

Misperceptions of Randomness: Revisiting the Hot Hand Fallacy

Phil166

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In their seminal, 1974 work, “Judgment under Uncertainty: Heuristics and Biases,” Amos Tversky and Daniel Kahneman asked the question, “how do people assess the probability of an uncertain event or the value of an uncertain quantity?” This question is fundamental to the human experience – day-to-day life is replete with ambiguity we must somehow navigate. In perceiving the distance of an object, or assessing the intended meaning of a speaker’s utterance, we converge on some pragmatic estimate of the “truth,” despite the constraints of limited cognitive resources and often-incomplete information. In his work, “Presumptive meanings,” the Linguist Stephen Levinson provided a guiding analogy using the Rembrandt sketch shown in Figure 1:

We interpret this sketch instantly and effortlessly as a gathering of people before a structure, probably a gateway; the people are listening to a single declaiming figure in the center. [...] But all this is a miracle, for there is little detailed information in the lines or shading (such as there is). Every line is a mere suggestion [...]. So here is the miracle: from a merest, sketchiest squiggle of lines, you and I converge to find adumbration of a coherent scene.



Figure 1. Rembrandt sketch

How do we converge on the scene of a speaker (likely Christ) before a gateway from mere squiggles? How do we navigate everyday uncertainty in the face of cognitive constraints and ambiguity in the environment? Tversky and Kahneman (1974) argued that, for a start, we likely rely on a set of heuristic principles. Such principles, they claim, are beneficial because they reduce the complexity of the tasks we face to “simpler judgmental operations.” But these heuristics may also lead to “severe and systematic errors.”

Current paper

In the following paper, we review one of the more popular errors studied within this framework – the divergence of people’s intuitive beliefs of randomness from the basic laws of chance. To do so, we revisit the controversial topic of the “Hot Hand” in Basketball, first orienting the reader to the basic framework provided by Tversky & Kahneman (1974) (T&K) and then turning to the original paper by Gilovich, Vallone & Tversky (1985), which investigated the Hot Hand phenomenon in basketball. We then review more recent work, which has called into question findings raised by the 1985 paper. Finally, we conduct a statistical analysis using a new data set of every shot taken during the 2014-15 NBA season, re-examining some of the analyses from Gilovich, Vallone & Tversky (1985) and assessing several of the shortcomings proposed by recent work. We end with a summary of our findings.

Tversky & Kahneman (1974)

Through a series of empirical investigations, T&K described a set of basic cognitive heuristics we may employ in everyday decision-making and cognition. In

particular, they described three such heuristics:

- 1) *Representativeness*
- 2) *Availability*
- 3) *Adjustment and Anchoring*

We will focus on the heuristic of *representativeness* as it plays a role in our current work. *Representativeness* encodes the idea that in answering questions such as “What is the probability that A belongs to class B?” or “What is the probability that A originates from process B?” people often do so by relying on the *degree to which A is representative of B*. This seems intuitive enough - if A originates from process B, it likely shares similar characteristics of B. However, T&K showed that *representativeness* can also lead to a number of misconceptions, some of which I enumerate here: judgments based on stereotyping, insensitivity to prior probabilities of outcomes, insensitivity to sample size, and insensitivity to predictability.

Representativeness

Critically, reliance on the *representativeness* heuristic can also lead to systematic *misconceptions of chance*. T&K found that people expected that sequences of events generated by a particular process should be *representative* of that process, even for short sequences. They gave an example of judging the origin of a coin toss. They presented participants with three sequences of coin tosses and asked which was more likely to be produced by an ordinary fair coin:

- 1) H-T-H-T-T-H
- 2) H-H-H-T-T-T

3) H-H-H-H-T-H

Respondents systematically believed that sequence (1) was more likely to have come from a fair coin than sequences (2) or (3). This is surprising for two reasons. First, sequence (2) has the same number of heads and tails as sequence (1) - it has the exact same likelihood of being generated by a fair coin as sequence (1). Second, while sequence (3) contains more heads than tails and does not appear to represent the fairness of a fair coin, this is from only a small ($n=6$) sample of tosses. T&K took these results as evidence that people expected that the “essential characteristics of the process [which generated the sequences] will be represented not only globally in the entire sequence, but also locally in each of its parts.”

This belief in the representativeness of local sequences is unwarranted statistically and forms the basis of the well-known “Gamblers Fallacy.” People generally believe that after a long sequence of an outcome, such as a long sequence of a particular color in a game of roulette, the next turn is more likely to be a different color. Simply put, if there has been a long run of red, you should bet black. T&K argued that misconceptions of this sort are widespread not only in “naïve” subjects, but also in experienced research psychologists. They dubbed this belief the “law of small numbers,” which reflects the expectation that “valid hypotheses about a population will be represented by a statistically significant result,” with little regard to the size of the sample.

T&K proposed that these systematic departures from the laws of probability were deeply rooted in our beliefs and perceptions of randomness. While tosses of a coin provide an easy characterization of chance events, we can apply the same

framework to any sequence of binary outcomes. Shots of a professional basketball player, for example, where success is a basket made and failure is a miss, fit this formulation nicely. We now turn to the original study, which adopted this formulation, to study the widespread perception of the Hot Hand or streakiness in basketball.

Gilovich, Vallone & Tversky (1985)

In “The Hot Hand in Basketball: On the Misperception of Random Sequences,” Gilovich, Vallone & Tversky (GVT) investigated the widespread belief that certain NBA players occasionally perform at levels well above what would be considered normative based on historical performance. A player who is “hot” is believed to have shifted into a heightened level of performance, possibly due to increased focus, attention or motor control, or to perform better following recent success. In both circumstances, the expected outcome of this heightened ability is more runs of successful shots (streak shooting). This can be realized as longer and more frequent runs, or as an increased probability of a made shot after making previous shots. Clearly, this perception of a Hot Hand is also consistent with the *misperceptions of random sequences* found in T&K (1974) and forms the basis of GVT’s investigation.

GVT explored the Hot Hand phenomenon in three studies. In the first, they surveyed a population of basketball fans to assess the degree to which people believed in the Hot Hand. In the second study, they examined shot data from an NBA team during the 1981 season, performing four analyses. Finally, GVT also conducted a controlled shooting experiment with players from an intercollegiate varsity basketball team. Given our use of NBA season data in the current study, we will

focus on the first two parts of the GVT paper in the next subsections and refer to elements of their third (the controlled shooting experiment) in a later section.

GVT Study 1

To assess the extent to which the Hot Hand phenomenon existed in the socio-psychological realm of the average basketball fan, GVT recruited one hundred basketball fans from Cornell and Stanford University. *Fans* were defined as people who played basketball “occasionally” and watched at least 5 games per year. Participants were given a questionnaire, which examined their beliefs regarding sequential dependence among shots. Results indicated a widespread belief in the Hot Hand phenomenon. 91% of participants believed that a player had a better chance of making shot after having just made two or three shots than after having missed two or three shots. 96% believed that players tend to make more shots after having made a series of shots in a row. 84% believed that a team should pass the ball to someone who has just made several shots in a row (i.e. distribute the ball to the Hot Hand).

GVT also asked participants to estimate the shooting percentage of a hypothetical player (with an overall 50% shooting percentage) after having just made a previous shot vs. after having just missed a previous shot. Participants believed that the shooting percentage for the hypothetical player would be higher on average (61%) after having made their previous shot, than having missed their previous shot (42%).

Clearly the perception of Hot Hand shooting in basketball was widespread in 1985 and recent studies indicate that it continues today (Aharoni & Sarig, 2011;

Attali, 2013; Cao, 2011). To test these intuitions formally, GVT conducted a statistical analysis of actual NBA shooting data, analyzing the records from 48 home games for the Philadelphia 76ers 1981 season. GVT conducted four analyses. First they examined the probability of a made shot conditioned on players' recent history of hits and misses. Second they examined the frequency of different sequences of hits and misses in players' shooting records, examining runs. Next, they tried to inspect "occasional" hot or cold streaks. Finally, they analyzed the stability of a players' performance across games.

GVT Study 2

GVT examined the conditional probabilities of nine players on the Philadelphia 76ers during the 1980-81 season. They compared the probability of making a shot conditioned on having missed the previous 1..3 shots and the probability of making a shot conditioned on having made the previous 1..3 shots. They found that for eight out of the nine players, the probability of making a shot after having made the previous shot was actually lower than after having missed the previous shot, a finding at odds with Hot Hand predictions. We conduct a modified version of this test in our analysis.

In their second analysis, GVT conducted a Wald-Wolfowitz run test comparing the observed number of runs for each player with their expected number of runs (given their shooting percentage). Five of the nine players actually had *more* runs than would be expected, a finding that also ran counter to Hot Hand predictions (fewer runs indicates more streaks of shorter lengths). We re-examine this in our new data set, examining runs data for over 300 players.

In their third analysis, GVT examined the idea that the previous two tests may not have had sufficient power to detect “occasional ‘hot’ stretches” embedded in longer stretches of normal performance. To do this they partitioned the entire shot history for each player into non-overlapping sets of four shots. They then counted the number of sets in which performance was “high” (three or four shots made) vs “moderate” (two shots made) vs “low” (one or zero shots made), to assess the hypothesis that if a player is occasionally hot then their record would have more “high” performance sets than expected by chance. They did not see evidence for streak shooting in this analysis, performing a series of χ^2 tests, none of which reached significance. They repeated this analysis four times, starting the partition of shot histories at different points, however all analyses failed to support Hot Hand predictions.

In their final analysis, GVT assessed the hypothesis that players might display more hot or cold nights, comparing their overall variability in per game shooting percentage with what would be expected on the basis of their overall history. To do so, GVT compared the standard deviation of each player’s per game shooting percentage and one derived from the player’s overall shooting percentage across their season history. If the Lexis ratio of these standard errors for a given player was greater than one, then shooting percentages were fluctuating more than would be expected by chance alone. GVT did not find evidence for significant variation in shooting percentage based on this analysis, providing evidence that players did not appear to have more hot or cold nights than expected by chance alone.

From these analyses GVT concluded that there was no evidence for streak

(Hot Hand) shooting in the NBA data they analyzed. Importantly, they did cite the possible confounds that accompany this type of data. For example, they were not able to account for the relative difficulty of separate shots, which depend on the nature of the shot (type of shot, distance) as well as elements of the defense (how closely the shooting player is guarded), for example.

Miller & Sanjurjo (2014)

Findings from GVT have roiled sports aficionados, especially basketball fans, for the last 30 years. Many basketball fans and professionals continue to believe in the Hot Hand (Aharoni & Sarig, 2011; Attali, 2013; Cao, 2011) while formal studies have had difficulty identifying the phenomenon, especially while using in-game basketball data. In “A Cold Shower for the Hot Hand Fallacy” (Miller & Sanjurjo, 2014) the authors avoided in-game basketball data entirely, instead conducting a controlled shooting experiment with semi-professional shooters. After orienting the reader to the experimental design in Miller & Sanjurjo (2014) (M&S) and their findings, we highlight the specific limitations of in-game data they propose.

Experimental set-up

M&S conducted a controlled shooting experiment in two phases. In the first phase they tested whether any individual shooter in their sample showed evidence for the Hot Hand. In the second phase, they tested whether they could identify a Hot Hand out of sample.

The M&S sample consisted of semi-professional players from the Santo Domingo de Betanzos team in Spain. In total, they collected shot data from eight players from this team in both Phase 1 and 2, due to availability and time

constraints. A shooting session for a player consisted of an initial warm-up phase in which the experimenter observed the player. The experimenter identified an approximate distance from which the shooter appeared to make 50 percent of his shots. (This was done to maximize the variance of shot outcomes.) This position was marked on the court. In a given session, the player would take 300 shots from the marked position with a monetary insensitive for specific shots made. After every shot a trained rebounder retrieved the ball and held it from a fixed location near the basket. To signal the beginning of the next shot (at which point the rebounder would pass the ball to the shooter), the experimenter initiated a computer-generated tone. On average, the next shot occurred approximately 7 seconds after the tone.

Phase 2 was conducted six months later using the same protocol as in Phase 1. M&S collected additional data for one of the players who was perceived by his teammates as the “hottest” shooter, in order to maximize the power of their tests for this particular shooter.

This experimental set-up differed from the original controlled shooting experiment (GVT’s study 3) in several ways. While shooters bet before every shot in GVT, possibly leading to interaction effects of betting and shooting, shooters in M&S only shot (no betting). The incentive schemes in M&S were constant throughout the experiment while they accumulated over time and changed according the players’ bets in GVT. Shooters in M&S always shot from the same location, whereas shooters in GVT were forced to move after every shot. M&S collected 300 shots per session, while GVT collected only 100.

Measures of interest

Given this system for data collection M&S assessed three statistics which measured (1) how often each player was “hot” (on a streak), (2) a player’s shooting percentage conditioned on having a recent streak of made shots, and (3) the length of a player’s longest streak. In general M&S, took a “streak” to be three or more made shots in a row. To measure statistic (1), which they call H_F , M&S examined the relative frequency of shots that immediately followed a streak by taking the ratio of the number of shot subsets that immediately followed a streak S_H and the number of shots in a set of shots $|S|$.

$$H_F = \frac{|S_H|}{|S|}$$

To measure whether the conditional hit rate of a player immediately following a streak of hits was better than would be expected for a player with a constant hit rate, M&S measured what they call the “hit streak momentum statistic” $|H_M|$ which was defined as the shooting percentage of the shot immediately following a streak of hits:

$$H_M = \frac{\sum_{s \in S_H} x_s}{|S_H|}$$

where S_H is the subset of shots that immediately follow a streak of hits, x_s is the value of the shot s (1 if the shot is made).

To measure whether hit streaks were longer than would be expected if the player had a constant hit rate, M&S measured a statistic they called the “hit streak length statistic,” $|H_L|$, which simply measured the length of the longest run of hits, assessing the probability of observing a streak of this length given the assumption

H_L is normally distributed.

$$H_L = \max_{H_0 \in \mathcal{H}_0} |H_0|$$

where H_0 is a run of hits (consecutive made shots flanked by misses) and \mathcal{H}_0 is the collection of all runs of hits such that $H_0 \in \mathcal{H}_0$.

M&S Findings

M&S argued that the three statistics enumerated above better captured the effects of Hot Hand shooting than in the controlled GVT experiment, through increase statistical power and improved identification. They conduct two analyses using these measures, one at the group level and one at the individual level. In particular, the authors identified one player for whom all the measures were significant, stating that the analysis of his data “demonstrates that an individual can have a substantial hot hand effect that ... systematically re-occurs across time.” M&S also saw evidence for Hot Hand effects with seven of the eight players having H_F and H_L statistics significantly above expected values. M&S went on to re-analyze data from previous controlled experiments of this sort, including the original GVT study, finding evidence for Hot Hand shooting in individual players.

M&S confounds of in-game data

On the basis of their empirical findings, M&S argued that the Hot Hand phenomenon likely exists, but that it is only detectable in highly controlled experimental domains. They suggested that their empirical findings form the basis for the widespread conception of the hot-hand and that it “likely exists in games and is detectable by decision makers.” M&S argue that any statistical analysis of in-game data is vulnerable to both false positive and false negatives without controlling for

such sources of variability. In particular, a shooter may appear “hot” when they have just taken a string of relatively easy shots, either due to a specific offensive strategy or the dynamics of a particular game (a blowout in which the other team has essentially forfeited). Likewise the effects of a “hot” shooter may be dampened by a defense that responds by guarding the player more closely, or the player attempting more difficult shots. Controlling for variables such as the quality of opponents, the fatigue or health of the player, the surrounding teammates, the league standing, and the current play, all represent possible sources of variation that could account for a player’s particular performance, potentially dampening or over-emphasizing Hot Hand effects.

Motivation for the current analysis

While M&S argued their study provided conclusive evidence for the existence of the Hot Hand phenomenon, it is at odds with a fundamental component of the phenomenon – most people who believe in the Hot Hand do not observe highly controlled shooting experiments and yet they are still convinced of the phenomenon. Not only that, they believe the phenomenon is incredibly robust. In GVT’s first study, the authors found that the average basketball fan believed an *arbitrary player* who shot with 50% accuracy should improve to 61% upon making their previous shot and decline to 42% having missed their previous shot. Finding evidence of occasional streakiness does not account for the magnitude of this belief in the Hot Hand nor how widespread it appears to be.

In our current analysis, we improve on some of the shortcoming from some of GVT’s in-game data, using a novel data set of every shot taken during the 2014-15

NBA season. We reassess some of the original analyses from the GVT study, investigate one of the potential confounds proposed by M&S, and finally, try to predict shot outcomes while controlling for *player*, *shot* and *game* level information.

Summary of the data

We scraped the entire shot history for the 2014-15 NBA season from *stats.nba.com* as well as player specific information for individual shooters. Table 1 includes a summary of the dataset along with custom feature engineering.

Player Data	Shot Data	Game Data
Age	Current hit streak*	Game ID
Height	Current miss streak*	Time
ID	Distance X	Quarter
Name	Distance Y	-
Position	Made/Missed	-
Team	Time since previous shot*	-
Weight	Shot Type*	-
Years in league	Shot Zone	-

Table 1. Data included in custom dataset. (*) Denotes features not included in scraped data.

Global Hot Hand

Perhaps the most striking finding from the original GVT study was the regular and robust belief that an arbitrary basketball player should be more likely to hit a shot having made their previous shot. To test this global assumption we examined the probability of making a shot conditioned on the outcome of the previous 1...4 shots. Figure 2 plots the conditional probabilities of made shots across all shots from the 2014-15 season. To examine this hypothesis simply, we conduct a two-sample t-test, comparing the probability of making a shot

conditioned on a previous made shot vs a previous miss. Results indicate that the probability of making a shot after a miss is significantly greater than after a make ($t(98635) = 1260.5, p < 0.001$). While this is clearly a crude measure, generalizing over all shots by all players, it makes a test of the *global assumption* that the probability of a made shot by an arbitrary player should increase after a make. Counter to Hot Hand predictions and in-line with GVT's earlier findings, the probability of a make actually *decreases* after a made shot. Of course, this may be an effect of some of the confounds suggested by M&S – perhaps players' shooting percentage decreases after a made shot because they are guarded more closely or take harder shots. We investigate one of these (whether streak shooting predicts shot distance) in a later analysis.

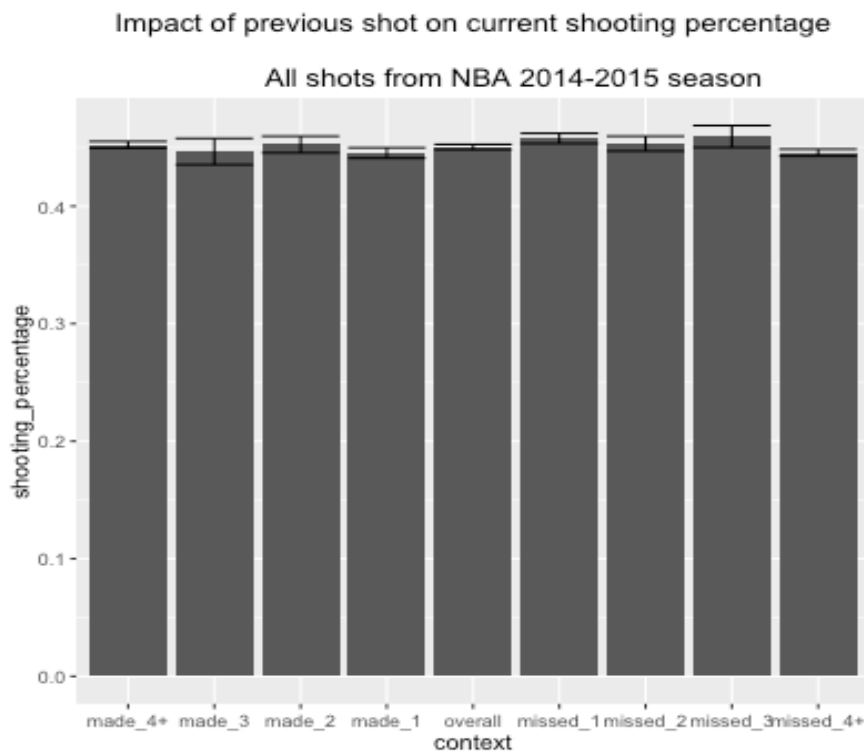


Figure 2. Probability of making a shot conditioned on the outcome of the previous shot across all players in the 2014-15 NBA season.

Runs Analysis

Moving from the global indications of the Hot Hand to individual players, we replicate the run-analysis GVT conducted. GVT ran Wald-Wolfowitz run tests for each player in their dataset. In this test, each sequence of hits or misses is counted as a run. The more a player's hits and misses cluster together, the fewer runs they have (and the longer their hot/cold streaks). GVT did not find any evidence for significant runs. We repeat this test, analyzing all players who have taken over 100 shots in the season. This left us with a sample of 353 players from the 2014-15 season. Figure 3 plots Z-scores for each player's run test. If a player has significantly *less* runs than expected we should expect *negative* Z-scores (observed runs – expected runs). Results indicate that only four players had significant Z-scores ($p < 0.01$), however these scores were all *positive* indicating that the player's actually had more runs than was expected by chance.

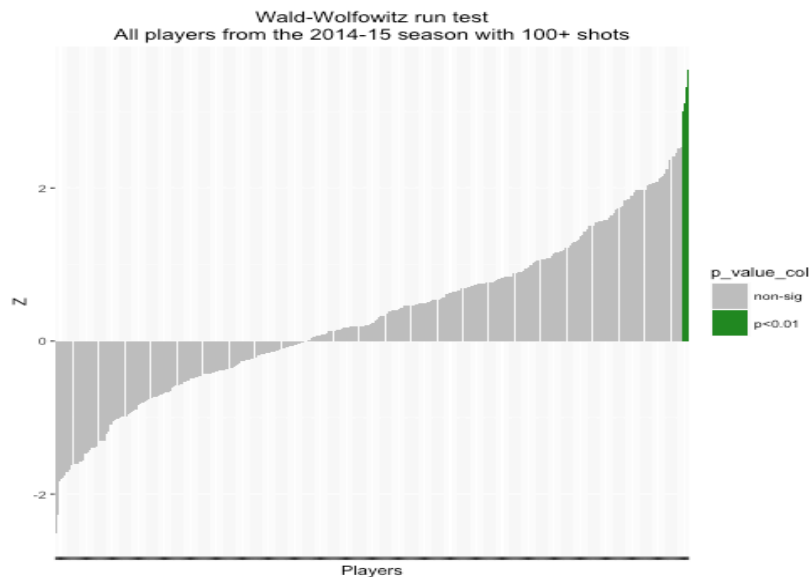


Figure 3. Wald-Wolfowitz run tests for all players from the 2014-15 NBA season who took at least 100 shots. Green coloring denotes a significant p-value ($p < 0.01$). Players with significantly more runs than expected by chance should display negative scores. Vertical axis plots z-scores. Player names are excluded from the horizontal axis for readability.

Do “Hot” players take harder shots?

M&S argue that a fundamental problem with assessing in-game shot data is the potential confounds. In particular, they suggest that a “hot” player will likely take harder shots, either because they believe they are more likely to hit the shot or because of increased pressure from the opposing team’s defense. We examine this hypothesis fitting a mixed effects model, regressing *shot distance* on *current streak* (hit or miss) with *player* and *position* random effects. Results indicate that players are significantly more likely to take longer shots as their made shot runs increase ($\beta = 0.272$, $t(196,900) = 12.4$, $p < 0.001$) and players are significantly more likely to take shorter shots as missed shot runs increase ($\beta = -0.315$, $t(196,900) = -17.3$, $p < 0.001$) (see Figure 4). This finding appears to support Hot Hand intuitions / M&S’s concerns with in-game data – players appear to adjust their shots to their (apparent) current level of performance. Of course, identifying this effect does not confirm or deny the presence of Hot Hand shooting. However, we attempt to control for it in our next analysis, including *shot distance* as a predictor in a regression model.

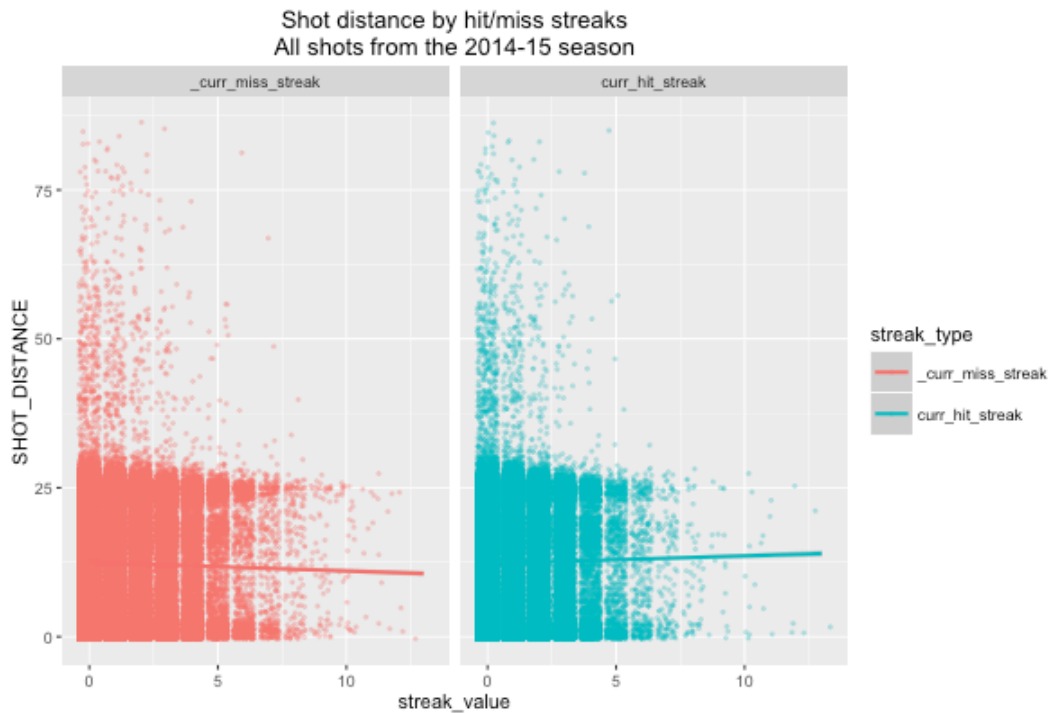


Figure 4. Impact of shot distance on recently made or missed shots. The vertical axis plots the shot distance (in feet from the hoop). The horizontal axis plots the current hit/miss streak value (a value of 5 in the right facet indicates that a player made their previous 5 shots). Regression lines are plotted with standard error shading.

Predicting shots, controlling for in-game variables

The fundamental analysis we're most interested is whether we can predict made shots. To this end, we fit a logistic regression model, including player and game specific factors. In particular, we saw that *shot distance* appeared to increase as players hit more shots, which indicates that players *may believe* they are hot or cold and choose shots accordingly. It does not provide direct evidence of the Hot Hand phenomenon, however. We regress shot outcome (made or missed) on the features included in table 2.

Model features

Shot features	Player features	Game features
Distance	Height	Quarter
Type	Weight	Time
Length of current hit or miss streak	Age	-
Point value	Years in league	-

Table 2. Features included in logistic regression model.

Controlling for the game, player and shot features, results indicate that there is approximately a 1% increase in the odds of making a shot following a *missed shot* ($\beta = 0.01, z = 2.22, p = 0.02$). The odds of making a shot following a *made* shot are not significant in our model ($\beta = -0.004, z = -0.779, p = 0.236$).

General Discussion

The Hot Hand phenomenon has received much attention from psychologists, statisticians, economists and sports fans alike for the last 40 years. In the flurry of claims for and against, one item has stood out – the discrepancy between the *magnitude* and *ubiquity* of the belief in the Hot Hand and *magnitude* of the evidence for it. While M&S claimed that their controlled shooting experiment and modified statistical tests provide evidence for the effect (an argument for existence), they do little to explain its ubiquity in basketball culture. This is the true challenge of any analysis of the Hot Hand phenomenon – to account for magnitude of its psychological effect. No study to date has yet provided evidence to counter the argument that the *ubiquity* of the Hot Hand phenomenon is not a misrepresentation of intuitive beliefs about representativeness and chance. We believe that such evidence likely needs to come from in-game data rather than controlled shooting

experiments, as game data is the overwhelming source of evidence for Hot Hand believers.

In our current study, we introduced a new data set including every shot from the 2014-15 season and player-specific information. We conducted several analyses -- employed two of the original tests by GVT, investigated a confound predicted by M&S, and tried to predict shot outcomes while controlling for *game*-, *player*- and *shot-level* information. Like the last 40 years of Hot Hand studies, our results are mixed. We certainly don't see evidence for an effect as sizeable as would be predicted by fan intuitions. However, we did see an effect of shot history on shot distance, a confound predicted by M&S. We attempted to control for a variety of in-game factors at the *game*, *player* and *shot* level, however we were still unable to detect a Hot Hand effect. We believe there is an important distinction to be made here -- simply detecting streak shooting is one challenge, explaining its ubiquity in basketball fan intuitions is another. As player tracking technology becomes more ubiquitous in sports like NBA basketball, we may soon have the ability to account for all the in-game confounds proposed by Hot Hand proponents such as M&S. Until then, the Hot Hand remains an elusive and enticing story of our perception (or misperception) of randomness and our ability to accommodate everyday uncertainty.

Note:

1) I've included a short summary of the analyses here:

https://htmlpreview.github.io/?https://github.com/benpeloquin7/hot_hand/blob/master/analysis/hot_hand_analysis.html

2) Data collection (scraping) code:

https://github.com/benpeloquin7/hot_hand/tree/master/scraping_code

3) Dataset:

https://github.com/benpeloquin7/hot_hand/tree/master/data

References

- Aharoni, G. & Sarig, G. (2011). Hot hands and equilibrium. *Applied Economics*, 44, 2309-2320.
- Attali, Y. (2013). Perceived hotness Affects Behavior of Basketball Players and Coaches, *Psychological Science*.
- Brocckocksy, A., Ezekowitz, J., & Stein, C. (2014). The Hot Hand: A New Approach to an Old "Fallacy". 2014 Research Paper Competition presented by ticketmaster and Harvard University.
- Cao, Z. (2011). Essays on Behavioral Economics. PhD thesis, Oregon State University.
- Gilovich, T., Vallone, R., & Tversky, A. (1985). The Hot Hand in Basketball: On the Misperception of Random Sequences. *Cognitive Psychology*. 17 (295-314).
- Miller, J. & Sanjurjo, A. (2014). A Cold Shower for the Hot Hand Fallacy. *IGIER Working Paper*.
- Tversky, A., & Kahneman, D. (1973). Availability: A heuristic for judging frequency and probability. *Cognitive Psychology*, 5, 207-232.
- Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science (Washington D.C.)*, 185, 1124-1131.