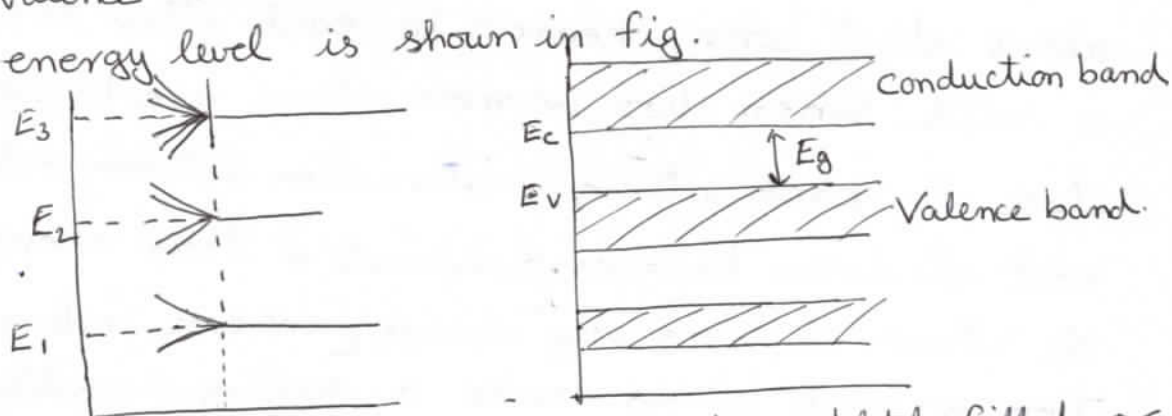


Band Theory of Solids / Energy bands in Solids

Consider Si or Ge crystal contains N atoms. Electrons of each atom will have discrete energies in different orbits. The electron energy will be same if all the atoms are isolated, i.e. separated from each other by a large distance. However, in a crystal, the atoms are close to each other (2 to 3 Å) and therefore the electrons interact with each other and also with the neighbouring atomic cores. The interaction will be more felt by the electrons in the outermost orbit while the inner orbit or core electrons energies may remain unaffected. Therefore, to understand electron energies in Si or Ge, we need to consider the changes in the energies of the electrons in the outermost orbit only. The number of outermost electrons in Si or Ge is 4 (2s and 2p). Hence, the total number of outer electrons in the crystal is $4N$. Suppose these atoms start coming nearer to each other to form a solid. When two or more atoms are brought close to each other, interaction between outermost electrons becomes dominant & their energy of ~~these electrons~~ may change (increase or decrease). Two or more atoms make a system. According to Pauli exclusion principle two electrons of

the same system cannot have the same energy. Thus each energy level splits into sub-energy levels. Smaller the separation between atoms higher will be splitting of energy level. If there are N atoms in an assembly or system, then energy level splits into N sub-energy levels. Energy difference between sub-energy levels is very small ($\sim 10^{-40} \text{ eV}$) and the combination of sub-energy levels formed by superposition of energy levels into a single broad energy level is known as band. The highest filled energy band which includes the electrons shared in covalent bond or electron transferred in ionic bond is called Valence band. The band above the Valence band which contains free electrons (in some situations) is called conduction band. Energy difference between the lowest energy level of conduction band and highest energy level of the valence band is called energy band gap. Splitting of energy level is shown in fig.



It is important to note that completely filled or completely empty band do not have any role in conduction. Only the partially filled bands are responsible for conduction.

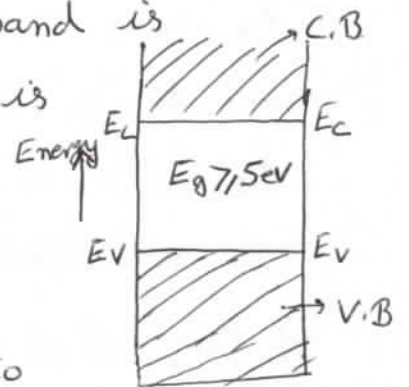
Insulators, Semiconductors and Conductors

- (i) Insulators: Substances which do not allow the current to pass through themselves are Insulators. Their resistivity is very high & is of the order of $10^7 \Omega\text{-m}$ to $10^{18} \Omega\text{-m}$ & their conductivity tends to zero & is not affected by temperature. Examples Mica, Plastic.
- (ii) Conductors: These are the substances which offers negligible resistance in the flow of current. They have sufficient number of free electrons. Resistivity of these materials is very small of the order of $10^{-6} \Omega\text{m}$ to $10^{-8} \Omega\text{-m}$, while conductivity is very high. On the rise of temperature of these materials conductivity decreases because thermal vibrations of ions are increased with the rise of temperature which causes more hinderance in flow of electrons. Thus electrical resistance is increased while conductivity is decreased.
Examples Cu, Ag, Au, Fe etc.
- (iii) Semiconductors: The substances whose conductivity lies between conductors and insulators are semiconductors. Their conductivity is more than insulators and smaller than conductors at room temperature. At absolute zero their conductivity is zero. On the rise of temperature of these materials more & more co-valent bonds are broken, thus large number of electrons available for conduction.

Band Structure

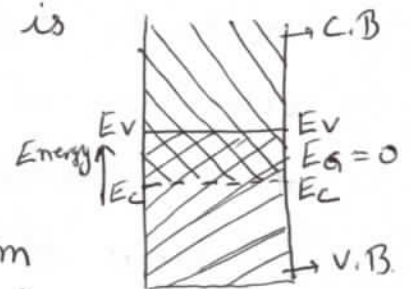
(i) Band structure of Insulators: Band structure of

Insulators is shown in fig. Valence band is completely filled while conduction band is completely vacant. Forbidden energy gap of these materials is fairly large ($\approx 7.5\text{eV}$). Hence electrons from valence band cannot jump easily to the conduction band. As no electrons are available in the conduction band of these materials, hence the conductivity of these materials is zero.



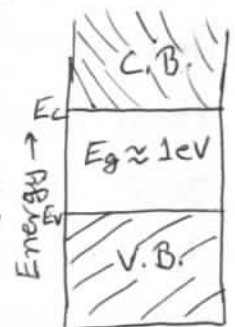
(ii) Band structure of conductors: As shown in fig,

the energy band gap of conductors is zero as there is overlapping of valence & conduction bands. Thus without supplying any energy from external source electrons transfer from valence to conduction band. Electrons are available both in conduction as well as in valence band. Because of presence of sufficient electrons in conduction band conductors behave as good conductors.



(iii) Band Structure of semiconductors:

Forbidden energy gap of semiconductors is very small is of the order of $\approx 1\text{eV}$. At absolute 0K, their valence band is completely filled & conduction band is completely empty. At room temperature some of the



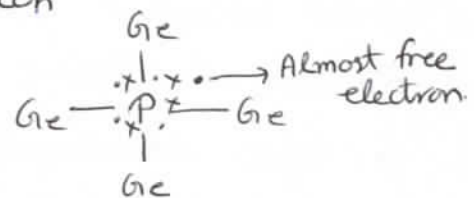
covalent bonds are broken and the free electrons are transferred to the conduction band. Due to these electrons conduction become possible in these materials.

Impurities: Conductivity of pure semiconductors at room temperature is very small and also it is not uniform. So they cannot be used for making electronic devices. Therefore, for increasing the conductivity of pure semiconductors elements of 3rd & 5th group of periodic table are added to them. Atoms of these elements are called impurities. They are of two types:

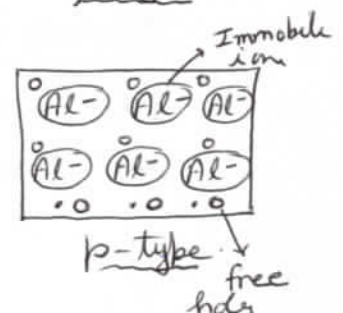
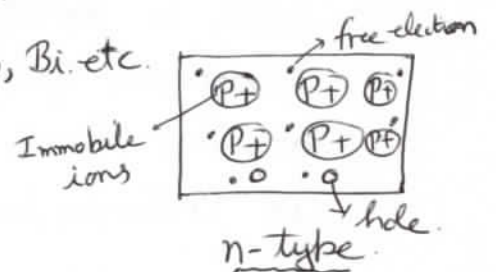
(i) Donor impurities: Pentavalent atoms added to intrinsic semiconductors are known as donor impurities. Donor impurities give free electrons to intrinsic semiconductors, so it becomes rich

in negative ^{free} charge & hence called n-type semiconductors.

Phosphorous atom would become phosphorous +ve ion when it transfers one electron to the semiconductor lattice. Eg. P, Ar, Sb, Bi. etc.



(ii) Acceptor impurity: Trivalent atoms added to the intrinsic semiconductors are known as acceptor impurities. These acceptor impurities provides hole to the semiconductor lattice. Thus pure semiconductor becomes

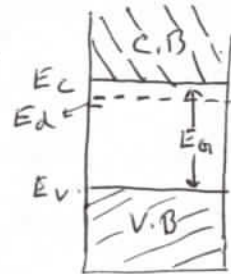


rich in free positive charge and hence named as p-type semiconductor. Example Al, B, In, Ga etc

When p-type impurity eg Al atom accepts an electron from the valence band it turns into Al^- ions. This Al^- ions is immobile ion as it is bonded completely from all four sides & its octate is completed.

Donor level in n-type semiconductor:

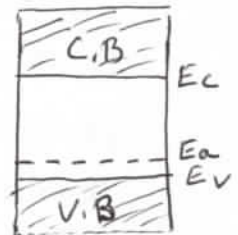
when donor impurity is added to pure semiconductor, donor level is created just below the conduction band. This level is shown by E_d (donor level). With a small rise in temperature, electrons jump from donor level E_d to conduction band E_c .



Acceptor level in p type semiconductor:

when acceptor impurity (i.e., trivalent atoms) are added to pure semiconductor, it converts into p-type semiconductor, and acceptor level E_a is created just above the valence band.

With a small rise in temperature, electrons jumps from valence band to acceptor level E_a and holes are produced in valence band.



Fermi level and Fermi Energy:

Electrons are fermions as their spin is $\frac{1}{2}\hbar$, $[\hbar = \frac{h}{2\pi}]$ so they would follow Fermi-Dirac statistics.

(4)

According to this statistics fermi function $f(E)$ gives the probability of finding the electron in energy state E at temperature T and is given by

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}},$$

where E , E_F , k & T are energy level, Fermi energy, Boltzmann's constant, & absolute temperature resp.

At $T = 0\text{ K}$, when $E > E_F$

$$f(E) = \frac{1}{1 + e^{+\infty}} = \frac{1}{\infty} = 0$$

For conduction band, $E > E_F$, therefore, the probability of filling the conduction band is zero i.e. it is completely unfilled at $T = 0\text{ K}$.

Further at $T = 0\text{ K}$ when $E < E_F$

$$f(E) = \frac{1}{1 + e^{-\infty}} = \frac{1}{1 + \frac{1}{e^{\infty}}} = \frac{1}{1 + 0} = 1.$$

For valance band $E < E_F$, therefore, probability of filling the valance band is 1 i.e. it will be completely filled.

Fermi level is the highest filled energy level in a conductor at 0 K , & Fermi energy is the maximum energy that an electron have in a conductor at absolute zero temperature.

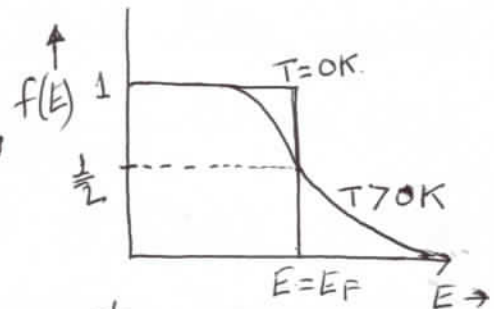
when $E = E_F$, $T \neq 0\text{ K}$ (i.e. $T > 0\text{ K}$), then

$$f(E) = \frac{1}{1 + e^{(E_F - E_F)/kT}} = \frac{1}{1 + e^0} = \frac{1}{1 + 1} = \frac{1}{2},$$

(P.T.O.)

Thus Fermi energy ^{of a conductor} is the energy where the probability of finding the electron is $\frac{1}{2}$ at temp. ($T=0K$).

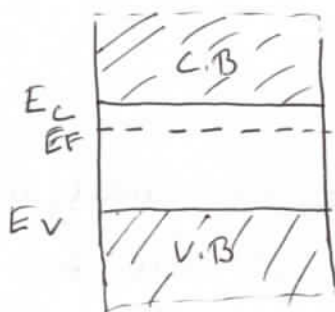
Position of Fermi level in n-type & p-type semiconductors,



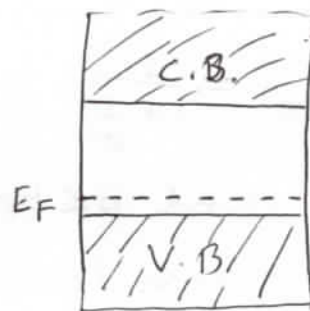
In n-type semiconductors,

concentration of electrons is greater than holes. In these substances Fermi level shifts in upward direction near to conduction edge E_c .

On the otherhand in case of p-type semiconductors concentration of holes is greater than that of electrons & the fermi level shifts near to the valence band.



n-type



p-type

With the rise in temperature ^{in semiconductor} $f(E)$ increases & hence the probability of occupancy of conduction bands increases and that of valence band decreases.

Drift velocity & Mobility: [S.K. Gupta]

When a potential difference is applied across a conductor the charge carriers acquire a drift vel. which is directly proportional to the applied electric field. If E be the applied electric field & v_d the drift velocity then

$v_d \propto E$ or, $v_d = \mu E$, where μ is called mobility of charge carriers. Thus the mobility is defined as average drift vel. per unit electric field,

Conductivity of Semiconductors (Temperature independent)

When a potential diff. of V volts is applied across the ends of a semiconductor block of length l & cross-section area A , then the electrons in the conduction band & holes in the valence band move in mutually opposite directions with their respective drift velocities v_e & v_h & constitute electron & hole currents. If I_e & I_h be the electron & hole currents respectively, then

Total current $I = I_e + I_h$ ———— (1)

If n_e ~~$\times A \times l$~~ be the number of electrons per unit volume or electron density in conduction band & n_h that of holes in valence band, then

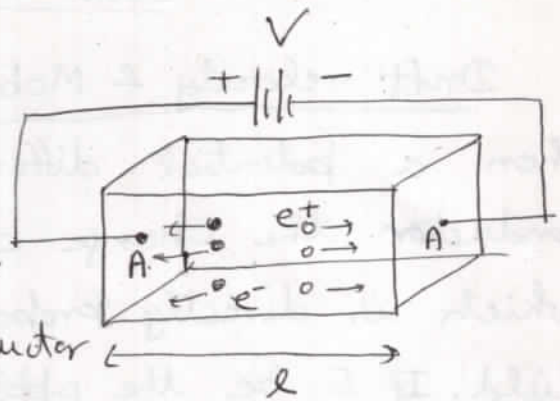
$$I_e = n_e A e v_e \quad \& \quad I_h = n_h A e v_h$$

$$\therefore I = I_e + I_h = n_e A e v_e + n_h A e v_h \\ = e A (n_e v_e + n_h v_h) \text{ ———— (2)}$$

(P. T. O.)

If ρ be the resistivity of the semiconductor material, then resistance offered by the semiconductor to the flow of current is

$$R = \frac{\rho l}{A}, \quad \rho = \text{electrical resistivity.}$$



A = cross-section area.
 l = length of specimen.

Since $V = IR$,

$$= eA(n_e v_e + n_h v_h) \frac{\rho l}{A} \quad [\text{using eqn (2)}]$$

$$V = e \rho l (n_e v_e + n_h v_h) \quad \text{--- (3)}$$

The Electric field set up across the semiconductor block is given by

$$E = \frac{V}{l} = e \rho (n_e v_e + n_h v_h) \quad [\text{using eqn (3)}]$$

$$\text{or, } \frac{E}{\rho} = e(n_e v_e + n_h v_h).$$

$$\text{or } \frac{1}{\rho} = e n_e \left(\frac{v_e}{E} \right) + e n_h \left(\frac{v_h}{E} \right) \quad \text{--- (4)}$$

We know drift velocity per unit electric field is defined as mobility, therefore

$\frac{v_e}{E} = \mu_e$ & $\frac{v_h}{E} = \mu_h$, where μ_e & μ_h are the mobility of electrons & holes respectively.

\therefore From eqn (4) we have

$$\frac{1}{\rho} = e n_e \mu_e + e n_h \mu_h \quad \text{--- (5)}$$

Since electrical conductivity (σ) is the reciprocal of of electrical resistivity ρ , therefore, conductivity of semiconductor is

$$\boxed{\sigma = \frac{1}{\rho} = e(n_e \mu_e + n_h \mu_h)} \quad \text{--- (6)}$$

From eqns. (5) & (6) it is clear that, the resistivity & conductivity of a semiconductor depends on the mobility of charge carriers as well as on their densities or concentrations.

Since n_e & n_h increases with rise of temperature, the conductivity of semiconductor increases with the rise of temperature, while the resistivity decreases with the rise of temperature.

Conductivity of Intrinsic semiconductor

In pure or intrinsic semiconductor n_e is exactly equal to n_h i.e.,

$$n_i = n_e = n_h$$

$$\therefore \text{Conductivity } (\sigma) = e(n_e \mu_e + n_h \mu_h)$$

$$\boxed{\sigma = n_i e (\mu_e + \mu_h)}$$

Conductivity of Extrinsic semiconductor

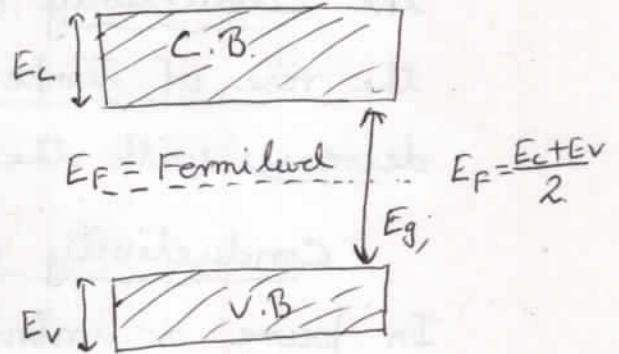
(i) In N type semiconductor $n_e \gg n_h$ $\therefore \sigma \approx n_e e \mu_e$, n_h may be neglected.

(ii) In P type semiconductors $n_h \gg n_e$ $\therefore \sigma \approx n_h e \mu_h$ or $\sigma \approx n_h e \mu_h$

Position of Fermi level in Intrinsic semiconductors

To find the position of Fermi energy level in the forbidden energy gap between the conduction & valence band of an intrinsic semiconductor, following assumptions are made:

- (1) The width of the conduction band & Valence band are small as compared to forbidden energy gap E_g bet. them.
- (2) Since width of both bands are small, so suppose the energy of all levels in conduction band is E_c & that of all energy levels in valence band is E_v .
- (3) At absolute zero temp. no conduction is possible because at 0K valence band is completely filled, while the conduction band is completely empty, i.e. semiconductor behaves like an insulator.



Let at temp. T K, the number of electrons in conduction band is n_c & the no. of electrons in valence band is n_v
 \therefore Total no. of electrons in both bands is,

$$N = n_c + n_v$$

The no. of electrons in conduction band is

$$n_c = N F(E_c) \quad \text{--- (1)}$$

where $F(E_c)$ is the probability of an electron having energy E_c

Acc. to Fermi-Dirac statistics

$$F(E_c) = \frac{1}{1 + e^{(E_c - E_F)/kT}} \quad \text{--- (2)}$$

where E_F is Fermi energy level, k = Boltzmann's const.

\therefore From eqn. (1) & (2) we have.

$$n_c = \frac{N}{1 + e^{(E_c - E_F)/kT}} \quad \text{--- (3)}$$

Similarly

$$n_v = \frac{N}{1 + e^{(E_v - E_F)/kT}} \quad \text{--- (4)}$$

(7)

$$\therefore N = n_c + n_v = \frac{N}{[1 + e^{(E_c - E_F)/kT}]} + \frac{N}{[1 + e^{(E_v - E_F)/kT}]}$$

$$\text{or, } 1 = \frac{1 + e^{(E_v - E_F)/kT} + 1 + e^{(E_c - E_F)/kT}}{[1 + e^{(E_c - E_F)/kT}][1 + e^{(E_v - E_F)/kT}]}$$

$$\text{or, } [1 + e^{(E_c - E_F)/kT}][1 + e^{(E_v - E_F)/kT}] = 2 + e^{(E_v - E_F)/kT} + e^{(E_c - E_F)/kT}$$

$$\text{or, } 1 + e^{(E_c - E_F)/kT} + e^{(E_v - E_F)/kT} + e^{(E_c - E_F + E_v - E_F)/kT} = 2 + e^{(E_v - E_F)/kT} + e^{(E_c - E_F)/kT}$$

$$\text{or, } e^{\frac{(E_c + E_v - 2E_F)}{kT}} = 1$$

Taking log of both sides

$$\left(\frac{E_c + E_v - 2E_F}{kT}\right) \log_e e = \log_e 1 = 0$$

$$\Rightarrow \left(\frac{E_c + E_v - 2E_F}{kT}\right) = 0 \Rightarrow \boxed{E_F = \frac{E_c + E_v}{2}}$$

Thus Fermi-level in an intrinsic semiconductor lies exactly in the middle of forbidden energy gap & also it is independent of temperature.

In n type semiconductor, the Fermi level lies above the middle of the energy gap because there are more ^{free} electrons in the conduction band than holes in the valence band; while in p-type semiconductor, the Fermi level lies below the middle of the forbidden energy gap, as the ^{number} electrons in the conduction band are less than the holes in the valence bands.

With the rise in temperature $f(E)$ increases & hence probability of occupancy of conduction band increases & that of valence band decreases.

(P. T. O.)

Variation of Conductivity with Temperature in intrinsic semiconductor.

There are two types of charge carriers in the intrinsic semiconductors (electrons & holes). Therefore total conductivity due to both is

$$\sigma_i = \sigma_e + \sigma_h = n_e \mu_e e + n_h \mu_h e \quad \text{--- (1)}$$

where σ denotes conductivity & i, e, h respectively denotes intrinsic, holes, & σ_e is conductivity due to free electrons & e is electronic charge on electron.

For intrinsic semiconductors

$$n_i = n_e = n_h \quad \text{--- (2)}$$

$$\therefore \text{From eqn (1), } \sigma_i = n_i e (\mu_e + \mu_h) \quad \text{--- (3)}$$

The mobility of electrons & holes depends on temp. as $\mu \propto T^{-3/2}$ or $\mu = AT^{-3/2}$, where $A = \text{proportionality const.}$

$\therefore \mu_e = \alpha T^{-3/2}$ & $\mu_h = \beta T^{-3/2}$, where α & β are proportionality const.

Putting values of μ_e & μ_h in eqn (3) we have

$$\sigma_i = n_i e (\alpha + \beta) T^{-3/2} \quad \text{or, } \sigma_i = n_i e \gamma T^{-3/2} \quad \text{--- (4)}$$

where $\gamma = (\alpha + \beta)$, a new constant.

The intrinsic concentration of holes or free electrons is of the form

$$n_i = C T^{3/2} e^{-E_g/2kT} \quad \text{--- (5)}$$

$$\text{where } C = 2 \left(\frac{2\pi k}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4}$$

where $C (= 4.83 \times 10^{21})$ is constant & E_g is the energy of the forbidden energy gap, $k = \text{Boltzmann's constant}$, & m_e & m_h are the effective mass of electrons & holes resp.

(8)

From eqns. (4) & (5) we have

$$\sigma_i = C e^{\gamma} e^{-E_g/2kT} \text{ or, } \sigma_i = P e^{-E_g/2kT} \quad (6)$$

where $P = C e^{\gamma}$.

Therefore electrical resistivity for an intrinsic semiconductor is

$$\rho_i = \frac{1}{\sigma_i} = \frac{1}{P} e^{E_g/2kT} \quad (7) \quad \text{or, } \rho_i = \frac{1}{P} e^{E_g/2kT} \quad (7)$$

where $P = \frac{1}{\rho_i} e^{E_g/2kT}$ ~~$P = \frac{1}{\rho_i} e^{E_g/2kT}$~~

Eqns (6) & (7) show that as the temp. increases the conductivity of semiconductor increases exponentially while the resistivity decreases exponentially whereas in metals/conductors conductivity decreases almost linearly with temperature.

If σ_{∞} is the extrapolated conductivity at $T = \infty$, then $\sigma_{\infty} = P$

Therefore eqn (6) may be written as

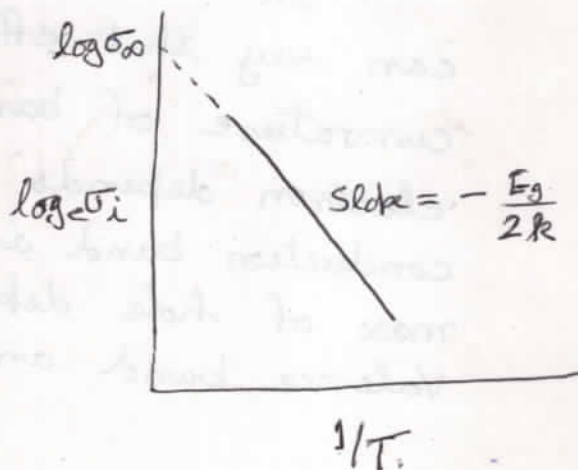
$$\sigma_i = \sigma_{\infty} e^{-E_g/2kT}$$

Taking log of both sides, we get

$$\log_e \sigma_i = \log_e \sigma_{\infty} - \frac{E_g}{2kT}$$

If we plot a graph between $\log_e \sigma_i$ & $\frac{1}{T}$, we get a straight line whose slope is ~~$-\frac{E_g}{2k}$~~ $-\frac{E_g}{2k}$.

This suggests a method for the determination of energy gap, E_g of an intrinsic semiconductor.



(P. T. O.)

Effective mass: When the electrons & holes move in solids then their mass is affected by the force between other atoms and hence their mass is altered. This altered mass of the electrons or holes is called effective mass.

$$E = \frac{1}{2} m v^2 = \frac{p^2}{2m} \quad \text{--- (1)}$$

$$\text{But } k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{p}{\left(\frac{h}{2\pi}\right)} = \frac{p}{\hbar} \quad \left(\because \hbar = \frac{h}{2\pi}\right)$$

$$\therefore p = \hbar k. \quad \text{--- (2)}$$

From eqn (1) & (2) we have

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow m = \frac{\hbar^2 k^2}{2E}$$

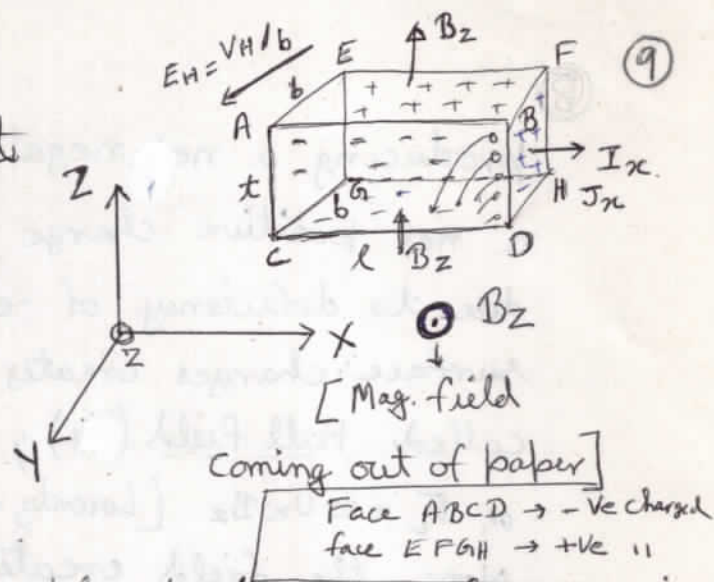
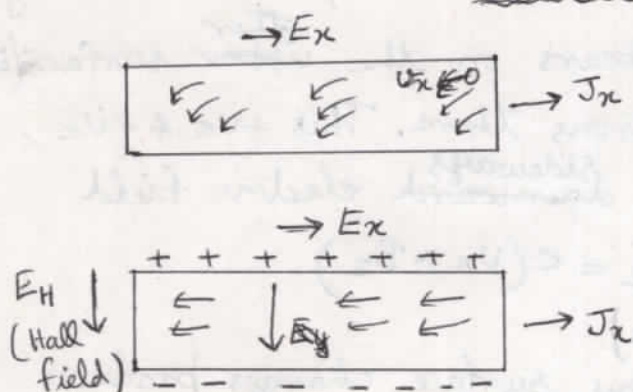
$$\therefore \frac{d^2 E}{dk^2} = \frac{\hbar^2}{m} \quad \text{or, } m = \frac{\hbar^2}{\left(\frac{d^2 E}{dk^2}\right)}$$

This m is known as effective mass & is represented by m^*

$$\therefore m^* = \frac{\hbar^2}{(d^2 E / dk^2)}$$

As $\frac{d^2 E}{dk^2}$ gives the curvature of band. So we can say that effective mass depends upon the curvature of band. The effective mass of electron depends upon the curvature of conduction band and is -ve while effective mass of hole depends upon curvature of Valence band and is +ve.

Hall effect



If a conducting material is placed in a uniform magnetic field & a current is passed along the length of the conductor (as shown in fig), a voltage is found to develop at right angles to both the direction of current flow & that of the mag. field. This voltage is known as Hall voltage, & its value will depend on mag. field strength & current. This effect is known as Hall effect.

Consider a specimen in form of rectangular cross-section carrying a current I_x in the x -direction. If a uniform mag. field B_z is applied along the z axis, ^{then} it is found that an e.m.f. develops along the y -axis i.e. in a direction perpendicular to I_x & B_z . This voltage is called Hall voltage & this effect is Hall effect.

If an electric current flowing in the +ve x direction, means that the conduction electrons are drifting with a velocity v_x in the -ve x -direction. When the mag. field is introduced, the Lorentz force F_L causes the electrons to bend ~~downwards~~ ^{sideways} as shown in fig. As a result, electrons accumulate on the ~~lower~~ ^{one} surface ($ABCD$),

producing a net negative charge there, simultaneously a net positive charge appears on the ^{other} ~~upper~~ surface (E_{RH}) due to deficiency of electrons there. This +ve & -ve surface charges creates a ^{sideways} ~~downward~~ electric field called Hall field (E_H).; $F_L = e(v_x \times B_z)$

or, $F_L = e v_x B_z$ (Lorentz force).

Now, the field created by surface charges produces a force which opposes this Lorentz force. The accumulation process continues until Hall force cancels the Lorentz force.

i.e. $F_H = F_L$ or, $e E_H = e v_x B_z$ or, $E_H = v_x B_z$ — (1)

If n is number of electrons per unit volume, then the current density J_x is given by

$$J_x = -n e v_x \text{ — (2)}$$

$$\therefore \frac{E_H}{J_x} = -\frac{B_z}{n e} \text{ or, } E_H = -\left(\frac{1}{n e}\right) J_x B_z \text{ — (3)}$$

Thus Hall field is proportional to both current & mag. field. The proportionality constant is called

Hall constant (R_H), or Hall co-efficient i.e. Acc. to fig on top of Page 9 is ↓

If t is thickness of sample,

$$R_H = -\frac{1}{n e} = \frac{E_H}{J_x B_z}$$

(H) then Hall voltage V_H ,

$$V_H = E_H b; J_x = \frac{I_x}{b t}, b = \text{width of sample}$$

The current density, $J_x = n e v_x$ $\therefore R_H = \frac{E_H}{J_x B_z} = \frac{V_H t}{I_x B_z}$

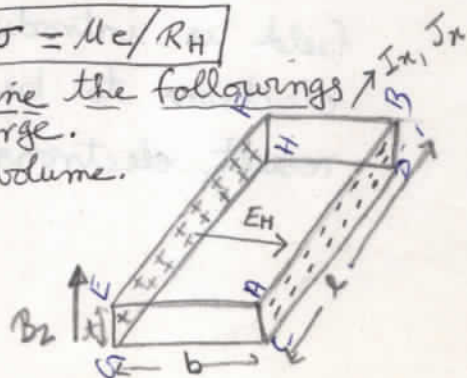
Electrical conductivity (σ) = $\frac{J_x}{E_x} = -n e \left(\frac{v_x}{E_x}\right) = n e \mu_c$

where $\mu_c = \frac{v_x}{E_x}$ = drift vel. per unit electric field, is known as mobility of charge carriers.

or, $\mu_c = \left(\frac{1}{n e}\right) \sigma$ or, $\mu_c = R_H \sigma R_H$ or, $\sigma = \mu_c / R_H$

By Hall voltage ^{or by R_H} we can determine the followings

- (i) The sign of current carrying charge.
- (ii) The no. of charge carriers per unit volume.
- (iii) The mobility of charge carriers.



$\sigma = n e \mu_c$
 $\sigma = n e \left(\frac{1}{R_H}\right)$
 $\therefore \mu_c = \sigma R_H$

Superconductivity

①

The phenomena of superconductivity was discovered by Kammerlingh Onnes in the year 1911. Onnes found that electrical resistance of some metals, alloys and compounds drops suddenly to zero when the specimen is cooled below a certain temperature. This phenomenon is called superconductivity and the cooled specimen is called superconductor. In superconducting state the resistivity of material is zero. The temperature T_c at which a material undergoes a transition from a state of normal conductivity to a superconducting state is called critical temperature.

§ For low T_c superconductors it varies from 0.01K to 23.3 K. Normally good conductors like Cu, Ag, Au, and compounds like Li, Na, K etc do not show

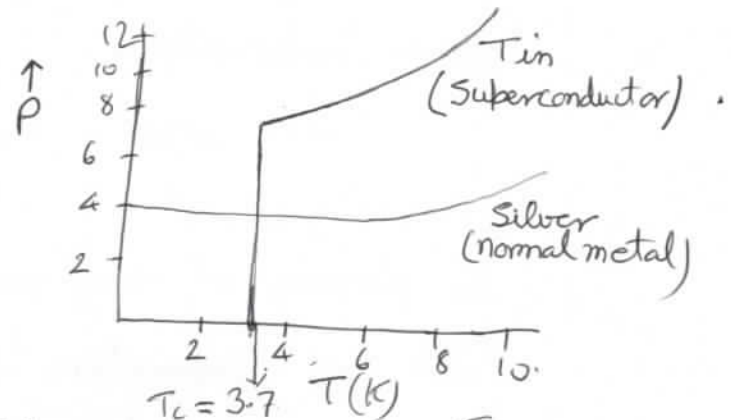


Fig. 1

superconductivity even at temperature 0 K. These materials are known as normal metals. ^{Resistivity of} Tin (Sn) [$T_c = 3.7$ K] and normal metal Silver (Ag) is shown in Fig. 1. Even at 0 K, silver offers some resistivity (or resistance) called residual resistance.

Effect of magnetic field on superconductivity

The superconducting state of a material mainly depends

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on certain range of temperature and magnetic field.

The superconducting state disappears if the temperature of the specimen is raised above its critical temperature T_c or if a strong magnetic field is applied. The maximum value of magnetic field at a given temperature ($T < T_c$) at which superconductivity just disappears is called the critical magnetic field H_c .

The relation between H_c , T and T_c is given by

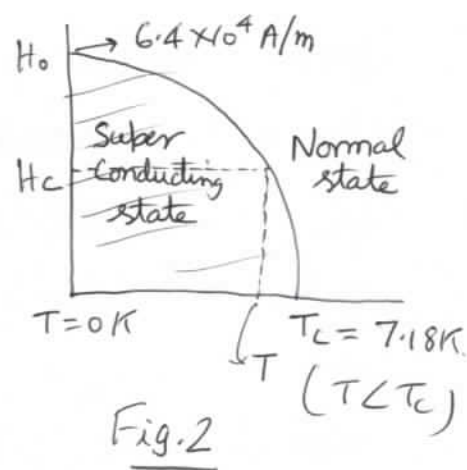
$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right], \text{ --- (1)}$$

where H_c is critical magnetic field at temperature T and H_0 is critical magnetic field at absolute zero temperature.

For Lead, $H_0 = 6.4 \times 10^4 \text{ A m}^{-1}$ and $T_c = 7.18 \text{ K}$

From fig.2, it is clear that in absence of any magnetic field, if temperature is less than T_c , the specimen is in superconducting state. When field strength becomes H_c , superconductivity disappears. At temperature

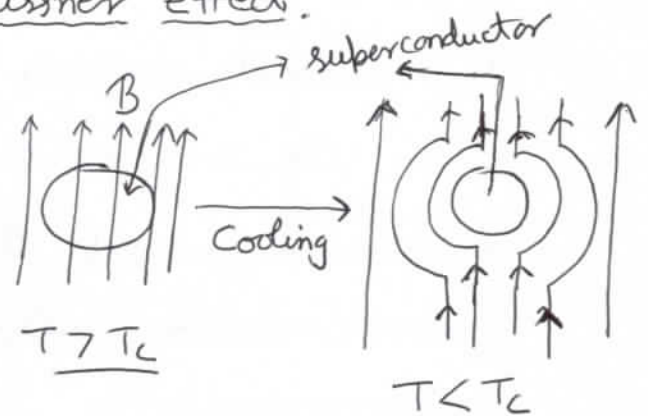
$T = T_c$, $H_c = 0$, i.e. when the specimen is at its critical temperature then magnetic field is not allowed and when temperature $T < T_c$, H_c has finite value given by eqn.(1).



Meissner effect [Flux expulsion or flux exclusion] (2)

Meissner in 1933, observed that if a superconductor is cooled in a magnetic field, below critical temperature corresponding to that field, then the lines of induction are expelled from the material. This effect is called Meissner effect.

"The expulsion of magnetic lines of force from a superconducting material when it is cooled below the transition temperature in a magnetic field is called Meissner effect."



$\Rightarrow \vec{B}$ (magnetic flux density) is always zero in superconducting state.

If I is intensity of magnetisation in the magnetic field H then

$$B = \mu_0 (H + I) = 0 \text{ or, } H = -I,$$

Therefore, magnetic susceptibility $X_m = \frac{I}{H} = -1$.

For normal diamagnets $|X_m| \ll 1$. And for ideal diamagnet $X_m = -1$. Therefore superconductors are ideal diamagnet.

Critical currents (I_c):

The magnetic field which destroys the superconductivity is not necessarily be an external applied field but it may be the magnetic field due to the current flowing in the superconducting wire. We know that a current carrying wire creates a magnetic field around it and if ^{mag.} field due to this current exceeds critical magnetic field H_c then its superconductivity disappears. "The maximum current that can be passed ^{through} a superconductor without destroying its superconductivity is called critical current I_c ."

Consider a superconducting wire of radius r and carrying current I , and if magnetic field produced just outside the wire be H , then according to Ampere's law

$$\oint H dl = I \quad \text{or} \quad H(2\pi r) = I$$

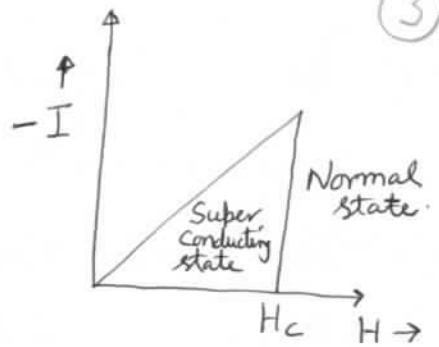
If $I = I_c$, then $H = H_c$ then

$$\boxed{I_c = 2\pi r H_c} \quad \text{i.e. If } I = I_c, \text{ superconductivity will be just destroyed.}$$

Type I and Type II superconductors

Type I or Soft superconductor: The superconductors which strictly follow the Meissner effect are called type I or soft superconductors. i.e. these superconductors expel

totally magnetic flux B sharply at or below critical magnetic field H_c . Examples tin, Mercury, lead etc.



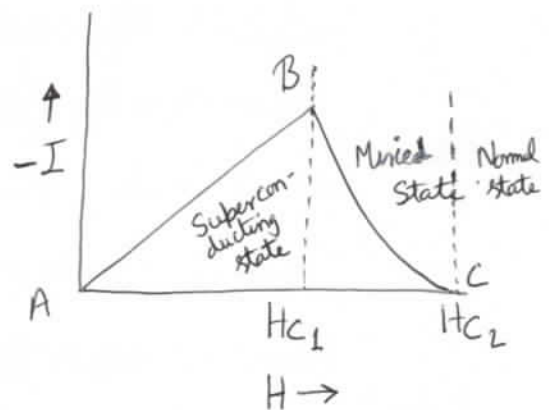
Type I superconductors

Characteristics of Type I superconductors:

- (i) Type I superconductors strictly follow Meissner effect.
- (ii) The value of critical magnetic field H_c of this superconductors are ^{comparatively} ~~comparatively~~ very low and is of the order of 0.1 tesla or 1000 gauss. The low value of H_c makes these materials unsuitable for ^{the} use in high field superconducting magnets.

Type II superconductors or Hard superconductors

Type II superconductors do not strictly follow Meissner effect i.e. these superconductors loses magnetisation gradually rather than suddenly. Type II superconductors have two critical magnetic fields called lower critical field H_{c1} and upper critical field H_{c2} .



The magnetic field strength upto lower critical magnetic field H_{c1} , the superconductor expels the magnetic field from its body completely and behaves as a perfect diamagnet. ^{or Type I superconductors.} It is represented in Figure by curve AB.

As the magnetic field increases from H_{c1} , the magnetic field lines begin to penetrate the material. The penetration increases until H_{c2} is reached. H_{c2} is called the upper critical magnetic field. At or above H_{c2} the superconductivity state is vanished completely i.e. external magnetic field has completely penetrated into the superconductor and destroys the superconductivity. That is, in region upto H_{c1} the specimen behaves as type I superconductor, in region from H_{c1} to H_{c2} the specimen behaves as mixture of normal and superconducting state. And after H_{c2} , the material turns to normal state. So between H_{c1} and H_{c2} Meissner effect is incomplete ^{and} the specimen is electrically superconductor ($\rho=0$) but not magnetically ($B \neq 0$). This region or state is known as mixed state or vortex state. Examples of Type II superconductors are Lead-indium alloys, Nb-Zr, Nb-Tc, Va-Ga etc.

Characteristics of Type II superconductors

- (i) They do not strictly follow Meissner effect
- (ii) It has two critical magnetic fields, lower critical magnetic field H_{c1} and upper critical magnetic field H_{c2} .

(4)

(iii) Type II superconductors have ^{very} high upper critical magnetic field H_{c2} upto 30 tesla. So they can be used in high magnetic field superconducting magnets.

Perfect Superconductors

- The perfect superconducting states are defined by zero resistivity ($\rho=0$) and perfect dia-magnetism below critical temperature, i.e.

$$\vec{E}=0 \text{ and } \vec{B}=0.$$

$$\left[\rho=0, J=\sigma E \text{ or } J=\frac{1}{\rho} E \right. \\ \left. \text{If } \rho=0 \text{ then } E=0 \right]$$

Applications of Superconductivity

- (1) Power Transmission: Electrical power transmission through any conductor is always accompanied by energy loss I^2R . If superconductor cables are used, the loss will be least and power transmission can be done at a lower voltage level.
- (2) Superconducting magnets: Solenoids made of superconducting wires can generate very strong magnetic field without consuming large amount of power. The cost of such magnets is quite less. High magnetic fields are required in many areas of research and in diagnostic equipments in medicine.
- (3) Magnetic levitation: In superconductor, the magnetisation

is in direction opposite to that of external magnetic field. This is known as diamagnetism. Thus when a superconductor is brought near a permanent magnet, there is a strong repulsive force between them. This force causes the lighter one to float over the other. This is called magnetic levitation. This concept has been used in the development of Maglev vehicle/train which can attain speeds of upto 500 Km/hour as there is no friction between the rails and the wheels.

(4) Superconducting Quantum Interference device (SQUID)

SQUID are superconducting rings that act as storage device for magnetic flux. They are capable to detect very small magnetic fields of the order of 10^{-14} Tesla and hence are used as diagnostic tools in detecting brain signals.

(5) Cryotron: Cryotron works as a switch. It works on the principle that superconductivity is destroyed when the magnetic field where it is placed exceeds the critical magnetic field.