1. Find demand for good y from the consumer's EMP

   - To find demand for good y, we plug the demand for good x into her utility target $u = 2x^{1/3} + y$

$$u = 2\left(\underbrace{\left(\frac{2p_y}{3p_x}\right)^{3/2}}_{x_E}\right)^{1/3} + y$$

   Solving for y, we obtain the consumer's demand for good y

$$y^E(p_x, p_y, u) = u - 2\left(\frac{2p_y}{3p_x}\right)^{1/2}$$

2. Calculate the CV for a price increase from $p_x = \$5$ to $p'_x = \$10$, where $u = 30$ and $p_y = \$1$.

   - The CV becomes the integral of the demand function for good x between prices $p_x = \$5$ and $p'_x = \$10$, that is

$$CV = \int_{p_x}^{p'_x} x^E(p_x, p_y, u)dp_x = \int_5^{10}\left(\frac{2}{3p_x}\right)^{3/2} dp_x$$

$$= 0.54\int_5^{10}\left(\frac{1}{p_x}\right)^{3/2} dp_x = 0.54\int_5^{10}(p_x)^{-3/2}\, dp_x$$

The integral of $(p_x)^a$ is $\frac{p_x^{a+1}}{a+1}$(power rule of integration), implying that the integral of $(p_x)^{-3/2}$ is $\frac{(p_x)^{-3/2+1}}{\frac{-3}{2}+1} = \frac{(p_x)^{-3/2+1}}{\frac{-1}{2}} = -2(p_x)^{-1/2}$. Therefore, the above integral becomes

$$CV = 0.54 \times | - 2(p_x)^{-1/2}|_5^{10}$$

$$CV = 0.54\left[-2(10^{-1/2} - 5^{-1/2})\right]$$

$$CV = -1.09 \times \left(\frac{1}{\sqrt{10}} - \frac{1}{\sqrt{5}}\right)$$

$$CV = 0.14$$

This means that we need to give the consumer \$0.14 to make her as well off as she is before the price increase.

3. Calculate the CV of the above price change in good x, but using the demand function of good y to see how the consumer's welfare in her purchases of good y is affected by a more expensive good x

   - The CV becomes the intergral of the demand function for good y between prices $p_x = \$5$ and $p'_x = \$10$, that is

$$CV = \int_{p_x}^{p'_x} y^E(p_x, p_y, u) dp_x = \int_5^{10} 30 - 2\left(\frac{2}{3p_x}\right)^{1/2} dp_x$$

$$\int_5^{10} 30 dp_x - 1.63 \int_5^{10} (p_x)^{-1/2} dp_x$$

The integral of $(p_x)^a$ is $\frac{p_x^{a+1}}{a+1}$(power rule of integration), implying that the integral of $(p_x)^{-1/2}$ is $\frac{(p_x)^{-1/2+1}}{\frac{-1}{2}+1} = \frac{(p_x)^{1/2}}{\frac{1}{2}} = 2(p_x)^{-1/2}$. Therefore, the above integral becomes

$$CV = |30p_x|_5^{10} - 1.63 \times |2(p_x)^{1/2}|_5^{10}$$

$$CV = 30 \times |p_x|_5^{10} - 3.26 \times |(p_x)^{1/2}|_5^{10}$$

$$CV = 30(10 - 5) - 3.26[10^{1/2} - 5^{1/2}]$$

$$CV = 146.98$$

This means that we need to give the consumer \$146.98 to make her as well off as she is before the price increase.