

Deep Independently Recurrent Neural Network (IndRNN)

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Abstract—Recurrent neural networks (RNNs) are known to be difficult to train due to the gradient vanishing and exploding problems and thus difficult to learn long-term patterns and construct deep networks. To address these problems, this paper proposes a new type of RNNs with the recurrent connection formulated as Hadamard product, referred to as independently recurrent neural network (IndRNN), where neurons in the same layer are independent of each other and connected across layers. The gradient vanishing and exploding problems are solved in IndRNN, and thus long-term dependencies can be learned. Moreover, an IndRNN can work with non-saturated activation functions such as ReLU (rectified linear unit) and be still trained robustly. Different deeper IndRNN architectures, including the basic stacked IndRNN, residual IndRNN and densely connected IndRNN, have been investigated, all of which can be much deeper than the existing RNNs. Furthermore, IndRNN reduces the computation at each time step and can be over 10 times faster than the commonly used Long short-term memory (LSTM). Experimental results have shown that the proposed IndRNN is able to process very long sequences and construct very deep networks. Better performances have been achieved on various tasks with IndRNNs compared with the traditional RNN and LSTM.



1 INTRODUCTION

LONG-TERM dependency is important for many applications. Especially for applications processing temporal sequences such as action recognition [1], [2], [3] and language processing [4], [5], the past information is important for the recognition of the future events. There are also applications exploring spatial context information such as scene segmentation [6] and spatial pooling [7]. To explore the long-term dependency, recurrent neural networks (RNNs) [8] have been widely used and have achieved impressive results. Compared with the feed-forward networks such as the convolutional neural networks (CNNs), a recurrent connection is added where the hidden state at the previous time step is used as an input to obtain the current state, in order to keep memory of the past information. The update of the hidden states at each time step follows:

$$\mathbf{h}_t = \sigma(\mathbf{W}\mathbf{x}_t + \mathbf{U}\mathbf{h}_{t-1} + \mathbf{b}) \quad (1)$$

where $\mathbf{x}_t \in \mathbb{R}^M$ and $\mathbf{h}_t \in \mathbb{R}^N$ are the input and hidden state at time step t , respectively. $\mathbf{W} \in \mathbb{R}^{N \times M}$, $\mathbf{U} \in \mathbb{R}^{N \times N}$ and $\mathbf{b} \in \mathbb{R}^N$ are the weights for the current input and the recurrent input, and the bias of the neurons, respectively. σ is an element-wise activation function of the neurons, and M and N are the dimension of the input and the number of neurons in this RNN layer, respectively.

Due to the recurrent connections with repeated multiplication of the recurrent weight matrix, training of the RNNs suffers from the gradient vanishing and exploding problem. Despite the efforts in initialization and training techniques [9], [10], [11], [12], it is still very difficult to learn long-term dependency. Several RNN variants such as the long short-term memory (LSTM) [13], [14] and the gated recurrent

unit (GRU) [15] have been proposed to address the gradient problems. However, the use of the hyperbolic tangent and the sigmoid functions as the activation function in these variants results in gradient decay over layers. Consequently, construction and training of a deep LSTM or GRU based RNN network is practically difficult.

On the other hand, the existing RNN models share the same component $\sigma(\mathbf{W}\mathbf{x}_t + \mathbf{U}\mathbf{h}_{t-1} + \mathbf{b})$ in (1), where the recurrent connection connects all the neurons through time. This makes it hard to interpret and understand the roles of each individual neuron (e.g., what patterns each neuron responds to) without considering the others. Moreover, with the recurrent connections, matrix product is performed at each time step and the computation cannot be easily paralleled, leading to a very time-consuming process when dealing with long sequences.

In this paper, we propose a new type of RNN, referred to as independently recurrent neural network (IndRNN). In the proposed IndRNN, the recurrent inputs are processed with the Hadamard product as $\mathbf{h}_t = \sigma(\mathbf{W}\mathbf{x}_t + \mathbf{u} \odot \mathbf{h}_{t-1} + \mathbf{b})$. This provides a number of advantages over the traditional RNNs including:

- Able to process longer sequences: the gradient vanishing and exploding problem is solved by regulating the recurrent weights, and long-term memory can be kept in order to process long sequences. Experiments have demonstrated that an IndRNN can well process sequences over 5000 steps.
- Able to construct deeper networks: multiple layers of IndRNNs can be efficiently stacked, especially with skip-connection and dense connection, to increase the depth of the network. An example of 21-layer residual IndRNN and deep densely connected IndRNN are demonstrated in the experiments.
- Able to be robustly trained with non-saturated functions such as ReLU: with the gradient backpropa-

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gation through time better behaved, non-saturated function such as ReLU can be used as the activation function and be trained robustly. IndRNN with ReLU is used throughout the experiments.

- Able to interpret the behaviour of IndRNN neurons independently without the effect from the others: since the neurons in one layer are independent of each other, each neuron's behaviour can be interpreted individually. Moreover, the relationship between the range of the memories and the recurrent weights are established through gradient backpropagation, and the memories learned by the task can be understood by visualizing the recurrent weights, which is shown in experiments.
- Reduced complexity. With the new recurrent connections based on element-wise vector product, which is much more efficient than the matrix product, the complexity of IndRNN is greatly reduced compared with the traditional RNNs (over 10 times faster than the cuDNN LSTM).

Experiments have demonstrated that IndRNN performs much better than the traditional RNN and LSTM models on the tasks of the adding problem, sequential MNIST classification, language modelling and action recognition. With the advantages brought by IndRNN, we are able to further show:

- Better performance can be achieved with deeper IndRNN architectures as verified for the sequential MNIST classification, language modelling and skeleton-based action recognition tasks.
- Better performance can be achieved by learning with longer dependency as verified for the language modelling tasks.

Part of this paper has appeared in the conference paper [16] where IndRNN is introduced and verified on some tasks without further analysing its advantage. Significant extension has been made in this paper. 1) New deep IndRNN architecture, densely connected IndRNN is proposed to enhance the feature reuse in addition to the residual IndRNN architecture. 2) The relationship between memory and recurrent weight is established through gradient backpropagation, and the learned memories are visualized for skeleton-based action recognition as an example. 3) More experiments are added including a new task, i.e., word-level language modelling. 4) Experiments are conducted to verify that IndRNN with longer temporal dependency and deeper architecture can achieve better performance. 5) A faster implementation with CUDA optimization is provided and made publicly available, which can be over 10 times faster than the cuDNN LSTM.

The rest of this paper is organized as follows. Section 2 describes the related work in the literature. Section 3 presents the proposed IndRNN with its gradient backpropagation through time process. It also describes the relationship between the recurrent weight and memory, and its complexity compared with the existing methods. Section 4 explains different deep IndRNN architectures and Section 5 presents the experimental results. Finally, conclusions are drawn at Section 6

2 RELATED WORK

It is known that a simple RNN suffers from the gradient vanishing and exploding problem due to the repeated multiplication of the recurrent weight matrix, which makes it very difficult to train and capture long dependencies. In order to solve the gradient vanishing and exploding problem, long short-term memory (LSTM) [17] was introduced, with a constant error carousel (CEC) to enforce a constant error flow through time. Multiplicative gates including input gate, output gate and forget gate are employed to control the information flow, resulting in many more parameters than the simple RNN. A well known LSTM variant is the gated recurrent unit (GRU) [15] composed of a reset gate and an update gate, which reduces the number of parameters to some extent. It has been reported in various papers [14] that GRU achieves similar performance as LSTM. There are also some other LSTM variants [13], [14], [18], [19] reported in the literature. However, these architectures [13], [14], [18], [19] generally take a similar form as LSTM and show a similar performance as well, so they are not discussed further. LSTM and its variants use gates on the input and the recurrent input to regulate the information flow through the network. However, the use of gates based on the recurrent input prevents parallel computation and thus increases the computational complexity of the whole network. To reduce the complexity and process the states of the network over time in parallel, QRNN (Quasi-Recurrent Neural Network) [20] and SRU (Simple Recurrent Unit) [21] were proposed where the recurrent connections are fixed to be identity connection and controlled by gates with only input information, thus making most of the computation parallel. While this strategy greatly simplifies the computational complexity, it reduces the capability of their RNNs since the recurrent connections are no longer trainable. By contrast, the proposed IndRNN solves the gradient exploding and vanishing problems without losing the power of trainable recurrent connections and without involving gate parameters. Moreover, IndRNN reduces computation and runs much faster than LSTM and even SRU [21].

There are also some simple RNN variants trying to solve the gradient vanishing and exploding problem by altering the recurrent connections. In [22], a skip RNN was proposed where a binary state update gate is added to control the network to skip the processing of one time step. In this way, fewer time steps may be processed and the gradient vanishing and exploding problem can be alleviated to some extent. In [23], [24], a unitary evolution RNN was proposed where the recurrent weights are empirically defined in the form of a unitary matrix. In this case, the norm of the back-propagated gradient can be bounded without exploding. In [25], a Fourier Recurrent Unit (FRU) was proposed where the hidden states over time are summarized with Fourier basis functions and the gradients are bounded. However, such methods usually introduce other transforms on the recurrent weight which complicates the recurrent units, making them hard to use and interpret. On the contrary, the proposed IndRNN further simplifies the recurrent connections and makes all the neurons in one layer independent from others, thus easier to interpret (e.g. understanding the long and short memories kept by the task).

There are also attempts at using the non-saturated activation functions such as ReLU (rectified linear unit) to reduce the gradient decay problem introduced by the activation function. While this reduces the gradient decay problem, it greatly aggravates the effect introduced by the repeated use of the recurrent weight matrix, making the network highly unstable. To alleviate this problem, works on initialization and training techniques, such as initializing the recurrent weights to a proper range or regulating the norm of the gradients over time, were proposed in the literature. In [11], an initialization technique was proposed for an RNN with ReLU activation, termed as IRNN, which initializes the recurrent weight matrix to be the identity matrix and bias to be zero. In [12], the recurrent weight matrix was further suggested to be a positive definite matrix with the highest eigenvalue being unity and all the remaining eigenvalues less than 1. In [9], the geometry of RNNs was investigated and a path-normalized optimization method for training was proposed for RNNs with ReLU activation. In [10], a penalty term on the squared distance between successive hidden states' norms was proposed to prevent the exponential growth of IRNN's activation. Although these methods help ease gradient exploding, they are not able to completely avoid the problem (e.g. the eigenvalues of the recurrent weight matrix may still be larger than 1 in the process of training and lead to gradient exploding). Moreover, the training of an RNN with ReLU is very sensitive to the learning rate. When the learning rate is large, the gradient is likely to explode. The proposed IndRNN solves gradient problems by constraining the recurrent weights, which are then mapped to the constraint on the maximum gradient. It can work with ReLU and be trained robustly. As a result, an IndRNN is able to process very long sequences (e.g. over 5000 steps as demonstrated in the experiments).

In addition to solving the gradient vanishing and exploding problem in order to process long sequences and learn long-term dependency, efforts are also put into investigating deeper RNN networks. Compared with the deep CNN architectures which could be over 100 layers such as the residual CNN [26] and the densely connected CNN [27], most of the existing RNN architectures only consist of several layers (2 or 3 for example [11], [28], [29]). This is partly due to the gradient vanishing and exploding problem which already results in the difficulty in training a single-layer RNN. A deeper RNN [30], [31] may further aggravate this problem. For LSTM models, since all the gate functions, input and output modulations employ sigmoid or hyperbolic tangent functions as the activation function, it suffers from the gradient decay over layers when multiple LSTM layers are stacked into a deep model. Currently, a few models were reported that employ residual connections [26] between LSTM layers to make the network deeper [32]. As shown in [33], residual networks behave like ensembles of relatively shallow networks. For the residual connections to work efficiently, each residual block desires a good gradient behaviour. Due to gradient decay problem in each LSTM, the deep LSTM model with the residual connections cannot significantly improve the performance, which is also observed in [34]. In [35], a recurrent highway network was proposed to extend LSTM where at each time step, multiple layers with highway connections are used to process the

recurrent state, which deepens the transition and also makes the process longer. All in all, RNN architectures that can be stacked with multiple layers and efficiently trained are still highly desired. The proposed IndRNN well addresses this issue and can construct very deep models with the use of ReLU, residual or dense connections (e.g. over 20 layers as demonstrated in the experiments).

Other than the above works making RNN process long sequences and construct deep networks, there are also other variants such as the multiplicative integration [36], [37], multidimensional [38], [39] or bidirectional extensions, which is beyond the scope of this paper and thus not further explained. One interesting result reported in [19] shows that the capacities of existing RNNs are very similar and the difference in their performances are driven primarily by differences in training effectiveness. Compared with the existing RNNs, the proposed IndRNN uses a simple formulation, and is easy and robust to train with different kinds of techniques such as the ReLU, batch normalization and dropout from CNN.

3 INDEPENDENTLY RECURRENT NEURAL NETWORK (INDRNN)

The proposed independently recurrent neural network (IndRNN) follows:

$$\mathbf{h}_t = \sigma(\mathbf{W}\mathbf{x}_t + \mathbf{u} \odot \mathbf{h}_{t-1} + \mathbf{b}) \quad (2)$$

where $\mathbf{x}_t \in \mathbb{R}^M$ and $\mathbf{h}_t \in \mathbb{R}^N$ are the input and hidden state at time step t , respectively. $\mathbf{W} \in \mathbb{R}^{N \times M}$, $\mathbf{u} \in \mathbb{R}^N$ and $\mathbf{b} \in \mathbb{R}^N$ are the weights for the current input and the recurrent input, and the bias of the neurons, respectively. \odot is the Hadamard product, σ is an element-wise activation function of the neurons, and N is the number of neurons in this RNN layer. Compared with the traditional RNN where the recurrent weight \mathbf{U} is a matrix and processes the recurrent input using matrix product, the recurrent weight \mathbf{u} in IndRNN is a vector and processes the recurrent input with element-wise vector product. Each neuron in one layer is independent from others, thus termed as "independently recurrent". For the n -th neuron, the hidden state $h_{n,t}$ can be obtained as

$$h_{n,t} = \sigma(\mathbf{w}_n\mathbf{x}_t + u_n h_{n,t-1} + b_n) \quad (3)$$

where \mathbf{w}_n , u_n and b_n are the n -th row of the input weight, recurrent weight and bias, respectively. The proposed IndRNN can be extended to a convolutional IndRNN where, instead of processing input of each time step using a fully connected weight ($\mathbf{W}\mathbf{x}_t$), it is processed with convolutional operation ($\mathbf{W} * \mathbf{x}_t$, where $*$ denotes the convolution operator).

Fig. 1b illustrates the proposed IndRNN unfolded in time. Each solid dot represents a neuron in a layer and each line represents a connection. It can be seen that the processing of each neuron in one layer is independent from each other as noted by different colors. Each neuron only receives information from the input and its own hidden state at the previous time step. That is, each neuron in an IndRNN deals with one type of spatial-temporal pattern independently. On the contrary, neurons in the simple RNN are connected together over time by the recurrent weight

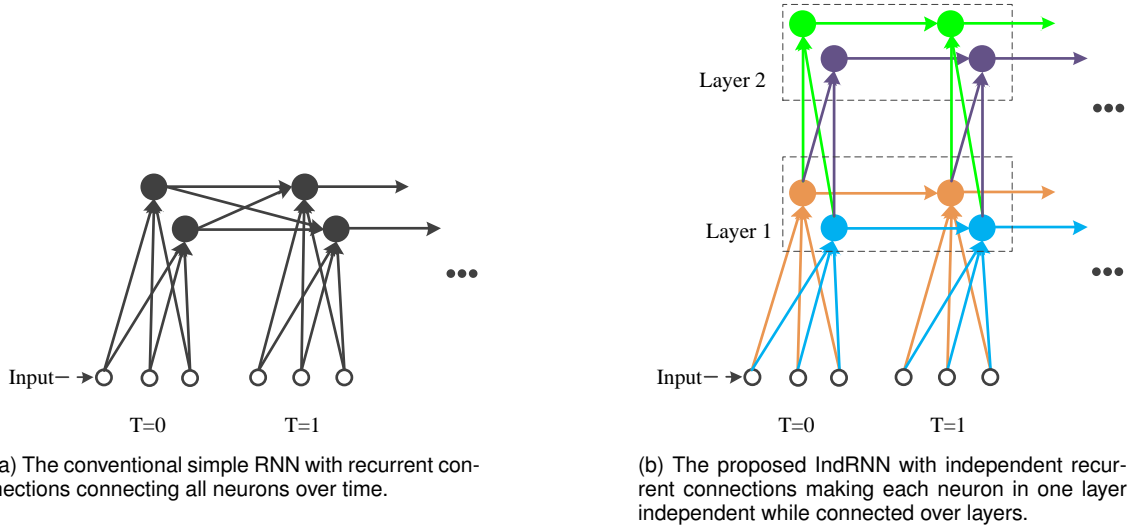


Fig. 1. Illustration of a conventional simple RNN and the proposed IndRNN unfolded in time. Each solid dot represents a neuron in a layer and each line represents a connection.

connection as shown in Fig. 1a. Fig. 1b also shows that one more IndRNN layer can explore the correlation (cross-channel information) among the neurons in one layer as the neuron in the following layer takes output of all the neurons in the previous layer as input. Conventionally, a RNN is treated as multiple layer perceptrons over time where the parameters are shared. Different from the conventional RNNs, the proposed IndRNN provides a new perspective of recurrent neural networks as independently aggregating spatial patterns (i.e. through w) over time (i.e. through u).

3.1 Backpropagation Through Time for An IndRNN

Since the neurons in each layer are independent of each other, the gradient backpropagation through time for neurons in one layer can also be performed independently. For the n -th neuron $h_{n,t} = \sigma(w_n x_t + u_n h_{n,t-1})$ where the bias is ignored, suppose the objective trying to minimize at time step T is J_n . Then the gradient back propagated to the time step t is

$$\begin{aligned} \frac{\partial J_n}{\partial h_{n,t}} &= \frac{\partial J_n}{\partial h_{n,T}} \frac{\partial h_{n,T}}{\partial h_{n,t}} = \frac{\partial J_n}{\partial h_{n,T}} \prod_{k=t}^{T-1} \frac{\partial h_{n,k+1}}{\partial h_{n,k}} \\ &= \frac{\partial J_n}{\partial h_{n,T}} \prod_{k=t}^{T-1} \sigma'_{n,k+1} u_n = \frac{\partial J_n}{\partial h_{n,T}} u_n^{T-t} \prod_{k=t}^{T-1} \sigma'_{n,k+1} \quad (4) \end{aligned}$$

where $\sigma'_{n,k+1}$ is the derivative of the element-wise activation function. It can be seen that the gradient only involves the exponential term of a scalar value u_n which can be easily regulated, and the gradient of the activation function which is often bounded in a certain range. Compared with the gradients of an RNN ($\frac{\partial J}{\partial h_T} \prod_{k=t}^{T-1} \text{diag}(\sigma'(h_{k+1})) U^T$ where $\text{diag}(\sigma'(h_{k+1}))$ is the Jacobian matrix of the element-wise activation function), the gradient of an IndRNN directly depends on the value of the recurrent weight (which is changed by a small magnitude according to the learning rate) instead of matrix product (which is mainly determined by its eigenvalues and can be changed significantly even though the change to each matrix entries is small [40]). Thus

the training of an IndRNN is more robust than a traditional RNN.

To solve the gradient exploding and vanishing problem over time, we only need to regulate the exponential term " $u_n^{T-t} \prod_{k=t}^{T-1} \sigma'_{n,k+1}$ " to an appropriate range (ignoring the gradient backpropagated from the objective at time step T). Denoting the largest gradient value without exploding by γ , the constraint " $u_n^{T-t} \prod_{k=t}^{T-1} \sigma'_{n,k+1} \leq \gamma$ " avoids the gradient exploding problem. Accordingly, the recurrent weight $|u_n|$ satisfies $|u_n| \leq \sqrt[T-t]{\frac{\gamma}{\prod_{k=t}^{T-1} \sigma'_{n,k+1}}}$. For the commonly used activation functions such as ReLU and tanh, their derivatives are no larger than 1, i.e., $|\sigma'_{n,k+1}| \leq 1$. Especially for ReLU, its gradient σ' is either 0 or 1. Therefore, this constraint can be simplified to $|u_n| \leq \sqrt[T-t]{\gamma}$. For especially long sequences, it can be further simplified to $|u_n| \leq 1$. On the other hand, to solve the gradient vanishing problem, the gradient needs to be larger than 0, which can be easily met by $|u_n| > 0$. Since neurons in one layer are independent from each other in IndRNN, the vanishing gradient of one neuron does not affect the learning of other neurons. Instead, it may be useful for the following layers by processing the current input with less information from the past. With many neurons in one layer, it is unlikely that all the recurrent weights are trained to be zero with the gradient of the whole network being 0. Therefore, for robust training of IndRNN, only the constraint $|u_n| \leq \sqrt[T-t]{\gamma}$ is needed and thus used in the experiments. Note that the regulation on the recurrent weight u is different from the gradient or gradient norm clipping technique. For the gradient clipping or gradient norm clipping [41], the calculated gradient has already exploded and is forced back to a predefined range. The gradients for the following steps may keep exploding. In this case, the gradient of the other layers relying on this neuron may not be accurate. On the contrary, the regulation proposed here essentially maintains the gradient in an appropriate range without affecting the gradient backpropagated through this neuron.

TABLE 1
Relationship between recurrent weight and memory through gradient.

Memory	Gradient	Recurrent weight
Long-term	effective after $T - t$ steps: $\frac{\partial J_{n,T}}{\partial h_{n,t}} > \epsilon$	$ u_n \in (\sqrt[T-t]{\frac{\epsilon}{\prod_{k=t}^{T-1} \sigma'_{n,k+1}}}, +\infty)$
Short-term	reduced after $T - t$ steps: $\frac{\partial J_{n,T}}{\partial h_{n,t}} \leq \epsilon$	$ u_n \in [0, \sqrt[T-t]{\frac{\epsilon}{\prod_{k=t}^{T-1} \sigma'_{n,k+1}}}]$
	not exploding: $\frac{\partial J_{n,T}}{\partial h_{n,t}} \leq \gamma$	$ u_n \in [0, \sqrt[T-t]{\frac{\gamma}{\prod_{k=t}^{T-1} \sigma'_{n,k+1}}}]$

3.2 Recurrent Weight and Memory

In this Subsection, the relationship between the recurrent weight and the memory keeping mechanism in IndRNN is thoroughly analyzed to explicitly show the range of the recurrent weights for keeping different ranges of memories.

One of the key roles of RNNs is to keep memories of the past. That is to say, the current state at time step T can be affected by the past state at time step t . In other words, a change of the past state (h_t) changes the current state (h_T). From the perspective of gradient backpropagation, the gradient from h_T propagated to h_t could still be effectively updating the weights. Assume the minimum effective gradient is ϵ , and the gradient follows $\frac{\partial J_{n,T}}{\partial h_{n,t}} > \epsilon$. Accordingly, a range for the recurrent weight to keep memory of $T - t$ time steps can be obtained, which is $|u_n| \in (\sqrt[T-t]{\frac{\epsilon}{\prod_{k=t}^{T-1} \sigma'_{n,k+1}}}, +\infty)$ according to (4) (ignoring the gradient backpropagated from the objective at time step T). On the contrary, when only short-term memory is required, the gradient from h_T propagated to h_t should not be able to effectively update the weights, and the gradient follows $\frac{\partial J_{n,T}}{\partial h_{n,t}} \leq \epsilon$. The range of the recurrent weight can be obtained accordingly as $|u_n| \in [0, \sqrt[T-t]{\frac{\epsilon}{\prod_{k=t}^{T-1} \sigma'_{n,k+1}}}]$.

In an extreme case, the short-term memory is no memory of the past at all, where the recurrent weight is 0 and the gradient backpropagated from the future time steps is zero. It is known that the short-term memories can be important for the performance of the network as well. Especially for a multiple layers RNN, the short-term memories from the earlier layers may be accumulated in the following layer with long-term memory, which is also the reason that the gradient constraint on the vanishing gradient in the above subsection can be alleviated. Together with the basic requirement of avoiding the gradient exploding described above, the relationships among recurrent weight and memory through the gradient can be depicted as Table 1. With this relationship between the recurrent weight and memory, the range of memories learned by the task can be visualized through the learned recurrent weights, which is illustrated in the experiments.

Without any prior knowledge, the short-term memories and long-term memories are assumed to be both important for sequence problems. Therefore, the recurrent weight can be initialized in the range $[0, \sqrt[T-t]{\frac{\gamma}{\prod_{k=t}^{T-1} \sigma'_{n,k+1}}}]$, where different neurons can keep memories of different lengths. For ReLU, it can be simplified to $|u_n| \in [0, \sqrt[T-t]{\gamma}]$. The training of the network can further refine the network to learn required memories for the task by updating the recurrent weights. On the other hand, for tasks such as classification after reading a sequence, it is ob-

vious that only the long-term memory is needed for the last layer as only the output at the last time step is useful. Therefore, the recurrent weights can be initialized in the range $[\sqrt[T-t]{\frac{\epsilon}{\prod_{k=t}^{T-1} \sigma'_{n,k+1}}}, \sqrt[T-t]{\frac{\gamma}{\prod_{k=t}^{T-1} \sigma'_{n,k+1}}}]$ (which is $[\sqrt[T-t]{\epsilon}, \sqrt[T-t]{\gamma}]$ for ReLU) to only keep long-term memory, which is also observed to improve performance in the experiments.

3.3 Complexity in terms of the number of parameters and computation

Regarding the number of parameters, for a N -neuron RNN network with input of dimension M , the number of parameters in a traditional RNN is $M \times N + N \times N + N$, while the number of parameters for LSTM is $4 * (M \times N + N \times N + N)$, which is much larger than the simple RNN. By contrast, IndRNN uses a simple formulation with less parameters, $M \times N + 2 \times N$. Even for a two-layer IndRNN where both layers consist of N neurons, the number of parameters is $M \times N + N \times N + 4 \times N$, which is of a similar order to the traditional RNN. Usually the number of parameters used in a network is large (e.g. much larger than 3), therefore, the number of parameters introduced by the recurrent weight vectors and extra bias vector ($3 \times N$) is negligible compared with the recurrent weight matrix ($N \times N$) used in the traditional RNN. And obviously, IndRNN uses much less parameters than LSTM.

From the perspective of computation, the processing of the input ($\mathbf{W}\mathbf{x}_t + \mathbf{b}$) is independent at different timesteps and can be implemented in parallel. For the processing of the recurrent input (which is usually the most time-consuming part in RNNs), IndRNN only applies one element-wise vector product operation, one adding operation and one activation calculation, involving less computation than the traditional RNNs with matrix product. Moreover, IndRNN works efficiently with ReLU, which is more efficient than other saturated functions such as the tanh and sigmoid used in the traditional RNNs. To further accelerate the computation on GPU, we implemented a fast CUDA optimized version based on PyTorch which is made publicly available. It combines the operations of element-wise vector product, adding and activation calculation to avoid the latency in calling each function separately.

Fig. 2 illustrates the computation complexity comparison in terms of computation time on different lengths of sequences including 256, 512 and 1024. The program is tested on a GPU P100 and the training time for each batch (averaged over 100 batches) including both the forward and backward time (milliseconds) are shown in the figure. In our experiments, we found that this is more accurate than using the forward and backward time separately for each

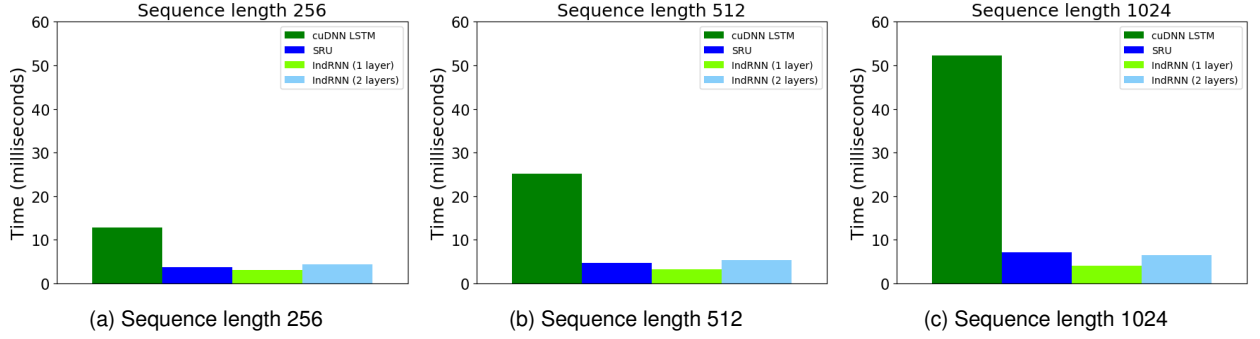


Fig. 2. Complexity comparison in terms of time training one batch (milliseconds) for processing sequences of different lengths. Note that in our experiments the time for one forward or backward process is not very accurate due to the precision of the system clock since the time taken by the forward process is very short. Therefore, the total time of training one batch (including fetching data, forward pass, loss calculation and backward pass) is averaged over 100 batches.

batch. The one-layer IndRNN and two-layer IndRNN are evaluated and the cuDNN LSTM and SRU [21] (both one layer) are used for comparison. The detailed setup is shown in Subsection 5.2.1. It can be seen that IndRNN (both 1-layer and 2-layer) takes much less time than LSTM. To be specific, for sequences of length 256, 512 and 1024, a 1-layer IndRNN is 4.3, 7.6 and 12.9 times faster than LSTM, respectively. Even a 2-layer IndRNN is 2.9, 4.8 and 8.0 times faster than a 1-layer LSTM for sequences of length 256, 512 and 1024, respectively. It is worth noting that while the time for processing longer sequences using LSTM is dramatically increasing, the time using IndRNN only increases a very small amount. This indicates that compared with the matrix product (in the weight processing of the input), the element-wise product and addition is much faster, making IndRNN highly efficient with a comparable computation as a feed-forward network. Moreover, it can also be seen that the one-layer IndRNN is faster than SRU [21] and even two-layer IndRNN is faster than SRU [21] for processing sequence of length 1000, which should be the case since SRU uses more computations at each time step.

4 DEEP INDEPENDENTLY RECURRENT NEURAL NETWORK

As described in the above Section, IndRNN can be stacked to construct multiple-layer IndRNN networks to better explore the cross-channel information. Moreover, since the gradient behaviour of IndRNN can be well regulated, it can work very robustly with ReLU and facilitates the gradient propagation across layers. Several deep IndRNN architectures are proposed in this paper, including the basic stacked IndRNN, residual IndRNN (res-IndRNN) and densely connected IndRNN (dense-IndRNN). First, the building block for the basic IndRNN architecture is shown in Fig. 3a, where “Weight” and “IndRec+ReLU” denote the processing of input and the recurrent process at each step with ReLU as the activation function. In addition, batch normalization, denoted as “BN”, can also be employed in the IndRNN network before or after the activation function as shown in Fig. 3a. By stacking this block, a deep basic IndRNN network can be constructed. Since the weight layer ($Wx_t + b$) is used to process the input, it is natural to extend it to multiple layers

to deepen the processing, making the architecture flexible with different depth to process input features.

The second type of deep IndRNN architecture is the residual IndRNN shown in Fig. 3b, which adds an identity shortcut (skip-connection) [26] to bypass the non-linear transformation of the input features. Denoting the non-linear transformation at layer l and time step t by $\mathcal{F}_{l,t}$, it follows $x_{l,t} = x_{l-1,t} + \mathcal{F}_{l,t}(x_{l-1,t})$, where $x_{l,t}$ is the output feature at layer l . Since the non-linear transformations containing IndRNN are shared over time, $\mathcal{F}_{l,t}$ can be simplified to \mathcal{F}_l and accordingly, $x_{l,t} = x_{l-1,t} + \mathcal{F}_l(x_{l-1,t})$. It can be seen that the skip-connection does not affect the processing in the time dimension, but makes the deeper feature a summation of the shallower feature and a residual feature processed with \mathcal{F}_l . In this case, the gradient can be directly propagated to the previous layers through the skip-connection, greatly alleviating the gradient decay problem across layers. Therefore, substantially deeper residual IndRNN can be constructed. Fig. 3b shows the non-linear transformation $\mathcal{F}_{l,t}$ of type “pre-activation” similarly as in [42], which uses the processing of a composite function of three consecutive operations: batch normalization (BN), the recurrent processing and activation calculation (IndRec+ReLU) over time, and the weight processing (Weight), denoted by “BN-IndRec+ReLU-Weight” for simplicity. Also in each residual block, different numbers of “BN-IndRec+ReLU-Weight” operation can be used, and two operations are shown in the figure.

The third type of deep IndRNN architecture is the densely connected IndRNN shown in Fig. 3c, which, instead of using a skip-connection, uses a concatenation operation to combine all the features of the previous layers [27]. It follows $x_{l,t} = \mathcal{C}(x_{l-1,t}, \mathcal{F}_l(x_{l-1,t}))$, where \mathcal{C} is concatenation operation. By substituting $x_{l-1,t}$ recursively, it can be easily seen that $x_{l,t}$ is a combination of all the features in the previous layers and the non-linear transformation of the features in the previous layer. This also forms an identity function among deeper layers and shallower layers, thus allowing gradients to be properly propagated. Compared with the residual IndRNN, it explicitly processes all the features from all the previous layers, encouraging the feature reuse and thus reducing the number of parameters. Different composite functions can be used as in the residual IndRNN, and Fig.

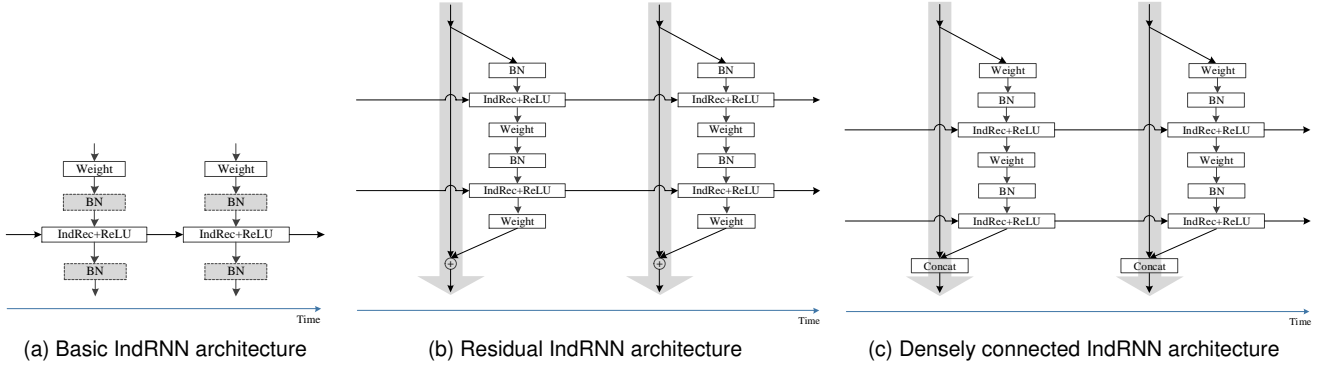


Fig. 3. Illustration of different deep IndRNN architecture.

3c shows an example with the composite function “Weight-BN-IndRec+ReLU”. As the number of layers concatenated together increases, the dimension of features also increases, leading to an increased number of parameters. Therefore, transition layers as in [27] are used to separate dense layers into dense blocks to reduce the dimension of features.

5 EXPERIMENTS

In this Section, first, the capability of IndRNN in processing long sequences and constructing deep networks is verified. Then the performance of the proposed IndRNN on various tasks are evaluated.

5.1 Training Setup

ReLU was used for activation throughout the experiments. For tasks with output at every timestep such as the language modeling problem in Subsection 5.3, the recurrent weights of all layers were initialized uniformly in the range $[0, \sqrt{(T-t)/\gamma}]$, while for tasks such as classification where only the output at the last time step in the last layer is used, the recurrent weights of the last layer were initialized uniformly in the range $[\sqrt{(T-t)/\epsilon}, \sqrt{(T-t)/\gamma}]$ as described in Subsection 3.2. To accelerate training, batch normalization was used except in the simple adding problem. Moreover, for classification tasks where the whole sequence is processed for output at the final time step, the statistics used in the batch normalization layer were obtained based on the samples at all time steps, while for other tasks such as language modeling which cannot access information from future time steps, the statistics are obtained based on all the samples in each time step. When dropout is applied, the dropout mask is shared over time to avoid the clipping of long memory. Weight decay of 10^{-4} is used for the weight parameters (without applying to the recurrent weight and bias). All the networks were trained using the Adam optimization method [43] with initial learning rate 2×10^{-4} . The learning rate is reduced by a factor of 5 when the accuracy (or loss) on the validation data no longer improves (drops) (with patience set to 100).

For the densely connected IndRNN, the network shape was simply following the conventional denseCNN [27]. Each dense layer (the non-linear transformation function) consists of two composite functions as shown in Fig. 3c and produces k feature maps, termed as the growth rate. The

first composite function is called the bottleneck layer and the number of neurons is set to four times ($4*k$) the growth rate. For each transition layer, the number of the neurons is set to be the half (50%) of the input feature maps. The dense block configuration is set to (8,6,4), where in the first, second and third dense block, 8, 6 and 4 dense layers are used, respectively. This keeps a relatively similar number of neurons in each dense block. Note that it is different from the denseCNN [27] because the tasks in the following experiments do not concern pooling (which reduces the size of the features). For the whole network, one IndRNN layer with six times ($6*k$) the growth rate are used to process the input first before going through the following dense layers. In the following, the residual IndRNN and the densely connected IndRNN are noted as res-IndRNN and dense-IndRNN, respectively.

5.2 Verification of processing long sequences and constructing deep networks

5.2.1 Adding Problem

The adding problem [17], [23] is commonly used to evaluate the performance of RNN models. Two sequences of length T are taken as input. The first sequence is uniformly sampled in the range $(0, 1)$ while the second sequence consists of two entries being 1 and the rest being 0. The output is the sum of the two entries in the first sequence indicated by the two entries of 1 in the second sequence. Three different lengths of sequences, $T = 100, 500$ and 1000 , were used for the experiments to show whether the tested models have the ability to model long-term memory.

The RNN models included in the experiments for comparison are the traditional RNN with tanh, LSTM, IRNN (RNN with ReLU). The proposed IndRNN was evaluated with ReLU activation function. Since GRU achieved similar performance as LSTM [14], it is not discussed here. RNN, LSTM, and IRNN are all one layer while the IndRNN model is two layers. 128 hidden units were used for all the models, and the number of parameters for RNN, LSTM, and two-layer IndRNN are 16K, 67K and 17K, respectively. It can be seen that the two-layer IndRNN has a comparable number of parameters to that of the one-layer RNN, while many more parameters are needed for LSTM. As discussed in Subsection 3.1, the recurrent weight is constrained in the range of $|u_n| \in (0, \sqrt{T/2})$ for the IndRNN.

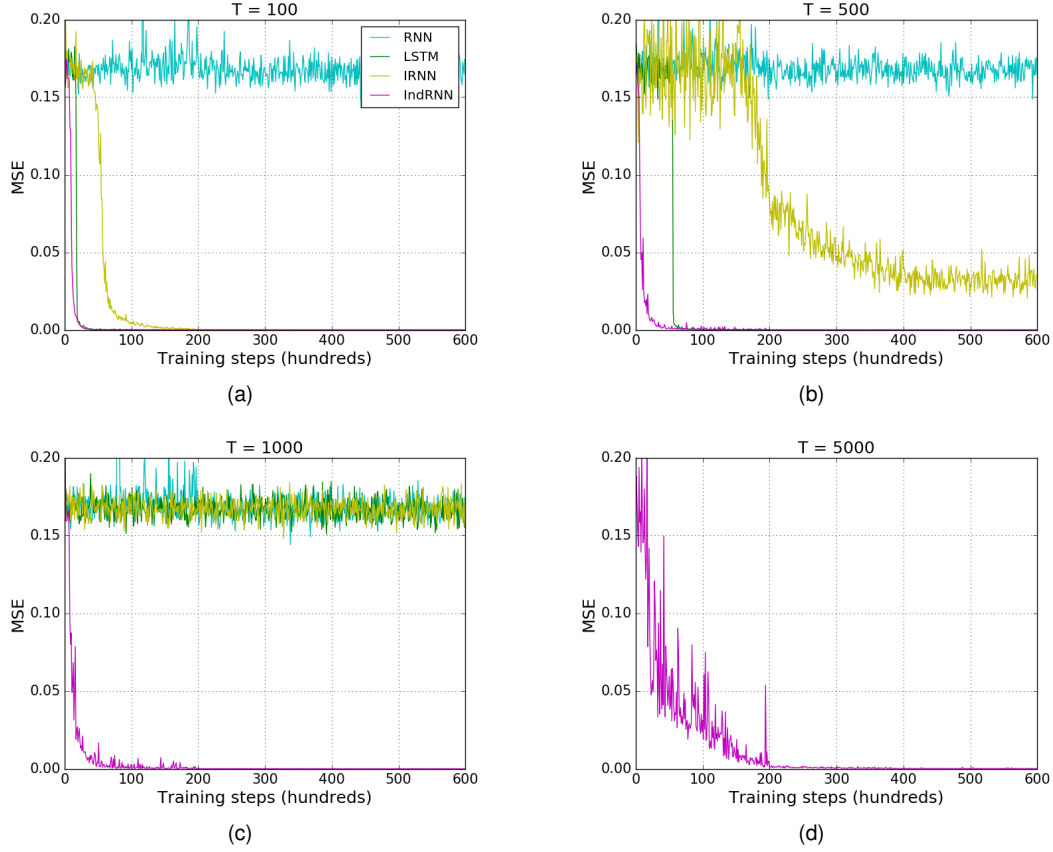


Fig. 4. Results of the adding problem for different sequence lengths. The legends for all figures are the same and thus only shown in (a).

Mean squared error (MSE) was used as the objective function and the Adam optimization method [43] was used for training. The baseline performance (predicting 1 as the output regardless of the input sequence) is mean squared error of 0.167 (the variance of the sum of two independent uniform distributions). The initial learning rate was set to 2×10^{-3} for models with tanh activation and set as 2×10^{-4} for models with ReLU activations. However, as the length of the sequence increases, the IRNN model does not converge and thus a smaller initial learning rate (10^{-5}) was used. The learning rate was reduced by a factor of 10 every 20K training steps. The training data and testing data were all generated randomly throughout the experiments, different from [23] which only used a set of randomly pre-generated data.

The results are shown in Fig. 4a, 4b and 4c. First, for short sequences ($T = 100$), most of the models (except RNN with tanh) performed well as they converged to a very small error (much smaller than the baseline). When the length of the sequences increases, the IRNN and LSTM models have difficulties in converging, and when the sequence length reaches 1000, IRNN and LSTM cannot minimize the error any more. However, the proposed IndRNN can still converge to a small error very quickly. This indicates that the proposed IndRNN can model a longer-term memory than the traditional RNN and LSTM.

From the figures, it can also be seen that the traditional RNN and LSTM can only keep a mid-range memory (about 500 - 1000 time steps). To evaluate the proposed

IndRNN model for very long-term memory, experiments on sequences with length 5000 were conducted where the result is shown in Fig. 4d. It can be seen that IndRNN can still model it very well. Note that the noise in the result of IndRNN is because the initial learning rate (2×10^{-4}) was relatively large and once the learning rate dropped, the performance became robust. This demonstrates that IndRNN can effectively address the gradient exploding and vanishing problem over time and keep a long-term memory.

The complexity of IndRNN was evaluated using the adding problem (with the similar configuration as above) compared with the cuDNN LSTM and SRU [21] which has been reported to be much faster than LSTM. Sequences of different lengths including 256, 512 and 1024 are all tested to be consistent with [21]. A GPU P100 is used as the test platform and the training time for each batch (averaged over 100 batches) including both the forward and backward time (milliseconds) are measured. The comparison is shown in Fig. 2. It can be seen that IndRNN (both 1-layer and 2-layer) takes much less time than LSTM, and reaches 12.9 times faster than LSTM when sequence length is 1024. It is also faster than SRU [21], and even a two-layer IndRNN is faster than SRU for processing sequence of length 1000. Moreover, while the time for processing longer sequences using LSTM is dramatically increasing, the time using IndRNN only increases a little bit, indicating that the recurrent connection is very efficient.

TABLE 2

Results (in terms of error rate (%)) for the sequential MNIST and permuted MNIST.

	MNIST	pMNIST
IRNN [11]	5.0	18
uRNN [23]	4.9	8.6
RNN-path [9]	3.1	-
LSTM [23]	1.8	12
LSTM+Recurrent dropout [44]	-	7.5
LSTM+Recurrent batchnorm [45]	-	4.6
LSTM+Zoneout [28]	-	6.9
LSTM+Recurrent batchnorm+Zoneout	-	4.1
IndRNN (6 layers)	1.0	4.0
IndRNN (12 layers)	0.63	3.16
res-IndRNN (12 layers)	0.61	2.98
dense-IndRNN	0.52	2.8

5.2.2 Sequential MNIST Classification

Sequential MNIST classification has been widely used to evaluate RNN models in capturing long dependencies. The pixels of MNIST digits [46] are presented sequentially to the networks and classification is performed after reading all pixels. Each digit image consists of 784 pixels (28×28), and to achieve a good classification accuracy, the models need to remember the patterns of such a long sequence, thus verifying the capability of the RNN models in capturing long-term dependency. To make the task even harder, the pixels are first processed with a fixed random permutation and then used for classification, known as the permuted MNIST classification. First, a six-layer IndRNN is used for test. Each layer contains 128 neurons, and batch normalization is inserted after each layer. Dropout with a dropping probability of 0.1 is used after each layer. 5% of the training data is reserved for validation. The results are shown in Table 2 in comparison with the existing methods. It can be seen that IndRNN achieved better performance than the existing RNN models.

To further verify that IndRNN can be stacked into very deep networks and still be effectively trained with a good performance, a deep IndRNN with 12 layers, a deep residual IndRNN (12 layers) and a densely connected IndRNN are also tested. For the deep IndRNN and the residual IndRNN, the settings are the same as the above 6-layer IndRNN, e.g., 128 neurons and dropout (0.1) applied after each layer. For the densely connected IndRNN, the growth rate is set to 16, and dropout is applied after the input (0.2), each dense layer (0.2), each bottleneck layer (0.1) and each transition layer (0.1). The results are also shown in Table 2. Compared with the 6-layer IndRNN, deeper IndRNN (12 layers) achieves better performance, less than 1% and 3% in terms of error rate for the normal MNIST classification and permuted MNIST classification, respectively. Moreover, the residual IndRNN and the densely connected IndRNN outperforms the simple stacked IndRNN by further reducing the error rate to 0.52% and 2.8%, validating its effectiveness.

5.3 Language Modeling

5.3.1 Char-level Penn Treebank

The character-level and word-level language modeling tasks are also used for evaluation. In this subsection, we first evaluate the performance using the character-level Penn

TABLE 3

Results of char-level PTB for our proposed IndRNN model in comparison with results reported in the literature, in terms of BPC.

	Test
RNN-tanh [10]	1.55
RNN-ReLU [9]	1.55
RNN-TRec [10]	1.48
RNN-path [9]	1.47
HF-MRNN [47]	1.42
LSTM [28]	1.36
LSTM+Recurrent dropout [44]	1.32
LSTM+Recurrent batchnorm [45]	1.32
LSTM+Zoneout [28]	1.27
HyperLSTM + LN [48]	1.25
Hierarchical Multiscale LSTM + LN [49]	1.24
Fast-slow LSTM [50]	1.19
Neural Architecture Search [51]	1.21
IndRNN (6 layers, 50 steps)	1.26
IndRNN (6 layers, 150 steps)	1.23
res-IndRNN (21 layers, 50 steps)	1.21
res-IndRNN (11 layers*, 150 steps)	1.19
dense-IndRNN (50 steps)	1.19
dense-IndRNN (100 steps*)	1.18

*Note that due to the limitation of GPU memory, an 11-layer residual IndRNN was used for time step 150 instead of 21 layers, and 100 steps are tested instead of 150 steps for the dense-IndRNN.

Treebank (PTB-c) dataset. The test setting is similar to [45]. Different network architectures are also tested, including a six-layer IndRNN, a 21-layer residual IndRNN (to demonstrate that the IndRNN network can be very deep with the residual connections) and a densely connected IndRNN. For the six-layer IndRNN and the 21-layer residual IndRNN, 2000 hidden neurons are used and dropout with dropping probabilities of 0.25 and 0.3 were used, respectively. While for the densely connected IndRNN, the growth rate is set to 256 and 600 hidden neurons are used for the embedding layer. Dropout is applied after each dense layer (0.5), each bottleneck layer (0.1) and each transition layer (0.4). Dropout of 0.3 is used at the last transition layer, which is connected to the output layer. The batch size is set to 128 and the sequence length $T = 50$ and $T = 150$ were both tested in training and testing.

Table 3 shows the performance of the proposed IndRNN models in comparison with the existing methods, in terms of bits per character metric (BPC). Compared with the traditional RNN and LSTM models, it can be seen that the proposed IndRNN model achieved better performance. By comparing the performance of the proposed IndRNN model with different depth, it can be seen that a deeper residual IndRNN or a densely connected IndRNN further improves the performance over the relatively shallow (6 layers) model. Moreover, an improvement can be further achieved with longer temporal dependencies (from time step 50 to 100 or 150), indicating that the proposed IndRNN can effectively learn long-term patterns.

5.3.2 Word-level Penn Treebank

In this subsection, the performance of the proposed IndRNN on the word-level Penn Treebank dataset is evaluated. The test setting is similar to [28]. A 12-layer residual IndRNN with 2000 neurons in each layer and a densely connected IndRNN with the growth rate of 256 were used for test.

TABLE 4

Results of word-level PTB for our proposed IndRNN model in comparison with results reported in the literature, in terms of perplexity.

	Test
RNN-LDA + KN-5 + cache [58]	92.0
Deep RNN [30]	107.5
CharCNN [59]	78.9
LSTM [28]	114.5
LSTM+Recurrent dropout [44]	87.0
LSTM+Zoneout [28]	77.4
LSTM+Variational Dropout [60]	73.4
Pointer Sentinel LSTM [61]	70.9
RHN [62]	65.4
Neural Architecture Search [51]	62.4
AWD-LSTM [54]	58.8
AWD-LSTM+Finetue [54]	57.3
AWD-LSTM+Finetue [54]+dynamic eval [57]	51.1
res-IndRNN (12 layers)	58.99
dense-IndRNN	56.37
dense-IndRNN+dynamic eval [57]	50.97

An embedding layer with 600 neurons is used. The weight tying [52], [53] of the input embedding and the final output weight is adopted and accordingly the last transition layer contains 600 neurons, which processes the IndRNN features from 2000 to 600. For the residual IndRNN, dropout with a dropping probability of 0.45 was used among IndRNN layers while 0.5 was used for the last IndRNN layer. For the densely connected IndRNN, dropout is applied after each dense layer (0.5), each bottleneck layer (0.1) and each transition layer (0.4). Dropout of 0.65 is used after the embedding layer and the last transition layer, and dropout (0.2) on the embedding weight as in [54] is also applied for both IndRNN networks. The batch size is set to 128. Sequence length $T = 50$ was used in training and testing.

The results are shown in Table 4 in comparison with the existing methods. It can be seen that the proposed IndRNN model achieved better performance than the traditional RNN and LSTM models including the AWD-LSTM [54], which is heavily optimized with weight drop, activation regularization, etc. The results can be further enhanced by post-processing with techniques such as neural cache models [55], mixture of ssoftmaxes (MOS) [56] and dynamic evaluation [57]. Here, dynamic evaluation [57] is used as an example and the others are not further discussed. The results are shown in the end of Table 4 with a perplexity of 50.97, which is also better than the AWD-LSTM [54] with the dynamic evaluation.

5.4 Skeleton based Action Recognition

In this subsection, the skeleton based action recognition is used to evaluate the performance of the proposed IndRNN. The widely used NTU RGB+D dataset [29] is adopted, which contains 56880 sequences of 60 action classes, including Cross-Subject (CS) (40320 and 16560 samples for training and testing, respectively) and Cross-View (CV) (37920 and 18960 samples for training and testing, respectively) evaluation protocols [29]. In each evaluation protocol, 5% of the training data (randomly selected) was used for evaluation as suggested in [29]. The joint coordinates of two subject skeletons were used as input. If only one is present, the second was set as zero. For this dataset, when multiple skeletons

TABLE 5

Results of all skeleton based methods on NTU RGB+D dataset.

Method	CS	CV
SkeletonNet(CNN) [63]	75.94%	81.16%
JDM+CNN [64]	76.20%	82.30%
Clips+CNN+MTLN [65]	79.57%	84.83%
Enhanced Visualization+CNN [66]	80.03%	87.21%
HCN [67]	86.5%	91.1%
TCN + TTN [68]	77.55%	84.25%
STGCN [69]	81.5%	88.3%
PB-GCN [70]	87.5%	93.2%
1 Layer PLSTM [29]	62.05%	69.40%
2 Layer PLSTM [29]	62.93%	70.27%
JL_d+RNN [71]	70.26%	82.39%
STA-LSTM [72]	73.40%	81.20%
Pose conditioned STA-LSTM [73]	77.10%	84.50%
1 Layer RNN (reported in [29])	56.02%	60.24%
2 Layer RNN (reported in [29])	56.29%	64.09%
1 Layer LSTM (reported in [29])	59.14%	66.81%
2 Layer LSTM (reported in [29])	60.09%	67.29%
IndRNN (4 layers)	78.58%	83.75%
IndRNN (6 layers)	81.80%	87.97%
dense-IndRNN	84.88%	90.43%
dense-IndRNN-aug	86.70%	93.97%

are present in the scene, the skeleton identity captured by the Kinect sensor may be changed over time. Therefore, an alignment process was first applied to keep the same skeleton saved in the same data array over time. 20 frames were sampled from each instance as one input in the same way as in [74] and batch size was set to 128. A four-layer IndRNN and a six-layer IndRNN with 512 hidden neurons were both tested first to demonstrate the effectiveness of the proposed IndRNN. Then a densely connected IndRNN is used to further validate the effectiveness of a deep architecture. The growth rate is set to 48. For the four-layer IndRNN and six-layer IndRNN, dropout [60] was applied after each IndRNN layer with a dropping probability of 0.25 and 0.1 for CS and CV settings, respectively. For the densely connected IndRNN, dropout is applied after the input (0.5), each dense layer (0.5), each bottleneck layer (0.1) and each transition layer (0.3).

The results are shown in Table 5 including comparisons with the existing methods. It can be seen that the proposed IndRNN greatly improves the performance over other RNN or LSTM models on the same task. For CS, RNN and LSTM of 2 layers can only achieve accuracies of 56.29% and 60.09% while a 4-layer IndRNN achieved 78.58%. For CV, RNN and LSTM of 2 layers only achieved accuracies of 64.09% and 67.29% while 4-layer IndRNN achieved 83.75%. As demonstrated in [29], [74], the performance of LSTM cannot be further improved by simply increasing the number of parameters or increasing the number of layers. However, by increasing the 4-layer IndRNN to a 6-layer IndRNN, the performance is further improved to 81.80% and 87.97% for CS and CV, respectively. Moreover, with a deep densely connected IndRNN, the performance is further improved to 84.88% and 90.43% for CS and CV, respectively, which is significantly better than the conventional RNN and LSTM methods. Considering that feature augmentation including the geometric features [71] and temporal feature augmentation [70] have shown to be useful, feature augmentation is further adopted on top of the densely

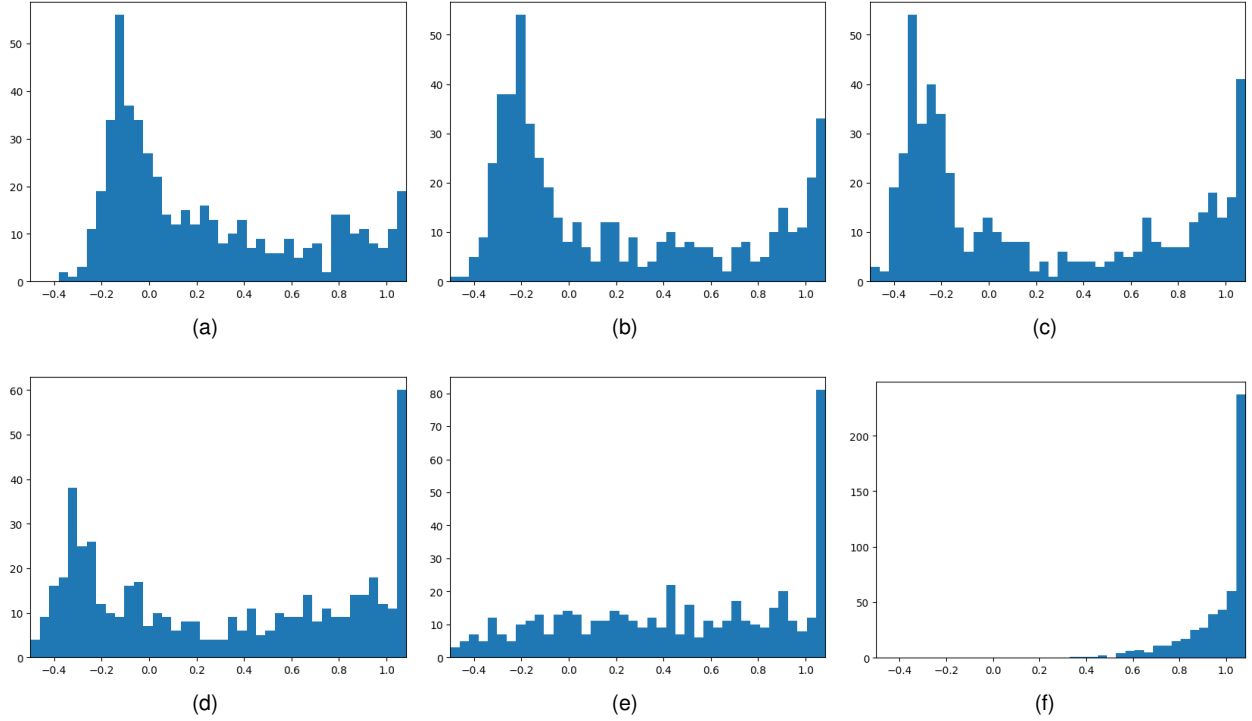


Fig. 5. Histograms of the learned recurrent weights for different layers, where (a)-(f) correspond to the 1st-6th layers. x axis and y axis represent the value of the recurrent weight and the frequency.

connected IndRNN. In addition, we also applied the data augmentation in the time dimension where sequences of length 10-30 are randomly sampled as input. The result is also shown in Table 5. It can be seen that the performance is further improved and better than the state-of-the-art methods.

5.4.1 Visualization of the Recurrent Weight and Memory

With the mapping between the recurrent weight and memory shown in subsection 3.2, the memory learned by the network for the task can be understood by visualizing the recurrent weights. Fig. 5 shows the histograms of the learned recurrent weights (the above six-layer IndRNN obtained under the CS setting for the NTU RGB+D dataset). It can be seen that for the first 5 layers, the learned recurrent weights severely deviate from the uniformly initialized recurrent weights. Especially for the higher layers such as layer 4 and layer 5, the recurrent weights are mostly around 1, learning the long-term memory. On the contrary, for shallower layers such as layer 0 and layer 1, in addition to the large recurrent weights around 1, there is also a group of weights being close to 0, learning the very short-term memory. This makes sense since the shallow layers process the very basic features in a short period while as the layers increase, high-level features in a longer period are processed. Surprisingly, the number of recurrent weights around 0.5 is small indicating that only a small number of neurons keep mid-range memory. Another interesting fact is that although the recurrent weights are initialized to be positive, part of the recurrent weights are learned to be negative. Note that for keeping different ranges of memories, only the absolute value of the recurrent weights

are relevant and thus the negative recurrent weights can still keep different ranges of memories. However, negative recurrent weights may cause oscillation as shown in the gradient backpropagation process. It is hard to intuitively understand how the negative recurrent weights work in the network yet. For the last layer, it can be seen that most of the recurrent weights stay closely to 1 to keep long-term memory, which agrees with the previous assumption shown at the end of Section 3.2.

6 CONCLUSION

In this paper, we proposed an independently recurrent neural network (IndRNN) with a new recurrent connection using the Hadamard product, making neurons in one layer independent of each other. It effectively solves the gradient vanishing and exploding problems by regulating the recurrent weights, which makes it able to be efficiently process very long sequences. Different deep IndRNN architectures, including the basic IndRNN, residual IndRNN and densely connected IndRNN, have been proposed, which can be much deeper than the traditional RNNs. With the gradient better regulated in IndRNN, non-saturated activation functions such as ReLU can be used and trained very robustly. Moreover, with the neurons in one layer independent from each other, each neuron can be better interpreted without effects from others. Also, the relationship between the memory and the recurrent weight has been established through gradient backpropagation, and the learned memories can be understood by investigating the recurrent weights. Experiments on multiple fundamental tasks have verified the advantages of the proposed IndRNN over existing RNN models.

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