# Minimum-Voltage Injection Method for Sensorless Control of PMSM for Low-Speed Operation

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***Abstract*— This paper aims to find a solution for low speed position calculation for sensorless permanent-magnet synchronous machines (PMSM). The method alters the field-oriented control (FOC) algorithm by injecting a voltage vector in every other period instead of the control loop. By measuring current variation produced by the injected voltage vector, it is possible to obtain accurate estimations of the rotor position. The FOC automatically adjusts to this voltage deviation and the current produced by it is low enough to be considered as loss. A phase-locked loop (PLL) control system is implemented to accurately follow the position. The proposed method does not required of any additional filter and its low computational burden allows for easy implementation. Simulation evaluation was conducted showing the validity of the method.**

***Index Terms*—Voltage injection, sensorless control, permanent magnet synchronous machine (PMSM), low-speed.**

## I. Introduction

SENSORLESS control of permanent-magnet synchronous machines (PMSM) provides many advantages to this already very promising type of motor for drive applications. Most notably, removing the mechanical position sensor may result in reduced costs, better reliability, and lower system volume, which translates to higher torque density.

For medium and high-speed operation, the back-EMF method is commonly used with largely successful results. This method relies on identifying the position of the rotor flux linkage by making use of the induced rotor voltage (back-EMF) obtained from stator current and voltage measurements. As the rotor flux linkage is, by definition, always aligned with the *d*-axis of the system, the method directly provides the required position information.

This method, however, is not well suited for estimation in low-speed operation, as the back-EMF term of the PMSM model is directly proportional to the rotational speed of the machine and thus can also become very small. Estimation methods used for low-speed operation are often based on voltage injection, relying on rotor saliency: the difference between *d*-axis and *q*-axis inductances.

Voltage injection methods can be broadly classified into two main categories, high-frequency injection and pulse or square voltage injection. This article focuses on a method belonging to the latter group. By injecting a constant voltage aligned with one of the two estimated rotor axes for half of each control period, the resultant current variation is directly linked to the rotor position estimation error. The main advantage of the presented method is the fact that it does not require any filtering, unlike high-frequency injection methods (which require high-pass filters to extract the current variation due to the injection voltage) and other injection techniques (which often require low-pass filters in the current control loop).

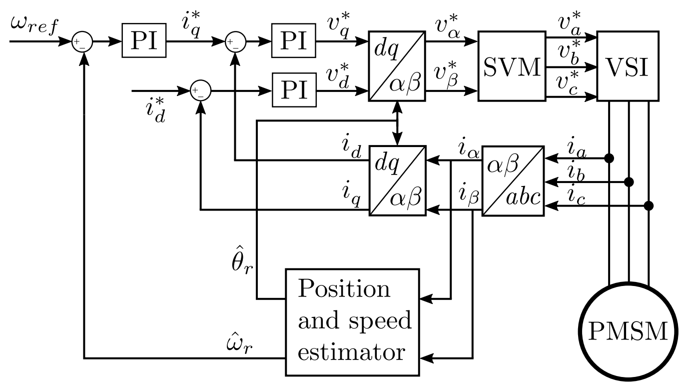


Fig. 1. Block diagram of speed control of a PMSM using sensorless FOC.

## II. PMSM control fundamentals

This section details the principles of field-oriented control, a commonly used scheme for speed control of permanent magnet synchronous motors.

### A. Basic modeling

Expressed in the rotor synchronous reference frame, or *dq*-reference frame, the voltage equations of a PMSM can be written as follows:

$$\begin{cases} v_d = Ri_d + L_d \frac{d}{dt} i_d - \omega_r L_q i_q \\ v_q = Ri_q + L_q \frac{d}{dt} i_q + \omega_r L_d i_d + \omega_r \lambda_{mpm} \end{cases} \tag{1}$$

Where $v_d$ and $v_q$ are the stator *d* and *q*-axis voltages, $i_d$ and $i_q$ are the stator *d* and *q*-axis currents, $R$ is the stator resistance, $L_d$ and $L_q$ are the stator *d* and *q*-axis inductances, $\omega_r$ is the rotational velocity of the rotor, and $\lambda_{mpm}$ is the maximum flux linkage provided by the rotor permanent magnets.

### B. Field-oriented control

The implemented control scheme to make use of the designed estimator is the classical field-oriented control (FOC). It is based on the well-known fact that torque production in a PMSM is directly proportional to *q*-axis current. A block diagram of sensorless FOC for PMSM speed control is shown in Figure 1.

A reference angular speed $\omega_{ref}$ and *d*-axis current $i_d^*$ must be provided first. By making use of the estimated rotor angular speed $\hat{\omega}_r$, the speed error is obtained and fed to a PI controller, which synthesizes the $q$-axis reference current $i_q^*$. With both current references known, current error is obtained by subtracting the measured $d$-axis and $q$-axis currents. It should be noted that the reference frame transformation used for the measured currents also relies on the estimated angular position. Using two more PI controllers, the current errors are converted to reference $d$ and $q$-axis voltages, which are themselves transformed to the $\alpha\beta$-reference frame and fed into a space-vector modulation (SVM) algorithm, that allows a voltage-source inverter (VSI) to synthesize the selected reference voltages. For the purposes of this article, voltage references are assumed to be tracked without any error or additional dynamics. Inverter output voltage is then fed to the three-phases of a PMSM. Phase currents are physically measured and transformed to the $\alpha\beta$-reference frame. The currents $i_\alpha$ and $i_\beta$ are used as inputs for the position and speed estimator, which obtains estimates for the rotor angular position $\hat{\theta}_r$ and its angular speed $\hat{\omega}_r$.

## III. The estimation method

The proposed method for position estimation is based on [1], which is in turn based on the more widespread INFORM method. The PMSM model presented in Equation 1 cannot be directly used for the purpose of rotor position and speed estimator, as it already assumes that the rotor position is known accurately to obtain the values of the variables in the $dq$-reference frame. The first step is then to obtain the machine model in the stationary $\alpha\beta$-reference frame. Defining the reference frame transformation matrix:

$$K_{dq\rightarrow\alpha\beta} = \begin{bmatrix} \cos(\theta_r) & -\sin(\theta_r) \\ \sin(\theta_r) & \cos(\theta_r) \end{bmatrix} \tag{2}$$

Where $\theta_r$ denotes the electrical angular position of the rotor with respect to the stationary horizontal $\alpha$-axis. Equation 1 can be expressed in vector form, noting that $\bar{f}_{dq} = f_d + jf_q$, with $j$ representing the imaginary unit. Multiplying this equation on both sides by the transformation matrix, the following voltage equation can be obtained in the $\alpha\beta$-reference frame:

$$\bar{v}_{\alpha\beta} = R\bar{i}_{\alpha\beta} + \frac{d}{dt}\bar{\lambda}_{\alpha\beta} \tag{3}$$

Where the total flux linkage $\bar{\lambda}_{\alpha\beta}$ is given by:

$$\begin{aligned} \bar{\lambda}_{\alpha\beta} = &\begin{bmatrix} L_1 + L_2\cos(2\theta_r) & L_2\sin(2\theta_r) \\ L_2\sin(2\theta_r) & L_1 - L_2\cos(2\theta_r) \end{bmatrix} \bar{i}_{\alpha\beta} \\ &+ \begin{bmatrix} \cos(\theta_r) \\ \sin(\theta_r) \end{bmatrix} \lambda_{mpm} \\ = &L_1\bar{i}_{\alpha\beta} + L_2\bar{i}^*_{\alpha\beta}e^{j2\theta_r} + \lambda_{mpm}e^{j\theta_r} \end{aligned} \tag{4}$$

Where $L_1 = (L_d + L_q)/2$ and $L_2 = (L_d - L_q)/2$. Combining Equations 3 and 4, the complete $\alpha\beta$ voltage equation can be obtained:

$$\begin{aligned} \bar{v}_{\alpha\beta} = &R\bar{i}_{\alpha\beta} + L_1\frac{d}{dt}\bar{i}_{\alpha\beta} + L_2\frac{d}{dt}\bar{i}^*_{\alpha\beta}e^{j\theta_r} \\ &+ j2\omega_r L_2\bar{i}_{\alpha\beta}e^{j2\theta_r} + j\omega_r\lambda_{mpm}e^{j\theta_r} \end{aligned} \tag{5}$$

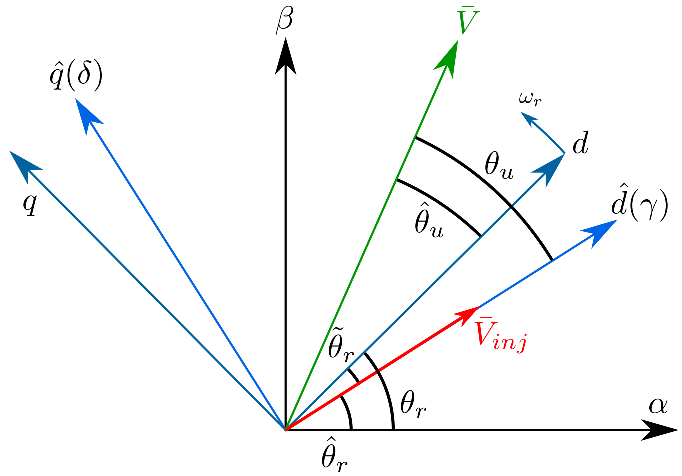


Fig. 2. Reference frames used in the modeling, estimation, and control scheme.

The presented estimation method is intended to be used for low-speed operation. Therefore, it is a reasonable assumption that:

$$L_1\frac{d}{dt}\bar{i}_{\alpha\beta} + L_2\frac{d}{dt}\bar{i}^*_{\alpha\beta}e^{j\theta_r} >> j2\omega_r L_2\bar{i}_{\alpha\beta}e^{j2\theta_r} + j\omega_r\lambda_{mpm}e^{j\theta_r} \tag{6}$$

As the two terms on the right-hand side are directly proportional to rotational speed. The injected voltage used for estimation should fulfill:

$$\bar{v}_{\alpha\beta} >> R\bar{i}_{\alpha\beta} \tag{7}$$

By making use of these two assumptions, Equation 3 may be simplified to:

$$\bar{v}_{\alpha\beta} \approx L_1\frac{d}{dt}\bar{i}_{\alpha\beta} + L_2\frac{d}{dt}\bar{i}^*_{\alpha\beta}e^{j2\theta_r} \tag{8}$$

Solving for $\frac{d}{dt}\bar{i}_{\alpha\beta}$:

$$\frac{d}{dt}\bar{i}_{\alpha\beta} = \frac{L_1}{L_1^2 - L_2^2}\bar{v}_{\alpha\beta} - \frac{L_2}{L_1^2 - L_2^2}\bar{v}^*_{\alpha\beta}e^{j2\theta_r} \tag{9}$$

By approximating $\frac{d}{dt}\bar{i}_{\alpha\beta}$ with $\frac{\Delta\bar{i}_{\alpha\beta}}{\Delta t}$, Equation 9 may be rewritten as:

$$\Delta\bar{i}_{\alpha\beta} = \Delta t(c_1 + c_2e^{j2(\theta_r - \theta_u)})\bar{v}_{\alpha\beta} \tag{10}$$

Where $c_1 = L_1/(L_1^2 - L_2^2)$, $c_2 = -L_2/(L_1^2 - L_2^2)$, and $\theta_u$ represents the angle at which the stator voltage vector $\bar{v}_{\alpha\beta}$ is injected. This equation can then be transformed to the estimated synchronous $dq$-reference frame, noted as $\gamma\delta$-reference frame. This reference frame transformation is obtained by rotating the $\alpha\beta$ frame to the estimated rotor angle $\hat{\theta}_r$:

$$\Delta\bar{i}_{\gamma\delta} = \Delta t(c_1 + c_2e^{2j(\theta_r - \hat{\theta}_r - \hat{\theta}_u)})\bar{v}_{\gamma\delta} \tag{11}$$

The voltage vector used for estimation can be injected either in the estimated $\gamma$-axis or $\delta$-axis, in order to further simplify Equation 11. In the case where injection is performed on the estimated $d$-axis ($\gamma$-axis), $\hat{\theta}_u = 0$ and $\bar{v}_{\gamma\delta} = V_me^{j0} = V_m$. Defining the estimated reference frame error angle $\tilde{\theta}_r = \theta_r - \hat{\theta}_r$, Equation 11 can be rewritten as:

$$\Delta\bar{i}_{\gamma\delta} = \Delta t(c_1 + c_2e^{j2\tilde{\theta}_r})V_m \tag{12}$$

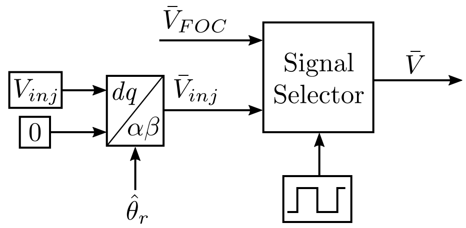


Fig. 3. Reference voltage generation.

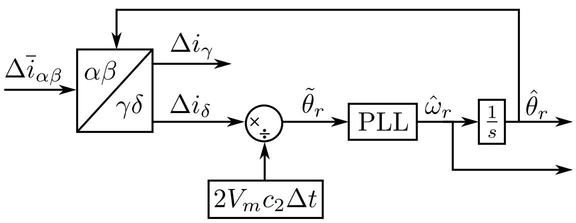


Fig. 4. Position and speed estimation scheme.

By separating the real ($\gamma$) and imaginary ($\delta$) parts of Equation 12:

$$Im(\Delta\bar{i}_{\gamma\delta}) = c_2 \Delta t V_m \sin(2\tilde{\theta}_r) \tag{13}$$

Linearizing Equation 13, while assuming a small value of the error angle $\tilde{\theta}_r$:

$$Im(\Delta\bar{i}_{\gamma\delta}) \approx 2c_2 \Delta t V_m \tilde{\theta}_r = 2k\tilde{\theta}_r \tag{14}$$

Where $k = c_2 \Delta t V_m$. These manipulations have served to show that the current variation in the estimated *q*-axis ($\delta$-axis) due to a constant voltage pulse injected in the estimated *d*-axis ($\gamma$-axis) is directly proportional to the angular position estimation error $\tilde{\theta}_r$. The two quantities can be simply related by a scalar constant $2k$. The used reference frames and applied voltages are depicted in Figure 2.

## IV. Implementation

The described method has been implemented using *Matlab/Simulink*. Once the model of the PMSM has been implemented based on Equation 1, as well as a standard FOC scheme as shown in Figure 1, the next step is to introduce the voltage injection in the control scheme. This process is depicted in Figure 3.

Initially, a constant injection voltage $V_{inj}$ is selected following the guideline provided by Equation 7. The injection voltage should therefore be much larger than the resistive losses that appear due to itself. On the other hand, too large a voltage would increase magnetization losses and lead to torque fluctuations, due to the injected voltage not appearing on the real *d*-axis [1]. This injection voltage on the estimated *d*-axis is transformed to the $\alpha\beta$-reference frame by making use of the estimated rotor angle, obtaining the $\alpha\beta$-vector $\bar{V}_{inj}$. This signal is combined with the $\alpha\beta$-reference voltage provided by the FOC scheme, $\bar{V}_{FOC}$, by means of a signal selector. The two signals must alternate with between each other, and they have been chosen to last for the same amount of time. The final voltage reference vector in the $\alpha\beta$-reference frame, $\bar{V}$ is then used as an input of the SVM algorithm.

TABLE I
Model parameters

| | |
|---|---|
| Pole pairs | 4 |
| Max. RMS phase voltage $[V]$ | 360 |
| Rated torque $[Nm]$ | 38 |
| Rated speed $[r/min]$ | 1750 |
| Rated current $[A]$ | 16 |
| Inertia $[Jm^2]$ | 0.001 |
| Switching frequency $[kHz]$ | 20 |
| Stator resistance $[\Omega]$ | 0.78 |
| d-axis inductance $[mH]$ | 10 |
| q-axis inductance $[mH]$ | 12.8 |
| d-axis injected voltage $[V]$ | 45 |
| PM flux linkage $[Wb]$ | 0.412 |

The estimation scheme is depicted as a block diagram in Figure 4. The scheme is executed once every two sampling periods, as the measured current increment $\Delta\bar{i}_{\alpha\beta}$ must be calculated only for the voltage injection periods. This also means that the time increment $\Delta t$ is equivalent to a single sampling period.

This method theoretically works injecting the voltage vector either in the d or in the q axis. The main drawback of *q*-axis injection is that the torque is affected by the produced *q*-current. If the voltage is injected in the *d*-axis, the current can just be considered as minor losses. The FOC is then able to inject an opposite voltage vector ($-V_{inj}$) in the *d*-axis, leading to an average *d*-current of 0A.

At the beginning of each estimation period, which is to say at the end of each injection period, the measured current increment in the $\alpha\beta$-reference frame is transformed to the estimated $\gamma\delta$-reference frame, by making use of the feedback angle estimate $\hat{\theta}_r$. As shown in the mathematical analysis of the method, voltage injection in the $\gamma$-axis results in a current change directly proportional to the angle estimation error in the $\delta$-axis. Using the $\delta$-axis current increment $\Delta i_\delta$ and scaling it by the constant $1/(2V_m c_2 \Delta t)$, the angle error $\tilde{\theta}_r$ is obtained. This value is fed to a PLL, which through a PI structure then synthesizes the estimated angular frequency $\hat{\omega}_r$. Integrating this signal results in the final value of $\hat{\theta}_r$.

## V. Results

Simulation results are obtained using a PMSM system with the characteristics displayed in Table I. A previously validated field oriented controller is used to implement the proposed method. As stated, the voltage vector is injected in the estimated *d*-axis and the response is measured in the estimated *q*-axis. Figures 5, 6, 7, 8 and 9 have been obtained with the real position ($\hat{\theta}$) as the reference for the *dq* transformations.

The data gathered for model validation includes 3-phase and dq currents, as well as speed and position real and estimated values. The error between the real and estimated rotor position is depicted at the bottom of each graph to show the transient performance in detail. Tests were conducted to measure the behaviour with step variations in load and in speed.

Since the method is intended for low speed position estimation, figure 5 shows the behaviour of the model at 100 rpm. Load torque is then risen from 0 Nm to $T_{rated} = 38$ Nm at 0.5 s and then removed at 1 s. It is seen how the estimator is able

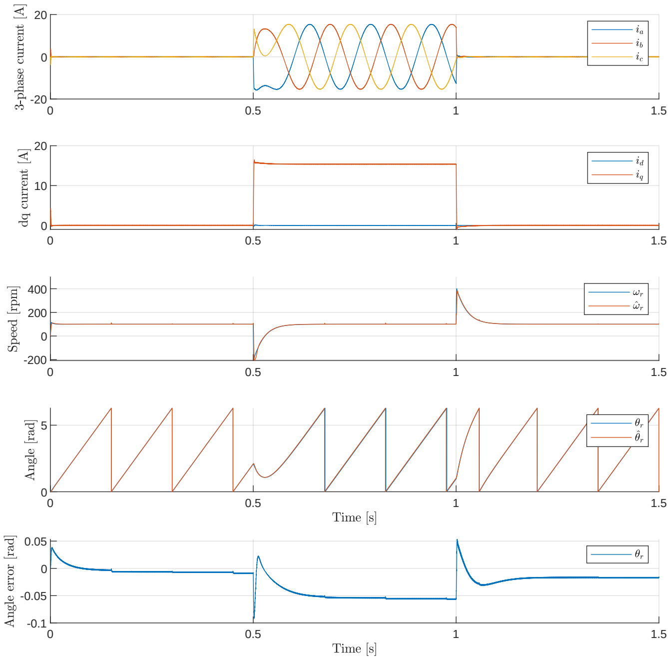


Fig. 5. 0 Nm to $T_{rated}$ load step at constant reference speed of 100 rpm. From top to bottom: 3-phase currents ($i_{abc}$), dq-currents ($i_{dq}$), real and estimated speeds, real and estimated positions and estimated position error.

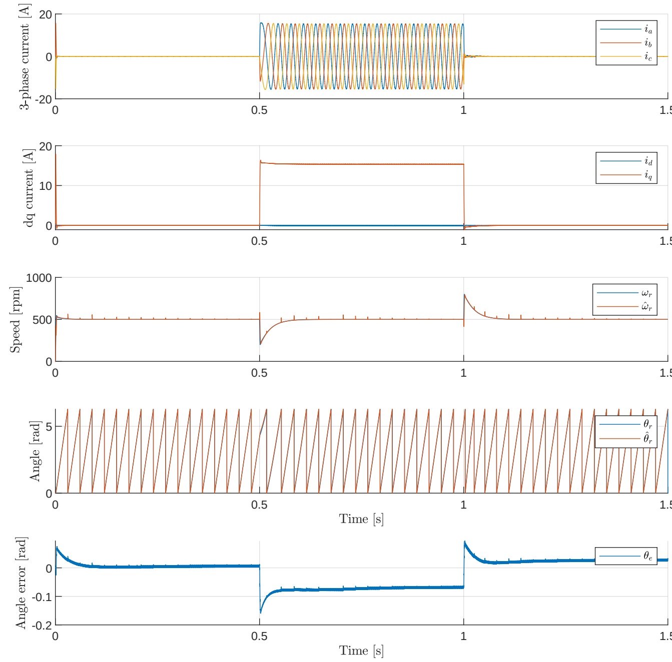


Fig. 6. 0 Nm to $T_{rated}$ load step at constant reference speed of 500 rpm. From top to bottom: 3-phase currents ($i_{abc}$), dq-currents ($i_{dq}$), real and estimated speeds, real and estimated positions and estimated position error.

to accurately follow the real value of the position. The FOC quickly adjusts the speed of the motor to settle at 100 rpm, to do so, $i_q$ reaches the expected current of 16 A. Nevertheless, it is seen how the speed drops to negative values when the load is introduced and reaches high speed when it is again removed, this effect is partly due to the low inertia of the motor.

A similar test was performed with a higher speed of 500 rpm as seen in figure 6. The results were also comparable, with slightly worse estimation of the speed since the assumption made in equation 6 gets less valid for higher speeds.

The proposed method has also been tested to see if the motor is able to maintain a zero speed with load torque applied to the rotor. Figure 7 shows this behaviour and it is seen how the system has no problem reacting in this scenario. Similar speed transients to those in figures 5 and 6 are seen. Also, at null speed, the PLL is unable to calculate the exact position of the rotor, but this situation does not affect the correct dynamics of the controller since it is able to understand that the motor is stopped.

The behaviour of the position estimators are also tested for speed variations. For this purpose two tests were conducted, taking the motor from a standstill position to both 100 and 500 rpm, figures 8 and 9 show these responses. Both of these cases show very similar results.

Firstly it is seen how the *dq*-currents are held constant throughout the simulation since the torque applied does not change, except for the spike produced by a sudden change in speed at $t = 0.2$ s. The position estimation is very accurate once the rotor starts moving and has the same problem as before when it is stopped. Other than these, the behaviour turns out as expected and the estimator is able to accurately follow during the simulation.

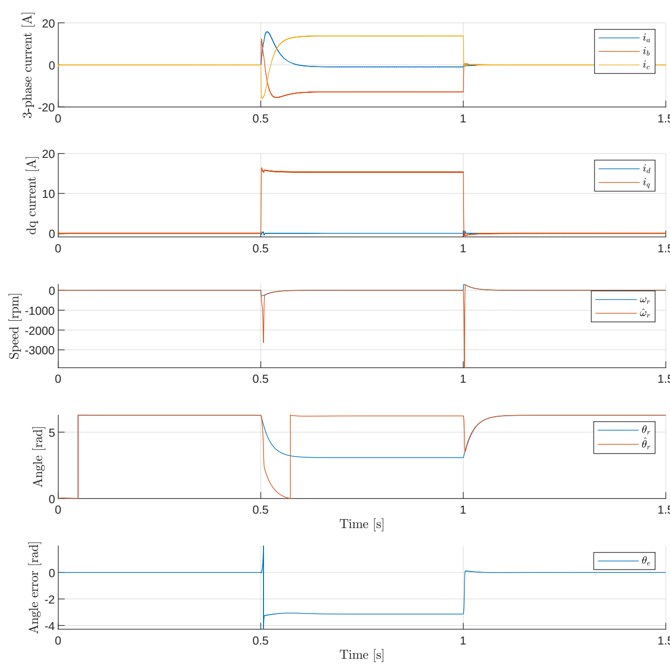


Fig. 7. 0 Nm to $T_{rated}$ load step at a constant null reference speed. From top to bottom: 3-phase currents ($i_{abc}$), dq-currents ($i_{dq}$), real and estimated speeds, real and estimated positions and estimated position error.

A final simulation is conducted in order to validate that the model is correctly working with the estimated position functioning as input for reference frame transformations. The same simulation as that shown in figure 5 is performed, but now with estimated position. The results are almost identical as those shown in figure 5. This validates that the estimation is accurate enough to control the position of the PMSM without

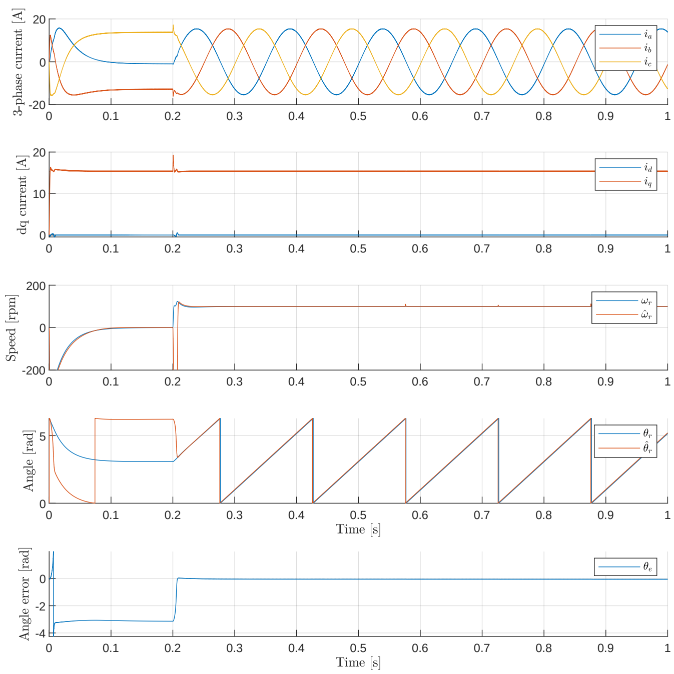


Fig. 8. 0 rpm to 100 rpm speed step at constant reference torque load of $T_{rated}$. From top to bottom: 3-phase currents ($i_{abc}$), dq-currents ($i_{dq}$), real and estimated speeds, real and estimated positions and estimated position error.

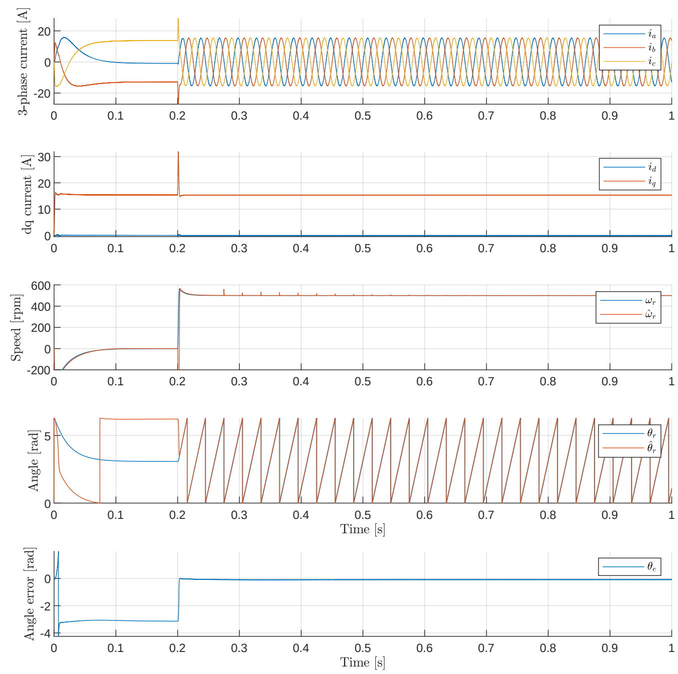


Fig. 9. 0 rpm to 500 rpm speed step at constant reference torque load of $T_{rated}$. From top to bottom: 3-phase currents ($i_{abc}$), dq-currents ($i_{dq}$), real and estimated speeds, real and estimated positions and estimated position error.

sensors.

## VI. Conclusions

This paper proves the effectiveness of a low speed position tracking method for sensorless estimation of a PMSM position. By replacing every other FOC output voltage with a constant $\gamma$-axis voltage vector, it is possible to find deviations in the estimated rotor position. Little power is lost during the implementation since relatively low voltage pulses are injected. The method produces negligible torque variations since the current is almost directly introduced in the d-axis voltage position. This simple method is easy to implement and it entails a fairly low computational burden, meaning that no additional hardware will be required in most cases. Nevertheless, by including voltage injection the FOC bandwidth is halved since it is only run in every other period. Nevertheless, this removes the need of filtering, both for injecting and for running the FOC. The method has been validated through simulations showing very accurate position tracking, the FOC was able to function at machine rated torque and at various low speeds. The low implementation cost and high accuracy of the proposed method allow for its easy use in low speed sensorless control of PMSM applications.

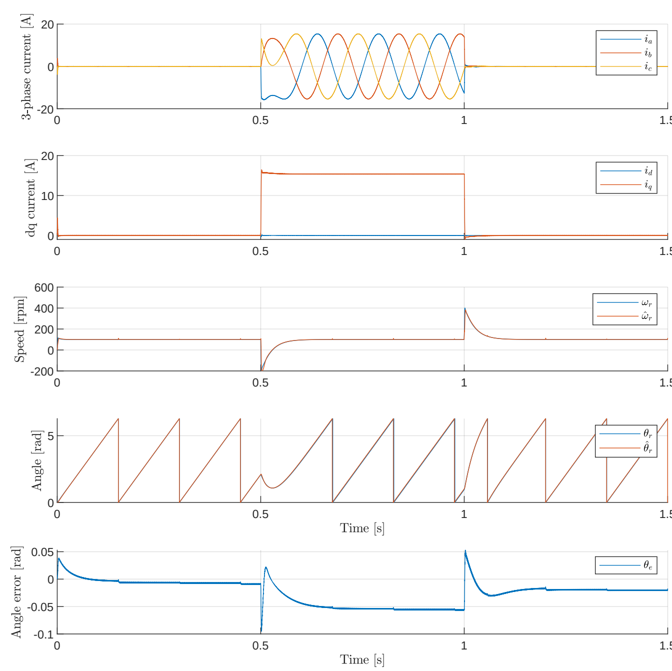


Fig. 10. 0 Nm to $T_{rated}$ load step at a constant reference speed of 100 rpm using estimated rotor position. From top to bottom: 3-phase currents ($i_{abc}$), dq-currents ($i_{dq}$), real and estimated speeds, real and estimated positions and estimated position error.

## References


[1] G. Xie et al. "Minimum-Voltage Vector Injection Method for Sensorless Control of PMSM for Low-Speed Operations". In: *IEEE Transactions on Power Electronics* 31.2 (Feb. 2016), pp. 1785–1794. ISSN: 1941-0107. DOI: 10.1109/TPEL.2015.2426200.