Discrete Mathematics
BCSC 0010
Module 2

Graph Theory

Types of Graphs

Types of Graph

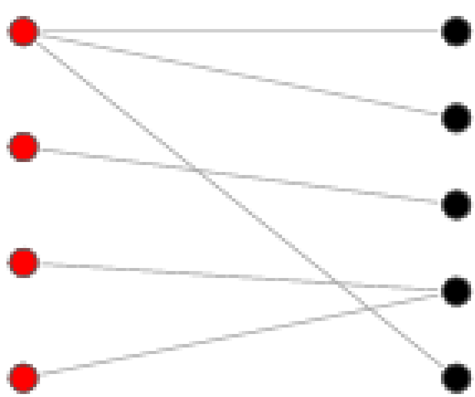
- Directed Graph
- Complete Graph
- Bipartite Graph
- Isomorphic Graph
- Euler Graph
- Hamiltonian Graph

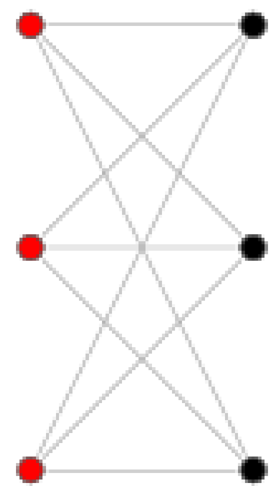
Bipartite Graph

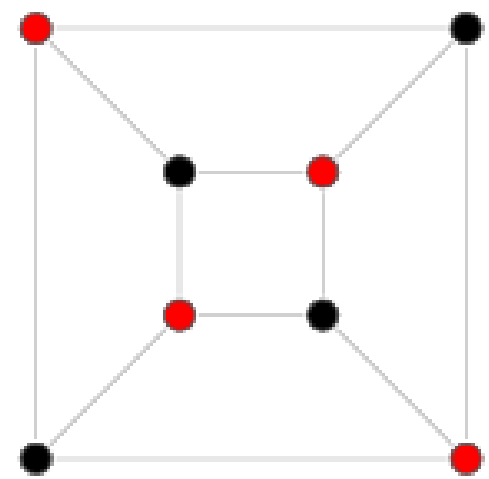
- A simple graph G is called ***bipartite*** *if its vertex set V can be partitioned into two disjoint* sets V1 and *V2 such that every edge in the graph connects a vertex in V1 and a vertex in V2*
- No edge in G connects either two vertices in V1 or two vertices in *V2*
- *When this* condition holds, we call the pair (V1, *V2) a bipartition of the vertex set V of G.*

Bipartite Graph

- Red vertices are in one set
- Black vertices are in another set





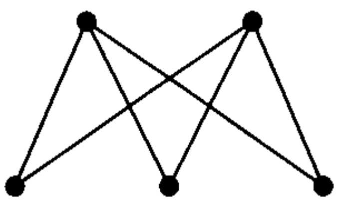


Complete Bipartite Graphs

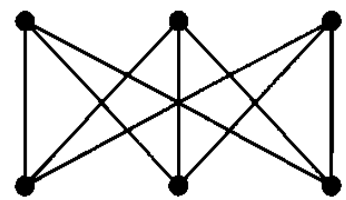
- **The complete bipartite graph** $K_{m,n}$ is the graph that has its vertex set partitioned into two subsets of m and n vertices, respectively such that every vertex of the first set is connected to every vertex of the second set.

Complete Bipartite Graphs

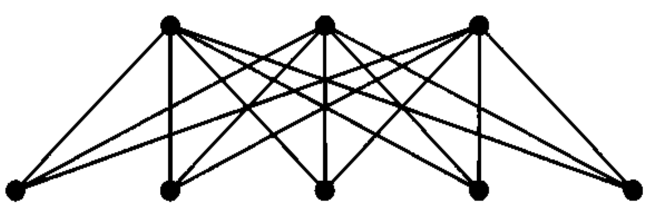
- The complete bipartite graphs *K2,3, K3,3, K3,5, and K2,6 are as follows:*



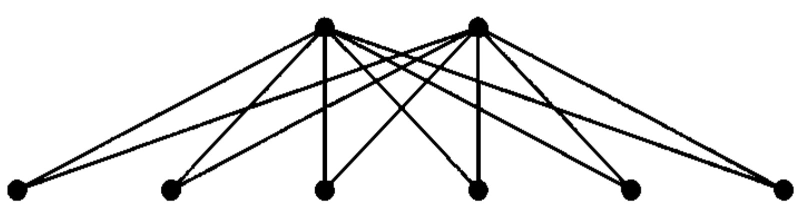
$K_{2,3}$



$K_{3,3}$



$K_{3,5}$



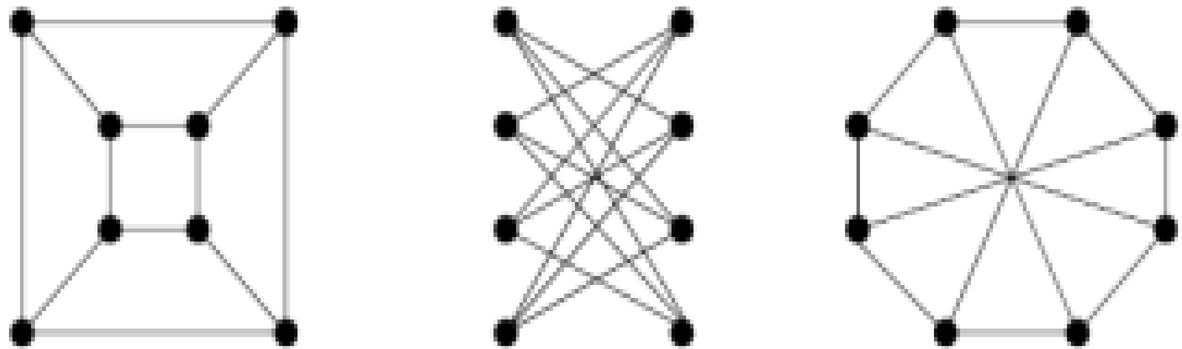
$K_{2,6}$

Isomorphism

- The simple graphs G1 = (V1, E1) and G2 = (V2, E2) are **isomorphic** if there is a one-to-one and onto function f from V1 to V2 with the property that a and b are adjacent in G1 if and only if f(a) and f(b) are adjacent in G2, for all a and b in V1. Such a function f is called an isomorphism.
- In other words, when two simple graphs are isomorphic, there is a one-to-one correspondence between vertices of the two graphs that preserves the adjacency relationship.

Isomorphism

- Isomorphic simple graphs must have the same number of edges
- the degrees of the vertices in isomorphic simple graphs must be the same.

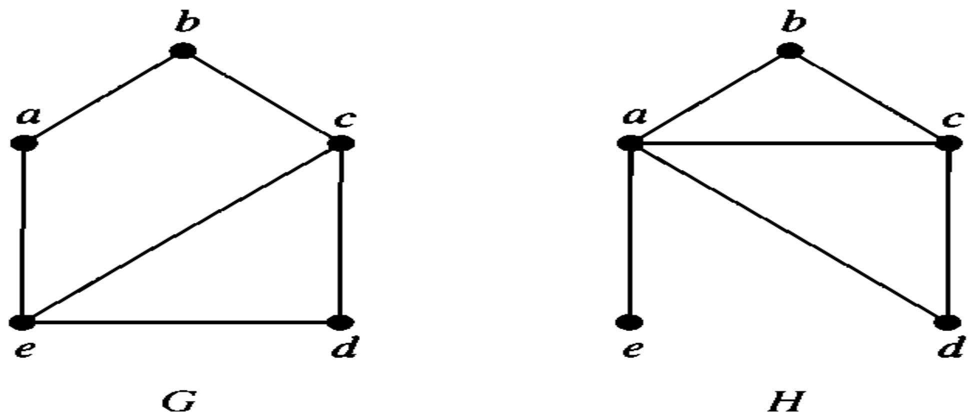


G_1 G_2 G_3

Isomorphism

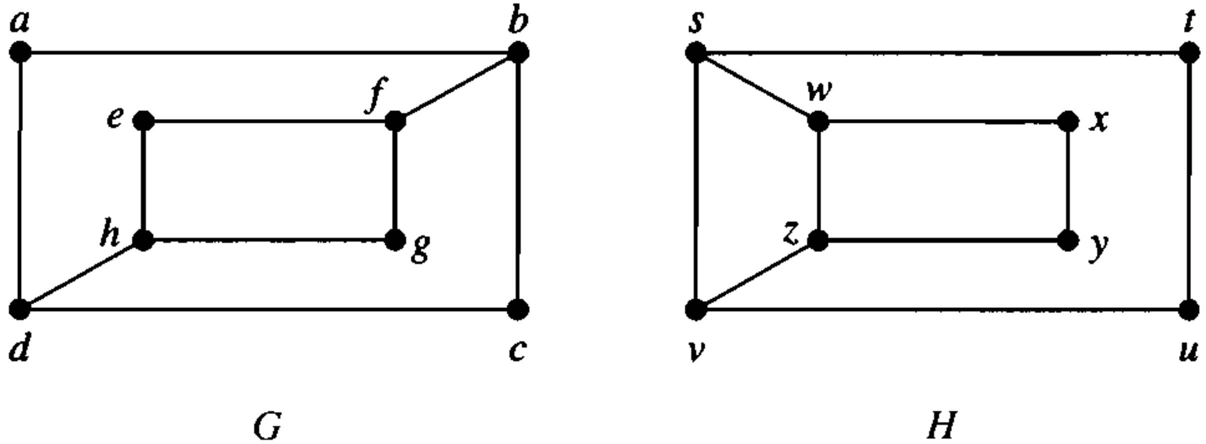
- The number of vertices, the number of edges, and the number of vertices of each degree are all **invariants under isomorphism**. If any of these quantities differ in two simple graphs, these graphs cannot be isomorphic.

Problem



Show that graphs displayed in Figure are not isomorphic.

Solution: *Both G and H have five vertices and six edges. However, H has a vertex of degree one,* namely, *e, whereas G has no vertices of degree one. It follows that G and H are* ***not isomorphic .***



- **Determine whether the graphs shown in Figure are isomorphic.**

- **Solution:** The graphs G and H both have eight vertices and 10 edges. They also both have four vertices of degree two and four of degree three. Because these invariants all agree
- However, G and H are not isomorphic.
- To see this, note that because deg(a) = 2 in G, a must correspond to either t, u, x, or y in H, because these are the vertices of degree two in H. However, each of these four vertices in H is adjacent to another vertex of degree two in H, which is not true for a in G.