15.4-5

Give an $O(n^2)$-time algorithm to find the longest monotonically increasing subsequence of a sequence of n numbers.

15.4-6 ★

Give an $O(n \lg n)$-time algorithm to find the longest monotonically increasing subsequence of a sequence of n numbers. (*Hint:* Observe that the last element of a candidate subsequence of length i is at least as large as the last element of a candidate subsequence of length $i - 1$. Maintain candidate subsequences by linking them through the input sequence.)

答题思路:

$O(n^2)$的算法可以通过将其排序以后放入另外一个数组，然后运用LCS算法寻找与原数组的最长公共子序列获得。

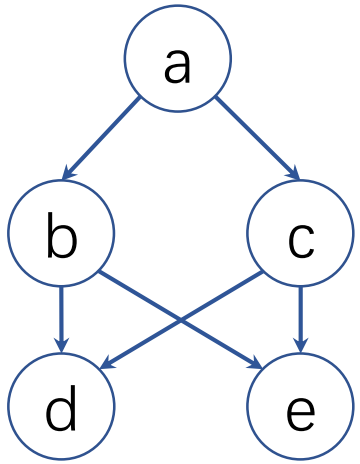
$O(n \lg n)$的算法需要设计动态规划算法，使用数组A[]，A[i]记录目前为止长度为i个的最长单调递增序列的最后一个元素的最小可能值，算法对输入数组B[]进行遍历，依次取出B[j]，然后在A[] 进行二分查找，使得A[k]<B[j]<A[k+1]，然后更新A[k+1] 的值为B[j]。如果B[j]比A[]中最大的数A[m]大，则更新A[m+1] 的值为B[j] 。时间复杂度自己分析。

22.2-6

Give an example of a directed graph $G = (V, E)$, a source vertex $s \in V$, and a set of tree edges $E_\pi \subseteq E$ such that for each vertex $v \in V$, the unique simple path in the graph (V, E_π) from s to v is a shortest path in G, yet the set of edges E_π cannot be produced by running BFS on G, no matter how the vertices are ordered in each adjacency list.

答题思路：

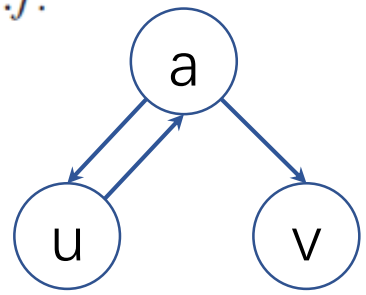
如右图所示，{ab, bd, ac, ce}就是这样一棵树。



22.3-9

Give a counterexample to the conjecture that if a directed graph G contains a path from u to v, then any depth-first search must result in $v.d \leq u.f$.

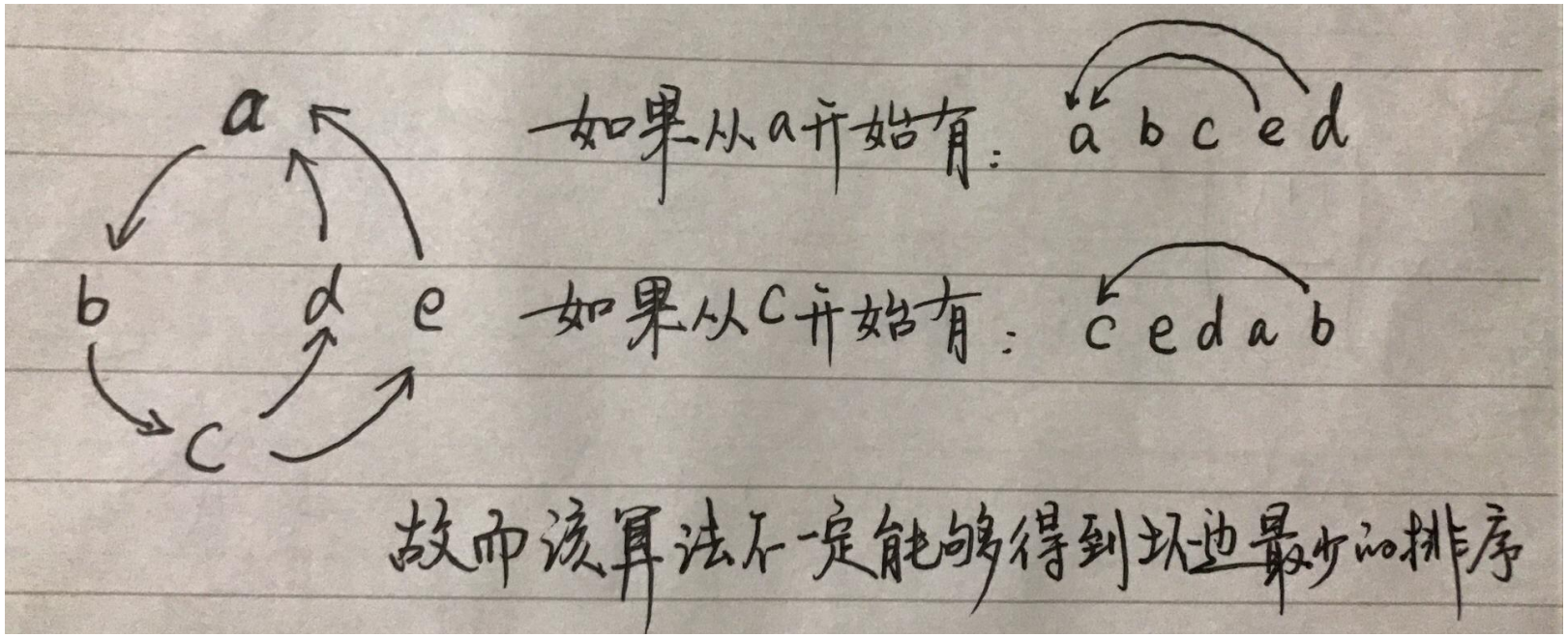
答题思路：如右图所示，uv之间有路径，但先访问a再访问u最后访问v即可得反例。



22.4-4

Prove or disprove: If a directed graph G contains cycles, then TOPOLOGICAL-SORT(G) produces a vertex ordering that minimizes the number of "bad" edges that are inconsistent with the ordering produced.

答题思路：不一定，取决于DFS过程中每棵树的根节点选择。



23.1-3

Show that if an edge (u, v) is contained in some minimum spanning tree, then it is a light edge crossing some cut of the graph.

23.1-4

Give a simple example of a connected graph such that the set of edges $\{(u, v) :$ there exists a cut $(S, V - S)$ such that (u, v) is a light edge crossing $(S, V - S)\}$ does not form a minimum spanning tree.

23.1-3答题思路：把(u,v)拿掉，则该MST分成两个部分，在这两个部分的割边集当中，(u,v)必然是一条轻边，否则假设有一条更小的(x,y)，将其替换掉(u,v)会得到更小的MST，矛盾。

23.1-4答题思路：任意一个边权相等的环路都构成范例，这时所有边都在轻边集合里。注意一个割集当中可以有多条轻边。

23.2-4

Suppose that all edge weights in a graph are integers in the range from 1 to $|V|$. How fast can you make Kruskal's algorithm run? What if the edge weights are integers in the range from 1 to W for some constant W?

答题思路:

如果边权为整数且不超过|V|，排序可以用计数排序在O(|E|)时间内完成，故而时间复杂度取决于后面的选边过程 O(|E| α(|V|))。

如果边权为整数且不超过W，排序可以用基数排序在 $O\left(|\mathrm{E}| \times log_{|E|}W\right)$ 时间内完成，当$O\left(|\mathrm{E}| \times log_{|E|}W\right) < O(|\mathrm{E}| \times \log|E|)$时，时间复杂度为 $O\left(|\mathrm{E}| \times \alpha(|\mathrm{V}|) + |\mathrm{E}| \times log_{|E|}W\right)$ 。