

ICT303 – Advanced Machine Learning and Artificial Intelligence

Topic 5: Convolutional Neural Networks (CNN) – Part I

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How to Get in Touch with the Teaching Team

- Internal and External Students

- Email: H.Laga@murdoch.edu.au.

- Important

- In any communication, please make sure that you
 - Start the subject of your email with ICT303
 - Include your student ID, name, and the lab slot in which you are enrolled.
 - We will do all our best to answer your queries within 24 hrs.

In this Lecture

- Introduction

- Recap of Linear regression, perceptron and MLP
- Why we need CNNs?

- Convolutional Neural Network (CNN)

- Convolutions
- Convolution layers

- Examples of CNNs

- LeNet

- Summary

- Learning objectives

- Understand CNN and the different components that compose it
- Implement CNNs in Python, NumPy and PyTorch

- Additional readings

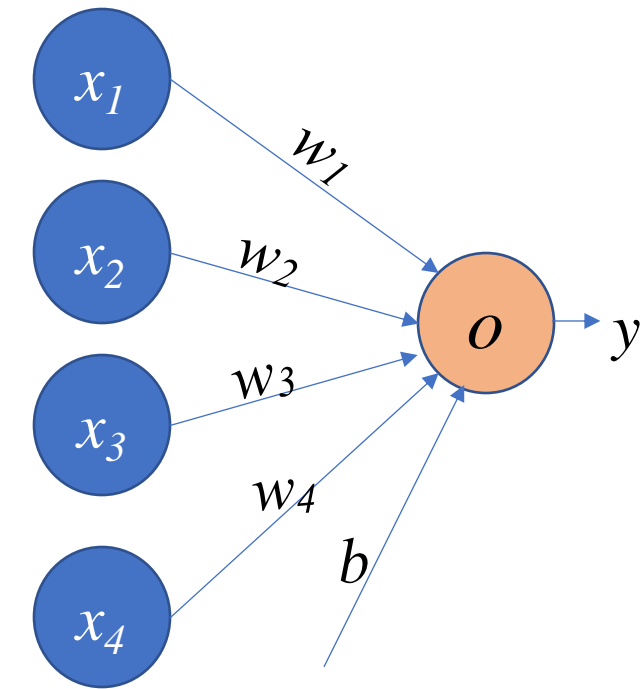
- Chapter 7 of the textbook, available at: <https://d2l.ai/>

Linear Methods for Regression and Classification (Recap)

- Linear regression

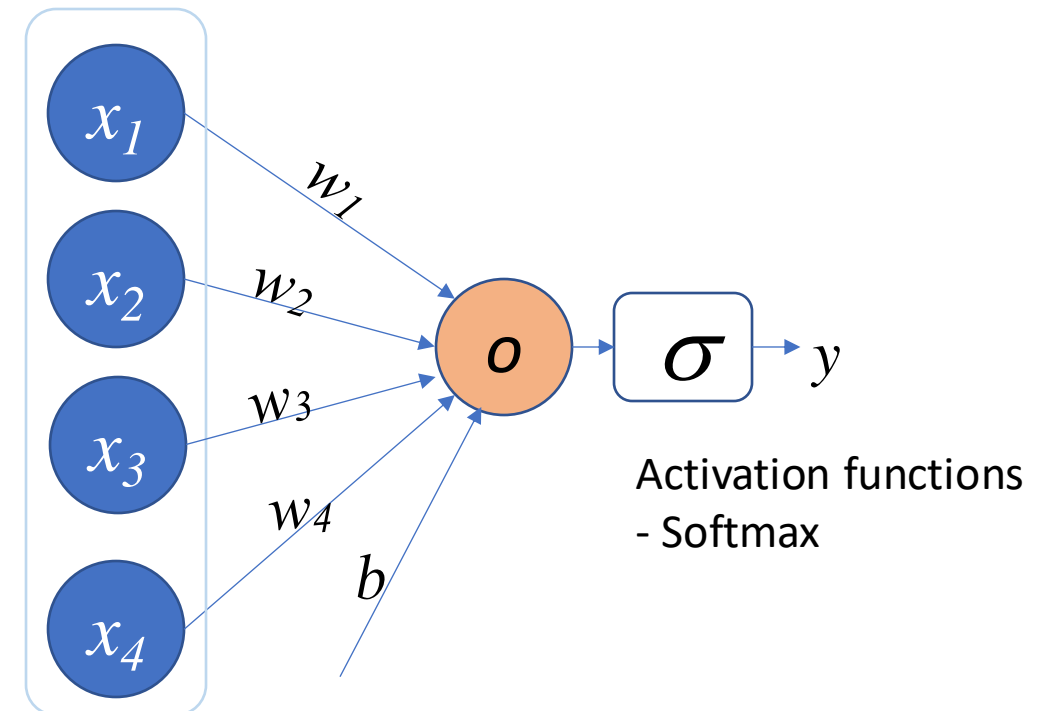
A **dense** (fully connected, or linear) layer has parameters

$\mathbf{W} \in \mathbb{R}^{m \times n}$, $\mathbf{b} \in \mathbb{R}^m$, it computes output $\mathbf{y} = \mathbf{W}\mathbf{x} + \mathbf{b} \in \mathbb{R}^m$



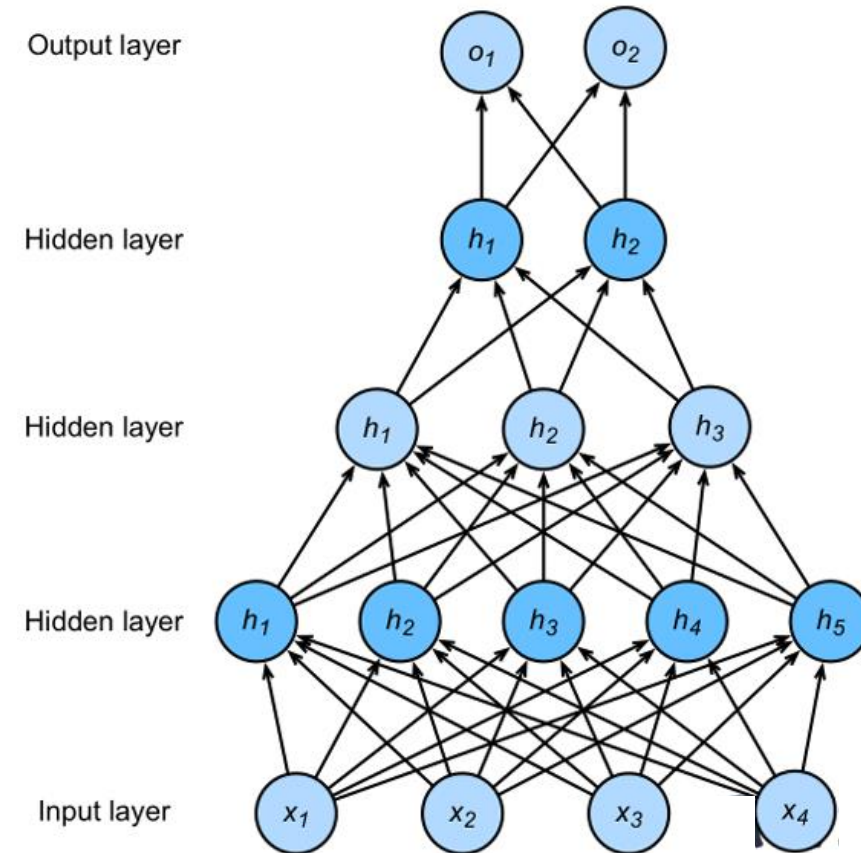
$$o = y = \langle W, X \rangle + b$$

- Classification



Multilayer Perceptron (MLP)

- Stacks multiple hidden layers (dense + activation) to get deeper models
- The activation function leads to non-linear models
 - Sigmoid, ReLU
- Hyper parameters
 - No. of hidden layers
 - No. of outputs of each hidden layers
- They are universal approximators
 - Can approximate any function



MLPs limitations

- Design an MLP composed of one single hidden layer
 - Input: RGB image of size 300 x 300 pixels
 - Output: The class of the object depicted in the image
- Problem
 - Consider the case where we:
 - have 1000 possible object classes (horse, dogs, tables,)
 - use a simple MLP composed of 1 input layer, two hidden layers, and one output layer
 - How many parameters the network will have?
 - If we randomly shuffle the image pixels, what would be the output produced by the network?

MLPs limitations

- **Design an MLP composed of one single hidden layer**
 - Input: RGB image of size 300 x 300 pixels
 - Output: The class of the object depicted in the image
- **Problem**
 - Consider the case where we:
 - have 1000 possible object classes (horse, dogs, tables,)
 - use a simple MLP composed of 1 input layer, two hidden layers, and one output layer
 - How many parameters the network will have?
 - If we randomly shuffle the image pixels, what would be the output produced by the network?

For 1 Single Hidden Layer (for 1 training example):

$$\begin{aligned}x &= (300 \times 300 \times 3, 1) \\ W &= (1000, 300 \times 300 \times 3) \\ b &= (1000, 1) \\ z &= Wx + b\end{aligned}$$

$$\begin{aligned}\text{Total Parameters} &= 1000 \times 300 \times 300 \times 3 + 1000 \\ &= 270\,001\,000\end{aligned}$$

For 2 Hidden Layers (needed for processing m training examples):

$$\begin{aligned}x &= (300 \times 300 \times 3, m) \\ W1 &= (n_1, 300 \times 300 \times 3) \\ b1 &= (n_1, m) \\ W2 &= (1000, n_1) \\ b2 &= (1000, m)\end{aligned}$$

$$\begin{aligned}\text{Total Parameters} &= n_1 \times 300 \times 300 \times 3 + 1000 \\ &\quad * n_1 + n_1 * m + 1000 * m\end{aligned}$$

From MLPS to Convolution Neural Networks (CNN)

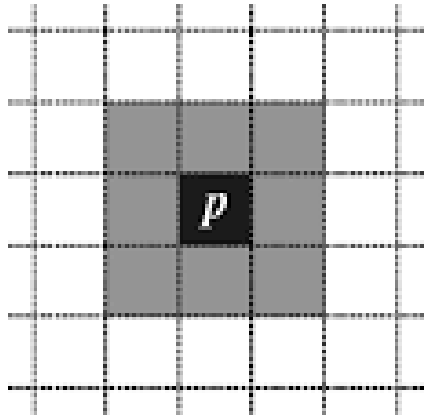
- Issues with MLPs

- Images are structured into grids of pixels
 - MLPs treat them as a flattened vector ignoring the local relations between pixels
 - If you take an image and randomly shuffle its pixels, the MLP will produce the same result!
- Have a very large number of parameters
 - Thus, they are difficult to train for complex problems

- Convolutional Neural Networks (CNN or ConvNets)

- A powerful family of neural networks specifically designed to
 - Capture the local spatial relationships between pixels
 - Reduce the number of parameters compared to MLPs
- Uses convolutions

Convolution



A pixel p and its
8 neighbors

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix},$$

A 3 x 3 filter

$$J(x, y) = aI(x-1, y-1) + bI(x, y-1) + cI(x+1, y-1) + \\ dI(x-1, y) + eI(x, y) + fI(x+1, y) + \\ gI(x-1, y+1) + hI(x, y+1) + iI(x+1, y+1)$$

Convolution - Example

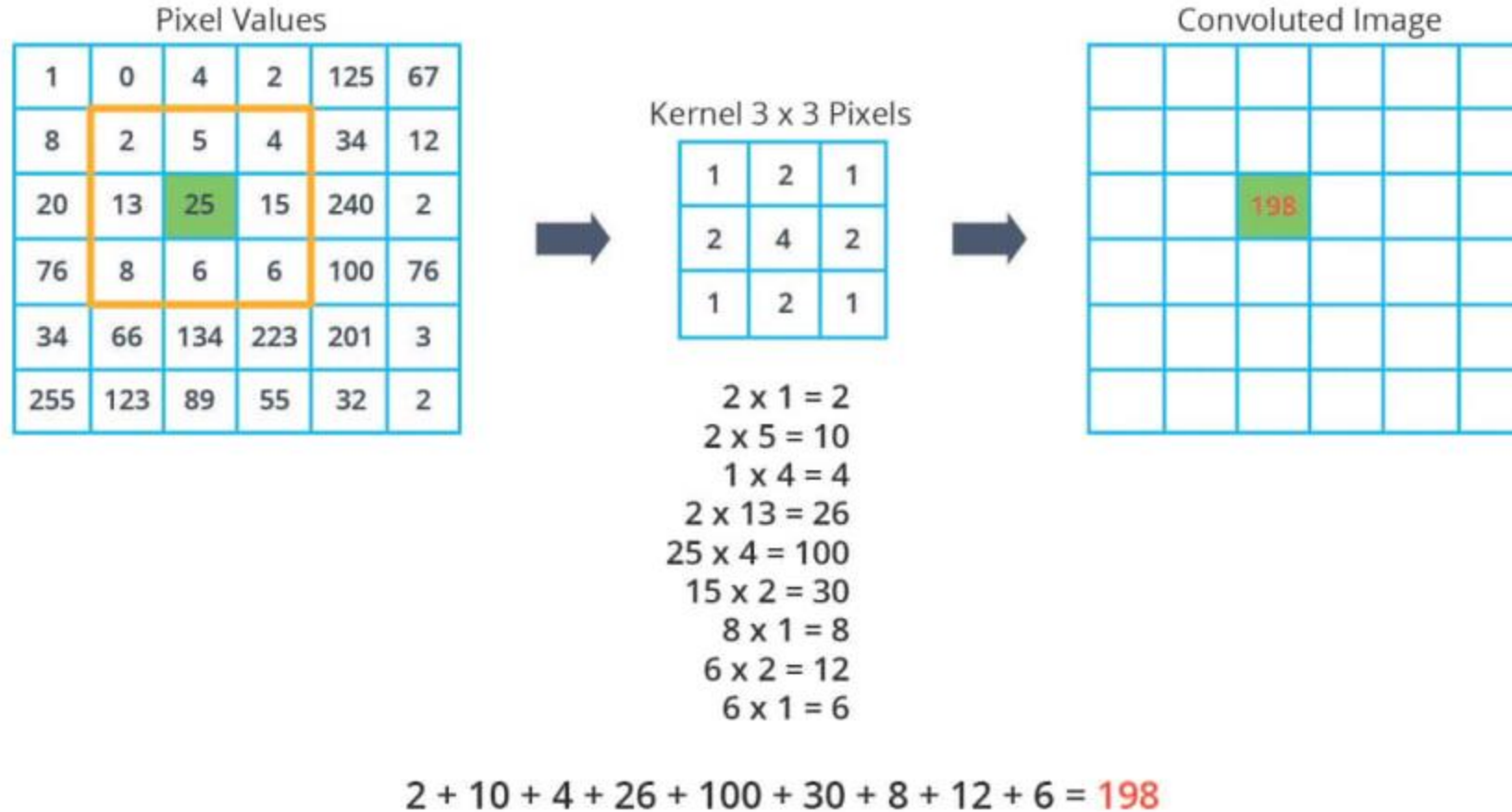


Image courtesy of: <https://dev.to/sandeepbalachandran/machine-learning-convolution-with-color-images-2p41>

Illustration of Convolution

Given this 5x5 image, convolve with a 3x3 filter

1	2	2	3	1
1	1	3	3	2
3	2	1	2	3
2	3	2	3	2
2	1	2	3	1

1	2	1
1	2	1
1	2	1

Illustration of Convolution

Given this 5x5 image, convolve with a 3x3 filter

1	2	1	3	1
1	2	1	3	2
1	2	1	2	3
2	3	2	3	2
2	1	2	3	1

Move the filter over the entire image from left to right,
top to bottom

$$1 \times 1 + 2 \times 2 + 1 \times 2 + 1 \times 1 + 2 \times 1 + 1 \times 3 + 1 \times 3 + 2 \times 2 + 1 \times 1$$

21		

1	2	2	3	1
1	1	3	3	2
3	2	1	2	3
2	3	2	3	2
2	1	2	3	1

Illustration of Convolution

Given this 5x5 image, convolve with a 3x3 filter

1	1	2	1	1
1	1	2	1	2
3	1	2	1	3
2	3	2	3	2
2	1	2	3	1

Move the filter over the entire image from left to right,
top to bottom

$$1 \times 2 + 2 \times 2 + 1 \times 3 + 1 \times 1 + 2 \times 3 + 1 \times 3 + 1 \times 2 + 2 \times 1 + 1 \times 2$$

21	25	

1	2	2	3	1
1	1	3	3	2
3	2	1	2	3
2	3	2	3	2
2	1	2	3	1

Illustration of Convolution

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1	1	1	2	1
3	2	1	2	1
2	3	2	3	2
2	1	2	3	1

Move the filter over the entire image from left to right,
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21	25	27

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21	25	27
25		

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21	25	27
25	26	

1	2	2	3	1
1	1	3	3	2
3	2	1	2	3
2	3	2	3	2
2	1	2	3	1

Illustration of Convolution

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2	1	2	3	1

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21	25	27
25	26	

1	2	2	3	1
1	1	3	3	2
3	2	1	2	3
2	3	2	3	2
2	1	2	3	1

Illustration of Convolution

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1	2	1	3	2
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21	25	27
25	26	29
24		

1	2	2	3	1
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21	25	27
25	26	29
24	24	

1	2	2	3	1
1	1	3	3	2
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21	25	27
25	26	29
24	24	27

1	2	2	3	1
1	1	3	3	2
3	2	1	2	3
2	3	2	3	2
2	1	2	3	1

Convolution - Example

- Edge detection with Sobel filter (or Sobel kernel)

- Convoluting the image I with this operator will result in J_v , the image of vertical edges

-1	0	1
-2	0	2
-1	0	1

- Convoluting the image I with this operator will result in J_h , the image of horizontal edges

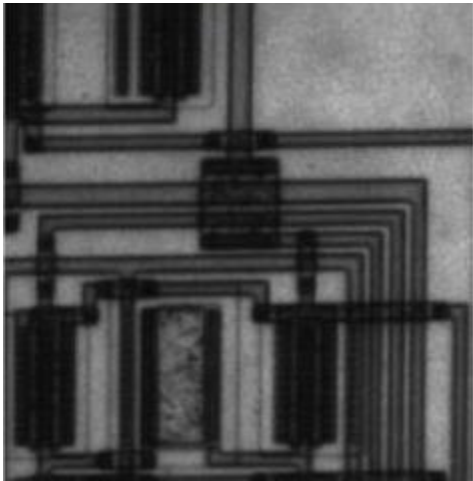
1	2	1
0	0	0
-1	-2	-1

- Combine the two filters to give a single measure of gradient magnitude

$$J(x, y) = \sqrt{J_h(x, y)^2 + J_v(x, y)^2}$$

Convolution - Example

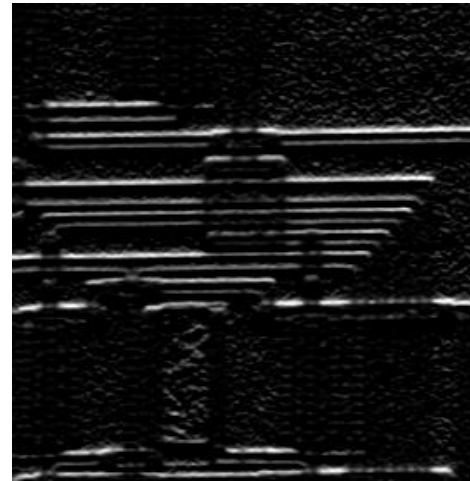
- Edge detection with Sobel filter (or Sobel kernel)



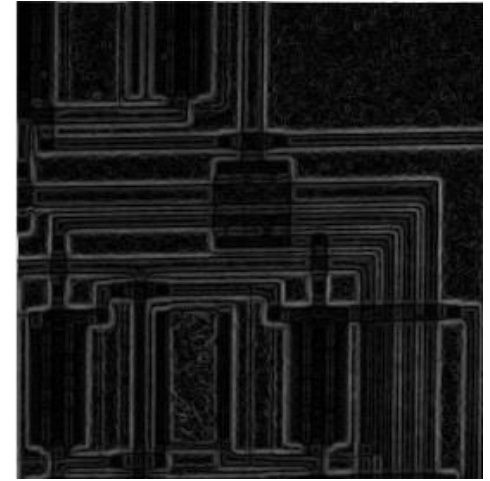
Input image



Vertical edges I_y



Horizontal edges I_x



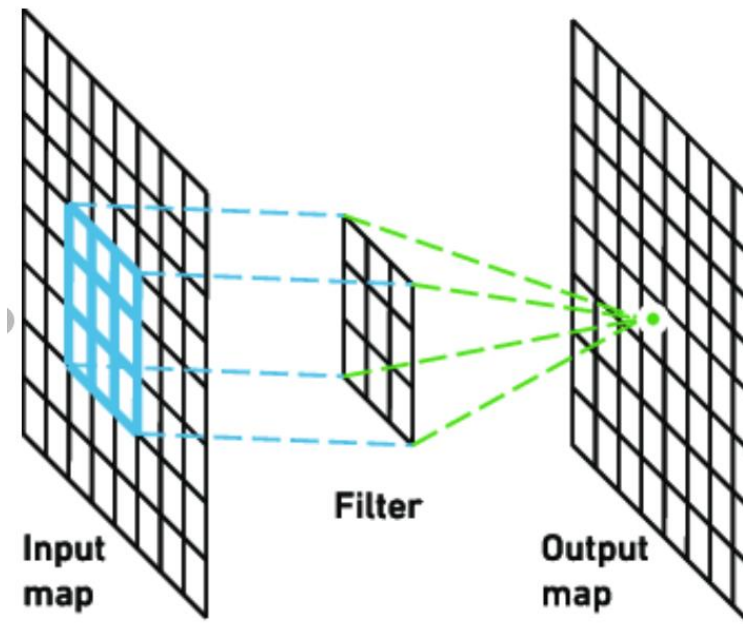
$$G = \sqrt{I_x^2 + I_y^2}$$

Convolution

- Question
 - Can you implement the Edge detector using neurons?

Convolution Layer

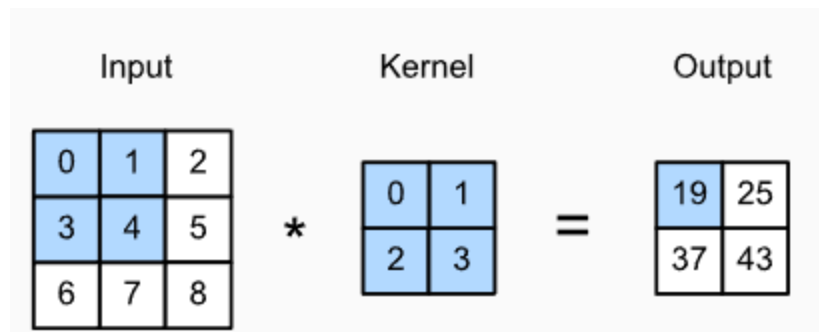
- At each image pixel p , unlike dense, or fully-connected, layer, convolutional layers take a weighted sum of the pixels around p
 - Think about it as a filter BUT the values of the filter (kernel) are unknown
 - The goal of training is to learn the values of the elements of the filter that best suit the task at hand



Convolution Layer – one filter case

- Greyscale image

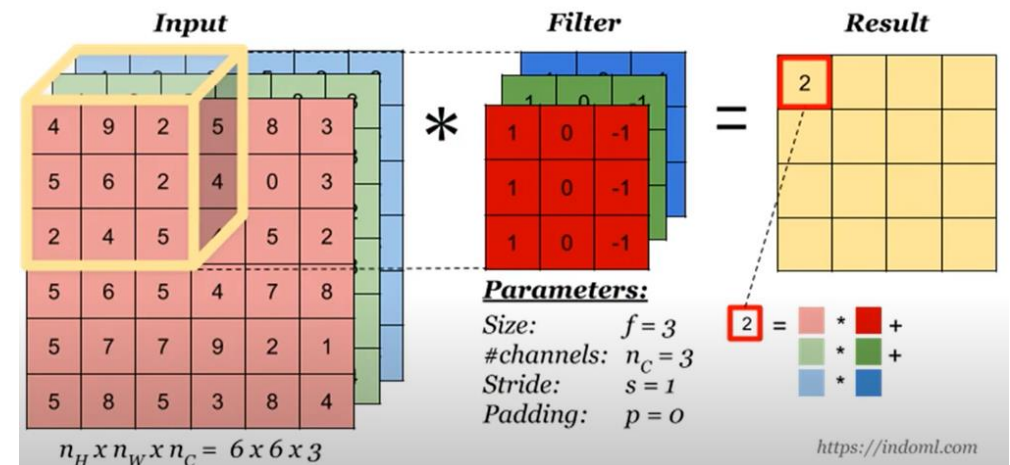
- Has $d = 1$ channel (for the grey level)
- The filter is then of size $s \times s$ (e.g., s can be 3, 5, 7, ..)



https://d2l.ai/chapter_convolutional-neural-networks/conv-layer.html

- Color image

- Has $d=3$ channels (one for R, one for G, and another for B)
- The filter is then of size $d \times s \times s$ (e.g., s can be 3, 5, 7, ..)



https://www.youtube.com/watch?v=3myNsOGhc3A&list=PLbNJQ-D5RH18U7BXm3NH8_6mnguQ45uRh&index=5

Convolution Layer – one filter case

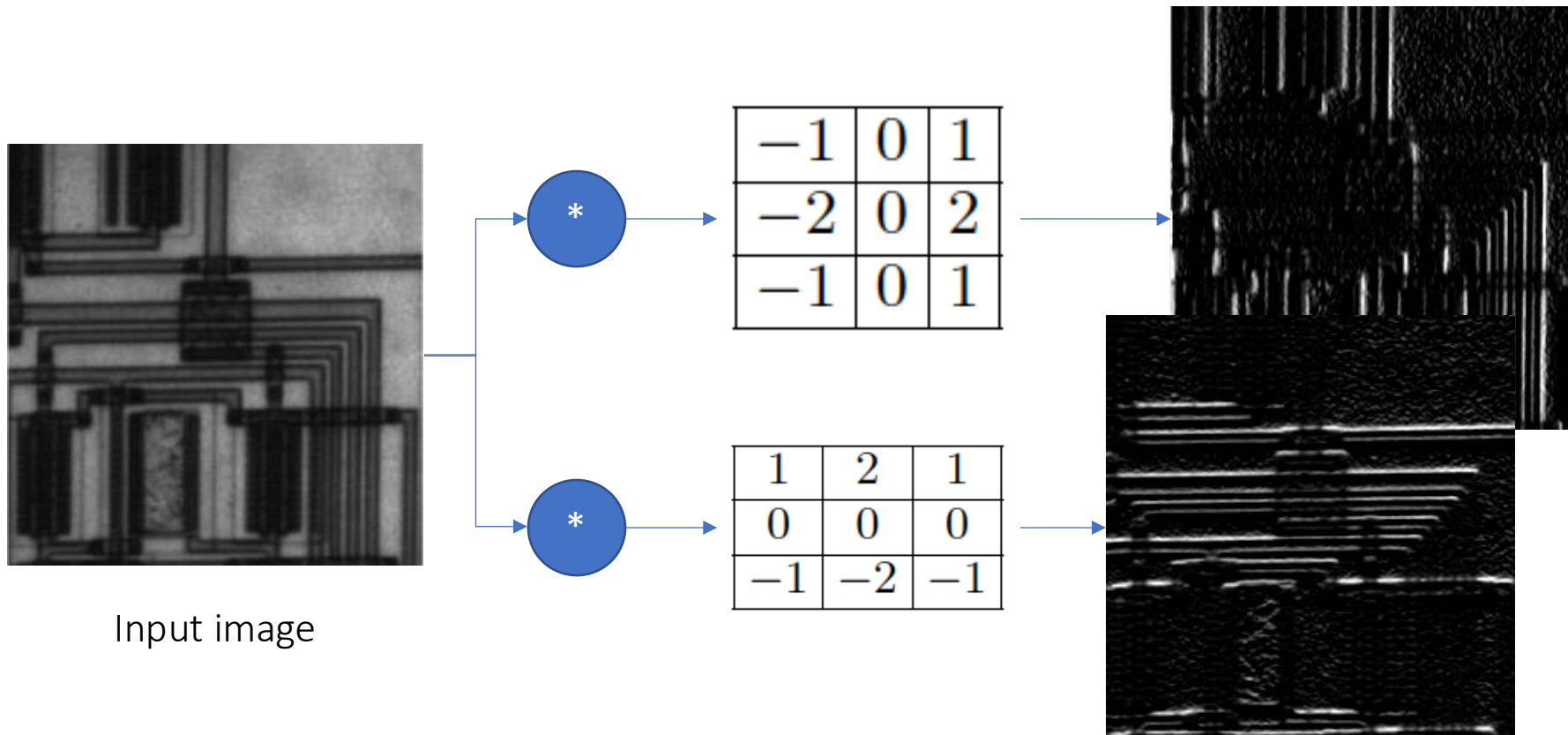
- Convolutional layer
 - Cross correlates the input image and the kernel and adds a bias to produce an output
- The two (learnable) parameters of a convolutional layer are
 - The kernel values
 - The bias
- The hyper parameters (not learnable, set by the user) are
 - The kernel size
 - It defines the receptive field size
 - Receptive field refers to all the elements from the previous layer that may affect the calculation of the output value
- The output of a convolutional layer is called feature map

Convolution Layer – Multiple Feature Maps

- Apply different kernels to the input
 - Each kernel will produce one output, called feature map
 - Stack the feature maps together to produce a multichannel output
- Hyper parameters
 - The kernel size
 - It defines the receptive field size
 - Receptive field refers to all the elements from the previous layer that may affect the calculation of the output value
 - No. of output channels at each layer
 - It defines the no. of different filters to learn at each layer

Convolution Layer – Multiple Feature Maps

- Example – Edge detection with Sobel filter (or Sobel kernel)



Convolution Layer – Practical Considerations

- **Padding**

- When you convolve an image with a filter, we will lose pixels at the boundary of the image

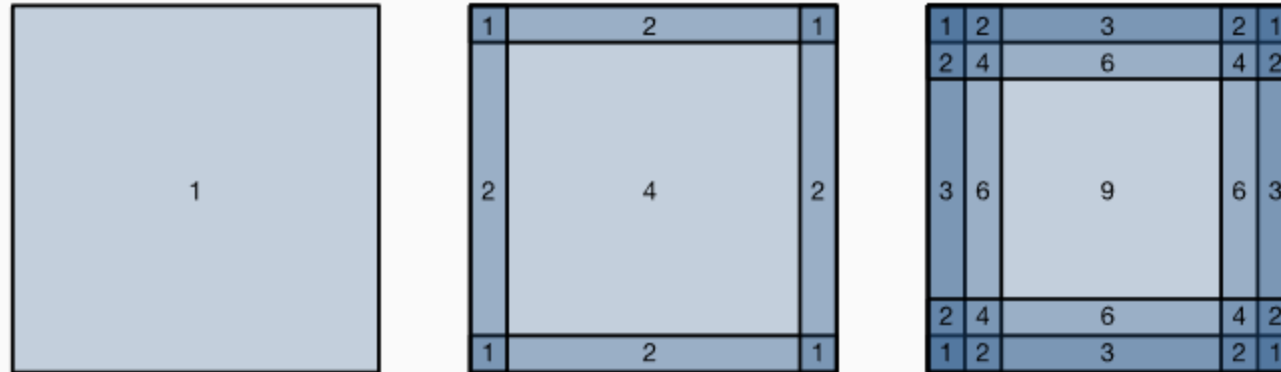


Fig. 7.3.1 Pixel utilization for convolutions of size 1×1 , 2×2 , and 3×3 respectively.

https://d2l.ai/chapter_convolutional-neural-networks/padding-and-strides.html

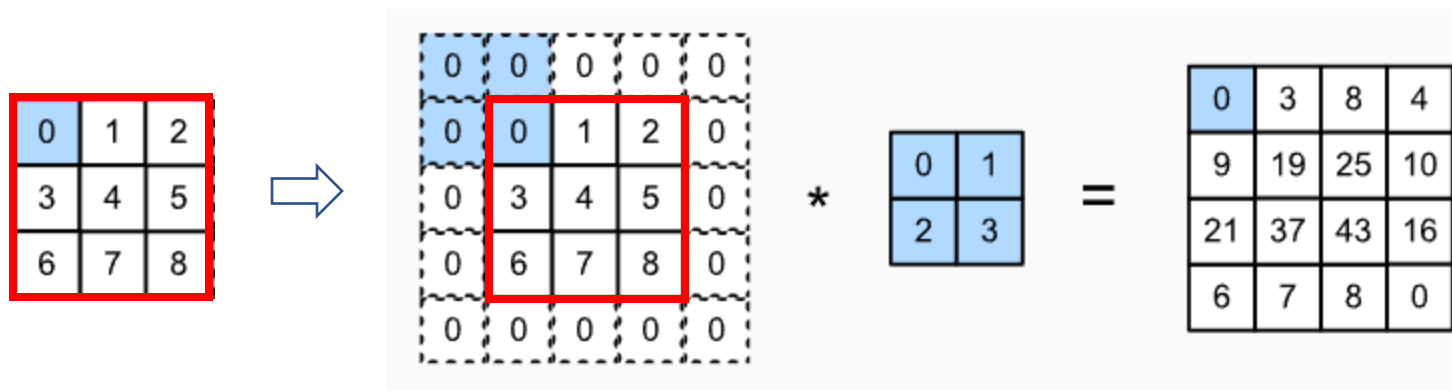
Convolution Layer – Practical Considerations

- **Padding**

- When you convolve an image with a filter, we will lose pixels at the boundary of the image

- **Solution → Padding**

- Add extra pixels (set to 0) of filler around the boundary of the input image



https://d2l.ai/chapter_convolutional-neural-networks/padding-and-strides.html

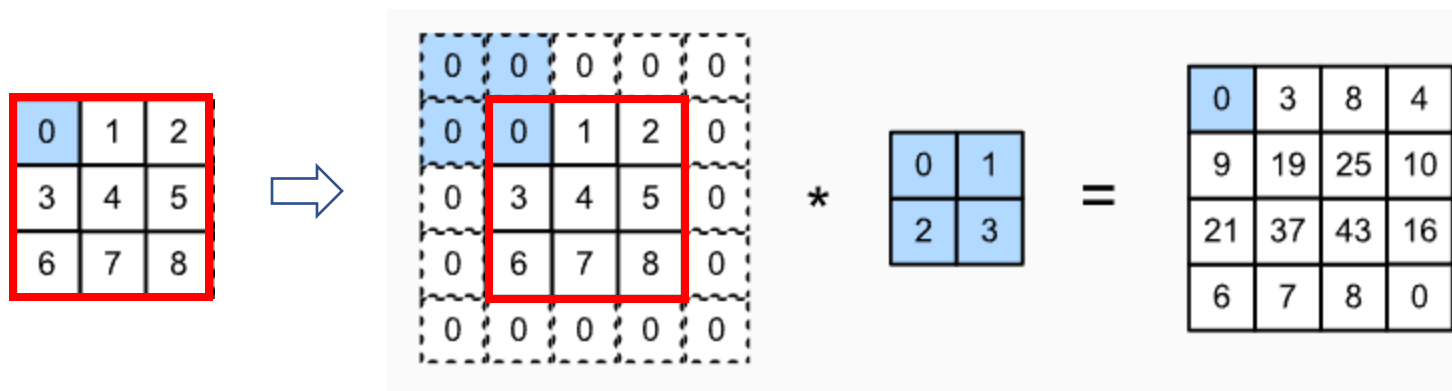
Convolution Layer – Practical Considerations

- **Padding**

- When you convolve an image with a filter, we will lose pixels at the boundary of the image

- **Solution → Padding**

- Add extra pixels (set to 0) of filler around the boundary of the input image
- Typically, we use kernels with **odd** height and width, e.g., 1, 3, 5, 7



https://d2l.ai/chapter_convolutional-neural-networks/padding-and-strides.html

Convolution Layer – Practical Considerations

- Typically,
 - The convolution window starts at the upper-left corner, and then we slide it over all locations both down and to the right
 - We slide the window one element at a time
 - For computational efficient (and to downsample)
 - Slide the window more than one elements, skipping the intermediate locations
 - The number of rows (and columns) skipped is called **stride**
- Hyper parameters
 - The kernel size
 - No. of output channels
 - Padding
 - The stride

Illustration of Convolution with Stride

Given this 5x5 image, convolve with a 3x3 filter

1	2	2	3	1
1	1	3	3	2
3	2	1	2	3
2	3	2	3	2
2	1	2	3	1

1	2	1
1	2	1
1	2	1

Illustration of Convolution with Stride

Given this 5x5 image, convolve with a 3x3 filter **with Stride = 2**

1	2	1	3	1
1	2	1	3	2
1	2	1	2	3
2	3	2	3	2
2	1	2	3	1

Move the filter over the entire image from left to right,
top to bottom

$$1 \times 1 + 2 \times 2 + 1 \times 2 + 1 \times 1 + 2 \times 1 + 1 \times 3 + 1 \times 3 + 2 \times 2 + 1 \times 1$$

21	

1	2	2	3	1
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Illustration of Convolution with Stride

Given this 5x5 image, convolve with a 3x3 filter **with Stride = 2**



1	2	1	2	1
1	1	1	2	1
3	2	1	2	1
2	3	2	3	2
2	1	2	3	1

Move the filter over the entire image from left to right,
top to bottom **skipping 2 columns or rows respectively**

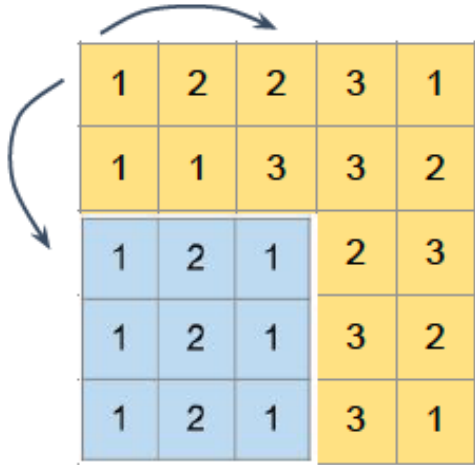
$$1 \times 2 + 2 \times 3 + 1 \times 1 + 1 \times 3 + 2 \times 3 + 1 \times 2 + 1 \times 1 + 2 \times 2 + 1 \times 3$$

21	28

1	2	2	3	1
1	1	3	3	2
3	2	1	2	3
2	3	2	3	2
2	1	2	3	1

Illustration of Convolution with Stride

Given this 5x5 image, convolve with a 3x3 filter **with Stride = 2**



1	2	2	3	1
1	1	3	3	2
1	2	1	2	3
1	2	1	3	2
1	2	1	3	1

Move the filter over the entire image from left to right,
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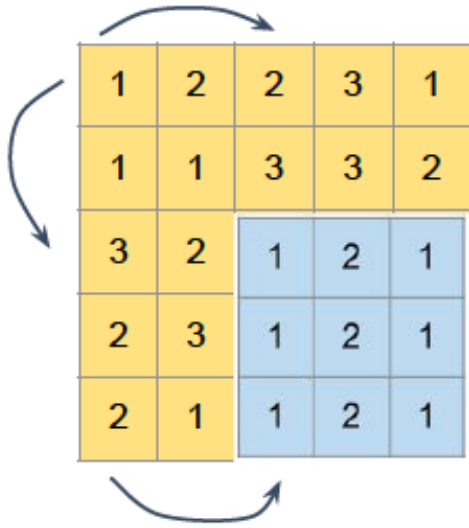
$$1 \times 3 + 2 \times 2 + 1 \times 1 + 1 \times 2 + 2 \times 3 + 1 \times 2 + 1 \times 2 + 2 \times 1 + 1 \times 2$$

21	28
24	

1	2	2	3	1
1	1	3	3	2
3	2	1	2	3
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Illustration of Convolution with Stride

Given this 5x5 image, convolve with a 3x3 filter **with Stride = 2**



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3	2	1	2	1
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2	1	1	2	1

Move the filter over the entire image from left to right,
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24	27

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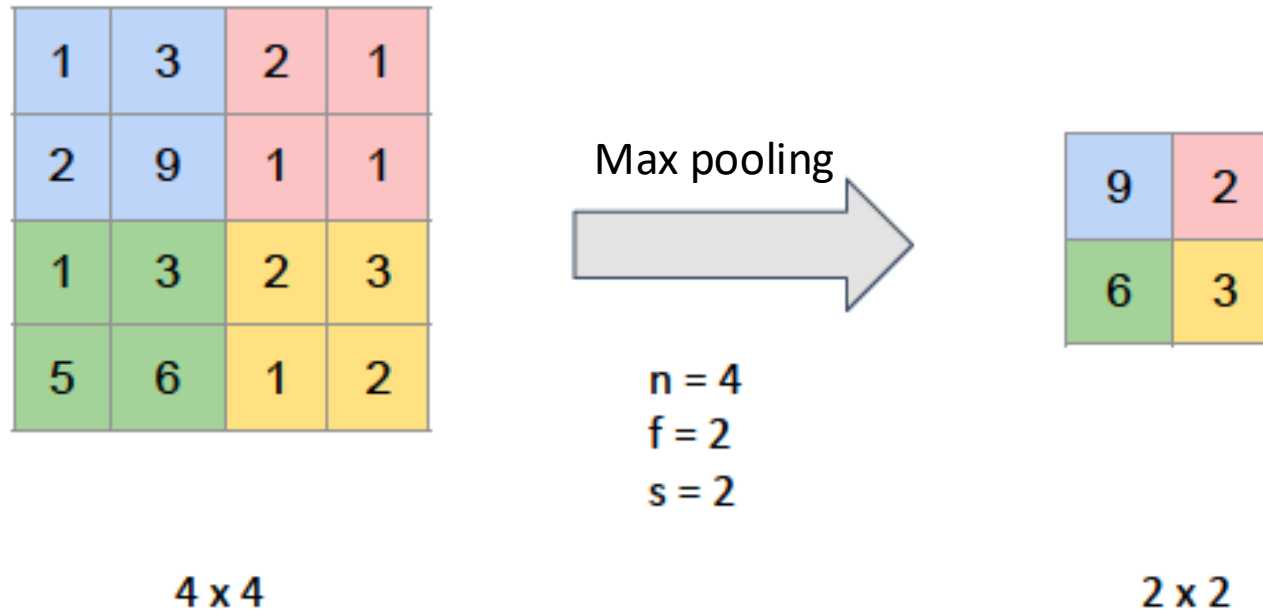
Pooling Layer

- Pooling is the operation of aggregating the values within a window around a pixel to produce one single value, e.g.,
 - Max pooling
 - takes the maximum of all the values within a window around a pixel
 - Min pooling
 - takes the minimum of all the values within a window around a pixel
 - Average pooling
 - takes the minimum of all the values within a window around a pixel

Pooling Layer

- Pooling is the operation of aggregating the values within a window around a pixel to produce one single value, e.g.,
 - Max pooling
 - takes the maximum of all the values within a window around a pixel
 - Min pooling
 - takes the minimum of all the values within a window around a pixel
 - Average pooling
 - takes the minimum of all the values within a window around a pixel
- It is important when stacking multiple layers
 - Reduces the size of the output of a layer before feeding it to the next layer
- Advantages
 - Reduce the size of the feature map
 - Although the size of the filters remain the same, the receptive field will increase with the depth of the network – thus subsequent layers will capture bigger context

Illustration of Max Pooling

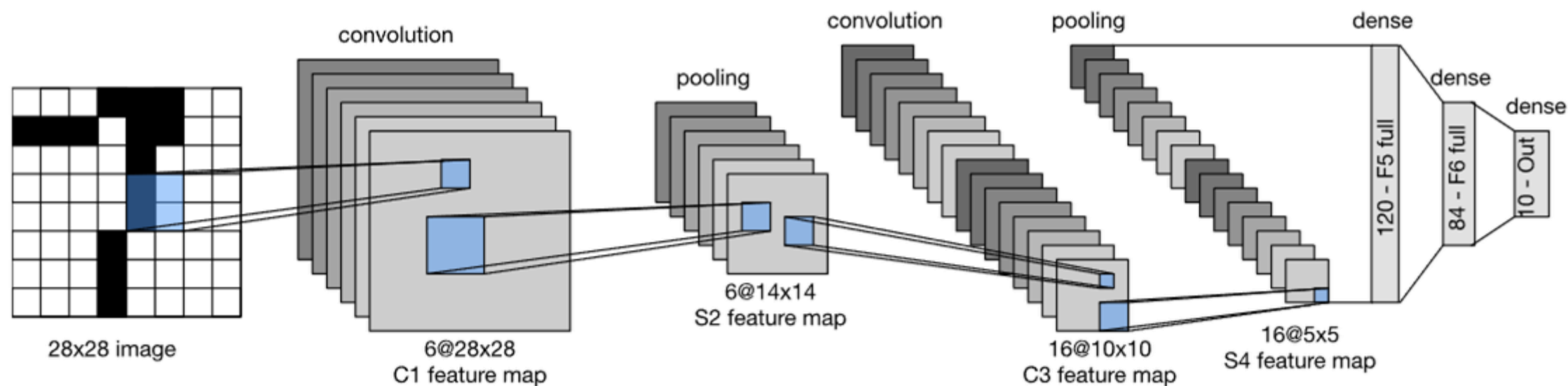


In this Lecture

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 - Linear regression, perceptron and MLP
 - Why we need CNNs
- Convolutional Neural Network (CNN)
 - Convolutions
 - Convolution layers
- Examples of CNNs
 - LeNet
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 - Understand CNN and the different components that compose it
 - Implement CNNs in Python, NumPy and PyTorch
- Additional readings
 - Chapter 7 of the textbook, available at: <https://d2l.ai/>

Convolutional Neural Networks: LeNet

- LeNet, the first published CNN (by Yann LeCun in 1998), used to recognize handwritten digits in images
 - Used to recognize digits for processing deposits in ATM machines (1990s)
 - Some ATM machines are still using this code!

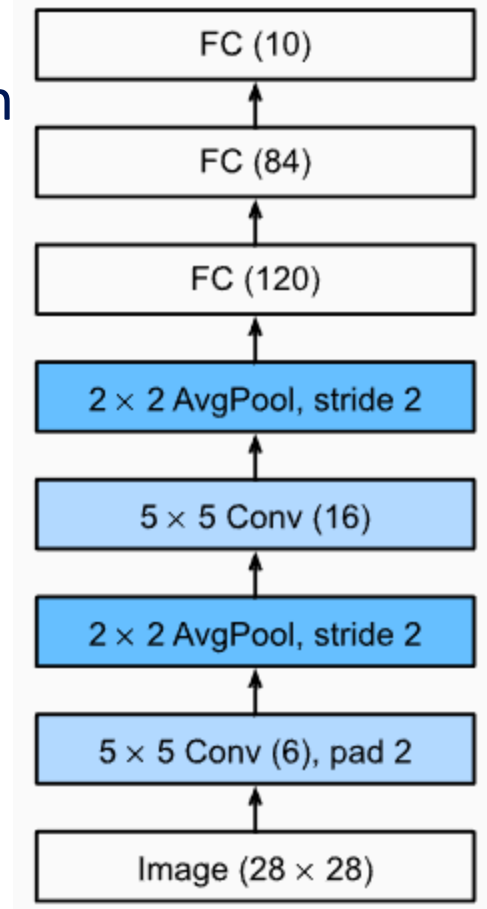


Convolutional encoder composed of 2 convolutional layers (5x5 kernel), each layer has sigmoid activation and is followed by 2x2 average pooling and stride 2

Two fully connected layers

Convolutional Neural Networks: LeNet

- LeNet, the first published CNN (by Yann LeCun in 1998), used to recognize handwritten digits in images
 - Also used to recognize digits for processing deposits in ATM machines
ATM machines are still using this code!
- In the lab,
 - You will create the network and train it on the MNIST dataset



Summary

- We derived the structure of convolutional layers
- In the lab
 - You will create and train LeNet
- Next week
 - Training (including data preparation), validation and testing
 - Tuning the hyper parameters
 - Modern Convolutional Neural Networks

Questions
