Parameter Estimation

1) Mean $\to \theta_1$

Variance $\to \theta_2$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

Take log.

$$\log L(\theta_1, \theta_2) = -\frac{n}{2} \log(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

for θ_1 diff $\log(L(\theta_1, \theta_2))$ wrt θ_1 & set it to zero.

$$\frac{\partial \log(L)}{\partial \theta_1} = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

M.L.E of θ_1 is sample mean

for θ_2 diff wrt θ_2 & put zero.

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

2) $B(m, \theta)$

$m \to$ no. of trials.

$\theta = (0,1)$ prob. of success.

$$L_\theta = \prod_{i=1}^{n} f(x_i; n, \theta)$$

PMF $f(x, n, \theta) = {}^{n}C_{x}\, \theta^{x} (1-\theta)$

$$L(\theta) = \prod_{i=1}^{n} ({}^{n}C_{x_i}) \cdot (\theta^{x_i}) (1-\theta)^{m-x_i}$$

Take log.

$$\log(L(\theta)) = \sum_{i=1}^{n} \log({}^{m}C_{x_i}) + \sum_{i=1}^{n} x_i \log(\theta) + \sum_{i=1}^{n} (m - x_i) \log(1-\theta)$$

$$\frac{\partial \log(L)}{\partial \theta} = \frac{1}{\theta} \sum_{i=1}^{n} x_i - \frac{1}{1-\theta} \sum_{i=1}^{n} (m - x_i) = 0$$

$$\frac{1}{\theta} \sum_{i=1}^{n} x_i = \frac{1}{1-\theta} \sum_{i=1}^{m} (m - x_i)$$

Multiply both sides $\theta(1-\theta)$

$$(1-\theta) \sum_{i=1}^{m} x_i = \theta \sum_{i=1}^{n} (m - x_i)$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{m}$$

MLE of θ for $B(n, \theta)$ is $\bar{x}$ where

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$