

# Logic in Computer Science Assignment 2

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## 1

### 1.1 Convert the following formula into CNF.

$$\neg(r \rightarrow (\neg((p \vee q) \wedge (\neg p \rightarrow (q \wedge r))))))$$

Let  $\phi = \neg(r \rightarrow (\neg((p \vee q) \wedge (\neg p \rightarrow (q \wedge r))))))$ .

First apply IMPL\_FREE:

$$\begin{aligned} \text{IMPL\_FREE}(\phi) &= \neg(r \rightarrow (\neg((p \vee q) \wedge (\neg p \rightarrow (q \wedge r)))))) \\ &= \neg(\text{IMPL\_FREE}(r \rightarrow (\neg((p \vee q) \wedge (\neg p \rightarrow (q \wedge r)))))) \\ &= \neg(\neg r \vee \text{IMPL\_FREE}(\neg((p \vee q) \wedge (\neg p \rightarrow (q \wedge r)))))) \\ &= \neg(\neg r \vee (\neg(\text{IMPL\_FREE}(p \vee q) \wedge \text{IMPL\_FREE}(\neg p \rightarrow (q \wedge r)))))) \\ &= \neg(\neg r \vee (\neg((p \vee q) \wedge (\text{IMPL\_FREE}(\neg p \rightarrow (q \wedge r)))))) \\ &= \neg(\neg r \vee (\neg((p \vee q) \wedge (\neg \neg p \vee \text{IMPL\_FREE}(q \wedge r)))))) \\ &= \neg(\neg r \vee (\neg((p \vee q) \wedge (\neg \neg p \vee (\text{IMPL\_FREE}q \wedge \text{IMPL\_FREE}r)))))) \\ &= \neg(\neg r \vee (\neg((p \vee q) \wedge (\neg \neg p \vee (q \wedge r)))))) \end{aligned}$$

Then apply NNF:

$$\begin{aligned} \text{NNF}(\text{IMPL\_FREE}(\phi)) &= \text{NNF}(\neg(\neg r \vee (\neg((p \vee q) \wedge (\neg \neg p \vee (q \wedge r)))))) \\ &= \text{NNF}(\neg r) \wedge \text{NNF}(\neg(\neg((p \vee q) \wedge (\neg \neg p \vee (q \wedge r)))))) \\ &= r \wedge \text{NNF}((p \vee q) \wedge (\neg \neg p \vee (q \wedge r))) \\ &= r \wedge (\text{NNF}(p \vee q) \wedge \text{NNF}(\neg \neg p \vee (q \wedge r))) \\ &= r \wedge ((\text{NNF}p \vee \text{NNF}q) \wedge (\text{NNF}(\neg \neg p) \vee \text{NNF}(q \wedge r))) \\ &= r \wedge ((p \vee q) \wedge (p \vee (q \wedge r))) \end{aligned}$$

Then apply CNF:

$$\begin{aligned}
 \text{CNF}(\text{NNF}(\text{IMPL\_FREE}(\phi))) &= \text{CNF}(r \wedge ((p \vee q) \wedge (p \vee (q \wedge r)))) \\
 &= r \wedge \text{CNF}((p \vee q) \wedge (p \vee (q \wedge r))) \\
 &= r \wedge \text{CNF}(p \vee q) \wedge \text{CNF}(p \vee (q \wedge r)) \\
 &= r \wedge \text{DISTR}(p, q) \wedge \text{DISTR}(\text{CNF}(p), \text{CNF}((q \wedge r))) \\
 &= r \wedge \text{DISTR}(p, q) \wedge \text{DISTR}(p, (q \wedge r)) \\
 &= r \wedge (p \vee q) \wedge \text{DISTR}(p, q) \wedge \text{DISTR}(p, r) \\
 &= r \wedge (p \vee q) \wedge (p \vee q) \wedge (p \vee r) \\
 &= r \wedge (p \vee q) \wedge (p \vee r)
 \end{aligned}$$

Above is the result of CNF algorithm. Further more, this result can be simplified by:

$$r \wedge (p \vee q) \wedge (p \vee r) = r \wedge (p \vee r)$$

## 1.2 Determine the satisfiability of the following formula with Horn algorithm.

$$(p \wedge q \wedge s \rightarrow \perp) \wedge (q \wedge s \rightarrow p) \wedge (\top \rightarrow s) \wedge (s \rightarrow q)$$

Marked initially:  $\top$

Since  $\top \rightarrow s$ , mark s. (Now marked: s,  $\top$ )

Since  $s \rightarrow q$ , mark q. (Now marked: s, q,  $\top$ )

Since  $q \wedge s \rightarrow p$ , mark p. (Now marked: s, p, q,  $\top$ )

Since  $p \wedge q \wedge s \rightarrow \perp$ , mark bot. (Now marked: s, p, q,  $\top$ ,  $\perp$ )

Since  $\perp$  is marked at last, from Horn's algorithm, this formula is not satisfiable.