

Разбор домашнего задания №4

Тренировки по ML

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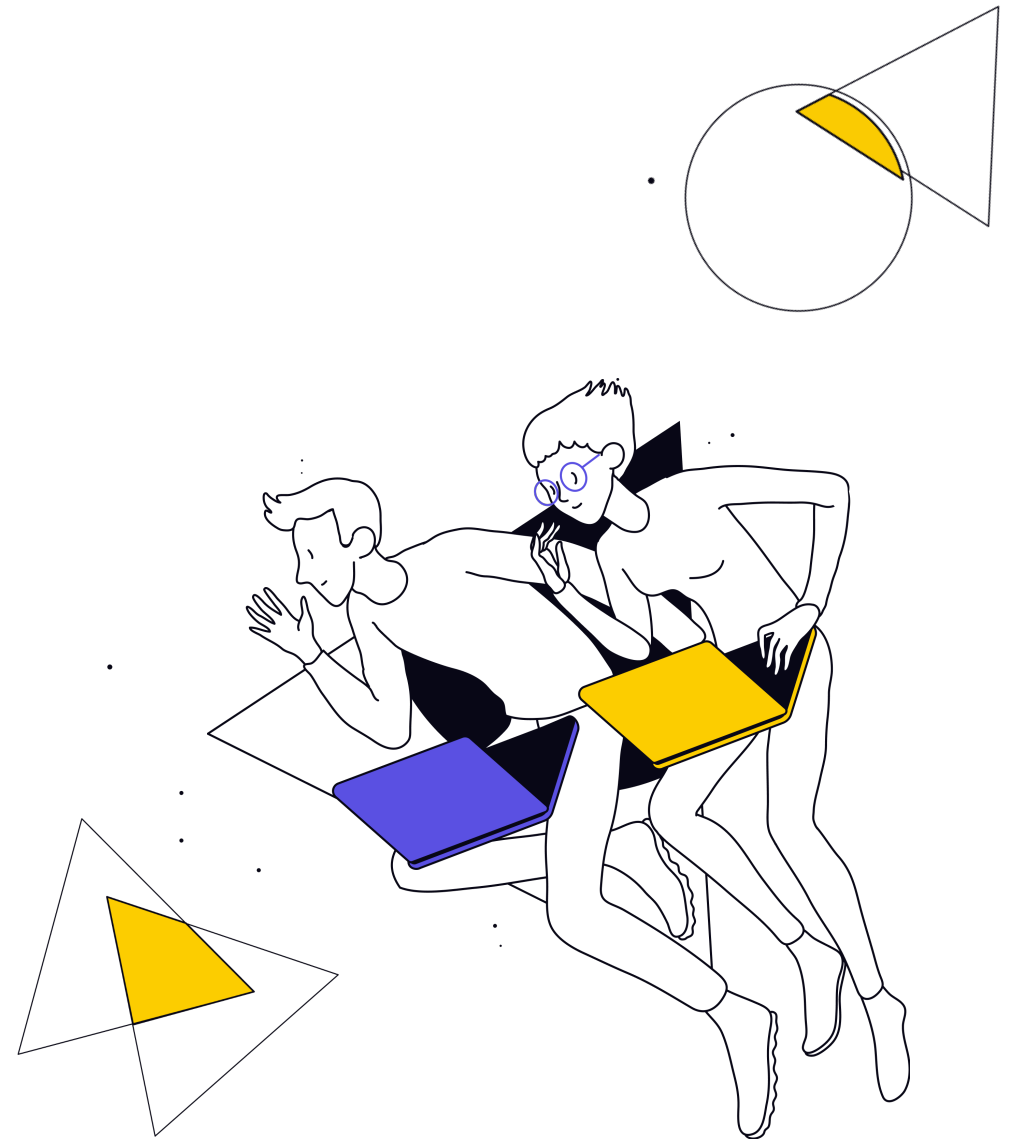


Outline

01 Reminder of models

02 Feature selection

03 Baseline review



Linear regression

Linear regression problem statement:

01 Dataset $\{(\mathbf{x}^{(i)}, y^{(i)})\}_{i=1}^n$, where $\mathbf{x}^{(i)} \in \mathbb{R}^p, y^{(i)} \in \mathbb{R}$

02 The model is linear:

$$\hat{y} = w_0 + \sum_{k=1}^p x_k w_k = // \mathbf{x} = [1; x_1, \dots, x_p] // = \mathbf{x}^\top \mathbf{w}$$

, where $\mathbf{w} = [w_0; w_1, \dots, w_p]$ is bias term

03 Least squares method (MSE minimization) provides a solution:

$$\hat{\mathbf{w}} = \arg \min_{\mathbf{w}} \|\mathbf{Y} - \mathbf{X}\mathbf{w}\|_2^2$$

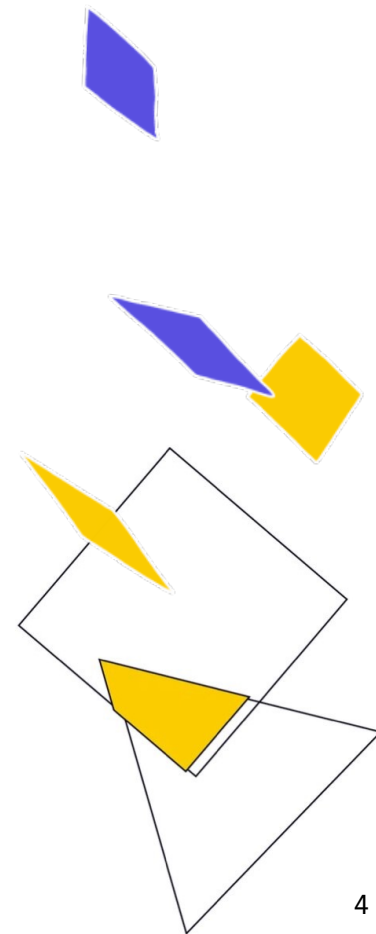
Logistic regression

$$X \in R^{n \times p}$$

$$Y \in C^n \quad \text{e.g. } C = \{-1, 1\}$$

$$|C| < +\infty$$

$$c(X) = \hat{Y} \approx Y$$



Logistic regression

Just to remind

$$\log L(w|X, Y) = \log P(X, Y|w) = \log \prod_{i=1}^n P(x_i, y_i|w)$$

Calculating probabilities for objects

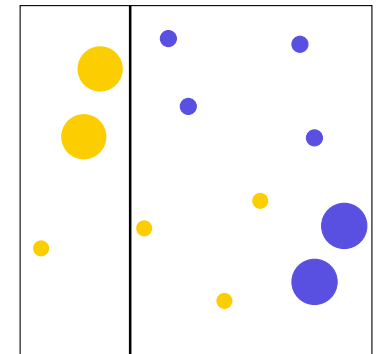
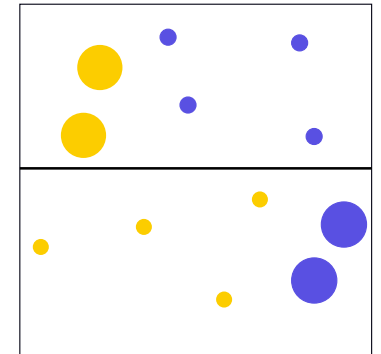
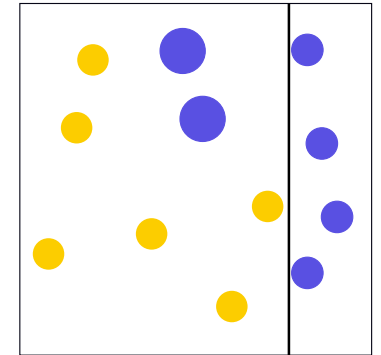
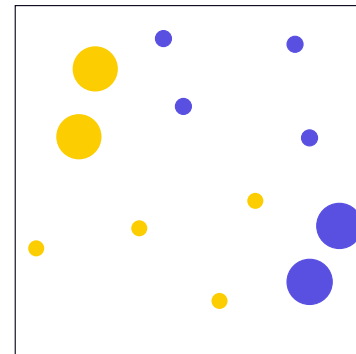
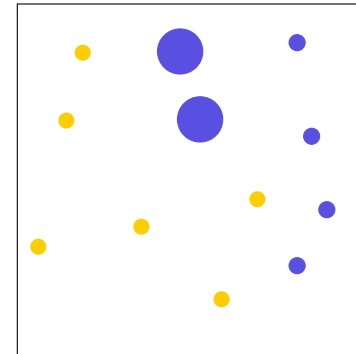
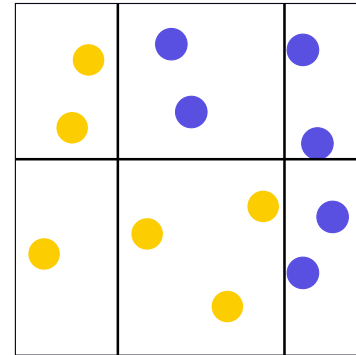
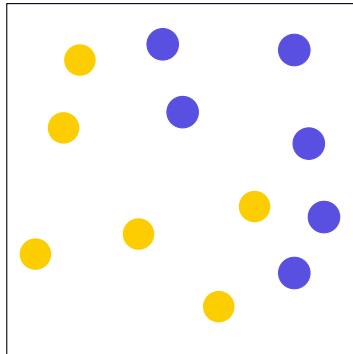
$$\text{if } y_i = 1 : P(x_i, 1|w) = \sigma_w(x_i) = \sigma_w(M_i)$$

$$\text{if } y_i = -1 : P(x_i, -1|w) = 1 - \sigma_w(x_i) = \sigma_w(-x_i) = \sigma_w(M_i)$$

$$\log L(w|X, Y) = \sum_{i=1}^n \log \sigma_w(M_i) = - \sum_{i=1}^n \log(1 + \exp(-M_i)) \rightarrow \max_w$$

Boosting: intuition

Binary classification
Use decision stumps



Gradient boosting

01
$$\hat{f}_{T-1}(\mathbf{x}) = \sum_{t=0}^{T-1} g_t(\mathbf{x}),$$

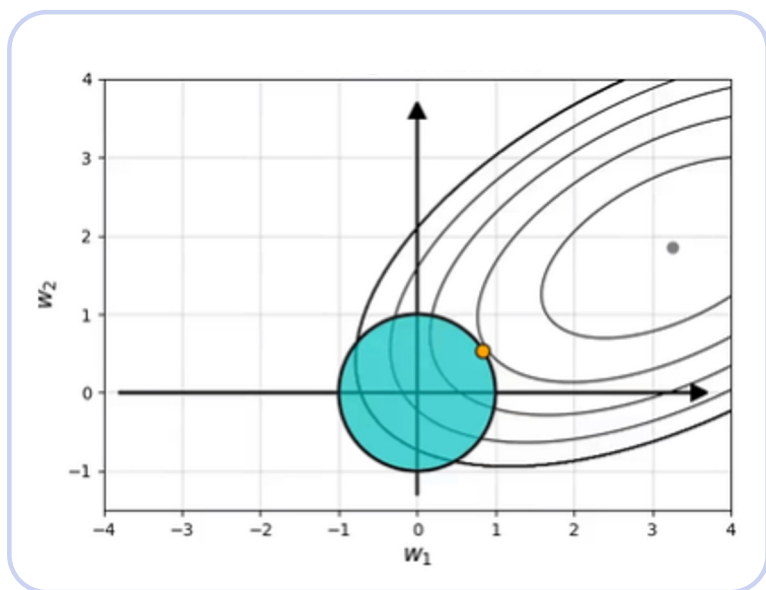
02
$$r_t^{(i)} = - \left[\frac{\partial L(y^{(i)}, f(\mathbf{x}^{(i)}))}{\partial f(\mathbf{x}^{(i)})} \right]_{f(\mathbf{x})=\hat{f}_T(\mathbf{x})}, \text{ for } i = 1, \dots, n,$$

03
$$\boldsymbol{\theta}_T = \arg \min_{\boldsymbol{\theta}} \sum_{i=1}^n \left(r_t^{(i)} - h(\mathbf{x}^{(i)}, \boldsymbol{\theta}) \right)^2,$$

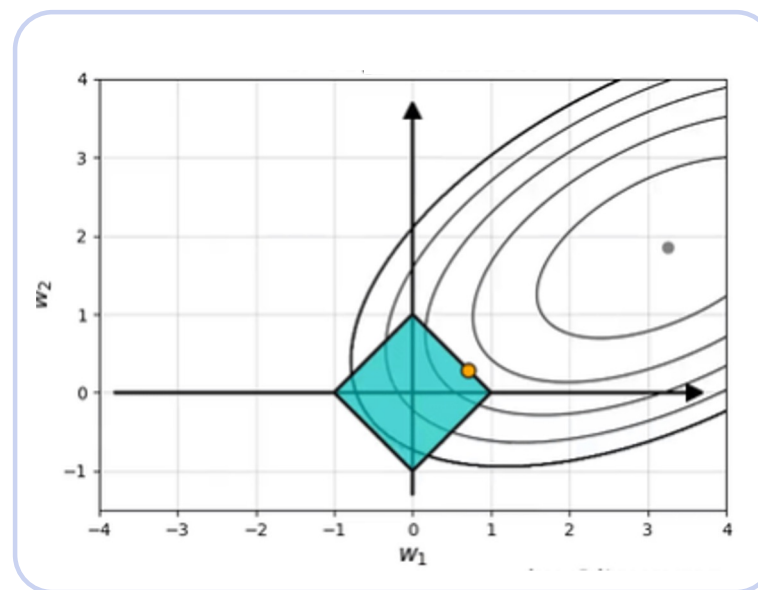
04
$$\rho_t = \arg \min_{\rho} \sum_{i=1}^n L \left(y^{(i)}, \hat{f}_{t-1}(\mathbf{x}^{(i)}) + \rho \cdot h(\mathbf{x}^{(i)}, \boldsymbol{\theta}_T) \right)$$

Regularization

L2 regularization $\|w\|_2^2$



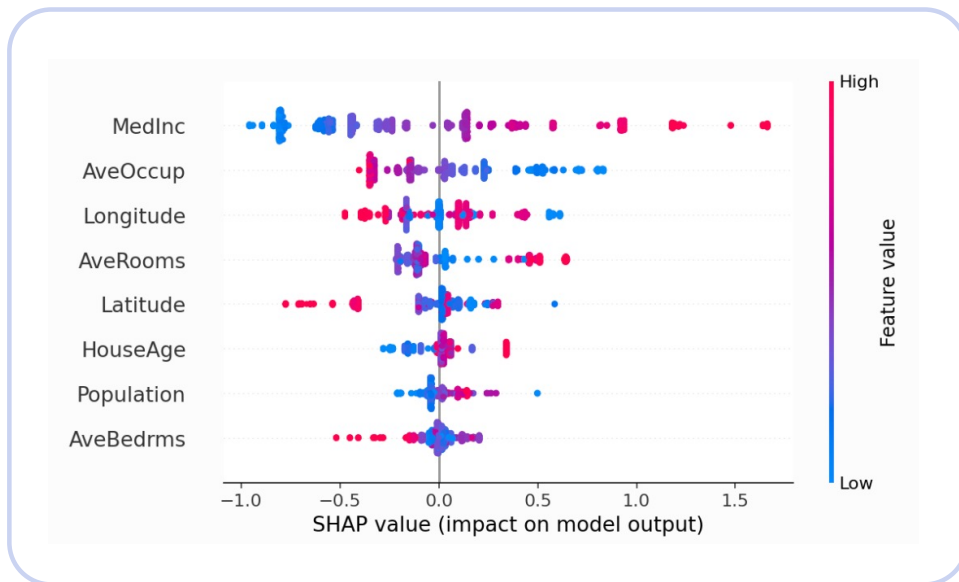
L1 regularization $\|w\|_1$



source: <https://people.eecs.berkeley.edu/~jrs/189/>

Shap values

$$\phi_i(p) = \sum_{S \subseteq N / \{i\}} \frac{|S|! (n - |S| - 1)!}{n!} (p(S \cup \{i\}) - p(S))$$



$p(S \cup \{i\})$ - this is the prediction of the model with the i-th feature

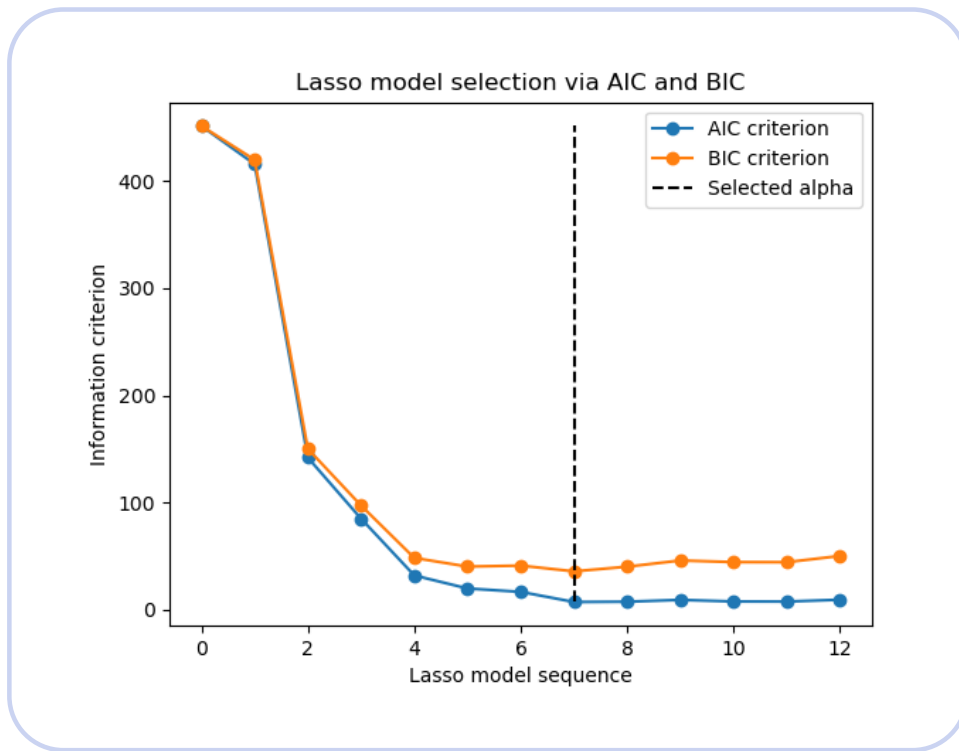
$p(S)$ - this is a prediction of the model without the i-th feature

n - number of features

S - an arbitrary set of features without the i-th feature

source: https://shap.readthedocs.io/en/latest/example_notebooks/overviews/

Information-criteria based model selection



$$AIC = -2 \log(\hat{L}) + 2d$$

$$BIC = -2 \log(\hat{L}) + \log(N) d$$

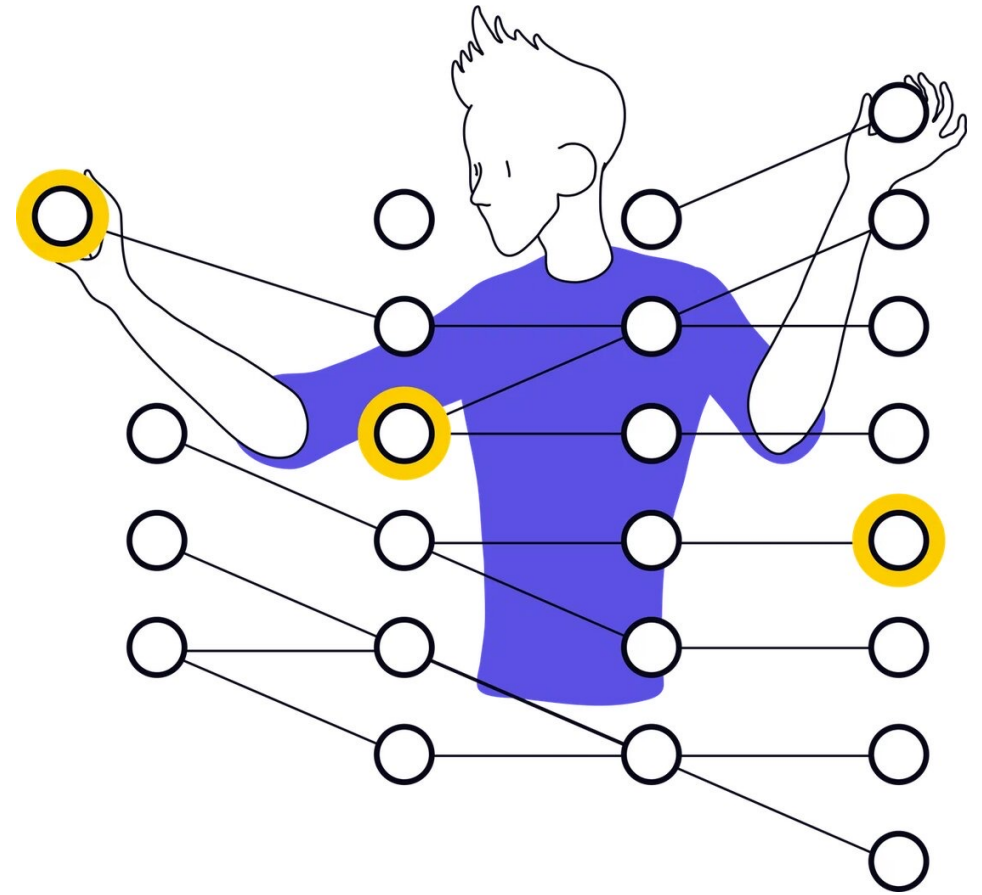
\hat{L} - is the maximum likelihood of the model

d - is the number of parameters

N - is the number of samples

source: https://scikit-learn.org/dev/modules/linear_model.html

Lifecode



Thanks for attention!

Questions?

