Electromagnetics

Here time varying electric & magnetic fields are considered.. These fields are mutually depend on each other & cannot exist without the other, ~~we~~ so they are called electromagnetic fields.

Michael Faraday in his experiment had shown that electric field is produced by changing magnetic field. & Maxwell had shown theoretically that a magnetic field is produced by changing electric field.

A complete set of relations giving the connection between the charges at rest (electrostatic), charges in motion (current electricity), electric fields & mag. fields (electromagnetism) were derived theoretically & summarised in 4 equations by Maxwell called Maxwell's Eqn..

Maxwell brought together & extended 4 basic laws in electromagnetism such as

(i) Gauss's law in electro-statics

(ii) " " " magnetism.

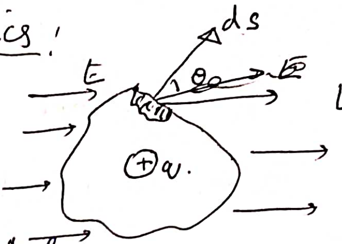
~~(iii) Ampere's law,~~

(iii) ~~(iv)~~ Faraday's law.

(iv) Ampere's law.

P.T.O.

① Gauss's law of electrostatics :



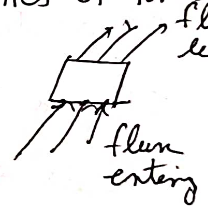
Gauss's law states that the total electric flux ϕ_E enclosed by a closed surface is $\frac{1}{\epsilon_0}$ times the total charge enclosed by that surface. i.e.

$\epsilon_0 \oint_s \vec{E}\cdot d\vec{s} = q$.. or $\epsilon_0 \phi_E = q$. or, $\phi_E = \frac{q}{\epsilon_0}$.

If ~~col~~ closed surface do not enclose any charge then $\phi_E = 0$.

② Gauss law in magnetism :

The mag. flux entering a closed surface is always equal to the mag. flux leaving the surface of the same volume because mag. lines of forces are continuous by nature.. So the net flux through a closed surface must be zero.



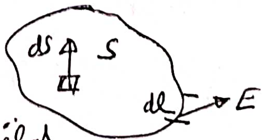
$$\oint \bar{B}\cdot d\bar{s} = \phi_B = 0.$$

③ Faraday's law of electromagnetic Induction :

Whenever the magnetic flux linked with a circuit is changed, an e.m.f.. is induced in the circuit.

If ϕ_B be the mag. flux linked with circuit ~~ano~~ at any instant & e be the induced e.m.f.

then $e = -\left(\frac{d\phi_B}{dt}\right)$.



The line integral of the electric field gives the induced e.m.f. in the closed circuit.

~~e = Edl~~

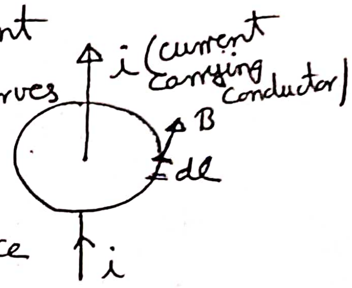
$e = \oint E\cdot dl$

$\therefore \oint E dl = -\frac{d\phi_B}{dt}$.

See page 1 (1)

Ampere's law : (Ampere's Circuital law)

According to Ampere's law, the line integral of magnetic flux density $\vec{B}$ along a closed curve is equal to μ_0 times the net current through the area bounded by the curves



$$\oint_l \bar{B} \cdot d\bar{l} = \mu_0 i \quad \text{——— (1)}$$

where μ_0 = permeability of free space

[Also $\vec{\nabla} \times \vec{H} = \vec{J}$, H = mag. field intensity & J = current density].

Ampere, in his experiment, assumed only conduction medium, where current flows through the wire. But Maxwell observed that electromagnetic phenomena takes place even in vacuum or in di-electric ~~(medium)~~ where electric flow is not possible.

Maxwell had shown that there are at least two ways of setting up a magnetic field :

(i) by a steady current

(ii) by a changing/varying electric field.

So a changing electric field is equivalent to a current which flows as long as the electric field is changing & it produces the same magnetic effect as an ordinary conduction current. This current is known as displacement current.

Displacement current have no significance for steady current, involving due to motion of true charges (True current).

Modified Ampere's law:

The Ampere's circuital law $\vec{\nabla} \times \vec{H} = \vec{J}$, was derived for steady current only & it does not hold good for time varying fields. This law contradicts the law of conservation of charge. This inconsistency was removed by Maxwell by introducing displacement current.

* If there exists an electric current as well as changing electric field then

~~Thus~~ * modified Ampere's law is given as

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$; where $\frac{\partial \vec{D}}{\partial t}$ is the displacement current density (J_D) ; $\vec{J}_D = \frac{\partial \vec{D}}{\partial t}$.

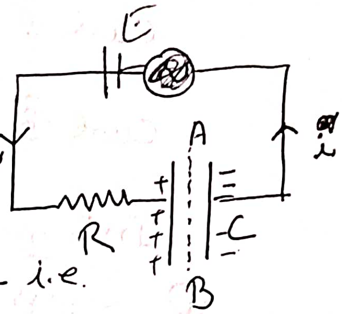
Example:

The study of discharge of condenser

The nature of current flowing through the conducting wire is different that through the capacitor. The current through conducting wire is conducting current, while the current through the capacitor is displacement current.

The conduction current is not continuous across the capacitor gap because no charge is transported across the gap. Displacement current is only current in the sence that it produces a magnetic field. It is not linked with the motion of charges. Displacement current is zero when applied voltage is constant.

Let at any instant q be the charge on capacitor plate. The conduction current (i_c) is defined as the rate of change of charge i.e.



$$i_c = \frac{dq}{dt} \quad \text{——} (1)$$

we know the electric displacement 'D' in dielectric is given as

$$D = \frac{q}{A} \quad \text{——} (2), \text{ where } A = \text{area of each plate.}$$

$$\text{or, } q = DA \quad \text{——} (3)$$

$\therefore$ From eqn.(1)

$$i_c = \frac{d}{dt}(DA) = A\frac{dD}{dt} \quad \text{——} (4)$$

Maxwell suggested that the term $A\left(\frac{dD}{dt}\right)$ should be considered as the current inside the di-electric. This current is called as displacement current & is denoted by i_d

i.e. $i_d = A\frac{dD}{dt}$ —(5) It is imp. that this current is zero when applied voltage is constant. It is flowing only when applied voltage is changing.

Consequently, displacement current density is given as

$J_d = \frac{dD}{dt}$. The vector D may vary with space, hence

$$J_d = \frac{\partial D}{\partial t} \quad \text{——} (6).$$

Inside the dielectrics there will be displacement current i_d, which is equal to conduction current i_c in the line.

P.T.O.

Maxwell's Eqns.

Differential form

$\operatorname{div} \vec{E} = \frac{\rho}{\epsilon_0}$ or, $\vec{\nabla}\cdot\vec{D} = \rho$ —— ①

The eqn $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$ is called Poisson equation..

$\operatorname{div} \vec{B} = 0$ or, $\vec{\nabla}\cdot\vec{B} = 0$ —— ②

$\operatorname{curl} \vec{E} = -\frac{\partial B}{\partial t}$ or, $\nabla\times\vec{E} = -\frac{\partial B}{\partial t}$ —— ③

$\operatorname{curl} \vec{B} = \mu_0\left(J + \epsilon_0 \frac{\partial E}{\partial t}\right)$ or $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ —— ④

or, $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial \vec{E}}{\partial t}$

Integral form:

① Gauss law

$$\epsilon_0 \oint \bar{E}\cdot d\bar{s} = q.$$

② Gauss law for magnetism

$$\oint_S \bar{B}\cdot d\bar{s} = 0.$$

③ Faraday's law

$\cancel{\oint_l \vec{E}\, dl = -\frac{d\phi_B}{dt}}$ or, $\oint_l \vec{E}\, dl = -\int_S \left(\frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{s}$

$$\oint_l H\cdot dl = \int (J + \frac{\partial D}{\partial t})\cdot dS$$

④ Ampere's law

$\cancel{\oint B\,dl = \mu_0 i + \mu_0\epsilon_0 \frac{d\phi_E}{dt}}$ or $\oint_l B\,dl = \int_S \mu_0\left(J + \frac{\partial D}{\partial t}\right) ds$

$\vec{B}$ = magnetic flux density
$\vec{D}$ = Electric " "
ρ = volume charge density

$\vec{E}$ = Electric field intensity.
$\vec{H}$ = Magnetic " intensity.
J = current density.

$\vec{D} = \epsilon_0 \bar{E}$; ϵ = permittivity of space
$\vec{B} = \mu H$, μ = permeability of medium
$\vec{J} = \sigma \vec{E}$; σ = conductivity.

Derivation of Maxwell's eqn in [Differential form]

1. Maxwell's 1st eqn : $\boxed{\bar{\nabla}\cdot\bar{D} = \rho}$

According to Gauss law the total electric flux out of a closed surface is equal to $\left(\frac{1}{\epsilon_0}\right)$ times the total charge enclosed. i.e.

$\oint_S \vec{E}\cdot d\vec{s} = \frac{q}{\epsilon_0}$. If ρ be the volume charge density & dv be the small volume considered, then

$\cancel{q = \int_V \rho dv}$ $q = \int_V \rho dv$

$\therefore \oint_S \bar{E}\cdot d\bar{s} = \frac{1}{\epsilon_0}\int_V \rho dv$ or, $\epsilon_0 \oint_S \bar{E}\cdot d\bar{s} = \int_V \rho dv$

or $\oint_S D\cdot ds = \int_V \rho dv$ — (1) Acc. to divergence theorem we have

$\oint_S D\cdot ds = \int_V (\bar{\nabla}\cdot\bar{D})\cdot dv$ ——(2)

From eqn (1) & (2) we have

$\int_V (\nabla\cdot D)\, dV = \int_V \rho dv$ or, $\boxed{\bar{\nabla}\cdot\bar{D} = \rho}$ or $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$.

Taking derivative of both sides we get

or $\text{div}\,\bar{E} = \frac{\rho}{\epsilon_0}$.

This eqn. $\text{div}\,E = \frac{\rho}{\epsilon_0}$ is called Poisson equation

2. Maxwell's 2nd eqn.

$\boxed{\bar{\nabla}\cdot\bar{B} = 0}$

Acc. to Gauss theory ~~of~~ for magnetism, the surface integral of mag. flux density over a closed surface is zero i.e.

P.T.O.

$\oint_S \bar{B}\cdot d\bar{S} = 0$ — (1) ; Transforming the surface integral into volume integral by Gauss's Divergence theorem, we have

$$\oint_S \vec{B}\cdot d\vec{S} = \int_V (\vec{\nabla}\cdot\vec{B})\, dv \quad \text{—— (2)}.$$

$\therefore$ From eqns (1) & (2) we have.

$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$ or $\operatorname{div}\vec{B} = 0$

This eqn. signifies that magnetic ~~lines of~~ flux are continuous, i.e. the number of magnetic lines of flux entering into any region is equal to the lines of flux leaving it. or, Magnetic poles cannot be isolated.

3. Maxwell's 3rd eqn:-

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}.$$

Acc. to Faraday's law of e.m. induction, when mag. flux through a circuit changes an e.m.f. 'e' is induced whose direction is always opposite to the change producing it. If ϕ_B ϕ_{mB} is the magnetic flux then

$e = -\frac{d\phi_B}{dt}$ —— (1)

$\cancel{e = + \frac{d\phi_{mB}}{dt}}$ —— (1)

The induced e.m.f. is equal to the induced electric field E around the circuit. i.e.

$e = \oint_l E\cdot dl$. —— (2)

The mag. flux through the circuit is

$$\phi_B = \int_S \bar{B}.d\bar{s} \quad \text{--- (3)}$$ where integral is over a bound surface of area s.

From above eqns. we have

$$\oint_l E.d\bar{l} = -\frac{d}{dt}\oint_S B.ds$$

$$\text{or,}\ \oint_l \bar{E}.d\bar{l} = -\oint_S \frac{\partial B}{\partial t}.ds \quad \text{--- (4)}$$

Here partial derivative is used because ~~Here~~ we are interested in change of $\bar{B}$ over small area elements $d\bar{s}$.

~~Now $\oint \bar{E}.d\bar{l}$ =~~ Applying Stoke's theorem

$$\oint \bar{E}.d\bar{l} = \int_S (\vec{\nabla}\times\vec{E}).d\vec{s}. \quad \text{--- (5)}$$

$\therefore$ From eqns (4) & (5) we have

$$\int_S (\vec{\nabla}\times\vec{E}).d\vec{s} = -\int_S \frac{\partial\vec{B}}{\partial t}.d\vec{s} \quad \text{--- (6)}$$

Eqn. (6) is true for all surfaces, therefore taking derivative ~~Integrating~~ of both sides we have

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}} \quad \text{--- (7)} \quad \text{or,} \quad \boxed{\text{Curl}\ \vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

This eqn. signifies that an electric field is produced by a changing magnetic field.

4. Maxwell's 4th eqn

$$\boxed{\nabla\times H = J + \frac{\partial D}{\partial t}}$$

Acc. to modified Ampere's law

$$\oint H.dl = \text{Total current} = I = i_c + i_d \quad \text{--- (1)}$$

i_c → Conduction current; i_d → Displacement current.

P.T.O.

Now,

$$i_c = \int_S J_c \cdot ds \quad \& \quad i_d = \int_S J_d \cdot ds \quad \text{——} (2)$$

where J_c = conduction current density,

& J_d = displacement " " ,

Putting these values in eqn. (1) we get

$$\oint H \cdot dl = \int_S J_c \cdot ds + \int_S J_d \cdot ds$$

$$\text{or, } \oint H \cdot dl = \int_S J \cdot ds + \int_S \frac{\partial D}{\partial t} ds$$

, D = electric flux density,

$J_c = J$.

$$\oint H \cdot dl = \int_S \left(J + \frac{\partial D}{\partial t} \right) \cdot ds \quad \text{——} (3)$$

Acc. to Stoke's ~~law~~ theorem

$$\oint H \cdot dl = \int_S (\nabla \times H) \cdot ds. \quad \text{——} (4)$$

From eqns. (3) & (4), we have

$$\int_S (\nabla \times H) \cdot ds = \int_S \left(J + \frac{\partial D}{\partial t} \right) \cdot ds$$

Taking derivatives of both sides we get

$$\boxed{(\vec{\nabla} \times \vec{H}) = \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right).}$$

This eqn. signifies that magnetic field is produced by conduction current & displacement current.

Sec 2.1

Continuity Eqn.

According to conservation of charge, the electric charge can neither be created nor be destroyed. As current is simply charge in motion, the total current flowing out of some volume V must be equal to the rate of decrease of charge within that volume V.

The current flow out through the closed surface is

$$\frac{dq}{dt} = i = \oint_S \vec{J}.d\vec{S} \qquad (1)$$

This current must be equal to ~~the rate at which charge flow out of it through the same vol. V~~ ~~i.e.~~ the rate of decrease of charge flowing through that region ⇒

$$i = \oint_S J\,dS = -\frac{dq}{dt} = -\int_V \left(\frac{\partial \rho}{\partial t}\right) dV \quad \left[\because \; q = \int_V \rho\, dV\right.$$

ρ = uniform volume charge density]

$$\therefore \oint_S J\,dS + \int_V \frac{\partial \rho}{\partial t}\, dV = 0$$

From divergence theorem,

$$\oint_S J\,dS = \int_V (\nabla.\vec{J})\, dV$$

Thus $\int_V (\nabla.J)\, dV + \int \left(\frac{\partial \rho}{\partial t}\right) dV = 0$

$$\text{or, } \int_V \left(\nabla.J + \frac{\partial \rho}{\partial t}\right) dV = 0$$

By taking derivative of both sides

$$\boxed{\nabla.\vec{J} + \frac{\partial \rho}{\partial t} = 0}$$ This is continuity eqn.

For static field $\frac{\partial \rho}{\partial t} = 0$ $\therefore$ Continuity eqn becomes $\boxed{\nabla.J = 0}$

P.T.O.

E.M. Waves in free space:

Free space is characterised by
$\rho = 0$, $\sigma = 0$, $J = 0$, $\mu = \mu_0$ & $\epsilon = \epsilon_0$. Therefore Maxwell's eqn. reduces to

$$\vec{\nabla}\cdot\vec{D} = \vec{\nabla}\cdot\vec{E} = 0 \quad \text{—— (1)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{—— (2)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t} \quad \text{—— (3)} \qquad \text{or, } \vec{\nabla}\times\vec{E} = -\mu_0\frac{\partial H}{\partial t} \quad \text{—— 3(a)}$$

$$\vec{\nabla}\times\vec{H} = \frac{\partial D}{\partial t} \quad \text{—— (4)} \qquad \text{or, } \vec{\nabla}\times\vec{H} = \epsilon_0\frac{\partial E}{\partial t} \quad \text{—— 4(a)}$$

$$D = \epsilon_0 E \quad \& \quad B = \mu_0 H.$$

Taking Curl of eqn 3(a) we get

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\mu_0\left(\nabla\times\frac{\partial H}{\partial t}\right)$$

or, $\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\mu_0\frac{\partial}{\partial t}(\nabla\times H)$ [by interchanging space & time derivatives]

or, $\nabla\times\nabla\times E = -\mu_0\frac{\partial}{\partial t}\left(\epsilon_0\frac{\partial E}{\partial t}\right)$, by using eqn 4(a)

$$\text{or, } \nabla\times\nabla\times E = -\mu_0\epsilon_0\,\partial^2 E/\partial t^2 \quad \text{—— (5)}$$

By Vector Identity we have $\nabla(\nabla\cdot E) - \nabla\cdot\nabla E$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \nabla(\underbrace{\nabla\cdot E}_{0 \text{ (by 1st law)}}) - \nabla^2 E = -\nabla^2 E = -\nabla^2 E$$

Vector triple product
$a\times(b\times c)$
$(1\cdot3)2 - (1\cdot2)3$
$(a\cdot c)b - (a\cdot b)c$

$\therefore$ Eqn (5) becomes

$$\boxed{\nabla^2\vec{E} = \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}} \quad \text{—— (6)}$$

Similarly by taking Curl of eqn. (4a) & using eqn. (3a) we get

$$\boxed{\nabla^2\vec{H} = \mu_0\epsilon_0\frac{\partial^2\vec{H}}{\partial t^2}} \quad \text{—— (7)}$$

The well known eqn. of an E.M. Wave propagating with a velocity v is given by

$$\nabla^2 u = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} \quad \text{—— (8)}$$, where u is any scalar or vector quantity

Comparing eqn (8) with eqn. (6) or (7) we have

$$\boxed{v = \frac{1}{\sqrt{\mu_0\epsilon_0}} = c} \text{ (for free space).}$$

Eqns (6) & (7) represent ~~plane~~ e.m. wave eqns. governing $\bar{E}$ & $\bar{H}$ in free space.

For free space $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3\times10^8$ m/sec $= c$ (vel. of light in vacuum). ← (unit

$\mu_0 = 4\pi\times10^{-7}$ N/A²; $\epsilon_0 = 8.85\times10^{-12}$ F/m or C²/(N-m²) ← *

∴ Eqns (6) & (7) can be represented in term of vel. as

$$\boxed{\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}} \quad \& \quad \boxed{\nabla^2 H = \frac{1}{c^2}\frac{\partial^2 H}{\partial t^2}}.$$

The same form is obtained for $\bar{B}$ & $\bar{D}$ also i.e.

$$\boxed{\vec{\nabla}^2 \bar{B} = \frac{1}{c^2}\frac{\partial^2 \bar{B}}{\partial t^2}} \quad \& \quad \boxed{\vec{\nabla}^2 \bar{D} = \frac{1}{c^2}\frac{\partial^2 \bar{D}}{\partial t^2}}.$$

* But refractive index 'n' of the medium

$n = \frac{c}{v} = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$ in absence of mag. medium $\mu = \mu_0$ ∴ $n = \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{\epsilon_r}$

∴ $n = \sqrt{\epsilon_r}$ ∴ $\boxed{\epsilon_r = n^2}$ → R.I. of the medium. (ϵ_r → Rel. permittivity)

$v = c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\left(\frac{N}{A^2}\right)\left(\frac{C^2}{N\cdot m^2}\right)}} = \frac{A\,m}{C}$

$A = \text{amp} = i = \frac{dq}{dt} = \frac{C}{sec}$

∴ $\frac{Am}{C} = \frac{C}{sec}\cdot\frac{m}{C} = m/sec =$ velocity

Propagation characteristics of E.M.W. in free space:

Transverse nature of E-M. Waves:

Plane e.m. waves for free space governing E & H are given by

$$\vec{\nabla}^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{—— (1)} \qquad \vec{\nabla}^2 \vec{H} = \frac{1}{c^2}\frac{\partial^2 \vec{H}}{\partial t^2} \quad \text{—— (2)}$$

The plane wave solutions of eqns (1) & (2) are.

$$E = E_0 e^{i(\bar{K}\cdot\bar{r} - \omega t)} \quad \text{—— (3)} \qquad \text{Here } i = \sqrt{-1}$$

$$H = H_0 e^{i(\bar{K}\cdot\bar{r} - \omega t)} \quad \text{—— (4)}$$

where E_0 & H_0 are complex amplitudes which is constant in space & time & $\vec{K}$ is a wave vector or propagation vector.

Now, $\vec{\nabla}\cdot\vec{E} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(E_0 e^{i(K\cdot r - \omega t)}\right)$

$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}E_{0x} + \hat{j}E_{0y} + \hat{k}E_{0z}\right)e^{i(K\cdot r - \omega t)}$

(P.T.O.)