Logic in Computer Science Assignment 5

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1 Check CTL

1.1 Check whether the following models satisfies the CTL formulas.

1.1.1 AG (Start → AF Heat)

Applying SAT algorithm:

$$
\begin{aligned}
&\text{SAT (AG (Start} \to \text{AF Heat))} = \text{SAT } (\neg\text{EF } \neg(\text{Start} \to \text{AF Heat})) \\
&\quad = S \setminus \text{SAT (EF } \neg(\text{Start} \to \text{AF Heat})) \\
&\quad = S \setminus \text{SAT } (E[\top \text{ U } \neg(\text{Start} \to \text{AF Heat})]) \\
&\quad = S \setminus \text{SAT }_{EU}(\top, \neg(\text{Start} \to \text{AF Heat})) \\
&\qquad\quad \text{SAT } (\top) = S \\
&\qquad\quad \text{SAT } (\neg(\text{Start} \to \text{AF Heat})) = S \setminus \text{SAT (Start} \to \text{AF Heat}) \\
&\qquad\quad = S \setminus \text{SAT } (\neg\text{Start} \vee \text{AF Heat}) \\
&\qquad\quad = S \setminus \text{SAT } (\neg\text{Start}) \cup \text{SAT (AF Heat)} \\
&\qquad\quad = S \setminus \{1,3,4\} \cup \text{SAT }_{AF}(\text{Heat}) \\
&\qquad\qquad\quad \text{SAT }_{AF}(\text{Heat}) = \{4,7\} \cup \{1,2,3,4,5,6,7\} \\
&\qquad\qquad\quad = \{1,2,3,4,5,6,7\} \\
&\qquad\quad = S \setminus \{,3,4\} \cup \{1,2,3,4,5,6,7\} \\
&\qquad\quad = S \setminus \{1,2,3,4,5,6,7\} \\
&\qquad\quad = \emptyset \\
&\quad \Rightarrow \text{SAT }_{EU}(\top, \neg(\text{Start} \to \text{AF Heat})) = \emptyset \\
&\Rightarrow \text{SAT (AG (Start} \to \text{AF Heat))} = S
\end{aligned}
$$

So the model satisfies the CTL formula.

1.1.2 AG ($A[\neg$`Error` U (AF `Close`)])

$$
\begin{aligned}
&\text{SAT (AG } (A\neg\texttt{Error U AF Close}))\\
&\quad = \text{SAT } (\neg\text{EF } \neg(A[\neg\texttt{Error U (AF Close)}]))\\
&\quad = S \setminus \text{SAT } (\text{EF } \neg(A[\neg\texttt{Error U (AF Close)}]))\\
&\quad = S \setminus \text{SAT } (E(\top \text{ U } ()\neg A[\neg\texttt{Error U } (AF\texttt{Close})]))\\
&\quad = S \setminus \text{SAT }_{EU}(\top, \neg A[\neg\texttt{Error U (AF Close)}])\\
&\qquad\quad \text{SAT } (\top) = S\\
&\qquad\quad \text{SAT } (\neg A[\neg\texttt{Error U (AF Close)}])\\
&\qquad\quad = S \setminus \text{SAT } (A[\neg\texttt{Error U (AF Close)}])\\
&\qquad\quad = S \setminus \text{SAT } (\neg(E[\texttt{Error U } (\texttt{Error} \wedge \neg(\text{AF Close}))] \vee \text{EG } \neg(\text{AF Close})))\\
&\qquad\quad = \text{SAT } (E[\texttt{Error U } (\texttt{Error} \wedge \neg(\text{AF Close}))] \vee \text{EG } (\neg(\text{AF Close})))\\
&\qquad\quad = \text{SAT } (E[\texttt{Error U } (\texttt{Error} \wedge \neg(\text{AF Close}))]) \cup \text{SAT } (\text{EG } (\neg(\text{AF Close})))\\
&\qquad\quad = \text{SAT }_{EU}(\texttt{Error}, \texttt{Error} \wedge \neg(\text{AF Close})) \cup \text{SAT } (\neg\text{AF } (\text{AF Close}))\\
&\qquad\quad = \text{SAT }_{EU}(\texttt{Error}, \texttt{Error} \wedge \neg(\text{AF Close})) \cup (S \setminus \text{SAT } (\text{AF } (\text{AF Close})))\\
&\qquad\quad = \text{SAT }_{EU}(\texttt{Error}, \texttt{Error} \wedge \neg(\text{AF Close})) \cup (S \setminus \text{SAT }_{AF}(\text{AF Close}))\\
&\qquad\qquad\quad \text{SAT } (\texttt{Error}) = \{2, 5\}\\
&\qquad\qquad\quad \text{SAT } (\texttt{Error} \wedge \neg(\text{AF Close}))\\
&\qquad\qquad\quad = \text{SAT } (\texttt{Error}) \cap \text{SAT } (\neg\text{AF } (\texttt{Close}))\\
&\qquad\qquad\quad = \{2, 5\} \cap (S \setminus \text{SAT AF Close})\\
&\qquad\qquad\quad = \{2, 5\} \cap (S \setminus \{1, 2, 3, 4, 5, 6, 7\})\\
&\qquad\qquad\quad = \emptyset\\
&\qquad\quad \Rightarrow \text{SAT }_{EU}(\texttt{Error}, \texttt{Error} \wedge \neg(\text{AF Close})) = \emptyset\\
&\qquad\qquad\quad \text{SAT } (\text{AF Close}) = \text{SAT }_{AF}(\texttt{Close})\\
&\qquad\qquad\quad = \{3, 4, 5, 6, 7\}\\
&\qquad\qquad\quad = \{1, 2, 3, 4, 5, 6, 7\}\\
&\qquad\quad = \emptyset \cup (S \setminus \{1, 2, 3, 4, 5, 6, 7\})\\
&\qquad\quad = \emptyset\\
&\quad = S \setminus \emptyset\\
&\quad = S
\end{aligned}
$$

So the model satisfies the CTL formula.