

Logic in Computer Science Assignment 1

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1 证明

1.1 $\neg(p \wedge q) \dashv\vdash \neg q \vee p$

正向:

$$1 \quad \neg(p \wedge q) \quad \text{premise}$$
$$2 \quad p \vee \neg p \quad \text{LEM}$$

$3 \quad p$	assumption
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$4 \quad q$	assumption
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$5 \quad p \wedge q$	$\wedge i \ 3, 4$
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$6 \quad \perp$	$\neg e \ 1, 5$
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$7 \quad \neg q$	$\neg i \ 4 - 6$
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$8 \quad \neg q \vee \neg p$	$\vee i_1 \ 7$
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$9 \quad \neg p$	assumption
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$10 \quad \neg q \vee \neg p$	$\vee i_2 \ 9$
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$$11 \quad \neg q \vee \neg p \quad \vee e \ 2, 3 - 8, 9 - 10$$

逆向:

$$1 \quad \neg q \vee \neg p \quad \text{premise}$$

$2 \quad \neg q$	assumption
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$3 \quad p \wedge q$	assumption
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$4 \quad q$	$\wedge e_2 \ 3$
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$5 \quad \perp$	$\neg e \ 2, 4$
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$6 \quad \neg(p \wedge q)$	$\neg i \ 3 - 5$
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$7 \quad \neg p$	assumption
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$8 \quad p \wedge q$	assumption
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$9 \quad p$	$\wedge e_1 \ 8$
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$10 \quad \perp$	$\neg e \ 7, 9$
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$11 \quad \neg(p \wedge q)$	$\neg i \ 8 - 10$
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$$12 \quad \neg(p \wedge q) \quad \vee e \ 1, 2 - 6, 7 - 11$$

1.2 $p \rightarrow q \dashv\vdash \neg q \rightarrow \neg p$

正向:

1	$p \rightarrow q$	premise
2	$\neg q$	assumption
3	$\neg p$	MT 1, 2
4	$\neg q \rightarrow \neg p$	\rightarrow i 2 – 3

逆向:

1	$\neg q \rightarrow \neg p$	premise
2	p	assumption
3	$\neg\neg p$	$\neg\neg$ i 2
4	$\neg\neg q$	MT 1, 3
5	q	$\neg\neg$ e 4
6	$p \rightarrow q$	\rightarrow i 2 – 5

1.3 $p \wedge q \rightarrow p \dashv\vdash r \vee \neg r$

正向:

$$1 \quad r \vee \neg r \quad \text{LEM}$$

逆向:

1	$p \wedge q$	assumption
2	p	\wedge e ₁ 1
3	$p \wedge q \rightarrow p$	\rightarrow i 1 – 2