Logic in Computer Science Assignment 2

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1.1 Convert the following formula into CNF.

$\neg(r \to (\neg((p \vee q) \wedge (\neg p \to (q \wedge r)))))$

Let $\phi = \neg(r \to (\neg((p \vee q) \wedge (\neg p \to (q \wedge r)))))$.
First apply `IMPL_FREE`:

$$\begin{aligned}
\texttt{IMPL_FREE}(\phi) &= \neg(r \to (\neg((p \vee q) \wedge (\neg p \to (q \wedge r))))) \\
&= \neg(\texttt{IMPL_FREE}(r \to (\neg((p \vee q) \wedge (\neg p \to (q \wedge r)))))) \\
&= \neg(\neg r \vee \texttt{IMPL_FREE}(\neg((p \vee q) \wedge (\neg p \to (q \wedge r))))) \\
&= \neg(\neg r \vee (\neg(\texttt{IMPL_FREE}(p \vee q) \wedge \texttt{IMPL_FREE}(\neg p \to (q \wedge r))))) \\
&= \neg(\neg r \vee (\neg((p \vee q) \wedge (\texttt{IMPL_FREE}(\neg p \to (q \wedge r)))))) \\
&= \neg(\neg r \vee (\neg((p \vee q) \wedge (\neg\neg p \vee \texttt{IMPL_FREE}(q \wedge r))))) \\
&= \neg(\neg r \vee (\neg((p \vee q) \wedge (\neg\neg p \vee (\texttt{IMPL_FREE}q \wedge \texttt{IMPL_FREE}r))))) \\
&= \neg(\neg r \vee (\neg((p \vee q) \wedge (\neg\neg p \vee (q \wedge r))))) \\
\end{aligned}$$

Then apply `NNF`:

$$\begin{aligned}
\texttt{NNF}(\texttt{IMPL_FREE}(\phi)) &= \texttt{NNF}(\neg(\neg r \vee (\neg((p \vee q) \wedge (\neg\neg p \vee (q \wedge r)))))) \\
&= \texttt{NNF}(\neg\neg r) \wedge \texttt{NNF}(\neg(\neg((p \vee q) \wedge (\neg\neg p \vee (q \wedge r)))))) \\
&= r \wedge \texttt{NNF}((p \vee q) \wedge (\neg\neg p \vee (q \wedge r))) \\
&= r \wedge (\texttt{NNF}(p \vee q) \wedge \texttt{NNF}(\neg\neg p \vee (q \wedge r))) \\
&= r \wedge ((\texttt{NNF}p \vee \texttt{NNF}q) \wedge (\texttt{NNF}(\neg\neg p) \vee \texttt{NNF}(q \wedge r))) \\
&= r \wedge ((p \vee q) \wedge (p \vee (q \wedge r)))
\end{aligned}$$

Then apply CNF:

$$\begin{aligned}
\texttt{CNF}(\texttt{NNF}(\texttt{IMPL_FREE}(\phi))) &= \texttt{CNF}(r \wedge ((p \vee q) \wedge (p \vee (q \wedge r)))) \\
&= r \wedge \texttt{CNF}((p \vee q) \wedge (p \vee (q \wedge r))) \\
&= r \wedge \texttt{CNF}(p \vee q) \wedge \texttt{CNF}(p \vee (q \wedge r)) \\
&= r \wedge \texttt{DISTR}(p, q) \wedge \texttt{DISTR}(\texttt{CNF}(p), \texttt{CNF}((q \wedge r))) \\
&= r \wedge \texttt{DISTR}(p, q) \wedge \texttt{DISTR}(p, (q \wedge r)) \\
&= r \wedge (p \vee q) \wedge \texttt{DISTR}(p, q) \wedge \texttt{DISTR}(p, r) \\
&= r \wedge (p \vee q) \wedge (p \vee q) \wedge (p \vee r) \\
&= r \wedge (p \vee q) \wedge (p \vee r) \\
&= r \wedge (p \vee q)
\end{aligned}$$

1.2 Determine the satisfiablity of the following formula with Horn algorithm.

$(p \wedge q \wedge s \rightarrow \bot) \wedge (q \wedge s \rightarrow p) \wedge (\top \rightarrow s) \wedge (s \rightarrow q)$

Marked initially: $\top$
Since $\top \rightarrow s$, mark s. (Now marked: s, $\top$)
Since $s \rightarrow q$, mark q. (Now marked: s, q, $\top$)
Since $q \wedge s \rightarrow p$, mark p. (Now marked: s, p, q, $\top$)
Since $p \wedge q \wedge s \rightarrow \bot$, mark bot. (Now marked: s, p, q, $\top$, $\bot$)
Since $\bot$ is marked at last, from Horn's algorithm, this formula is not satisfiable.