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Saathi

Assignment -1

1.

1.1.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$$

applying $R_2 \to R_2 - R_1$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

applying $R_3 \to R_3 + R_2$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

∴ Since there are 2 non-zero row remaining in this echelon form of A.

* $[\rho(A) = 2]$

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1.2 $A = \begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix}$

applying $R_1 \to \frac{1}{9} R_1$

$$A = \begin{bmatrix} 1 & 7/9 & 1/3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix}$$

applying $R_2 \to R_2 - 5 \times R_1$

$R_3 \to R_3 - 6 \times R_1$

$$A = \begin{bmatrix} 1 & 7/9 & 1/2 \\ 0 & -44/9 & 7/3 \\ 0 & 10/3 & 0 \end{bmatrix}$$

∴ Since, there are 3 non-zero row remaining in this echelon form of A

* $[\rho(A) = 3]$

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13. $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

applying $R_2 \to R_2 - 2 \times R_1$

$R_3 \to R_3 - 3 \times R_1$

$R_4 \to R_4 - 6 \times R_1$

$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & +5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$

applying $R_2 \to \frac{1}{5} \times R_2$

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$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & 3/5 & 7/5 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$

applying $R_3 \to R_3 - 4 \times R_2$

$R_4 \to R_4 - 9 \times R_2$

$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & 3/5 & 7/5 \\ 0 & 0 & 33/5 & 22/5 \\ 0 & 0 & 33/5 & 22/5 \end{bmatrix}$

applying $R_4 \to R_4 - R_3$

$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & 3/5 & 7/5 \\ 0 & 0 & 33/5 & 22/5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\therefore$ Since, there are 3 non-zero row remaining in this echelon form of A.

$\therefore \rho(A) = 3$

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2.

2.1 $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix}$

* $\boxed{b = 2}$

2.2 $A = \begin{bmatrix} 2 & -1 & 4 \\ 9 & 7 & 3 \\ 5 & b & -5 \end{bmatrix}$

if 3×3 matrix has a rank less than 3, then determent is 0

$\Delta = 0$

$|A| = \begin{vmatrix} 2 & -1 & 4 \\ 9 & 7 & 3 \\ 5 & b & -5 \end{vmatrix} = 0$

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$2(-35-3b) + 1(-45-15)$
$+ 4(9b-35) = 0$

$-70-6b-60+36b-140 = 0$
$6b = -140-130+36b$
$30b = 270$
$b = 9$

2.3 $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & b \end{bmatrix}$

if 3×3 matrix has a rank less than 3 then determent is 0

$\Delta = 0$

$|A| = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & b \end{vmatrix}$

applying $C_3 \to C_3 + C_1$

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$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 4 & 8 \\ 0 & -6 & b \end{vmatrix} = 0$$

$4b - 48 = 0$

$* \; [b = 12]$

3.

3.1. $x - y + 2z = 3$ — (i)

$x + 2y + 3z = 5$ — (ii)

$3x - 4y - 5z = -13$ — (iii)

aug $A = [A : B]$

$$\text{aug } A = \left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ 1 & 2 & 3 & 5 \\ 3 & -4 & -5 & -13 \end{array}\right]$$

applying $R_2 \to R_2 - R_1$

$R_3 \to R_3 - 3R_1$

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$$\text{aug } A = \left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ 0 & 3 & 1 & 2 \\ 0 & -1 & -11 & -22 \end{array}\right]$$

$$\text{aug } A = \left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ 0 & -1 & -11 & -22 \\ 0 & 3 & 1 & 2 \end{array}\right]$$

applying $R_3 \to R_3 + 3R_2$

$$\text{aug } A = \left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ 0 & -1 & -11 & -22 \\ 0 & 0 & -32 & -64 \end{array}\right]$$

$\rho(A) = \rho(\text{aug } A) = 3$

Hence they are consistent

$\cancel{\frac{x}{1} = \frac{y}{} = \frac{z}{}}$

$y = x + 2z - 3$ — (iv)

Put eq (iv) in (ii) and (iii)

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$$x+2(x+2z-3)+3z=5$$

$$3x+7z=11 \quad \text{— (5)}$$

$$3x-4(x+2z-3)-5z=-13$$

$$-x-13z=-25$$

$$x+13z=25 \quad \text{— (6)}$$

Solving eq^n (5) and (6)

$$3x+7z=11$$
$$x+13z=25 \times 3$$

$$3x+7z=11$$
$$3x+39z=75$$
$$- \quad - \quad -$$

$$-32z=-64$$

$$z=\frac{64}{32}=2$$

$$x=1, \; y=0, \; z=+2$$

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3.2. $x_1+2x_2-x_3=1$

$3x_1-2x_2+2x_3=2$ — (i)

$7x_1-2x_2+3x_3=5$ — (ii) — (iii)

$$\text{aug}A = [A:B]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 3 & -2 & 2 & 2 \\ 7 & -2 & 3 & 5 \end{array}\right]$$

applying $R_2 \to R_2-3R_1$

$R_3 \to R_3-7R_1$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -8 & 5 & -1 \\ 0 & -16 & 10 & -2 \end{array}\right]$$

applying $R_3 \to R_3-2R_2$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -8 & 5 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(\text{aug}A) = 2 <$ no. of variables

then the system is consistent and has infinitely many solⁿ and these solutions and

Q3.3.

$x+y+z+t=2$

$x-y+z+t=0$

$2x-3y+2z+t=-2$

$x+y+t=1$

$\text{aug}A = [A:B]$

$$\text{aug}A = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & 1 & 0 \\ 2 & -3 & 2 & 1 & -1 \\ 1 & 1 & 0 & 1 & 1 \end{array}\right]$$

applying $R_2 \to R_2 - R_1$

$R_3 \to R_3 - 2R_1$

$R_4 \to R_4 - R_1$

$$\text{aug}A = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -2 & 0 & 0 & -2 \\ 0 & -5 & 0 & -1 & -5 \\ 0 & 0 & -1 & 0 & -1 \end{array}\right]$$

applying $R_2 \to \frac{R_2}{2}$

$$\text{aug}A = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & -5 & 0 & -1 & -5 \\ 0 & 0 & -1 & 0 & -1 \end{array}\right]$$

applying $R_3 \to R_3 + 5R_2$

$$\text{aug}A = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 5 & 4 & 5 \\ 0 & 0 & -1 & 0 & -1 \end{array}\right]$$

$$\text{aug}A = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 5 & 4 & 5 \end{array}\right]$$

applying $R_4 \to R_4 - 5R_3$

$$\text{aug}A = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 4 & 0 \end{array}\right]$$

$$\rho(A) \neq \rho(\text{aug}(A))$$

Hence these eq^n are inconsistent

4.

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + az = b$$

i. no sol^n

$$\text{aug}A = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & a & b \end{array}\right]$$

$$\text{aug}A = \left[\begin{array}{ccc|c} 1 & 3/2 & 5/2 & 9/2 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & a & b \end{array}\right]$$

applying $R_2 \to R_2 - 7R_1$

$R_3 \to R_3 - 2R_1$

$$\text{aug}A = \left[\begin{array}{ccc|c} 1 & 3/2 & 5/2 & 9/2 \\ 0 & -15/2 & -39/2 & -47/2 \\ 0 & 0 & a-5 & b-9 \end{array}\right]$$

if the system has no sol^n then $\rho(A) \neq \rho(\text{aug}(A))$

$\therefore$ i. $a = 5$ and $b \neq 9$ | ii. $a \neq 5$ and $b \neq 9$

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(i) Unique Sol^n $\rightarrow$

For unique Sol^n

$\rho(A) = \rho(\text{aug}(A)) =$ no. of variables

$\therefore a \neq 5$ and $b \neq 9$

(ii) Infinitely many Sol^n $\rightarrow$

For infinitely many Sol^n

$\rho(A) = \rho(\text{aug}(A)) <$ no. of variables

$\therefore a = 5$ and $b = 9$

5. $-2x + y + z = a$

$x - 2y + z = b$

$x + y - 2z = c$

no Sol^n unless $a + b + c = 0$

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$$\text{aug } A = \left[\begin{array}{ccc|c} -2 & 1 & 1 & a \\ 1 & -2 & 1 & b \\ 1 & 1 & -2 & c \end{array}\right]$$

$$\text{aug } A = \left[\begin{array}{ccc|c} 1 & -2 & 1 & b \\ -2 & 1 & 1 & a \\ 1 & 1 & -2 & c \end{array}\right]$$

applying $R_2 \rightarrow R_2 + 2R_1$
$R_3 \rightarrow R_3 - R_1$

$$\text{aug}A = \left[\begin{array}{ccc|c} 1 & -2 & 1 & b \\ 0 & 3 & -3 & (a+b) \\ 0 & 3 & -3 & c-b \end{array}\right]$$

applying $R_3 \rightarrow R_3 - R_2$

$$\text{aug}A = \left[\begin{array}{ccc|c} 1 & -2 & 1 & b \\ 0 & 3 & -3 & -(a+b) \\ 0 & 0 & 0 & a+b+c \end{array}\right]$$

~~there for no~~ therefore,

for no Sol^n $a + b + c \neq 0$

$\rho(A) \neq \rho(\text{aug}A)$

For $a=1, b=1, c=-2$

$$\text{aug}A = \begin{bmatrix} 1 & -2 & 1 & : & 1 \\ -2 & 1 & 1 & : & 1 \\ 1 & 1 & -2 & : & -2 \end{bmatrix}$$

applying $R_2 \to R_2 + 2R_1$

$R_3 \to R_3 - R_1$

$$= \begin{bmatrix} 1 & -2 & 1 & : & 1 \\ 0 & -3 & 3 & : & 3 \\ 0 & 3 & -3 & : & -3 \end{bmatrix}$$

applying $R_3 \to R_3 + R_2$

$$\text{aug}A = \begin{bmatrix} 1 & -2 & 1 & : & 1 \\ 0 & -3 & 3 & : & 3 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$\rho(A) = \rho(\text{aug}A) = 2 \quad 2$

no. of variable

Hence, this system of eq^n has infinitely many sol^n

6.

$x - y + z = 0$

$4x - 3y + 2z = 0$

$2x - 3y + 4z = 0$

$$\text{aug}A = \begin{bmatrix} 1 & -1 & 1 & : & 0 \\ 4 & -3 & 2 & : & 0 \\ 2 & -3 & 4 & : & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 4 & -3 & 2 \\ 2 & -3 & 4 \end{bmatrix}$$

applying $R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - 2R_1$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & -1 & 2 \end{bmatrix}$$

Applying $R_3 \to R_3 + R_2$

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$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 <$ no. of variables
so the ~~has~~ system has an infinite number of solⁿ

6.2. $4x+2y+z+3w=0$
$6x+3y+4z+7w=0$
$2x+y+w=0$

$$A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \end{bmatrix}$$

applying $R_2 \to R_2 - 2R_1$ $R_3 \to R_3 - 3R_1$

$$A = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 4 & 4 \end{bmatrix}$$

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applying $R_3 \to R_3 - 4R_2$

$$A = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 <$ no. of variables
So the system has an infinite number of solⁿ

7. $2x+by+3z=0$
$x+3y+bz=0$
$2x+y+2z=0$

$$A = \begin{bmatrix} 2 & b & 3 \\ 1 & 3 & b \\ 2 & 1 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

i. if the system has trivial solⁿ then,

$X = 0$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad x = y = z = 0$$

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ii. Non-trivial solⁿ →

if the system is non-trivial then

$|A| = 0$

$$\therefore |A| = \begin{vmatrix} 2 & b & 3 \\ 1 & 3 & b \\ 2 & 1 & 2 \end{vmatrix} = 0$$

$$2(6-b) - b(2-2b) + 3(1-6) = 0$$

$$12 - 2b - 2b + 2b^2 - 15 = 0$$

$$2b^2 - 4b - 3 = 0$$

$\cancel{2}b^2 \ldots$

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$$b = \frac{+4 \pm \sqrt{16 + 4(2)(3)}}{4}$$

$$= \frac{4 \pm \sqrt{40}}{4}$$

$$= \frac{4 \pm 2\sqrt{10}}{4} = \frac{2 \pm \sqrt{10}}{2}$$

$$b = \frac{2 \pm \sqrt{10}}{2}$$

Q.2 $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Characteristic eqn $(A-\lambda)X = 0$

$\therefore \quad |A-\lambda| = \begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$

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$$-\lambda^3 + 7\lambda^2 - 36 = 0$$

$$\lambda = 6, 3, -2$$

eigenvector for $\lambda = 6$

$$= \begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix}$$

The null space of this matrix is

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$$

This is eigenvector

eigenvector for $\lambda = 3$

$$= \begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix}$$

The null space of this matrix is $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$ This is the eigenvector

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eigen vector For $\lambda = -2$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix}$$

The null Space of this matrix is

$$\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

This is eigen vector

Q.3 $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

$A - \lambda I = 0$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 \\ 1 & -\lambda \end{vmatrix}$$

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$\lambda(\lambda-1) - 2 = 0$

$\lambda^2 - \lambda - 2 = 0$

$\lambda^2 - 2\lambda + \lambda - 2 = 0$

$\cancel{2\lambda(\lambda}$ $\lambda(\lambda-2) + 1(\lambda-2) = 0$

$\lambda = -1, 2$

Eigen vector for $\lambda = -1$

The null space of this matrix is

$$\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$$

This is the eigenvector

eigen vector for $\lambda = 2$

The null space of this matrix is

$$\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$$ This is the eigen vector

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9.

9.1.

$$A = \begin{bmatrix} -1 & 2i & 4+i \\ -2i & 2 & -2 \\ 4-i & -2 & 5 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} -1 & -2i & 4-i \\ 2i & 2 & -2 \\ 4+i & -2 & 5 \end{bmatrix}$$

$$(\bar{A})' = \begin{bmatrix} -1 & 2i & 4+i \\ -2i & 2 & -2 \\ 4-i & -2 & 5 \end{bmatrix}$$

$$A = A^*$$

This matrix is Hermitian matrix

9.2 $A = \begin{bmatrix} i & 2+3i & 4i \\ -2+3i & 0 & 5 \\ 4i & -5 & -3i \end{bmatrix}$

$\Rightarrow \quad A^* = \begin{bmatrix} -i & -(2+3i) & -4i \\ -(2+3i) & 0 & -5 \\ -4i & 5 & 3i \end{bmatrix}$

$= -A$

$A^* = -A$

Hence, this matrix is skew ~~matrix~~ hermition matrix

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Q.3

$$A = \begin{bmatrix} \frac{1+i}{2} & -\frac{1-i}{2} \\ \frac{1-i}{2} & \frac{1+i}{2} \end{bmatrix}$$

$$A^* = \frac{1}{2}\begin{bmatrix} 1-i & 1+i \\ 1+i & 1-i \end{bmatrix}$$

$$A \cdot A^* = \frac{1}{4}\begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}\begin{bmatrix} 1-i & 1+i \\ 1+i & 1-i \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1+1+1+1 & 0 \\ 0 & 1+1+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A \cdot A^* = A^* \cdot A = I$$

Hence this matrix is Unitary matrix

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given → $A = A^*$ and $B = B^*$

10. To prove → $AB - BA$ is skew hermition matrix

$$\therefore (AB - BA)^*$$

$$= (AB)^* - (BA)^*$$

$$= B^* A^* - A^* B^*$$

$$(AB - BA)^* = BA - AB$$

$$= -(AB - BA)$$

$\therefore$ $AB - BA$ is skew-hermitian

11. $\frac{d^4 x}{dt^4} + 4x = \sin x$

$$D^4 x + 4x = \sin x$$

$$(D^4 + 4)x = \sin x$$

A.e → $m^4 + 4 = 0$

$$(m^2)^2 + (2)^2 = (m^2+2)^2 - 4m^2 = 0$$

$$(m^2 + 2 + 2m)(m^2 + 2 - 2m) = 0$$

$$m^2 + 2m + 2 = 0$$

$$m^2 - 2m + 2 = 0$$

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$$m = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$= \frac{-2 \pm 2i}{2} = -1 \pm i$$

$$m = \pm 1 \pm i$$

$$m = 1+i, -1+i, 1-i, -1-i$$

$$C.F \rightarrow (c_1 \cos x + c_2 \sin x) e^{x} + (c_3 \cos x + c_4 \sin x) e^{-x}$$

$$P.I \rightarrow \frac{\sin x}{D^4 + 4} \quad : D^2 \rightarrow -1$$

$$\frac{\sin x}{(-1)^2 + 4} = \frac{\sin x}{5}$$

$$\therefore y = (c_1 \cos x + c_2 \sin x) e^{x} + (c_3 \cos x + c_4 \sin x) e^{-x} + \frac{\sin x}{5}$$

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13. $x^2 \frac{d^3y}{dx^3} + 3x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

$x^3 \frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

$x^3\frac{d^3y}{dx^3} = D(D-1)(D-2)y$

$x^2\frac{d^2y}{dx^2} = D(D-1)y$

$x\frac{dy}{dx} = Dy$ put $x = e^z$

$(D(D-1)(D-2) + 3D(D-1) + D)y = 0$

$(D(D^2-3D+2) + 3D^2 - 3D + D)y = 0$

$(D^3 - 3D^2 + 2D - 3D + D + 3D^2)y = 0$

$D^3 y = 0$

A.E $m^3 = 0$

$m = 0, 0, 0$

$\therefore y = (c_1 + c_2 z + c_3 z^2)e^0$

$= c_1 + c_2 \log x + c_3 (\log x)^2$

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14. $\frac{d^2y}{dx^2} - 9y = x$

$(D^2-9)y = x$

A.E $\rightarrow$ $m^2 - 9 = 0$

$m = \pm 3$

C.F $\rightarrow$ $c_1 e^{-3x} + c_2 e^{3x}$

P.I $\rightarrow$ $\frac{x}{(D^2-9)} = -(9-D^2)^{-1}x$

$= -\frac{1}{9}\left(1-\frac{D^2}{9}\right)^{-1} x$

$= -\frac{x}{9}\left(1+\frac{D^2}{9}+\cdots\right)$

$= -x/9$

$\therefore y = C.F + P.I$

$= c_1 e^{-3x} + c_2 e^{3x} - x/9$

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15. $(2D^2+5D+3)y = \cos x$

A.E $\rightarrow$ $2m^2+5m+3=0$

$2m^2+3m+2m+3=0$

$m(2m+3)+1(2m+3)=0$

$m=-1, -3/2$

C.F $\rightarrow$ $c_1e^{-x}+c_2e^{-\frac{3}{2}x}$

P.I $\rightarrow$ $\dfrac{\cos x}{2D^2+5D+3}$ $\quad$ $\cancel{D^2 \rightarrow -x}$

$D^2 \rightarrow -1$

$= \dfrac{\cos x}{-2+5D+3} = \dfrac{\cos x}{5D+1}$

$= \dfrac{(5D-1)\cos x}{(25D^2-1)}$

$= \dfrac{5D\cos x - \cos x}{-26}$

$= \dfrac{-5\sin x - \cos x}{-26}$

$= \dfrac{5\sin x + \cos x}{26}$

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$\therefore y = C.F + P.I$

$= c_1e^{-x} + c_2e^{-3/2x} + \dfrac{5\sin x + \cos x}{26}$

16. $(D^2+1)^2y = e^x$

A.E $\rightarrow$ $(m^2+1)^2=0$

$(m^2+1)(m^2+1)=0$

$m=\pm i$

$m = i, i, -i, -i$

$\alpha=0, \beta=1$

C.F $\rightarrow$ $c_1\cos x + c_2\sin x + c_3\cos x + c_4\sin x$

P.I $\rightarrow$ $\dfrac{e^x}{(D^2+1)^2} = \dfrac{e^x}{4}$

$\therefore y = C.F + P.I$

$y = (c_1\cos x + c_2\sin x) + (c_3\cos x + c_4\sin x) + \dfrac{e^x}{4}$

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