myITS learning

SET

Logic and Discrete Structure

Dr. Rarasmaya Indraswari, S.Kom.

Outline

- Definition of Set
- Characteristics of Set
- Declare the Set’s Elements
- *Sets of Sets*
- Himpunan Kosong (*Empty Set*)
- Kesamaan Himpunan (*Set Equality*)
- *Subsets* and *Proper Subsets*
- *Set cardinality*
- *Power sets*
- *Tuples*
- *Cartesian products*

Definition of Set

- A **set** is a **collection** of **different objects**.
- Object in a set are called element or member.
- Example:
  - *People in a class*: { Alice, Bob, Chris }
  - *Colors of a rainbow*: { red, orange, yellow, green, blue, purple }
  - *States of matter* { solid, liquid, gas, plasma }
  - *States in the US*: { Alabama, Alaska, Virginia, … }
  - Sets can contain non-related elements: { 3, a, red, Virginia }

Characteristics of Set

- Order does not matter
  - We often write them in order because it is easier for humans to understand it that way
  - {1, 2, 3, 4, 5} is equivalent to {3, 5, 2, 4, 1}
- Sets are notated with curly brackets { }
- Sets do not have duplicate elements
  - Consider the set of vowels in the alphabet.
    - It makes no sense to list them as {a, a, a, e, i, o, o, o, o, o, u}
    - What we really want is just {a, e, i, o, u}
  - Consider the list of students in this class
    - Again, it does not make sense to list somebody twice
- Note that a list is like a set, but order does matter, and duplicate elements are allowed
  - We won't be studying lists much in this class

Declare the Set's Elements

1. Enumeration

Each member of the set is listed in detail.

Example:

- The set of the first four natural numbers: $A = \{1, 2, 3, 4\}$.
- The set of the first five positive even numbers: $B = \{2, 4, 6, 8, 10\}$.
- $R = \{ a, b, \{a, b, c\}, \{a, c\} \}$
- $C = \{a, \{a\}, \{\{a\}\} \}$
- $K = \{ \{\} \}$

Declare the Set's Elements

2. Ellipsis

Members of a set can be written with an ellipsis (...) if the element pattern is already known.

For example, B = {0, 1, 2, 3, …}

But this can be ambiguous or confusing.

- For example, the set C = {3, 5, 7, …} what number comes next?
- If the set is an odd integer greater than 2 then the next is 9
- If the set is a prime number greater than 2 then the next is 11

Declare the Set’s Elements

3. **Standard Symbols**

P = the set of positive integers = { 1, 2, 3, ... }

N = the set of natural numbers = { 1, 2, ... }

Z = the set of integers = { ..., -2, -1, 0, 1, 2, ... }

Q = the set of rational numbers

R = the set of real numbers

C = the set of complex numbers

U = **universal** set

For example, U = {1, 2, 3, 4, 5} and A is a subset of U, with A = {1, 3, 5}.

Declare the Set's Elements

4. **Set Builder Notation**

Notation: { x | conditions that must be met by x }

Example

i. A is the set of positive integers smaller than 5

A = { x | x is a positive integer smaller than 5}

or $A = \{ x \mid x \in P, x < 5 \}$ that equivalent with $A = \{1, 2, 3, 4\}$

ii. M = { x | x is a DSI student that takes LSD course}

Declare the Set’s Elements

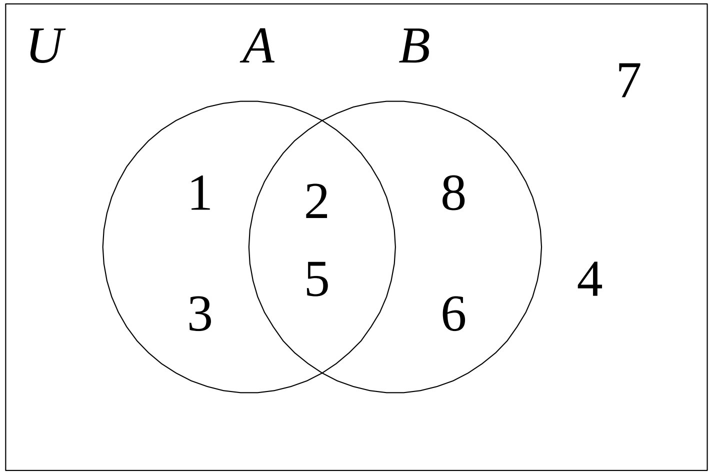
5. **Venn Diagram**

Example

Let U = {1, 2, ..., 7, 8},

A = {1, 2, 3, 5} and B = {2, 5, 6, 8}.

The Venn Diagram:



Sets of sets

- A member of a set can be another set.
  - S = { {1}, {2}, {3} }
  - T = { {1}, {{2}}, {{{3}}} }
  - V = { {{1}, {{2}}}, {{{3}}}, {{1}, {{2}}, {{{3}}}} }
    - V only have 3 elements!

- Note that 1 ≠ {1} ≠ {{1}} ≠ {{{1}}}
  - Each is a different element.

Himpunan Kosong (*empty set*)

- A set whose members have 0 members (has no members) is called an empty or null set
  - Expressed with symbol $\varnothing$
  - Therefore, $\varnothing = \{\ \}$ ← **IMPORTANT**
  - If you're confused, try replacing $\varnothing$ with $\{\ \}$
- Since the empty set is a set, the empty set can also be a member of another set.
  - $\{ \varnothing, 1, 2, 3, x \}$ is a valid set
- Note that $\varnothing \neq \{ \varnothing \}$
  - The first set is a set with 0 element
  - The second set is a set with 1 element (which is empty set)
- Try replacing $\varnothing$ with $\{\ \}$, therefore: $\{\ \} \neq \{ \{\ \} \}$
  - It can be seen that the two are not equivalent

Kesamaan Himpunan (*Set Equality*)

- Two sets are said to be the same (equivalent) if they both have the same elements.
  - $\{1, 2, 3, 4, 5\} = \{5, 4, 3, 2, 1\}$
    - Remember that the element's order can be reversed!
  - $\{1, 2, 3, 2, 4, 3, 2, 1\} = \{4, 3, 2, 1\}$
    - Because a set cannot have duplicate elements!
- Two sets are not equivalent if their members are different:
  - $\{1, 2, 3, 4, 5\} \neq \{1, 2, 3, 4\}$

Himpunan Bagian (*Subsets*)

- If all the elements of set S are also elements of set T, then S is a subset of T
  - For example, if S = {2, 4, 6} and T = {1, 2, 3, 4, 5, 6, 7}, then S is a subset of T
  - Expressed with the symbol $S \subseteq T$
    - or $\{2, 4, 6\} \subseteq \{1, 2, 3, 4, 5, 6, 7\}$
- If S is **not a subset** of T, it is expressed with: $S \not\subseteq T$
  - For example, $\{1, 2, 8\} \not\subseteq \{1, 2, 3, 4, 5, 6, 7\}$
- Note that each set is a subset of itself!
  - S = {2, 4, 6}, since all elements of S are elements of S, S is a subset of itself
  - This is like saying that 5 is less than or equal to 5
  - Therefore, for every set S, $S \subseteq S$

Himpunan Bagian (*Subsets*)

- The **empty set** is a subset of all sets (including itself!)
  - Remember that all sets are subsets of themselves
- All sets are subsets of its universal set.
- *A horrible way* to define a subset:
  - $\forall x\,(x \in A \rightarrow x \in B)$
  - ***English translation***: *for all possible values of x, (meaning for all possible elements of a set), if x is an element of A, then x is an element of B*

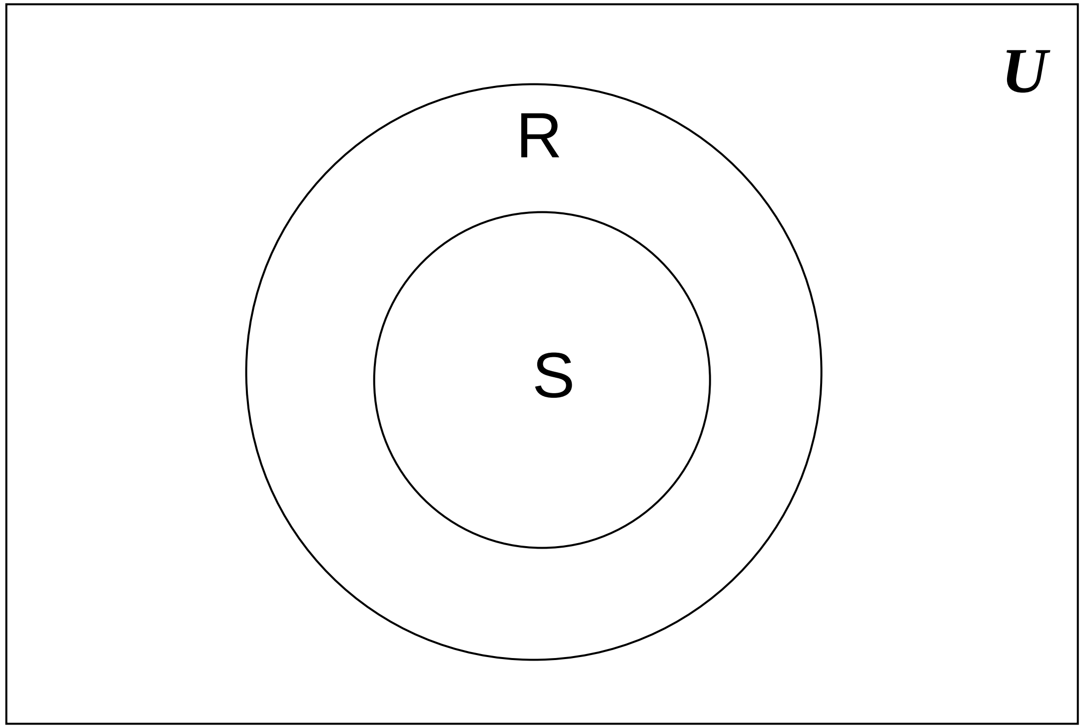
Proper Subsets

If S is a subset of T, and S ≠ T, then S is a **proper subset** of T.

- Let T = {0, 1, 2, 3, 4, 5}
- If S = {1, 2, 3}, S ≠ T, and S is a subset of T
- Notation to express that S is a **proper subset** of T is **$S \subset T$**
- For example, R = {0, 1, 2, 3, 4, 5}. R = T, so that R is a *subset* (but not a *proper subset*) of T
  - Can be written as: $R \subseteq T$ and $R \not\subset T$
- Let Q = {4, 5, 6}. Q is neither a *subset* or *proper subset* of T
- The difference of "***subset***" and "***proper subset***" is like the difference between "**less than or equal to (≤)** " and "**less than (<)**".
- The empty set is a proper subset of all sets other than the empty set (because it is the same as the empty set itself).

Venn Diagram for Proper Subset

$$S \subset R$$



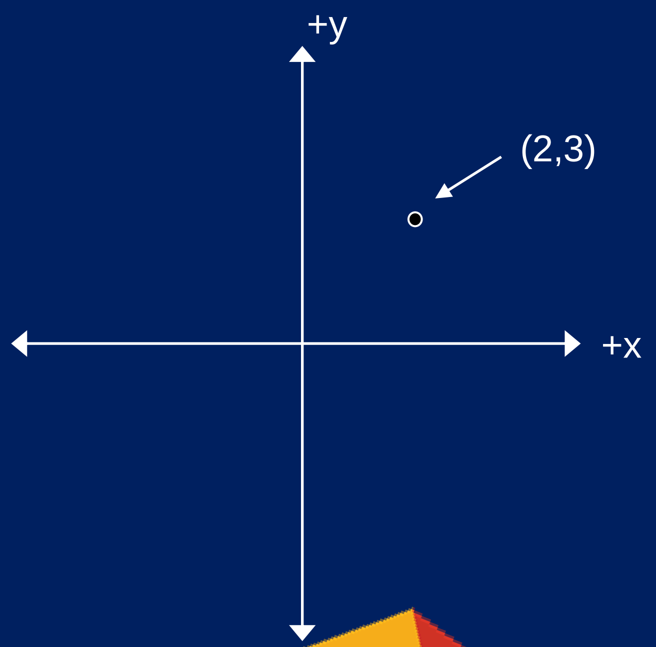
Kardinalitas Himpunan (*Set Cardinality*)

- The cardinality of a set is the number of elements in a set.
  - The notation for the cardinality of set A is $|A|$
- For example:
  - If $R = \{1, 2, 3, 4, 5\}$. Then $|R| = 5$
  - $|\varnothing| = 0$
  - If $S = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$. Then $|S| = 4$
- The notation for the cardinality of sets is the same as the notation for the length of geometric vectors.

Power Set

- Let S = {0, 1}. How many possible subsets of S are there?
  - The possibilities are: ∅ (because it is a subset of all sets), {0}, {1}, and {0, 1}
  - **Power set** of S (expressed as P(S)) is the number of all possible subsets of S
  - P(S) = { ∅, {0}, {1}, {0,1} }
    - Note that |S| = 2 and |P(S)| = 4
- Let T = {0, 1, 2}. Then P(T) = { ∅, {0}, {1}, {2}, {0,1}, {0,2}, {1,2}, {0,1,2} }
  - Note that |T| = 3 and |P(T)| = 8
- P(∅) = { ∅ }
  - Note that |∅| = 0 and |P(∅)| = 1
- If a set has *n* elements, its power set will have 2^n elements.

Tuples



- In 2-dimensional space, tuples are expressed as pairs of coordinate points (x, y) to represent a location.
- In 3-dimensional space, the coordinate pair (1,2,3) is not the same as (3,2,1), which represents the coordinate pair of 3 numbers (x, y, z).
- In *n*-dimensional space, it is an *n*-tuple of corresponding number coordinates.
- Note that tuples are expressed sequentially (**ordered**), unlike sets.
  - The x value is always written first.

Cartesian Product

- Cartesian product is a set of all *n*-tuples where each "part" is taken from a certain set.
  - Expressed with A x B, and use parentheses (not curly brackets)
  - For example, 2-D Cartesian coordinates are the set of all ordered pairs **Z** x **Z**
    - **Note that Z** is a set of all integers
    - Shows all coordinates that exist in 2-D space
  - For example: Let A = { a, b } and B = { 0, 1 }, determine its *Cartesian product*.
    - C = A x B = { (a,0), (a,1), (b,0), (b,1) }
- The definition of Cartesian product can be written as follows:
  - A x B = { (*a*,*b*) | *a* ∈ A and *b* ∈ B }

Cartesian Product

- All possible grades in a class are the Cartesian product of the set S which consists of all the students in the class and the set G which consists of all the possible grades obtained.
  - Let S = { Ali, Bobi, Cici } and G = { A, B, C }
  - D = { (Ali, A), (Ali, B), (Ali, C), (Bobi, A), (Bobi, B), (Bobi, C), (Cici, A), (Cici, B), (Cici, C) }
  - The final grade obtained is a subset of D: { (Ali, C), (Bobi, B), (Cici, A) }
    - Subsets of a Cartesian product are called relations (explained in the next chapter)
- Cartesian product can also be performed on more than two sets.
- 3-D coordinates are elements of the Cartesian product Z x Z x Z

SET OPERATIONS

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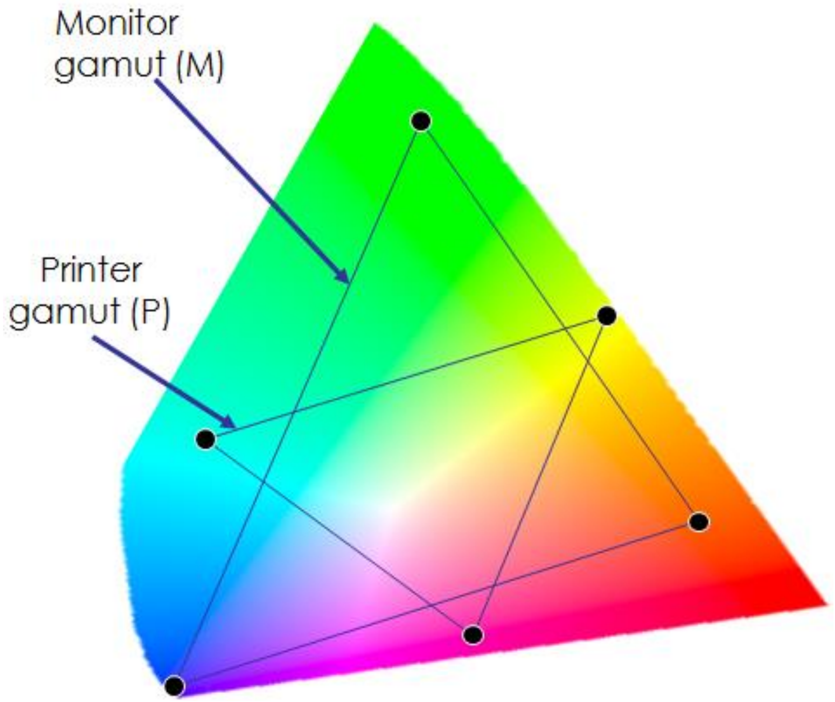
Outline

- Gabungan (*Union*)
- Irisan (*Intersection*)
- Himpunan Saling Lepas (*Disjoint*)
- Selisih (*Difference*)
- *Symmetric Difference*
- *Complement*
- *Set Identities*
- *How to Proof Set Identities*

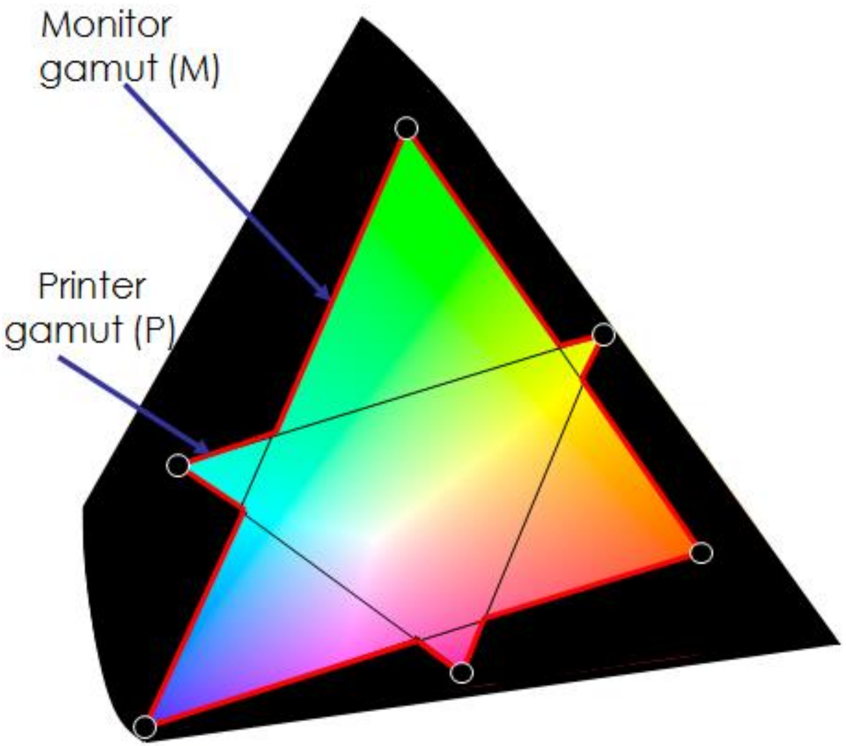
Color Set

The triangle in the following image shows a combination of color ranges (gamut) – which is a collection of various colors.



Gabungan (*Union*)

- *A union of the sets contains all the elements in **EITHER** set.*
- Symbol for *union* is ∪
- For example:
  - $C = M \cup P$

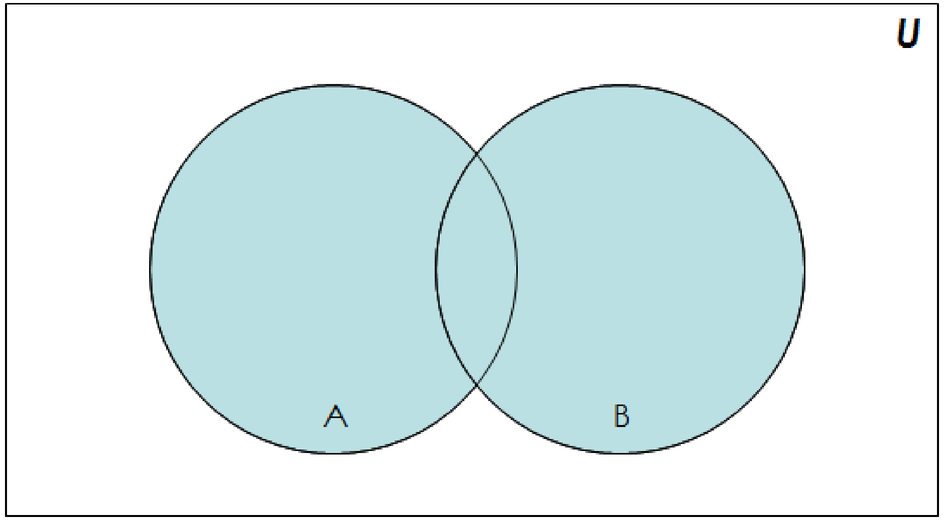


Gabungan (*Union*)

- The formal definition for the union of two sets is:

  $A \cup B = \{ x \mid x \in A \textbf{ or } x \in B \}$
- For example:
  - {New York, Washington} $\cup$ {3, 4} = {New York, Washington, 3, 4}
  - $\{1, 2\} \cup \varnothing = \{1, 2\}$
  - $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$

A U B

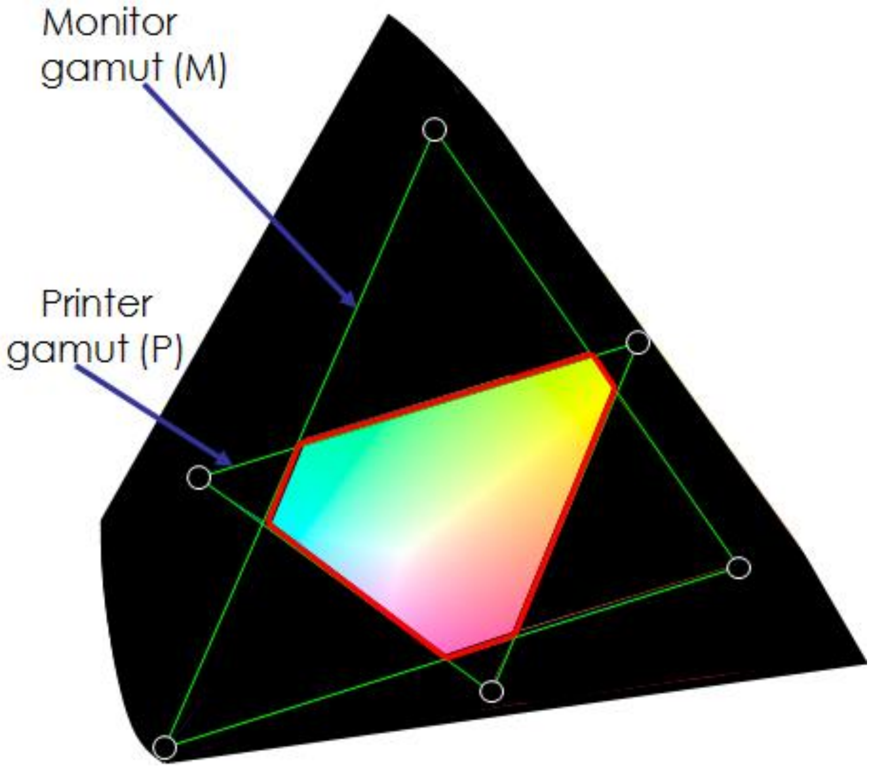


Characteristics of Union Operations

- $A \cup \varnothing = A$ — Identity law
- $A \cup \boldsymbol{U} = \boldsymbol{U}$ — Domination law
- $A \cup A = A$ — Idempotent law
- $A \cup B = B \cup A$ — Commutative law
- $A \cup (B \cup C) = (A \cup B) \cup C$ — Associative law

Irisan (*Intersection*)

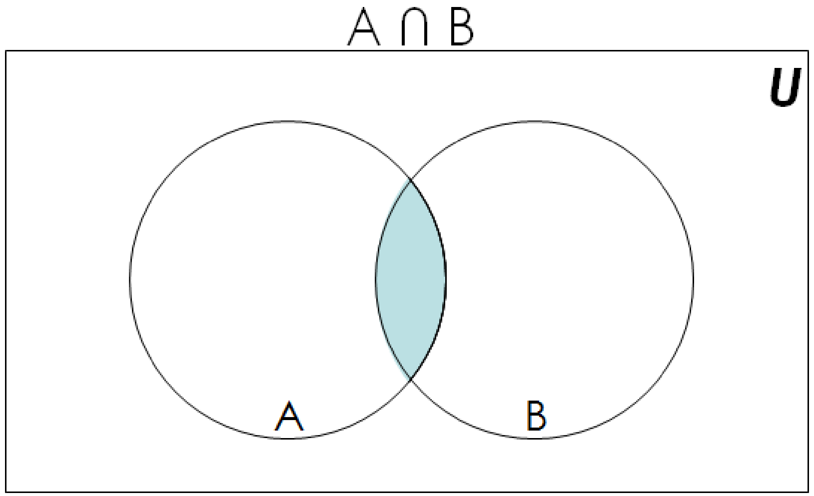
- *An intersection of the sets contains all the elements in **BOTH** sets.*
- Symbol of *intersection* is $\cap$
- For example:
  $C = M \cap P$



Irisan (*Intersection*)

- The formal definition for the intersection of two sets is :

  $A \cap B = \{ x \mid x \in A \textbf{ and } x \in B \}$



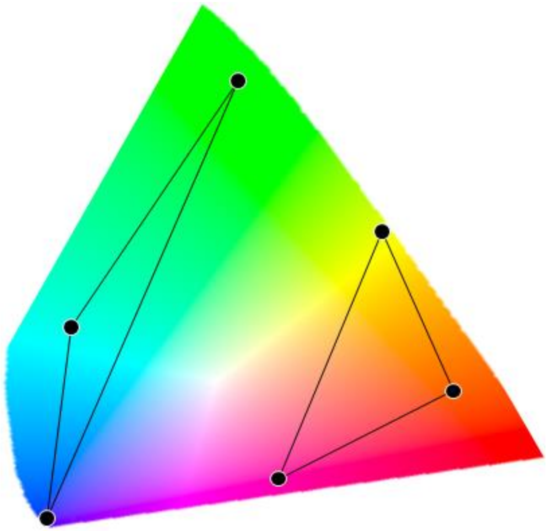
- For example:
  - $\{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$
  - $\{\text{New York, Washington}\} \cap \{3, 4\} = \varnothing$
    - Do not have the same elements.
  - $\{1, 2\} \cap \varnothing = \varnothing$
    - The intersection of a set with an empty set is an empty set.

Characteristics of Intersection Operations

- $A \cap \boldsymbol{U} = A$ Identity law
- $A \cap \varnothing = \varnothing$ Domination law
- $A \cap A = A$ Idempotent law
- $A \cap B = B \cap A$ Commutative law
- $A \cap (B \cap C) = (A \cap B) \cap C$ Associative law

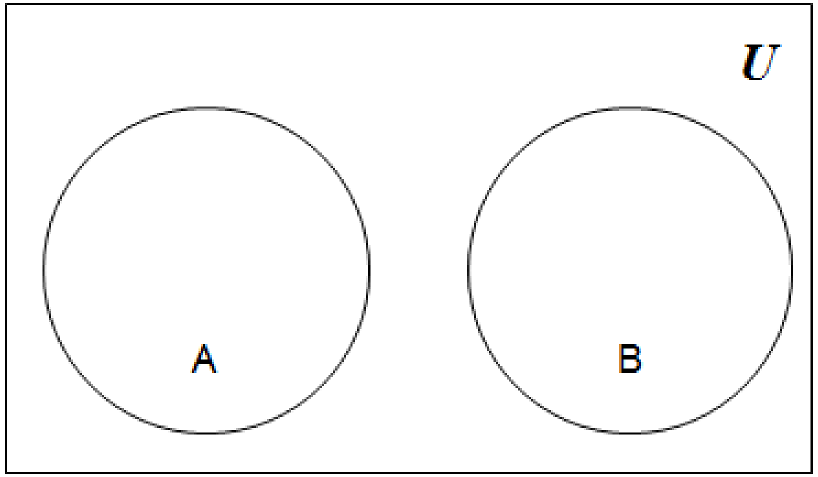
Himpunan Saling Lepas (*Disjoint*)

- Two sets are said to be disjoint (mutually exclusive) if they do not have the same elements.
- Formally, two sets are said to be disjoint if their intersection is an empty set.
- Example: set of even numbers and odd numbers.



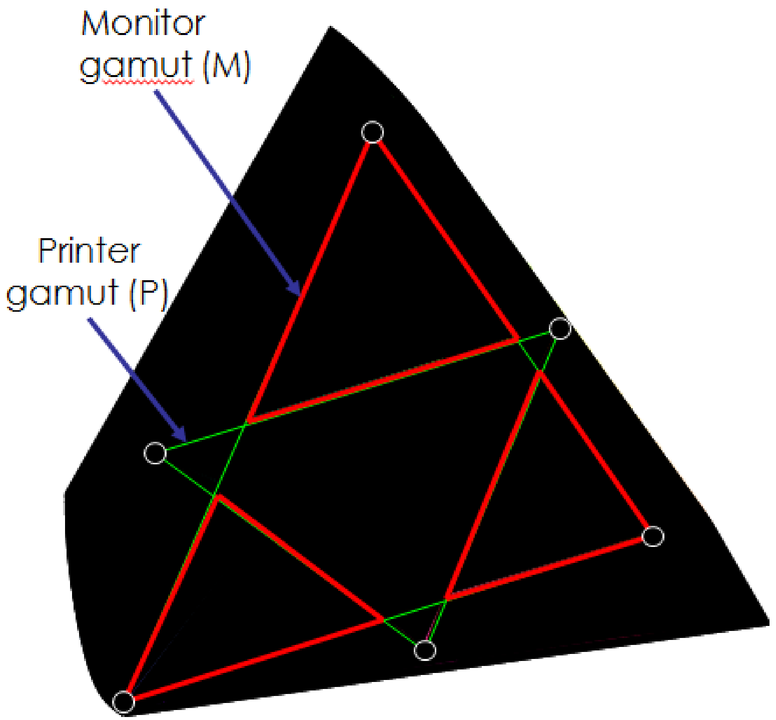
Himpunan Saling Lepas (*Disjoint*)

- Formal definition of *disjoint*: Two sets are said to be disjoint (mutually exclusive) if they do not have the same elements.
- For example:
  - {1, 2, 3} and {3, 4, 5} is not a *disjoint*
  - {New York, Washington} and {3, 4} is a *disjoint*
  - {1, 2} and ∅ is a *disjoint*
    - Their intersection is an empty set.
  - ∅ and ∅ is a *disjoint*!
    - Their intersection is an empty set.



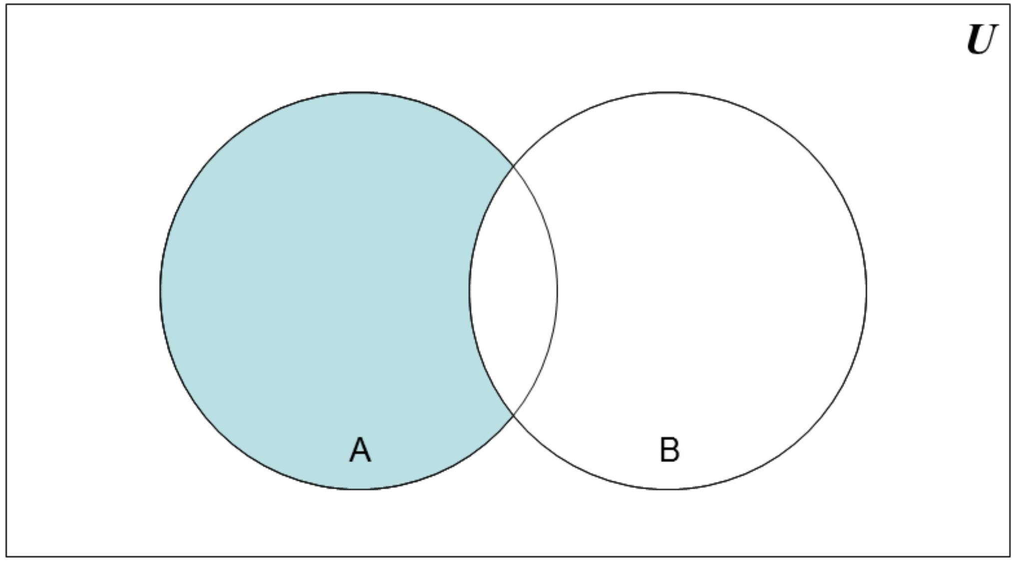
Selisih (*Difference*)

- *A difference of two sets is the elements in one set that are **NOT** in the other.*
- Symbol for difference is a **minus sign.**
- For example:
  - C = M – P
- And vice versa:
  - C = P – M

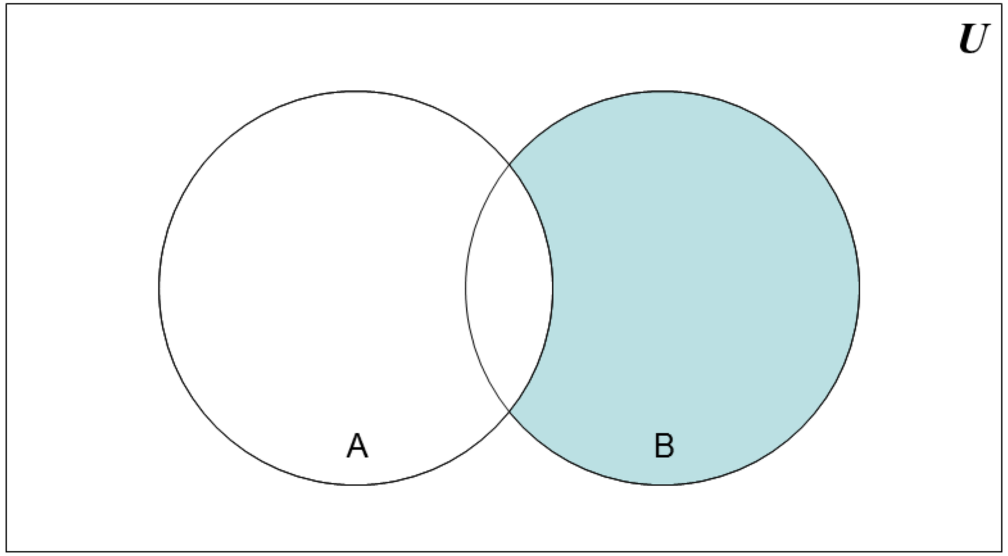


Selisih (*Difference*)

A - B



B - A



Selisih (*Difference*)

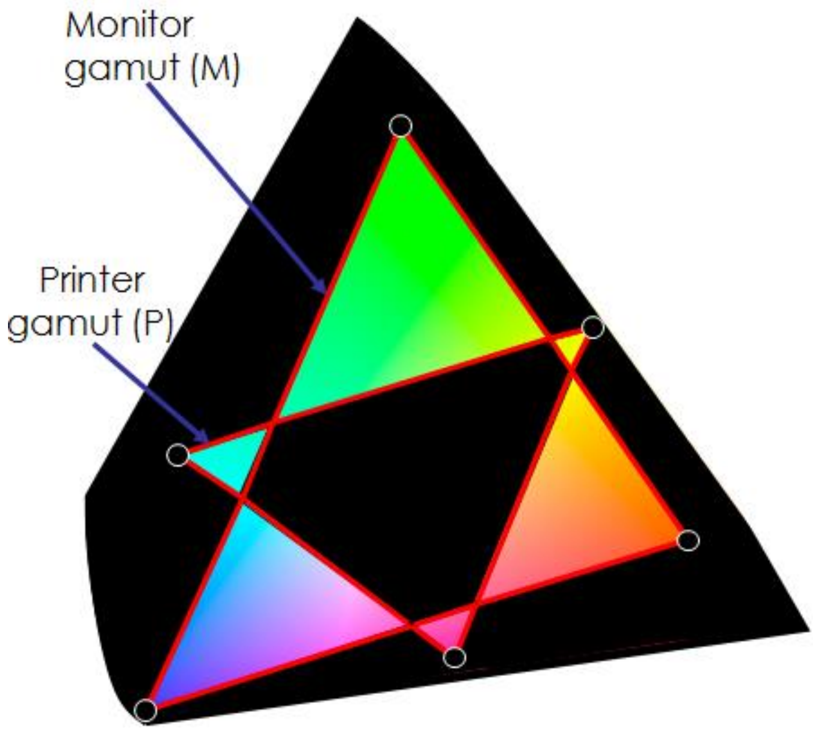
- Formal definition for the difference of two sets:
  $A - B = \{ x \mid x \in A \textbf{ and } x \notin B \}$

  $A - B = A \cap \overline{B}$ ← *Important*!
- For example:
  - {1, 2, 3} - {3, 4, 5} = {1, 2}
  - {New York, Washington} - {3, 4} = {New York, Washington}
  - {1, 2} - $\varnothing$ = {1, 2}
    - The difference between any set S and an empty set is the set S itself.

Symmetric Difference

- *A symmetric difference of the sets contains all the elements in either set but **NOT both.***
- Symbol for *symmetric difference* is $\oplus$
- For example:

  $C = M \oplus P$



Symmetric Difference

- Formal definition for the symmetric difference of two sets:

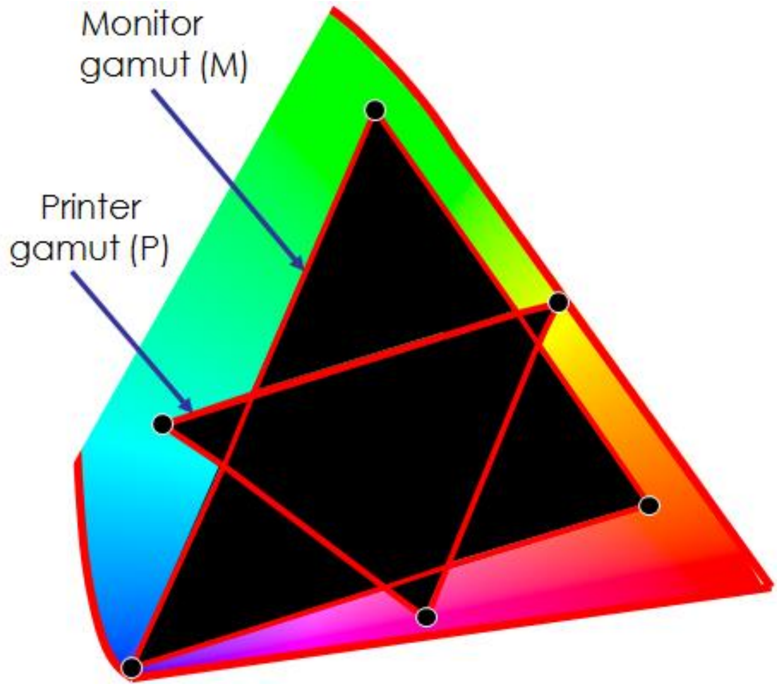
  $A \oplus B = \{ x \mid (x \in A \text{ or } x \in B) \text{ and } x \notin A \cap B\}$

  $A \oplus B = (A \cup B) - (A \cap B)$ ← *Important*!

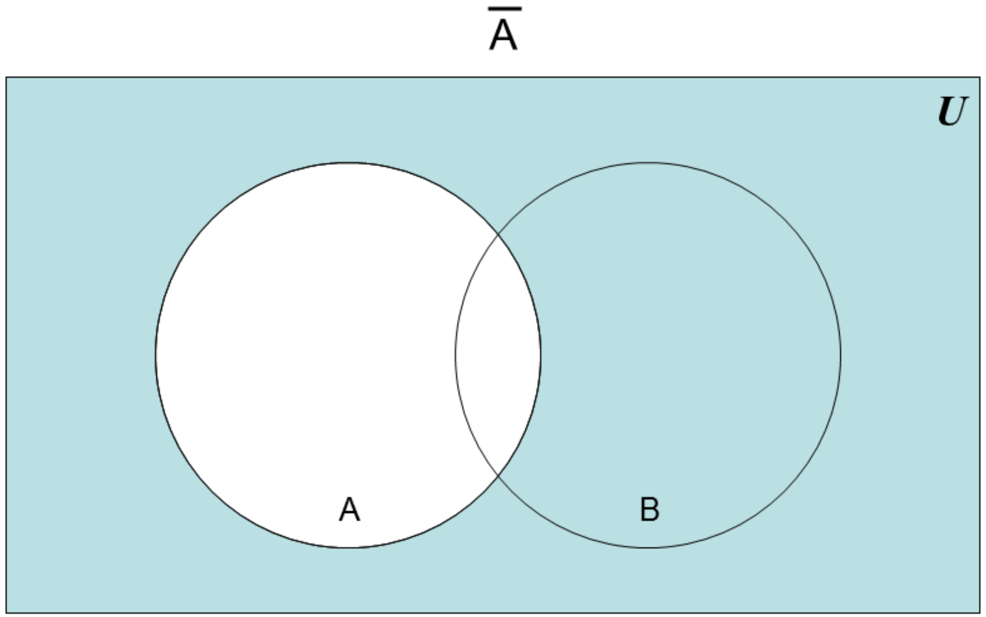
- For example:
  - $\{1, 2, 3\} \oplus \{3, 4, 5\} = \{1, 2, 4, 5\}$
  - {New York, Washington} $\oplus$ {3, 4} = {New York, Washington, 3, 4}
  - $\{1, 2\} \oplus \varnothing = \{1, 2\}$
    - The symmetric difference between any set S and an empty set is the set S itself.

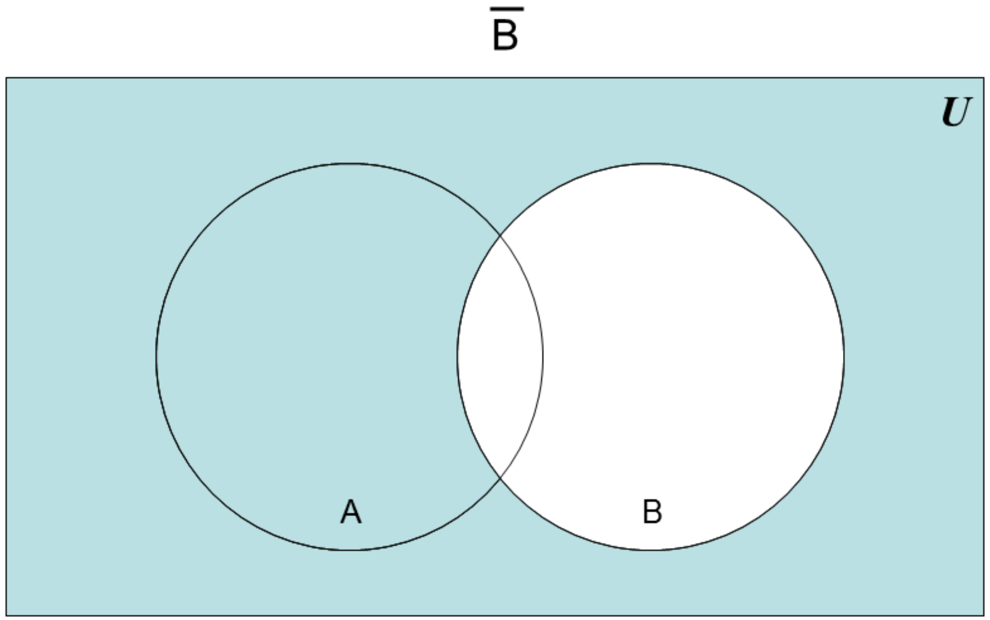
Complement

- *A complement of a set is all the elements that are NOT in the set.*
- The symbol for complement is a line above the set name
- $\overline{P}$ or $\overline{M}$
- Alternate symbol:
  - P^C or M^C



Complement





Complement

- Formal definition for the complement of a set: $\overline{A} = \{ x \mid x \notin A \} = A^c$
  - or $\boldsymbol{U}$ – A, with $\boldsymbol{U}$ is the universal set
- For example (assuming that $\boldsymbol{U} = \mathbf{Z}$)
  - $\overline{\{1, 2, 3\}} = \{ \ldots, -2, -1, 0, 4, 5, 6, \ldots \}$
- Characteristics of complement operations:

$\overline{\overline{A}} = A$	Complementation law
$A \cup \overline{A} = \boldsymbol{U}$	Complement law
$A \cap \overline{A} = \emptyset$	Complement law

Kesamaan Himpunan (*Set Identities*)

- Set identities are the basic laws of how set operations work.
  - Some laws have been explained on the previous slide.
- Same as in common logic, just need to replace:
  - $\cup$ with $\vee$
  - $\cap$ with $\wedge$
  - $\varnothing$ with F
  - $\boldsymbol{U}$ with T

Summary of Set Identities

$A \cup \emptyset = A$ $A \cap U = A$	Identity Law	$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination law
$A \cup A = A$ $A \cap A = A$	Idempotent Law	$(A^c)^c = A$	Complement Law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative Law	$(A \cup B)^c = A^c \cap B^c$ $(A \cap B)^c = A^c \cup B^c$	De Morgan's Law
$A \cup (B \cup C)$ $= (A \cup B) \cup C$ $A \cap (B \cap C)$ $= (A \cap B) \cap C$	Associative Law	$A \cap (B \cup C) =$ $(A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) =$ $(A \cup B) \cap (A \cup C)$	Distributive Law
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption Law	$A \cup A^c = U$ $A \cap A^c = \emptyset$	Complement Law

Proving the Equivalence of Sets

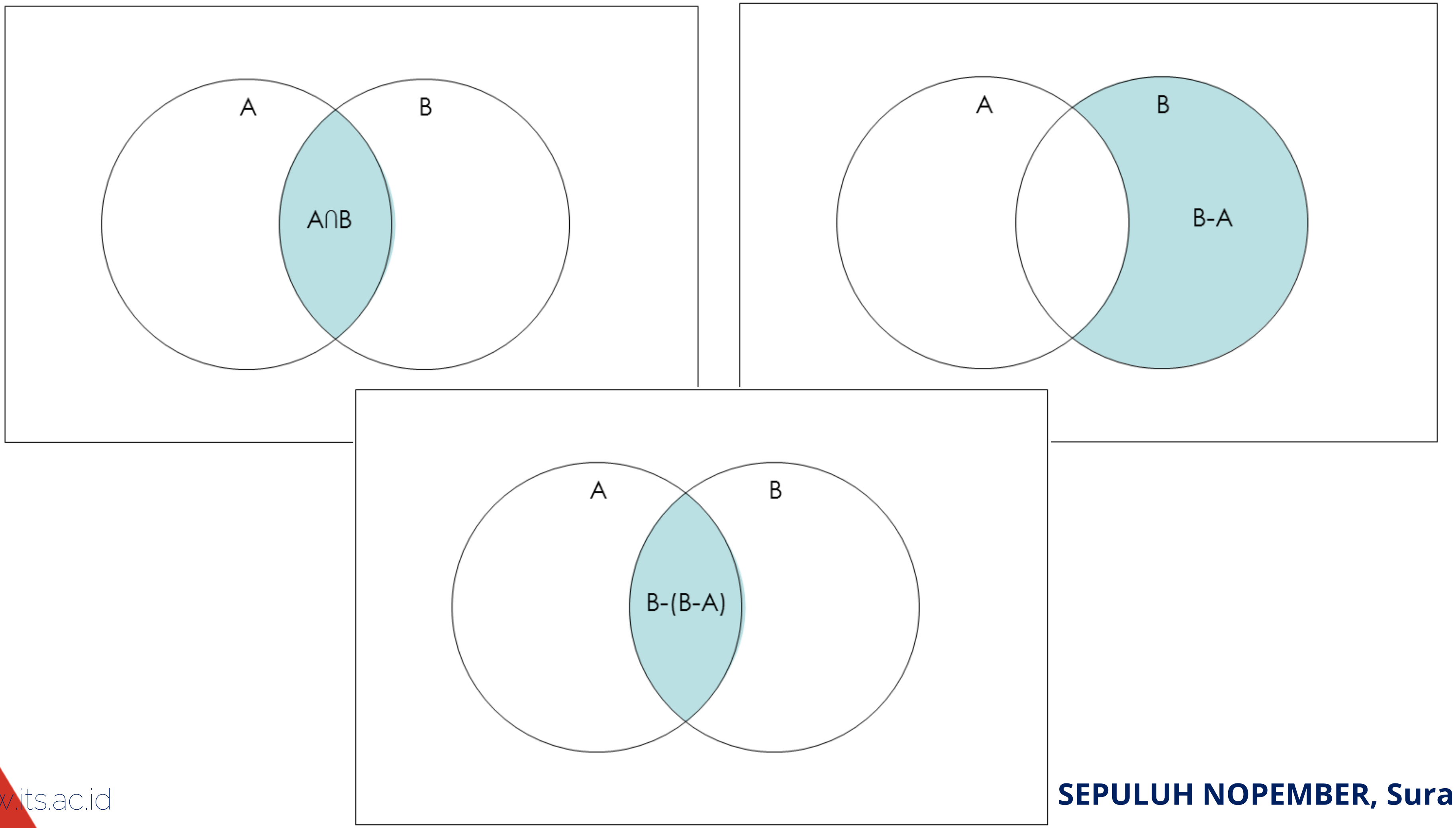
- For example, prove that:

  $$A \cap B = B - (B - A)$$

- There are 4 prove methods:
  - Using membership tables
  - Using the laws of set identities
  - Using the set builder notation and logical equivalences
  - Proving that one set is a subset of another set
    - Such as proving that 2 numbers are equivalent by showing that one number is less than or equal to the other number.

What we are going to prove?

$$A \cap B = B - (B - A)$$



Membership Tables

- Membership tables show all the combinations of sets an element can belong to
  - 1 means the element belongs, 0 means it does not
- Consider the following membership table:

A	B	A U B	A ∩ B	A - B
1	1	1	1	0
1	0	1	0	1
0	1	1	0	0
0	0	0	0	0

- The top row is all elements that belong to both sets A and B
  - Thus, these elements are in the union and intersection, but not the difference

Membership Tables

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- Consider the following membership table:

A	B	A U B	A ∩ B	A - B
1	1	1	1	0
1	0	1	0	1
0	1	1	0	0
0	0	0	0	0

- The second row is all elements that belong to set A but not set B
  - Thus, these elements are in the union and difference, but not the intersection

Membership Tables

- Membership tables show all the combinations of sets an element can belong to
  - 1 means the element belongs, 0 means it does not
- Consider the following membership table:

A	B	A U B	A ∩ B	A - B
1	1	1	1	0
1	0	1	0	1
0	1	1	0	0
0	0	0	0	0

- The third row is all elements that belong to set B but not set A
  - Thus, these elements are in the union, but not the intersection or difference

Membership Tables

- Membership tables show all the combinations of sets an element can belong to
  - 1 means the element belongs, 0 means it does not
- Consider the following membership table:

A	B	A U B	A ∩ B	A - B
1	1	1	1	0
1	0	1	0	1
0	1	1	0	0
0	0	0	0	0

- The bottom row is all elements that belong to neither set A or set B
  - Thus, these elements are neither the union, the intersection, nor difference

Proof by membership tables

- The following membership table shows that A ∩ B = B - (B - A)

A	B	A ∩ B	B-A	B-(B-A)
1	1	1	0	1
1	0	0	0	0
0	1	0	1	0
0	0	0	0	0

- Because the two indicated columns have the same values, the two expressions are identical
- This is similar to Propositional logic!

Using Laws of Set Identities

Prove that A ∩ B = B - (B - A)

B - (B - A)	$= B - (B \cap \overline{A})$	Definition of difference
	$= B \cap \overline{(B \cap \overline{A})}$	Definition of difference
	$= B \cap (\overline{B} \cup \overline{\overline{A}})$	DeMorgan's law
	$= B \cap (\overline{B} \cup A)$	Complementation law
	$= (B \cap \overline{B}) \cup (B \cap A)$	Distributive law
	$= \varnothing \cup (B \cap A)$	Complement law
	$= (B \cap A)$	Identity law
	$= A \cap B$	Commutative law

Proof by set builder notation and logical equivalences

- First, translate both sides of the set identity into set builder notation
- Then modify one side to make it identical to the other
  - Do this using logical equivalences

Proof by set builder notation and logical equivalences

$B-(B-A)$	Original statement
$=\{x \mid x \in B \wedge x \notin (B-A)\}$	Definition of difference
$=\{x \mid x \in B \wedge \neg(x \in (B-A))\}$	Negating "element of"
$=\{x \mid x \in B \wedge \neg(x \in B \wedge x \notin A)\}$	Definition of difference
$=\{x \mid x \in B \wedge (x \notin B \vee x \in A)\}$	DeMorgan's Law
$=\{x \mid (x \in B \wedge x \notin B) \vee (x \in B \wedge x \in A)\}$	Distributive Law
$=\{x \mid (x \in B \wedge \neg(x \in B)) \vee (x \in B \wedge x \in A)\}$	Negating "element of"
$=\{x \mid F \vee (x \in B \wedge x \in A)\}$	Negation Law
$=\{x \mid x \in B \wedge x \in A\}$	Identity Law
$= A \cap B$	Definition of intersection

Proof by showing each set is a subset of the other

Assume that an element is a member of one of the identities
Then show it is a member of the other
Repeat for the other identity

We are trying to show:
$(x \in A \cap B \rightarrow x \in B-(B-A)) \wedge (x \in B-(B-A) \rightarrow x \in A \cap B)$
This is the biconditional:
$x \in A \cap B \leftrightarrow x \in B-(B-A)$
Not good for long proofs

Proof by showing each set is a subset of the other

- Assume that $x \in B-(B-A)$
  - By definition of difference, we know that $x \in B$ and $x \notin B-A$
- Consider $x \notin B-A$
  - If $x \in B-A$, then (by definition of difference) $x \in B$ and $x \notin A$
  - Since $x \notin B-A$, then only one of the inverses has to be true (DeMorgan's law): $x \notin B$ or $x \in A$
- So we have that $x \in B$ and ($x \notin B$ or $x \in A$)
  - It cannot be the case where $x \in B$ and $x \notin B$
  - Thus, $x \in B$ and $x \in A$
  - This is the definition of intersection
- Thus, if $x \in B-(B-A)$ then $x \in A \cap B$

Proof by showing each set is a subset of the other

- Assume that $x \in A \cap B$
  - By definition of intersection, $x \in A$ and $x \in B$
- Thus, we know that $x \notin B-A$
  - B-A includes all the elements in B that are also not in A not include any of the elements of A (by definition of difference)
- Consider B-(B-A)
  - We know that $x \notin B-A$
  - We also know that if $x \in A \cap B$ then $x \in B$ (by definition of intersection)
  - Thus, if $x \in B$ and $x \notin B-A$, we can restate that (using the definition of difference) as $x \in B-(B-A)$
- Thus, if $x \in A \cap B$ then $x \in B-(B-A)$

Example

Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

Solution:

$$
\begin{aligned}
\overline{A \cap B} &= \{x \mid x \notin A \cap B\} && \text{by definition of complement} \\
&= \{x \mid \neg(x \in (A \cap B))\} && \text{by definition of does not belong symbol} \\
&= \{x \mid \neg(x \in A \wedge x \in B)\} && \text{by definition of intersection} \\
&= \{x \mid \neg(x \in A) \vee \neg(x \in B)\} && \text{by the first De Morgan law for logical equivalences} \\
&= \{x \mid x \notin A \vee x \notin B\} && \text{by definition of does not belong symbol} \\
&= \{x \mid x \in \overline{A} \vee x \in \overline{B}\} && \text{by definition of complement} \\
&= \{x \mid x \in \overline{A} \cup \overline{B}\} && \text{by definition of union} \\
&= \overline{A} \cup \overline{B} && \text{by meaning of set builder notation}
\end{aligned}
$$

Exercise

1. If a set has *n* elements, what is the cardinality of its power set?
2. If A ⊕ B = A, what kind of sets A and B are??
3. Using one of the methods for proving the equivalence of sets, show that

$$\overline{A \cup (B \cap C)} = (\bar{C} \cup \bar{B}) \cap \bar{A}$$

Answers

1. 2^n elements
2. B =Ø
3. $$\begin{aligned}\overline{A \cup (B \cap C)} &= \overline{A} \cap \overline{(B \cap C)} && \text{by the first De Morgan law}\\ &= \overline{A} \cap (\overline{B} \cup \overline{C}) && \text{by the second De Morgan law}\\ &= (\overline{B} \cup \overline{C}) \cap \overline{A} && \text{by the commutative law for intersections}\\ &= (\overline{C} \cup \overline{B}) \cap \overline{A} && \text{by the commutative law for unions.}\end{aligned}$$

THANK YOU