APPENDIX 12I ENSEMBLE AVERAGE OF EE^* AND SCATTERING COEFFICIENTS

To find the ensemble average of EE^* we must first determine the ensemble average of ZZ^*, i.e.

$$\langle ZZ^*\rangle = \frac{1}{(2\pi)^2}\iiiint_{-\infty}^{\infty}\langle z(x,y)z(x',y')\rangle \times \exp\left[j(k_x'-k_x)x'+j(k_y'-k_y)y' - jk_x(x-x')-jk_y(y-y')\right] \times dx\,dy\,dx'\,dy'. \tag{12I.1}$$

Since $\langle z(x,y)z(x',y')\rangle$ is the surface correlation function, for a stationary process we can denote it by $\sigma_1^2\rho(x-x', y-y')$, where σ_1^2 is the variance of surface heights and ρ is the surface correlation coefficient. Now let $u=x-x'$ and $v=y-y'$. Then $du\,dv=dx\,dy$, and after integrating with respect to x' and y' we obtain

$$\begin{aligned}\langle ZZ^*\rangle &= \iint_{-\infty}^{\infty}\sigma_1^2\rho(u,v)e^{-jk_xu-jk_yv}du\,dv\,\delta(k_x'-k_x)\delta(k_y'-k_y)\\ &=2\pi\sigma_1^2W(k_x,k_y)\delta(k_x'-k_x)\delta(k_y'-k_y),\end{aligned} \tag{12I.2}$$

where $W(k_x,k_y)=(1/2\pi)\iint_{-\infty}^{\infty}\rho(u,v)e^{-jk_xu-jk_yv}du\,dv$ is recognized as the Fourier transform of the correlation coefficient and hence is the surface roughness spectrum.

In Section 12-5.3 we have shown that

$$E_{pq}=\frac{1}{2\pi}\iint_{-\infty}^{\infty}(j2k\cos\theta\alpha_{pq}Z)f\,dk_x\,dk_y,$$

where $Z=Z(k_x+k\sin\theta, k_y)$. Thus,

$$\langle E_{pq}E_{pq}^*\rangle=\frac{1}{(2\pi)^2}\iiiint_{-\infty}^{\infty}|2k\cos\theta\alpha_{pq}|^2\langle ZZ^*\rangle ff^*\,dk_x\,dk_y\,dk_x'\,dk_y'. \tag{12I.3}$$

Upon substituting (12I.2) in (12I.3) and integrating with respect to k_x' and k_y', we get

$$\langle E_{pq}E_{pq}^*\rangle=\frac{|2k\sigma_1\cos\theta\alpha_{pq}|^2}{2\pi}\iint_{-\infty}^{\infty}W(k_x+k\sin\theta,k_y)\,dk_x\,dk_y. \tag{12I.4}$$

By comparing (12I.4) with (12H.1) it follows that

$$f(k_x,k_y)=|2k\sigma_1\cos\theta\alpha_{pq}|^2W(k_x+k\sin\theta,k_y)/2\pi. \tag{12I.5}$$

Thus, for horizontally incident plane waves, the general form of the scattering coefficient from (12H.4) is

$$\begin{aligned}\sigma_{pq}^0&=4\pi k_s^2\cos^2\theta_s f(k_x,k_y)\eta/\eta_s\\ &=8\eta|kk_s\sigma_1\cos\theta\cos\theta_s\alpha_{pq}|^2W(k_x+k\sin\theta,k_y)/\eta_s.\end{aligned} \tag{12I.6}$$

Inside medium 1, $k=k_s$, $\eta=\eta_s$, $k_x=-k\sin\theta_s\cos\phi_s$, and $k_y=-k\sin\theta_s\sin\phi_s$. For scattering in the transmitted region, $k_s=k'$, $\eta_s/\eta=(\mu_r/\varepsilon_r)^{1/2}$, $k_x=-k'\sin\theta_s'\cos\phi_s$ and $k_y=-k'\sin\theta_s'\sin\phi_s$.

For vertically polarized incident wave the field amplitudes are given for the magnetic fields. (12I.6) will apply after we convert the incident and scattered fields back to electric. This involves multiplying the magnetic fields by the corresponding intrinsic impedances. Within medium 1 the intrinsic impedance is the same for both the incident and the scattered fields. Hence, (12I.6) continues to apply. However, in the transmitted medium, (12I.6) should be multiplied by η_r^2, i.e.,

$$\sigma_{pq}^0=8\eta_r|kk'\sigma_1\cos\theta\cos\theta_s\alpha_{pq}'|^2W(k_x+k\sin\theta,k_y), \tag{12.7}$$

where $\eta_r=(\mu_r/\varepsilon_r)^{1/2}$, $k_x=-k'\sin\theta_s'\cos\phi_s$, and $k_y=-k'\sin\theta_s'\sin\phi_s$.

APPENDIX 12J BASIC MATHEMATICAL RESULTS

12J.1 Green's Vector Theorem

This theorem relates the vector fields inside a region to their values on the boundary surface of the region. It can be derived from the divergence theorem, which states that

$$\int_V\nabla\cdot\mathbf{A}\,dv=\oint_s\mathbf{A}\cdot d\mathbf{s}, \tag{12J.1}$$

where s is the closed surface forming the boundary of the volume V.

Let $\mathbf{A}=\mathbf{P}\times\nabla\times\mathbf{Q}$. Then

$$\int_V\nabla\cdot(\mathbf{P}\times\nabla\times\mathbf{Q})\,dv=\oint_s(\mathbf{P}\times\nabla\times\mathbf{Q})\cdot d\mathbf{s}. \tag{12J.2}$$

In view of the vector identity

$$\nabla\cdot(\mathbf{a}\times\mathbf{b})=\mathbf{b}\cdot\nabla\times\mathbf{a}-\mathbf{a}\cdot\nabla\times\mathbf{b},$$

we can write (12J.2) as

$$\int[(\nabla\times\mathbf{Q})\cdot(\nabla\times\mathbf{P})-\mathbf{P}\cdot\nabla\times(\nabla\times\mathbf{Q})]\,dv=\oint_s(\mathbf{P}\times\nabla\times\mathbf{Q})\cdot d\mathbf{s}. \tag{12J.3}$$

Since **P** and **Q** are arbitrary differentiable vectors, (12J.3) yields another identity upon interchanging **P** and **Q**:

$$\int[(\nabla\times\mathbf{P})\cdot(\nabla\times\mathbf{Q})-\mathbf{Q}\cdot\nabla\times(\nabla\times\mathbf{P})]\,dv=\oint_s(\mathbf{Q}\times\nabla\times\mathbf{P})\cdot d\mathbf{s}. \tag{12J.4}$$

The vector Green's theorem is obtained by taking the difference between (12J.3) and (12J.4), i.e.,

$$\int_v[\mathbf{Q}\cdot\nabla\times\nabla\times\mathbf{P}-\mathbf{P}\cdot\nabla\times\nabla\times\mathbf{Q}]\,dv$$
$$=\oint_s[\mathbf{P}\times\nabla\times\mathbf{Q}-\mathbf{Q}\times\nabla\times\mathbf{P}]\cdot d\mathbf{s}. \tag{12J.5}$$

It is known that the scalar Green's theorem can be applied to find the field at a point in a source-free region in terms of its value and its derivative on the closed boundary surface of the region. By analogy (12J.5) can also be applied for the same purpose to vector fields. This has been done by both Stratton (1941) and Silver (1947), who derived (12.10). A different and more rigorous approach to obtain (12.10) in a different form has been reported by Sancer (1968).

12J.2 Dyadic Green's Function

The dyadic Green's function, G, is the solution to the vector equation

$$\nabla\times\nabla\times\mathsf{G}-k^2\mathsf{G}=-\mathsf{I}\delta(\mathbf{r}-\mathbf{r}') \tag{12J.6}$$

where I is the unit dyadic or the identity matrix and $\delta(\mathbf{r}-\mathbf{r}')$ is the Dirac delta function. Expand the curl in (12J.6) and rewrite it as

$$\nabla^2\mathsf{G}+k^2\mathsf{G}=\mathsf{I}\delta(\mathbf{r}-\mathbf{r}')+\nabla(\nabla\cdot\mathsf{G}). \tag{12J.7}$$

Take the divergence of (12J.6):

$$\begin{aligned}-k^2\nabla\cdot\mathsf{G}&=-\nabla\cdot\mathsf{I}\delta(\mathbf{r}-\mathbf{r}')\\&=-\nabla\delta(\mathbf{r}-\mathbf{r}').\end{aligned} \tag{12J.8}$$

Substituting (12J.8) in (12J.7), we get

$$\nabla^2\mathsf{G}+k^2\mathsf{G}=(\mathsf{I}+\nabla\nabla/k^2)\delta(\mathbf{r}-\mathbf{r}').$$

The form of the above equation suggests that G can be written in terms of the scalar Green's function χ as

$$\mathsf{G}=(I+\nabla\nabla/k^2)\chi, \tag{12J.9}$$

where χ satisfies the scalar wave equation,

$$\nabla^2\chi+k^2\chi=\delta(\mathbf{r}-\mathbf{r}').$$

12J.3 Surface-Integral Representation for a Vector Field

Let V be a region of source-free space enclosed by the surface S. In (12J.5) let **Q** be the electric field **E** and **P** be $\mathsf{G}\cdot\hat{\mathbf{a}}$ where $\hat{\mathbf{a}}$ is an arbitrary constant vector. From (12J.6), we obtain

$$\nabla\times\nabla\times(\mathsf{G}\cdot\hat{\mathbf{a}})-k^2\mathsf{G}\cdot\hat{\mathbf{a}}=-\hat{\mathbf{a}}\delta(\mathbf{r}-\mathbf{r}'). \tag{12J.10}$$

The electric field satisfies the vector wave equation

$$\nabla\times\nabla\times\mathbf{E}-k^2\mathbf{E}=0. \tag{12J.11}$$

Substitution of (12J.10) and (12J.11) in (12J.5) reduces the volume integral to

$$\int_v\left[\mathbf{E}\cdot(k^2\mathsf{G}\cdot\hat{\mathbf{a}}-\hat{\mathbf{a}}\delta(\mathbf{r}-\mathbf{r}'))-(\mathsf{G}\cdot\hat{\mathbf{a}})\cdot(k^2\mathbf{E})\right]dv=-\hat{\mathbf{a}}\cdot\mathbf{E}(\mathbf{r}). \tag{12J.12}$$

Hence, **E**(**r**) is expressed in terms of the surface integral given by (12J.5).

Consider the problem of a plane wave, $\mathbf{E}^i$, incident upon S from the region outside of V. (12J.12) is valid for any electric field. Thus, when **E**(**r**) represents the transmitted scattered field, we get

$$-\hat{\mathbf{a}}\cdot\mathbf{E}^s(\mathbf{r})=\int_s[(\mathsf{G}\cdot\hat{\mathbf{a}})\times\nabla\times\mathbf{E}^s(\mathbf{r}')-\mathbf{E}^s(\mathbf{r}')\times\nabla\times(\mathsf{G}\cdot\hat{\mathbf{a}})]\cdot d\mathbf{s}(\mathbf{r}'), \tag{12J.13}$$

where the ∇ acts on the $\mathbf{r}'$ variable. This shows that the scattered field inside a region V can be expressed in terms of its derivative and itself on the boundary surface of V. Such a result depends on the fact that the Green's function has its source located in V and $\mathbf{E}^s$ is also in V.

When we interpret **E** in (12J.12) as the incident field, then

$$\int_V\mathbf{E}^i(\mathbf{r}')\cdot\hat{\mathbf{a}}\delta(\mathbf{r}-\mathbf{r}')\,dv=0,$$

since the incident field is outside of V while the source point of the Green's function is inside of V. Hence, for the incident field

$$0=\int_S \left[(\mathbf{G}\cdot\hat{\mathbf{a}})\times\nabla\times\mathbf{E}^i(\mathbf{r}')-\mathbf{E}^i(\mathbf{r}')\times\nabla\times(\mathbf{G}\cdot\hat{\mathbf{a}})\right]\cdot ds(\mathbf{r}'). \tag{12J.14}$$

It follows from (12J.13) and (12J.14) that by adding (12J.13) to (12J.14), it is possible to express $E^s(\mathbf{r}')$ in terms of $\mathbf{E}^s+\mathbf{E}^i$, which is the total field. It should be emphasized that when $\nabla\times(\mathbf{E}^s+\mathbf{E}^i)$ is an approximate result or when the integrand of the surface integral is an approximate result, it may happen that (12J.14) does not vanish. In this case $\mathbf{E}^s$ in the surface integral given by (12J.13) should not be replaced by the total field.

When we interpret $\mathbf{E}(\mathbf{r})$ as the scattered field from S, it is more convenient to apply (12J.5) to V_∞ and $S+S_\infty$ as shown in Figure 12J.1. Analogous to (12J.12) the volume integral over V_∞ yields $-\hat{\mathbf{a}}.\ \mathbf{E}^s(\mathbf{r})$, where $\mathbf{r}$ is inside of V_∞. The surface integral from (12J.5) over S_∞ vanishes when S_∞ recedes to infinity, since the scattered field must vanish at infinity. Upon directing the unit surface normal on S towards V_∞ instead of V, we get

$$\hat{\mathbf{a}}\cdot\mathbf{E}^s(\mathbf{r})=\int_S \left[(\mathbf{G}\cdot\hat{\mathbf{a}})\times\nabla\times\mathbf{E}^s(\mathbf{r}')-\mathbf{E}^s(\mathbf{r}')\times\nabla\times(\mathbf{G}\cdot\hat{\mathbf{a}})\right]\cdot\hat{\mathbf{n}}\, dS(\mathbf{r}'). \tag{12J.15}$$

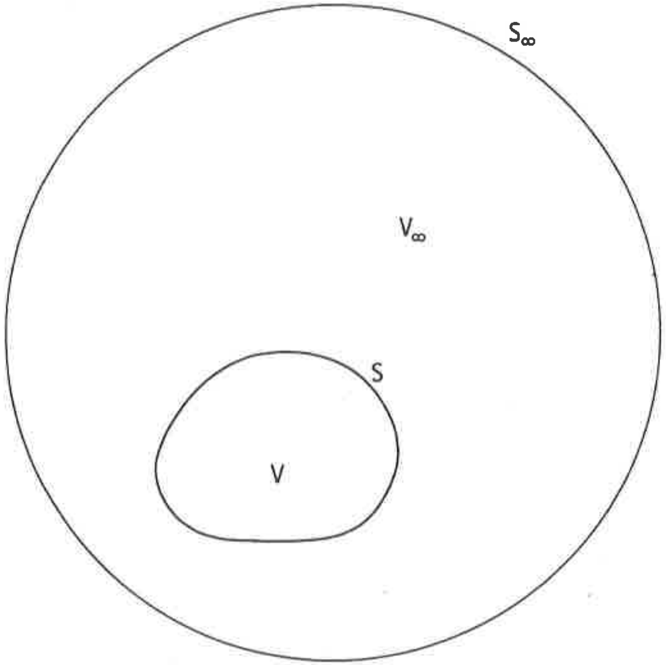

Fig. 12J.1 Geometry for illustrating Green's second theorem.

12J.4 Reduction of the Surface Integral

Equation (12J.15) shows the surface integral in terms of the dyadic Green's function. It is possible to rewrite it in terms of the scalar Green's function by first noting the following two relations:

$$\hat{\mathbf{a}}\cdot\mathbf{G}=\mathbf{G}\cdot\hat{\mathbf{a}}, \qquad \text{since} \quad \mathbf{G} \text{ is symmetric;} \tag{12J.16}$$

$$\begin{aligned}\nabla\times(\mathbf{G}\cdot\hat{\mathbf{a}}) &= \nabla\times\left[\hat{\mathbf{a}}+\hat{\mathbf{a}}\cdot\nabla\nabla/k^2\right]\chi\\ &=\nabla\times(\hat{\mathbf{a}}\chi)\\ &=\nabla\chi\times\hat{\mathbf{a}}.\end{aligned} \tag{12J.17}$$

The integrand of (12J.15) can now be written as

$$\begin{aligned}&(\hat{\mathbf{a}}\cdot\mathbf{G})\cdot(\nabla\times\mathbf{E}^s)\times\hat{\mathbf{n}}-\mathbf{E}^s\times(\nabla\chi\times\hat{\mathbf{a}})\cdot\hat{\mathbf{n}}\\ &\quad=-\hat{\mathbf{a}}\cdot\left[\mathbf{G}\cdot\hat{\mathbf{n}}\times\nabla\times\mathbf{E}^s+(\hat{\mathbf{n}}\times\mathbf{E}^s)\times\nabla\chi\right]\\ &\quad=-\hat{\mathbf{a}}\cdot\left[(\mathbf{I}+\nabla\nabla/k^2)\chi\cdot\hat{\mathbf{n}}\times\nabla\times\mathbf{E}^s+(\hat{\mathbf{n}}\times\mathbf{E}^s)\times\nabla\chi\right]\\ &\quad=-\hat{\mathbf{a}}\cdot\left[\hat{\mathbf{n}}\times(-j\omega\mu\mathbf{H}^s)\chi-j\omega\mu\nabla\nabla\chi\right.\\ &\qquad\left.\cdot(\hat{\mathbf{n}}\times\mathbf{H}^s)/k^2+(\hat{\mathbf{n}}\times\mathbf{E}^s)\times\nabla\chi\right]\end{aligned} \tag{12J.18}$$

The surface integral of the middle term in (12J.18) can be written, upon adding and subtracting $\nabla\chi\hat{\mathbf{n}}\cdot(\nabla\times\mathbf{H}^s)$, as

$$I_0=\frac{j}{\omega\varepsilon}\left\{\int\hat{\mathbf{a}}\cdot\left[\nabla\nabla\chi\cdot(\hat{\mathbf{n}}\times\mathbf{H}^s)-\nabla\chi\hat{\mathbf{n}}\cdot(\nabla\times\mathbf{H}^s)\right]ds\right.$$
$$\left.+\int\hat{\mathbf{a}}\cdot\nabla\chi\hat{\mathbf{n}}\cdot(\nabla\times\mathbf{H}^s)\,ds\right\}. \tag{12J.19}$$

The integrand of the first integral in (12J.19) can be simplified by using the following vector identities:

$$\begin{aligned}\nabla(\hat{\mathbf{a}}\cdot\nabla\chi)&=(\hat{\mathbf{a}}\cdot\nabla)\nabla\chi+(\nabla\chi\cdot\nabla)\hat{\mathbf{a}}+\hat{\mathbf{a}}\times(\nabla\times\nabla\chi)\\ &\quad+\nabla\chi\times(\nabla\times\hat{\mathbf{a}})\\ &=(\hat{\mathbf{a}}\cdot\nabla)\nabla\chi,\end{aligned}$$

$$\nabla\times\left[(\hat{\mathbf{a}}\cdot\nabla\chi)\mathbf{H}^s\right]=(\hat{\mathbf{a}}\cdot\nabla\chi)(\nabla\times\mathbf{H}^s)+\nabla(\hat{\mathbf{a}}\cdot\nabla\chi)\times\mathbf{H}^s,$$

as follows:

$$\begin{aligned}&(\hat{\mathbf{a}}\cdot\nabla)\nabla\chi\cdot(\hat{\mathbf{n}}\times\mathbf{H}^s)-(\hat{\mathbf{a}}\cdot\nabla\chi)\hat{\mathbf{n}}\cdot(\nabla\times\mathbf{H}^s)\\ &\quad=-\nabla(\hat{\mathbf{a}}\cdot\nabla\chi)\cdot(\mathbf{H}^s\times\hat{\mathbf{n}})-(\hat{\mathbf{a}}\cdot\nabla\chi)(\nabla\times\mathbf{H}^s)\cdot\hat{\mathbf{n}}\\ &\quad=-\left[\nabla(\hat{\mathbf{a}}\cdot\nabla\chi)\times\mathbf{H}^s+(\hat{\mathbf{a}}\cdot\nabla\chi)\nabla\times\mathbf{H}^s\right]\cdot\hat{\mathbf{n}}\\ &\quad=-\left[\nabla\times(\hat{\mathbf{a}}\cdot\nabla\chi)\mathbf{H}^s\right]\cdot\hat{\mathbf{n}}.\end{aligned} \tag{12J.20}$$

Equation (12J.20) shows that the first integral in (12J.19) has the form of the Stokes theorem when S is an open surface and can be converted to a line integral. However, if the surface S is closed, then the total length of this line integral goes to zero. In this case only the second integral in (12J.19) is nonzero, and since $\nabla\times\mathbf{H}^s = j\omega\varepsilon\mathbf{E}^s$, we get

$$I_0 = -\hat{\mathbf{a}}\cdot\int \nabla\chi(\hat{\mathbf{n}}\cdot\mathbf{E}^s)\,ds. \tag{12J.21}$$

Hence, when $\mathbf{E}^s$ and $\mathbf{H}^s$ are nonzero every where over a closed surface S, substitution of (12J.18) in (12J.15) and use of (12J.21) for the integral of the middle term in (12J.18) yields

$$\mathbf{E}^s(\mathbf{a}) = \oint_s [\,j\omega\mu(\hat{\mathbf{n}}\times\mathbf{H}^s)\chi - (\hat{\mathbf{n}}\times\mathbf{E}^s)\times\nabla\chi - \nabla\chi(\hat{\mathbf{n}}\cdot\mathbf{E}^s)]\,ds. \tag{12J.22}$$

On the other hand, when $\mathbf{H}^s$ and $\mathbf{E}^s$ are nonzero only over a portion of the surface S, then the surface acts like an open surface. In this case I_0 is not given by (12J.21). Instead, it is

$$\begin{aligned} I_0 &= \frac{-j}{\omega\varepsilon}\int \nabla\times(\hat{\mathbf{a}}\cdot\nabla\chi)\mathbf{H}^s\cdot\hat{\mathbf{n}}\,ds - \hat{\mathbf{a}}\cdot\int\nabla\chi(\hat{\mathbf{n}}\cdot\mathbf{E}^s)\,ds \\ &= -\hat{\mathbf{a}}\cdot\left[\frac{j}{\omega\varepsilon}\oint\nabla\chi\mathbf{H}^s\cdot d\mathbf{l} + \int\nabla\chi(\hat{\mathbf{n}}\cdot\mathbf{E}^s)\,ds\right]. \end{aligned} \tag{12J.23}$$

This means that for open surfaces, there is an additional line integral which should be added on to (12J.22), i.e.

$$\begin{aligned} \mathbf{E}^s(\mathbf{r}) &= \int_s [\,j\omega\mu(\hat{\mathbf{n}}\times\mathbf{H}^s)\chi - (\hat{\mathbf{n}}\times\mathbf{E}^s)\times\nabla\chi - \nabla\chi(\hat{\mathbf{n}}\cdot\mathbf{E}^s)]\,ds \\ &\quad - \frac{j}{\omega\varepsilon}\oint\nabla\chi\mathbf{H}^s\cdot d\mathbf{l} \end{aligned} \tag{12J.24}$$

In both (12J.24) and (12J.22) the operator V acts on the integration variable $\mathbf{r}'$.

12J.5 The Far-Zone Scattered Field

The scalar Green's function can be written as

$$\chi = -\frac{\exp(-jk|\mathbf{r}-\mathbf{r}'|)}{4\pi|\mathbf{r}-\mathbf{r}'|}.$$

Let us denote the field point by $\mathbf{r}$ and the source point by $\mathbf{r}'$. In the far zone

$|\mathbf{r}-\mathbf{r}'|\simeq|\mathbf{r}| = r$ in the denominator and $|\mathbf{r}-\mathbf{r}'|\simeq r-\hat{\mathbf{r}}\cdot\mathbf{r}'$ in the phase. Hence,

$$\chi \simeq -\frac{\exp(-jkr + jk\hat{\mathbf{r}}\cdot\mathbf{r}')}{4\pi r}, \tag{12J.25}$$

$$\begin{aligned} \nabla\chi &= -\frac{e^{-jkr}}{4\pi r}\nabla(e^{j\mathbf{k}\cdot\mathbf{r}'}) \\ &= -\frac{j\mathbf{k}}{4\pi r}\exp(-jkr + j\mathbf{k}\cdot\mathbf{r}'), \end{aligned} \tag{12J.26}$$

where $\mathbf{k} = k\hat{\mathbf{r}}$.

The far-field expression for (12J.24) can be found by substituting (12J.25) and (12J.26) in (12J.24):

$$\begin{aligned} \mathbf{E}^s(\mathbf{r}) &= \frac{je^{-jkr}}{4\pi r}\int_s [(\hat{\mathbf{n}}\times\mathbf{E}^s)\times\mathbf{k} - \omega\mu(\hat{\mathbf{n}}\times\mathbf{H}^s) + \mathbf{k}(\hat{\mathbf{n}}\cdot\mathbf{E}^s)]\,e^{j\mathbf{k}\cdot\mathbf{r}'}\,ds \\ &\quad - \frac{e^{-jkr}}{4\pi r\omega\varepsilon}\mathbf{k}\oint e^{j\mathbf{k}\cdot\mathbf{r}'}\mathbf{H}^s\cdot d\mathbf{l}. \end{aligned} \tag{12J.27}$$

To show that (12J.27) reduces to (12.10) consider the last integral in (12J.27). By Stokes's theorem,

$$\begin{aligned} \oint e^{j\mathbf{k}\cdot\mathbf{r}'}\mathbf{H}^s\cdot d\mathbf{l} &= \int_s \nabla\times(\mathbf{H}^s e^{j\mathbf{k}\cdot\mathbf{r}'})\cdot d\mathbf{s} \\ &= \int_s [(\nabla e^{j\mathbf{k}\cdot\mathbf{r}'})\times\mathbf{H}^s + e^{j\mathbf{k}\cdot\mathbf{r}'}\nabla\times\mathbf{H}^s]\cdot d\mathbf{s} \\ &= \int_s [\,j\mathbf{k}\times\mathbf{H}^s + j\omega\varepsilon\mathbf{E}^s]\,e^{j\mathbf{k}\cdot\mathbf{r}'}\cdot d\mathbf{s}. \end{aligned} \tag{12J.28}$$

Substituting (12J.28) in the last term in (12J.27), we get

$$\frac{je^{-jkr}}{4\pi r}\int\left[\frac{\mathbf{k}(\mathbf{k}\times\mathbf{H}^s\cdot\hat{\mathbf{n}})}{\omega\varepsilon} + \mathbf{k}(\hat{\mathbf{n}}\cdot\mathbf{E}^s)\right]e^{j\mathbf{k}\cdot\mathbf{r}'}\,ds \tag{12J.29}$$

Substitution of (12J.29) in (12J.27) yields

$$\begin{aligned} \mathbf{E}^s(\mathbf{r}) &= \frac{je^{-jkr}}{4\pi r}\int\{(\hat{\mathbf{n}}\times\mathbf{E}^s)\times\mathbf{k} - \omega\mu(\hat{\mathbf{n}}\times\mathbf{H}^s) \\ &\qquad + [\mathbf{k}\cdot(\mathbf{n}\times\mathbf{H}^s)]\mathbf{k}/\omega\varepsilon\}\,e^{j\mathbf{k}\cdot\mathbf{r}'}\,ds \\ &= K\hat{\mathbf{r}}\times\int[(\hat{\mathbf{n}}\times\mathbf{E}^s) - \hat{\mathbf{r}}\times(\hat{\mathbf{n}}\times\mathbf{H}^s)\eta]\,e^{j\mathbf{k}\cdot\mathbf{r}'}\,ds, \end{aligned} \tag{12J.30}$$

where $K = -jke^{-jkr}/4\pi r$ and $\hat{\mathbf{r}}$ is the unit vector pointing in the direction of observation.