{0}^{1}\int_{0}^{1}(xye^{x+y})\,dy\,dx$

(d) $\int_{-1}^{0}\int_{1}^{2}(-x\log y)\,dy\,dx$

5. Use Cavalieri's principle to show that the volumes of two cylinders with the same base and height are equal (see Figure 5.1.10).

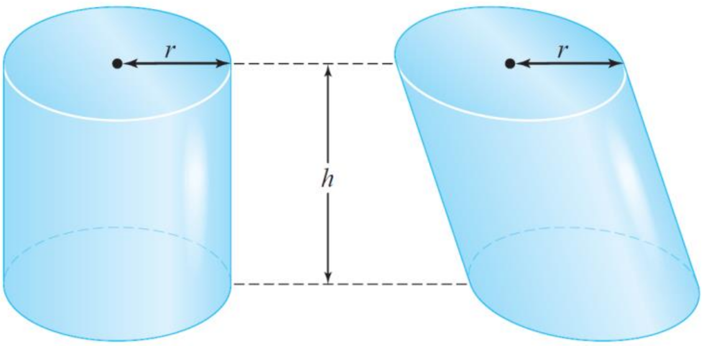

figure **5.1.10** Two cylinders with the same base and height have the same volume.

R is the rectangle $[0, 2] \times [-1, 0]$

9. $\iint_R (x^2y^2+x)\,dy\,dx$

10. $\iint_R \left(|y|\cos\frac{1}{4}\pi x\right)\,dy\,dx$

11. $\iint_R \left(-xe^x\sin\frac{1}{2}\pi y\right)\,dy\,dx$

13. Evaluate the iterated integral:

$$\int_0^1 \int_0^1 (3x + 2y)^7 \; dx\, dy.$$

Part B

13. Consider the integral in 2(a) as a function of m and n; that is,

$$f(m, n) := \iint\limits_R x^m y^n \, dx\, dy.$$

Evaluate $\lim_{m,n\to\infty} f(m, n)$.

14. Let:

$$f(m, n) := \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \cos nx \sin my \, dx\, dy.$$

Show that $\lim_{m,n\to\infty} f(m, n) = 0$.

15. Let $f\colon [0, 1] \times [0, 1] \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} 1 & x \text{ rational} \\ 2y & x \text{ irrational.} \end{cases}$$

Show that the iterated integral $\int_0^1 \left[\int_0^1 f(x, y)\, dy \right] dx$ exists but that f is not integrable.

Part C

3. Evaluate the following iterated integrals and draw the regions D determined by the limits. State whether the regions are x-simple, y-simple, or simple.

(a) $\displaystyle\int_0^1 \int_0^{x^2} dy\, dx$

(b) $\displaystyle\int_1^2 \int_{2x}^{3x+1} dy\, dx$

(c) $\displaystyle\int_0^1 \int_1^{e^x} (x + y)\, dy\, dx$

(d) $\displaystyle\int_0^1 \int_{x^3}^{x^2} y\, dy\, dx$

11. Let D be the region given as the set of (x, y), where $1 \le x^2 + y^2 \le 2$ and $y \ge 0$. Is D an elementary region? Evaluate $\iint_D f(x, y)\, dA$, where $f(x, y) = 1 + xy$.

19. Let D be a region given as the set of (x, y) with $-\phi(x) \le y \le \phi(x)$ and $a \le x \le b$, where ϕ is a nonnegative continuous function on the interval $[a, b]$. Let $f(x, y)$ be a function on D such that $f(x, y) = -f(x, -y)$ for all $(x, y) \in D$. Argue that $\iint_D f(x, y)\, dA = 0$.

Part D

5. Change the order of integration and evaluate:

$$\int_0^1 \int_{\sqrt{y}}^1 e^{x^3}\, dx\, dy.$$

7. If $f(x, y) = e^{\sin(x+y)}$ and $D = [-\pi, \pi] \times [-\pi, \pi]$, show that

$$\frac{1}{e} \le \frac{1}{4\pi^2} \iint_D f(x, y)\, dA \le e.$$

9. If $D = [-1, 1] \times [-1, 2]$, show that

$$1 \le \iint_D \frac{dx\, dy}{x^2 + y^2 + 1} \le 6.$$

10. Using the mean-value inequality, show that

$$\frac{1}{6} \le \iint_D \frac{dA}{y - x + 3} \le \frac{1}{4},$$

where D is the triangle with vertices $(0, 0)$, $(1, 1)$, and $(1, 0)$.

Part E

15. $\int_0^1 \int_1^2 \int_2^3 \cos\left[\pi(x+y+z)\right] dx\, dy\, dz$

16. $\int_0^1 \int_0^x \int_0^y (y+xz)\, dz\, dy\, dx$

17. $\iiint_W (x^2+y^2+z^2)\, dx\, dy\, dz$; W is the region bounded by $x+y+z=a$ (where $a>0$), $x=0$, $y=0$, and $z=0$.

18. $\iiint_W z\, dx\, dy\, dz$; W is the region bounded by the planes $x=0$, $y=0$, $z=0$, $z=1$, and the cylinder $x^2+y^2=1$, with $x \geq 0$, $y \geq 0$.

19. $\iiint_W x^2 \cos z\, dx\, dy\, dz$; W is the region bounded by $z=0$, $z=\pi$, $y=0$, $y=1$, $x=0$, and $x+y=1$.

For the regions in Exercises 25 to 28, find the appropriate limits $\phi_1(x)$, $\phi_2(x)$, $\gamma_1(x, y)$, and $\gamma_2(x, y)$, and write the triple integral over the region W as an iterated integral in the form

$$\iiint_W f\, dV = \int_a^b \left\{ \int_{\phi_1(x)}^{\phi_2(x)} \left[\int_{\gamma_1(x,y)}^{\gamma_2(x,y)} f(x, y, z)\, dz \right] dy \right\} dx.$$

25. $W = \{(x, y, z) \mid \sqrt{x^2+y^2} \leq z \leq 1\}$

26. $W = \{(x, y, z) \mid \frac{1}{2} \leq z \leq 1 \text{ and } x^2+y^2+z^2 \leq 1\}$

27. $W = \{(x, y, z) \mid x^2+y^2 \leq 1,\ z \geq 0 \text{ and } x^2+y^2+z^2 \leq 4\}$

28. $W = \{(x, y, z) \mid |x| \leq 1,\ |y| \leq 1,\ z \geq 0 \text{ and } x^2+y^2+z^2 \leq 1\}$