Prem-1

Algebra of set

① Idempotent laws

$A \cup A = A$,

$A \cap A = A$.

② Associative law (sequence is not matter).

$(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

③ Commutative law.

$A \cup B = B \cup A$.

$A \cap B = B \cap A$.

④ Distributive law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Ex: $A = \{1, 2, 3\}$

$B = \{3, 4\}$

$C = \{4, 5, 6\}$

$A \cup (B \cap C) =$

$A \cup \{4\}$

$\Rightarrow \{1, 2, 3, 4\}$

$(A \cup B) \cap (A \cup C)$

$\Rightarrow \{1, 2, 3, 4\} \cap \{1, 2, 3, 4, 5, 6\}$

$\Rightarrow \{1, 2, 3, 4\}$.

⑤ Identity law

$A \cup \phi = A$

$A \cap \phi = \phi$

⑥ Complement law

A, A^c

$\rightarrow A \cup A^c = U$

$\rightarrow A \cap A^c = \phi$

$(A^c)^c = A$

$(U)^c = \phi$

$\phi^c = U$

Demorgan's law

$(A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$

$x \in (A \cup B)^c$

$x \notin (A \cup B)$

$x \notin A$ and $x \notin B$

$x \in A^c$ and $x \in B^c$

$x \in A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$

$x \in (A \cap B)^c$

$x \notin (A \cap B)$

$x \notin A$ or $x \notin B$

$x \in A^c$ or $x \in B^c$

$x \in A^c \cup B^c$

$U = \{1, 2, 3, 4, 5, 6, \ldots\}$

$A = \{1, 2, 3.\}$

$B = \{3, 4, 5, 6, \ldots\}$

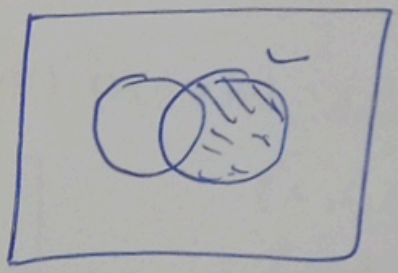
$A \cup B = 1, 2, 3 \ldots$

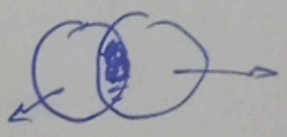
$(A \cup B)^c = \{7\}$

$A^c = 4, 5, 6, 7$

$B^c = 1, 2, 7$

$A^c \cap B^c = 7$





Lec-3

Dimension of a set
→ Fundamental of counting principle
→ power set.

Dimension of a set

→ It also known as cardinality / dimension of a set.
→ It measure total no. of element present in a set A.

$$A = \{1, 2, 3, 4, 5\}$$

$$n(A) = 6$$

Fundamental of counting principle.

① If A & B are finite sets

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

② If A & B are finite disjoint set.

$$n(A \cup B) = n(A) + n(B)$$

③ A, B & C are finite set.

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C) - n(B \cap C)$$

n = {1, 2, 3}

⇒ ϕ, {1}, {2}, {3}, {1, 2}, {2, 3}, {3, 1}, {1, 2, 3}

power set → If we have given a set 'n', then the collection of all subset of 'n' is called as power set of 'n'.

power(Y) = 2^n | n = no of element of n.

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Inclusion Exclusion principle in set theory.

(Q1) A computer company must hire 20 programmers to handle system programming jobs & 30 programmers for applications programming & 5 are expected to perform jobs of both types. How many programmers must be hired?

Ans $n(A) = 20$, $n(B) = 30$.

$n(A \cap B) = 5$

$n(A \cup B) = ?$, $n(A) + n(B) - n(A \cap B)$

$\Rightarrow 20 + 30 - 5 \Rightarrow (45)$

(Q2) Out of 250 candidates who failed in an examination, revealed that 128 failed in math, 87 in physics & 134 in chemistry, 31 failed in math, and in physics, 54 failed. 54 failed in math & chemistry., 30 failed in chemistry & in physics.

Find how many candidates failed.

(a) In all the 3 subjects

(b) In math but not in physics.

128
134
87
349

$A \cup B \cup C = 250$

$n(A) = 128$ (m)

$n(B) = 87$ (p)

$n(C) = 134$ (c)

$(m \cap p) = 31$

$(m \cap c) = 54$

$(p \cap c) = 30$

$A \cup B \cup C = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$

$250 = 128 + 87 + 134 - 31 - 54 - 30 + |A \cap B \cap C|$

$= 349 - 115 + |A \cap B \cap C|$

$\Rightarrow 365 - 349 = |A \cap B \cap C| = (16)$

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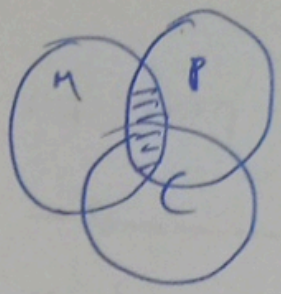
$n(m \cap \bar{p})$

$n(m - p)$

↳ $n(m) - n(m \cap p)$

⇒ $128 - 31$

⇒ 97



(Q5) In a class of 80 students, the girls & boys are in the ratio 3:5. The students can speak only Hindi or only English or both Hindi & English. The number of boys & the number of girls who can speak only Hindi are equal & each of them is 40% of total no of girls.
60% of girls can speak both the languages & 75% of the boys can speak only English.

(a) How many girls can speak only English?

(b) How many boys can speak only Hindi?

(c) How many students (boys & girls together) can speak both languages?

(d) How many boys can speak either only hindi or only english.

(45) Suppose 120 out of 120 computer science students study at least one of the following languages. French, German, & Russian. It is given 65 studen study French, 45 studet study german, 42 students study Russian, 20 studet study French & german, 25 students study French & Russian & 15 studet study German & Russian.

we want to find the no of studet who study.

(i) only French & German but not Russian.

(ii) only French & Russian but not german.

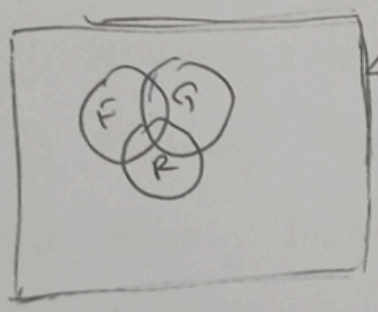
(iii) only german & Russian but not French.

(iv) only French

(v) only german

(vi) only Russian

(vii) none of the three languages



120

French = F
german = G
Russ = R

$|F \cup G \cup R| = 120$

$|F| = 65$ $\quad |F \cap G| = 20$

$|G| = 45$ $\quad |F \cap R| = 25$

$|R| = 42$ $\quad |G \cap R| = 15$

$|F \cap G \cap R| = ?$

$120 = 65 + 45 + 42 - 20 - 25 - 15 + |F \cap G \cap R|$

$120 = 152 - 60 = 92$

$\Rightarrow |F \cap G \cap R| = 92$

Q-5

(i) $|F \cap \bar{S}| - |F \cap S \cap R|$

$\Rightarrow 20 - 8 = 12$

(ii) $|F \cap R| - |F \cap R \cap S|$

$= 25 - 8 = 17$

(iii) $|S \cap R| - |F \cap R \cap \bar{S}|$

$\Rightarrow 15 - 8 = 7$

(iv) Only F?

$|F| - |F \cap S| - |F \cap R| + |F \cap S \cap R|$

$\Rightarrow 65 - 20 - 25 + 8 = 73 - 45 = 28$

(v) Only Science.

$|S| - |S \cap F| - |S \cap R| + |F \cap S \cap R|$

$\Rightarrow 45 - 20 - 15 + 8 \Rightarrow 45 - 35 + 8 = 18$

(vi) Only Reasoning (10)

$|R| -$

(vii) Num of the $120 - 100 = 20$