Logic in Computer Science Assignment 3

10185101210 陈俊潼

November 2020

1 Prove

Prove the following theorems with deduction rules.

1.1 $\forall x(P(x) \to \neg Q(x)) \vdash \neg(\exists x(P(x) \land Q(x)))$

Proof:

	1	$\forall x(P(x) \to \neg Q(x))$	premise
	2	$\exists x(P(x) \land Q(x))$	assumption
x_0	3	$P(x_0) \land Q(x_0)$	assumption
	4	$P(x_0)$	$\land e_1$ 3
	5	$Q(x_0)$	$\land e_2$ 3
	6	$P(x_0) \to \neg Q(x_0)$	$\forall$e 1
	7	$\neg Q(x_0)$	$\to$ e 6
	8	$\perp$	$\neg$i 5, 7
	9	$\perp$	$\exists$e 2, 3 − 8
	10	$\neg(\exists x(P(x) \land Q(x)))$	$\neg$i 2, 9

1.2 $\forall x(P(x) \leftrightarrow x = b) \vdash P(b) \land \forall x \forall y(P(x) \land P(y) \to x = y)$

Proof:

	1	$\forall(P(x) \to x = b)$	premise
	2	$\forall(x = b \to P(x))$	premise
	3	$b = b$	$=$ i
	4	$b = b \to P(b)$	$\forall$e 2
	5	$P(b)$	$\to$ e 4
x_0	6	$P(x_0) \to x_0 = b$	$\forall$e 1
y_0	7	$P(y_0) \to y_0 = b$	$\forall$e 1
	8	$P(x_0) \land P(y_0)$	assumption
	9	$P(x_0)$	$\land$e$_1$ 8
	10	$x_0 = b$	$\to$ e 6
	11	$P(y_0)$	$\land$e$_2$ 8
	12	$y_0 = b$	$\to$ e 7
	13	$x_0 = y_0$	$=$ e $10, 12$
	14	$P(x_0) \land P(y_0) \to x_0 = y_0$	$\to$ i $8, 9 - 13$
	15	$\forall y(P(x_0) \land P(y) \to x_0 = y)$	$\forall$i $7 - 14$
	16	$\forall x \forall y(P(x) \land P(y) \to x = y)$	$\forall$i $6 - 15$
	17	$P(b) \land \forall x \forall y(P(x) \land P(y) \to x = y)$	$\land$i $5, 16$