Exercises - Special relativity - Dynamics

2.1 4-vectors and velocity addition. In A's frame, B moves to the right with speed u, and C moves to the left with speed v. What is the speed of B with respect to C? Use the properties of 4-vectors to derive the velocity-addition formula.

2.2 4-vectors and relative velocity. In the lab frame, two particles move with velocity v along the paths shown in Fig. 1. The angle between the trajectories is 2θ. What is the velocity of one particle as viewed by the other?

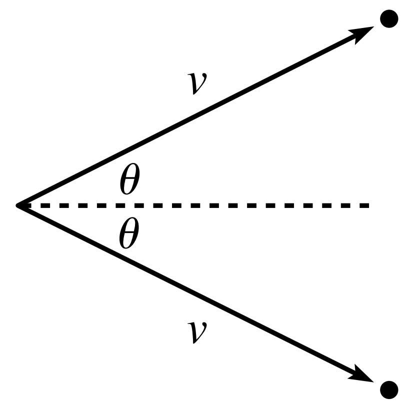

Figure 1: Exercise 2.

2.3 Colliding photons. Two photons each have energy E. They collide at an angle θ and create a particle of mass M. What is M?

Hint: All you need to know about photons here is that they are massless particles.

2.4 Relativistic collision. A particle with mass m and energy E approaches an identical particle at rest. They collide elastically in such a way that they both scatter at an angle θ relative to the incident direction (Fig. 2). What is θ in terms of E and m? What is θ in the relativistic and nonrelativistic limits?

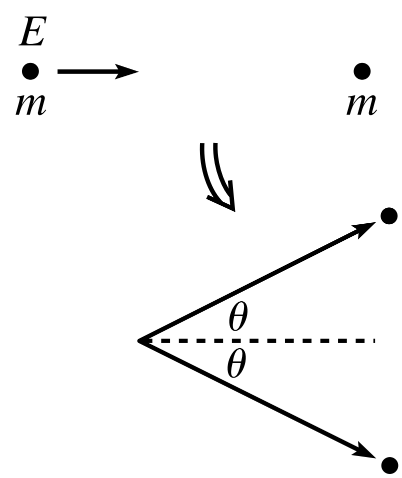

Figure 2: Exercise 4.

2.5 Decay at an angle. A particle with mass M and energy E decays into two identical particles. In the lab frame, one of them is emitted at a 90 degree angle, as shown in Fig. 3. What are the energies of the created particles?

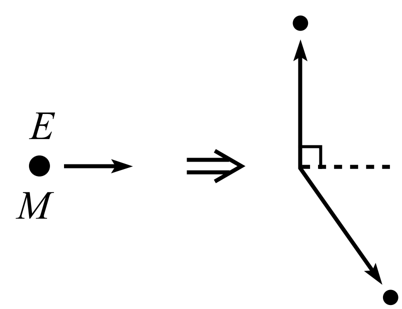

Figure 3: Exercise 5.

2.6 Relativistic collision. A particle of mass M and energy E collides head-on elastically with a stationary particle of mass m. Show that the final energy of mass M is

$$E' = \frac{2mM^2 + E\left(m^2 + M^2\right)}{2Em + m^2 + M^2} \tag{1}$$

Hint: This problem gets a bit messy, but you can save yourself some of the mess by noting that $E' = E$ must be a solution of the equation you get for E'. (Why?)

Obtain and discuss the following limits for E':

(a) $E \approx M$, that is, the particle is barely moving.

(b) $M = m$.

(c) $m \gg E > M$.

(d) $E > M \gg m$ and $M^2 \gg Em$.

(e) $E > M \gg m$ and $Em \gg M^2$.

(f) $E \gg m \gg M$.

2.7 4-VECTORS AND RELATIVISTIC LAWS OF MOTION. Let V and F be, respectively, the velocity and force 4-vectors of a particle. Using that $V^\mu V_\mu = -1$ and assuming that the mass of the particle is constant, prove that $V^\mu F_\mu = 0$ and, from here, that $dE/dt = \vec{f} \cdot \vec{v}$, where $\vec{f}$ and $\vec{v}$ are the Newtonian force and velocity, respectively.[1]

[1] This expression implies that the change in total energy of the particle equals the work per unit time exerted by the force, just as in Newtonian physics.