

22.1-6

Most graph algorithms that take an adjacency-matrix representation as input require time $\Omega(V^2)$, but there are some exceptions. Show how to determine whether a directed graph G contains a *universal sink*—a vertex with in-degree $|V| - 1$ and out-degree 0—in time $O(V)$, given an adjacency matrix for G .

22.2-6

Give an example of a directed graph $G = (V, E)$, a source vertex $s \in V$, and a set of tree edges $E_\pi \subseteq E$ such that for each vertex $v \in V$, the unique simple path in the graph (V, E_π) from s to v is a shortest path in G , yet the set of edges E_π cannot be produced by running BFS on G , no matter how the vertices are ordered in each adjacency list.

22.3-9

Give a counterexample to the conjecture that if a directed graph G contains a path from u to v , then any depth-first search must result in $v.d \leq u.f$.

22.4-4

Prove or disprove: If a directed graph G contains cycles, then $\text{TOPOLOGICAL-SORT}(G)$ produces a vertex ordering that minimizes the number of “bad” edges that are inconsistent with the ordering produced.

23.1-3

Show that if an edge (u, v) is contained in some minimum spanning tree, then it is a light edge crossing some cut of the graph.

23.1-4

Give a simple example of a connected graph such that the set of edges $\{(u, v) : \text{there exists a cut } (S, V - S) \text{ such that } (u, v) \text{ is a light edge crossing } (S, V - S)\}$ does not form a minimum spanning tree.

23.2-4

Suppose that all edge weights in a graph are integers in the range from 1 to $|V|$. How fast can you make Kruskal's algorithm run? What if the edge weights are integers in the range from 1 to W for some constant W ?