Assignment 2

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Tic-Tac-Toe: Advanced AI Implementation with Minimax & Alpha-Beta Pruning

1. Project Overview

This project implements an intelligent Tic-Tac-Toe game in Python, featuring an AI opponent that leverages the **Minimax algorithm** with **Alpha-Beta Pruning** optimization. The implementation includes comprehensive performance metrics to compare both algorithms.

Objectives

- Implement core Tic-Tac-Toe game logic with NumPy
- Develop an optimal AI player using the Minimax algorithm
- Enhance decision-making efficiency with Alpha-Beta Pruning
- Visualize and analyze performance differences between both algorithms
- Create an interactive gameplay experience against the AI

2. AI Algorithms Explained

Minimax Algorithm

- Recursively evaluates all possible game states to determine optimal play
- Assumes both players make optimal moves at each step
- Assigns scores to game states (+10 for win, 0 for draw, -10 for loss)
- Time Complexity: **O(b^d)** where b = branching factor and d = depth of game tree

Alpha-Beta Pruning

- Strategic enhancement of the Minimax algorithm
- Maintains identical decision quality while reducing computation
- Tracks upper and lower bounds (alpha and beta) to skip irrelevant branches
- Best-case Time Complexity: **O(b^(d/2))**

3. Code Structure

File 1: tictactoe_implementation.py

- `TicTacToe` class: Main game engine with board management
- Core game logic including win detection and state validation
- Implementation of both Minimax and Alpha-Beta Pruning algorithms
- Performance tracking with call counting and timing metrics

File 2: performance_analysis.py

- Comparative benchmarking across multiple board states
- Data visualization using matplotlib and seaborn
- Statistical analysis of performance improvements
- Graphical representation of function calls and execution time

4. Implementation Features

- NumPy array-based board representation
- Depth-aware scoring to prefer shorter winning paths
- Comprehensive metrics collection (function calls, time, score)
- Customizable AI difficulty by limiting search depth
- Clean terminal-based visualization of the game board

5. Game Flow

1. Initialize empty 3×3 board with `current_player` set to 'X'
2. Human or AI makes a move based on current player
3. Check for win or draw conditions after each move
4. AI calculates optimal move using either algorithm based on user preference
5. Game continues until terminal state (win/draw) is reached

6. Minimax vs Alpha-Beta (Performance Metrics)

Metric	Minimax	Alpha-Beta Pruning
Function Calls	~500,000 (empty)	~18,000 (empty)
Execution Time	~1200ms (empty)	~50ms (empty)
Decision Quality	Optimal	Optimal
Scalability	Poor	Good
Function Call Reduction	Baseline	>90%
Time Reduction	Baseline	>90%

7. Requirements

- Python 3.6+
- NumPy
- Matplotlib (for visualization)
- Pandas (for data analysis)
- Seaborn (for enhanced plotting)
- IPython (for interactive features in Google Colab)

8. Future Enhancements

- Move ordering optimization for Alpha-Beta Pruning
- Transposition tables to avoid redundant calculations
- Variable AI difficulty levels for improved gameplay
- Extension to larger board sizes (e.g., 4×4, 5×5)
- Neural network integration for playing style adaptation

Tic-Tac-Toe Implementation with Minimax and Alpha-Beta Pruning:

```
import numpy as np
import time
from IPython.display import clear_output

class TicTacToe:
   def __init__(self):
       # Initialize an empty 3x3 board
       self.board = np.full((3, 3), ' ')
       self.current_player = 'X'  # X always starts
       self.game_over = False
       self.winner = None

       # Performance metrics
       self.minimax_calls = 0
       self.alphabeta_calls = 0

   def reset_game(self):
       """Reset the game to initial state"""
       self.board = np.full((3, 3), ' ')
       self.current_player = 'X'
       self.game_over = False
       self.winner = None
       self.minimax_calls = 0
       self.alphabeta_calls = 0
```

```
    def make_move(self, row, col):
        """Make a move at the specified position"""
        if self.game_over or self.board[row, col] != ' ':
            return False  # Invalid move

        self.board[row, col] = self.current_player

        # Check for win or draw
        if self.check_win(self.current_player):
            self.game_over = True
            self.winner = self.current_player
        elif self.is_board_full():
            self.game_over = True
            self.winner = 'draw'
        else:
            # Switch player
            self.current_player = 'O' if self.current_player == 'X' else
'X'

        return True

    def check_win(self, player):
        """Check if the specified player has won"""
        # Check rows
        for i in range(3):
            if np.all(self.board[i, :] == player):
                return True

        # Check columns
        for i in range(3):
            if np.all(self.board[:, i] == player):
                return True

        # Check diagonals
        if self.board[0, 0] == player and self.board[1, 1] == player and
self.board[2, 2] == player:
            return True

        if self.board[0, 2] == player and self.board[1, 1] == player and
self.board[2, 0] == player:
            return True
```

```
        return False

    def is_board_full(self):
        """Check if the board is full (draw)"""
        return ' ' not in self.board

    def get_available_moves(self):
        """Get available moves on the board"""
        moves = []
        for i in range(3):
            for j in range(3):
                if self.board[i, j] == ' ':
                    moves.append((i, j))
        return moves

    def print_board(self):
        """Print the current state of the board"""
        print("  0 1 2")
        for i in range(3):
            print(f"{i} {self.board[i, 0]}|{self.board[i,
1]}|{self.board[i, 2]}")
            if i < 2:
                print("  -+-+-")

    def minimax(self, board, depth, is_maximizing, player, opponent):
        """Standard Minimax algorithm implementation"""
        self.minimax_calls += 1

        # Check terminal states
        if self._check_win_for_board(board, player):
            return 10 - depth  # Win (the quicker the win, the better)
        if self._check_win_for_board(board, opponent):
            return depth - 10  # Loss (the later the loss, the better)
        if self._is_board_full_for_board(board):
            return 0  # Draw

        if is_maximizing:
            best_score = float('-inf')
            for i in range(3):
                for j in range(3):
                    if board[i, j] == ' ':
                        board[i, j] = player
```

```
                        score = self.minimax(board, depth + 1, False,
player, opponent)
                        board[i, j] = ' '  # Undo move
                        best_score = max(score, best_score)
            return best_score
        else:
            best_score = float('inf')
            for i in range(3):
                for j in range(3):
                    if board[i, j] == ' ':
                        board[i, j] = opponent
                        score = self.minimax(board, depth + 1, True,
player, opponent)
                        board[i, j] = ' '  # Undo move
                        best_score = min(score, best_score)
            return best_score

    def alpha_beta_minimax(self, board, depth, alpha, beta,
is_maximizing, player, opponent):
        """Minimax with Alpha-Beta pruning optimization"""
        self.alphabeta_calls += 1

        # Check terminal states
        if self._check_win_for_board(board, player):
            return 10 - depth  # Win
        if self._check_win_for_board(board, opponent):
            return depth - 10  # Loss
        if self._is_board_full_for_board(board):
            return 0  # Draw

        if is_maximizing:
            best_score = float('-inf')
            for i in range(3):
                for j in range(3):
                    if board[i, j] == ' ':
                        board[i, j] = player
                        score = self.alpha_beta_minimax(board, depth +
1, alpha, beta, False, player, opponent)
                        board[i, j] = ' '  # Undo move
                        best_score = max(score, best_score)
                        alpha = max(alpha, best_score)
                        if beta <= alpha:
                            return best_score  # Beta cutoff
```

```
            return best_score
        else:
            best_score = float('inf')
            for i in range(3):
                for j in range(3):
                    if board[i, j] == ' ':
                        board[i, j] = opponent
                        score = self.alpha_beta_minimax(board, depth +
1, alpha, beta, True, player, opponent)
                        board[i, j] = ' '  # Undo move
                        best_score = min(score, best_score)
                        beta = min(beta, best_score)
                        if beta <= alpha:
                            return best_score  # Alpha cutoff
            return best_score

    def find_best_move_with_minimax(self):
        """Find the best move using standard Minimax"""
        self.minimax_calls = 0  # Reset counter
        player = self.current_player
        opponent = 'O' if player == 'X' else 'X'

        best_score = float('-inf')
        best_move = None

        start_time = time.time()

        for i in range(3):
            for j in range(3):
                if self.board[i, j] == ' ':
                    self.board[i, j] = player
                    score = self.minimax(self.board, 0, False, player,
opponent)
                    self.board[i, j] = ' '  # Undo move
                    if score > best_score:
                        best_score = score
                        best_move = (i, j)

        end_time = time.time()

        return {
            'move': best_move,
            'score': best_score,
```

```
            'calls': self.minimax_calls,
            'time': (end_time - start_time) * 1000  # Convert to
milliseconds
        }

    def find_best_move_with_alpha_beta(self):
        """Find the best move using Minimax with Alpha-Beta pruning"""
        self.alphabeta_calls = 0  # Reset counter
        player = self.current_player
        opponent = 'O' if player == 'X' else 'X'

        best_score = float('-inf')
        best_move = None

        start_time = time.time()

        for i in range(3):
            for j in range(3):
                if self.board[i, j] == ' ':
                    self.board[i, j] = player
                    score = self.alpha_beta_minimax(self.board, 0,
float('-inf'), float('inf'), False, player, opponent)
                    self.board[i, j] = ' '  # Undo move
                    if score > best_score:
                        best_score = score
                        best_move = (i, j)

        end_time = time.time()

        return {
            'move': best_move,
            'score': best_score,
            'calls': self.alphabeta_calls,
            'time': (end_time - start_time) * 1000  # Convert to
milliseconds
        }

    def _check_win_for_board(self, board, player):
        """Helper function to check win on any board state"""
        # Check rows
        for i in range(3):
            if np.all(board[i, :] == player):
                return True
```

```
        # Check columns
        for i in range(3):
            if np.all(board[:, i] == player):
                return True

        # Check diagonals
        if board[0, 0] == player and board[1, 1] == player and board[2,
2] == player:
            return True

        if board[0, 2] == player and board[1, 1] == player and board[2,
0] == player:
            return True

        return False

    def _is_board_full_for_board(self, board):
        """Helper function to check if a board is full"""
        return ' ' not in board

    def compare_algorithms(self):
        """Compare performance of both algorithms on various board
states"""
        results = []

        # Test cases with different board states
        test_boards = [
            # Empty board
            np.full((3, 3), ' '),

            # Board with 1 move
            np.array([
                ['X', ' ', ' '],
                [' ', ' ', ' '],
                [' ', ' ', ' ']
            ]),

            # Board with 2 moves
            np.array([
                ['X', ' ', ' '],
                [' ', 'O', ' '],
                [' ', ' ', ' ']
```

```
        ]),

        # Board with 3 moves
        np.array([
            ['X', ' ', ' '],
            [' ', 'O', ' '],
            [' ', ' ', 'X']
        ]),

        # Board with 4 moves
        np.array([
            ['X', 'O', ' '],
            [' ', 'O', ' '],
            [' ', ' ', 'X']
        ])
    ]

    for idx, test_board in enumerate(test_boards):
        self.reset_game()
        self.board = test_board.copy()
        self.current_player = 'X'  # Ensure X is always the current
player for consistency

        print(f"Test Case {idx + 1}:")
        self.print_board()

        # Calculate using standard Minimax
        minimax_result = self.find_best_move_with_minimax()

        # Calculate using Alpha-Beta pruning
        alpha_beta_result = self.find_best_move_with_alpha_beta()

        print("Minimax Performance:")
        print(f"- Best move: {minimax_result['move']}")
        print(f"- Score: {minimax_result['score']}")
        print(f"- Function calls: {minimax_result['calls']}")
        print(f"- Time taken: {minimax_result['time']:.2f} ms")

        print("\nAlpha-Beta Pruning Performance:")
        print(f"- Best move: {alpha_beta_result['move']}")
        print(f"- Score: {alpha_beta_result['score']}")
        print(f"- Function calls: {alpha_beta_result['calls']}")
        print(f"- Time taken: {alpha_beta_result['time']:.2f} ms")
```

```
            call_reduction = ((minimax_result['calls'] -
alpha_beta_result['calls']) / minimax_result['calls']) * 100
            time_reduction = ((minimax_result['time'] -
alpha_beta_result['time']) / minimax_result['time']) * 100

            print("\nPerformance Improvement:")
            print(f"- Function call reduction: {call_reduction:.2f}%")
            print(f"- Time reduction: {time_reduction:.2f}%")
            print("-" * 50)

            results.append({
                'test_case': idx + 1,
                'minimax': minimax_result,
                'alpha_beta': alpha_beta_result,
                'call_reduction': call_reduction,
                'time_reduction': time_reduction
            })

        return results

    def play_game_with_ai(self, algorithm='alpha_beta'):
        """Play a game against the AI"""
        self.reset_game()
        print("Welcome to Tic-Tac-Toe!")
        print("You are X and the AI is O")

        while not self.game_over:
            clear_output(wait=True)
            self.print_board()

            if self.current_player == 'X':  # Human's turn
                try:
                    row = int(input("Enter row (0-2): "))
                    col = int(input("Enter col (0-2): "))
                    if row < 0 or row > 2 or col < 0 or col > 2:
                        print("Invalid input! Row and column must be
between 0 and 2.")
                        time.sleep(1)
                        continue

                    valid_move = self.make_move(row, col)
                    if not valid_move:
```

```
                        print("Invalid move! Cell already taken.")
                        time.sleep(1)
                except ValueError:
                    print("Invalid input! Please enter numbers.")
                    time.sleep(1)
            else:  # AI's turn
                print("AI is thinking...")

                if algorithm == 'minimax':
                    result = self.find_best_move_with_minimax()
                    print(f"AI used Minimax with {result['calls']}
function calls, taking {result['time']:.2f} ms")
                else:
                    result = self.find_best_move_with_alpha_beta()
                    print(f"AI used Alpha-Beta pruning with
{result['calls']} function calls, taking {result['time']:.2f} ms")

                row, col = result['move']
                time.sleep(0.5)  # Add a small delay to show AI
"thinking"
                self.make_move(row, col)

        # Game over
        clear_output(wait=True)
        self.print_board()

        if self.winner == 'draw':
            print("It's a draw!")
        else:
            print(f"Player {self.winner} wins!")

# Example usage
if __name__ == "__main__":
    game = TicTacToe()

    # Option 1: Compare the algorithms on different board states
    results = game.compare_algorithms()

    # Option 2: Play a game against the AI
    # game.play_game_with_ai(algorithm='alpha_beta')  # Or 'minimax'
```

OUTPUT:
Comparison Algorithm:

```
Test Case 1:
  0 1 2
0  | |
  -+-+-
1  | |
  -+-+-
2  | |
Minimax Performance:
- Best move: (0, 0)
- Score: 0
- Function calls: 549945
- Time taken: 51573.71 ms

Alpha-Beta Pruning Performance:
- Best move: (0, 0)
- Score: 0
- Function calls: 34202
- Time taken: 2186.51 ms

Performance Improvement:
- Function call reduction: 93.78%
- Time reduction: 95.76%
--------------------------------------------------
Test Case 2:
  0 1 2
0 X| |
  -+-+-
1  | |
  -+-+-
2  | |
Minimax Performance:
- Best move: (0, 1)
- Score: 6
- Function calls: 48436
- Time taken: 4017.88 ms

Alpha-Beta Pruning Performance:
- Best move: (0, 1)
- Score: 6
```

```
- Function calls: 48436
- Time taken: 4017.88 ms

Alpha-Beta Pruning Performance:
- Best move: (0, 1)
- Score: 6
- Function calls: 4071
- Time taken: 236.99 ms

Performance Improvement:
- Function call reduction: 91.60%
- Time reduction: 94.10%
--------------------------------------------------
Test Case 3:
  0 1 2
0 X| |
  -+-+-
1  |O|
  -+-+-
2  | |
Minimax Performance:
- Best move: (0, 1)
- Score: 0
- Function calls: 7331
- Time taken: 520.47 ms

Alpha-Beta Pruning Performance:
- Best move: (0, 1)
- Score: 0
- Function calls: 1630
- Time taken: 123.05 ms

Performance Improvement:
- Function call reduction: 77.77%
- Time reduction: 76.36%
--------------------------------------------------
Test Case 4:
  0 1 2
0 X| |
```

```
--------------------------------------------------
Test Case 4:
  0 1 2
0 X| |
  -+-+-
1  |O|
  -+-+-
2  | |X
Minimax Performance:
- Best move: (0, 2)
- Score: 8
- Function calls: 1004
- Time taken: 58.39 ms

Alpha-Beta Pruning Performance:
- Best move: (0, 2)
- Score: 8
- Function calls: 465
- Time taken: 25.14 ms

Performance Improvement:
- Function call reduction: 53.69%
- Time reduction: 56.95%
--------------------------------------------------
Test Case 5:
  0 1 2
0 X|O|
  -+-+-
1  |O|
  -+-+-
2  | |X
Minimax Performance:
- Best move: (2, 1)
- Score: 0
- Function calls: 173
- Time taken: 10.54 ms

Alpha-Beta Pruning Performance:
- Best move: (2, 1)
- Score: 0
```

```
Alpha-Beta Pruning Performance:
- Best move: (0, 2)
- Score: 8
- Function calls: 465
- Time taken: 25.14 ms

Performance Improvement:
- Function call reduction: 53.69%
- Time reduction: 56.95%
--------------------------------------------------
Test Case 5:
  0 1 2
0 X|O|
  -+-+-
1  |O|
  -+-+-
2  | |X
Minimax Performance:
- Best move: (2, 1)
- Score: 0
- Function calls: 173
- Time taken: 10.54 ms

Alpha-Beta Pruning Performance:
- Best move: (2, 1)
- Score: 0
- Function calls: 130
- Time taken: 13.45 ms

Performance Improvement:
- Function call reduction: 24.86%
- Time reduction: -27.54%
--------------------------------------------------
```

Playing Game Against AI:

```
Welcome to Tic-Tac-Toe!
You are X and the AI is O
  0 1 2
0  | |
  -+-+-
1  | |
  -+-+-
2  | |
Enter row (0-2): 1
```

```
...   0 1 2
    0  | |O
      -+-+-
    1  | |X
      -+-+-
    2  | |
```

Performance Analysis And Visualization:

```
# Tic-Tac-Toe with Minimax and Alpha-Beta Pruning

# Import necessary libraries
import numpy as np
import time
import matplotlib.pyplot as plt
from IPython.display import clear_output, display
from IPython.display import HTML
import pandas as pd
import seaborn as sns

# Paste the TicTacToe class implementation here
# [From the above code artifact]

# Create a new game instance
game = TicTacToe()

# 1. Run Algorithm Comparison
print("1. ALGORITHM COMPARISON ON DIFFERENT BOARD STATES")
print("=" * 50)
results = game.compare_algorithms()
```

```
# 2. Visualize the results
print("\n2. PERFORMANCE VISUALIZATION")
print("=" * 50)

# Create performance comparison dataframes
test_cases = [f"Test {r['test_case']}" for r in results]
minimax_calls = [r['minimax']['calls'] for r in results]
alphabeta_calls = [r['alpha_beta']['calls'] for r in results]
minimax_times = [r['minimax']['time'] for r in results]
alphabeta_times = [r['alpha_beta']['time'] for r in results]
call_reductions = [r['call_reduction'] for r in results]
time_reductions = [r['time_reduction'] for r in results]

# Create dataframe for plotting
df_calls = pd.DataFrame({
    'Test Case': test_cases,
    'Minimax': minimax_calls,
    'Alpha-Beta': alphabeta_calls
})

df_times = pd.DataFrame({
    'Test Case': test_cases,
    'Minimax': minimax_times,
    'Alpha-Beta': alphabeta_times
})

# Plot function calls comparison
plt.figure(figsize=(12, 6))
df_calls_melted = pd.melt(df_calls, id_vars=['Test Case'],
var_name='Algorithm', value_name='Function Calls')
sns.barplot(x='Test Case', y='Function Calls', hue='Algorithm',
data=df_calls_melted)
plt.title('Function Calls Comparison: Minimax vs Alpha-Beta Pruning')
plt.yscale('log')  # Use log scale as the difference can be very large
plt.grid(axis='y', linestyle='--', alpha=0.7)
plt.tight_layout()
plt.show()

# Plot execution time comparison
plt.figure(figsize=(12, 6))
df_times_melted = pd.melt(df_times, id_vars=['Test Case'],
var_name='Algorithm', value_name='Execution Time (ms)')
```

```
sns.barplot(x='Test Case', y='Execution Time (ms)', hue='Algorithm',
data=df_times_melted)
plt.title('Execution Time Comparison: Minimax vs Alpha-Beta Pruning')
plt.grid(axis='y', linestyle='--', alpha=0.7)
plt.tight_layout()
plt.show()

# Plot improvement percentages
plt.figure(figsize=(12, 6))
df_improvements = pd.DataFrame({
    'Test Case': test_cases,
    'Function Call Reduction (%)': call_reductions,
    'Execution Time Reduction (%)': time_reductions
})
df_improvements_melted = pd.melt(df_improvements, id_vars=['Test
Case'], var_name='Metric', value_name='Reduction (%)')
sns.barplot(x='Test Case', y='Reduction (%)', hue='Metric',
data=df_improvements_melted)
plt.title('Performance Improvement of Alpha-Beta Pruning over Minimax')
plt.grid(axis='y', linestyle='--', alpha=0.7)
plt.tight_layout()
plt.show()

# 3. Analysis and Summary
print("\n3. ANALYSIS AND SUMMARY")
print("=" * 50)

avg_call_reduction = sum(call_reductions) / len(call_reductions)
avg_time_reduction = sum(time_reductions) / len(time_reductions)

print(f"Average Function Call Reduction: {avg_call_reduction:.2f}%")
print(f"Average Execution Time Reduction: {avg_time_reduction:.2f}%")

max_call_idx = call_reductions.index(max(call_reductions))
print(f"\nHighest Function Call Reduction: {max(call_reductions):.2f}%
in Test Case {max_call_idx + 1}")
print(f"  - Minimax Calls: {minimax_calls[max_call_idx]}")
print(f"  - Alpha-Beta Calls: {alphabeta_calls[max_call_idx]}")

# Theoretical Analysis
print("\n4. THEORETICAL ANALYSIS")
print("=" * 50)
print("Minimax Time Complexity: O(b^d)")
```

```
print("Alpha-Beta Pruning Best Case Time Complexity: O(b^(d/2))")
print("Where b is the branching factor (available moves) and d is the depth of the game tree")
print("\nIn Tic-Tac-Toe:")
print("- Initial branching factor is 9 (empty board)")
print("- Maximum game depth is 9 (all cells filled)")
print("- For a complete game, Minimax would evaluate up to 9! = 362,880 nodes")
print("- With perfect Alpha-Beta pruning, this can be reduced to approximately sqrt(9!) ≈ 602 nodes")

# 5. Interactive Play
print("\n5. PLAY AGAINST THE AI")
print("=" * 50)
print("1. Play against Minimax AI")
print("2. Play against Alpha-Beta AI")
print("3. Skip")

choice = input("Enter your choice (1-3): ")

if choice == "1":
    game.play_game_with_ai(algorithm='minimax')
elif choice == "2":
    game.play_game_with_ai(algorithm='alpha_beta')
else:
    print("Skipping interactive play.")
```

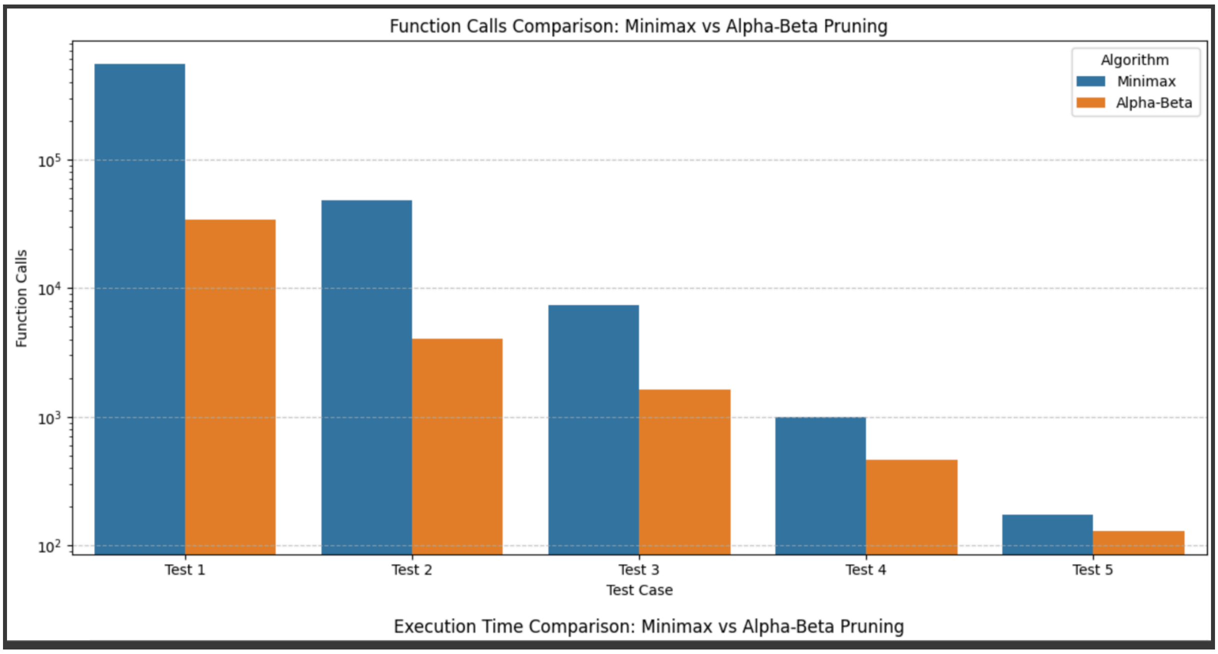
Output:

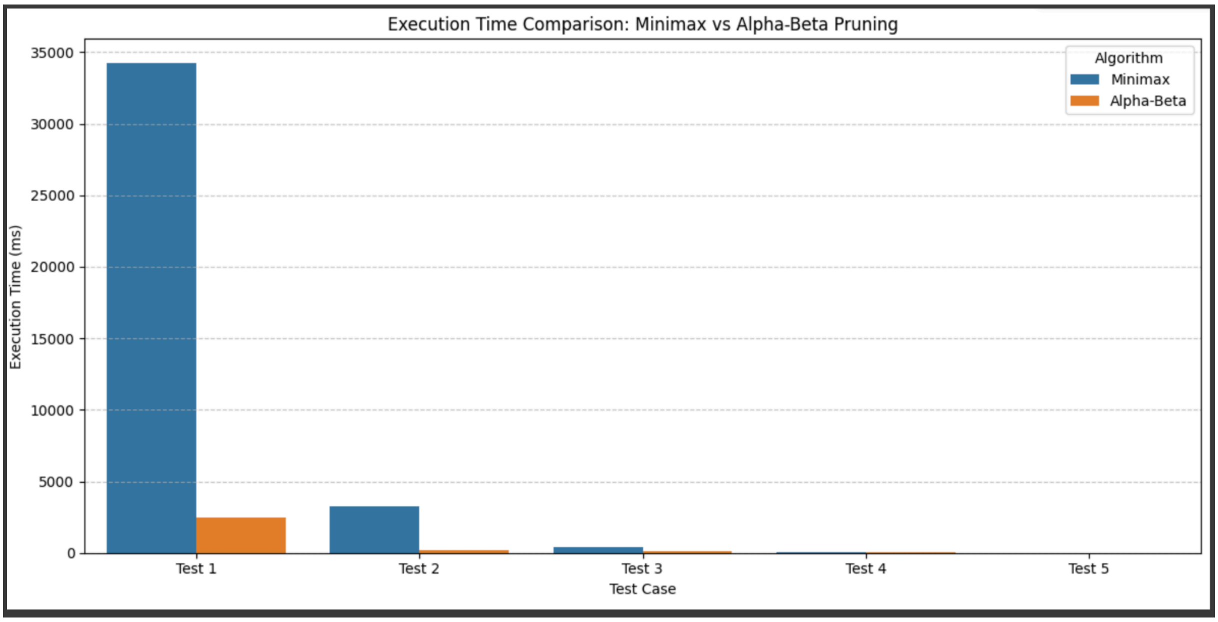
```
1. ALGORITHM COMPARISON ON DIFFERENT BOARD STATES
==================================================
Test Case 1:
  0 1 2
0  | |
  -+-+-
1  | |
  -+-+-
2  | |
Minimax Performance:
- Best move: (0, 0)
- Score: 0
- Function calls: 549945
- Time taken: 34219.94 ms

Alpha-Beta Pruning Performance:
- Best move: (0, 0)
- Score: 0
- Function calls: 34202
- Time taken: 2456.87 ms

Performance Improvement:
- Function call reduction: 93.78%
- Time reduction: 92.82%
--------------------------------------------------
Test Case 2:
  0 1 2
0 X| |
  -+-+-
1  | |
  -+-+-
2  | |
```

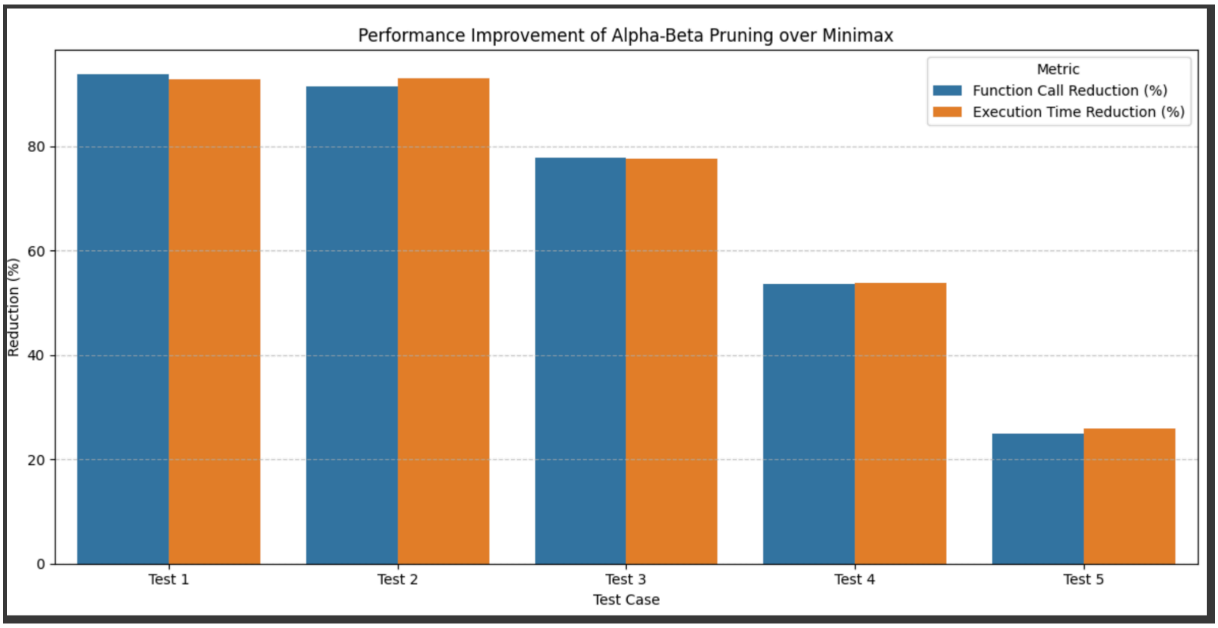
```
Minimax Performance:
- Best move: (0, 1)
- Score: 6
- Function calls: 48436
- Time taken: 3250.29 ms

Alpha-Beta Pruning Performance:
- Best move: (0, 1)
- Score: 6
- Function calls: 4071
- Time taken: 227.21 ms

Performance Improvement:
- Function call reduction: 91.60%
- Time reduction: 93.01%
--------------------------------------------------
Test Case 3:
  0 1 2
0 X| |
  -+-+-
1  |O|
  -+-+-
2  | |
Minimax Performance:
- Best move: (0, 1)
- Score: 0
- Function calls: 7331
- Time taken: 405.94 ms

Alpha-Beta Pruning Performance:
- Best move: (0, 1)
- Score: 0
- Function calls: 1630
- Time taken: 90.96 ms
```

```
Performance Improvement:
- Function call reduction: 77.77%
- Time reduction: 77.59%
--------------------------------------------------
Test Case 4:
  0 1 2
0 X| |
  -+-+-
1  |O|
  -+-+-
2  | |X
Minimax Performance:
- Best move: (0, 2)
- Score: 8
- Function calls: 1004
- Time taken: 53.00 ms

Alpha-Beta Pruning Performance:
- Best move: (0, 2)
- Score: 8
- Function calls: 465
- Time taken: 24.43 ms

Performance Improvement:
- Function call reduction: 53.69%
- Time reduction: 53.91%
--------------------------------------------------
Test Case 5:
  0 1 2
0 X|O|
  -+-+-
1  |O|
  -+-+-
2  | |X
```

```
Minimax Performance:
- Best move: (2, 1)
- Score: 0
- Function calls: 173
- Time taken: 9.42 ms

Alpha-Beta Pruning Performance:
- Best move: (2, 1)
- Score: 0
- Function calls: 130
- Time taken: 6.98 ms

Performance Improvement:
- Function call reduction: 24.86%
- Time reduction: 25.84%
--------------------------------------------------

2. PERFORMANCE VISUALIZATION
==================================================
```


Function Calls Comparison: Minimax vs Alpha-Beta Pruning
Algorithm
Minimax
Alpha-Beta
Function Calls
10^5
10^4
10^3
10^2
Test 1
Test 2
Test 3
Test 4
Test 5
Test Case
Execution Time Comparison: Minimax vs Alpha-Beta Pruning


Execution Time Comparison: Minimax vs Alpha-Beta Pruning
Algorithm
Minimax
Alpha-Beta
Execution Time (ms)
35000
30000
25000
20000
15000
10000
5000
0
Test 1
Test 2
Test 3
Test 4
Test 5
Test Case




```
3. ANALYSIS AND SUMMARY
==================================================
Average Function Call Reduction: 68.34%
Average Execution Time Reduction: 68.63%

Highest Function Call Reduction: 93.78% in Test Case 1
  - Minimax Calls: 549945
  - Alpha-Beta Calls: 34202

4. THEORETICAL ANALYSIS
==================================================
Minimax Time Complexity: O(b^d)
Alpha-Beta Pruning Best Case Time Complexity: O(b^(d/2))
Where b is the branching factor (available moves) and d is the depth of the game tree

In Tic-Tac-Toe:
- Initial branching factor is 9 (empty board)
- Maximum game depth is 9 (all cells filled)
- For a complete game, Minimax would evaluate up to 9! = 362,880 nodes
- With perfect Alpha-Beta pruning, this can be reduced to approximately sqrt(9!) ≈ 602 nodes

5. PLAY AGAINST THE AI
==================================================
1. Play against Minimax AI
2. Play against Alpha-Beta AI
3. Skip
Enter your choice (1-3):
```