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Engineering Mathematics

- **Fitting of a straight line:-**

Let the equation of straight line of best fit be:-

$$y = a + bx \quad \text{—} \; (1)$$

Normal equations are

$$\Sigma y = na + b\Sigma x \; - \; (2)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \; \text{—} \; (3)$$

Solving equation (2) and (3) we find the values ~~are~~ a & b are:-

Putting the values of a and b in eq (1) which is required equation of straight line.

In case of change of origin:-

(1) If n is odd then

$$u = \frac{x - (\text{Middle term})}{\text{Interval } (h)}$$

(2) If n is even then

$$u = \frac{x - (\text{Mean of two middle term})}{\frac{1}{2} \times h \,(\text{Interval})}$$

By the method of least squares find the straight line that best fits the following data

x	1	2	3	4	5
y	14	27	40	55	68

Soln: let the straight line of best fit be:-

$$y = a + bx \quad \text{—— (1)}$$

Normal equations are

$$\Sigma y = na + b\Sigma x \quad \text{—— (2)}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \text{—— (3)}$$

x	y	xy	x^2
1	14	14	1
2	27	54	4
3	40	120	9
4	55	220	16
5	68	340	25
$\Sigma x = 15$	$\Sigma y = 204$	$\Sigma xy = 748$	$\Sigma x^2 = 55$

Putting these values in eq (2) and (3)

$$204 = 5a + 15b \quad \text{—— (4)}$$

$$748 = 15a + 55b \quad \text{— 5}$$

~~Multi & (4) 4 by~~ 3

$$a = 0, \; b = 13.6$$

Putting the values of and b in equation (1)

$$y = 0 + 13.6x$$

$$y = 13.6x$$

which is required equation of straight line

Q2: Show that the line of fit, to following data is given by :-

$$y = 0.7x + 11.285$$

x	0	5	10	15	20	25
y	12	15	17	22	24	30

→ interval of 5

Soln: $n = 6$ (even)

$h = 5$ (interval)

$$u = \frac{x - (\text{Mean of two middle terms})}{1/2 \times h}$$

$$u = \frac{x - [(10+15)/2]}{1/2 \times 5} = \frac{x - 12.5}{2.5}$$

$$v = (y - 20)$$ (20 ⇒ is the term taken after the two middle terms)

Let the equation of straight line be

$$v = a + bu \quad (1)$$

Normal equations are:-

$$\Sigma v = na + b\Sigma u \quad (2)$$
$$\Sigma uv = a\Sigma u + b\Sigma v^2 \quad (3)$$

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x	y	u	v	uv	u^2
0	12	-5	-8	40	25
5	15	-3	-5	15	9
10	17	-1	-3	3	1
15	22	1	2	2	1
20	24	3	4	12	9
25	30	5	10	50	25
Σx		Σu	Σv	Σuv	Σu^2
75		0	0	122	70

Putting these values in eq ② & ③, we get :-

$a = 0, \quad b = 1.743$

Putting the values of ~~and~~ a and b in eq ①, we get

$$v = 0 + 1.743u$$

$$v = 1.743u$$

$$y - 20 = 1.743\left(\frac{x - 12.5}{2.5}\right)$$

$$\boxed{y = 0.7x + 11.285}$$

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H.W.

Q.1 Fit a straight line to the following data

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

Ans. $y = 0.72 + 1.33x$

Q.2

x	71	68	73	69	67	65	66	67
y	69	72	70	70	68	67	68	64

Ans. $\boxed{y = 39.54 + 0.4242x}$

Co-relation :-

Karl Pearsons coefficient of correlation or Product Moment Correlation Coefficient.

Correlation coefficients between two variables x is denoted by $r(x,y)$ or r_{xy} is a numerical measure of linear relationship between them is defined as

$$r_{xy}$$

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n\,\sigma_x\,\sigma_y}$$

where $= \bar{x} = \frac{\Sigma x}{n}$, $\bar{y} = \frac{\Sigma y}{n}$

σ_x = Standard deviation of x

σ_y = Standard deviation of y

$n \rightarrow$ Number of pairs of values of x & y

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Alternate form of $r(x, y)$:-

$$r(x,y) = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

Let us define two variables u and v, as

$$u = \frac{(x-a)}{h}, \quad v = \frac{y-b}{k}$$

where a, b, h, k are constraints, then

$$r(u,v) = \frac{n\sum uv - (\sum u)(\sum v)}{\sqrt{n\sum u^2 - (\sum u)^2}\sqrt{n\sum v^2 - (\sum v)^2}}$$

Q. Find the coefficient of correlation between the values of x and y

x	1	3	5	7	8	10
y	8	12	15	17	18	20

x	y	x^2	y^2	xy
1	8	1	64	8
3	12	9	144	36
5	15	25	225	75
7	17	49	289	119
8	18	64	324	
10	20	100	400	

Karl Pearson's coefficient of correlation is given by:-

$$r(x,y) = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

$$r(x,y) = \frac{6\times 582 - 34\times 90}{\sqrt{6\times 248 - (34)^2}\sqrt{6\times 1446 - (90)^2}}$$

$$r(x,y) = 0.9879$$

Find the coefficient of correlation for the following data.

x	10	14	18	22	26	30
y	8	12	24	6	30	36

$$U = \frac{x-a}{h} = \frac{x-22}{4}$$

$$V = \frac{y-b}{k} = \frac{y-24}{6}$$

Q. Fit a straight line to the following data regarding x as the independent variable.

x	1	2	3	4	5	6
y	1200	900	600	200	110	50

Let the equation of straight line of best fit be:-

$$y = a + bx \quad \text{—(1)}$$

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Normal Equations :- $\Sigma y = na + b\Sigma x$ — (2)

$\Sigma xy = a\Sigma x + b\Sigma x^2$ — (3)

$a = 1361.9$, $b = -243.42$

Putting values of a & b in $y = a + bx$

$$y = 1361.97 - 243.42x$$

Q. Fit a straight line to the following data

x	71	68	73	69	67	65	66	67
y	69	72	70	70	68	67	68	64

$a = 35.94$ $b = 0.42$

Find the coefficient of correlation for the ~~table~~ following table :-

x	10	14	18	22	26	30
y	18	12	24	6	30	36

Rearranging :-

6 | 12 | 18 | 24 | 30 | 36

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x	y	$u = \left(\frac{x-22}{4}\right)$	$v = \left(\frac{y-24}{6}\right)$	u^2	v^2	uv
10	18	-3	-1	9	1	3
14	12	-2	-2	4	4	4
18	24	-1	0	1	0	0
22	6	0	-3	0	9	0
26	30	1	1	1	1	1
30	36	2	2	4	4	4
		$\Sigma u = -3$	$\Sigma v = -3$	$\Sigma u^2 = 19$	$\Sigma v^2 = 19$	$\Sigma uv = 12$

Correlation coefficient is given by :-

$$r_{uv} = \frac{n\Sigma uv - (\Sigma u)(\Sigma v)}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\ \sqrt{n\Sigma v^2 - (\Sigma v)^2}}$$

$$r_{uv} = 0.6$$

$$r_{xy} = r_{uv} = 0.6$$

Rank Correlation

$$r = 1 - \left[\frac{6\,\Sigma D_i^2}{n(n^2-1)}\right]$$

Case I :- when ranks are given

Case II :- when ranks are not given

Case III :- when ranks are equal

Complete the rank correlation coefficient for following data.

rson:	A	B	C	D	E	F	G	H	I	J
aths	9	10	6	5	7	2	4	8	1	3
ysics	1	2	3	4	5	6	7	8	9	10

Soln:

Person	Rank in Math (R_1)	Rank in Phy (R_2)	$D = R_1 - R_2$	D^2
A	9	1	8	64
B	10	2	8	64
C	6	3	3	9
D	5	4	1	1
E	7	5	2	4
F	2	6	-4	16
G	4	7	-3	9
H	8	8	0	0
I	1	9	-8	64
J	3	10	-7	49

$\Sigma D^2 = 280$

Rank Correlation coefficient

$$r = 1 - \frac{6\Sigma D^2}{n(n^2-1)} = 1 - \frac{6 \times 280}{10(10^2-1)}$$

$$= -0.696$$

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Case II When ranks are not given:-

Q. The marks secured by recruits in the ~~students~~ selection test (X) and in the proficiency test (Y) are given below

Serial No.	1	2	3	4	5	6	7	8	9
X	10	15	12	17	13	16	24	14	22
Y	30	42	45	46	33	34	40	35	39

Calculate rank correlation coefficient

X	Rank in (x) (R_1)	Y	Rank in (Y) (R_2)	$D^2 = (R_1 - R_2)$
10	9	30	9	0
15	5	42	3	4
12	8	45	2	36
17	3	46	1	4
13	7	33	8	1
16	4	34	7	9
24	1	40	4	9
14	6	35	6	0
22	2	39	5	9

$\Sigma D^2 = 72$

$$r = 1 - \left[\frac{6 \Sigma D^2}{n(n^2-1)}\right]$$

$$1 - \left[\frac{6 \times 72}{}\right] = 0.4$$

Case III When ranks are equal or Tied ranks :-

$$r = 1 - \frac{6\left[\Sigma D^2 + \frac{1}{12}m(m^2-1) + \frac{1}{12}m(m^2-1)\right]}{n(n^2-1)}$$

where, $m \rightarrow$ repeated ranks
$D \rightarrow$ Difference between two Ranks
$n \rightarrow$ no. of observations

Q. Obtain Rank correlation coefficient for the following data

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

$\frac{2+3}{2} = 2.5$

x	Rank(x) (R_1)	y	Rank (y) R_2	$D = (R_1 - R_2)$	D^2
68	4	62	5	-1	1
64	6	58	7	-1	1
75	2.5	68	3.5	-1	1
50	9	45	10	-1	1
64	6	81	1	5	25
80	1	60	6	-5	25
75	2.5	68	3.5	-1	1
40	10	48	9	1	1
55	8	50	8	0	0
64	6	70	2	4	16

$\Sigma D^2 = 72$

$m_1 = 2, \quad m_2 = 3, \quad m_3 = 2$

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$$r = 1 - \frac{6\left[\Sigma D^2 + \frac{1}{12} m_1 (m_1^2 - 1)) + \frac{1}{12} m_2 (m_2^2 - 1) + \frac{1}{12} m_3 (m_3^2 - 1)\right]}{n(n^2-1)}$$

$$= 1 - \frac{6\left[\frac{1}{12} 2(2^2-1) + \frac{1}{12} 3(3^2-1) + \frac{1}{12} 2, (2^2-1)\right]}{10-(10^2-1)}$$

$r = 0.545$

Lines of Regression:-

For 10 observations of price (x) and supply (y) the following data were observed

$\Sigma x = 130$, $\Sigma y = 220$, $\Sigma x^2 = 2288$, $\Sigma y^2 = 5506$ and $\Sigma xy = 3467$

Obtain two lines of regression and estimate the supply when price is 16 units.

Regression line of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

Regression line of x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

Q. In a partially destroyed laboratory record of an analysis of a correlation data, the following results only are legible.

Variance of $x = 9$ ~~mean~~ $\longrightarrow$ $(\sigma x^2 = 9)$

Regression equations are:-

$$8x + 10y + 66 = 0$$
$$40x - 18y = 214$$

~~where~~ what were

(1) The mean value of x and y

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(2). The standard deviation of y and coefficient of correlation between x and y.

Since equation (1) and (2) passing through the points $(\bar{x}, \bar{y})$, then

$$8\bar{x} - 10\bar{y} = -66 \quad \text{--- (3)}$$
$$40\bar{x} - 18\bar{y} = 214 \quad \text{--- (4)}$$

$$\bar{x} = 13, \ \bar{y} = 17$$

From eq. (1) & (2), we get

$$-10y = -8x - 66$$
$$y = 0.8x + 6.6$$

$\to b_{yx}$

From eq (2), we get

$$40x - 18y = 214$$

$$x = \frac{18}{40}y + \frac{214}{40}$$

$$x = 0.45y + 5.35$$

$\downarrow$

b_{xy}

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = 0.8$$

$$b_{yx} = r\frac{\sigma_x}{\sigma_y} = 0.45$$

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$r = \sqrt{b_{yx} \cdot b_{xy}} = \sqrt{0.8 \times 0.45} = r = 0.6$

Binomial Distribution:- Let there be n independent trials in an experiment, Let a random variable X denote number of successes in these ~~the~~ n-trials. Let p be the probability of successes and q that of a failure in a single trial so that $p+q=1$, Let the trial be independent and p be constant for every trial, then the probability of r successes in n-trials, that is

$$\boxed{P(x=r) = {}^nC_r\, p^r q^{n-r}}$$ where $p+q=1$
$r = 0, 1, 2, \ldots\ldots n$

Mean and variance of binomial distribution :-

Mean of binomial distribution $= np$

Variance of binomial distribution $= npq$

Variance $(\sigma^2) = npq$

$\sigma = \sqrt{npq}$

Q. If 10% of the ~~balls~~ bolts, ~~provided~~ produced by a machine are defective, determine the probability that out of 10 bolts ~~are~~ chosen at random.

(1) one
(2) None
(3) at most two bolts will be defective

p (defective) $= \frac{10}{100} = \frac{1}{10}$

q (non-defective) $= 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$

$n = 10$

Using binomial distribution

$$P(x=r) = {}^nC_r \, p^r \, q^{n-r}$$

① $r = 1$

$$P(x=1) = {}^{10}C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{10-1} = (0.9)^9 = 0.3874$$

② None that is $r = 0$.

$$P(X=0) = {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{1}{10}\right)^{10-0} = \left(\frac{9}{10}\right)^{10} = 0.3486$$

③ Probability that at most 2 bolts will be defective
that is $P(r \le 2) = P(0) + P(1) + P(2)$

$$P(r \le 2) = {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10-0} + {}^{10}C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{10-9} +$$

$$ {}^{10}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{10-2}$$

$$= 0.9297$$

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Q. Fit a binomial distribution to the following frequency data

x	0	1	2	3	4
y	30	62	46	10	2

x	f	fx
0	30	0
1	62	62
2	46	92
3	10	30
4	2	8
	$\Sigma f = 150$	$\Sigma fx = 192$

Mean of the observations $= \dfrac{\Sigma fx}{\Sigma f} = \dfrac{192}{150} = 1.28$

Mean of binomial distribution $= np$

Hence $np = 1.28$

$4p = 1.28$

$p = 0.32$

$q = 1-p = 1-0.32 = 0.68$

Hence binomial distribution is $N(p+q)^n$

$= 150(0.32+0.68)^4$

$= 150$ Ans

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During war, 1 ship out of 9 was sunk on an average in making a certain toyes. What was the probability that exactly 3 out of a convoy of 6 ships would arrive safely

Solution :- $q = 1/9$, $p = 1 - \frac{1}{9} = 8/9$

$n = 6$.

We know that binomial distribution,

$$P(x = r) = {}^{n}C_{r}\, p^{r}\, q^{n-r} \quad (r = 0, 1, 2, \ldots n)$$

$$P(x = 3) = {}^{6}C_{3}\,(8/9)^{3}\,(1/9)^{6-3} = \frac{10240}{9^{6}}$$

If the probability of hitting a target is 10% and 10 shots are fired independently, what is the probability that the target will be hit atleast once.

$p = 10\% = \frac{10}{100} = 1/10$ $\qquad q = 1 - p = \frac{1}{10} = \frac{9}{10}$

$n = 10$

Probability that the target will atleast hit once =

$$= P(r > 1)$$

$$= 1 - P(r = 0)$$

$$= 1 - {}^{n}C_{r}\, p^{r}\, q^{n-r}$$

$$= 1 - {}^{10}C_{0} \left(\frac{1}{10}\right)^{0} \left(\frac{2}{10}\right)^{10-0}$$

$$= 0.6513$$

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Poisson Distribution:- If the parameters n and p of a binomial distribution are known we can find the distribution. But in situation where n is very large and p is very small, application of binomial distribution is very laborious. However if we assume that as $n \to \infty$ and $p \to 0$, such that np always finite say λ, we get the poisson distribution to the Binomial distribution, i.e.

$$\boxed{P(x=r) = \frac{e^{-\lambda}\lambda^{r}}{r!}}, \quad r=0,1,2,\ldots$$

$$\lambda = np$$

Mean and variance of poisson distribution

Mean of Poisson distribution $= \lambda$
Variance of Poisson distribution $= \lambda$

Recurrence formula for Poisson distribution

We know that, Poisson distribution

$$P(r) = \frac{e^{-\lambda}\lambda^{r}}{r!} \quad \text{— (1)}$$

$$P(r+1) = \frac{e^{-\lambda}\lambda^{r+1}}{(r+1)!} \quad \text{— (2)}$$

Here

$$\frac{P(r+1)}{P(r)} = \frac{\frac{e^{-\lambda}\lambda^{r+1}}{(r+1)!}}{\frac{e^{-\lambda}\lambda^{r}}{r!}} = \frac{\lambda r!}{(r+1)!} = \frac{\lambda r!}{(r+1)r!}$$

$$= \frac{\lambda}{r+1}$$

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$$\boxed{P(r+1) = \frac{\lambda}{r+1} P(r)} \quad r = 0, 1, 2, 3,$$

This is called recurrence formula for Poisson distribution

Q. Using poisson distribution, find the probability that the ace of spades, will be drawn from a pack of well shuffled cards at least once in 104 consecutive trials.

$p = 1/52, \; n = 104, \; \lambda = np = 104 \times \frac{1}{52} = 2$

~~Prob~~ Probability at least once $= P(r \geq 1) = 1 - P(r = 0)$

$$= 1 - \frac{e^{-2}(2)^0}{0!}$$

$$= 1 - 0.135335 = 0.864$$

H.W. Binomial Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or six.
Ans:- 233

Q. Fit a poisson distribution to the following data and calculate theoretical frequencies.

Deaths	0	1	2	3	4
Frequencies	122	260	15	2	1

Deaths (x)	Frequencies (f)	fx
0	122	0
1	260	260
2	15	30
3	2	6
4	1	4
	$\Sigma f = N = 400$	$\Sigma fx = 300$

Mean of poisson distribution

$$\lambda = \frac{\Sigma fx}{\Sigma f \text{ or } N} = \frac{300}{400} = 0.75$$

Required poisson distribution is given by:-

$$= NP(X=r) = 400\,\frac{e^{-\lambda}\lambda^{r}}{r!} = 400\,\frac{e^{-0.75}(0.75)^{r}}{r!}$$

r	NP(r)	Theoretical frequencies
0	$\{400e^{-0.75}\cdot(0.75)^0\}/0!$	$= 188.84 = 188$
1	$\{400.e^{-0.75}(0.75)^1\}/1!$	$= 141.70 = 141$
2	$\{400.e^{-0.75}(0.75)^2\}/2!$	$= 53.141 = 53$
3	$\{400.e^{-0.75}(0.75)^3\}/3!$	$= 13.28 = 13$
4	$\{400.e^{-0.75}(0.75)^4\}/4!$	$= 2.49 = 2$

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A manufacturer knows, that the condensors, he makes contains an an average 1% of defective, he packs them in boxes 100. What is the probability that a box picked at random will contain ~~four or~~ 4 or more, faulty condensors.

$$p = 1\% = 0.01, \quad n = 100, \quad \lambda = np = 0.01 \times 100 = 1$$

Using poisson distribution: $P(X=r) = \dfrac{e^{-\lambda}\lambda^r}{r!}$

$r = 0, 1, 2, \ldots$

$$P(r) = \frac{e^{-1}(1)^r}{r!} = \frac{e^{-1}}{r!}$$

$$P(4 \text{ or more faulty condensors}) = 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$= 1 - \left[\frac{e^{-1}}{0!} + \frac{e^{-1}}{1!} + \frac{e^{-1}}{2!} + \frac{e^{-1}}{3!}\right]$$

$$= 1 - e^{-1}\left[1 + 1 + \frac{1}{2} + \frac{1}{6}\right]$$

$$= 0.019$$

The frequency of accidents per shift in a factory is shown in the following table.

Accident per shift	Frequency
0	193
1	100
2	24
3	3
4	1
Total	320

If the probability of a bad reaction from a certain injection ~~reaction~~ is 0.0002. Determine the chance that out of 1000 individuals more than, two will get a bad reaction.

~~Chi-Square (χ^2) Test:-~~

Q. Suppose that a book of 600 pages contains 40 printing mistakes. Assume that these errors are randomly distributed throughout the book and r, the number of errors per page, has a Poisson distribution, what is the probability that 10 pages, selected at random will be free from errors.

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$$p = \frac{40}{600} = \frac{1}{15}, \quad n = 10$$

$$\lambda = np = 10 \times \frac{1}{15} = 0.66$$

Using Poisson distribution

$$P(r) = \frac{e^{-\lambda} \lambda^{r}}{r!}, \quad r = 0, 1, 2, \ldots$$

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$$P(0) = \frac{e^{-0.67}(0.67)^0}{0!} = 0.51$$

In a certain factory turning out razor blades, there is a chance of 0.002 for any blade, to be defective. The blades are supplied in packets of 10. Calculate the approximate number of packets containing no defective one defective and two defective blades in a consignment of 10,000 packets (given $e^{-0.02} = 0.9802$)

$p = 0.002$

$n = 10$ ~~,000~~

$\lambda = np = 0.002 \times 10 = 0.02$

$N = 10,000$

Probability of having no defective

$$\Rightarrow P(0) = \frac{e^{-0.02}(0.02)^0}{0!} = 0.9802$$

Approximate number of packets having zero defective in the consignment $= 0.9802 \times 10,000 = 9802$

Probability of having one defective

$$P(1) = \frac{e^{-0.02}(0.02)^1}{1!} = 0.0196$$

Approximate number of packets having one defective in the consignment

$= 0.0196 \times 10,000 = 196$

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Probability of having two defective

$$P(2) = \frac{e^{-0.02} \times (0.02)^2}{2!} = 0.000196$$

Approximate number of packets having two defective blades in the consignment $= 0.000196 \times 10,000$
$= 1.96 \equiv 2$

Suppose that number of telephone calls on an operator recieved from 9:00 to 9:05, follow a Poisson distribution with mean 3, Find the probability that interval

(1) The operator will recieve no calls on that time tomorrow.

$\lambda = 3$

$$P(0) = \frac{e^{-3}(3)^0}{0!} = 0.04978$$

Chi-square (χ^2) Test