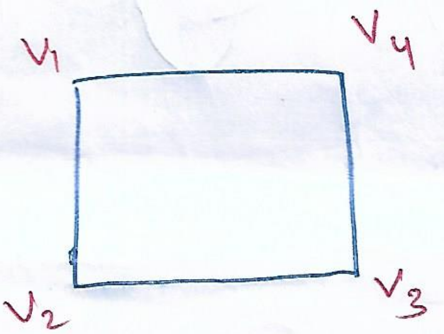
part-1

## Bipartite graph

'g' is bipartite if its vertex set 'V' can be partitioned into '2' disjoint non-empty sets $S_1$ & $S_2$ such that every edge in the graph connects a vertex in '$S_1$' & a vertex in '$S_2$'. so that no edge in 'g' connects either two vertices in '$S_1$' or two vertices in $S_2$.

Ex




→ convert this into '2' nonempty disjoint set

$V_1, V_2, V_3, V_4$

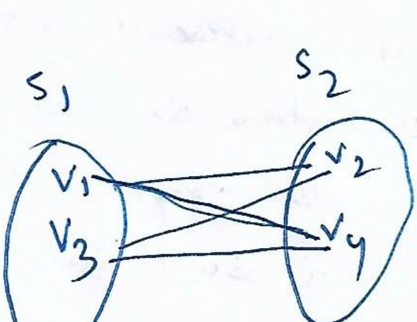
$S_1$ ~~( $V_1$, $V_3$)~~

$S_2$ ~~( $V_2$, $V_4$)~~

$S_1 = \{ V_1, V_3 \}$

$S_2 = \{ V_2, V_4 \}$

← disjoint means no vert are common

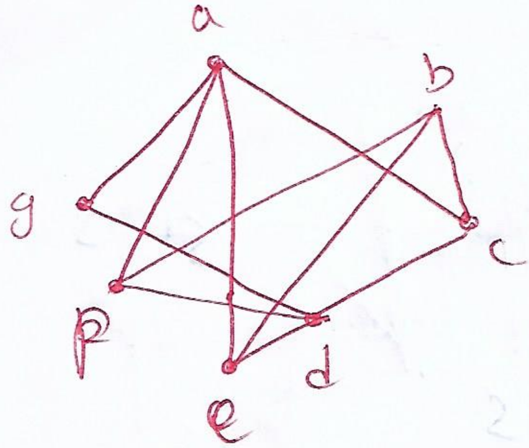



① draw all edges

② check, are they belongs to diff set or not.

③ ~~chee~~ If yes, then can Bipartite graph.

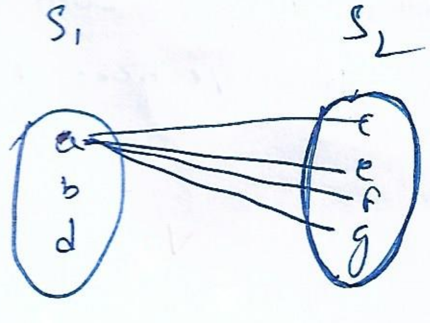
pacre·2



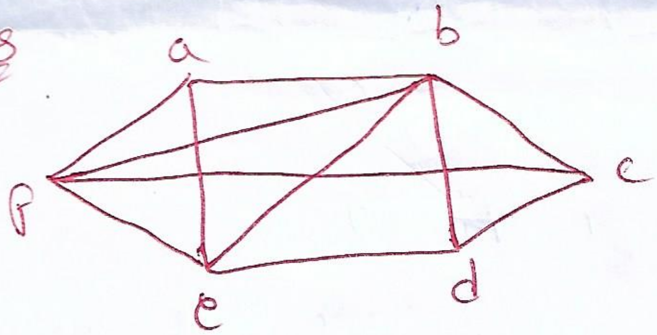

$V(G) = \underline{a}, \underline{b}, c, \underline{d}, e, f, g.$

$S_1 = \{a, b, d\}$

$S_2 = \{c, e, f, g\}$




eg




Check for the given graph is bipartite graph or not?

$S_1 = \{a, \underline{d, c}\}$ ↳ edge

$S_1 = \{a, e\}$

$S_2 = \{b\}$

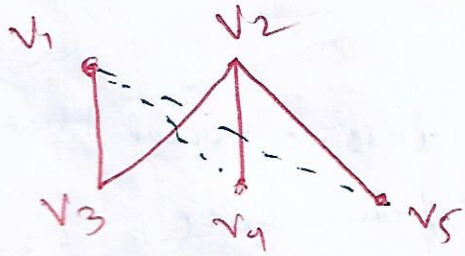
$S_3 = \{f, d\}$

So it is not a bipartite graph. because its vertex can't be ~~partitioned~~ partition into two sets, such that edges can't connect two vertices from same subset.

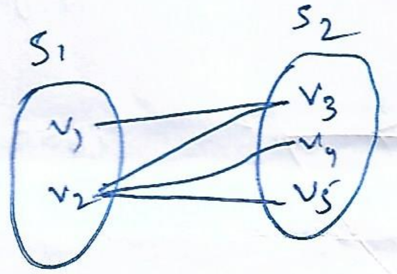
π-3

## Complete Bipartite graph.

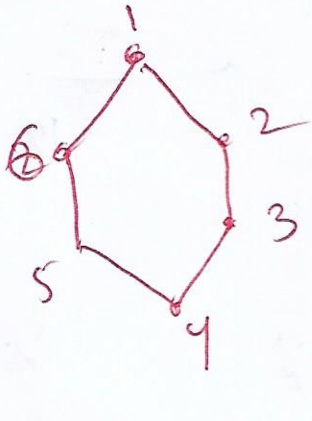
If a graph has its vertex set partitioned into 2 subsets of 'm' & 'n' vertices respectively.
There is an edge b/w two vertices if & only if one vertex is in first subset & other in second subset.




$S_1 = \{ V_1, V_2 \}$
$S_2 = \{ V_3, V_4, V_5 \}$




Some more example.




$S_1 = \{ 1, 3, 5 \}$
$S_2 = \{ 2, 4, 6 \}$

page-4

## Euler graph

① euler path

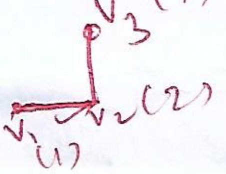
↳ it is a path that traverse each edge exactly once & only once.

→ A graph that contains an euler path is called as euler graph.

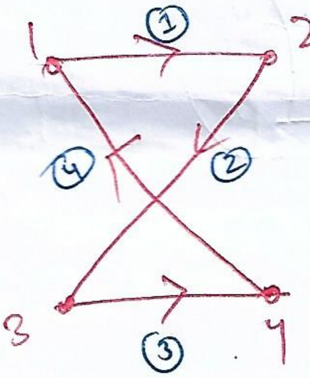
② euler circuit

↳ first & last vertex are same : it is a circuit that traverse each edge exactly once & only once.

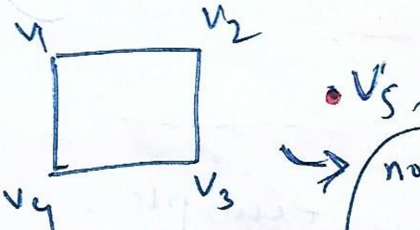
(euler path)




Eg




1→2→3→4




→ not an euler graph as it is not connected

### Some properties of Euler graph.

① A connected graph is a Euler graph iff, it has atmost 2 odd degree vertices.

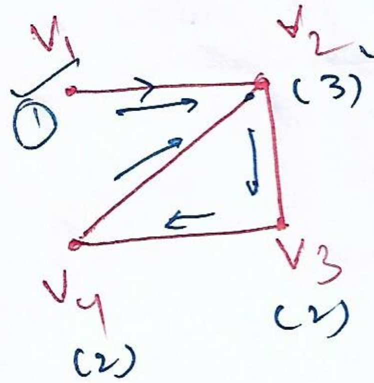
② In euler circuit, each vertex is of even degree.

E+P

③ in euler path max$^n$ ; 2 vertexes has odd degree.

1-S

eg 2

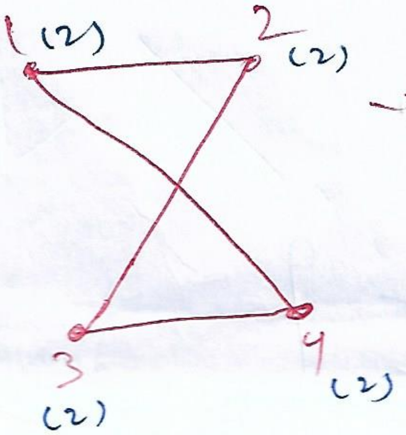



$v_1 \to v_2 \to v_3 - v_4 \to v_2$

↳ euler path

→ euler ~~graph~~ path

↳ 2 vertex has odd degree.

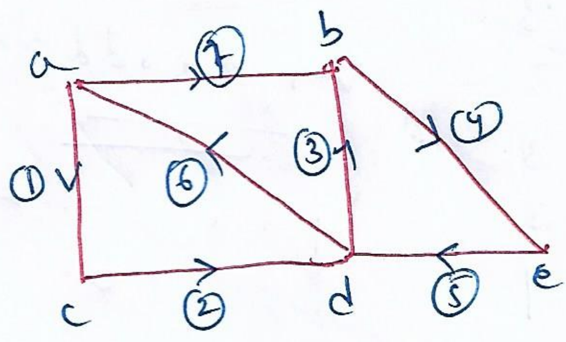



→ euler circuit

each vertex of even degree

---

Q5 Find euler path




1. all edge cover.
2. no repeate edges
3. vertex may repeate.

a - c - d - b - e - d - a - b

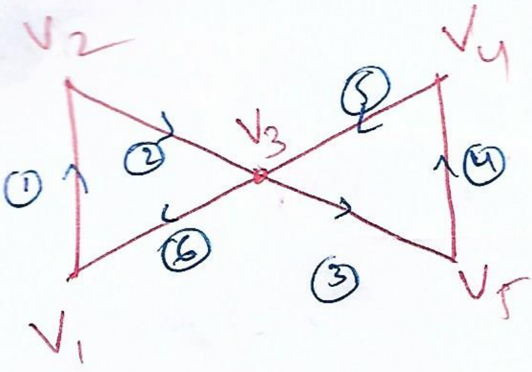
2 vertex has odd degree

- a → 3
- b → 3
- c → 2
- d → 4
- e → 2

cound → euler graph

page-6

(2)




→ It is a Euler circuit.

→ initial $v_1$ & end $v_1$

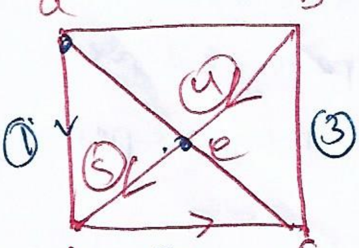
$v_1 \to v_2 - v_3 - v_5 - v_4 - v_3 - v_1$

degree → even

$v_1 \to 2$
$v_2 \to 2$
$v_3 \to 4$
$v_4 - 2$
$v_5 - 2$

even degree

(3)




a → 3
b → 3
c → 3
d → 3
e → 4

many are odd degree

4 vertices with odd degree

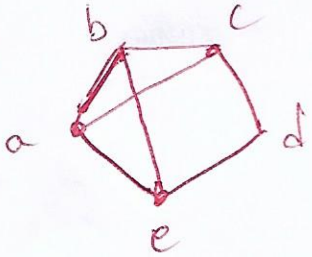
(1) So no Euler path

(2) all not even degree

no Euler circuit

ase - 7

Q)




a -> 3
b -> 3
c -> 3
d -> 2
e -> 3

Check for Euler cnt. & if it is not Euler circuit,

~~check~~ check for Euler path.

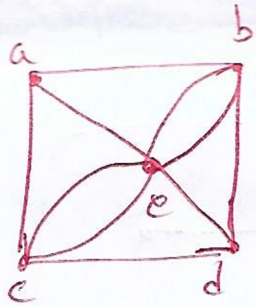
* all are not even degree. (So not Euler circuit)

* Euler path

↳ rule max$^n$ - 2 vertex are odd, but here more than '2'.

So, it is neither Euler path or circuit.

Q)



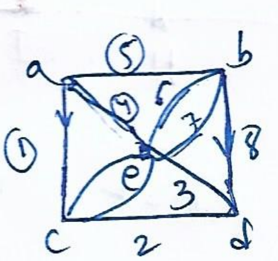

-> a - 3
b - 4
c - 4
d - 3
e - 6

(not a Euler circuit) -> not an even no

but -> Euler ~~cct~~ path

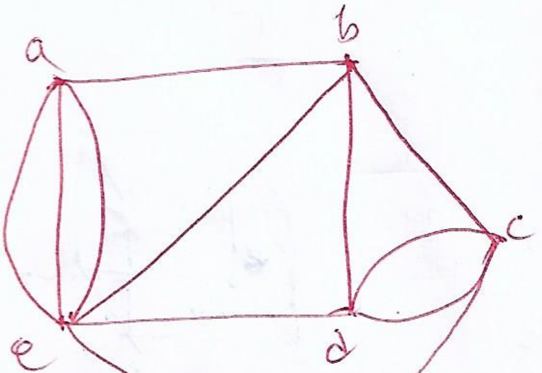
as max$^n$ 2 are odd n

path -> a-e-c-e-b-e-d-b-a-c




a ->

Q)

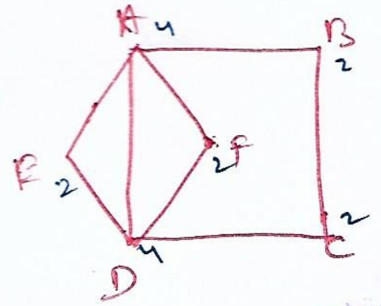



a -> 4
b - 4
c - 4
d -> 4
e - 6

Euler circuit

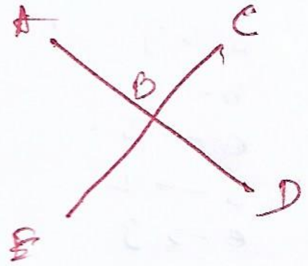
↳ a-b-c-d-c-
e-d-b-e-a-c-

page-8 check for Euler path & Circuit

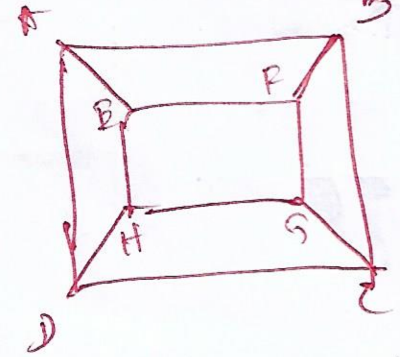
1



Euler ~~graph path~~

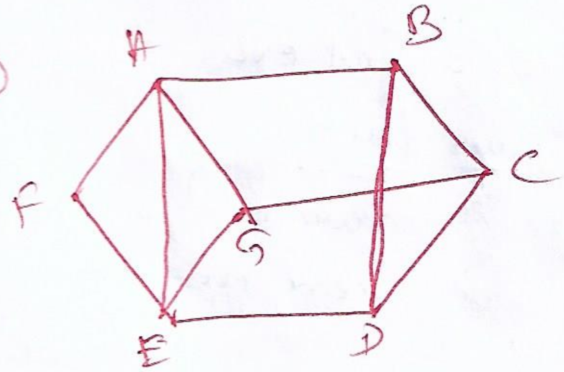
Euler Ckt

2


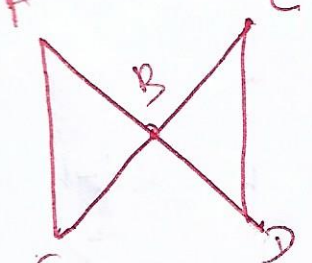

not Euler graph

3


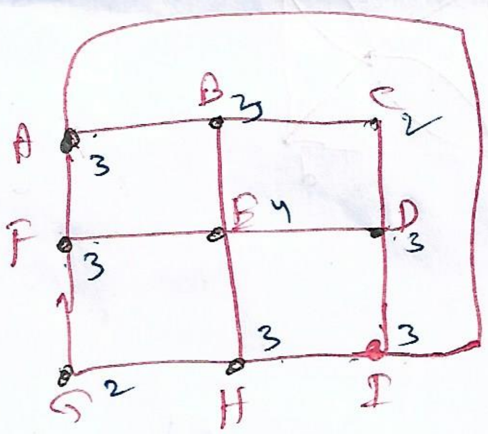

not -> graph

4



not graph

5


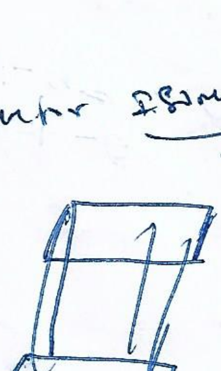

Euler graph

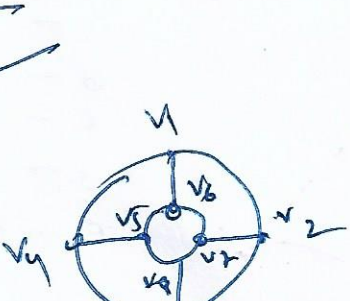
6


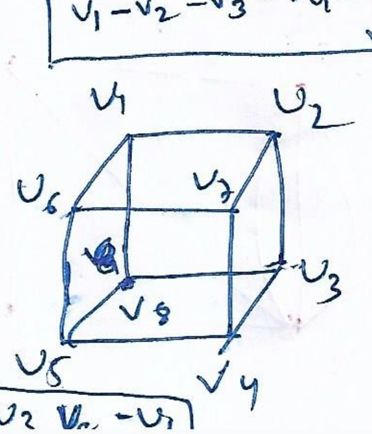

not Euler graph

$v_1 - v_2 - v_3 - v_4 - v_5 - v_8 - v$
$v_6 - v_7$

check for Isomorphic






$v_1 - v_2 - v_7\ v_6\ v_5\ v_4\ v_2\ v_8 - v_1$

se-9

## HAMILTON GRAPH

-> Hamiltonian path contains each vertex exactly once.

Exp


$v_1 - v_2 - v_3 - v_4 - v_5$.

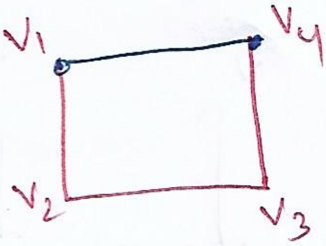
-> In Hamitonian circuit. ~~Contains~~ 1st & last vertex are same. & no vertex are repeated. (cover all vertex)

Exp -> * on removing any one edge from a hamilton circuit. then we are left with Hamelton path.

Theorem:-

Let G be a graph of 'n' vertices, then G has a hamiltonian circuit ~~path~~ if two vertices 'u' & 'v' ~~of~~ have satisfied below condition.

i.e $\boxed{deg(u) + deg(v) \geq n}$

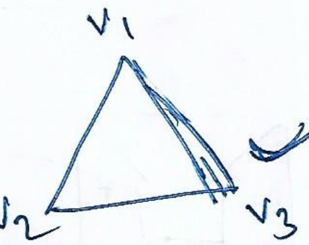



Hamiltonian path

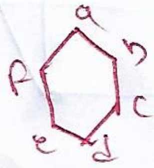
↳ ~~Vp~~ node = 4 (vertices)

$v_1 = 3$
$v_2 = 2$
$v_3 = 2$
$v_4 = 2$

add anytime $\geq n$



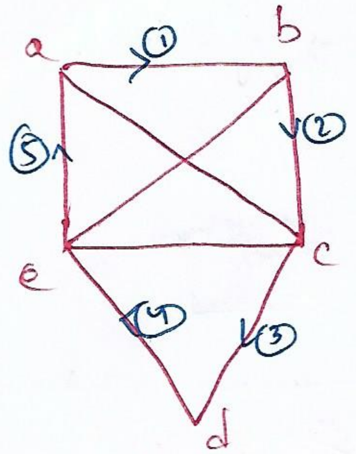

* it is a partiality true statement.

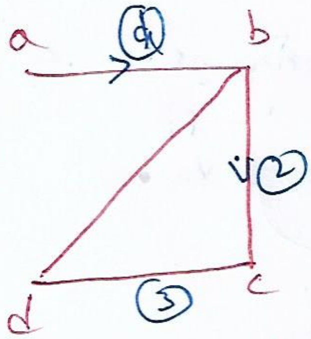


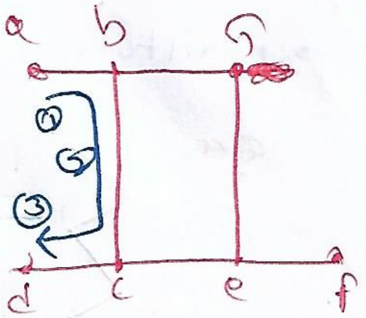

$v + u \geq n$ (X)

page-10

exp







①

→ Hamilton ckt.
→ all node.
→ starting & ending are same.

1-2-3-4-5
[a-b-c-d-e-a].

Hamilton path.

①-2-3 (a-b-c-d)
(a-b-d-c)

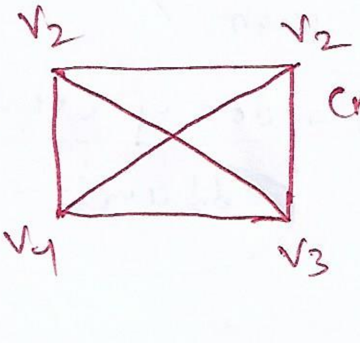
not path or ckt as node repeat.

---

## planar graph

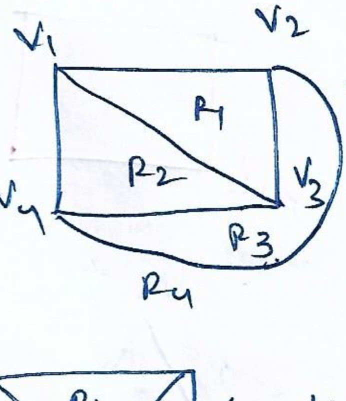
→ A graph is called planar, if it can be drawn in the plane without any edges crossing.

exp

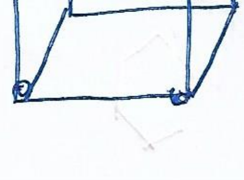


(non planar)

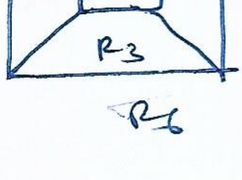
↔



planar graph



→



← planar graph.

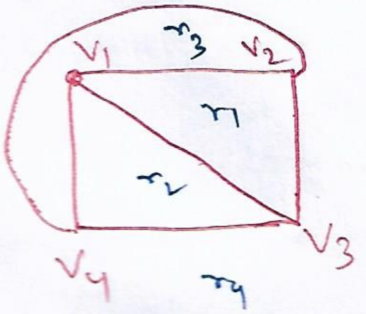
-11

# Euler's formula on planar graph.

Let 'G' be a connected planar graph with 'e' edges & vertices 'V'. Let 'r' be the no of regions in planar graph 'g', then

$$\boxed{r = e - v + 2}$$

Exp




edge = 6

v = 4

$$\boxed{r = 6 - 4 + 2 = 4}$$

(Q5) If there are 20 vertices, each of degree '3', then into how many regions does can be generate in this planner graph splits.

$\underline{V = 20}$

$\sum \deg(V) = \underline{2 \times e}$

$2 \times e = 20 \times 3 = 60$

$\Rightarrow \boxed{e = 30}$

$r = e - v + 2$

$= 30 - 20 + 2$

$\boxed{r = 12}$

3
3
3
3
12

page-12

proof Euler's formula using mathematical induction method.

need to proof $\boxed{r = e - v + 2}$

① basic $n = 1$, means for edge $= 1$

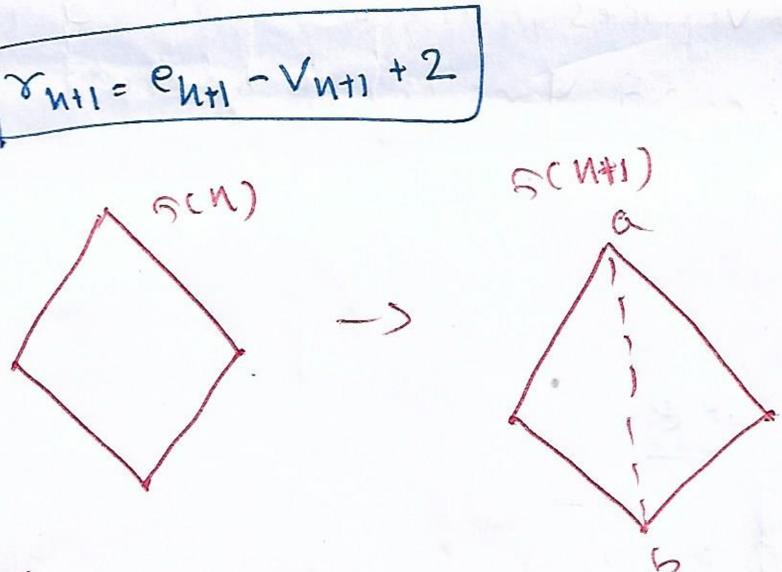

$r = 1 - 2 + 2 = 1$

Step-2 (n)

$\boxed{r_n = e_n - v_n + 2}$

$P(n+1)$ $\boxed{r_{n+1} = e_{n+1} - v_{n+1} + 2}$

let




case-1

$\boxed{r_{n+1} = e_{n+1} - v_{n+1} + 2}$

$e_{n+1} = e_n + 1$

$v_{n+1} = v_n$

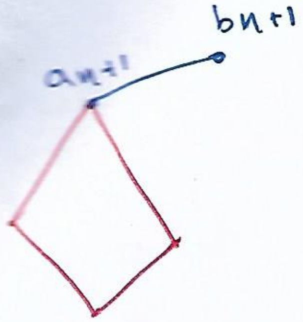
$r_{n+1} = r_n + 1$

$\Rightarrow r_n + \cancel{1} = e_n + \cancel{1} - v_n + 2$

$\boxed{r_n = e_n - v_n + 2}$

n-13

Case-2




$$r_{n+1} = r_n$$

$$v_{n+1} = v_n + 1$$

$$e_{n+1} = e_n + 1$$

$$r_{n+1} = e_{n+1} - v_{n+1} + 2$$

$$\Rightarrow r_n = e_n + \cancel{1} - v_n - \cancel{1} + 2$$

$$\boxed{r_n = e_n - v_n + 2}$$