

Logic in Computer Science Assignment 1

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1 证明

1.1 $\neg(p \wedge q) \dashv\vdash \neg q \vee p$

正向:

$$1 \quad \neg(p \wedge q) \quad \text{premise}$$
$$2 \quad p \vee \neg p \quad \text{LEM}$$

| | |
|-------------|------------|
| $3 \quad p$ | assumption |
|-------------|------------|

| | |
|-------------|------------|
| $4 \quad q$ | assumption |
|-------------|------------|

| | |
|----------------------|-------------------|
| $5 \quad p \wedge q$ | $\wedge i \ 3, 4$ |
|----------------------|-------------------|

| | |
|-----------------|-----------------|
| $6 \quad \perp$ | $\neg e \ 1, 5$ |
|-----------------|-----------------|

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|------------------|------------------|
| $7 \quad \neg q$ | $\neg i \ 4 - 6$ |
|------------------|------------------|

| | |
|------------------------------|----------------|
| $8 \quad \neg q \vee \neg p$ | $\vee i_1 \ 7$ |
|------------------------------|----------------|

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|------------------|------------|
| $9 \quad \neg p$ | assumption |
|------------------|------------|

| | |
|-------------------------------|----------------|
| $10 \quad \neg q \vee \neg p$ | $\vee i_2 \ 9$ |
|-------------------------------|----------------|

$$11 \quad \neg q \vee \neg p \quad \vee e \ 2, 3 - 8, 9 - 10$$

逆向:

$$1 \quad \neg q \vee \neg p \quad \text{premise}$$

| | |
|------------------|------------|
| $2 \quad \neg q$ | assumption |
|------------------|------------|

| | |
|----------------------|------------|
| $3 \quad p \wedge q$ | assumption |
|----------------------|------------|

| | |
|-------------|------------------|
| $4 \quad q$ | $\wedge e_2 \ 3$ |
|-------------|------------------|

| | |
|-----------------|-----------------|
| $5 \quad \perp$ | $\neg e \ 2, 4$ |
|-----------------|-----------------|

| | |
|----------------------------|------------------|
| $6 \quad \neg(p \wedge q)$ | $\neg i \ 3 - 5$ |
|----------------------------|------------------|

| | |
|------------------|------------|
| $7 \quad \neg p$ | assumption |
|------------------|------------|

| | |
|----------------------|------------|
| $8 \quad p \wedge q$ | assumption |
|----------------------|------------|

| | |
|-------------|------------------|
| $9 \quad p$ | $\wedge e_1 \ 8$ |
|-------------|------------------|

| | |
|------------------|-----------------|
| $10 \quad \perp$ | $\neg e \ 7, 9$ |
|------------------|-----------------|

| | |
|-----------------------------|-------------------|
| $11 \quad \neg(p \wedge q)$ | $\neg i \ 8 - 10$ |
|-----------------------------|-------------------|

$$12 \quad \neg(p \wedge q) \quad \vee e \ 1, 2 - 6, 7 - 11$$

1.2 $p \rightarrow q \dashv\vdash \neg q \rightarrow \neg p$

正向:

| | | |
|---|-----------------------------|-----------------------|
| 1 | $p \rightarrow q$ | premise |
| 2 | $\neg q$ | assumption |
| 3 | $\neg p$ | MT 1, 2 |
| 4 | $\neg q \rightarrow \neg p$ | \rightarrow i 2 – 3 |

逆向:

| | | |
|---|-----------------------------|-----------------------|
| 1 | $\neg q \rightarrow \neg p$ | premise |
| 2 | p | assumption |
| 3 | $\neg\neg p$ | $\neg\neg$ i 2 |
| 4 | $\neg\neg q$ | MT 1, 3 |
| 5 | q | $\neg\neg$ e 4 |
| 6 | $p \rightarrow q$ | \rightarrow i 2 – 5 |

1.3 $p \wedge q \rightarrow p \dashv\vdash r \vee \neg r$

正向:

$$1 \quad r \vee \neg r \quad \text{LEM}$$

逆向:

| | | |
|---|----------------------------|---------------------------|
| 1 | $p \wedge q$ | assumption |
| 2 | p | \wedge e ₁ 1 |
| 3 | $p \wedge q \rightarrow p$ | \rightarrow i 1 – 2 |