Series Solution

Homogeneous Second order linear diff. Eq^n with variable coefficient →

$$P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)\,y = 0 \quad \text{---(i)}$$

here $P_0(x)$, $P_1(x)$, $P_2(x)$ are polynomials in power of x.

or we can write

$$\frac{d^2y}{dx^2} + \frac{P_1(x)}{P_0(x)}\frac{dy}{dx} + \frac{P_2(x)}{P_0(x)}\,y = 0$$

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)\,y = 0$$

this eq^n is called Normal form / canonical form / standard form of H.L.D.E. ~~with~~

eg. $$(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$$\frac{d^2y}{dx^2} + \frac{x}{1+x^2}\frac{dy}{dx} + \frac{1}{1+x^2}\,y = 0$$

Here $P(x) = x/1+x^2$

$Q(x) = 1/1+x^2$

REDMI NOTE 8
AI QUAD CAMERA

Linear Partial Differential Equations with

Classification of point $x=a$ →

Point $x=a$

- **Ordinary Point (OP)**
  If at $x=a$
  $P_0(a) \neq 0$

- **Singular Pt. (SP)**
  If at $x=a$
  $P_0(a) = 0$

  - **Regular Singular pt. (RSP)**
    If both
    $$\lim_{x \to a} (x-a) \frac{P_1(x)}{P_0(x)} = \text{finite}$$
    $$\lim_{x \to a} (x-a)^2 \frac{P_1(x)}{P_0(x)} = \text{finite}$$

  - **Irregular singular pt. ISP**
    otherwise

eg. Find the ordinary pt., singular pt., Regular & irregular singular pt. of the diff. Eq^n.

$$x^3(x-1) \frac{d^2y}{dx^2} + (x-1)\frac{dy}{dx} + 4xy = 0$$

(i) For ordinary pt. → $P_0(a) \neq 0$

$a^3(a-1) \neq 0$ i.e. $a \neq 0, a \neq 1$

so every value of x is an ordinary pt. except $x=0$, $x=1$

(ii) For singular pt. - $P_0(a) = 0$, $a^3(a-1) = 0$

singular pts $x=0$, $x=1$

(iii) at $x=0$ point $\lim_{x \to a} (x-a) \frac{P_1(x)}{P_0(x)} = \lim_{x \to 0} (x-0) \frac{(x-1)}{x^3(x-1)} = \frac{1}{x^2} = \infty$

at $x=0$ irregular S.P.

at $a=1$ pt. → R.S.P.

REDMI NOTE 8
AI QUAD CAMERA