

DP-3

✓1) Subset Sum Problem

$s = \{6, 2, 3, 1\}$ $\text{sum} = 5$

i/p:-

arr[] ✓

n ✓

sum ✓

o/p:-

yes/no

$0/1$
 $6 \quad 3 \quad 2 \quad 1 \rightarrow n$
 $2 \quad 2 \quad 2 \quad 2$
 $2^4 = 16$

Optimal SS:

of elements
in set/array

$ss(i, \text{sum}) =$

target sum \rightarrow

$ss(i-1, \text{sum})$
 $||$
 $ss(i-1, \text{sum} - a_i)$

$\times 6 \quad 3 \quad 2 \quad 1,$
 $\underbrace{\hspace{1.5cm}}$
 $\{2,$

$\text{sum} = 3$
 $\Rightarrow \text{true}$

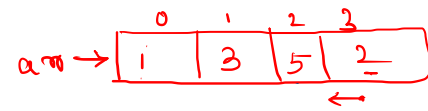
$;$ sum == 0 $\rightarrow \{ \}$
 $;$ i == 0
 $;$ $a_i > \text{sum}$
 $;$ o/w

// SubSet sum problem

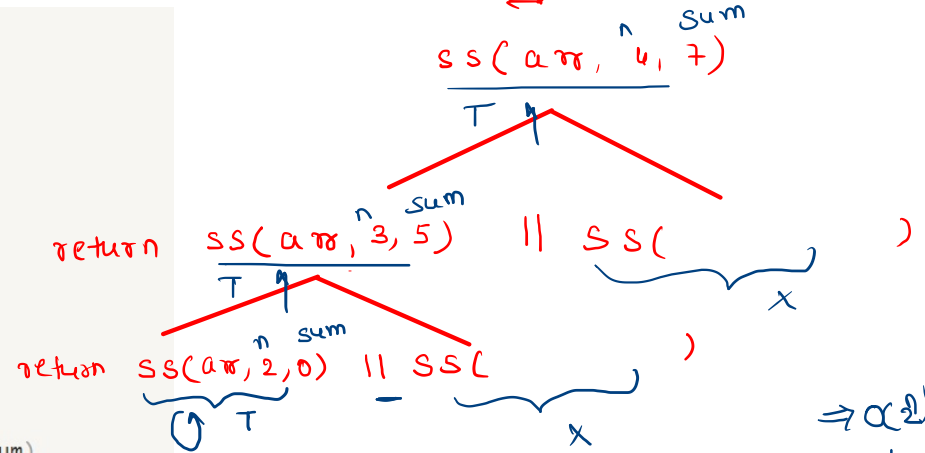
1) Recursive code

```
function ss(arr[], n, sum)
{
    ✓ if(sum==0)
        return true // empty set
    if(n==0) ✗ ✗
        return false
    if(arr[n-1]>sum) ✗ ✗
        return ss(arr, n-1, sum)
    → else
        return ss(arr, n-1, sum-arr[n-1]) || ss(arr, n-1, sum)
}
```

↳ logical OR operator

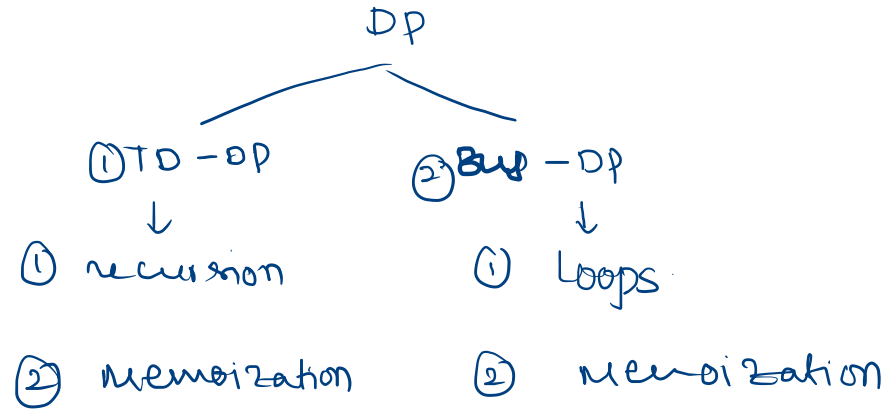


n=4, sum=7



⇒ O(2^n)
↳ exponential

✓	A	B	A B
	F	F	F
	<u>F</u>	<u>T</u>	T
	<u>T</u>	<u>F</u>	T
	<u>T</u>	<u>T</u>	T



dp [n+1] [sum+1]
↑ ↑

o/p

arr[]

n → # of elements

sum → weight

(1cs) ✓

2) TD-DP code for Subset Sum problem

```
dp[n+1][sum+1]={-1} // fill all the values with -1
function ss(arr[], n, sum)
{
    if(sum==0)
        return true // empty set
    if(n==0)
        return false
    if(dp[n][sum] != -1)
        return dp[n][sum]
    if(arr[n-1] > sum)
        return dp[n][sum] = ss(arr, n-1, sum)
    else
        return dp[n][sum] = ss(arr, n-1, sum-arr[n-1]) || ss(arr, n-1, sum)
}
```

BUP - DP for ss - problem.

arr \rightarrow

0	1	2	3
5	3	1	2

, sum = 6
n = 4

dp[n+1][sum+1]

5 x 7

$j \downarrow$
 \vdots
 \checkmark

\rightarrow

sum $\rightarrow j$

	0	1	2	3	4	5	6
$i \rightarrow 0$	T	F	F	F	F	F	F
1	T						
2	T						
3	T						
4	T						

of elements in array

\downarrow
n
 i

True/False

0
 \rightarrow T/F

5 x 7

3) BUP-DP code for Subset Sum problem

```
function ss(arr[],n,sum)
{
    dp[n+1][sum+1];
    for(i=0;i<=sum;i++)//first row
        dp[0][i]=false

    for(i=0;i<=n;i++)//first col
        dp[i][0]=true

    for(i=1;i<=n;i++)
    {
        for(j=1;j<=sum;j++)
        {
            if(arr[i-1]>j)
                dp[i][j]=dp[i-1][j]
            else
                dp[i][j]=dp[i-1][j-arr[i-1]] || dp[i-1][j]
        }
    }
    return dp[n][sum]
}
```

$n \rightarrow i$
 $sum \rightarrow j$

$$\checkmark \quad s = \{ 2, 3, 1, \underline{5} \} \quad \text{sum} = 5$$

✓ 2) Total number of subsets with the given sum

$$\{ 2, 3 \} \Rightarrow 2+3=5 \checkmark$$

$$\{ 5 \} \Rightarrow \textcircled{a} \underline{1}$$

	0	1	2	3	4	5	6
0	1	0	0	0	0	0	0
1	1						
2	1						
3	1						
4	1						

5X7
 → Final Count

4) BUP-DP code for number of subsets whose sum is given sum

```
function ss(arr[],n,sum)
{
    dp[n+1][sum+1];
    for(i=0;i<=sum;i++)//first row
        dp[0][i]=0 .

    for(i=0;i<=n;i++)//first col
        dp[i][0]=1

    for(i=1;i<=n;i++)
    {
        for(j=1;j<=sum;j++)
        {
            if(arr[i-1]>j)
                dp[i][j]=dp[i-1][j]
            else
                dp[i][j]=dp[i-1][j-arr[i-1]]+ dp[i-1][j]
        }
    }
    return dp[n][sum]
}
```

$n \rightarrow i$
 $sum \rightarrow j$

of subsets

(*)

3) Un Bounded KS

Rod-cutting
(*)



Input : $W = 8$

$val[] = \{10, 40, 50, 70\}$

$wt[] = \{1, 3, 4, 5\}$

Output : 110

We get maximum value with one unit of weight 5 and one unit of weight 3.

Bounded KS → 0/1
(*) UB KS →

	o_1	o_2	o_3	o_4	$C = 7$
P_i	5	7	9	10	✓
w_i	<u>2</u>	<u>1</u>	<u>3</u>	<u>4</u>	

$(22) = 10 + 5 + 7$

UB → 49



```
function ks(p[], wt[], w, n)
{
    dp[n+1][w+1];

    for(i=0; i<=w; i++) // 1st row all zeros
        dp[0][i]=0
    for(i=0; i<=n; i++)
        dp[i][0]=0

    for(i=1; i<=n; i++)
    {
        for(j=1; j<=w; j++)
        {
            if(wt[i-1]>j)
                dp[i][j]=dp[i-1][j]
            else
                dp[i][j]=max( p[i-1]+ dp[i][w-wt[i-1]], dp[i-1][j] )
        }
    }

    return dp[n][w]
}
```