Logic in Computer Science Assignment 6

10185101210 陈俊潼

January 2021

1 Prove

1.1 Prove the following progam `MAX` is partial correctness.

$$\{n > 0 \wedge dom(f) = [0 \dots n-1] \wedge ran(f) \in N\}$$
$$Max$$
$$\{y = max\Big(ran(f)\Big)\}$$

Proof:

First we add an empty else statement and a pair of brace for the original code to make it more clear in the process of proving. So the refined version of the `MAX` program would be:

```
y = f[0]
i = 1
while (i < n)
{
    if (y < f[i])
    {
        y = f[i])
    }
    else
    {
    }
    i = i + 1
}
```

Then analysing the algorithm, we assume the loop invariant to be `y = max(ran(f[0:i]))`. Now we can proof its partial correctness as follows:

$(\!|\ n > 0 \land dom(f) = [0 \ldots n-1] \land ran(f) \in N\ |\!)$

$(\!|\ \top\ |\!)$ Implied

$(\!|\ f[0] = \max(f[0])\ |\!)$ Implied

$(\!|\ f[0] = \max(\mathrm{ran}(f[0:1]))\ |\!)$ Implied

`y = f[0]`

$(\!|\ y = \max(\mathrm{ran}(f[0:1]))\ |\!)$ Assignment

`i = 1`

$(\!|\ y = \max(\mathrm{ran}(f[0:i]))\ |\!)$ Assignment

`while (i < n)`

`{`

$(\!|\ y = \max(\mathrm{ran}(f[0:i])) \land i < n\ |\!)$ Invariant Hyp. $\land$ Guard

$(\!|\ y = \max(\mathrm{ran}(f[0:i]))\ |\!)$ Implied

$(\!|\ \Big(y < f[i] \to \max(\mathrm{ran}(f[0:i])) < f[i]\Big) \land$
$\neg(y < f[i]) \to y = \max(\mathrm{ran}(f[0:i]))\ |\!)$ Implied

`if (y < f[i])`

`{`

$(\!|\ \max(\mathrm{ran}(f[0:i])) < f[i]\ |\!)$ If-Statement

$(\!|\ f[i] = \max(\mathrm{ran}(f[0:i]),\ f[i])\ |\!)$ Implied

`y = f[i]`

$(\!|\ y = \max(\mathrm{ran}(f[0:i]),\ f[i])\ |\!)$ Assignment

$(\!|\ y = \max(\mathrm{ran}(f[0:i+1]))\ |\!)$ Implied

`}`

`else`

`{`

$(\!|\ y = \max(\mathrm{ran}(f[0:i]))\ |\!)$ If-Statement

$(\!|\ y = \max(\mathrm{ran}(f[0:i+1]))\ |\!)$ Implied

`}`

$(\!|\ y = \max(\mathrm{ran}(f[0:i+1]))\ |\!)$ If-Statement

`i = i + 1`

$(\!|\ y = \max(\mathrm{ran}(f[0:i]))\ |\!)$ Assignment

`}`

$(\!|\ y = \max(\mathrm{ran}(f[0:i])) \land \neg(i < n)\ |\!)$ Partial-While

$(\!|\ y = \max(\mathrm{ran}(f[0:i])) \land i \geq n\ |\!)$ Implied

$(\!|\ y = \max(\mathrm{ran}(f[0:n]))\ |\!)$ Implied

$(\!|\ y = \max(\mathrm{ran}(f))\ |\!)$ Implied