華東師範大學
EAST CHINA NORMAL UNIVERSITY

Logic in Computer Science

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PART 1

Introduction

Logic in Computer Science

Applications of logic in computer science include:

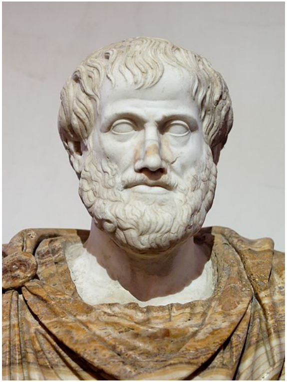
- Artificial intelligence:
  Represent knowledge and draw conclusions from it
- Databases:
  Extract data from a database
- Hard and software design:
  Does a system exhibit any undesired properties?
- Programming languages:
  Infer the type of a variable, function, or method

Logic in Computer Science

- Logic offers the “right” methods and tools: precise language, and methods to derive valid conclusions (convincingly)
- This course:
  1. Propositional logic:
  2. First-order logic:

Propositional Logic

Formal Nature of Logical Reasoning



Aristotle (384–322 BCE)

Formal Nature of Logical Reasoning

Assumption 1:	**All** humans **are** mortal.
Assumption 2:	Sokrates **is** a human.
Conclusion:	**Therefore**, Sokrates **is** mortal.

This argument is valid by pure formal reasons. Only its form is relevant:

Assumption 1:	**All** *A* **are** *B*.
Assumption 2:	*C* **is** *A*.
Conclusion:	**Therefore**, *C* **is** *B*.

Examples

Assumption 1:	**All** substitution ciphers **are** insecure.
Assumption 2:	The Caesar cipher **is** a substitution cipher.
Conclusion:	**Therefore**, the Caesar cipher **is** insecure.

Assumption 1:	**All** humans **have** four eyes.
Assumption 2:	I **am** a human.
Conclusion:	**Therefore**, I **have** four eyes.

The Need for a Precise Language

- The following argument is invalid:

Assumption 1:	**Some** humans **are** mortal.
Assumption 2:	Sokrates **is** a human.
Conclusion:	**Therefore**, Sokrates **is** mortal.

Why?

- Natural language can be imprecise:

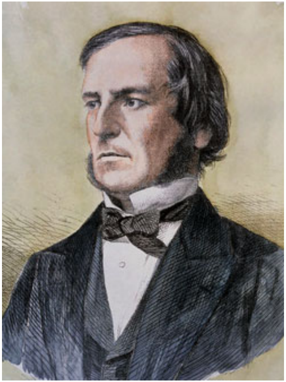
  "Mary does not believe that Eric can pass *any* test."

- Natural language can lead to paradoxes:

  "This sentence is false."

Reasoning as Calculation

Gottfried Wilhelm Leibniz
1646–1716



George Boole
1815–1864

Propositional Logic

- Precise language:
  - Symbols p, q, r, . . . for basic propositions
  - Symbols can be connected by logical operators for "and", "or", "not", "implies", "equivalent" with precise meaning to form complex propositions
- Reasoning via algebraic manipulation of formulas
- Nowadays standard tool in automated reasoning, hardware verification, configuration problems, etc.

Example

p: app records video

q: app has permission to access camera

r: camera indicator light is on

Properties of our system:

$p \leftrightarrow (q \wedge r)$

Question:

Does this imply $\neg r \rightarrow \neg p$?

First-order Logic

Why First-order Logic?

How to formalise this argument in propositional logic?

Assumption 1:	**All** humans **are** mortal.
Assumption 2:	Sokrates **is** a human.
Conclusion:	**Therefore**, Sokrates **is** mortal.

More natural in first-order logic:

$$\frac{\begin{array}{l}\forall x\big(\mathrm{Human}(x) \to \mathrm{Mortal}(x)\big)\\ \mathrm{Human}(\mathrm{Sokrates})\end{array}}{\mathrm{Mortal}(\mathrm{Sokrates})}$$

Another Example

Every print job is eventually printed always.

$$\forall \mathit{job}\ \forall \mathit{time} \Big(\text{Submitted}(\mathit{job}, \mathit{time}) \rightarrow \exists \mathit{time}'\ \text{Printed}(\mathit{job}, \mathit{time}') \Big)$$

Logic as a Foundation for Mathematics

In the mid 19th century, Mathematics (Geometry, Calculus) had rather shaky foundations.

- What does it mean that $\sum_{i=0}^{\infty} a_i$ exists?
- Is Euclidean geometry the only possible geometry?

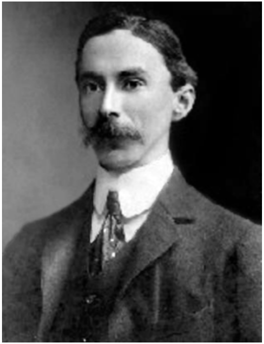
Logic as a Foundation for Mathematics

Gottlob Frege (1848–1925)

Logic as a Foundation for Mathematics

- Frege proposed logic as a foundation for mathematics.
- Invented the basics for first-order logic:
    - Constants: $\pi, \ldots$
    - Predicates: e.g., $<$ to assert $4 < 8$
    - Functions: e.g., $+$ to form $1 + 1$
    - Logical connectives: $\wedge$ (“and”), $\vee$ (“or”), $\neg$ (“not”), etc.
    - Quantifiers: $\exists$ (“exists”), $\forall$ (“for all”)

Inconsistency in Frege's System



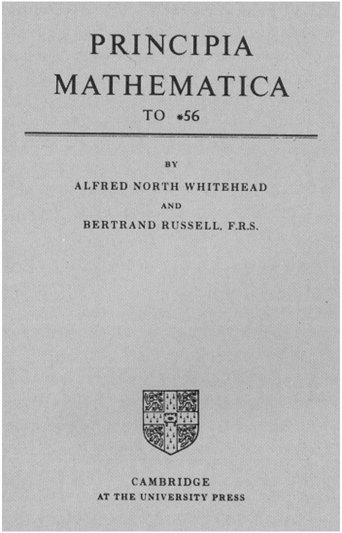
Bertrand Russell
1872–1970

- Frege's system is inconsistent.
- It allows to form the "set":

$$Y := \{X \mid X \text{ is a set with } X \notin Y\}$$

- But: Y is not a set!

The Quest for a Consistent System



- Published 1910, > 2000 pages
- Attempt to repair Frege's system
- To develop logic as a foundation for mathematics

Is Mathematics Automatable?

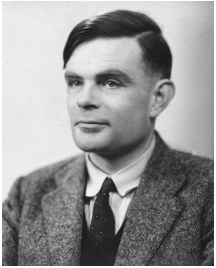
David Hilbert
1862–1943

- Can we prove that mathematics (Principia Mathematica) is consistent?
- Can we develop a complete formal system for mathematics?
- Is mathematics decidable:
  Is there a mechanical way to determine whether a given mathematical statement is true?

Mathematics is Not Automatable

Kurt Gödel
1906–1978

Alonzo Church
1903–1995



Alan Turing
1912–1954

- Gödel (1931): Consistency of mathematics is not provable.
- Church, Turing (1930s): No mechanical procedure that can decide whether a statement in first-order logic is valid.

Beginning of Computer Science

Hilbert's questions required to answer the following ones:

- What is a mechanical procedure?
- What is a solvable problem?
- What is an algorithm?

First-order Logic in Computer Science

- Artificial intelligence:
  Representation of knowledge and inference
- Database query languages:
  First-order logic as the core of SQL
- Verification:
  Verify the correctness of a specification
- …

Learning Outcomes

At the end of this module you should be able to:

1. Translate natural language descriptions and reasoning processes to and from logical equivalents in the propositional and predicate logic.
2. Evaluate first-order logic formulae in relational stuctures and understand the relationship to relational calculus.
3. State and apply a proof system for propositional and first-order logic.

Scores

- Presents 20%
- Assignments 20%
- Final Exam 60%

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