

CHAPTER 6

TOPOLOGY CONTROL IN WIRELESS AD HOC NETWORKS

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6.1 INTRODUCTION

Recent years have seen a great amount of research in wireless networks, especially ad hoc wireless networks due to their potential applications in various situations such as battle-field, emergency relief, and so on. There are no wired infrastructures or cellular networks in *ad hoc* wireless network. In this chapter, we assume that each wireless node has an omnidirectional antenna and a single transmission of a node can be received by *any* node within its vicinity, which we assume is a disk centered at this node. We also discuss specifically the topology control when directional antennas are used. Each mobile node has a transmission range. Node v can receive the signal from node u if node v is within the transmission range of the sender u . Otherwise, they communicate through multihop wireless links by using intermediate nodes to relay messages. Consequently, each node in the wireless network also acts as a router, forwarding data packets for other nodes. In addition, we assume that each node has a low-power Global Position System (GPS) receiver, which provides the position information of the node itself. If GPS is not available, the distance between neighboring nodes can be estimated on the basis of incoming signal strengths and the direction of arrival. Relative coordinates of neighboring nodes can be obtained by exchanging such information between neighbors [1].

It is common to separate the network design problem from the management and control of the network in the communication network literature. The separation is very convenient and helps to significantly simplify these two tasks, which are already very complex on their own. Nevertheless, there is a price to be paid for this modularity, as the decisions made at the network-design phase may strongly affect the network management and con-

trol phase. In particular, if the issue of designing efficient routing schemes is not taken into account by the network designers, then the constructed network might not be suited for supporting a good routing scheme. For example, a backbone like network topology is more suitable for a hierarchical routing method than a flat network topology.

Wireless ad hoc networks need some special treatment as they intrinsically have their own special characteristics and some unavoidable limitations compared with wired networks. For example, wireless nodes are often powered by batteries only, and they often have limited memories. A transmission by a wireless device is often received by many nodes within its vicinity, which possibly causes signal interferences at these neighboring nodes. On the other hand, we can also utilize this property to save the communications needed to send some information. Unlike most traditional static communication devices, the wireless devices often are moving during the communication. Therefore, it is more challenging to design a network topology for wireless ad hoc networks that is suitable for designing an efficient routing scheme to save energy and storage memory consumption than for the traditional wired networks.

To simplify the question so we can derive some meaningful understanding of wireless ad hoc networks, we assume that the wireless nodes are quasistatic for a period of time. Then, in technical terms, the question we deal with is whether it is possible (and if possible, then how) to design a network that is a subgraph of the unit disk graph, such that it ensures both attractive network features such as bounded node degree, low stretch factor, and linear number of links, and attractive routing schemes such as localized routing with guaranteed performance.

Unlike the wired networks that typically have fixed network topologies, each node in a wireless network can potentially change the network topology by adjusting its transmission range and/or selecting specific nodes to forward its messages, thus controlling its set of neighbors. The primary goal of topology control in wireless networks is to maintain network connectivity, optimize network lifetime and throughput, and make it possible to design power-efficient routing. Not every connected subgraph of the unit disk graph plays the same important role in network design. One of the perceptible requirements of topology control is to construct a subgraph such that the shortest path connecting any two nodes in the subgraph is not much longer than the shortest path connecting them in the original unit disk graph. This aspect of path quality is captured by the *stretch factor* of the subgraph. A subgraph with constant stretch factor is often called a *spanner*, and a spanner is called a *sparse spanner* if it has only a linear number of links. In this chapter we review and study how to construct a sparse network topology efficiently for a set of static wireless nodes.

Restricting the size of the network has been found to be extremely important in reducing the amount of routing information. The notion of establishing a subset of nodes that perform the routing has been proposed in many routing algorithms [2–5]. These methods often construct a virtual backbone by using the connected dominating set [6–8], which is often constructed from a dominating set or a maximal independent set. For a full review of the state of the art in constructing the backbone, see Li [9].

The other imperative requirement for network topology control in wireless ad hoc networks is the fault tolerance. To guarantee a good fault tolerance, the underlying network structure must be k -connected for some $k > 1$, i.e., given any pair of wireless nodes, there need to be at least k disjoint paths to connect them. By setting the transmission range sufficiently large, the induced unit disk graph will be k -connected without doubt. Since energy conservation is important to increase the life of the wireless device, the question is how

to find the minimum transmission range such that the induced unit disk graph is multiply connected.

Many routing algorithms [3, 5, 11–14] have been proposed recently for wireless ad hoc networks. The routing protocols proposed may be categorized as table-driven protocols or demand-driven protocols. A good survey may be found in [15]. Route discovery can be very expensive, thus reducing the response time of the network. On the other hand, explicit route maintenance can be even more costly in the explicit communication of substantial routing information and the usage of scarce memory of wireless network nodes. The geometric nature of the multihop ad-hoc wireless networks allows a promising solution: localized routing protocols.

Localized routing does not require the nodes to maintain routing tables, a distinct advantage given the scarce storage resources and the relatively low computational power available to the wireless nodes. More importantly, given the numerous changes in topology expected in ad-hoc networks, no recomputation of the routing tables is needed and, therefore, we expect a significant reduction in overhead. Thus, localized routing is scalable. Localized routing is also uniform, in the sense that all the nodes execute the same protocol when deciding to which other node to forward a packet.

But localized routing is challenging to design, as even guaranteeing the successful arrival at the destination of the packet is a nontrivial task. This task was successfully solved by Bose et al. [16] (see also [17]). Mauve et al. [18] conducted an excellent survey of position-based localized routing protocols.

6.1.1 Organization

The rest of the chapter is organized as follows. In Section 6.2, we review in detail the geometry structures that are suitable for topology control in wireless ad hoc networks, especially the structures with bounded stretch factors, bounded node degree, or planar structures. We also review the current status of controlling the transmission power so the total or maximum transmission power is minimized without sacrificing the network connectivity. After reviewing the geometric structures, we review the so-called localized routing methods in Section 6.3. Location service protocols are also discussed. In Section 6.4, we review the current status of applying stochastic geometry to study the connectivity, capacity, and so on of wireless networks. We conclude in Section 6.5 by pointing out some possible research questions.

6.2 NETWORK TOPOLOGY CONTROL

We consider a wireless ad hoc network consisting of a set V of n wireless nodes distributed in a two-dimensional plane. By a proper scaling, we assume that all nodes have the maximum transmission range equal to one unit. These wireless nodes define a *unit disk graph* $UDG(V)$ in which there is an edge between two nodes if and only if their Euclidean distance is at most one. In this chapter, we concentrate on how to apply some structural properties of a point set for wireless networks as we treat wireless devices as two-dimensional points.

Due to the limited resources of the wireless nodes, it is preferred that the underlying network topology be constructed in a localized manner. Stojmenovic et al. first defined what a localized algorithm is in [16, 19]. Here, a distributed algorithm constructing a

graph G is a *localized algorithm* if every node u can exactly decide all edges incident on u based only on the information of all nodes within a constant number of hops of u (plus a constant number of additional nodes' information if necessary).

Energy conservation is a critical issue in ad hoc wireless network for the node and network life, as the nodes are powered by batteries only. In the most common power-attenuation model, the power, denoted by $p(e)$, needed to support a link $e = uv$ is $\|uv\|^\beta$, where $\|uv\|$ is the Euclidean distance between u and v , and β is a real constant between 2 and 5, depending on the wireless transmission environment. This power consumption is typically called *path loss*. Practically, there is some other overhead cost for each device to receive and then process the signal. For simplicity, this overhead cost can be integrated into one cost c , which is almost the same for all nodes. Without specification, it is assumed that $c = 0$.

Let $G = (V, E)$ be a n -vertex connected weighted graph over V . The distance in G between two vertices $u, v \in V$ is the total weight of the shortest path between u and v and is denoted by $d_G(u, v)$. A subgraph $H = (V, E')$, where $E' \subseteq E$, is a t -*spanner* of G if for every $u, v \in V$, $d_H(u, v) \leq t \cdot d_G(u, v)$. The value of t is called the *stretch factor*. If the weight is the length of the link, then t is called the length stretch factor; if the weight is the power to support the communication of the link, then t is called the power stretch factor.

Recently, topology control for wireless ad hoc networks has attracted considerable attention [20–28]. Rajaraman [29] conducted an excellent survey recently.

6.2.1 Known Structures

Several geometrical structures have been studied recently both by computational geometry scientists and network engineers. Here we review the definitions of some of them that could be used in wireless networking applications.

Let V be the set of wireless nodes in a two dimensional plane. The *relative neighborhood graph*, denoted by $\text{RNG}(V)$, is a geometric concept proposed by Toussaint [30]. It consists of all edges uv such that there is no point $w \in V$ with uw and wv satisfying $\|uw\| < \|uv\|$ and $\|wv\| < \|uv\|$. Let $\text{disk}(u, v)$ be the disk with diameter uv . Then, the *Gabriel graph* [31] $\text{GG}(V)$ contains an edge uv from G if and only if $\text{disk}(u, v)$ contains no other vertex $w \in V$ inside. It is easy to show that $\text{RNG}(V)$ is a subgraph of the Gabriel graph $\text{GG}(V)$. For the unit disk graph, the relative neighborhood graph and the Gabriel graph only contain the edges in UDG satisfying the respective definitions.

The relative neighborhood graph has length stretch factor as large as $n - 1$ [32]. Li et al. [33] showed that its power stretch factor could also be as large as $n - 1$. The Gabriel graph has length stretch factor between $\sqrt{n}/2$ and $4\pi\sqrt{2n} - 4/3$ [32]. Li et al. [33] showed that the power stretch factor of any Gabriel graph is exactly one when the overhead cost $c = 0$.

The *Yao graph* with an integer parameter $k \geq 6$, denoted by $\vec{Y}G_k(V)$, is defined as follows. At each node u , any k equally separated rays originated at u define k cones. In each cone, choose the shortest edge uv , if there is any, and add a directed link \vec{uv} . Ties are broken arbitrarily or by the smallest ID. The resulting directed graph is called the Yao graph. See Figure 6.1 for an illustration. Let $Y\vec{G}_k(V)$ be the undirected graph by ignoring the direction of each link in $\vec{Y}G_k(V)$. If we add the link \vec{vu} instead of the link \vec{uv} , the graph is denoted by $\vec{Y}G_k(V)$, which is called the *reverse* of the Yao graph. Some researchers used a similar construction called a θ -graph [34]. The difference is that, in each cone, it chooses the edge that has the shortest projection on the axis of the cone instead of the shortest edge. Here, the axis of a cone is the angular bisector of the cone. For more detail, please

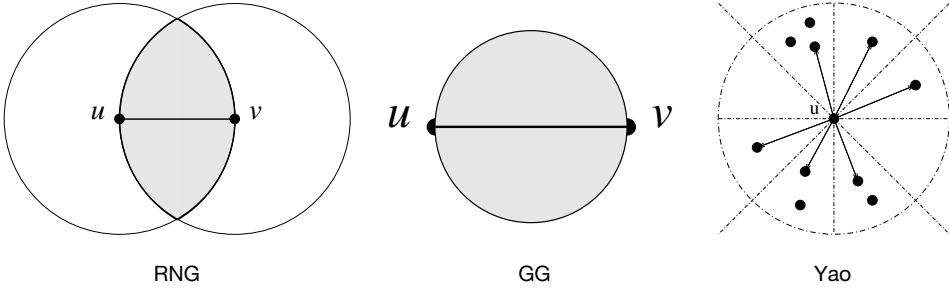


Figure 6.1. The definitions of RNG, GG, and Yao on point set. Left: The lune using uv is empty for RNG. Middle: The diametric circle using uv is empty for GG. Right: The shortest edge in each cone is added as a neighbor of u for Yao.

refer to [34]. Recently, the Yao structure has been rediscovered by several researchers for use in topology control in wireless ad hoc networks of directional antennas.

The Yao graph $YG_k(V)$ has length stretch factor $1/(1 - 2 \sin \pi/k)$. Li et al. [33] proved that the power stretch factor of the Yao graph $YG_k(V)$ is at most $1/[1 - (2 \sin \pi/k)^2]$.

Li et al. [35] extended the definitions of these structures on top of any given graph G . They proposed to apply the Yao structure on top of the Gabriel graph structure [the resulting graph is denoted by $\overrightarrow{YG}_k(V)$], and apply the Gabriel graph structure on top of the Yao structure [the resulting graph is denoted by $\overrightarrow{GYG}_k(V)$]. These structures are sparser than the Yao structure and the Gabriel graph and they still have a constant bounded-power stretch factor. These two structures are connected graphs. Wattenhofer et al. [28] also proposed a two-phased approach that consists of a variation of the Yao graph followed by a variation of the Gabriel graph. Unfortunately, there are some bugs in their proofs, which were discussed in detail in [33].

Li et al. [23] proposed a structure that is similar to the Yao structure for topology control. Each node u finds a power $p_{u,\alpha}$ such that in every cone of degree α surrounding u , there is some node that u can reach with power $p_{u,\alpha}$. Here, nevertheless, we assume that there is a node reachable from u by the maximum power in that cone. Notice that the number of cones to be considered in the traditional Yao structure is a constant k . However, unlike the Yao structure, for each node u , the number of cones needed to be considered in the method proposed in [23] is about $2n$, where each node v could contribute two cones on both sides of segment uv . Then the graph G_α contains all edges uv such that u can communicate with v using power $p_{u,\alpha}$. They proved that, if $\alpha \leq 5\pi/6$ and the UDG is connected, then graph G_α is a connected graph. On the other hand, if $\alpha > 5\pi/6$, they showed that the connectivity of G_α is not guaranteed by giving some counterexample [23]. Unlike the Yao structure, the final topology G_α is not necessarily a bounded degree graph.

6.2.2 Bounded Node Degree

Notice that although the directed graphs $\overrightarrow{YG}_k(V)$, $\overrightarrow{GYG}_k(V)$, and $\overrightarrow{YG}_k(V)$ have a bounded power-stretch factor and a bounded out-degree k for each node, some nodes may have a very large in-degree. The node configuration given in Figure 6.2 will result a very large in-degree for node u . The bounded out-degree gives us advantages when applied to several routing algorithms. However, an unbounded in-degree at node u will often cause large over-

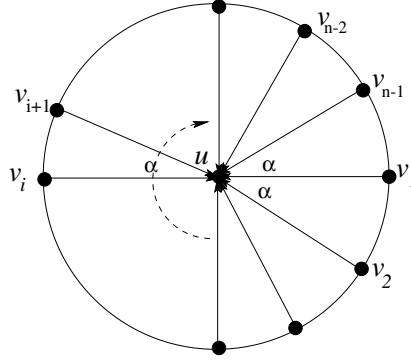


Figure 6.2. Node u has degree (or in-degree) $n - 1$.

head at u . Therefore, it is often imperative to construct a sparse network topology such that both the in-degree and the out-degree are bounded by a constant so as to be power-efficient.

Sink Structure. Arya et al. [36] gave an ingenious technique to generate a bounded degree graph with constant length-stretch factor. In [33], Li et al. applied the same technique to construct a sparse network topology with a bounded degree and a bounded power-stretch factor from $YG(V)$. The technique is to replace the directed star consisting of all links toward a node u by a directed tree $T(u)$ of a bounded degree with u as the sink. Tree $T(u)$ is constructed recursively. The algorithm is as follows.

Algorithm: Constructing- $T(u)$ Tree[$u, I(u)$]

1. Choose k equal-sized cones $C_1(u), C_2(u), \dots, C_k(u)$, centered at u .
2. Node u finds the nearest node $y_i \in I(u)$ in $C_i(u)$, for $1 \leq i \leq k$, if there is any. Link $y_i \vec{u}$ is added to $T(u)$ and y_i is removed from $I(u)$. For each cone $C_i(u)$, if $I(u) \cap C_i(u)$ is not empty, call Tree[$y_i, I(u) \cap C_i(u)$] and add the created edges to $T(u)$.

The union of all trees $T(u)$ is called the *sink structure* $\vec{YG}_k^*(V)$. Node u constructs the tree $T(u)$ and then broadcasts the structure of $T(u)$ to all nodes in $T(u)$. Since the total number of edges in the Yao structure is at most $k \cdot n$, where k is the number of cones divided, the total number of edges of $T(u)$ of all node u is also at most $k \cdot n$. Thus, the total communication cost is at most $k \cdot n$. Li et al. [33] proved that its power stretch factor is at most $\{1/[1 - (2 \sin \pi/k)^\beta]\}^2$, the maximum degree of the graph $\vec{YG}_k^*(V)$ is at most $(k + 1)^2 - 1$, and the maximum out-degree is k .

Notice that the sink structure and the Yao graph structure do not have to have the same number of cones, and the cones centered at different nodes do not need to be aligned. For setting up a power-efficient wireless network, each node u finds all its neighbors in $YG_k(V)$, which can be done in linear time proportional to the number of nodes within its transmission range.

YaoYao Structure. Li et al. [35] proposed another structure called YaoYao. Assume that each node v_i of V has a unique identification number $ID(v_i) = i$. The identity of a directed link \vec{uv} is defined as $ID(\vec{uv}) = (\|uv\|, ID(u), ID(v))$.

Node u chooses a node v from each cone, if there is any, so the directed link \vec{vu} has the smallest $ID(\vec{vu})$ among all directed links \vec{wu} in $YG(V)$ in that cone. The union of all chosen directed links is the final network topology, denoted by $\vec{Y\bar{Y}}_k(V)$. If the directions of all links are ignored, the graph is denoted as $YY_k(V)$. They [35] proved that the directed graph $\vec{Y\bar{Y}}_k(V)$ is strongly connected if $UDG(V)$ is connected and $k > 6$.

It was proved in [37] that $\vec{Y\bar{Y}}_k(V)$ is a spanner in a civilized graph. Here a unit disk graph is a civilized graph if the distance between any two nodes in this graph is larger than a positive constant λ . In [38], the civilized unit disk graph is called the λ -precision unit disk graph. Notice that the wireless devices in wireless networks can not be too close or overlapped. Thus, it is reasonable to model the wireless ad hoc networks as a civilized unit disk graphs.

The experimental results obtained by Li et al. [35] showed that this sparse topology has a small power-stretch factor in practice. They [35] conjectured that $\vec{Y\bar{Y}}_k(V)$ also has a constant bounded power-stretch factor theoretically in any unit disk graph. The proof of this conjecture or the construction of a counterexample remain to be determined.

Symmetric Yao Graph. In [35], Li et al. also considered another undirected structure called a *symmetric Yao graph* $YS_k(V)$. An edge uv is selected to graph $YS_k(V)$ if and only if both directed edges \vec{uv} and \vec{vu} are in the *Yao graph* $\vec{Y\bar{Y}}_k(V)$. Then it is obvious that the maximum node degree is k .

Li et al. [33] showed that the graph $YS_k(V)$ is strongly connected if $UDG(V)$ is connected and $k \geq 6$. The experiment by Li et al. also showed that $YS_k(V)$ has a small power-stretch factor in practice. However, it was shown recently in [21] that $YS_k(V)$ is not a spanner theoretically. The basic idea of the counterexample is similar to the counterexample for RNG proposed by Bose et al. [32].

High-Degree Yao Graph. Recently, Li et al. [39] proposed an efficient scatternet formation method based on the Yao structure.

The first step, which is optional, of the scatternet formation algorithm is to construct some subgraph satisfying some properties such as planar properties. In the second step, which is mandatory, of the algorithm, the degree of each node is limited to seven by applying the Yao structure, and the master-slave relations are formed in created subgraphs. Each node creates a key, which could be identity (ID), or degree, or the combination of both, for comparison with its neighbors.

In each iteration, undecided nodes with higher keys than any of their undecided neighbors (such nodes are referred as *active* nodes in the following) apply the Yao structure to limit the degree. They [39] described in detail how to assign master-slave relations. The active node then switches to a decided state. Assume that an active node u is a node that applies the Yao construction. Then node u divides the region surrounding it into seven equal angles centered at u , and chooses the closest node from each region, if there is any, with ties broken arbitrarily. All remaining connections at u are simply deleted from the graph. Notice that the elimination of any such edge uv by u immediately reduces the degree of v , that is, node v has to remove link uv also. However, in order to avoid excessive information exchange between neighbors, the originally decided keys (that is, original degrees) are used in all comparisons. We call the final structure as $YH_k(S)$.

This structure $YH_k(S)$ is different from all previous structures. First of all, $YS_k(S) \subseteq YH_k(S)$ since any edge uv from $YS_k(S)$ will not be removed by either node u or node v in the construction of $YH_k(S)$. It is not difficult to construct an example, for example, that in

Figure 6.3, such that $YS_k(S) \neq YH_k(S)$. The right two figures of Figure 6.3 also show that $YH_k(S)$ is different from $YY_k(S)$.

6.2.3 Planar Spanner

The Gabriel graph was used as a planar subgraph in the Face routing protocol [16, 40, 41] and the GPSR routing protocol [17]. The right-hand rule is used to guarantee the delivery of the packet in [16]. The relative neighborhood graph RNG was used for efficient broadcasting (minimizing the number of retransmissions) in the one-to-one broadcasting model in [42]. Since RNG and GG have large stretch factors in the worst case, some other structure is needed if we want to bound the distance traveled from the source to the destination. One of the known planar spanners is the Delaunay triangulation.

Assume that there are no four vertices of V that are cocircular. A triangulation of V is a *Delaunay triangulation*, denoted by $Del(V)$, if the circumcircle of each of its triangles does not contain any other vertices of V in its interior. A triangle is called the *Delaunay triangle* if its circumcircle is empty of vertices of V . The *Voronoi region*, denoted by $Vor(p)$, of a vertex $p \in V$, is a collection of two-dimensional points such that every point is closer to p than to any other vertex of V . The *Voronoi diagram* for V is the union of all Voronoi regions $Vor(p)$, where $p \in V$. The Delaunay triangulation $Del(V)$ is also the dual of the Voronoi diagram: two vertices p and q are connected in $Del(V)$ if and only if $Vor(p)$ and $Vor(q)$ share a common boundary. The shared boundary of two Voronoi regions $Vor(p)$ and $Vor(q)$ is on the perpendicular bisector line of segment pq . The boundary segment of a Voronoi region is called the *Voronoi edge*. The intersection point of two Voronoi edges is called the *Voronoi vertex*. The Voronoi vertex is the circumcenter of some Delaunay triangle.

Given a set of nodes V , it is well known that the Delaunay triangulation $Del(V)$ is a planar t -spanner of the completed graph $K(V)$ [43–45]. However, it is not appropriate to require the construction of the Delaunay triangulation in the wireless communication environment because of the possible massive communications it requires. Given a set of points V , let $UDel(V)$ be the graph formed by edges of $Del(V)$ with length at most one unit, that is, $UDel(V) = Del(V) \cap UDG(V)$. Li et al. [46] considered the *unit Delaunay triangulation* $UDel(V)$ for the planar spanner of UDG. Using the approach from [45], Li et al. [46] proved that $UDel(V)$ is a t -spanner of the unit disk graph $UDG(V)$.

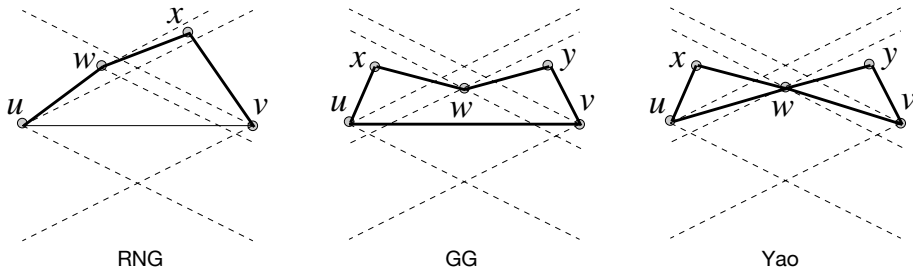


Figure 6.3. Left: The graph $YH_k(S)$ (represented by four solid lines) is different from $YS_k(S)$ (represented by three thick solid lines). Here the dashed lines define some cones around the nodes. Middle and Right: The graph $YH_k(S)$ is different from $YY_k(S)$. Here the node degrees are in decreasing order as w, u, v, x , and y .

6.2.3.1 Localized Delaunay Triangulation. Li et al. [46] gave a localized algorithm that constructs a sequence graphs, called a *localized Delaunay graph* $LDe\ell^{(1)}(V)$, which are supergraphs of $UDel(V)$. We begin with some necessary definitions before presenting the algorithm. Triangle Δuvw is called a *k-localized Delaunay triangle* if the interior of the circumcircle of Δuvw , denoted by $disk(u, v, w)$ hereafter, does not contain any vertex of V that is a k -neighbor of u , v , or w ; and all edges of the triangle Δuvw have length no more than one unit. The *k-localized Delaunay graph* over a vertex set V , denoted by $LDe\ell^{(k)}(V)$, has exactly all unit Gabriel edges and edges of all k -localized Delaunay triangles.

When it is clear from the context, we will omit the integer k in our notation of $LDe\ell^{(k)}(V)$. As shown in [46], the graph $LDe\ell^{(1)}(V)$ may contain some edges intersecting, although they showed that $LDe\ell^{(1)}(V)$ can be decomposed to two planar graphs, that is, has thickness 2. They proved that $LDe\ell^{(k)}(S)$ is a planar graph for any $k \geq 2$. Although the graph $UDel(V)$ is a t -spanner for $UDG(V)$, it is unknown how to construct it locally. Li et al. [46] present an efficient algorithm to extract a planar graph $PLDe\ell(V)$ out of $LDe\ell^{(1)}(V)$. They provide a novel algorithm to construct $LDe\ell^{(1)}(V)$ using linear communications and then make it planar in linear communication cost. The final graph still contains $UDel(V)$ as a subgraph. Thus, it is a t -spanner of the unit-disk graph $UDG(V)$.

The basic approach of their method is to let each node u compute the Delaunay triangulation $Del[N_1(u)]$ of its 1-neighbors $N_1(u)$, including u itself. Node u then sends messages to its neighbors asking if the triangles in $Del[N_1(u)]$ can be accepted by $LDe\ell^{(1)}(V)$. Its neighbor v accepts the triangle if it is in $Del[N_1(v)]$. The novel part is to bound the communications by only letting u query for triangle Δvuw if $\angle vuw$ is at least $\pi/3$. It was proved that the graph constructed by the above algorithm is $LDe\ell^{(1)}(V)$. As $Del[N_1(u)]$ is a planar graph, and a proposal is made only if $\angle wuv \geq \pi/3$, node u broadcasts at most six proposals. And each proposal is replied to by at most two nodes. Therefore, the total communication cost is $O(n)$. They also gave an algorithm to extract from $LDe\ell^{(1)}(V)$ a planar subgraph, denoted by $PLDe\ell(V)$. They showed that $PLDe\ell(V)$ is a supergraph of $LDe\ell^{(2)}(V)$.

Recently, Călinescu [47] proposed an efficient approach to collect $N_2(u)$ using total $O(n)$ communications based on an efficient construction of the connected dominating set [7, 37]. Using the collected two-hop information, we then can construct the local Delaunay triangulation $LDe\ell^{(2)}(V)$, which is guaranteed to be a planar graph. The cost of updating the structure $LDe\ell^{(2)}(V)$ in a mobile environment could be more expensive than that of updating the structure $LDe\ell^{(1)}(V)$ from the definition of these two structures. It remains open whether we can update these two structures using asymptotically same communication costs.

6.2.3.2 Restricted Delaunay Graph. Gao et al. [48] also proposed another structure, called a *restricted Delaunay graph* RDG, and showed that it has good spanning ratio properties and is easy to maintain locally. A restricted Delaunay graph of a set of points in the plane is a planar graph and contains all the Delaunay edges with length at most one. In other words, they call any planar graph containing $UDel(V)$ a restricted Delaunay graph. They described a distributed algorithm to maintain the RDG such that at the end of the algorithm, each node u maintains a set of edges $E(u)$ incident to u . Those edges $E(u)$ satisfy that (1) each edge in $E(u)$ has length at most one unit; (2) the edges are consistent, that is, an edge $uv \in E(u)$ if and only if $uv \in E(v)$; (3) the graph obtained is planar; and (4) $UDel(V)$ is in the union of all edges $E(u)$.

The algorithm works as follows. First, each node u acquires the position of its one-hop neighbors $N_1(u)$ and computes the Delaunay triangulation $Del[N_1(u)]$ on $N_1(u)$, including u itself. In the second step, each node u sends $Del[N_1(u)]$ to all of its neighbors. Let $E(u) = \{uv \mid uv \in Del[N_1(u)]\}$. For each edge $uv \in E(u)$, and for each $w \in N_1(u)$, if u and v are in $N_1(w)$ and $uv \notin Del[N_1(w)]$ then node u deletes edge uv from $E(u)$.

They proved that when the above steps are finished, the resulting edges $E(u)$ satisfy the four properties listed above. However, unlike the local Delaunay triangulation, the computation cost and communication cost of each node needed to obtain $E(u)$ is not optimal within a small constant factor. The communication cost could be as large as $\Theta(n^2)$, and the computation cost could be as large as $\Theta(n^3)$.

6.2.3.3 Partial Delaunay Triangulation. Stojmenovic and Li [39] also proposed a geometry structure, namely the partial Delaunay triangulation (*PDT*), that can be constructed in a localized manner. Partial Delaunay triangulation contains a Gabriel graph as its subgraph, and itself is a subgraph of the Delaunay triangulation; more precisely, the subgraph of the unit Delaunay triangulation $UDel(V)$. The algorithm for the construction of *PDT* goes as follows.

Let u and v be two neighboring nodes in the network. Edge uv belongs to $Del(V)$ if and only if there exists a disk with u and v on its boundary, which does not contain any other point from the set V . First test whether $disk(u, v)$ contains any other node from the network. If it does not, the edge belongs to *GG* and therefore to *PDT*. If it does, check whether nodes exist on both sides of line uv or on only one side. If both sides of line uv contain nodes from the set inside $disk(u, v)$ then uv does not belong to $Del(V)$.

Suppose now that only one side of line uv contains nodes inside the circle $disk(u, v)$, and let w be one such point that maximizes the angle $\angle u w v$. Let $\alpha = \angle u w v$. Consider now the largest angle $\angle u x v$ on the other side of the mentioned circle $disk(u, v)$, where x is a node from the set S . If $\angle u w v + \angle u x v > \pi$, then edge uv is definitely not in the Delaunay triangulation $Del(V)$. The search can be restricted to common neighbors of u and v , if only one-hop neighbor information is available, or to neighbors of only one of the nodes if two-hop information (or exchange of the information for the purpose of creating *PDT* is allowed) is available. Then whether edge uv is added to *PDT* is based on the following procedure.

Assume that only $N_1(u)$ is known to u , and there is one node w from $N_1(u)$ that is inside $disk(u, v)$ with the largest angle $\angle u w v$. Edge uv is added to *PDT* if the following conditions hold: (1) there is no node from $N_1(u)$ that lies on the different side of uv with w and inside the circumcircle passing through u, v , and w ; (2) $\sin \alpha > d/R$, where R is the transmission radius of each wireless node, d is the diameter of the circumcircle $disk(u, v, w)$, and $\alpha = \angle u w v$ (here $\alpha \geq \pi/2$).

Assume only one-hop neighbors are known to u and v , and there is one node w from $N_1(u) \cup N_1(v)$ that is inside $disk(u, v)$ with the largest angle $\angle u w v$ (Figure 6.4). Edge uv is added to *PDT* if the following conditions hold: (1) there is no node from $N_1(u) \cup N_1(v)$ that lies on the different side of uv with w and inside the circumcircle passing u, v , and w ; (2) $\cos \alpha/2 > d/2R$, where R is the transmission radius of each wireless node and $\alpha = \angle u w v$.

Obviously, the partial Delaunay triangulation is a subgraph of $UDel(V)$. The spanning ratio of the partial Delaunay triangulation could be very large.

Hu [22] proposed a structure using the Delaunay triangulation to bound the node degree of each wireless node to be at most Δ . The centralized algorithm starts with the Delaunay triangulation $Del(S)$ of the set of wireless nodes and then removes the edges in

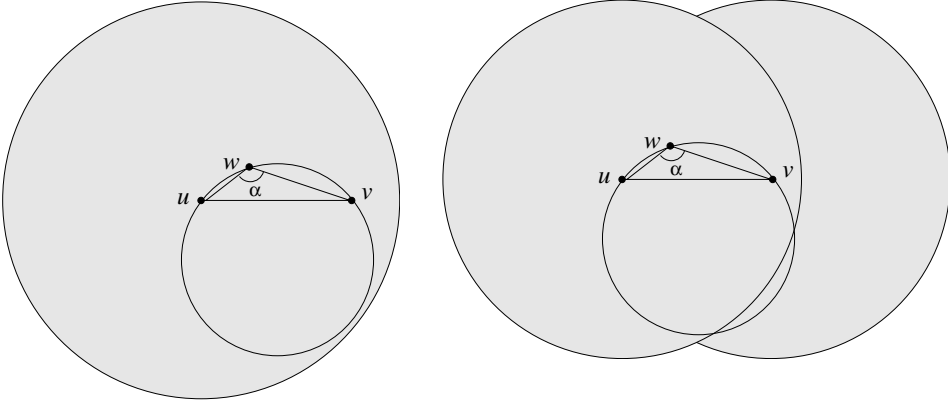


Figure 6.4. Left: Only one-hop information is known to u . Then $\text{disk}(u, v, w)$ must be covered by the transmission range of u (denoted by the shaded region) and is empty of neighbors of u . Right: Node u knows $N_1(u)$ and node v knows $N_1(v)$. The circumcircle $\text{disk}(u, v, w)$ is covered by the union of the transmission ranges of u and v and is empty of other vertices.

$\text{Del}(S)$ that are longer than the transmission range (normalized to one unit here). Then it processes the remaining edges in the order of the decreasing length and removes an edge if it causes either end node to have degree larger than Δ . Finally, it processes in the order of increasing length all possible edges that are not in the graph, and adds an edge if it does not cause a violation of the degree constraint. The worst time complexity of the above approach is $O(n^2 \log n)$. In [22], a distributed implementation was also proposed. Unfortunately, it is not correct since it requires each node u to find Delaunay edges uv with $\|uv\| \leq 1$. However, if we replace the computation of the Delaunay edges with length at most one unit by the computation of the local Delaunay triangulation, the method still produce a planar structure with bounded degree.

Recently, Li and Wang [49] proposed a novel localized method to construct a bounded-degree planar spanner for wireless ad hoc networks using $O(n)$ total communications.

Although all the structures discussed so far are flat structures, there are another set of structures, called hierarchical structures, that are used in wireless networks. Instead of all nodes being involved in relaying packets for other nodes, the hierarchical routing protocols pick a subset of nodes that serve as the routers, forwarding packets for other nodes. The structure used to build this virtual backbone is usually the connected dominating set. See a recent survey [9] on methods of constructing connected dominating sets efficiently in wireless ad hoc networks.

Figure 6.5 gives some concrete examples of the geometry structures introduced before. Here we randomly generate 100 nodes in a 200 m by 200 m square. The transmission radius of each node is set as 50 m. Notice that the graph LDel shown in Figure 6.5 is not a planar graph.

6.2.4 Transmission Power Control

In the previous sections, we have assumed that the transmission power of every node is equal and is normalized to one unit. We relax this assumption for a moment in this sub-

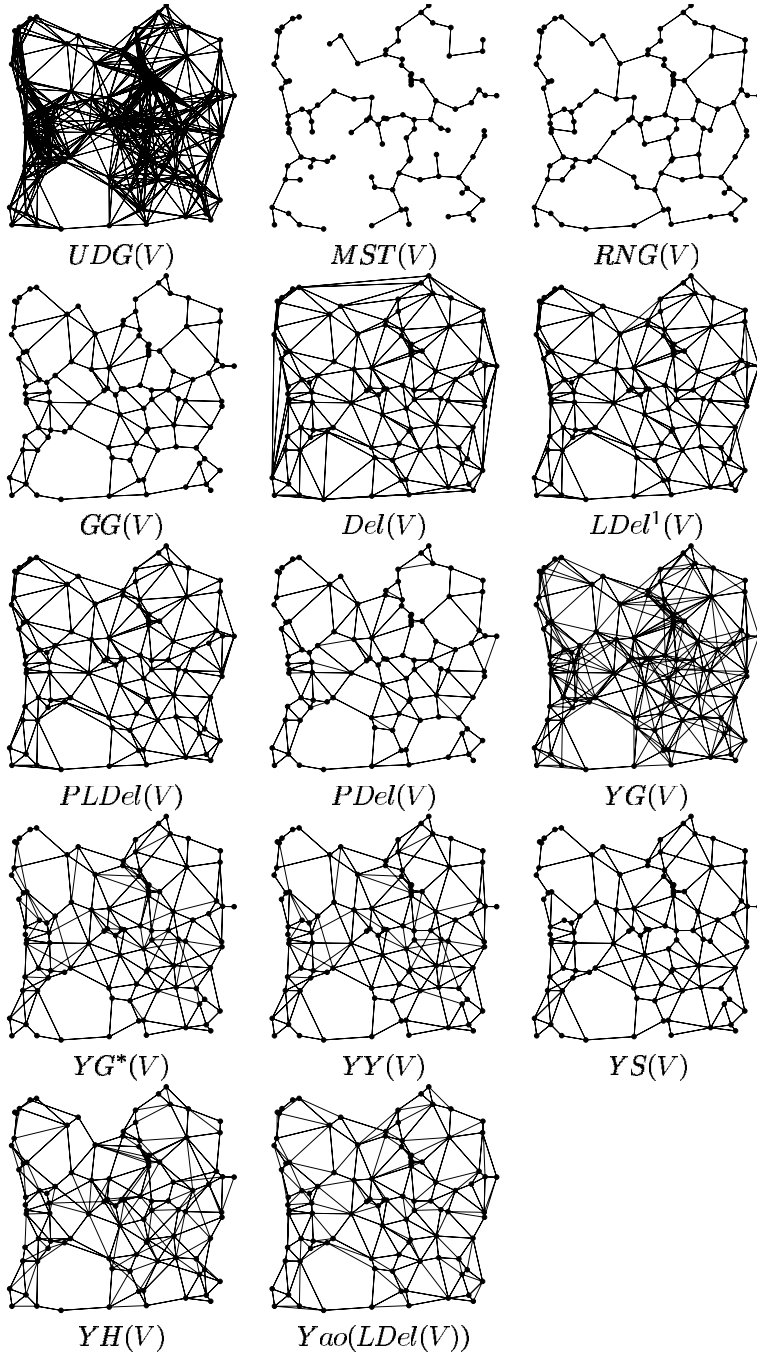


Figure 6.5. Different topologies from $UDG(V)$.

section. In other words, we assume that each node can adjust its transmission power according to its neighbors' positions for possible energy conservation. A natural question is then how to assign the transmission power for each node such that the wireless network is connected with optimization criteria being minimizing the maximum or total transmission power assigned.

A transmission power assignment on the vertices in V is a function f from V into real numbers. The *communication graph*, denoted by G_f , associated with a transmission power assignment f , is a directed graph with V as its vertices and has a directed edge $\vec{v_i v_j}$ if and only if $\|v_i v_j\|^\beta + c \leq f(v_i)$. We call a transmission power assignment f *complete* if the communication graph G_f is strongly connected. Recall that a directed graph is strongly connected if, for any given pair of ordered nodes s and t , there is a directed path from s to t .

The *maximum cost* of a transmission power assignment f is defined as $mc(f) = \max_{v_i \in V} f(v_i)$. And the *total cost* of a transmission power assignment f is defined as $sc(f) = \sum_{v_i \in V} f(v_i)$. The min-max assignment problem is then to find a complete transmission power assignment f whose cost $mc(f)$ is the least among all complete assignments. The min-total assignment problem is to find a complete transmission power assignment f whose cost $sc(f)$ is the least among all complete assignments.

Given a graph H , we say the power assignment f is induced by H if

$$f(v) = \max_{(v,u) \in E} \|vu\|^\beta + c$$

where E is the set of edges of H . In other words, the power assigned to a node v is the largest power needed to reach all neighbors of v in H .

Transmission power control has been well studied by peer researchers in the recent years. Monks et al. [50] conducted simulations that show that implementing power control in a multiple access environment can improve the throughput of the nonpower-controlled IEEE 802.11 by a factor of 2. Therefore it provides a compelling reason for adopting the power controlled MAC protocol in wireless networks.

The min-max assignment problem was studied by several researchers [26, 51]. Let $EMST(V)$ be the Euclidean minimum spanning tree over a point set V . Both [26] and [51] use the power assignment induced by $EMST(V)$. It was proved in [26] that the longest edge of the Euclidean minimum spanning tree $EMST(V)$ is always the critical link for min-max assignment. Here, for an optimum transmission power assignment f_{opt} , call a link uv the *critical link* if $\|uv\|^\beta + c = mc(f_{opt})$. Both algorithms presented in [26] and [51] compute the minimum spanning tree from the fully connected graph with possible very large communication cost. Notice that the best distributed algorithm [52–54] can compute the minimum spanning tree in $O(n)$ rounds using $O(m + n \log n)$ communications for a general graph with m edges and n nodes. Using the fact that the relative neighborhood graph, the Gabriel graph, and the Yao graph all have $O(n)$ edges and contain the Euclidean minimum spanning tree, a simple $O(n \log n)$ time-complexity centralized algorithm can be developed and can be implemented efficiently in a distributed manner.

The min-total assignment problem was studied by Kiroustis et al. [55] and by Clementi et al. [56–58]. Kiroustis et al. [55] first proved that the min-total assignment problem is *NP-hard* when the mobile nodes are deployed in a three-dimensional space. A simple two-approximation algorithm based on the Euclidean minimum spanning tree was also given in [55]. The algorithm guarantees the same approximation ratio in any dimensions.

Clementi et al. [56–58] proved that the min–total assignment problem is still NP-hard when nodes are deployed in a two-dimensional space.

So far, we have generated asymmetric communication graphs from the power assignment. For the symmetric communication, several methods also guarantee a good performance. It is easy to show that the minimum spanning tree method still gives the optimum solution for the min–max assignment and a two-approximation for the min–total assignment. Recently, Călinescu et al. [59] gave a method that achieves a better approximation ratio ($5/3$) by using an idea from the minimum Steiner tree. Like the minimum spanning tree method, it works for any power definition.

6.3 LOCALIZED ROUTINGS

The geometric nature of the multihop ad hoc wireless networks allows a promising idea: localized routing protocols. A routing protocol is *localized* if the decision to which node to forward a packet is based only on:

- The information in the header of the packet. This information includes the source and the destination of the packet, but more data could be included, provided that its total length is bounded.
- The local information gathered by the node from a small neighborhood. This information includes the set of one-hop neighbors of the node, but a larger neighborhood set could be used provided it could be collected efficiently.

Randomization is also used in designing the protocols. A routing is said to be *memory-less* if the decision as to which node to forward a packet is solely based on the destination, current node, and its neighbors within some constant hops. Localized routing is sometimes called in the literature *stateless* [17], *online* [60, 61], or *distributed* [62].

6.3.1 Location Service

In order to make the localized routing work, the source node has to learn the current (or approximately current) location of the destination node. Notice that for sensor networks collecting data, the destination node is often fixed; thus, location service is not needed in these applications. However, the help of a *location service* is needed in most application scenarios. Mobile nodes register their locations to the location service. When a source node does not know the position of the destination node, it queries the location service to get that information. In cellular networks, there are dedicated position servers. It will be difficult to implement the centralized approach of location services in wireless ad-hoc networks. First, for centralized approach, each node has to know the position of the node that provides the location services, which is a chicken-and-egg problem. Second, the dynamic nature of the wireless ad hoc networks makes it very unlikely that there is at least one location server available for each node. Thus, we will concentrate on distributed location services.

For the wireless ad hoc networks, the location service provided can be classified into four categories: *some-for-all*, *some-for-some*, *all-for-some*, and *all-for-all*. *Some-for-all* service means that some wireless nodes provide location services for all wireless nodes. Other categorizations are defined similarly.

An example of all-for-all services is the location services provided in the Distance Routing Effect Algorithm for Mobility (DREAM) by Basagni et al. [63]. Each node stores a database of the position information for all other nodes in the wireless networks. Each node will regularly flood packets containing its position to all other nodes. A frequency of the flooding and the range of the flooding is used as a control of the cost of updating and the accuracy of the database.

Using the idea of *quorum* developed in the databases and distributed systems, Hass and Liang [64] and Stojmenovic [65] developed quorum-based location services for wireless ad hoc networks. Given a set of wireless nodes V , a quorum system is a set of subset (Q_1, Q_2, \dots, Q_k) of nodes whose union is V . These subsets could be mutually disjoint or often have equal numbers of intersections. When one of the nodes requires the information of the other, it suffices to query one node (called the representative node of Q_i) from each quorum Q_i . A virtual backbone is often constructed between the representative nodes using a nonposition-based methods such as those in [7, 6]. The updating information of a node v is sent to the representative node (or the nearest if there are many) of the quorum containing v . The difficulty of using quorums is that the mobility of the nodes requires the frequent updating of the quorums. The quorum-based location service is often of the *some-for-some* type.

The other promising location service is based on the quadtree partition of the two-dimensional space [66]. It divides the region containing the wireless network into a hierarchy of squares. The partition of the space in [66] is uniform. However, we notice that the partition could be nonuniform if the density of the wireless nodes were not uniform for some applications. Each node v will have the position information of all nodes within the same *smallest* square containing v . This position information of v is also propagated to up-layer squares by storing it in the node with the nearest identity to v in each up-layer square containing v . Using the nearest identity over the smallest identity can avoid the overload of some nodes. The query is conducted accordingly. It is easy to show that it takes about $O(\log n)$ time to update the location of v and to query another node's position information.

6.3.2 Localized Routing Protocols

We summarize some localized routing protocols proposed in the networking and computational geometry literature (see Figure 6.6).

The following routing algorithms on the graphs were proposed recently.

Compass Routing. Let t be the destination node. Current node u finds the next relay node v such that the angle $\angle vut$ is the smallest among all neighbors of u in a given topology. See [67].

Random Compass Routing. Let u be the current node and t be the destination node. Let v_1 be the node on the above of line ut such that $\angle v_1ut$ is the smallest among all such neighbors of u . Similarly, we define v_2 to be nodes below line ut that minimizes the angle $\angle v_2ut$. Then node u randomly chooses v_1 or v_2 to forward the packet. See [67].

Greedy Routing. Let t be the destination node. Current node u finds the next relay node v such that the distance $\|v\|$ is the smallest among all neighbors of u in a given topology. See [16].

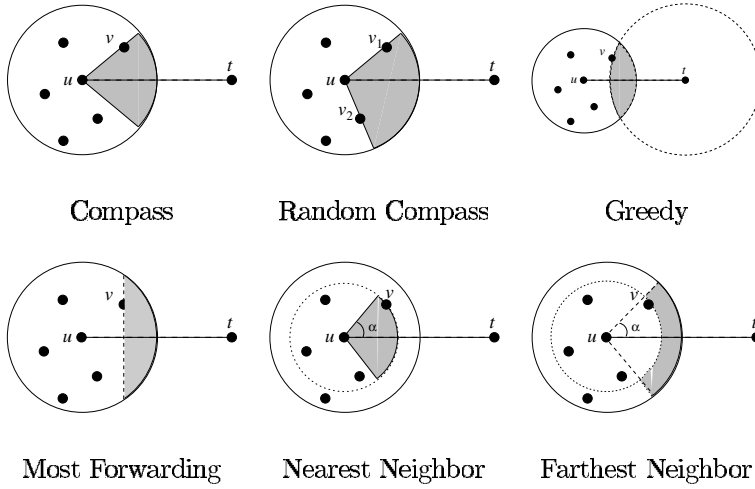


Figure 6.6. Various localized routing methods. Shaded area is empty and v is next node.

Most Forwarding Routing (MFR). Current node u finds the next relay node v such that $\|v't\|$ is the smallest among all neighbors of u in a given topology, where v' is the projection of v on segment ut . See [62].

Nearest Neighbor Routing (NN). Given a parameter angle α , node u finds the nearest node v as forwarding node among all neighbors of u in a given topology such that $\angle vut \leq \alpha$.

Farthest Neighbor Routing (FN). Given a parameter angle α , node u finds the farthest node v as forwarding node among all neighbors of u in a given topology such that $\angle vut \leq \alpha$.

Greedy-Compass. Current node u first finds the neighbors v_1 and v_2 such that v_1 forms the smallest counterclockwise angle $\angle tuv_1$ and v_2 forms the smallest clockwise angle $\angle tuv_2$ among all neighbors of u with the segment ut . The packet is forwarded to the node of $\{v_1, v_2\}$ with minimum distance to t . See [61, 68]

Notice that it is shown in [16, 67] that compass routing, random compass routing, and greedy routing guarantee to deliver the packets from the source to the destination if Delaunay triangulation is used as the network topology. They proved this by showing that the distance from the selected forwarding node v to the destination node t is less than the distance from current node u to t . However, the same proof cannot be carried over when the network topology is Yao graph, Gabriel graph, relative neighborhood graph, and localized Delaunay triangulation. When the underlying network topology is a planar graph, the right-hand rule is often used to guarantee the packet delivery after simple localized routing heuristics fail [16, 62, 17].

It was proved in [68] that greedy routing guarantees the delivery of the packets if the Delaunay triangulation is used as the underlying structure. Compass routing guarantees the delivery of the packets if the regular triangulation is used as the underlying structure. There are triangulations (not Delaunay) that defeat these two schemes. Greedy-compass

routing guarantees the delivery of the packets as long as there is a triangulation used as the underlying structure. Every oblivious routing method is defeated by some convex subdivisions.

Localized routing protocols support mobility by eliminating the communication-intensive task of updating the routing tables. But mobility can affect the localized routing protocols, in both the performance and the guarantee of delivery. There has been no work so far to design protocols with guaranteed delivery when the network topology changes during the routing.

6.3.3 Quality Guaranteed Protocols

With respect to localized routing, there are several ways to measure the quality of the protocol. Given the scarcity of the power resources in wireless networks, minimizing the total power used is imperative. A stronger condition is to minimize the total Euclidean distance traversed by the packet. Morin et al. [61, 68] also studied the performance ratio of previously studied localized routing methods. They proved that none of the previous proposed heuristics guarantees a constant ratio of the traveled distance of a packet compared with the minimum. They gave the first localized routing algorithm such that the traveled distance of a packet from u to v is at most a constant factor of $\|uv\|$ when the Delaunay triangulation is used as the underlying structure.

Bose and Morin [68] basically use the binary search method to find which path of the tunnel connecting the source u and the destination v is better. However, their algorithm (called DTR hereafter) needs the Delaunay triangulation, which is expensive to construct in wireless ad hoc networks, as the underlying structure. In [69], they further extend their method to any triangulations satisfying the diamond property. Let $G(V, r_n)$ be the graph defined over V , which has an edge uv iff $\|uv\| \leq r_n$. Li et al. [70] showed that the sum of all edges in $Del(V)$ is no more than r_n with high probability, where r_n is the transmission radius needed by each node so the induced unit disk graph $G(V, r_n)$ is connected with high probability.

To make $G(V, r_n)$ connect with probability $1 - (1/n)$, we need $n\pi r_n^2 \geq 2 \ln n$, see [71]. They [70] showed that, with probability at least $1 - (1/\beta)$, the longest edge D_n of Delaunay triangulation is $D_n \leq \sqrt{3(\ln n + \ln \beta + \ln 3)/(n\pi)}$. Thus, the required transmission range so that local Delaunay triangulation $PLDel$ equals the Delaunay triangulation Del is just $\sqrt{3/2}$ of the minimum transmission range to have a connected network with high probability. This implies that the localized Delaunay triangulation can be used to approximate the Delaunay triangulation almost always when the network $G(V, r_n)$ is connected and when V is randomly deployed. Consequently, the method by Bose et al. [68] can be used on local Delaunay triangulation almost always.

Table 6.1 illustrates the delivery rates. For routing methods NN and FN, we choose the next node within $\pi/3$ of the destination direction. Interestingly, we found that when the Yao graph is used, the delivery rates are high in all methods. The reason these methods delivered the packets when the Yao structure is used could be that there is a node within the transmission range in the direction of the destination with high probability when $N_1(u)$ is large enough. Table 6.2 illustrates the maximum spanning ratios of the path traversed by the packet from source s to destination t to $\|st\|$. Although the maximum spanning ratio by DTR is larger than most previous methods, DTR is the only known method guaranteeing a constant spanning ratio.

Table 6.1. The Delivery Rate

	Yao	RNG	GG	Del	LDel ⁽²⁾	PLDel
NN	100	20.4	83.3	100	100	98.4
FN	94.5	25.8	70.2	94.6	100	95.2
MFR	98.6	54.5	90	97.4	95	97.2
Cmp	97.1	23.2	66.2	100	100	100
RCmp	95.4	51.4	66.2	85.7	86.9	89.9
Grdy	100	78.3	100	100	100	100
GCmp	95	26.5	76.6	100	98.4	100
DTR				100	100	100

Table 6.2. The Maximum Spanning Ratio

	Yao	RNG	GG	Del	LDel ⁽²⁾	PLDel
NN	1.7	1.3	1.6	1.5	1.6	1.6
FN	3.3	1.6	1.8	1.9	2.3	2.4
MFR	4.7	1.7	2.3	1.8	1.8	3.1
Cmp	11	2.2	6.2	16	18	18
RCmp	27	20	19	31	27	21
Grdy	1.7	2.0	1.8	1.6	1.5	1.6
GCmp	2.5	1.6	2.7	1.9	1.9	1.9
DTR				8.6	8.6	8.4

6.4 STOCHASTIC GEOMETRY

One of the remaining fundamental and critical issues is to have multiple disjoint paths connecting every pair of nodes without sacrificing the spectrum-reusing property. As power is a scarce resource in wireless networks, it is important to save the power consumption without losing the network connectivity. The universal minimum power used by all wireless nodes such that the induced network topology is connected is called the *critical power*. Although determining the critical power for static wireless ad hoc networks has been well studied [26, 51, 72], it remains to study the critical power for connectivity for mobile wireless networks. As the wireless nodes move around, it is impossible to have a unanimous critical power to guarantee the connectivity for all instances of the network configuration. Thus, we need to find a critical power, if possible, at which each node has to transmit to guarantee the connectivity of the network almost surely, i.e., with high probability of almost one.

For simplicity, we assume that the wireless devices are distributed in a unit-area square (or disk) according to some distribution function, e.g., uniform distribution or Poisson process. A point set process is said to be a *random uniform point process*, denoted by \mathcal{X}_n , in a unit-area square $C = [-0.5, 0.5] \times [-0.5, 0.5]$ if it consists of n independent points each of which is uniformly and randomly distributed over C .

The standard probabilistic model of a *homogeneous Poisson process* with density n , denoted by \mathcal{P}_n , is characterized by the property that the number of nodes in a region is a random variable depending only on the area (or volume in higher dimensions) of the region. In other words,

- The probability that there are exactly k nodes appearing in any region Ψ of area A is $[(n/A)^k/k!] \cdot e^{-nA}$.
- For any region Ψ , the conditional distribution of nodes in Ψ given that exactly k nodes in the region is *joint uniform*.

Hereafter, we assume that the movement of wireless devices still keeps them in the same distribution (uniform or Poisson process). Gupta and Kumar [72] showed that there is almost surely a critical power at which the wireless nodes are randomly and uniformly distributed in a unit area disk. The result by Penrose [71] implies the same conclusion. Moreover, Penrose [71] gave the probability of the network to be connected if the transmission radius is set as a positive real number r and n goes to infinity.

The theoretical value of the transmission ranges gives us insight into how to set the transmission radius to achieve the k -connectivity with a certain probability. These results also apply to mobile networks in which the moving of wireless nodes always generates randomly (or Poisson process) distributed node positions. They also have applications in system design of large scale wireless networks. For example, for setting up a sensor network monitoring a certain region, we should deploy how many sensors in order to have a multiple connected network, knowing that each sensor can transmit a range r_0 ? Notice that most results hold only when the number of wireless devices n goes to infinity, which is difficult to deploy practically. Li et al. [73] conducted extensive simulations to study the transmission radius achieving k -connectivity with certain probability for practical settings.

Let $G(V, r)$ be the graph defined on V with edges $uv \in E$ if and only if $\|uv\| \leq r$. Here $\|uv\|$ is the Euclidean distance between nodes u and v . Let $\mathcal{G}(\mathcal{X}_n, r_n)$ be the set of graphs $G(V, r_n)$ for n nodes V that are uniformly and independently distributed in a two-dimensional unit-area disk \mathcal{D} with center at the origin. The problem considered by Gupta and Kumar [72] is then to determine the value of r_n such that a random graph in $\mathcal{G}(\mathcal{X}_n, r_n)$ is asymptotically connected with probability one as n goes to infinity. Let $P_k(\mathcal{X}_n, r_n)$ be the probability that a graph in $\mathcal{G}(\mathcal{X}_n, r_n)$ is k -connected.

Fault tolerance is one of the central challenges in designing wireless ad hoc networks. To make fault tolerance possible, first of all, the underlying network topology must have multiple disjoint paths to connect any two given wireless devices. Here, the path could be vertex disjoint or edge disjoint. Considering the communication nature of the wireless networks, the vertex disjoint multiple paths are often used in the literature. Here, we are interested in what is the condition of r_n such that the underlying network topology $G(V, r_n)$ is k -connected almost surely when V is uniformly and randomly distributed over a two-dimensional domain Ω . A graph is called k -vertex connected (k -connected for simplicity) if, for each pair of vertices, there are k mutually vertex disjoint paths (except end-vertices) connecting them. Equivalently, a graph is k -connected if there is no set of $k - 1$ nodes whose removal will partition the network into at least two components. Thus, a k -connected wireless network can sustain the failure of $k - 1$ nodes.

The *vertex connectivity*, denoted by $\kappa(G)$, of a graph G is the maximum k such that G is k -vertex connected. The *edge connectivity*, denoted by $\xi(G)$, of a graph G is the maximum k such that G is k -edge connected. The minimum degree of a graph G is denoted by $\delta(G)$ and the maximum degree of a graph G is denoted by $\Delta(G)$. Clearly, for any graph G , $\kappa(G) \leq \xi(G) \leq \delta(G) \leq \Delta(G)$.

A graph property is called *monotone increasing* if G has such a property and all graphs on the same vertex set containing G as a subgraph have this property. Let Q be any monot-

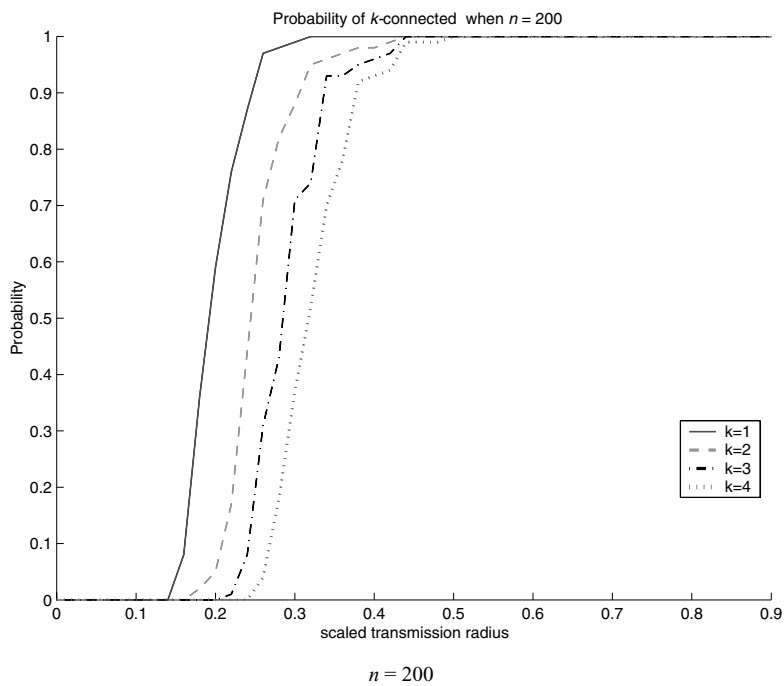
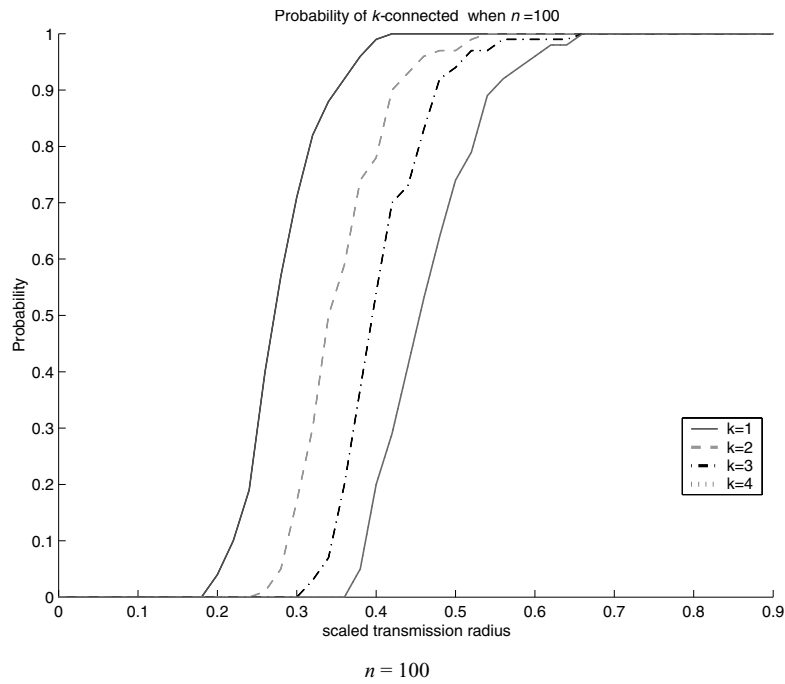


Figure 6.7. Transition phenomena of graph $G(V, r)$ being k -connected.

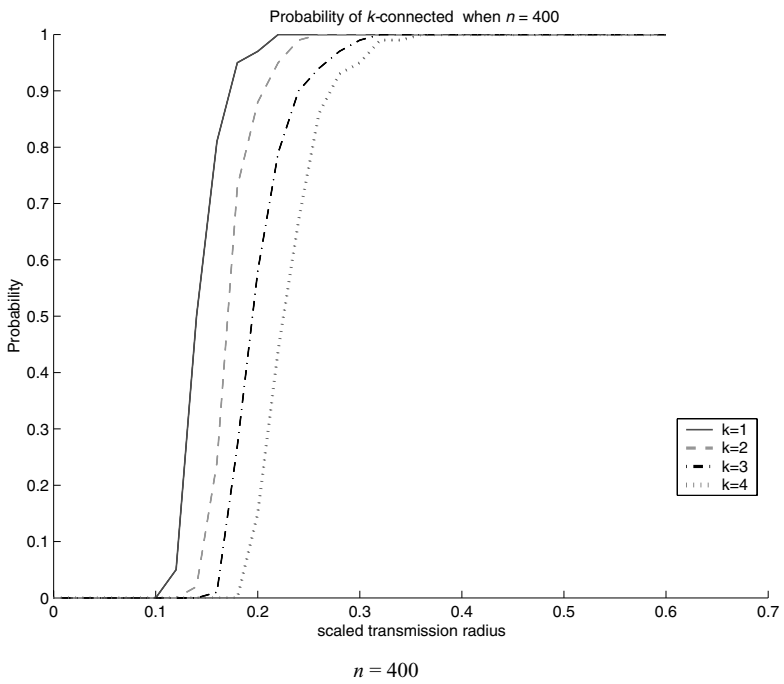
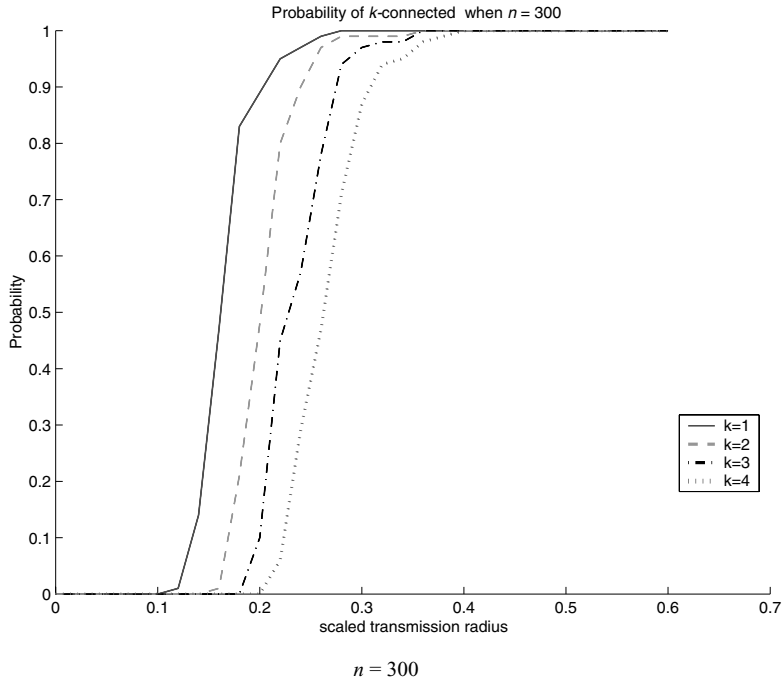


Figure 6.7. Continued.

one increasing property of graphs, for example, the connectivity, the k -edge connectivity, the k -vertex connectivity, the minimum node degree at least k , and so on. The *hitting radius* $\varrho(V, Q)$ is the infimum of all r such that graph $G(V, r)$ has property Q . For example, $\varrho(V, \kappa \geq k)$ is the minimum radius r such that $G(V, r)$ is at least k -vertex connected; $\varrho(V, \delta \geq k)$ is the minimum radius r at which the graph $G(V, r)$ has the minimum degree k . Obviously, for any V ,

$$\varrho(V, \kappa \geq k) \geq \varrho(V, \delta \geq k)$$

Penrose [74] showed that these two hitting radii are asymptotically the same for n points V randomly and uniformly distributed in a unit square and n goes infinity.

The connectivity of random graphs, especially the geometric graphs and their variations, have been considered in the random graph theory literature [75], in the stochastic geometry literature [71, 74, 76–78], and the wireless ad hoc network literature [72, 79–86].

Let us first consider the connectivity problem. Given n nodes V randomly and independently distributed in a unit-area disk \mathcal{D} , Gupta and Kumar [72] showed that $G(V, r_n)$ is connected almost surely if $n\pi \cdot r_n^2 \geq \ln n + c(n)$ for any $c(n)$ with $c(n) \rightarrow \infty$ as n goes infinity. Notice that this bound is tight, as they also proved that $G(X_n, r_n)$ is asymptotically disconnected with positive probability if $n\pi \cdot r_n^2 = \ln n + c(n)$ and $\limsup_n c(n) < +\infty$. Notice that they actually derived their results for a homogeneous Poisson process of points in \mathcal{D} instead of the independent and uniform point process. They showed that the difference between them is negligible. Penrose [71] showed that the same result holds if the geometry domain in which the wireless nodes are distributed is a unit-area square C instead of the unit-area disk \mathcal{D} .

Independently, Penrose [71] showed that the longest edge M_n of the minimum spanning tree of n points randomly and uniformly distributed in a unit area square C satisfies

$$\lim_{n \rightarrow \infty} Pr(n\pi M_n^2 - \ln n \leq \alpha) = e^{-e^{-\alpha}}$$

for any fixed real number α . Here $Pr(X)$ is the probability of event X . Remember that the longest edge of EMST is always the critical power [26, 51]. Thus, the result in [71] is actually stronger than that in [72] since it will give the probability that the network is connected. For example, if we set $\alpha = \ln \ln n$, we have $Pr(n\pi M_n^2 \leq \ln n + \ln \ln n) = e^{-1/\ln n}$. It implies that the network is connected with probability at least $e^{-1/\ln n}$ if the transmission radius of each node r_n satisfies $n\pi r_n^2 = \ln n + \ln \ln n$. Notice that $e^{-1/\ln n} > 1 - (1 - \ln n)$ from $e^{-x} > 1 - x$ for $x > 0$. By setting $\alpha = \ln n$, the probability that the graph $G(V, r_n)$ is connected is at least $e^{-1/n} > 1 - (1/n)$, where $n\pi r_n^2 = 2 \ln n$. Notice that the above probability is only true when n goes to infinity. When n is a finite number, then the probability of the graph being connected is smaller. In [73], Li et al. presented the experimental study of the probability of the graph $G(V, r_n)$ being connected for finite number n .

One closely related question is the *coverage* problem: disks of radius r are placed in a two-dimensional unit-area disk \mathcal{D} with centers from a Poisson point process with intensity n . A result shown by Hall [87] implies that, if $n\pi \cdot r^2 = \ln n + \ln \ln n + c(n)$ and $c(n) \rightarrow \infty$, then the probability that there is a vacancy area in \mathcal{D} is 0 as n goes infinity; if $c(n) \rightarrow -\infty$, the probability that there is a vacancy in \mathcal{D} is at least 1/20. This implies that the hitting radius r_n such that $G(V, r_n)$ is connected satisfies $\pi \cdot r_n^2 \leq 4[\ln n + \ln \ln n + c(n)/n]$ for $c(n) \rightarrow +\infty$.

Let $\mathcal{B}[n, p(n)]$ be the set of graphs on n nodes in which each edge of the completed graph K_n is chosen independently with probability $p(n)$. Then it has been shown that the probability that a graph in $\mathcal{B}[n, p(n)]$ is connected goes to one if $p(n) = [\ln n + c(n)/n]$ for any $c(n) \rightarrow \infty$. Although their asymptotic expressions are the same as those by Gupta and Kumar [72], we cannot apply this to the wireless model as, in wireless networks, the existences of two edges are not independent, and we do not choose edges from the completed graph using the Bernoulli model.

For geometric graphs, it was proved by Penrose [74] that, given any metric l_p with $2 \leq p \leq \infty$ and any positive integer k ,

$$\lim_{n \rightarrow \infty} \Pr[\varrho(\mathcal{X}_n, \kappa \geq k) = \varrho(\mathcal{X}_n, \delta \geq k)] = 1$$

The result is analogous to the well-known results in the graph theory [75] that a graph becomes k -vertex connected when it achieves the minimum degree k if we add the edges randomly and uniformly from $\binom{n}{2}$ possibilities.

This result by Penrose [74] says that a graph of $G(\mathcal{X}_n, r)$ becomes k -connected almost surely at the moment it has minimum degree k by letting r go from 0 to ∞ . However, this result does not imply that, to guarantee a graph over n points k -connected almost surely, we only have to connect every node to its k -nearest neighbors. Let V be n points randomly and uniformly distributed in a unit square (or disk). Xue and Kumar [86] proved that, to guarantee a geometry graph over V connected, the number of nearest neighbors that every node has to connect is asymptotically $\Theta(\ln n)$. Dette and Henze [76] studied the maximum length of the graph by connecting every node to its k nearest neighbors asymptotically. Li et al. [73] showed that, given n random points V over a unit-area square, to guarantee a geometry graph over V k -connected, the number of nearest neighbors that every node has to connect is asymptotically $\ln n + (2k - 3)\ln \ln n$.

Similarly, instead of considering \mathcal{X}_n , Penrose also considered a homogeneous Poisson point process with intensity n on the unit-area square C . Penrose gave loose upper and lower bounds on the hitting radius $r_{n,k} = \varrho(\mathcal{P}_n, \delta \geq k)$ as $(\ln n / 2^{d+1}) \leq nr_{n,k}^d \leq d! 2 \ln n$ for a homogeneous Poisson point process on a d -dimensional unit cube. This result is too loose. More importantly, the parameter k does not appear in this estimation at all. In [73], a tighter bound on $r_{n,k}$ was derived for two-dimensional n points V randomly and uniformly distributed in C such that the graph $G(V, r_{n,k})$ is k -connected with high probability.

Bettstetter [79] conducted the experiments to study the relations of the k -connectivity and the minimum node degree using a toroidal model. Li et al. [73] also conducted experiments to study the probability that a graph has minimum degree k and has vertex connectivity k simultaneously using a Euclidean model. Surprisingly, they found that this probability is sufficiently close to 1 even when n is at the scale of 100. This observation implies a simple method (by just computing the minimum vertex degree) to approximate the connectivity of a random geometry graph. Recently, Bahramgiri et al. [20] showed how to decide the minimum transmission range of each node such that the resulting directed communication graph is k -connected. Here, it is assumed that the unit disk graph, by setting each node with the maximum transmission range, is k -connected. Lukovszki [88] provided a method to construct a spanner that can sustain k -nodes or k -links failures.

Penrose [71, 74] also studied the k -connectivity problem for d -dimensional points distributed in a unit-area cube using the toroidal model instead of the Euclidean model as

one way to eliminate the boundary effects. He showed that the hitting radius $r_{n,k}$ such that the graph $G(V, r_{n,k})$ is k -connected satisfies

$$\lim_{n \rightarrow \infty} \Pr[n\pi r_{n,k}^2 \leq \ln n + (k-1) \ln \ln n - \ln(k-1)! + \alpha] = e^{-e^{-\alpha}}$$

Detle and Henze [76] studied the largest length, denoted by $\ell_{n,k}$ here, of the k th nearest neighbor link for n points drawn independently and uniformly from the d -dimensional unit-length cube or the d -dimensional unit volume sphere. They gave asymptotic results of this length according to $k < d$, $k = d$, or $k > d$. For unit-volume cube, they use the norm l_∞ instead of l_2 . For the unit-volume sphere, their result implies that when $d = 2$ and $k > 2$,

$$\lim_{n \rightarrow \infty} \Pr[n\pi \ell_{n,k}^2 \leq \ln n + (2k-3) \ln \ln n - 2 \ln(k-1)! - 2(k-2) \ln 2 + \ln \pi + 2\alpha] = e^{-e^{-\alpha}}$$

Notice that, Penrose [74] had showed that when the domain is a unit-area square, the probability that a random geometry graph $G(V, r_{n,k})$ is k -connected and has minimum vertex degree k goes to 1 as n goes to infinity. Following from a combination of [76] and [74], Li et al. [73] showed that if the transmission range $r_{n,k}$ satisfies $n\pi \cdot r_{n,k}^2 \geq \ln n + (2k-1) \ln \ln n - 2 \ln k! + \alpha + 2 \ln(8k/2^k \sqrt{\pi})$, then $G(V, r_{n,k})$ is $(k+1)$ -connected with probability at least $e^{-e^{-\alpha}}$ as n goes to infinity.

6.5 CONCLUSION

Wireless ad hoc networks have attracted considerable attention recently due to their potential wide applications in various areas and, moreover, the ubiquitous computing. In this chapter, we presented an overview of the recent progress in topology control and localized routing in wireless ad hoc networks. Nevertheless, there are still many excellent results that are not covered in this survey due to space limitations.

There are many interesting open questions for topology control in wireless ad hoc networks. First, we would like to know whether the YaoYao structure $YY_k(V)$ [and, similarly, the structure $YH_k(V)$] is a length spanner. Second, when the overhead cost c of signal transmission is not negligible, are the structures reviewed here still power spanners? Third, how can one control the network topology when different nodes have different transmission ranges such that the topology has some nice properties? Fourth, can we design a localized routing protocol that achieves constant ratio of the length of the found path to the minimum? The answer is probably negative; see [89].

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