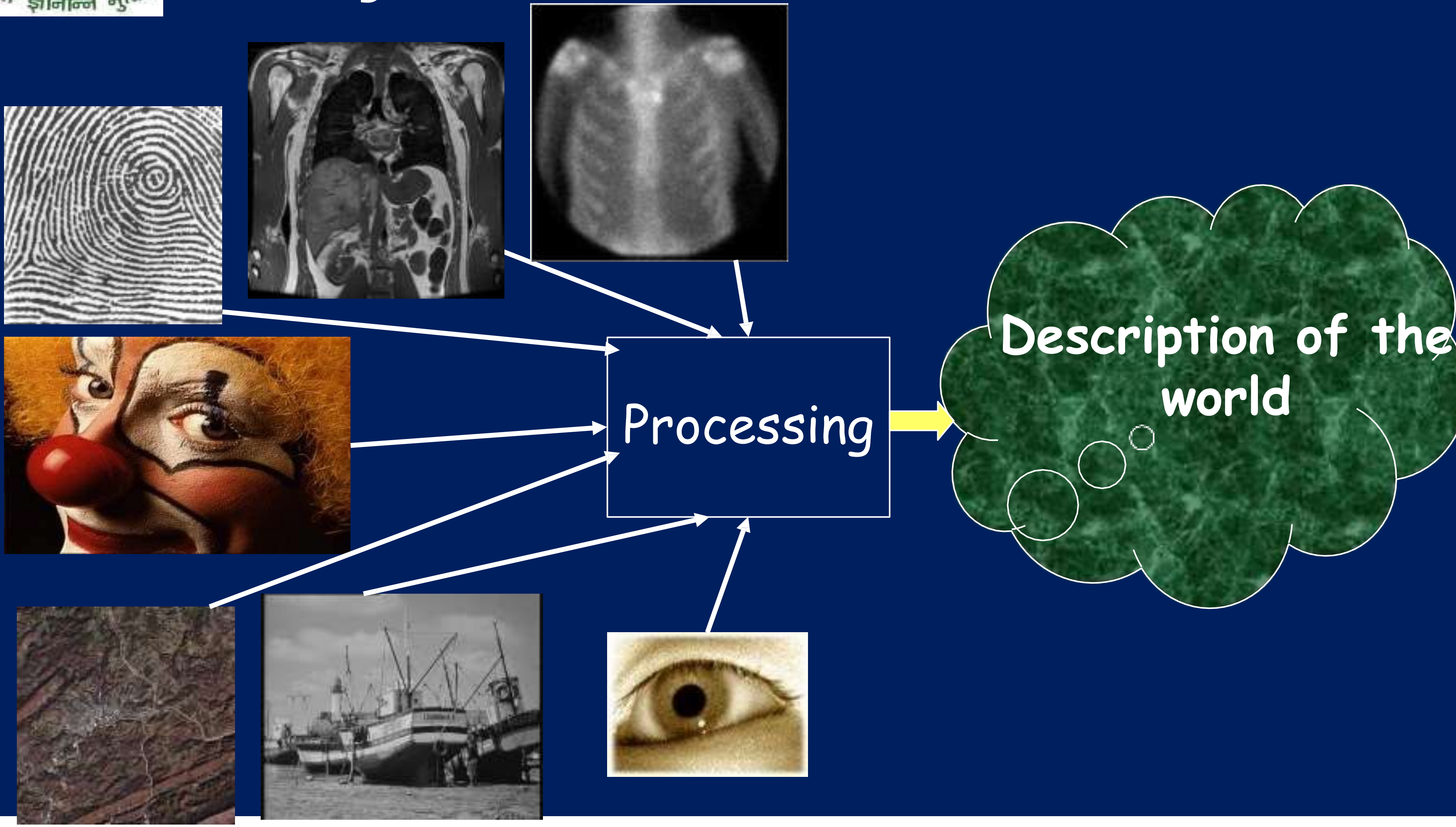
MCSC0009: Image Processing And Analysis



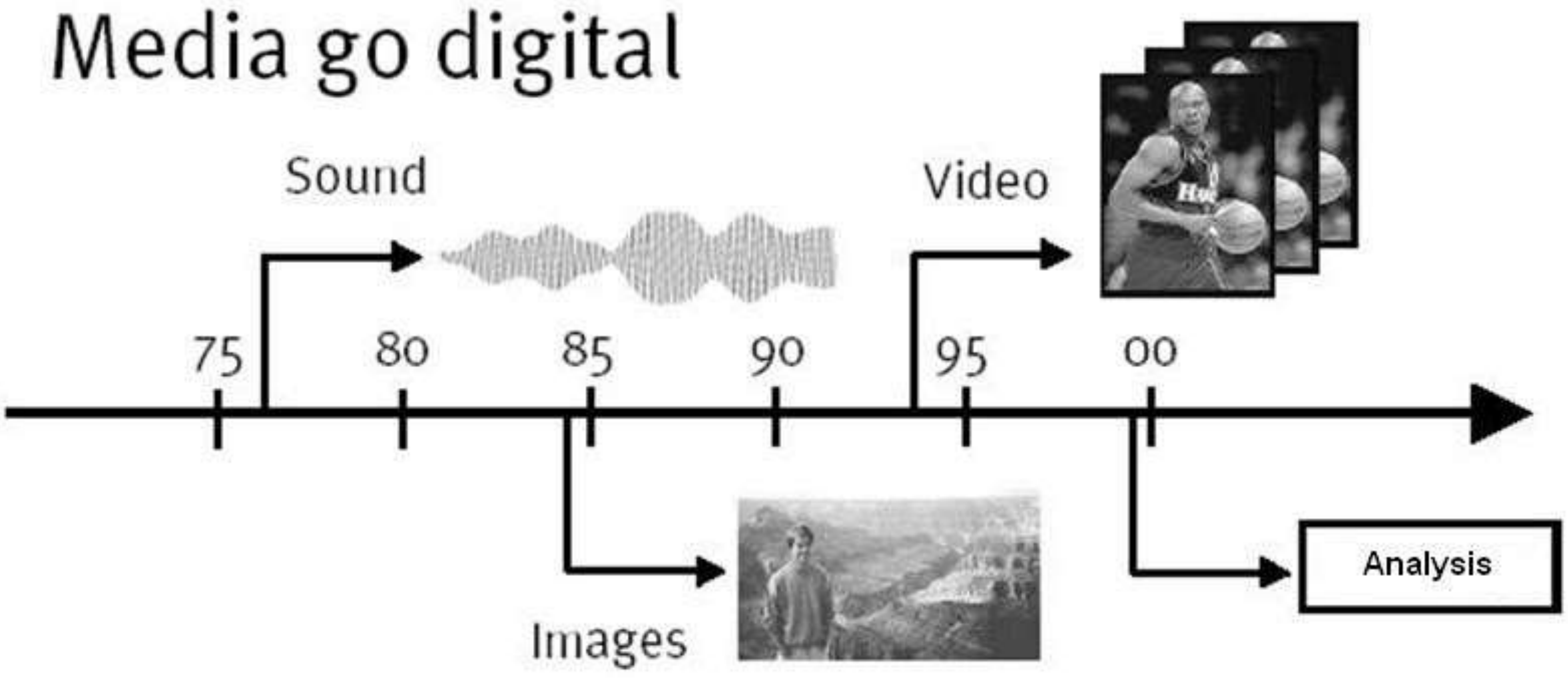
MCS 2004 General Information

- **Suggested textbook:**
  - R.C. Gonzalez and R.E. Woods, "**Digital Image Processing**", 3rd edition, Prentice-Hall' 2011
  - Chanda, Bhabatosh, Majumder, Dwijesh Dutta "**Digital Image Processing And Analysis**" 2nd edition, PHI Learning
- **Prerequisites**
  - Knowledge of the following three areas:
    - Linear algebra, Elementary probability theory, Digital Signal Processing

Teaching Objectives

- **By the end of this semester, you will**
  - Know basics of Digital Image Processing and Computer Vision including Image Acquisition, Perception, Transformation, Enhancement, Compression, Segmentation, Analysis, and so on ……
  - Be able to use MATLAB to implement basic image processing algorithms and get familiar with some functions provided by MATLAB image processing toolbox

Media



More than 80% of information is received by visual perception

Media …

- Early days of computing, data was numerical.
- Later, textual data became more common.
- Today, many other forms of data exist: voice, music, speech, images, computer graphics, etc
- Each of these types of data are signals.
- Loosely defined, a signal is a function that conveys information.

A Picture in worth a

1,000 words

Digital Image Processing

- A major portion of the information received by a human from the environment is visual

- Processing visual information by computer is drawing a lot of attention by the researchers.

- The process of receiving & analyzing (processing) visual information by digital computer is called

Digital Image Processing

What is a Digital Image?

- A **digital image** is a representation of a two-dimensional image as a finite set of digital values, called picture elements or pixels

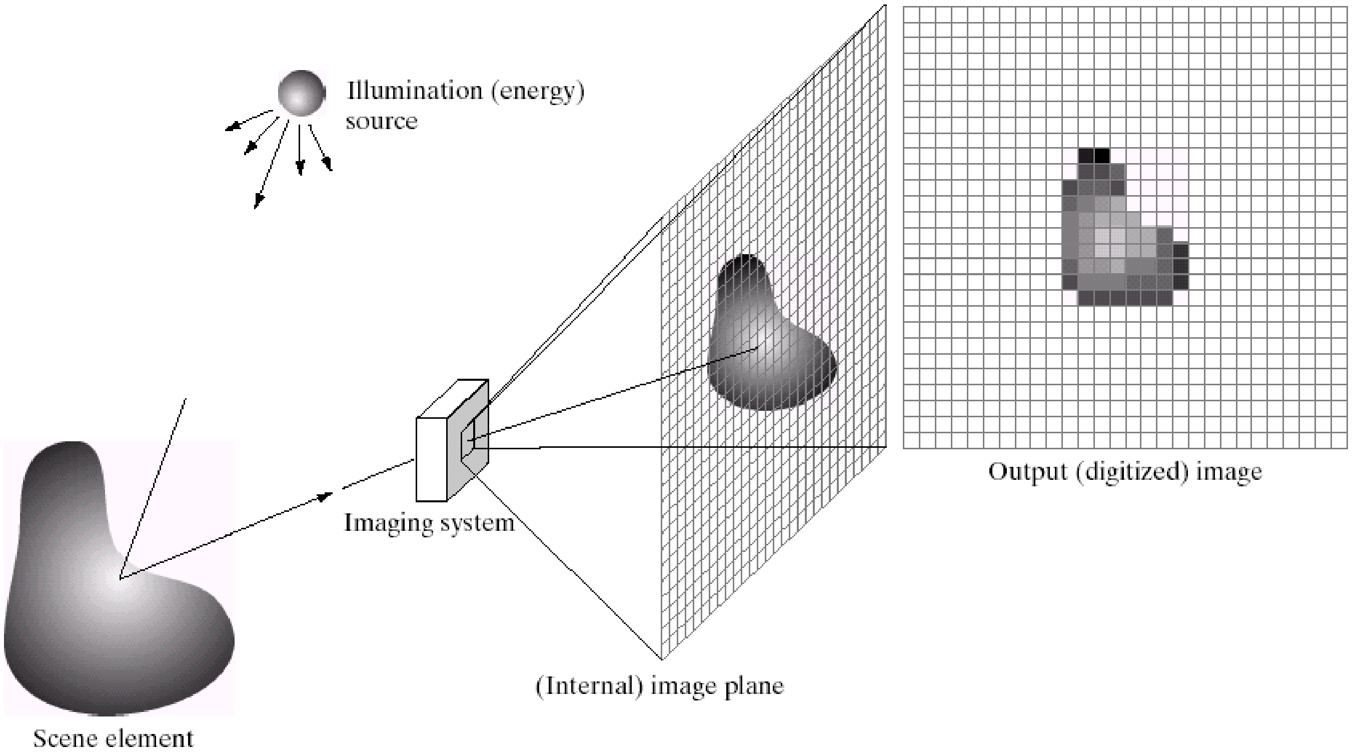
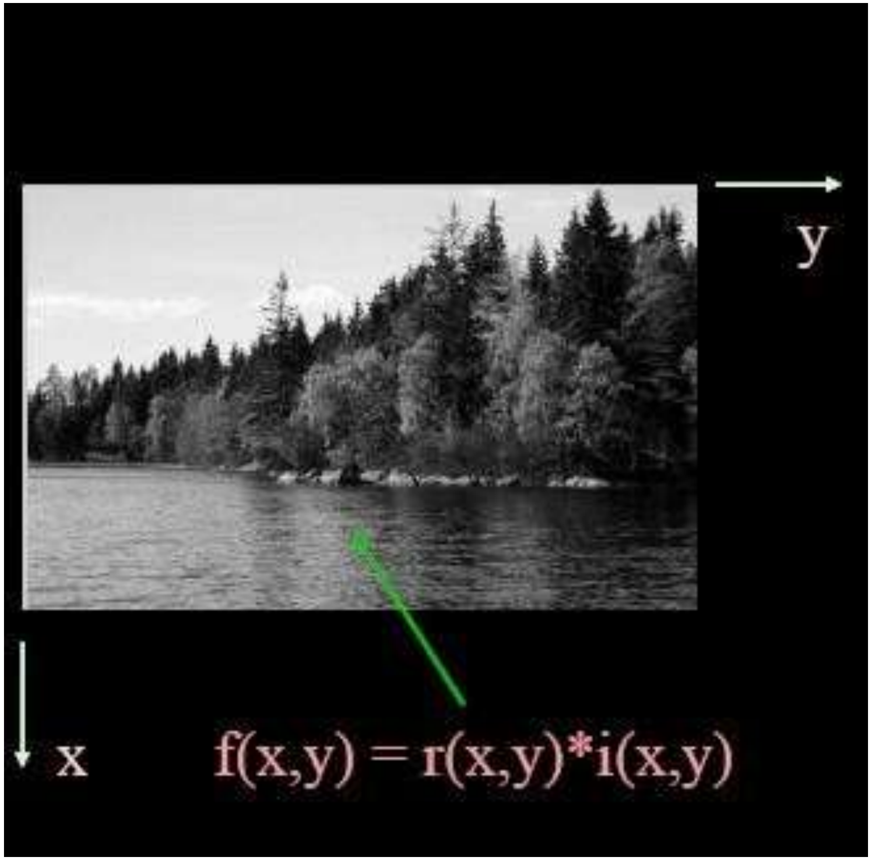


Image Representation



r(x,y) – reflectance of surface (0-1)
i(x,y) – intensity of light (0-infinite)

- An image is a 2-D light intensity function f(x,y)
- A digital image f(x,y) is discretized both in spatial coordinates and brightness
- It can be considered as a matrix whose row, column indices specify a point in the image and the element value identifies gray level value at that point
- These elements are referred to as pixels or pels

Image Representation …

An image is a two-dimensional function **f(x,y)**, where x and y are the **spatial** (plane) coordinates, and the amplitude of **f** at any pair of coordinates (x,y) is called the intensity of the image at that level.

If x,y and the amplitude values of f are finite and discrete quantities, we call the image a digital image. A digital image is composed of a finite number of elements called pixels, each of which has a particular location and value.

Image Representation …

- Spatial discretization by grids
- Intensity discretization by quantization

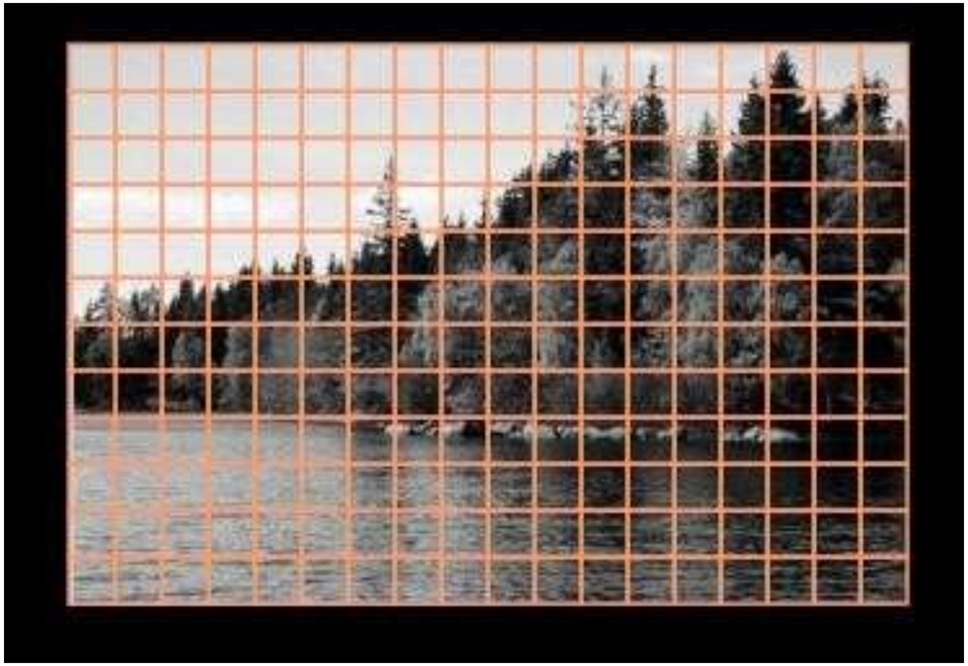


Image Representation …

$$I = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2)\ldots\ldots\ldots & f(0,N-1) \\ f(1,0) & f(1,1) & f(1,2)\ldots\ldots\ldots & f(1,N-1) \\ f(2,0) & f(2,1) & f(2,2)\ldots\ldots\ldots & f(2,N-1) \\ \vdots & \vdots & \vdots & \vdots \\ f(M-1,0) & f(M,1) & f(M,2)\ldots & f(M-1,N-1) \end{bmatrix}$$

- Image Size : 256x256, 512x512, 1024x1024 etc
- Quantization: 8 bits

Image Representation …

Image Representation …

Pixel intensity value

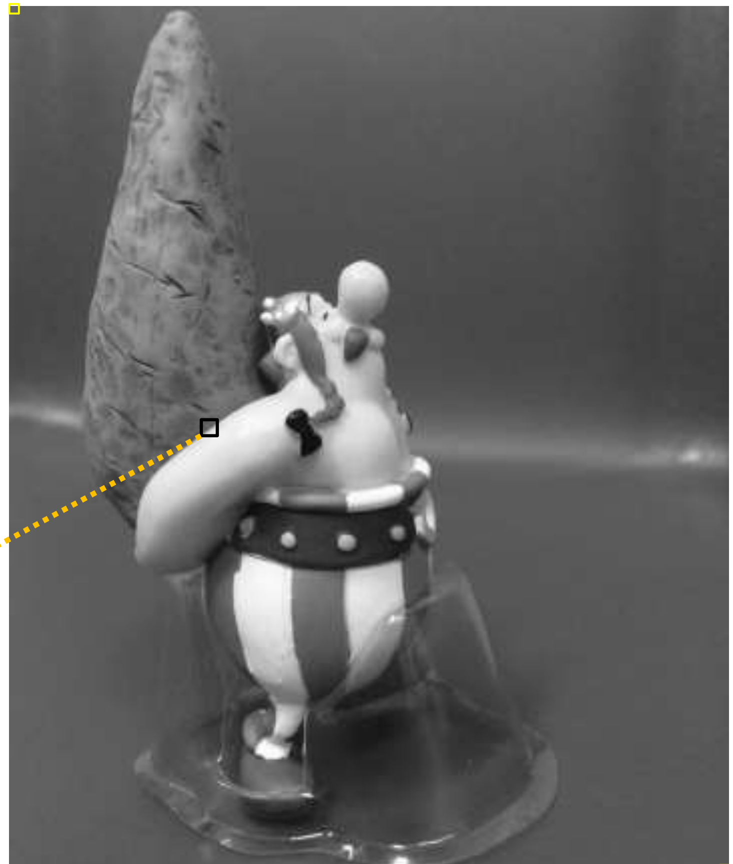
f(1,1) = 103

Pixel location

rows columns

f(645:650,1323:1328) =

83	82	82	82	82	82
82	82	82	81	81	81
82	82	81	81	80	80
82	82	81	80	80	79
80	79	78	77	77	77
80	79	78	78	77	77



f(2724,2336) = 88

Consider the following image (2724x2336 pixels) to be 2D function or a matrix with rows and columns

In 8-bit representation Pixel intensity values change between 0 (Black) and 255 (White)

Digital Image Processing …

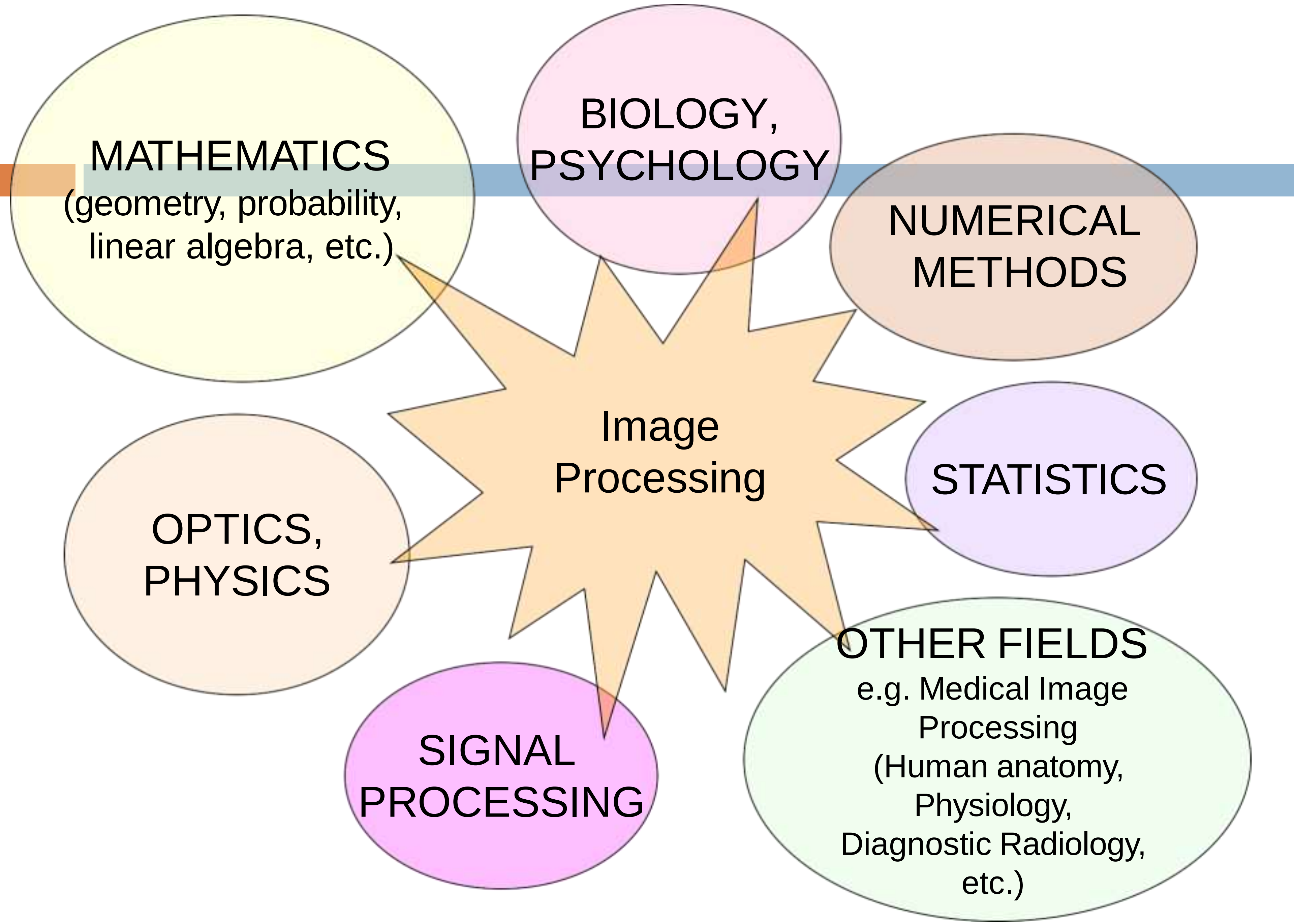
- **More specifically, DIP is**

- Improvement of pictorial for human information interpretation.
- Processing of image data for storage, transmission, and representation for autonomous machine perception.
- **No clear boundary as to where image processing ends and fields such as image analysis and computer vision start**

Image Processing Vs Computer Vision

The continuum from image processing to computer vision can be broken up into low-, mid- and high-level processes

Low Level Process	**Mid Level Process**	**High Level Process**
Input: Image **Output:** Image	**Input:** Image **Output:** Attributes	**Input:** Attributes **Output:** Understanding
Examples: Noise removal, image sharpening	**Examples:** Object recognition, segmentation	**Examples:** Scene understanding, autonomous navigation


MATHEMATICS
(geometry, probability,
linear algebra, etc.)
BIOLOGY,
PSYCHOLOGY
NUMERICAL
METHODS
Image
Processing
STATISTICS
OPTICS,
PHYSICS
SIGNAL
PROCESSING
OTHER FIELDS
e.g. Medical Image
Processing
(Human anatomy,
Physiology,
Diagnostic Radiology,
etc.)

Quantization

Quantization is **a lossy compression technique which is achieved by compressing a range of values to single quantum**. In other words, we can also say that it is a process of converting a continuous range of values into a finite range of discrete values.

Sampling

The sampling rate determines the spatial resolution of the digitized image, while the quantization level determines the number of grey levels in the digitized image. A magnitude of the sampled image is expressed as a digital value in image processing. The transition between continuous values of the image function and its digital equivalent is called quantization.

History of Digital Image Processing

First Digital Photograph

In 1957 when Russell Kirsch made a 176×176 pixel **digital image**

History of Digital Image Processing

Early 1920s: One of the first applications of digital imaging was in the news-paper industry

- The Bartlane cable picture transmission service
- Images were transferred by submarine cable between London and New York
- Pictures were coded for cable transfer and reconstructed at the receiving end on a telegraph printer

Early digital image

History of Digital Image Processing …

Mid to late 1920s: Improvements to the Bartlane system resulted in higher quality images

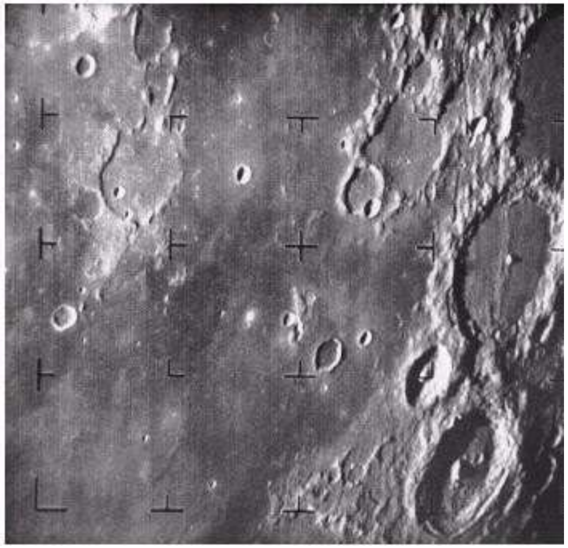
- New reproduction processes based on photographic techniques
- Increased number of tones in reproduced images

Improved digital image

History of Digital Image Processing …

1960s: Improvements in computing technology and the onset of the space race led to a surge of work in digital image processing

- **1964:** Computers used to improve the quality of images of the moon taken by the *Ranger 7* probe
- Such techniques were used in other space missions including the Apollo landings

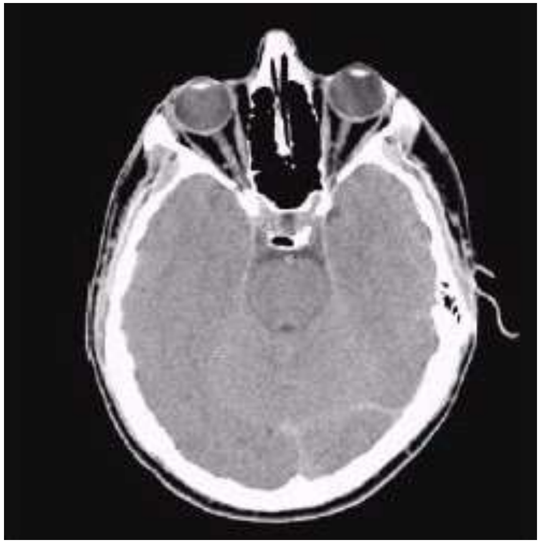


A picture of the moon taken by the Ranger 7 probe minutes before landing

History of Digital Image Processing …

1970s: Digital image processing begins to be used in medical applications

- **1979:** Sir Godfrey N. Hounsfield & Prof. Allan M. Cormack share the Nobel Prize in medicine for the invention of tomography, the technology behind Computerised Axial Tomography (CAT) scans



Typical head slice CAT image

History of Digital Image Processing …

1980s - Today: The use of digital image processing techniques has exploded and they are now used for all kinds of tasks in all kinds of areas

- Image enhancement/restoration
- Artistic effects
- Medical visualisation
- Industrial inspection
- Law enforcement
- Human computer interfaces

Why do we need Image Processing?

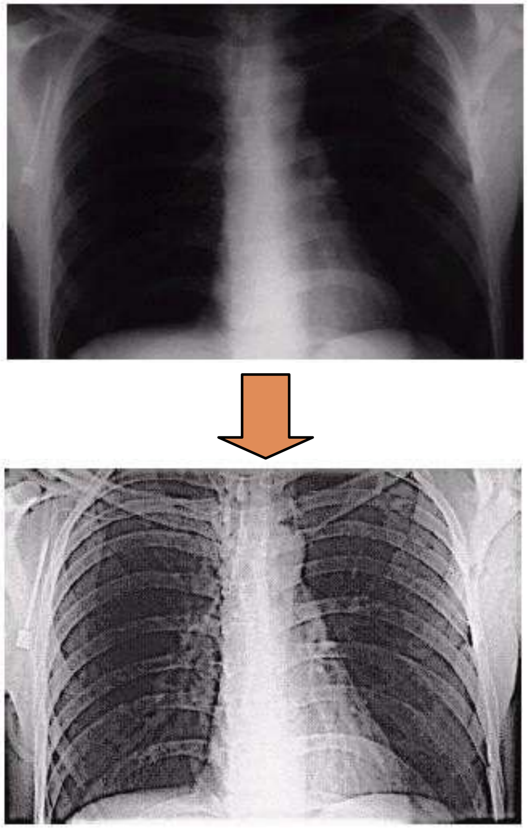
- What do we mean by *Digital Image Processing*
  - **Processing digital images by a digital computer**

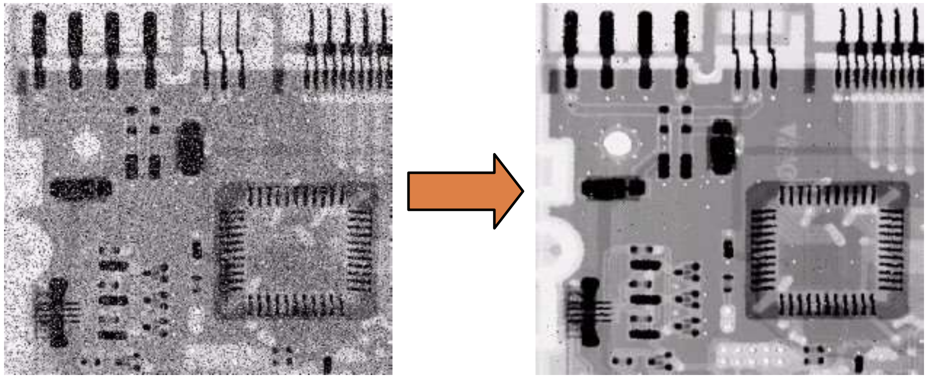
It is Motivated by three major applications-

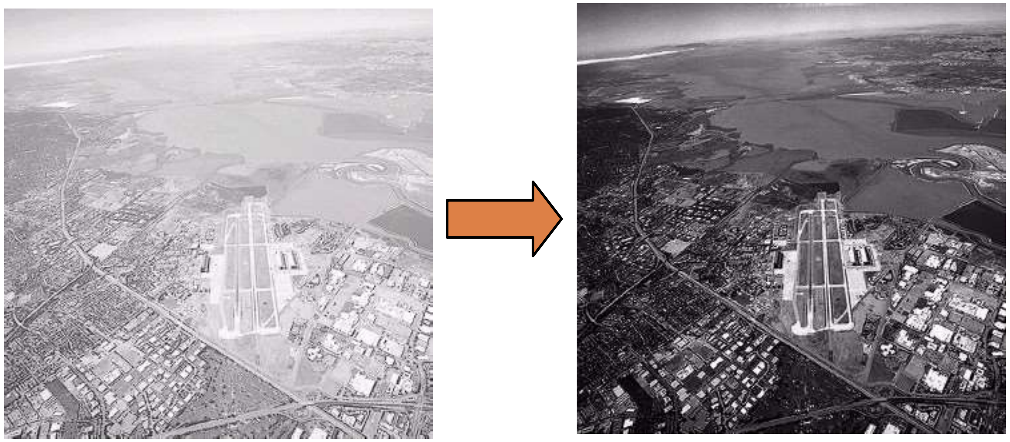
- Improvement of pictorial information for human perception
- Image processing for autonomous machine application
- Efficient storage and transmission

Examples: Image Enhancement

One of the most common uses of DIP techniques: improve quality, remove noise etc







Human Perception

Employ methods capable of **enhancing pictorial information** for human interpretation and analysis

- Typical applications:
  - Noise filtering
  - Content enhancement
    - Contrast enhancement
    - Deblurring
  - Remote sensing

Filtering

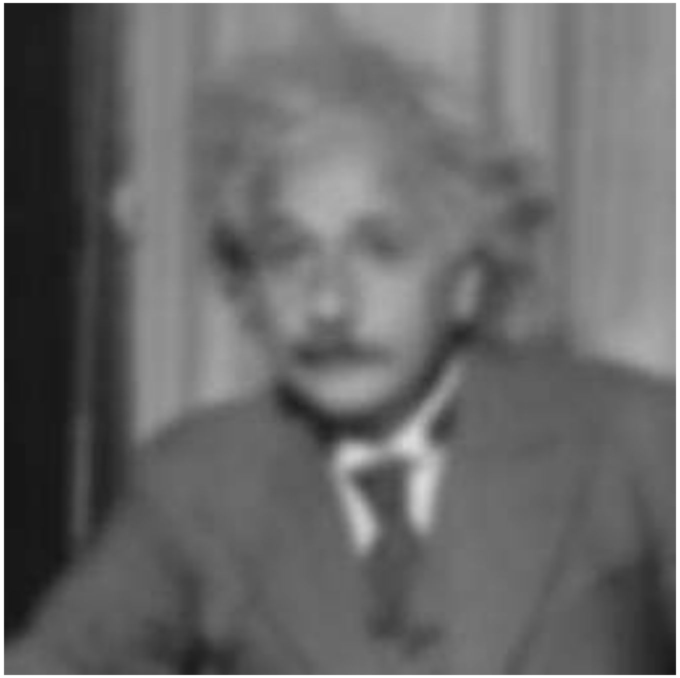
Filtering

Noisy Image

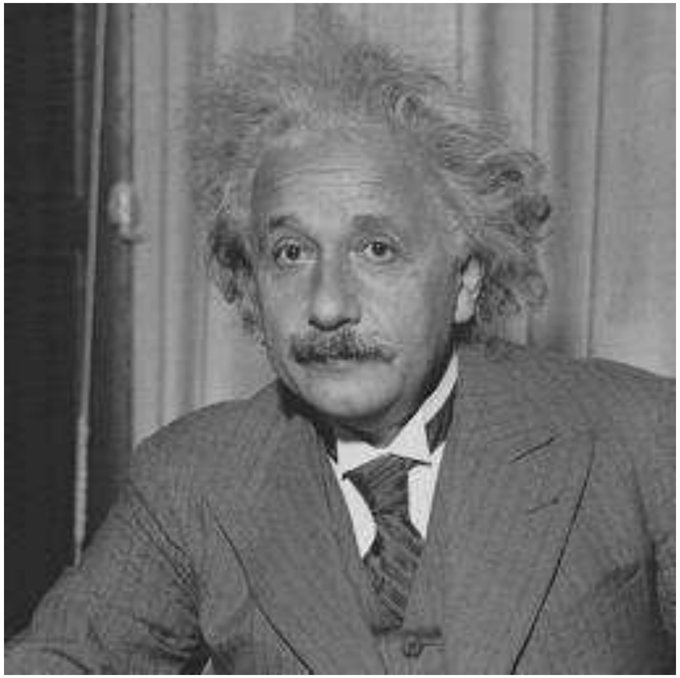
Filtered Image

Image Enhancement

Image Deblurring



Deblurring

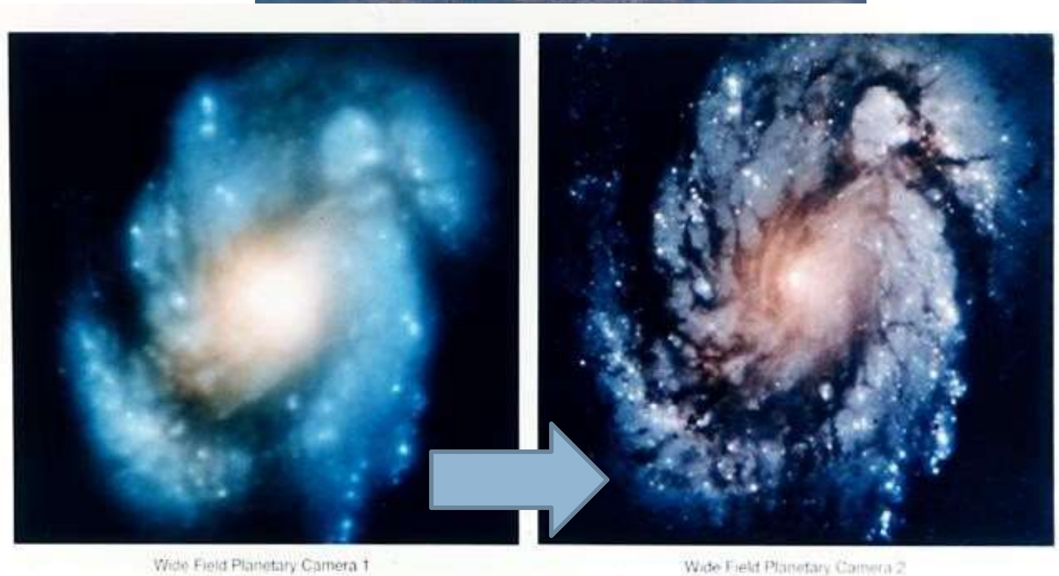


Blurred

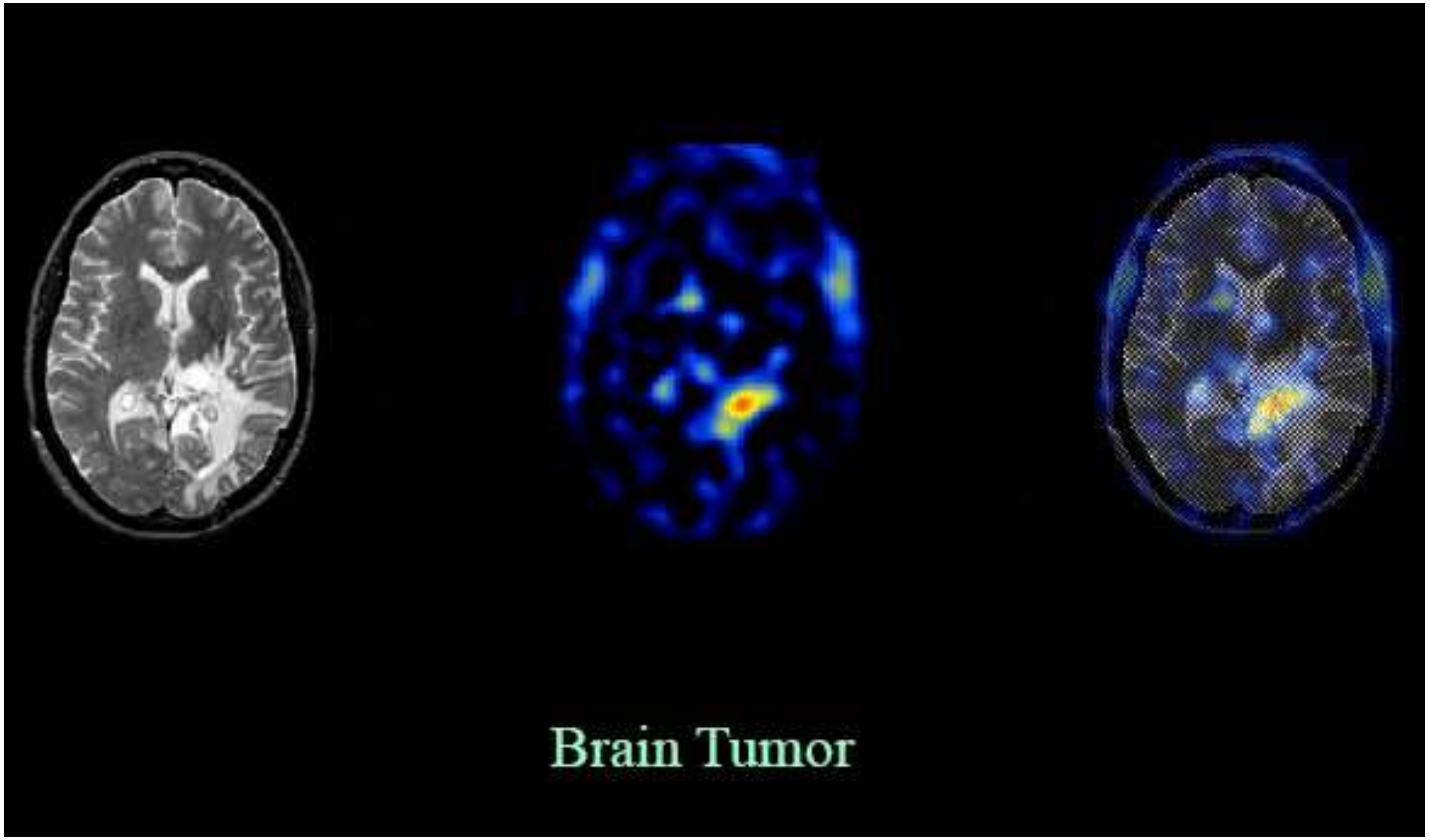
Deblurred

The Hubble Telescope

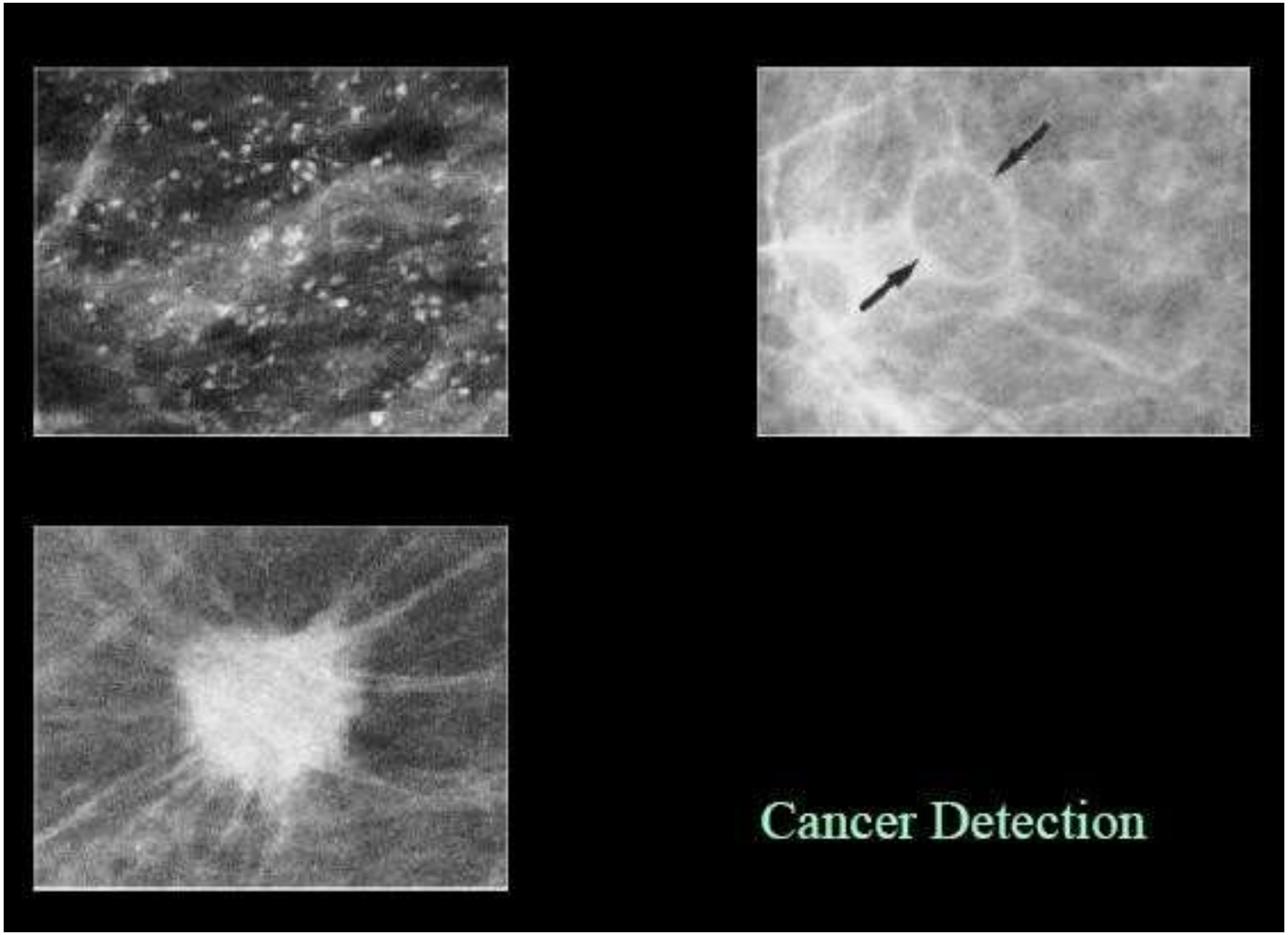
- Launched in 1990 the Hubble telescope can take images of very distant objects
- However, an incorrect mirror made many of Hubble's images useless
- Image processing techniques were used to fix this



Medical Imaging

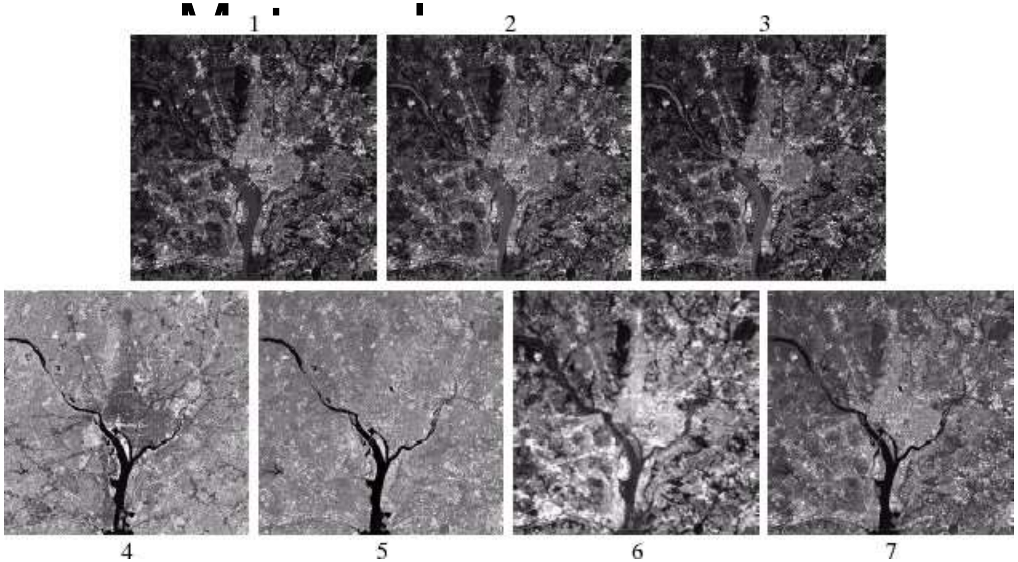


Medical Imaging



GIS

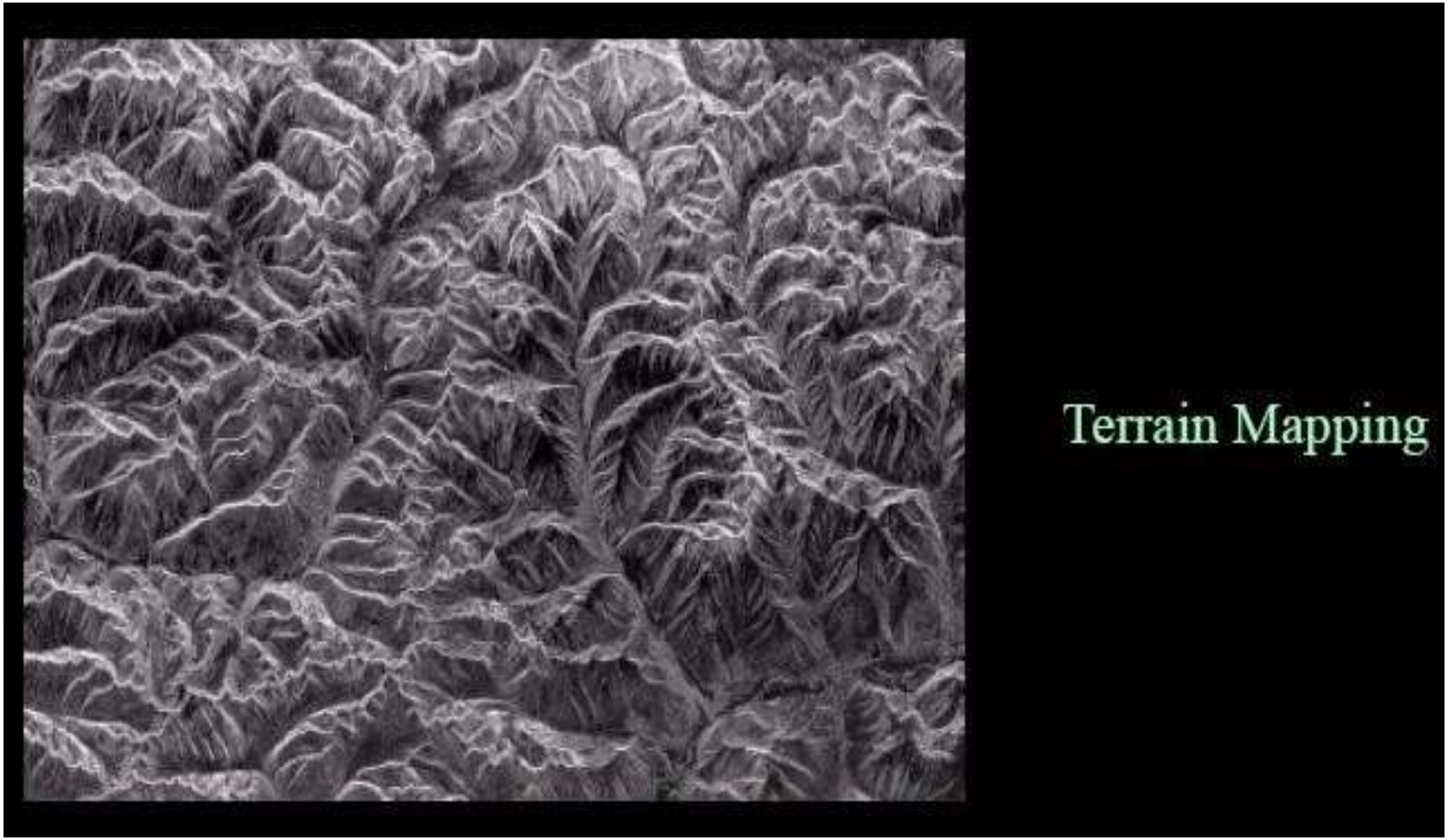
- Geographic Information Systems
  - Digital image processing techniques are used extensively to manipulate satellite imagery
  - Terrain classification



Remote Sensing

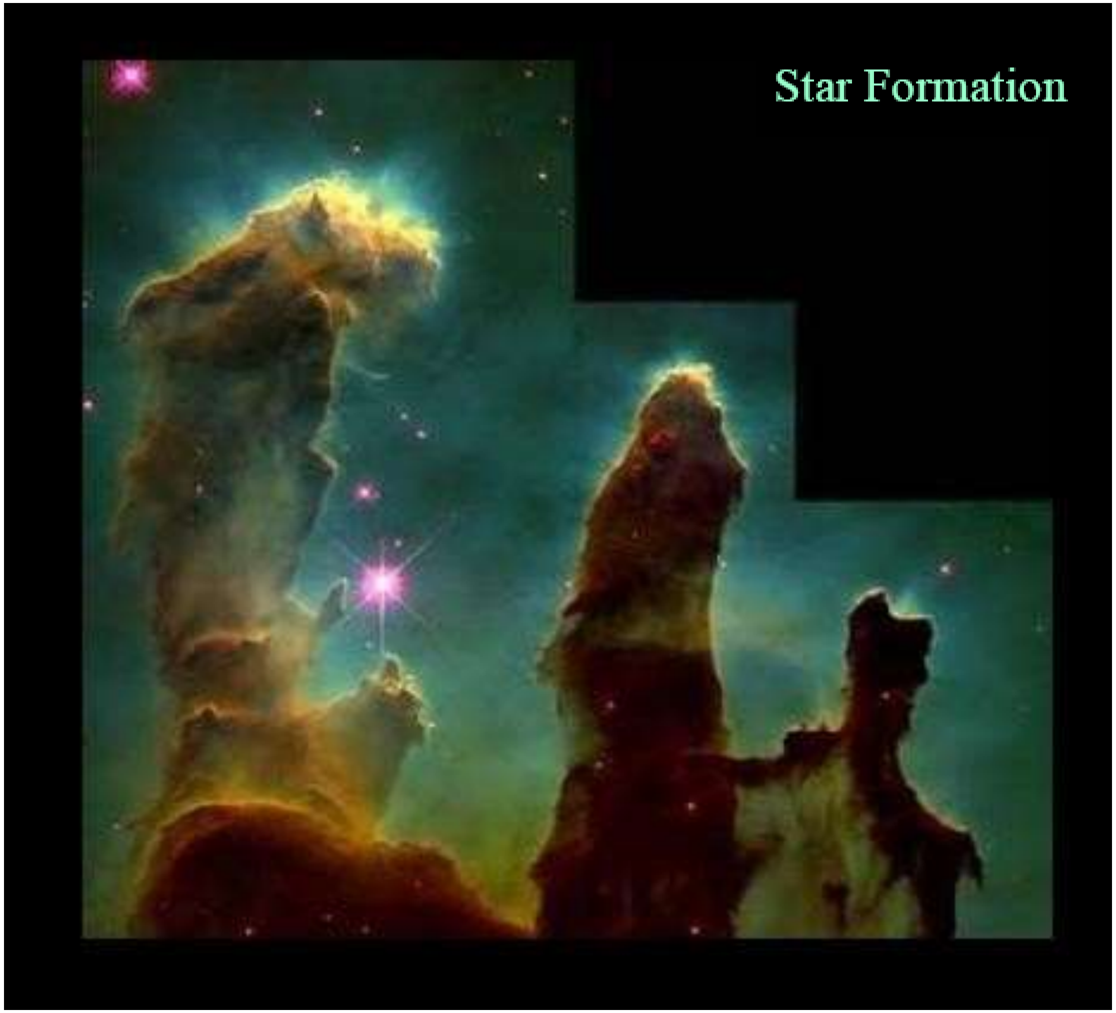
Satellite Image, Kolkata

Remote Sensing …



Weather Forecasting

Astronomy



Artistic Effects

- Artistic effects are used to make images more visually appealing, to add special effects and to make composite images

Image Compression

- An image usually contains lot of redundancy that can be exploited to achieve compression
  - Pixel redundancy
  - Coding redundancy
  - Psychovisual redundancy
- Applications:
  - Reduced storage
  - Reduction in bandwidth

Image Compression …

Image Compression …

Medium quality (Q = 25), filesize 9553 B. Low quality (Q = 10), filesize 4787 B.

Image Compression …

Low quality (Q = 10), filesize 4787 B. Lowest quality (Q = 1), filesize 1523 B.

What is Computer Vision?

- Deals with the development of the **theoretical** and **algorithmic** basis by which useful information about the 3D world can be automatically extracted and analyzed from a **single** or **multiple** 2D images of the world.

Why is Computer Vision Difficult?

- It is a **many-to-one** mapping
  - A variety of surfaces with different *material* and *geometrical* properties, possibly under different *lighting* conditions, could lead to identical images
  - Inverse mapping has non unique solution (a lot of information is **lost** in the transformation from the 3D world to the 2D image)
- It is **computationally intensive**
- We **do not understand** the recognition problem

Recognition Cues

Scene interpretation, even of complex, cluttered scenes is a straightforward task for humans.

Recognition Cues (cont'd)

How are we able to discern reality and an image of reality?
What clues are present in the image?
What knowledge do we use to process this image?

The role of color

What is this object?
Does color play a role in recognition?
Might this be easier to recognize from a different view?

The role of texture

- Characteristic image texture can help us readily recognize objects.

The role of shape

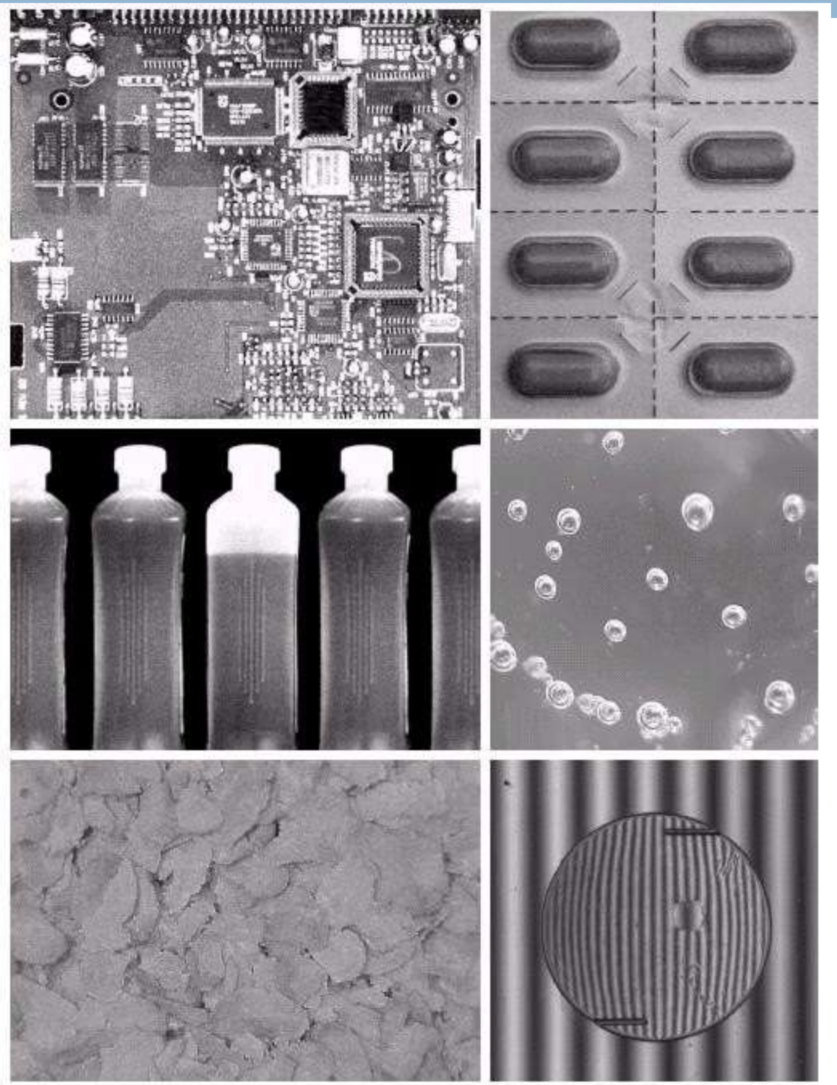
The role of grouping

Computer Vision Applications

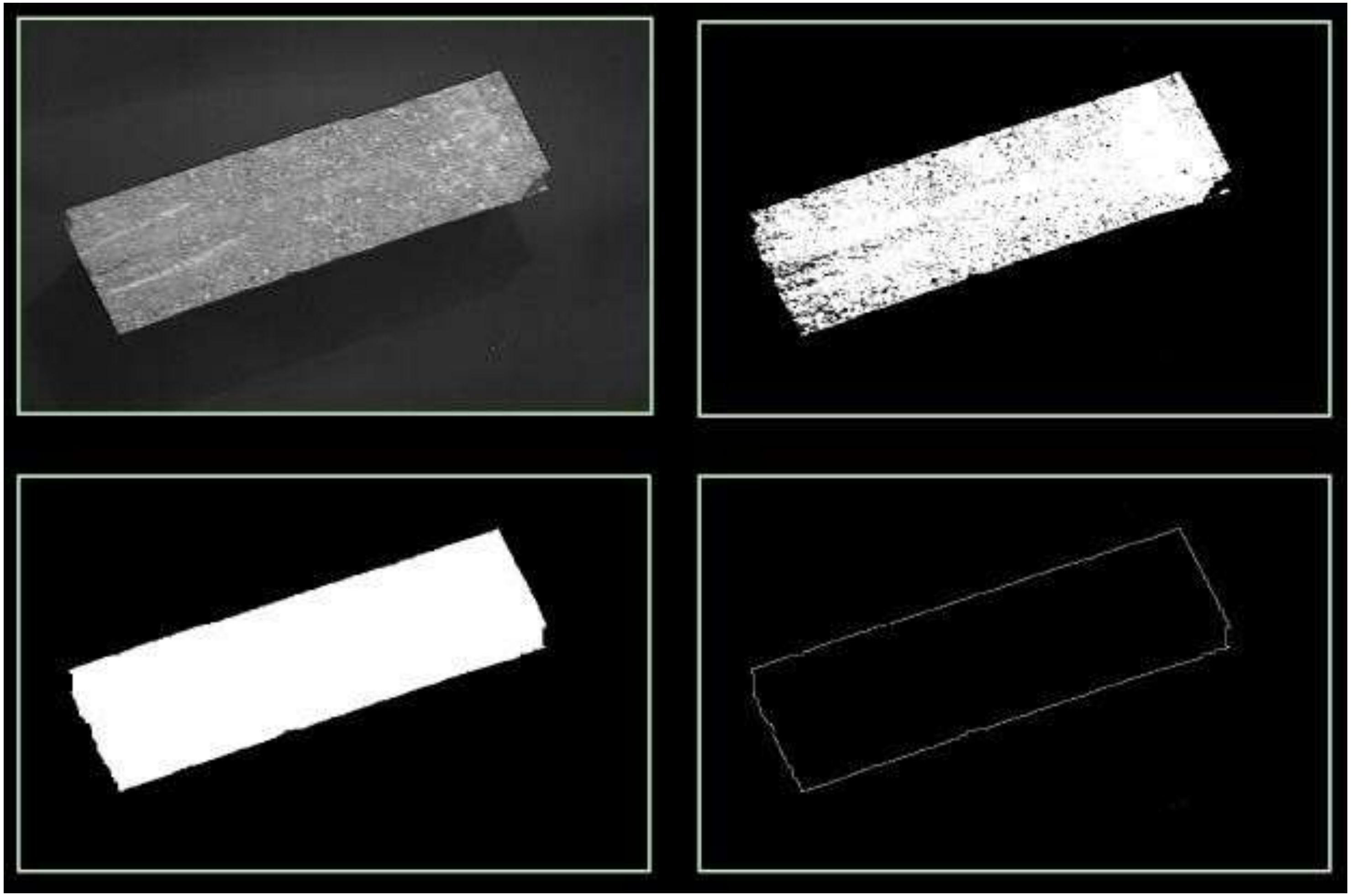
- Industrial inspection/quality control
- Surveillance and security
- Face recognition
- Gesture recognition
- Space applications
- Medical image analysis
- Autonomous vehicles
- Virtual reality and much more ……

Examples: Industrial Inspection

- Human operators are expensive, slow and unreliable
- Make machines do the job instead
- Industrial vision systems are used in all kinds of industries
- Can we trust them?

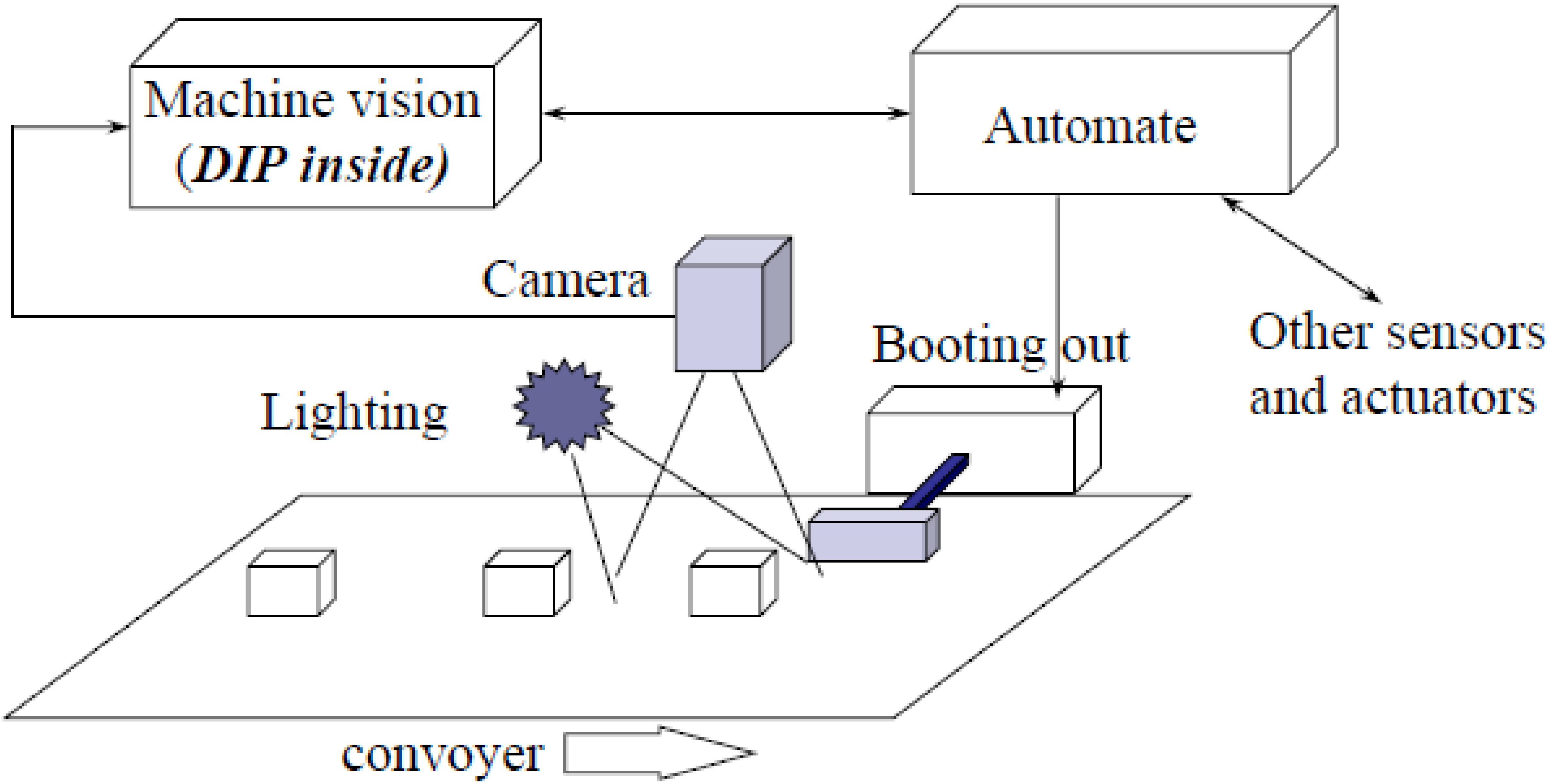


Automated Inspection …



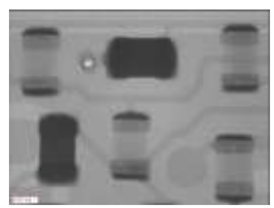
Automated Inspection …

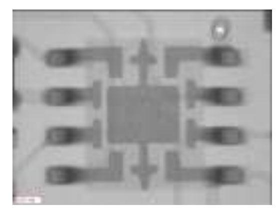
- Industrial inspection, computer vision



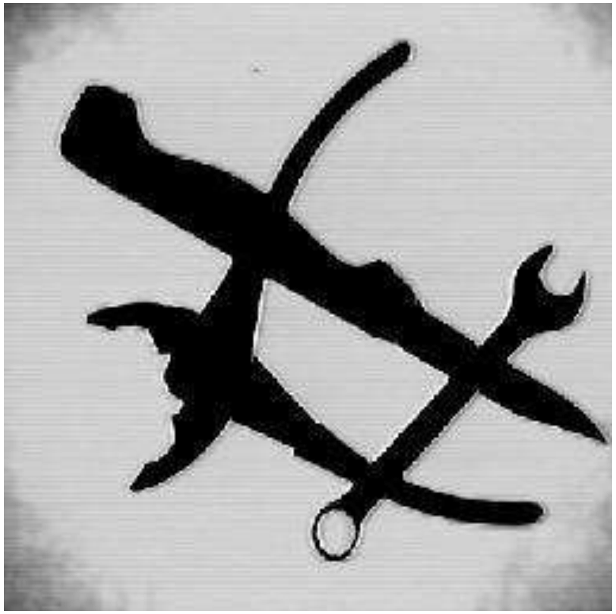
Examples: PCB Inspection

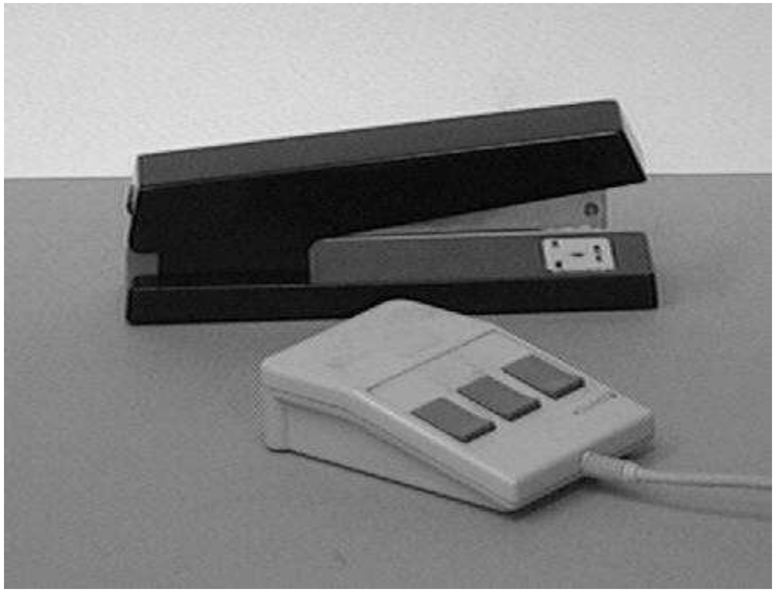
- Printed Circuit Board (PCB) inspection
  - Machine inspection is used to determine that all components are present and that all solder joints are acceptable
  - Both conventional imaging and x-ray imaging are





Object Recognition



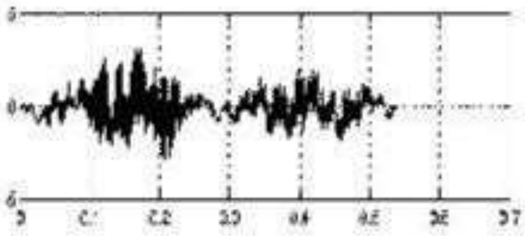


Indexing into Databases (cont'd)

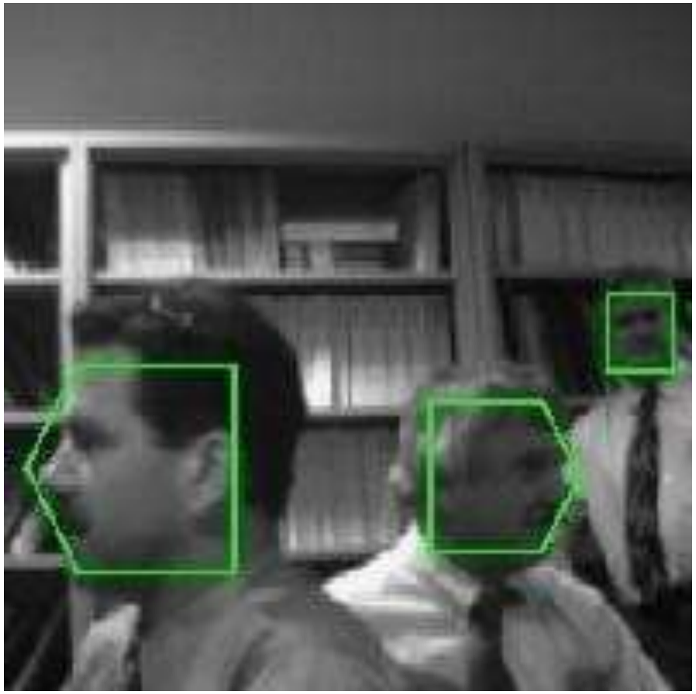
- Color, texture

$T = 33.6\text{s}$, found 2 of 2

Biometrics



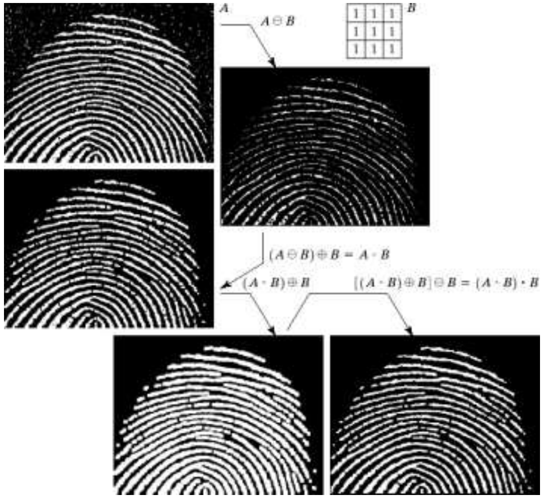
Face Detection



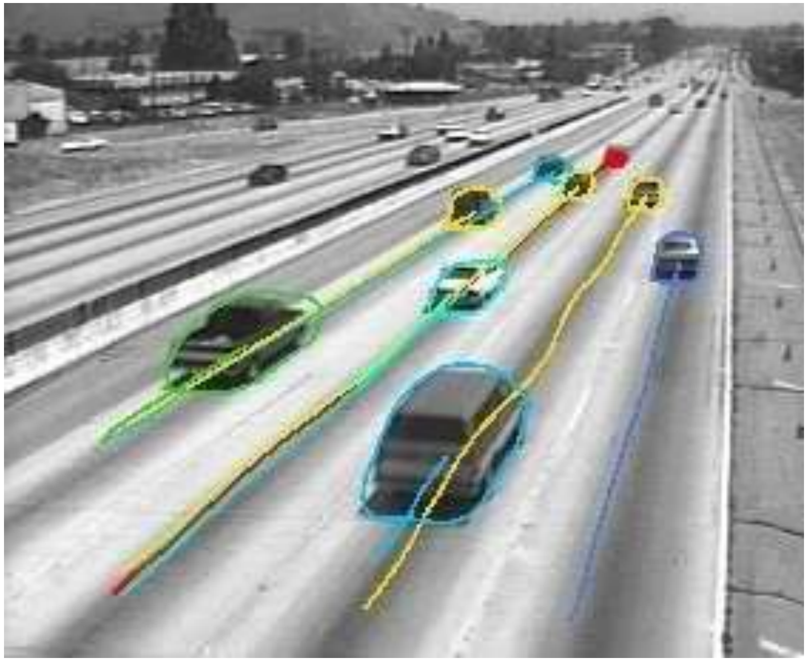
Face Recognition

Examples: Law Enforcement

- Image processing techniques are used extensively by law enforcers
  - Number plate recognition for speed cameras/automated toll systems
  - Fingerprint recognition
  - Enhancement of CCTV images

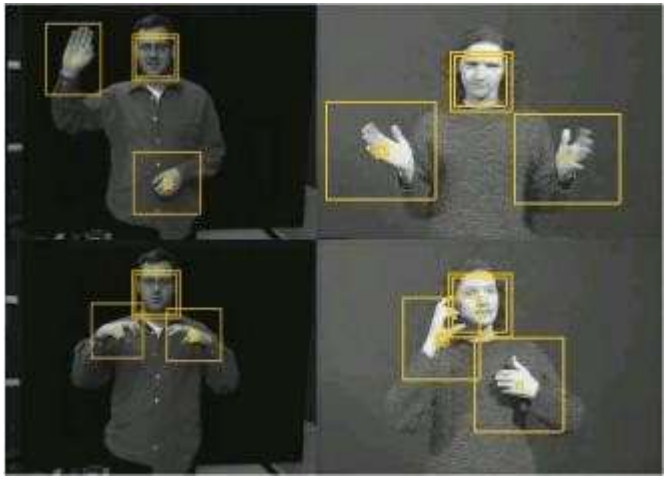


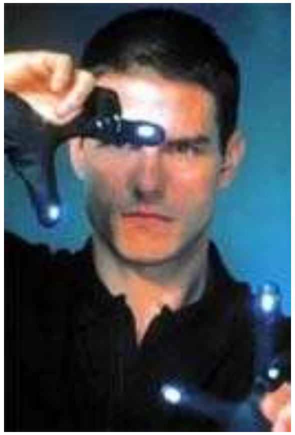
Traffic Monitoring



Examples: HCI

- Try to make human computer interfaces more natural
  - Face recognition
  - Gesture recognition
- These tasks can be difficult



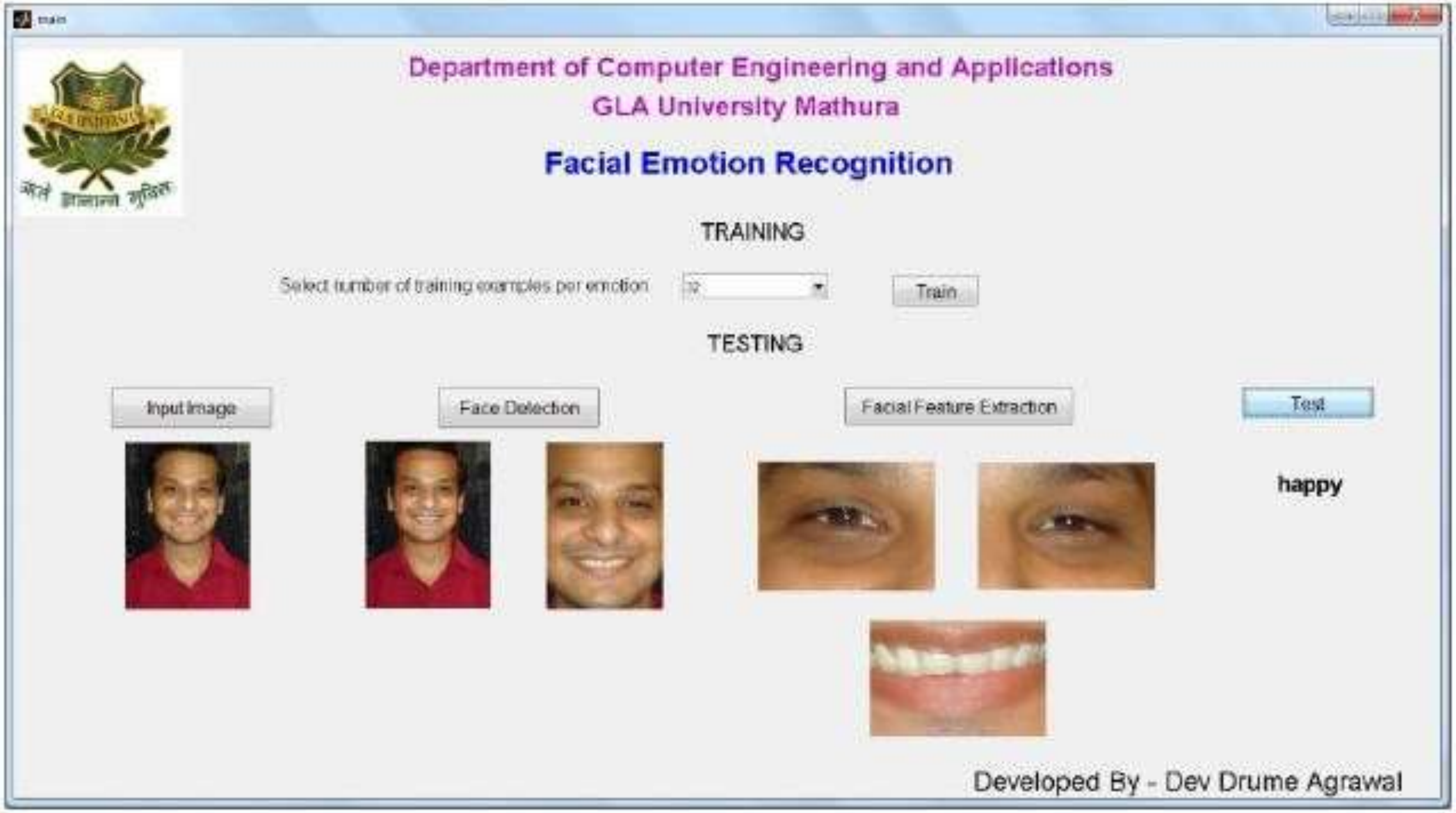


Computer Vision Applications …

- Video surveillance is the task of analyzing video data to identify unusual or suspicious activities in security-sensitive areas such as banks, department stores, parking lots.
- Manual surveillance requires the system to be monitored continuously by a person and is costly and problematic.

Computer Vision @ GLA

Computer Vision @ GLA …

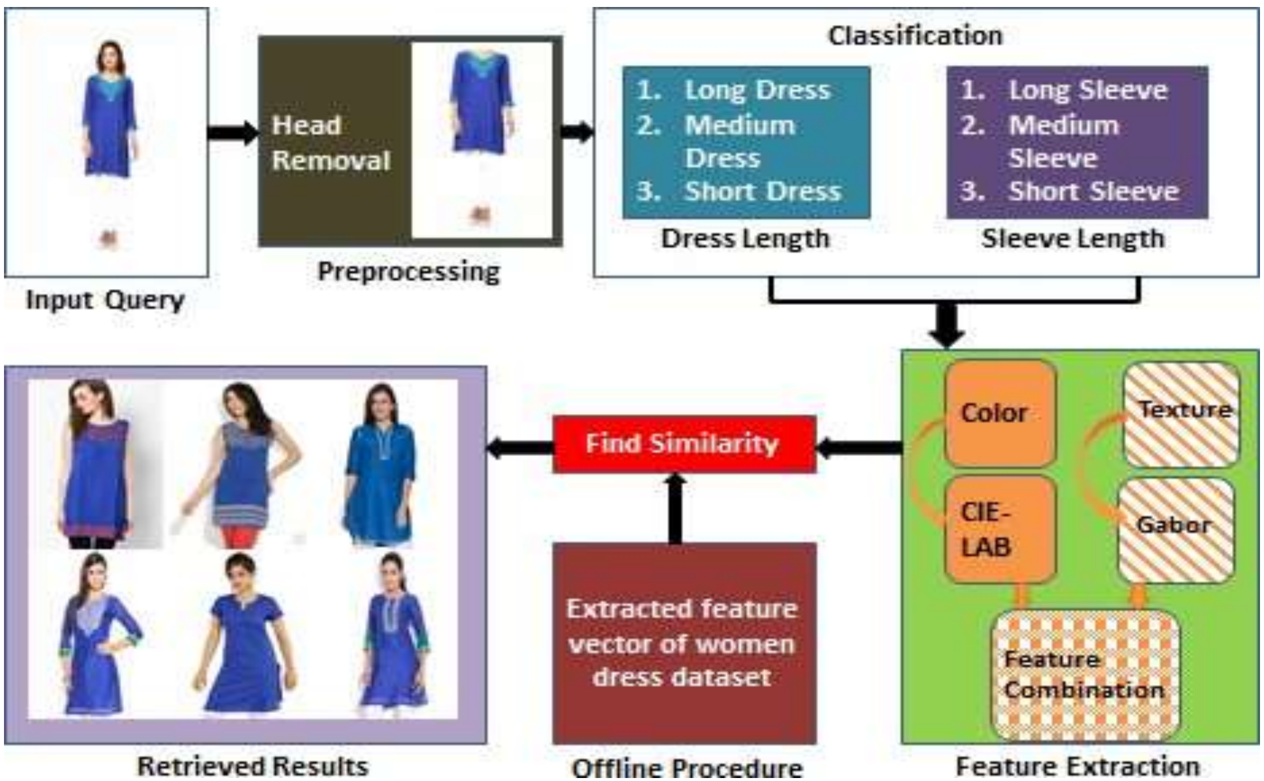


Computer Vision @ GLA …

Computer Vision @ GLA …

Computer Vision @ GLA …

Clothing Image Retrieval



Computer Vision @ GLA …

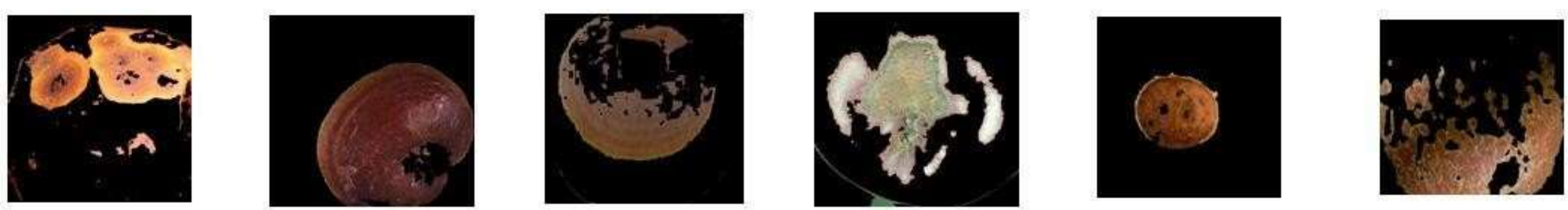
- **Applications in Agriculture**
  - Fruit and Vegetable Classification

Computer Vision @ GLA …

- **Applications in Agriculture …**

  Automatic Detection and Classification of Fruit Diseases

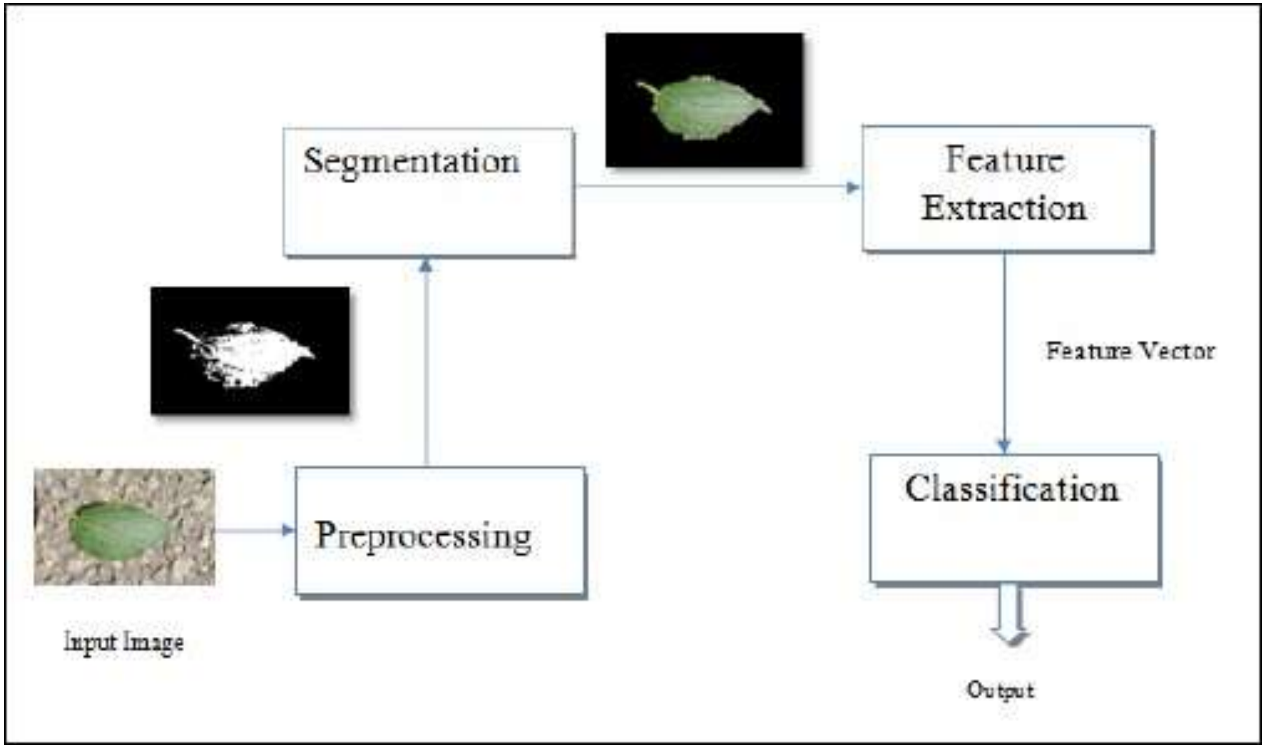
(a)



(b)

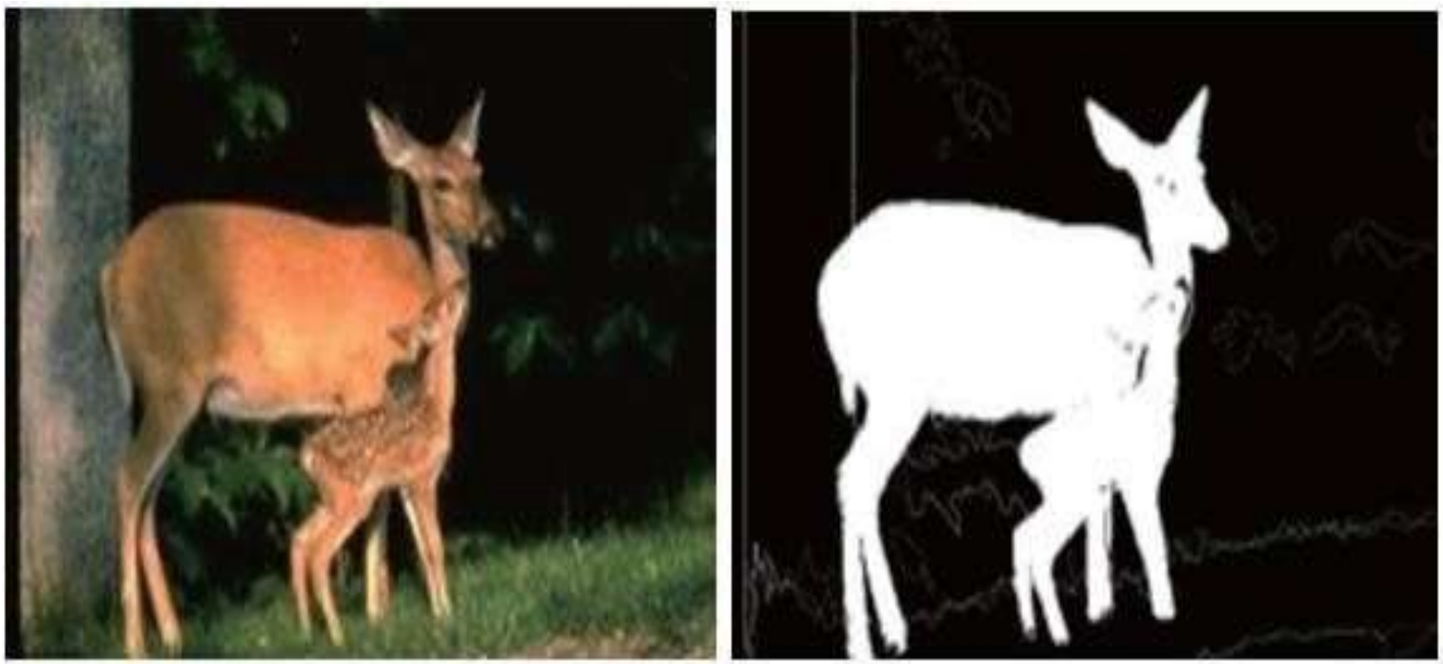
Computer Vision @ GLA …

Plant Identification system



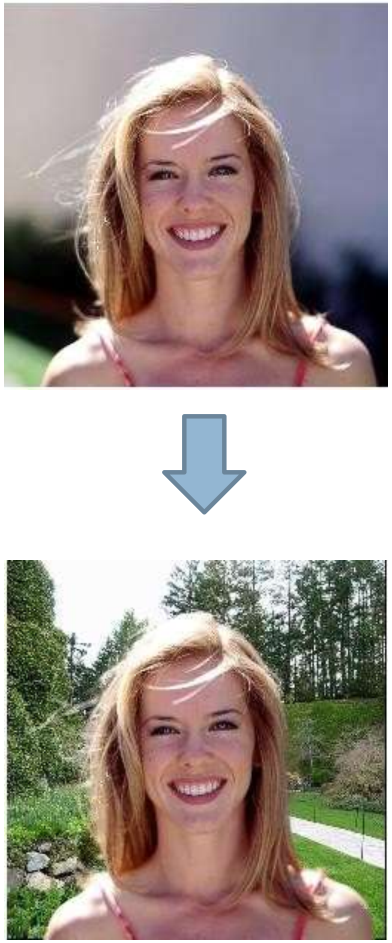
Computer Vision @ GLA …

Salient Region Detection



Computer Vision @ GLA …

Digital image matting is a way through which we can extract foreground object from the given image.



Computer Vision @ GLA …

Image Forgery Detection

Computer Vision @ GLA …

- **Object Tracking**

Computer Vision @ GLA …

Human Activity Identification

Computer Vision @ GLA …

Abandoned Object Detection

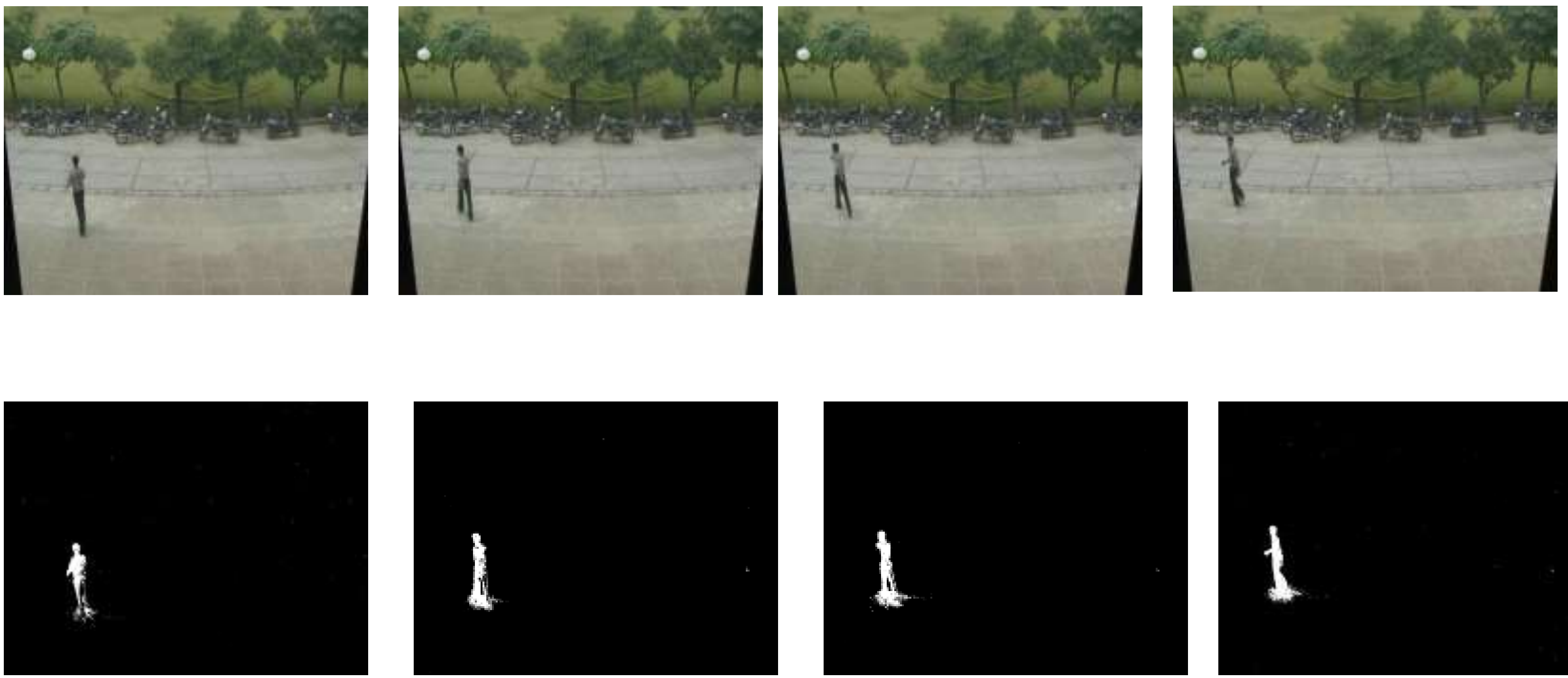
Machine Vision Applications ..

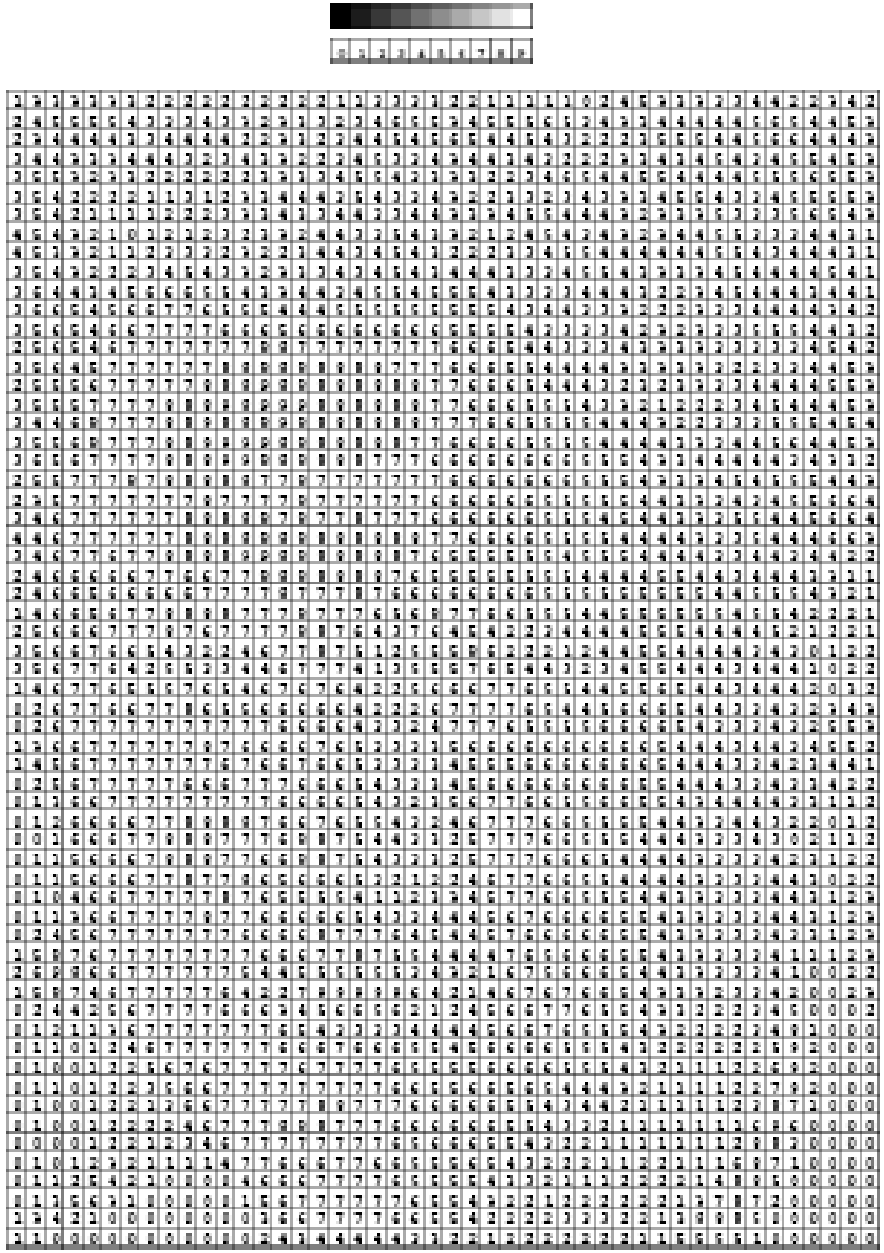
Video Sequence Processing

- The major emphasis of image sequence processing is detection of moving parts
- This has various applications
  - Detection and tracking of moving targets for security surveillance purpose
  - To find out the trajectory of a moving target
  - Monitoring the movements of organ boundaries in medical applications etc.

Machine Vision Applications ..

Movement Detection





Importance of prior information

Digital Image Processing

00000000	01110010	01100100	…
00011001	01000010	01100101	…
00011111	00110010	01100101	…
00000000	01110010	01100100	…
00011001	01000010	01100101	…
00011111	00110010	01100101	…
00000000	01110010	01100100	…
00011001	01000010	01100101	…
00011111	00110010	01100101	…
00000000	01110010	01100100	…
00011001	01000010	01100101	…
00011111	00110010	01100101	…

…
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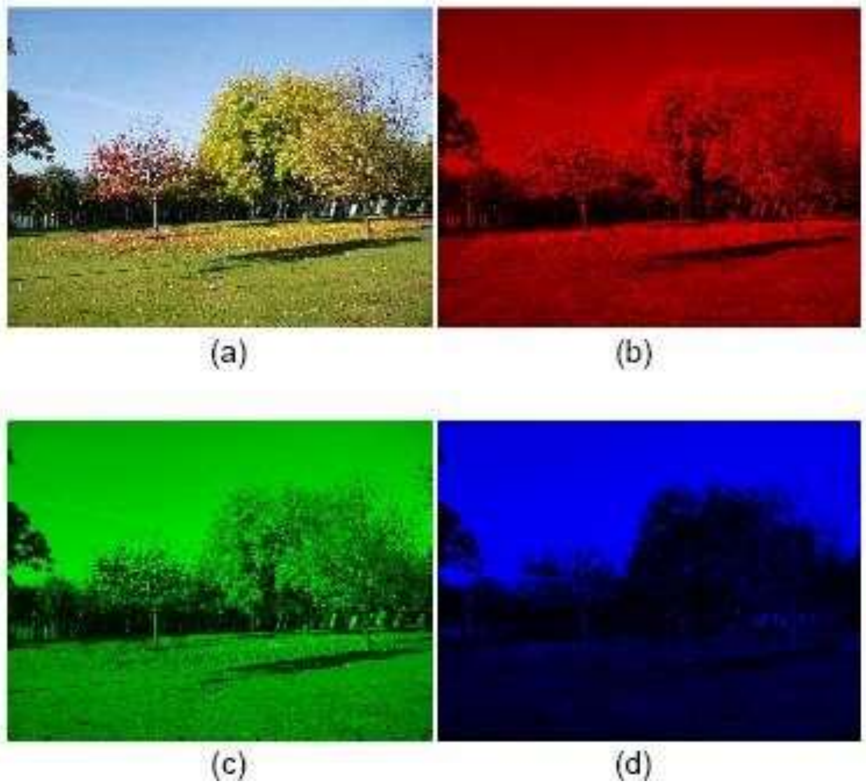
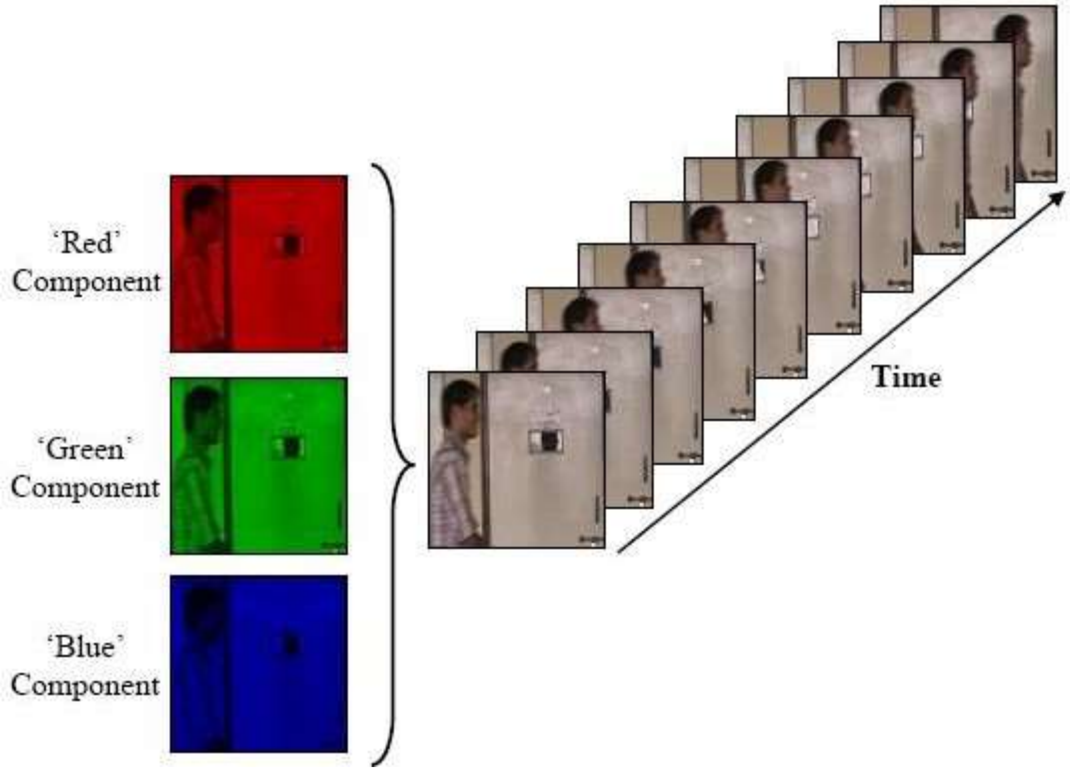
What we see

What the computer sees!!!

Digital image is a representation of a two-dimensional image using ones and zeros (binary).



RGB image along with its R, G, B components: a) RGB image b) R component c) G component d) B component



Video structure and representation

Digital Image Processing …

Radiant (light) energy is recorded at corresponding points on a plane to form an image.

B/W images can be represented as a function of two variable,

$$f(x, y),$$

where $f(x, y)$ is the brightness (or grayness or intensity) of the image at the coordinate (x, y)

In colour image, the intensity is measured in 3 wavelengths (red, green, blue), so

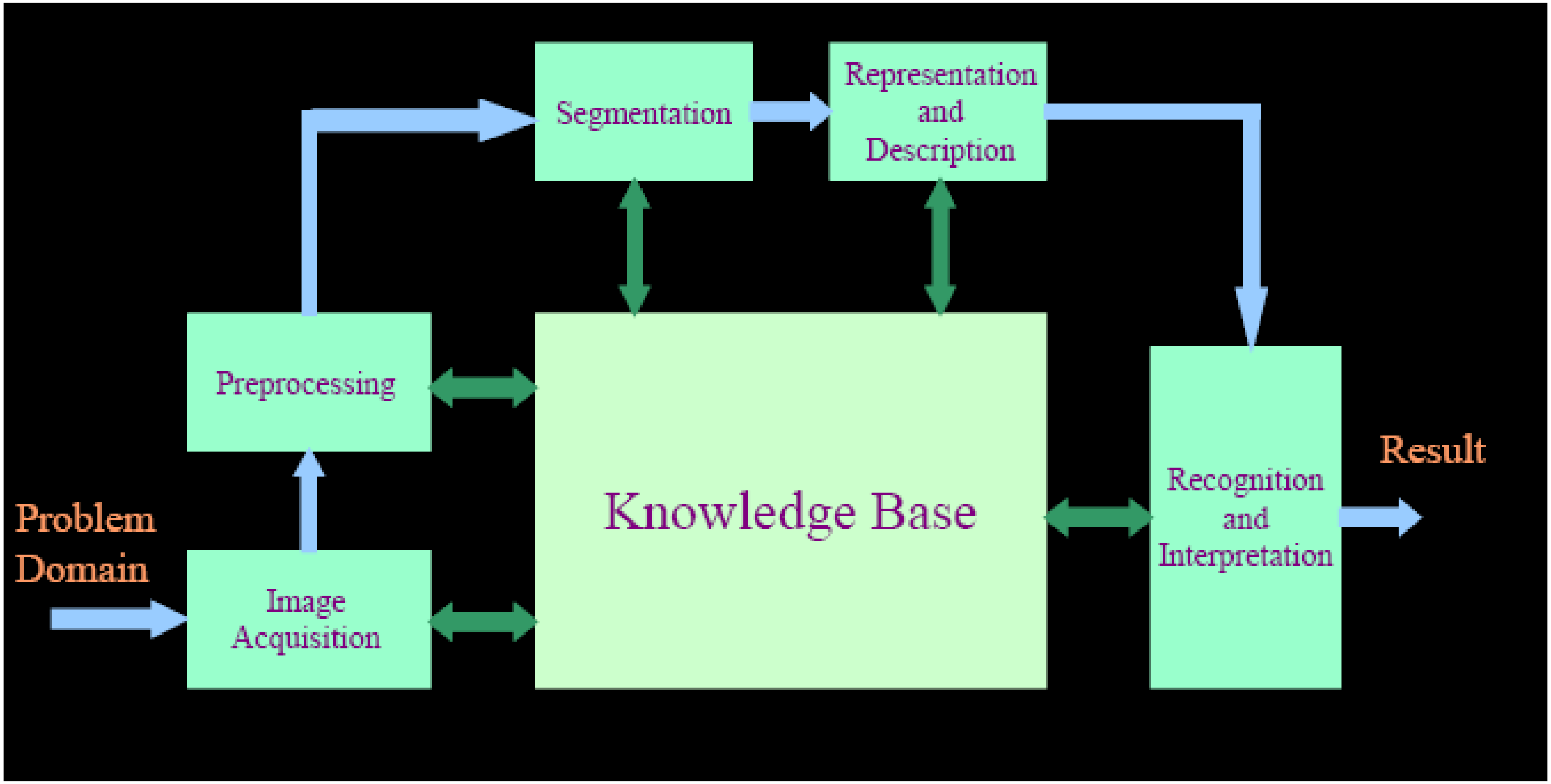
$$f(x, y) = [f_R(x, y), f_G(x, y), f_B(x, y)].$$

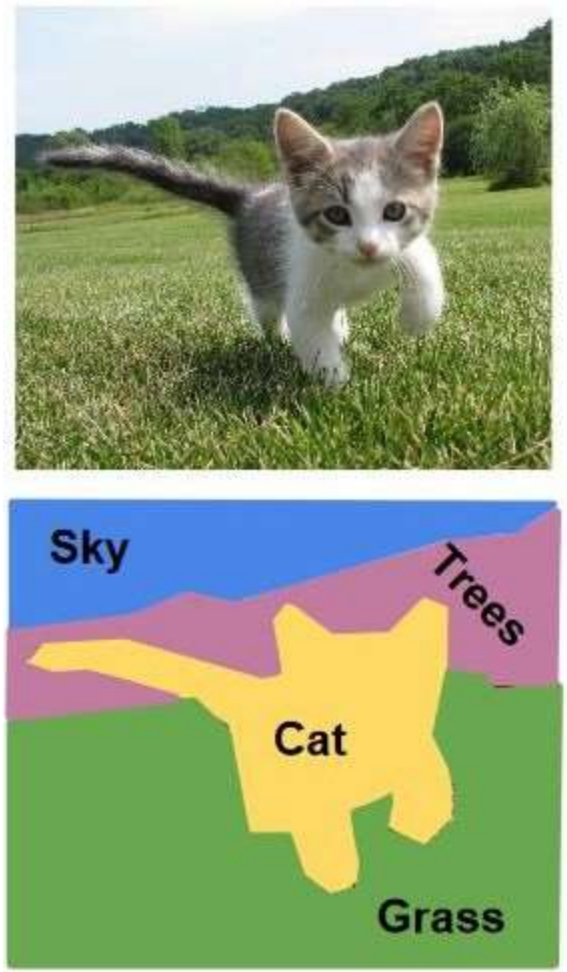
Steps in Digital Image Processing

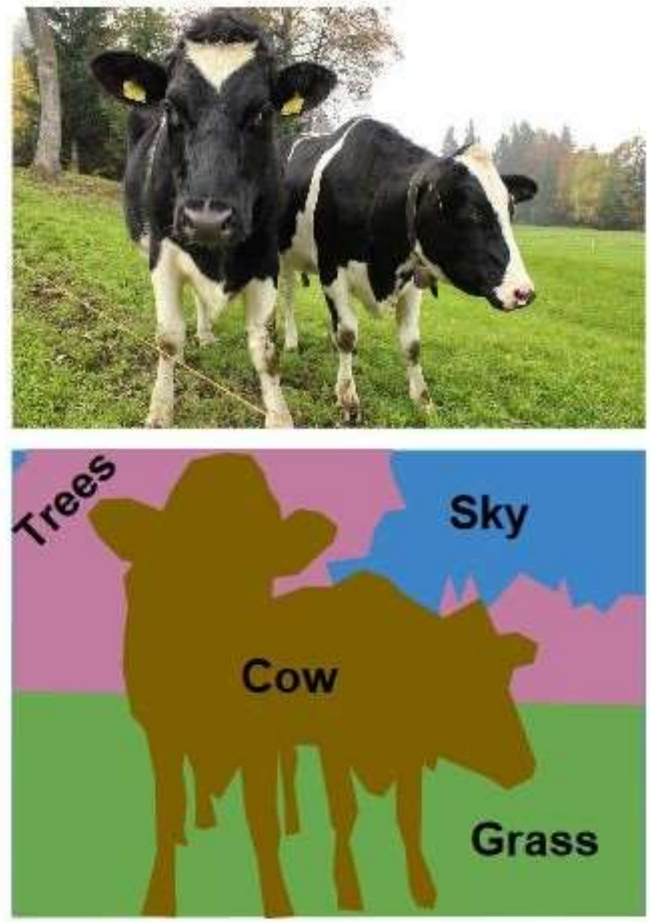
Digital Image Processing involves following basic tasks

- **Image Acquisition:-** An imaging sensor and the capability to digitize the signal produced by the sensor
- **Preprocessing:-** Enhances the image quality, filtering, contrast enhancement etc.
- **Segmentation:-** Partitions an input image into constituent parts of objects
- **Description/ Feature Selection:-** Extracts description of image objects suitable for further computer processing
- **Recognition & Interpretation:-** Assigning a label to the object based on the information provided by its descriptor. Interpretation assigns meaning to a set of labeled objects.
- **Knowledge Base:-** Knowledge Base helps for efficient processing as well as inter module cooperation

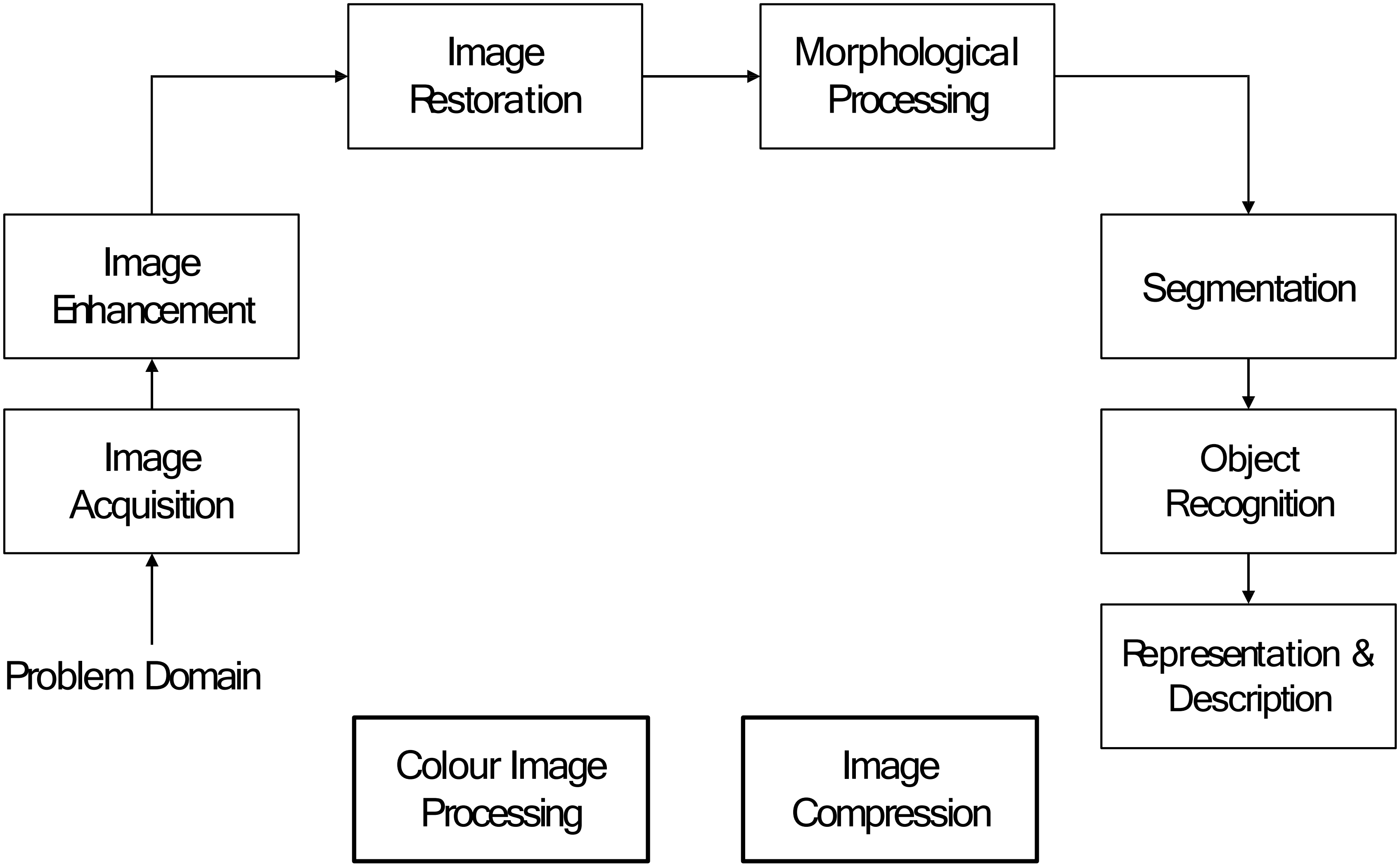
Steps in Digital Image Processing



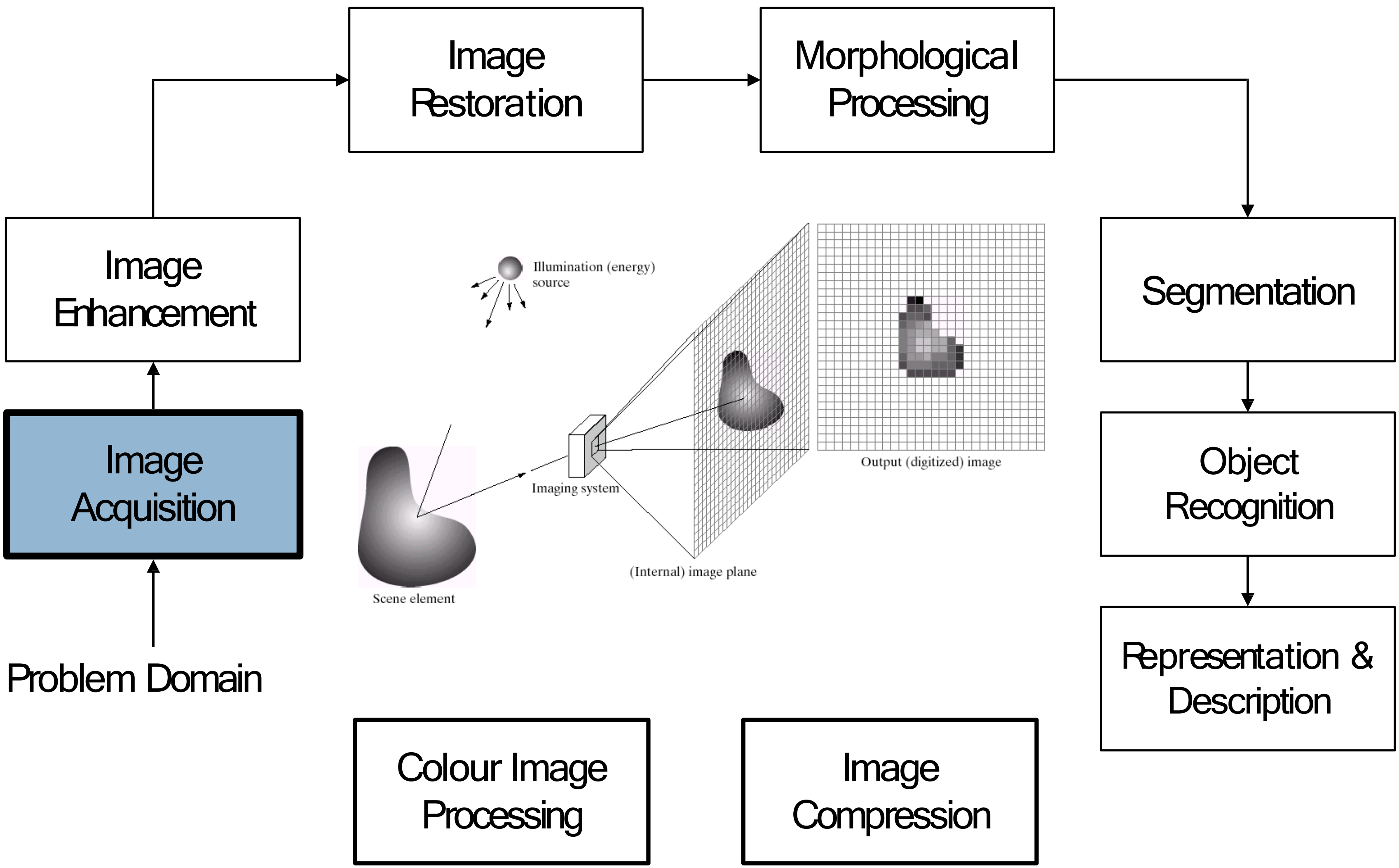

Sky
Trees
Cat
Grass


Trees
Sky
Cow
Grass

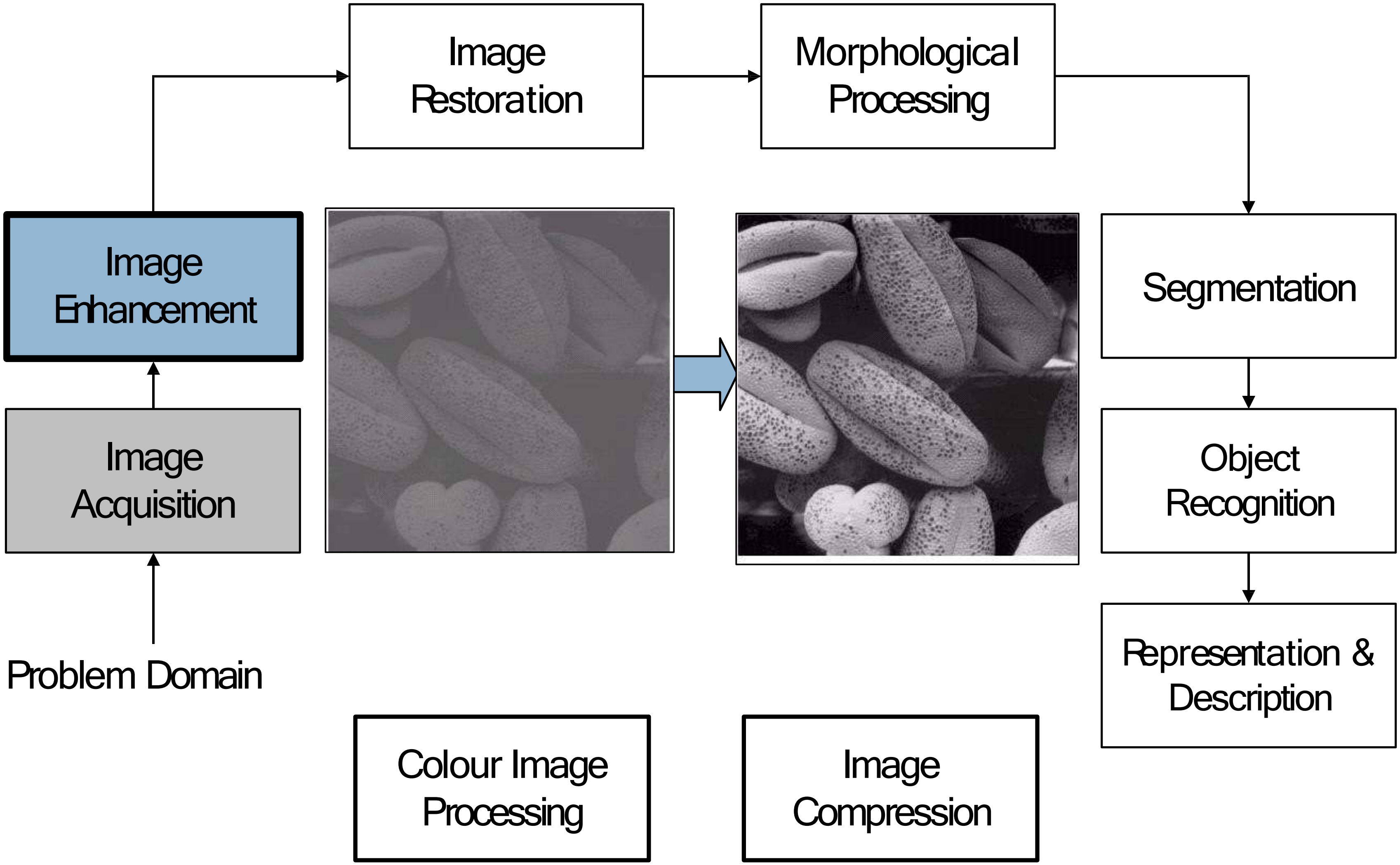
Key Stages in Digital Image Processing



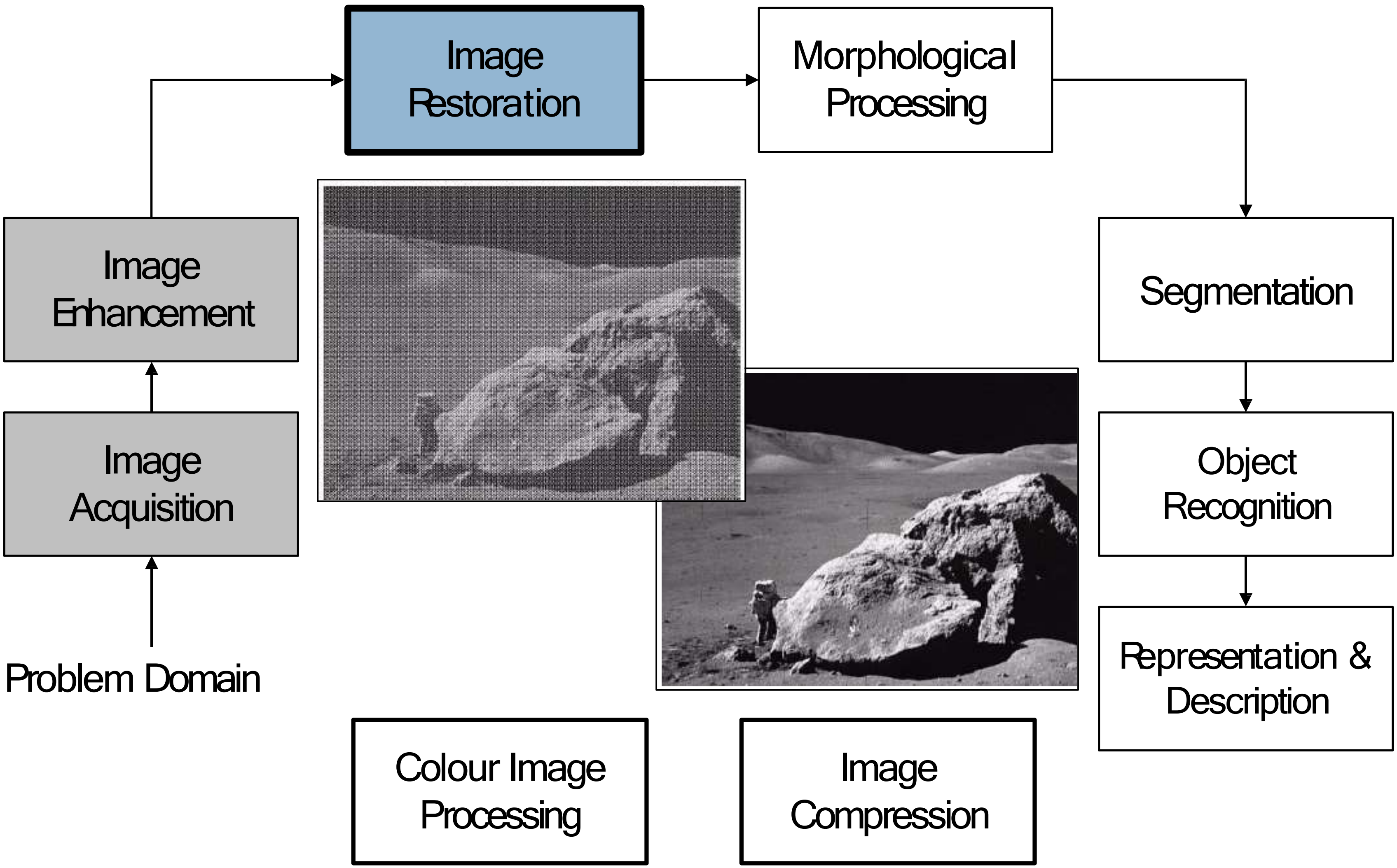
Key Stages in Digital Image Processing: Image Aquisition



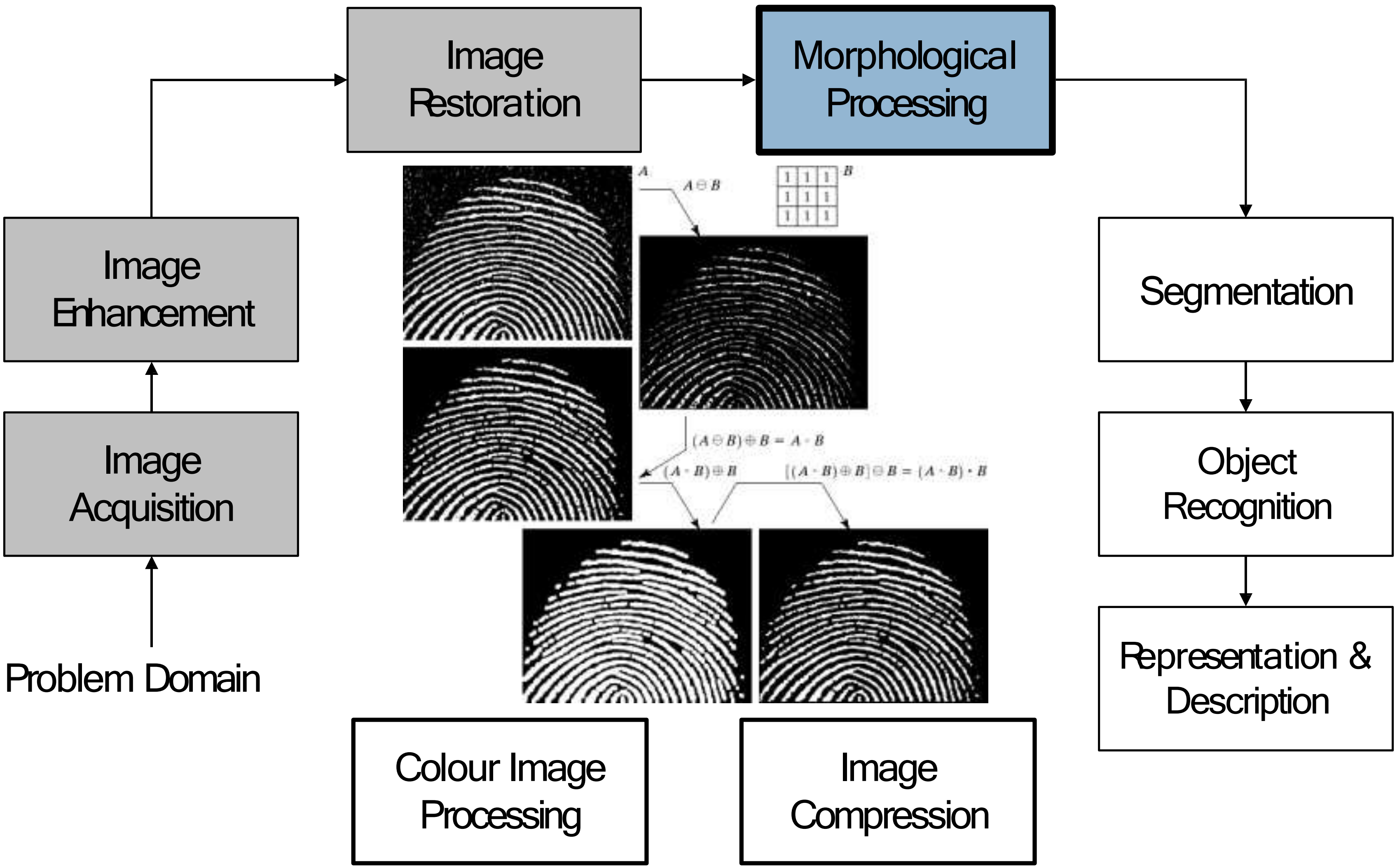
Key Stages in Digital Image Processing: Image Enhancement



Key Stages in Digital Image Processing: Image Restoration



Key Stages in Digital Image Processing: Morphological Processing



Key Stages in Digital Image Processing: Segmentation

Problem Domain

Image Acquisition

Image Enhancement

Image Restoration

Morphological Processing

Segmentation

Object Recognition

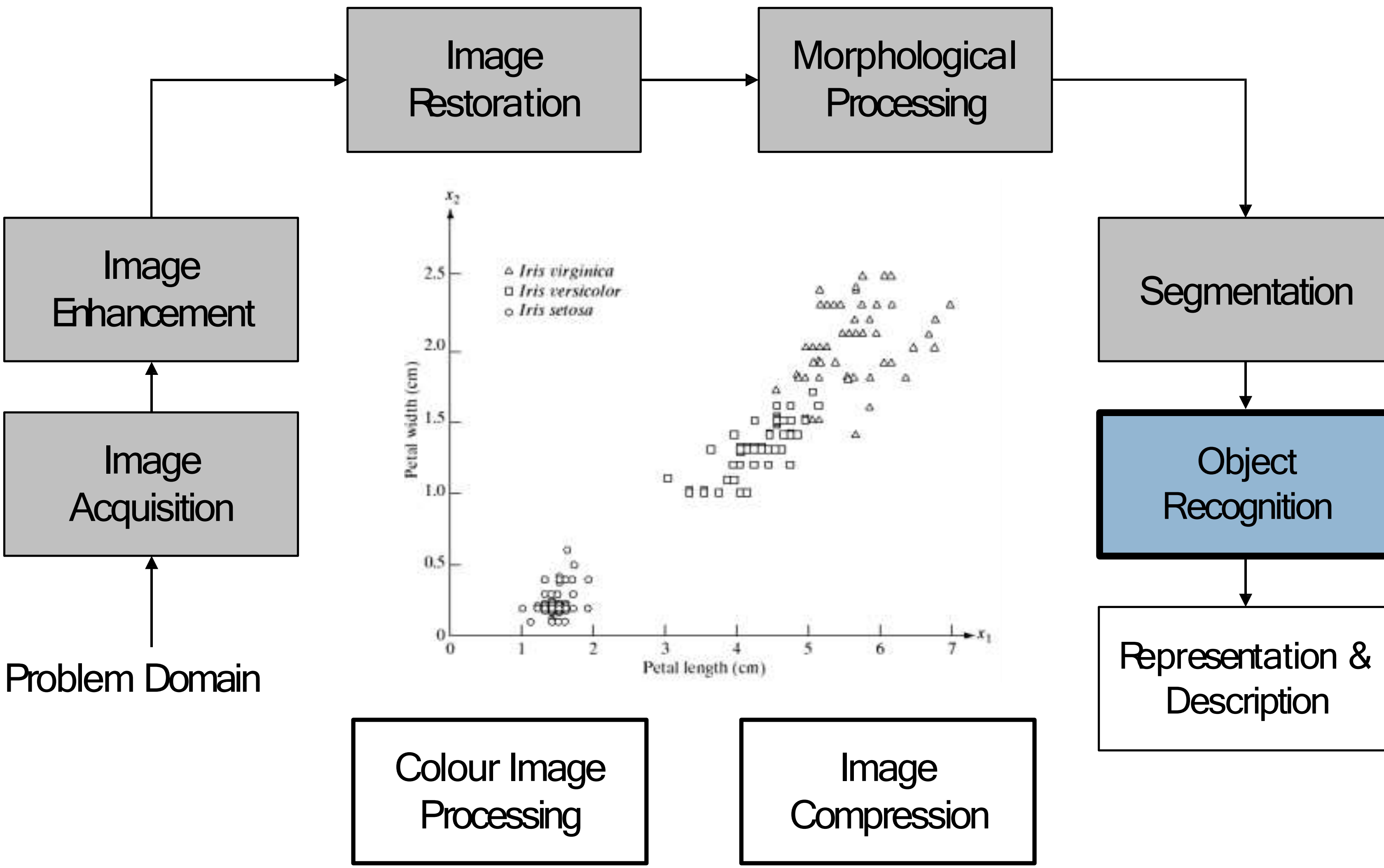
Representation & Description

Colour Image Processing

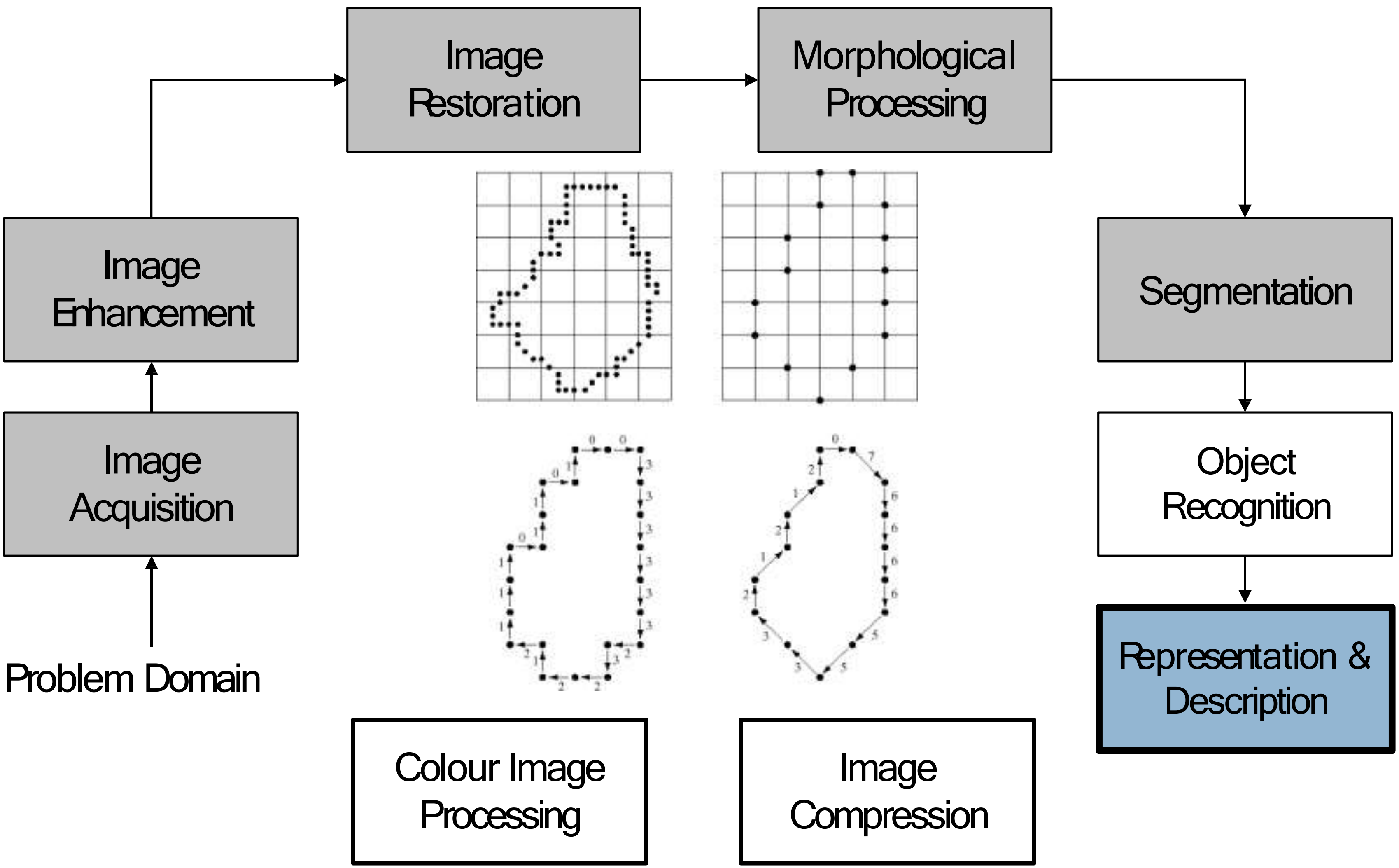
Image Compression

(a) (b)

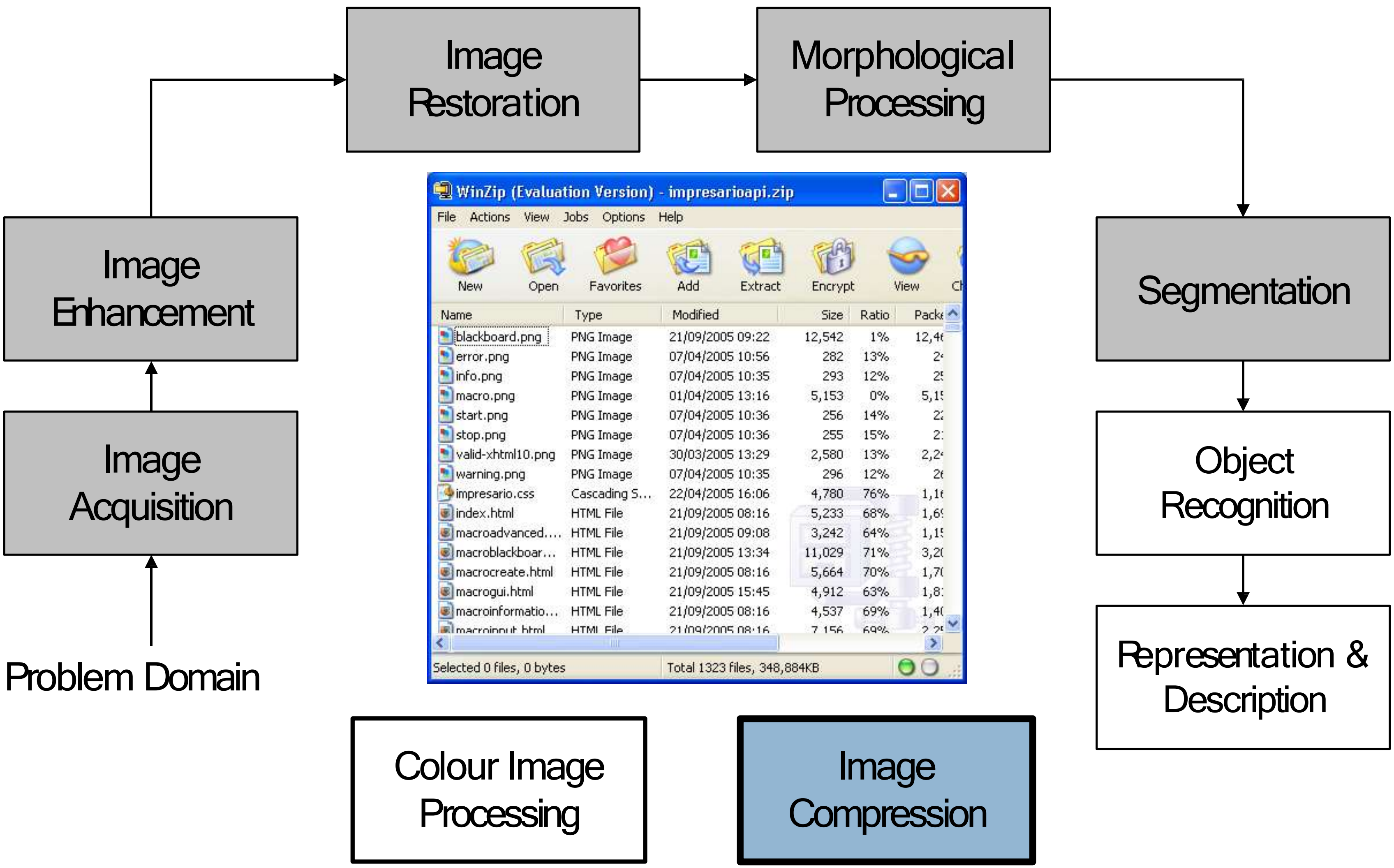
Key Stages in Digital Image Processing: Object Recognition



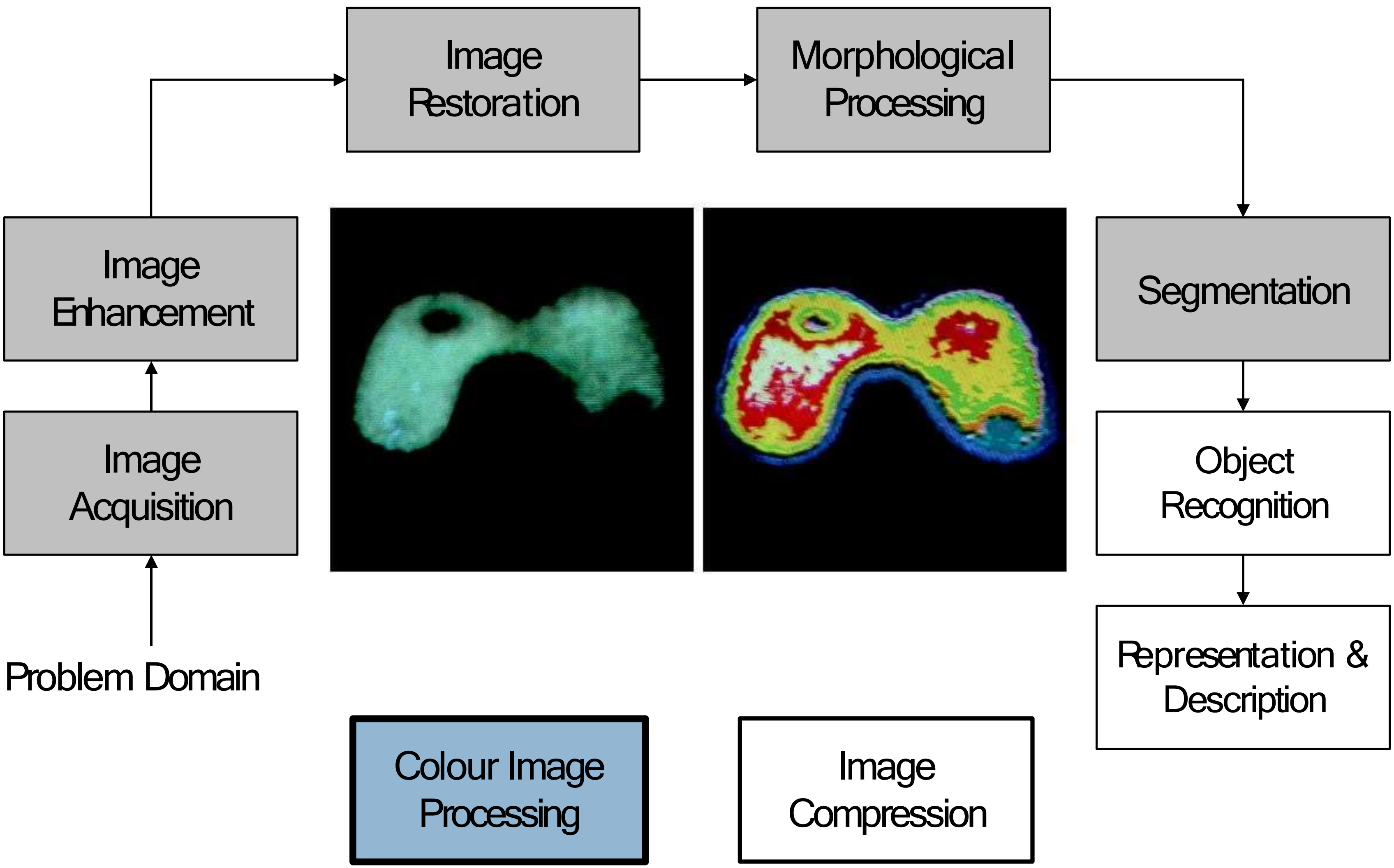
Key Stages in Digital Image Processing: Representation & Description



Key Stages in Digital Image Processing: Image Compression



Key Stages in Digital Image Processing: Colour Image Processing



Another Example

Image processing stages – acquisition

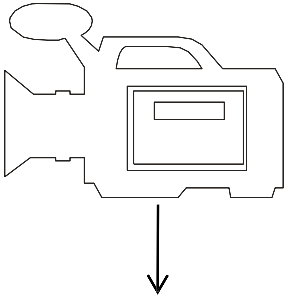


Image processing stages – filtering

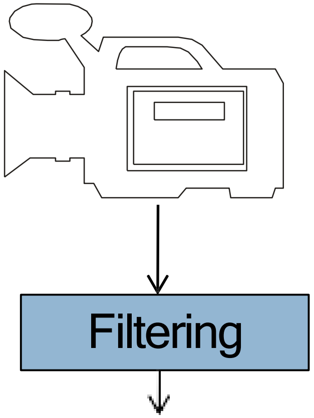
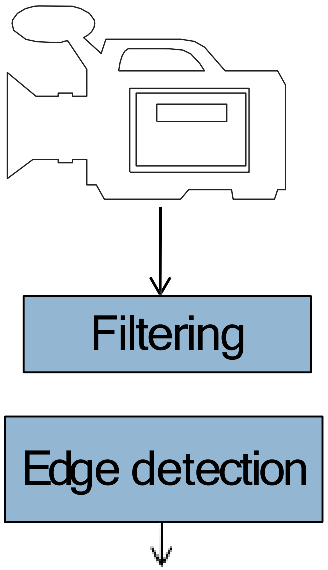


Image processing stages – edge detection



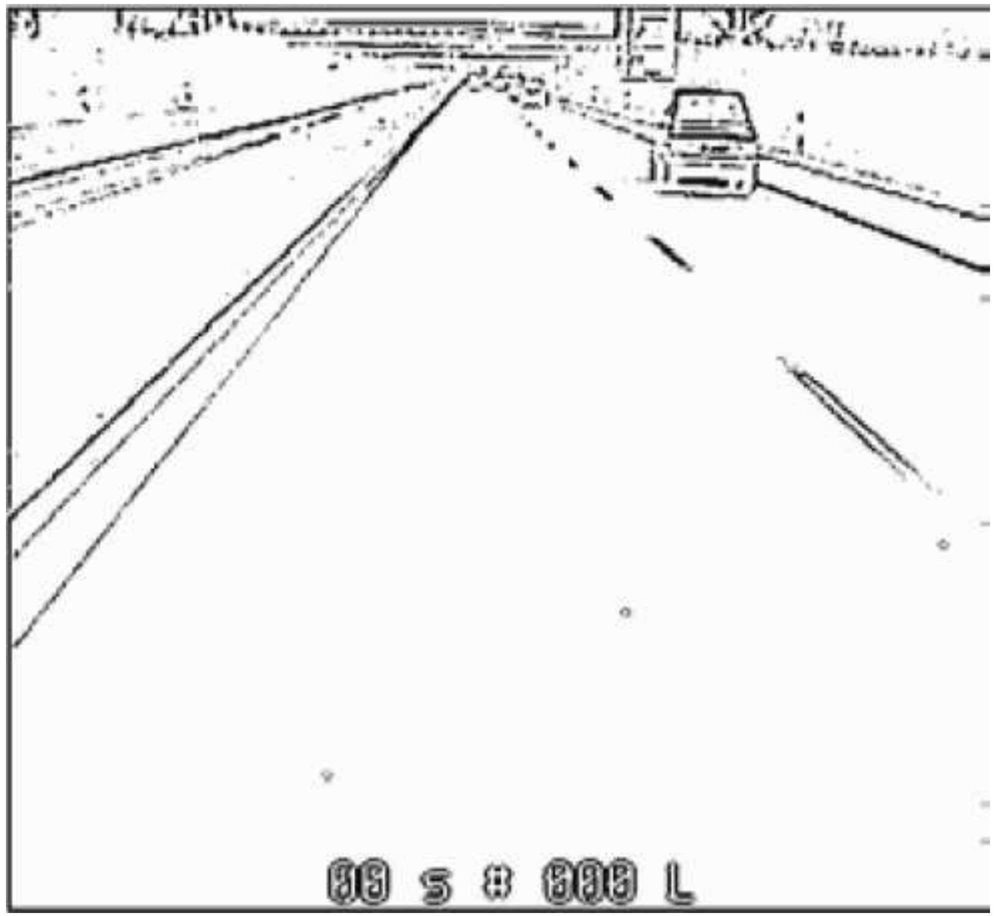
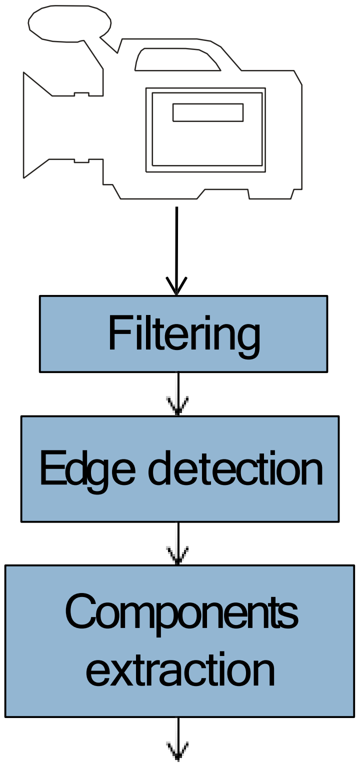
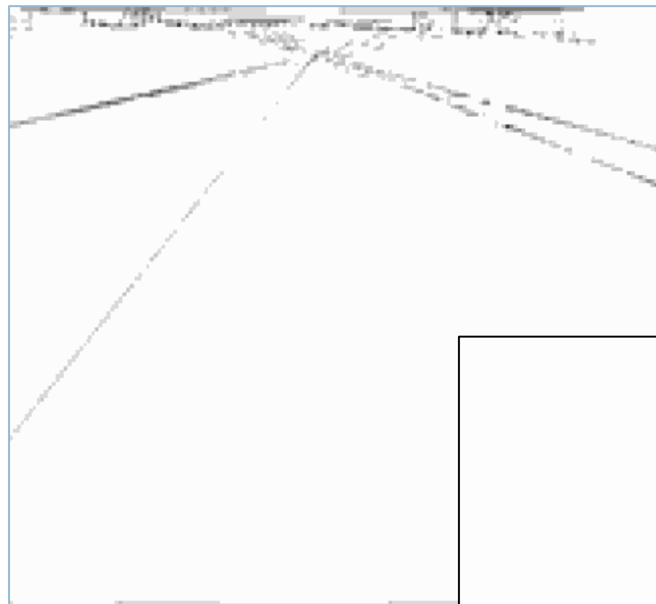


Image processing stages – components extraction





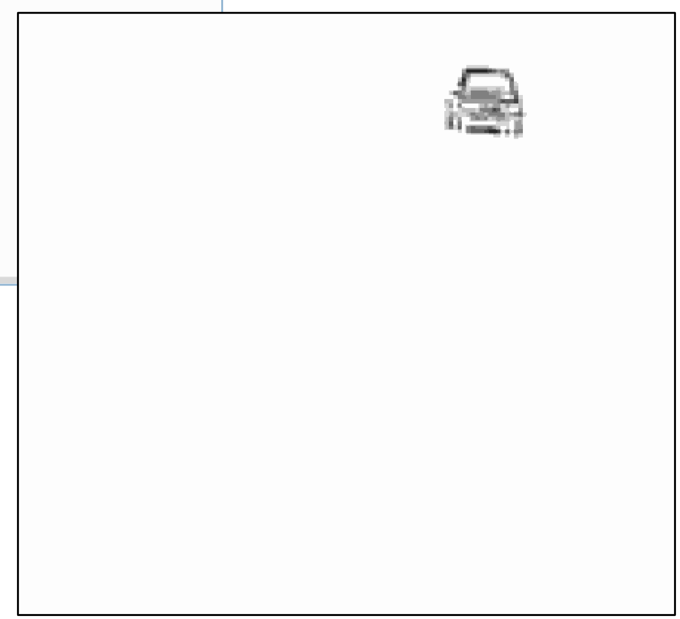
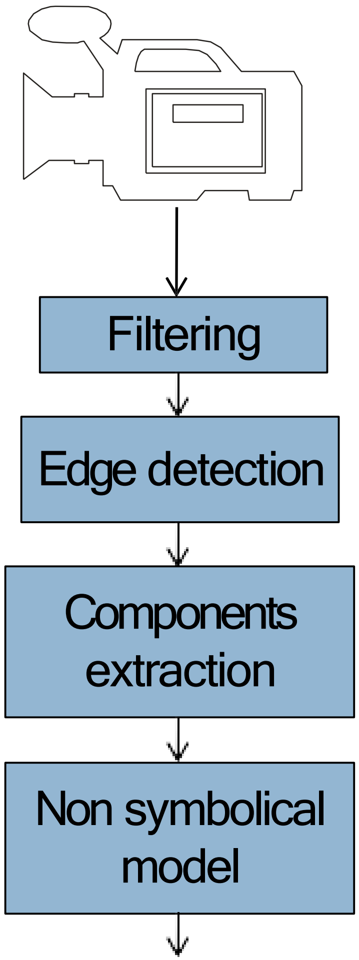
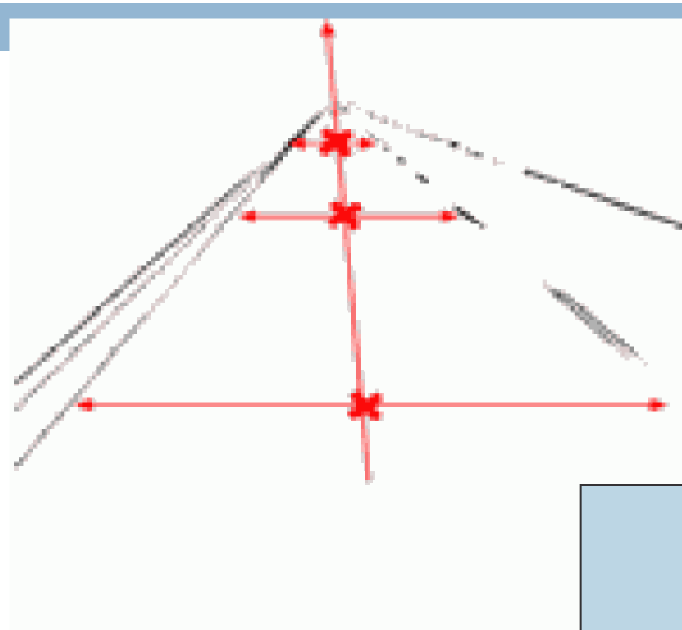
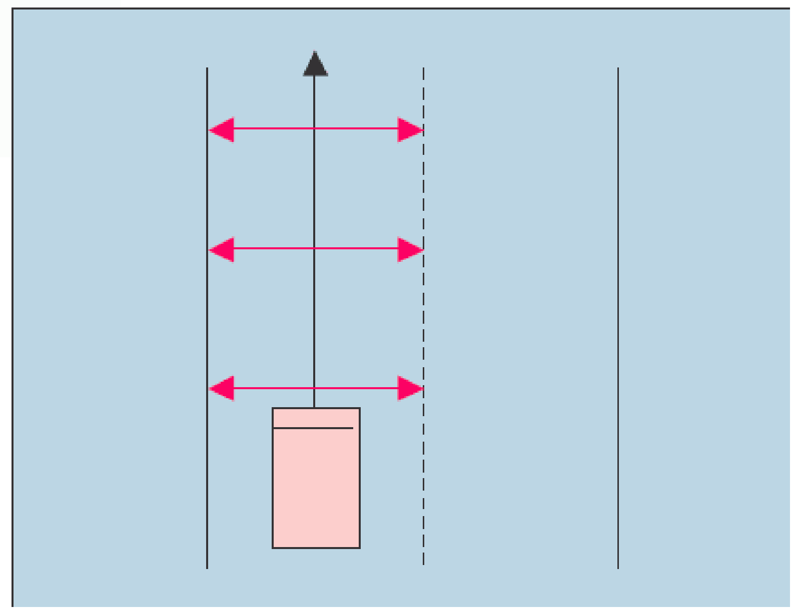


Image processing stages – non-symbolical



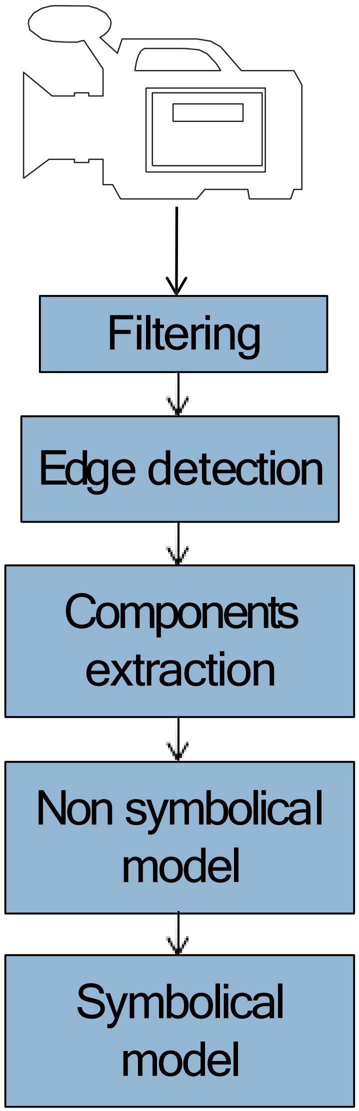


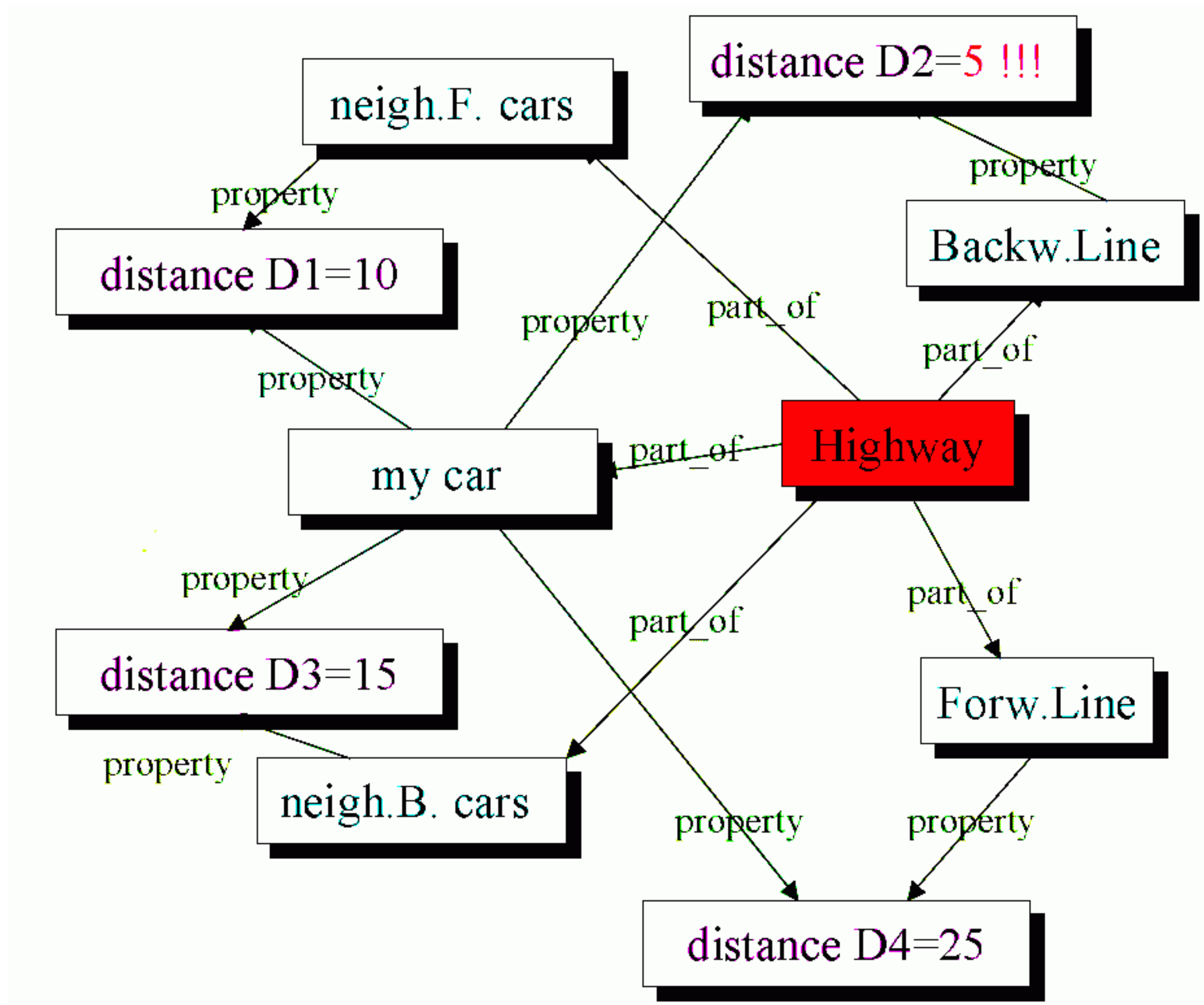
The image



Iconical mode

Image processing stages – symbolical model





Summary

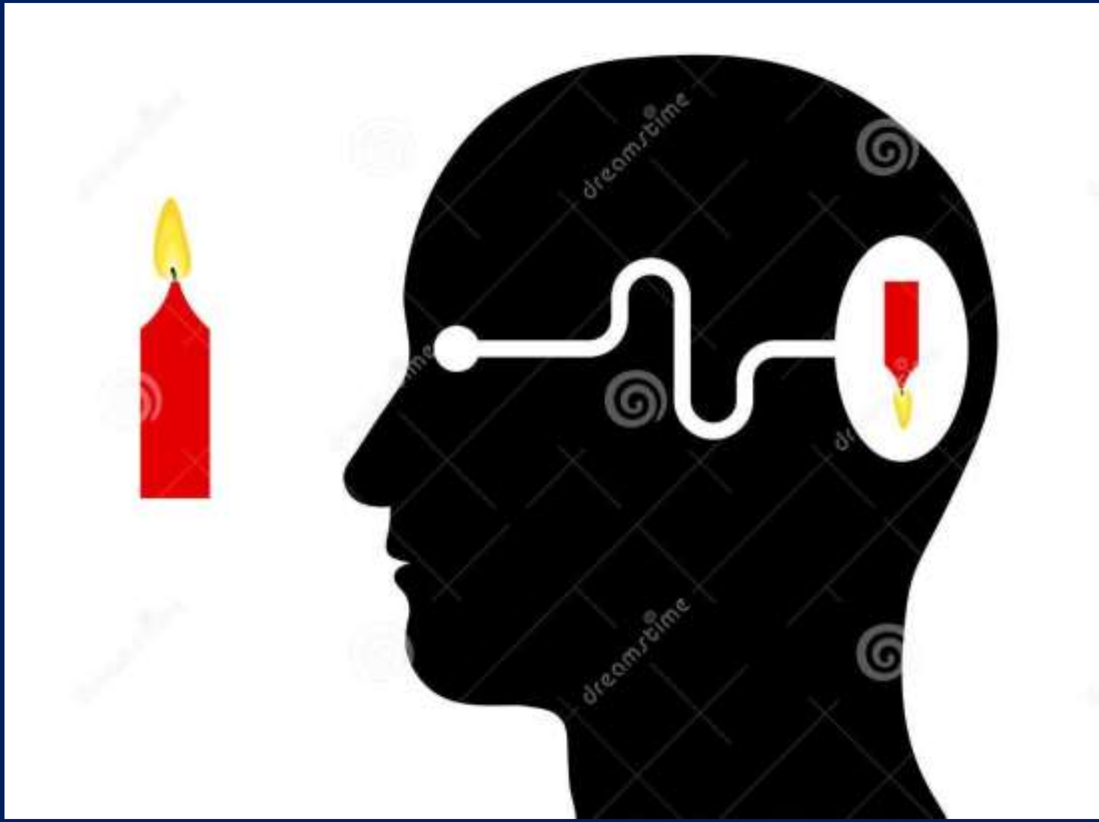
- We have looked at:
  - What is a digital image?
  - What is digital image processing?
  - History of digital image processing
  - State of the art examples of digital image processing
  - Key stages in digital image processing
- Next time we will start to see how it all works…

Any Questions ?

MCSC0009:

Image Processing And Analysis

Elements of Visual Perception



DIP

Mathematical	Probabilistic
Formulations	

However, the choice of technique is often based on subjective, visual judgment

Hence, understanding Human Visual Perception VERY IMPORTANT !!!

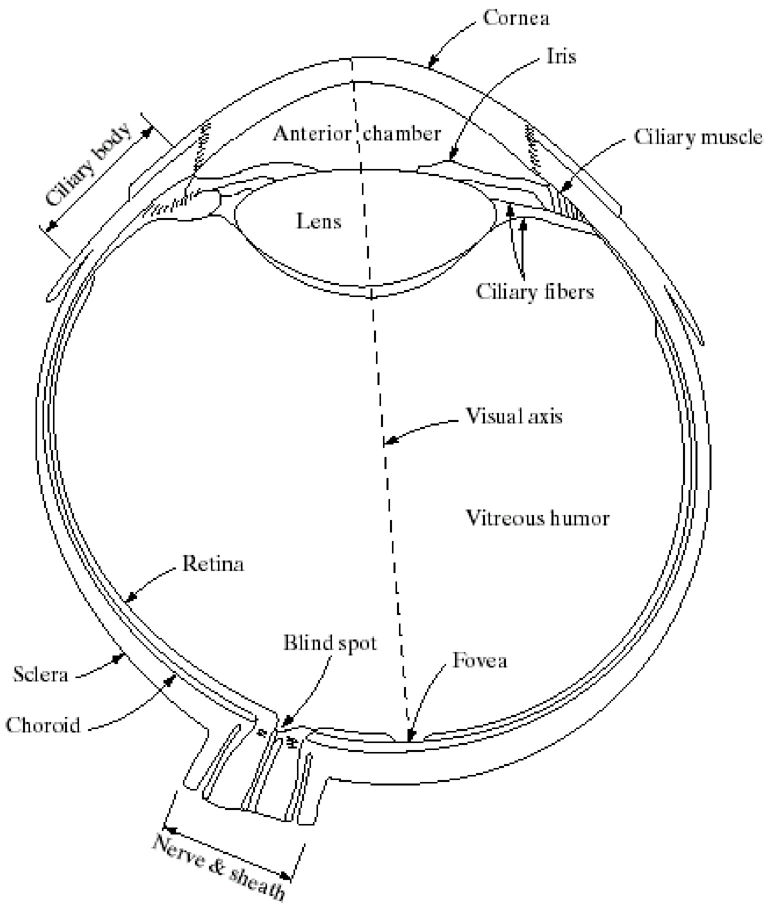
The best vision model we have!

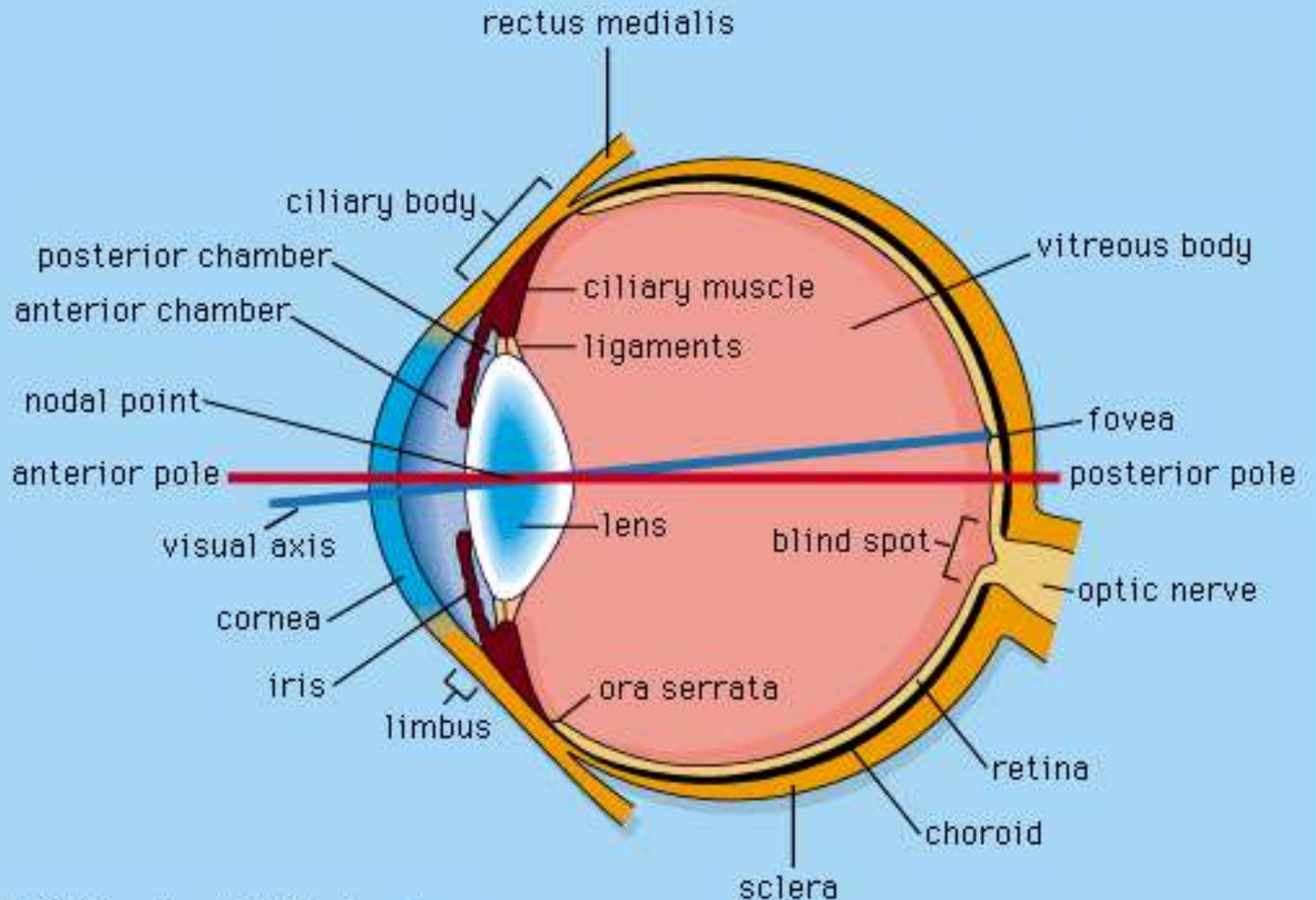
Knowledge of how images form in the eye can help us with processing digital images

Structure (anatomy) of the human eye

- Avg diameter : 20mm

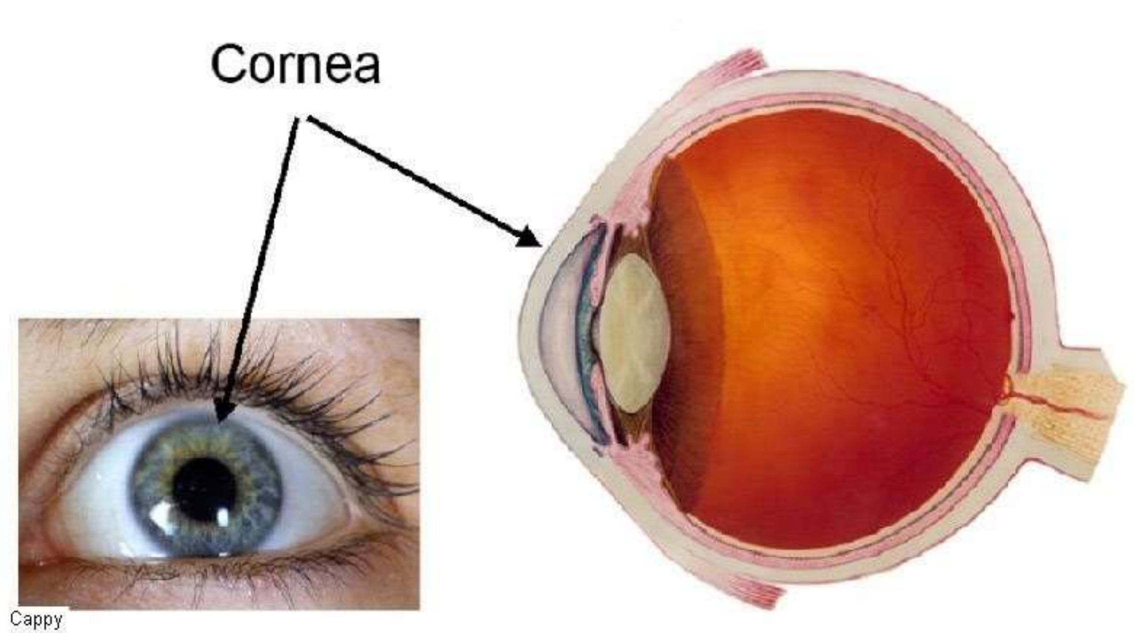
- 3 membranes enclose the eye
  - Cornea & sclera
  - Choroid
  - Retina




rectus medialis
ciliary body
posterior chamber
anterior chamber
nodal point
anterior pole
visual axis
cornea
iris
limbus
ciliary muscle
ligaments
lens
ora serrata
vitreous body
fovea
posterior pole
blind spot
optic nerve
retina
choroid
sclera

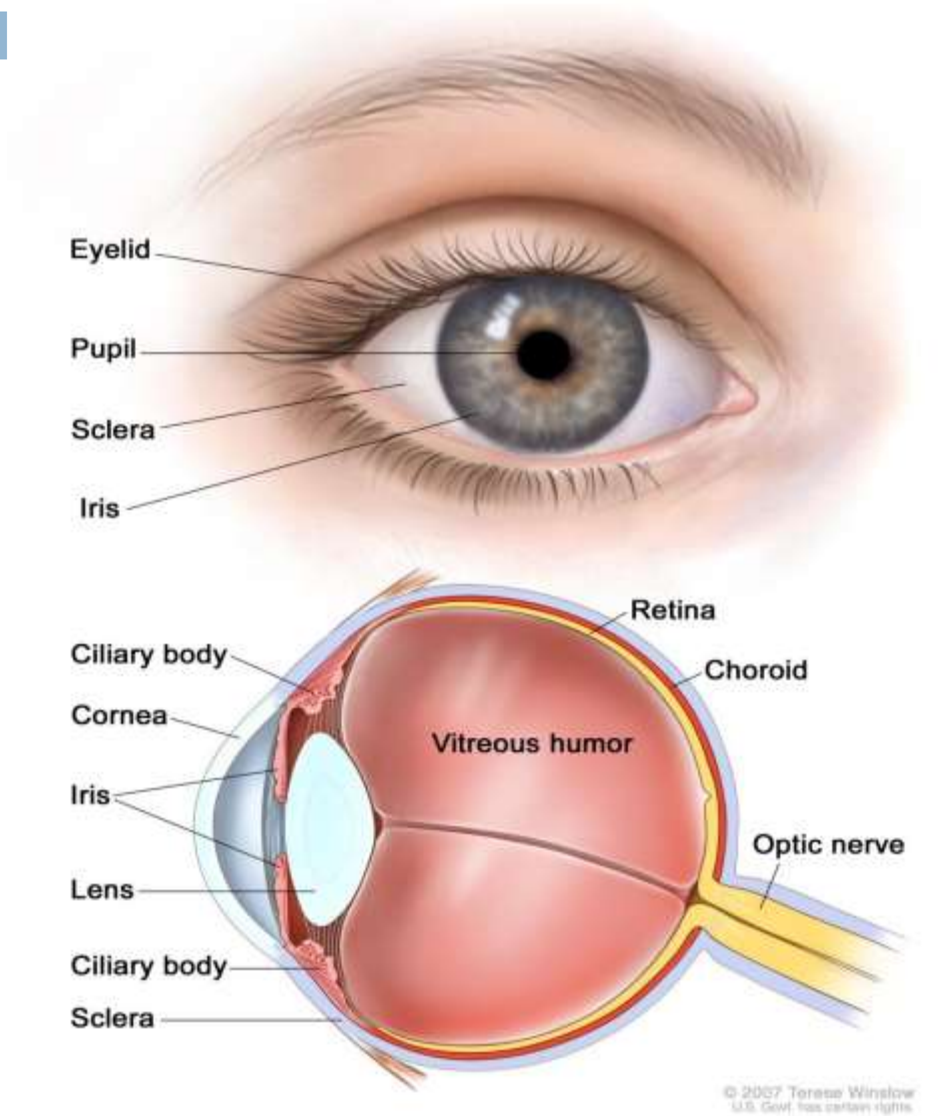
Structure of the human eye …

- **Cornea**
  - Tough & Transparent tissue
  - Covers the frontal surface of the eye



Structure of the human eye …

- **Sclera**
  - Continuous with cornea
  - Opaque membrane
  - Encloses the remainder of the optic globe



Structure of the human eye …

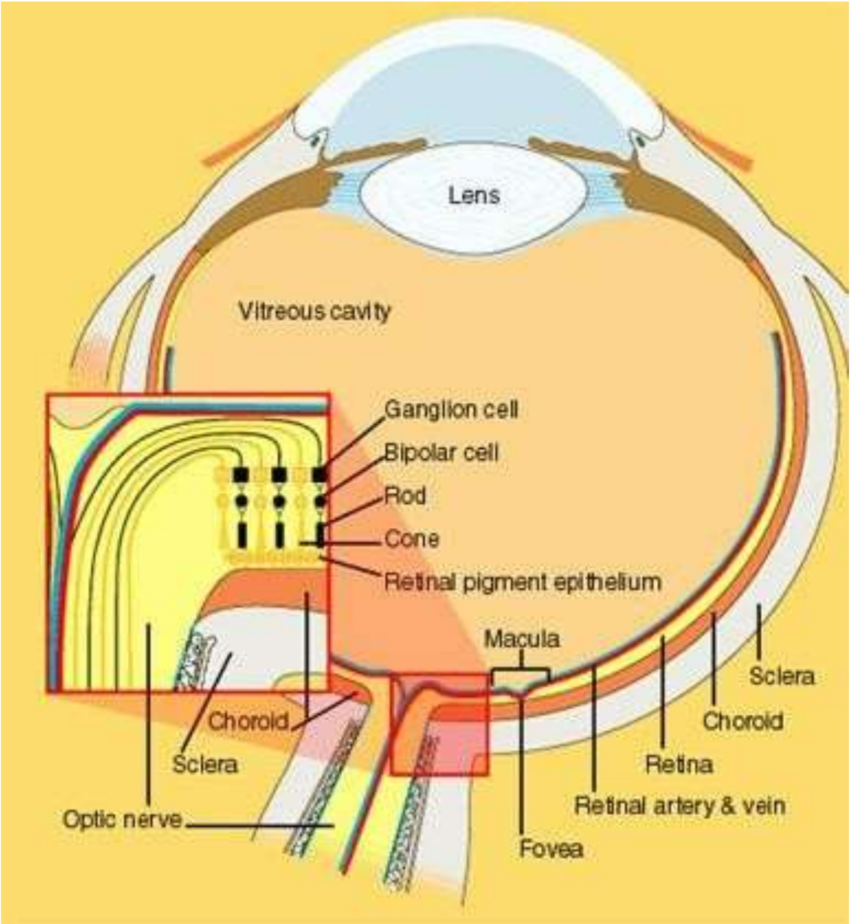
Choroid

- The choroid contains blood vessels for eye nutrition and is heavily pigmented to reduce extraneous light entrance and backscatter.
- It is divided into the ciliary body and the iris diaphragm, which controls the amount of light that enters the pupil (2 mm ~ 8 mm).

- **The lens is made up of fibrous cells and is suspended by fibers that attach it to the ciliary body.**
- **It is slightly yellow and absorbs approx. 8% of the visible light spectrum.**

Structure of the human eye …

- **Retina**
  - Light from an object is imaged on the retina
  - The retina lines the entire posterior portion.
  - Discrete light receptors are distributed over the surface of the retina:
    - cones (6-7 million per eye) and
    - rods (75-150 million per eye)



Structure of the human eye …

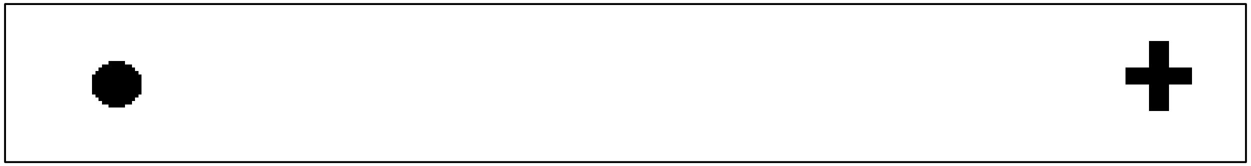
- **Cones**
  - Cones provide color vision and respond to higher levels of illumination
  - The density of the cones is higher in the fovea
  - Each one is connected to its own nerve end.
  - Cone vision is called *photopic* (or bright-light vision).
  - **Muscles controlling the eye rotate the eye ball until the image of an object of interest falls on the fovea.**

Structure of the human eye …

- **Rods**
  - Rods are distributed over the retinal surface
  - Rods give a general, overall picture of the field of view and are not involved in color vision.
  - Rods are important for black and white vision in dim light
  - Discriminate between different shades of darks and light
  - Rods provide visual response called Scotopic Vision
  - **Objects seen by moon light appear as colourless forms because only rods are stimulated.**

Blind-Spot Experiment

- Draw an image similar to that below on a piece of paper (the dot and cross are about 6 inches apart)

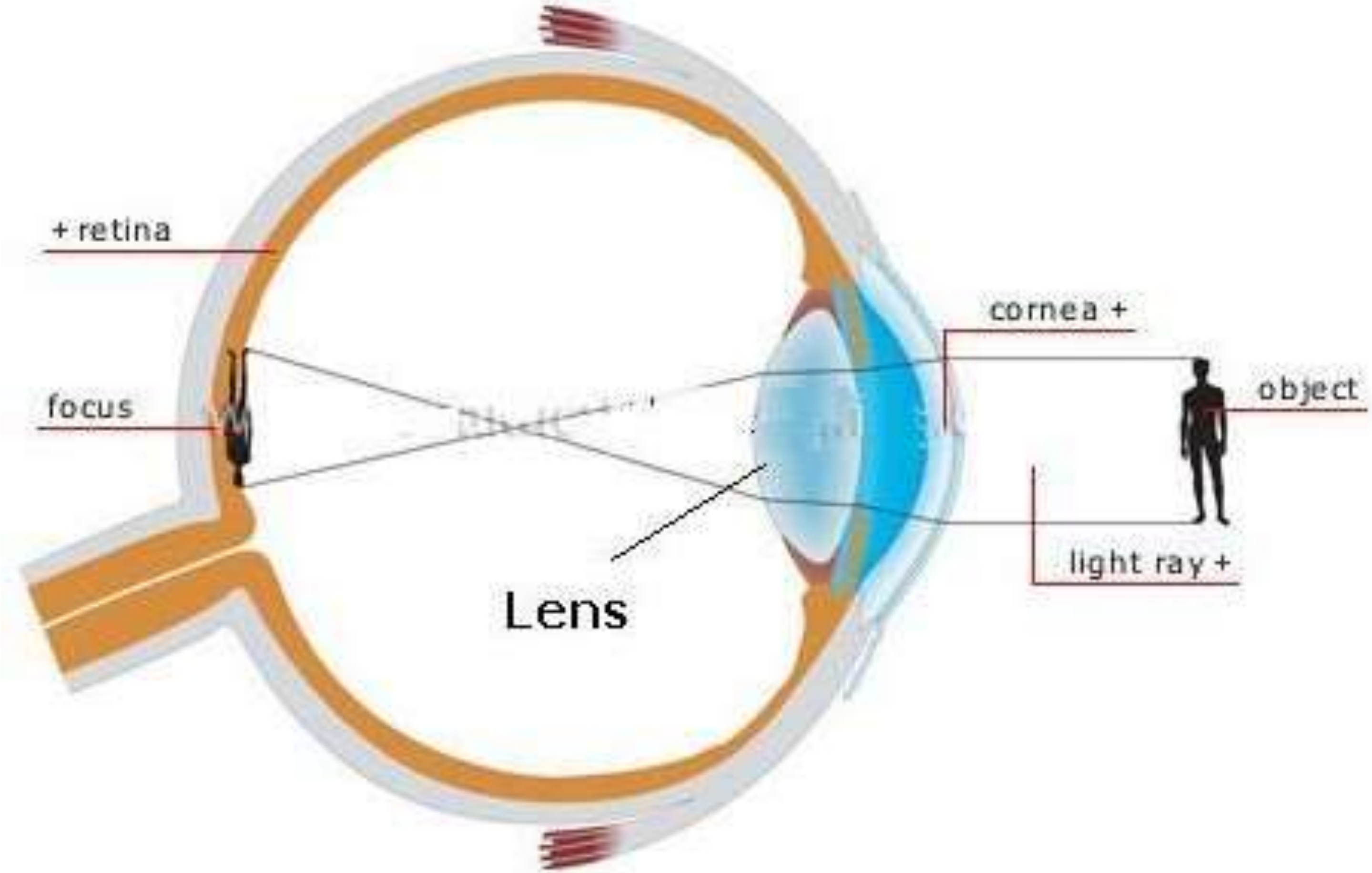


- Close your right eye and focus on the cross with your left eye
- Hold the image about 20 inches away from your face and move it slowly towards you
- The dot should disappear!

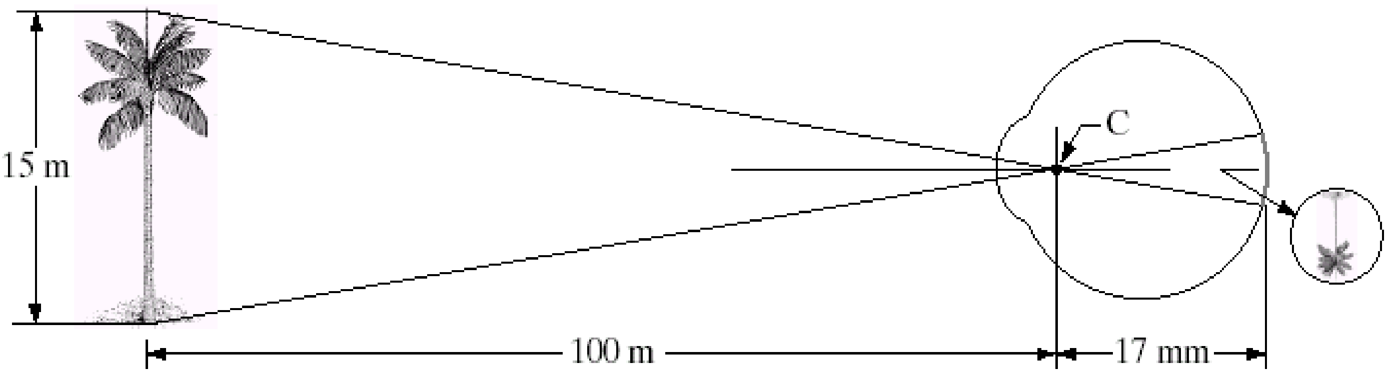
Image Formation in the Eye

- **The eye lens (as compared to an optical lens) is flexible.**
- **It gets controlled by the fibers of the ciliary body and to focus on distant objects it gets flatter (and vice versa).**
- Distance between the center of the lens and the retina (*focal length*):
  - varies from 17 mm to 14 mm (refractive power of lens goes from minimum to maximum).
- Objects farther than 3 m use minimum refractive lens powers (Focal Length 17 mm) and vice versa.

Image Formation in the Eye



Graphical representation of the eye looking at a palm tree. Point C is the optical center of the lens.



Size of retinal image (h) $15 / 100 = h / 17$

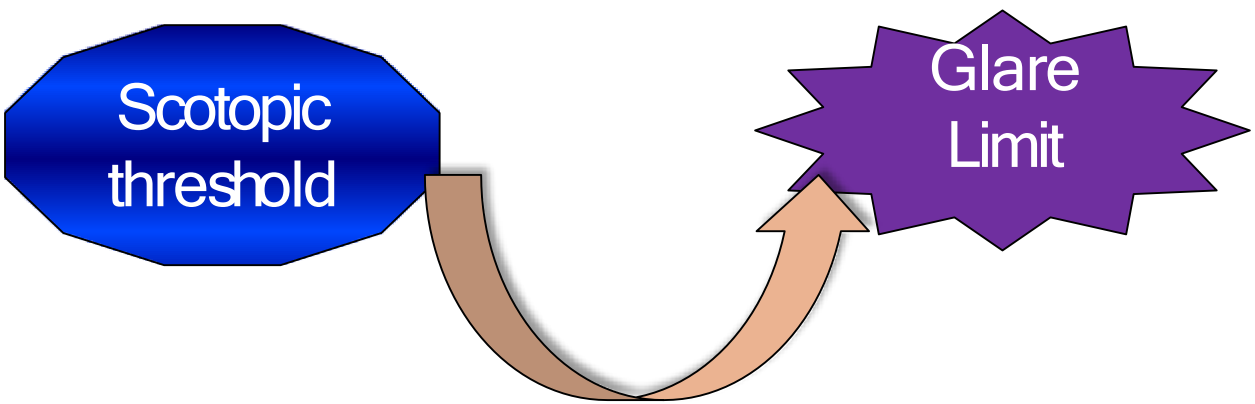
$h = 2.55$ mm

- Retinal image reflected primarily in the fovea
- Perception takes place by relative excitation of light receptors
- Receptors transform radiant energy into electrical impulses which are decoded by the brain

Brightness Adaptation & Discrimination

Range of light intensity levels to which human visual system (HVS) can adapt

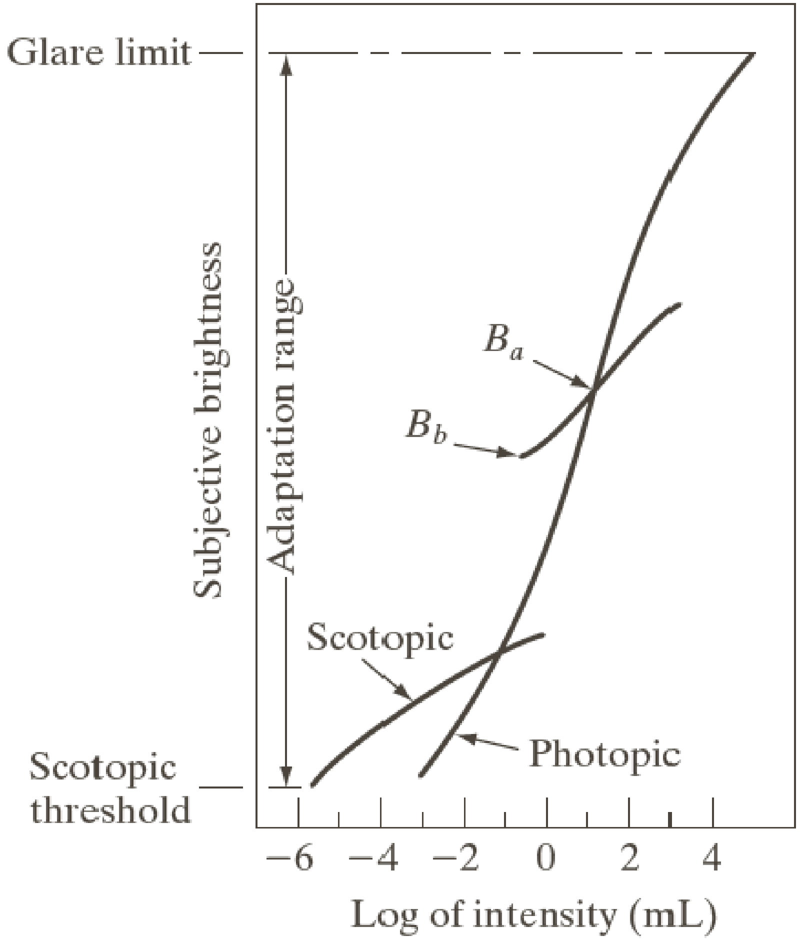
of the order of 10^{10}



Subjective brightness (i.e. intensity as perceived by the HVS) is a **logarithmic function** of the light intensity incident on the eye.

Visual Phenomena: Brightness adaptation

- The subjective brightness of human visual system has an impressive dynamic range.
- Cannot accomplish this range **simultaneously**



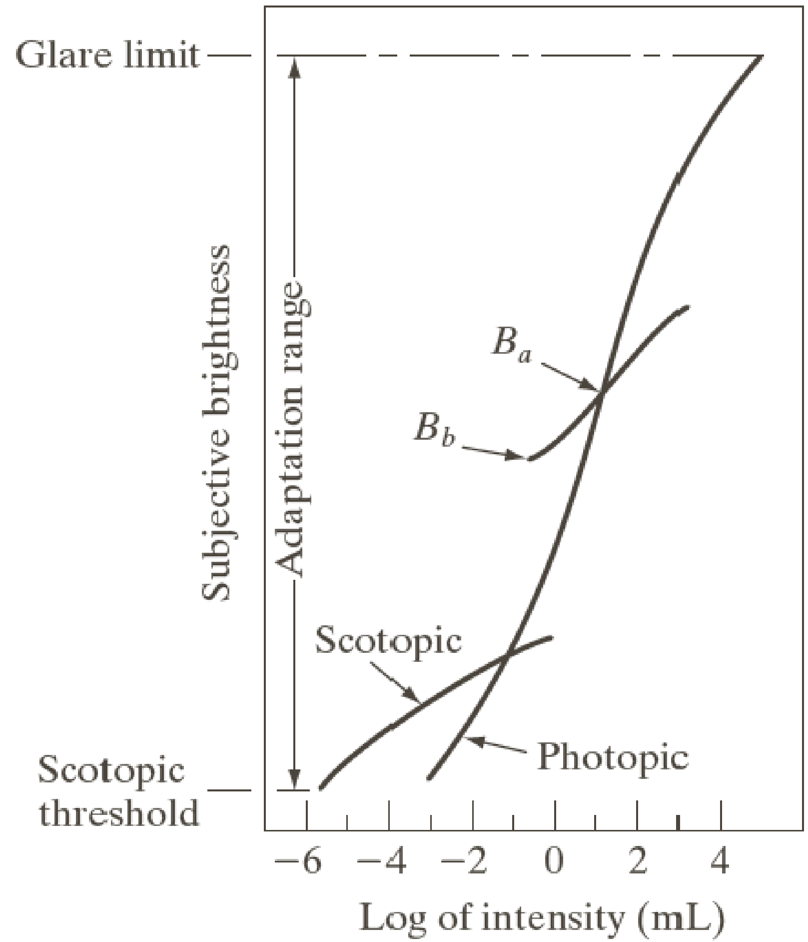
Visual Phenomena: Brightness adaptation

- The HVS accomplishes this wide variation by changes in its overall sensitivity.

Brightness Adaptation

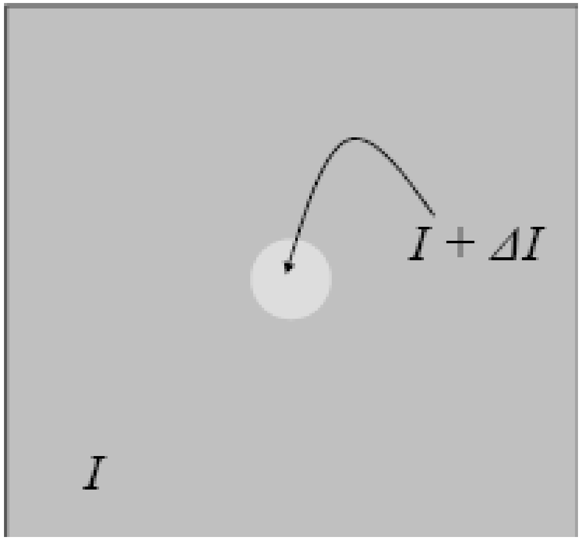
Brightness Adaptation Level

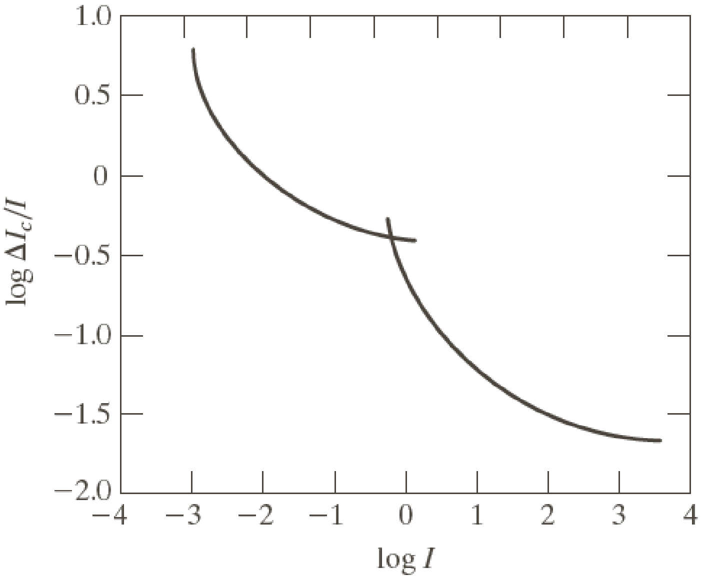
For any given set of conditions, the current



Brightness discrimination

- **Weber ratio** (the experiment) $\Delta I_c / I$
  - I: the background illumination
  - ΔI_c : the increment of illumination
  - Small Weber ratio indicates good discrimination
  - Larger Weber ratio indicates poor discrimination





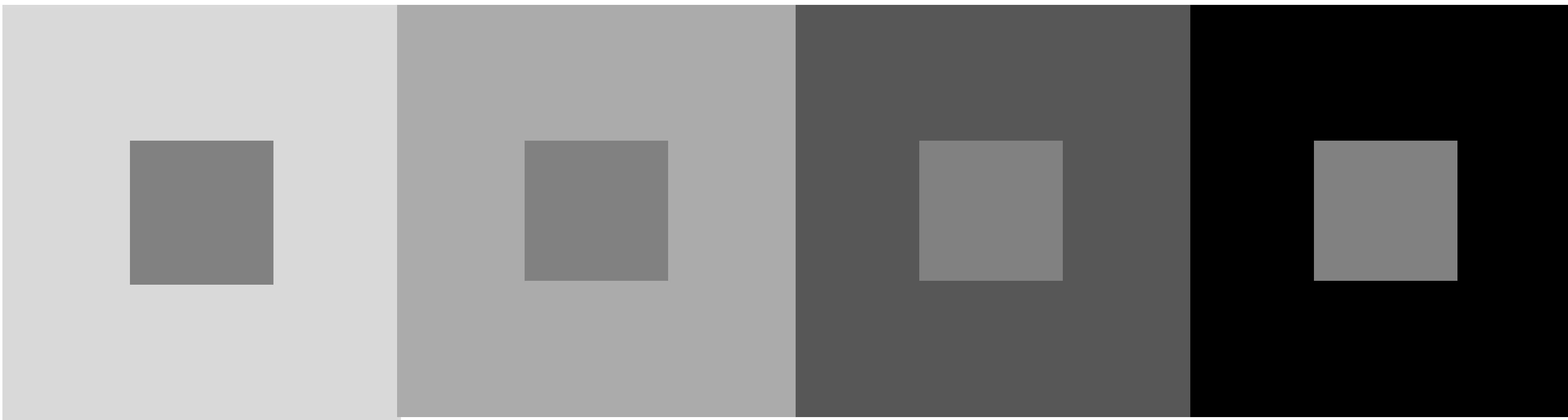
Psychovisual effects

- The perceived brightness is not a simple function of intensity
  - **Mach band pattern**
    - The visual system tends to undershoot or overshoot around the boundary of regions of different intensities
  - **Simultaneous contrast**
    - A region's perceived brightness does not depend simply on its intensity.
  - **Optical illusion**
    - Eye fills in nonexisting information or wrongly perceives geometrical properties of objects

Psychovisual effects : Mach band pattern

- The Mach band effect is illustrated in the figure above.
- The intensity is uniform over the width of each bar.
- However, the visual appearance is that each strip is darker at its right side than its left.

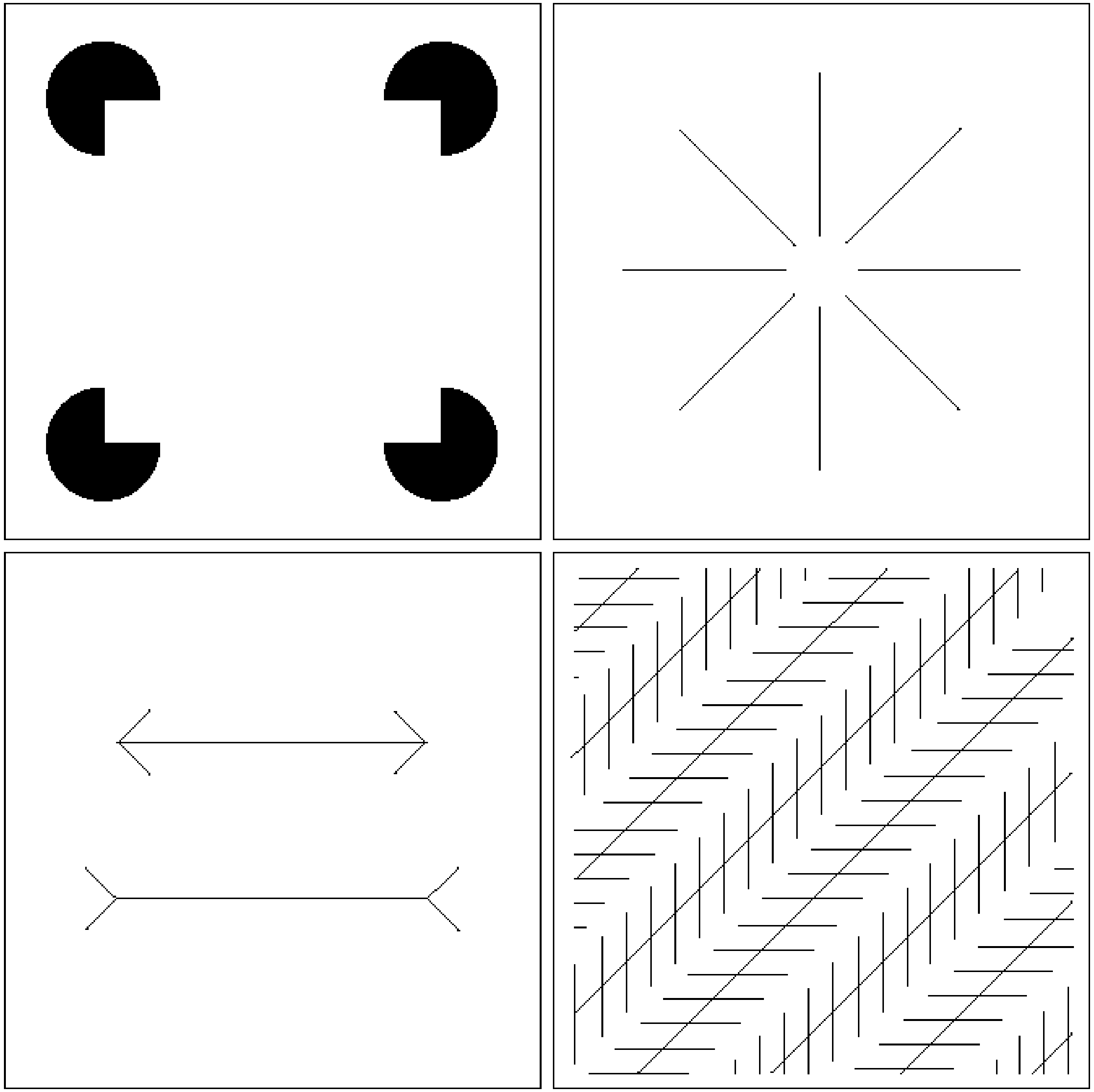
Psychovisual effects :Simultaneous contrast



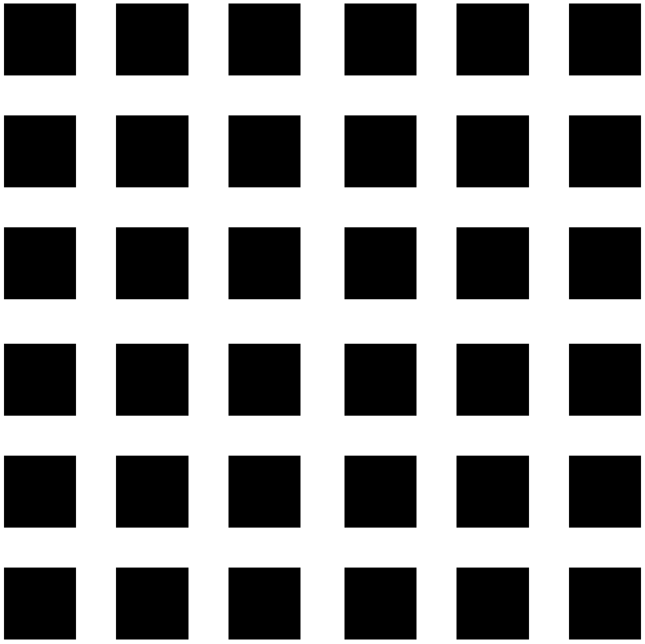
- The simultaneous contrast phenomenon is illustrated above.
- The small squares in each image are the same intensity.
- Because of the different background intensities, the small squares do not appear equally bright.

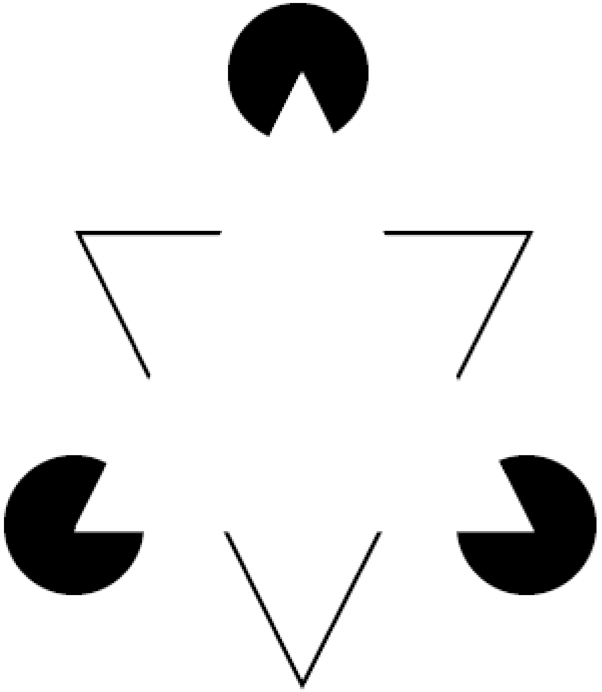
Psychovisual effects :Optical illusion

- Eye fills in non existing information or wrongly perceives geometrical properties of an object

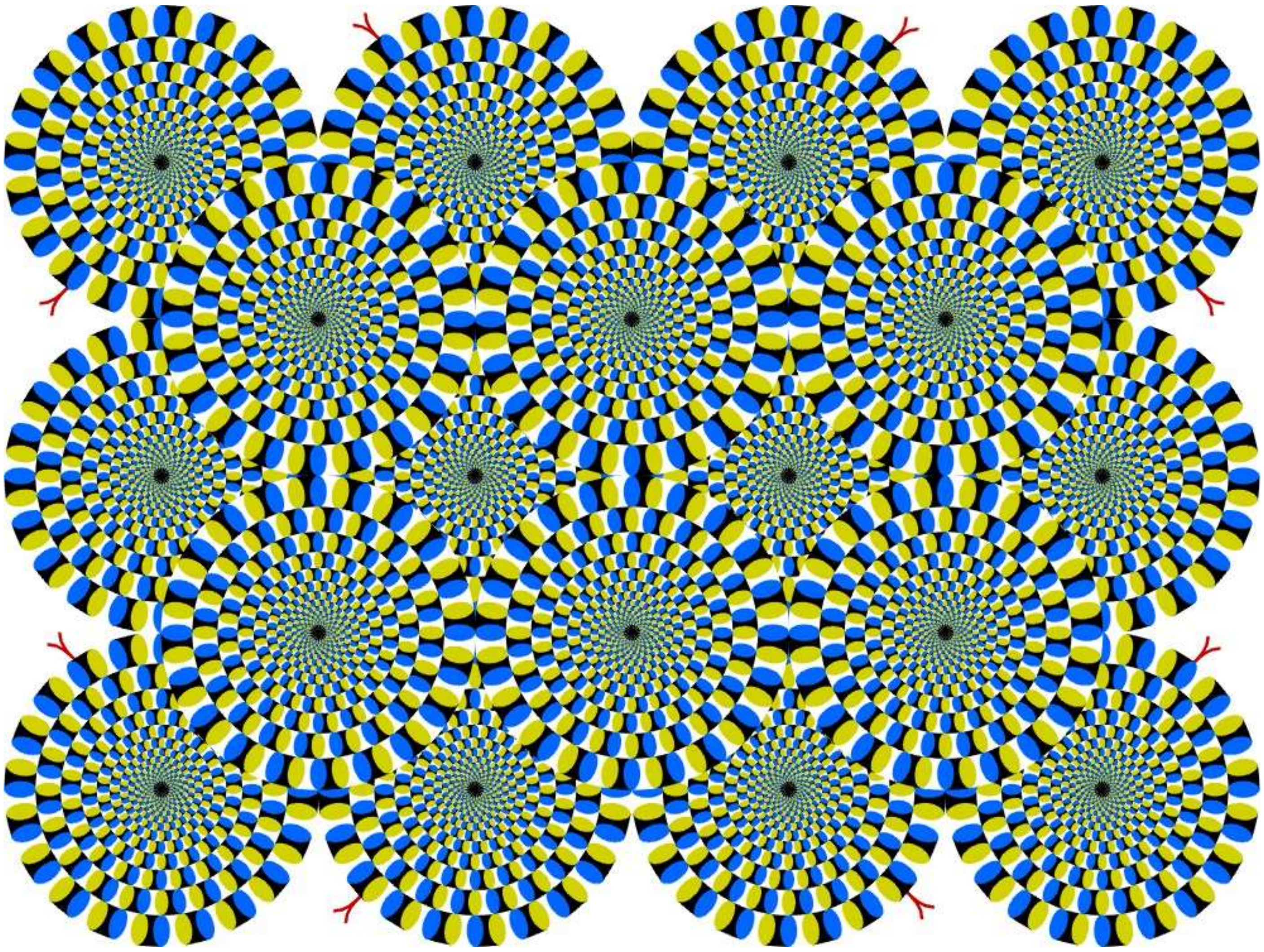


Psychovisual effects :Optical illusion …

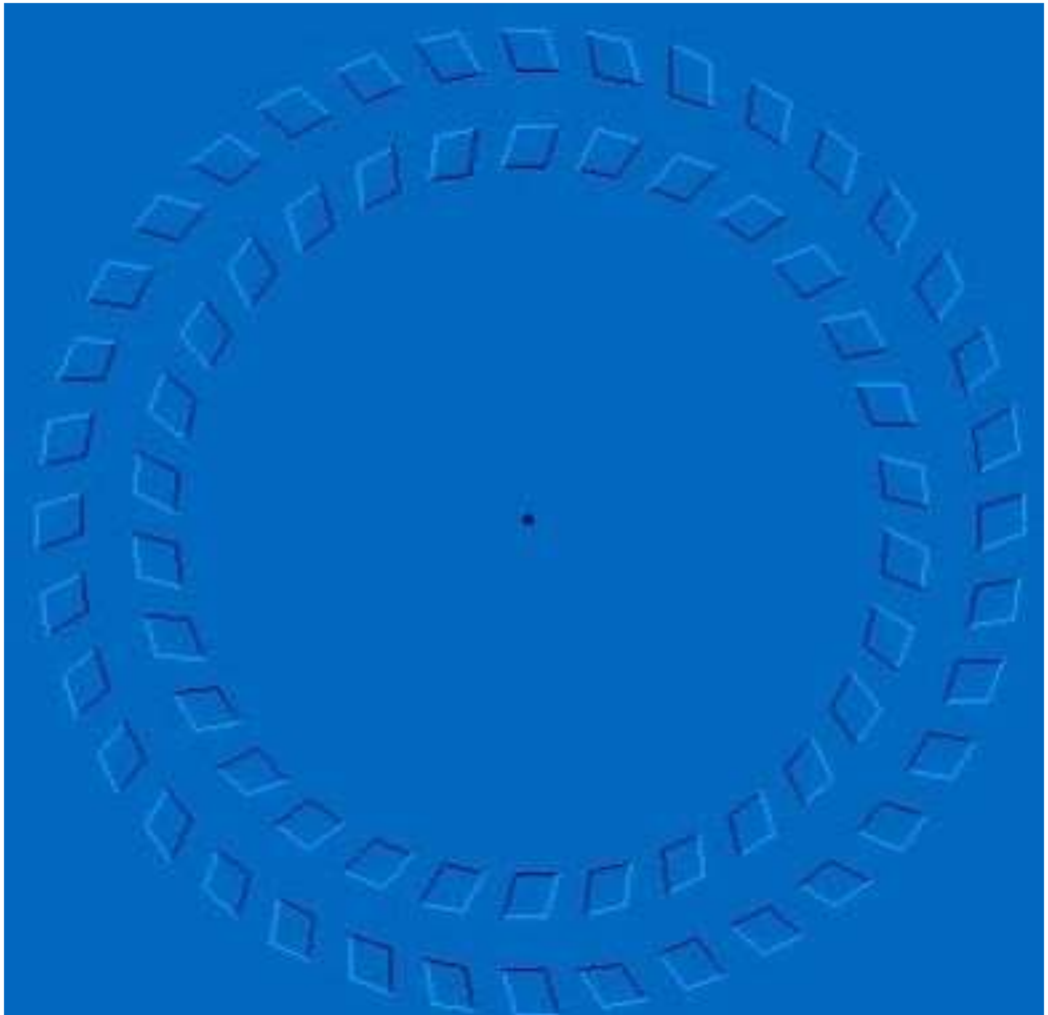




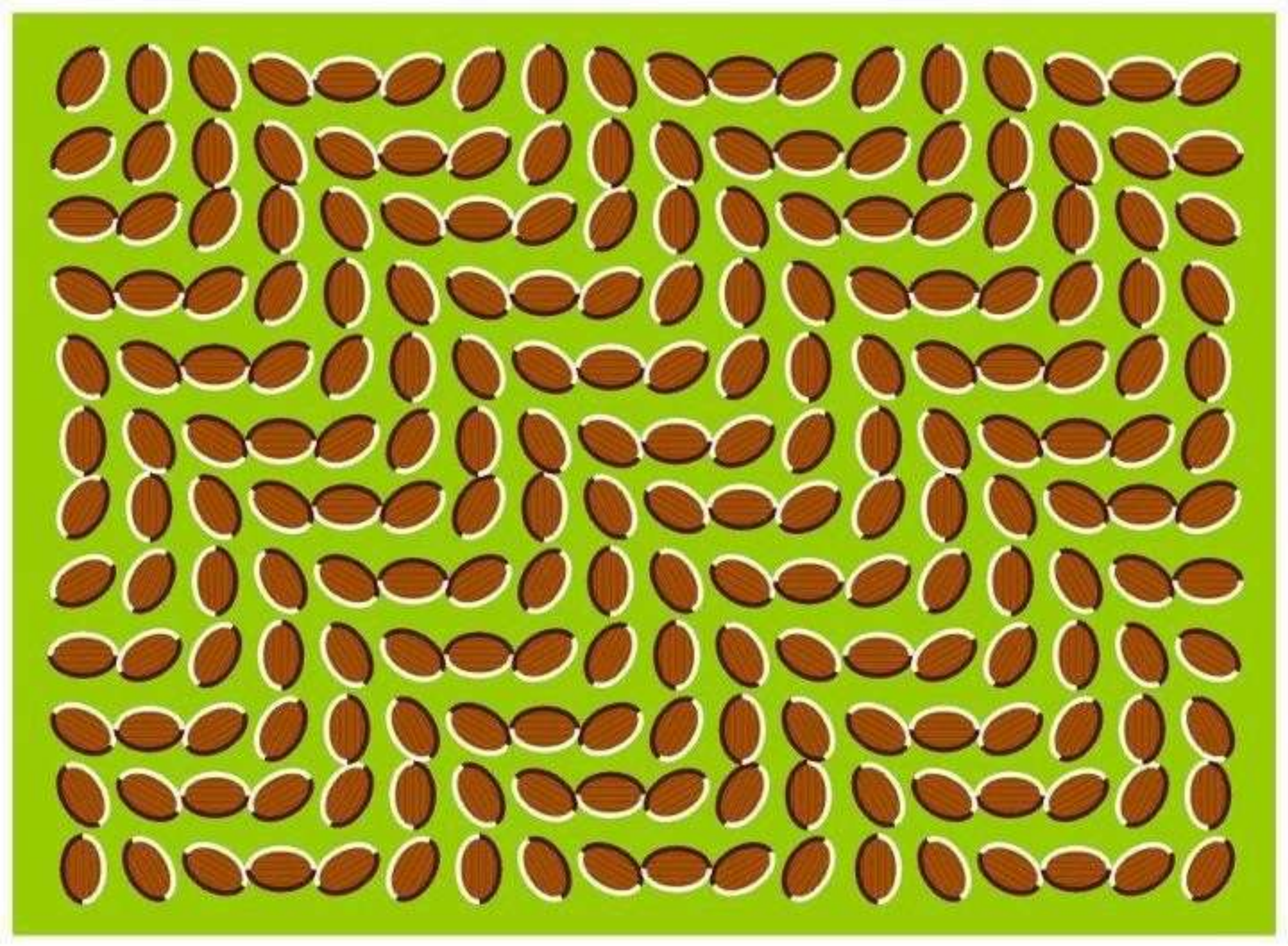
Psychovisual effects :Optical illusion …



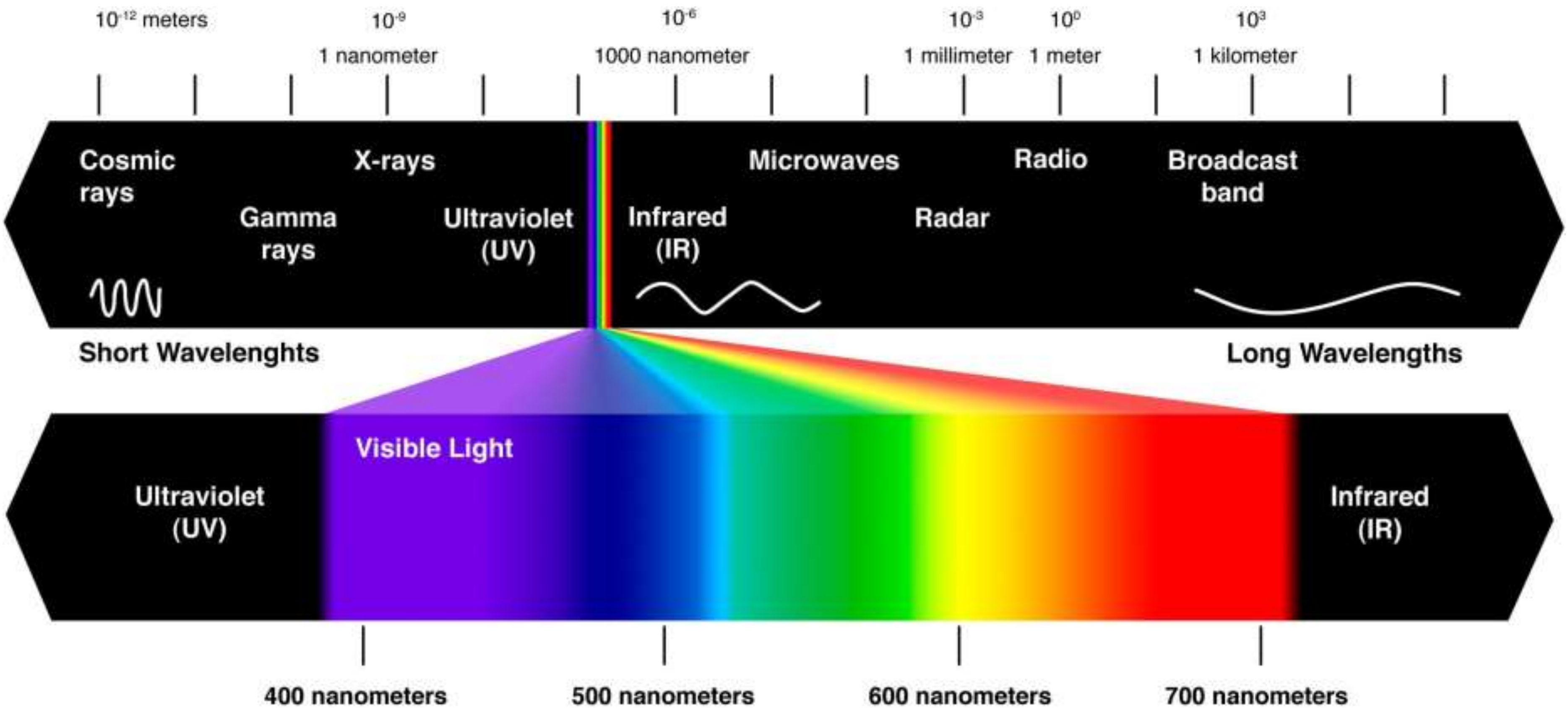
Psychovisual effects :Optical illusion …



Psychovisual effects :Optical illusion …

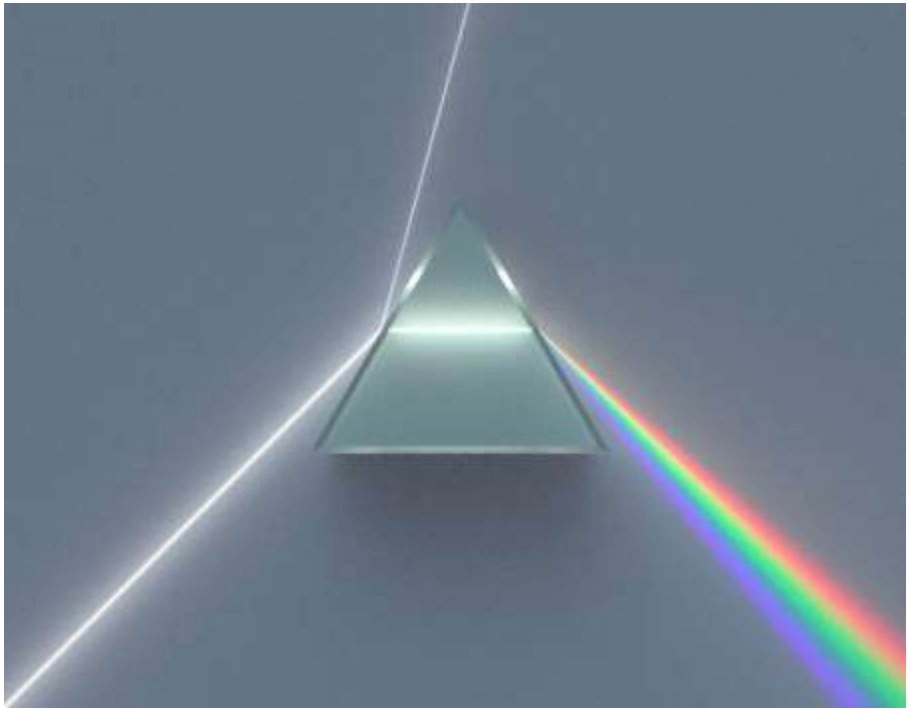


Electromagnetic spectrum

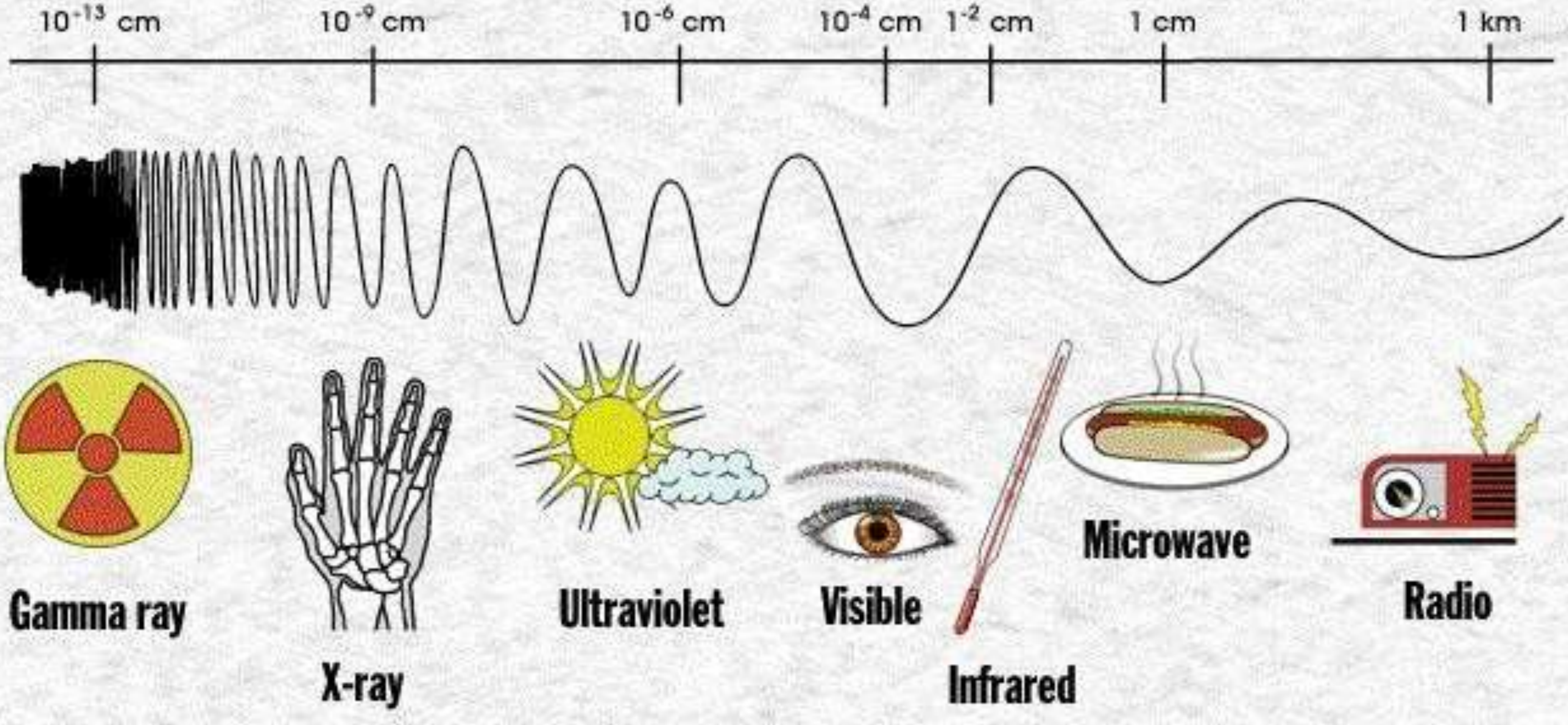


Spectrum of Colors

Sir Isaac Newton



The Electromagnetic Spectrum



Electromagnetic spectrum

- Speed = frequency x wavelength
- i.e $\lambda = c/\nu$
- Speed of light is 3×10^8 m/sec.
- The energy E, of the various components of the electromagnetic spectrum is given as:

$$E = h\nu$$ where h is Planck's constant

Chromatic Light

- **Radiance**
  - Total amount of energy that flows from the light source
  - Measured in Watts (W)
- **Luminance**
  - Measures the amount of energy an observer perceives from a light source.
  - Measured in lumens (lm)
- **Brightness**
  - Subjective descriptor - practically impossible to measure.
  - It embodies the notion of intensity.
  - Key factor in describing colour sensation.

Any Questions ?

MCSC0009:
Image Processing And Analysis

Image Sensing, Acquisition and Formation

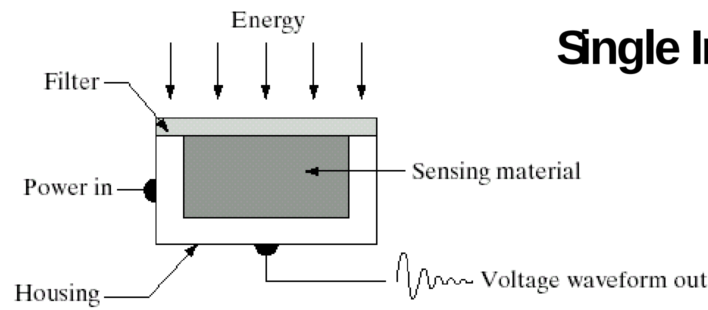
Sensors

Sensors are use to transform illumination energy into digital images.

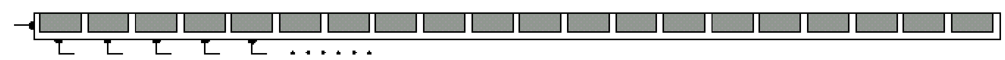
Sensors are three types:

- Single Imaging Sensor
- Line sensor
- Array Sensor

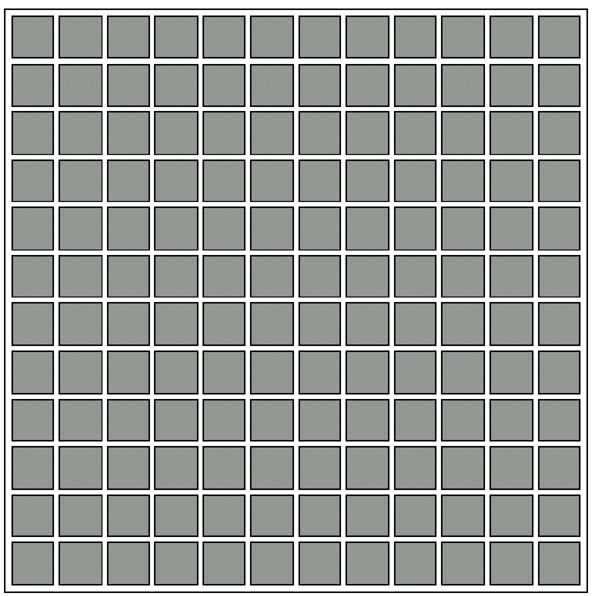
Single Imaging Sensor



Line sensor

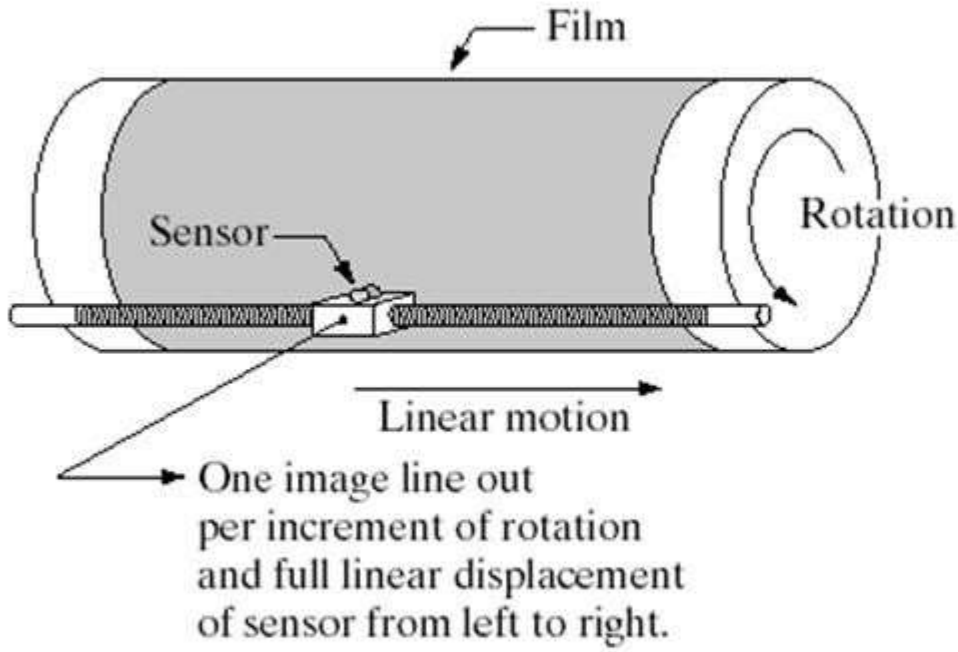


Array Sensor



Sensors: Single Sensors

- **Image acquisition using a single sensor**
- To generate a 2-D image using a single sensor, there has to be relative displacements in both the x- and y-directions between the sensor and the area to be imaged.

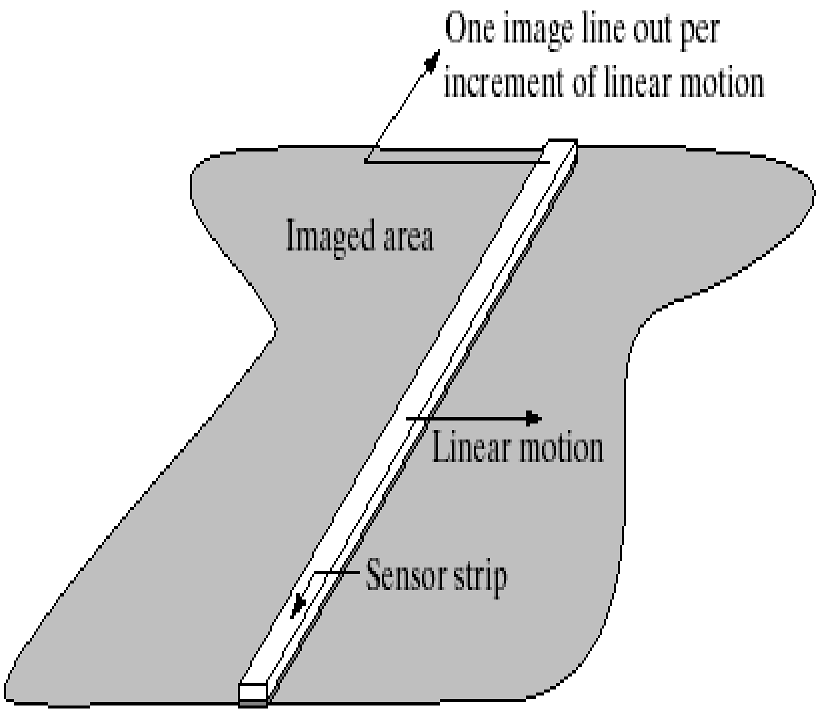


Combining a single sensor with motion to generate a 2-D image.

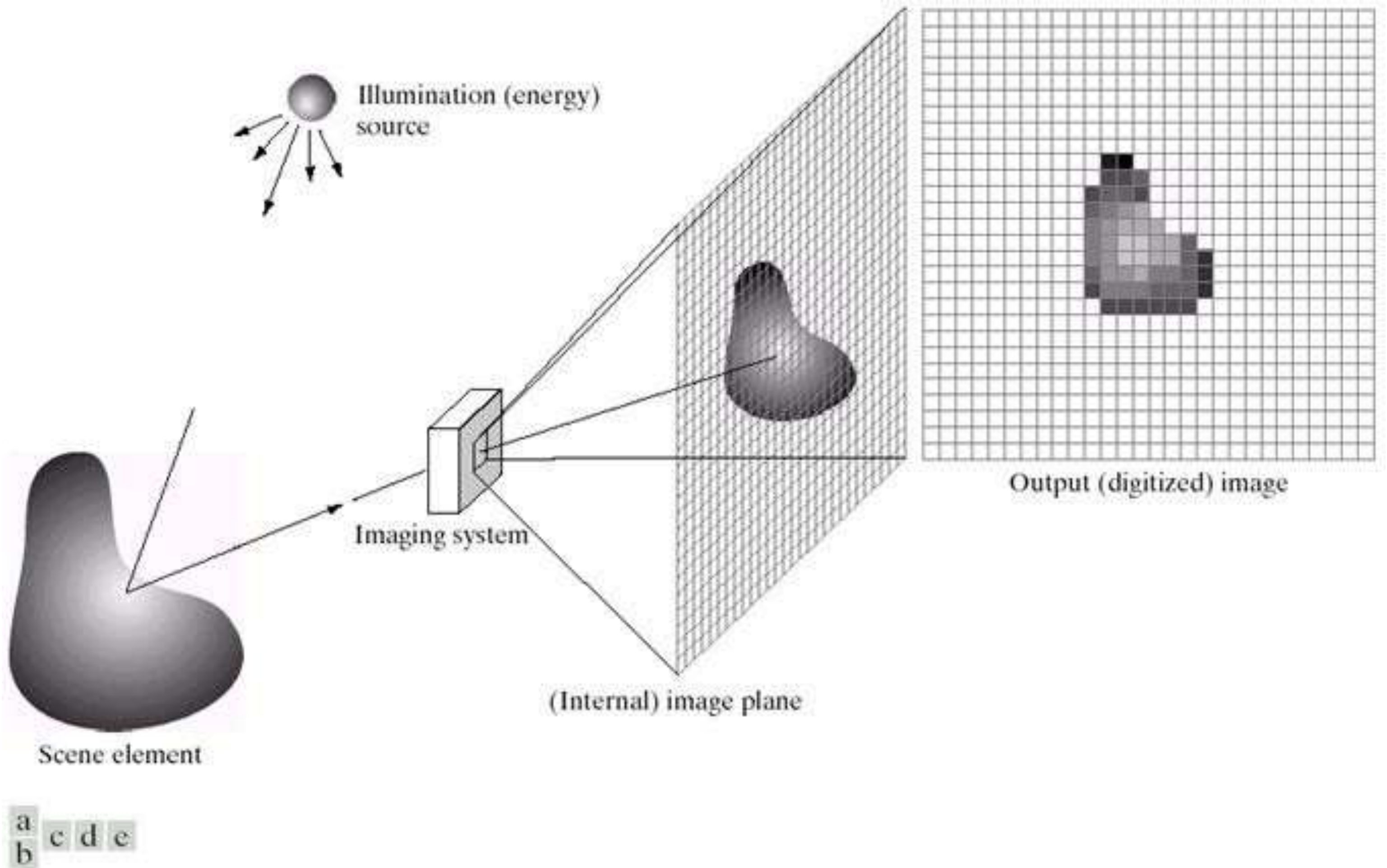
Sensors: Linear Sensor

A geometry that is used much more frequently than single sensors consists of an in-line arrangement of sensors in the form of a sensor strip.

The strip provides imaging elements in one direction. Motion perpendicular to the strip provides imaging in the other direction



Sensors: Array Sensor



An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

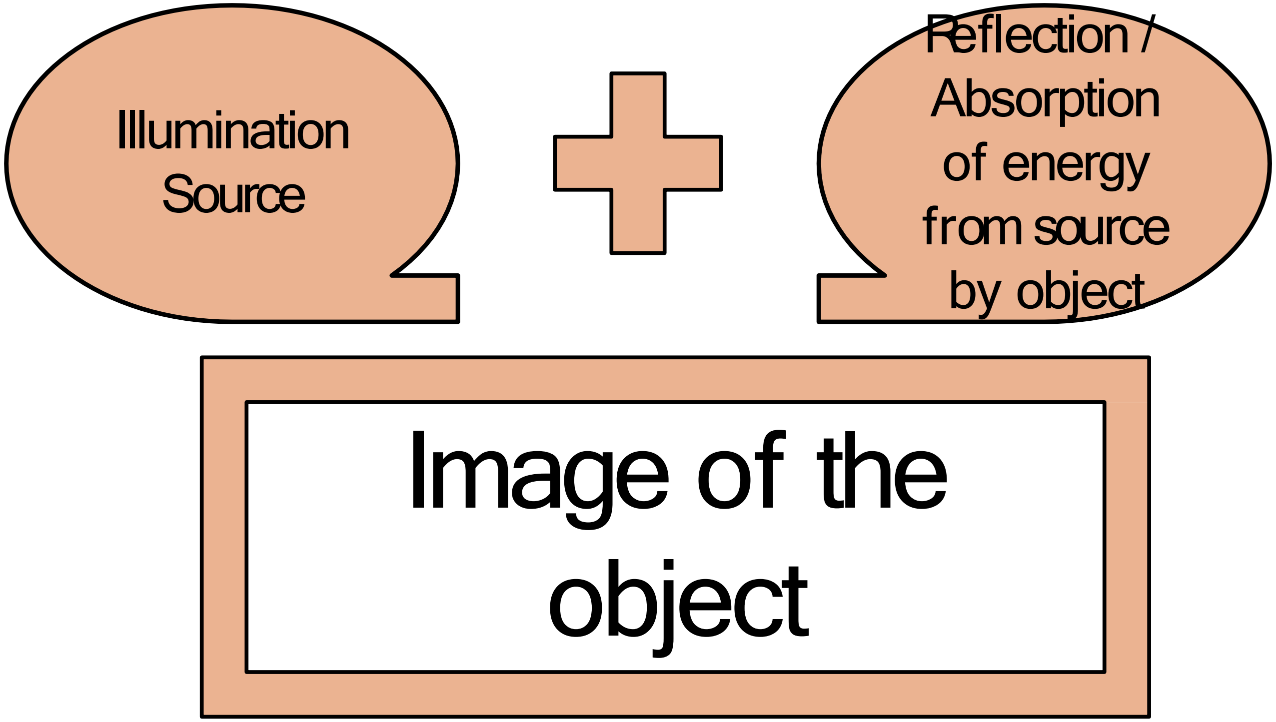
A simple image formation model

- Image: a 2-D light-intensity function f(x,y)
- f(x,y): the intensity is called the **gray level** for monochrome image
- $0 < f(x,y) < \infty$

- Nature of f(x,y):
  - The amount of source light incident on the scene being viewed
  - The amount of light reflected by the objects in the scene

A simple image formation model …

f(x, y) is characterized by two components



i(x, y) illumination component	r(x, y) reflectance component

A simple image formation model…

$$f(x, y) = i(x, y)\, r(x, y)$$

where

$$0 < i(x, y) < \infty$$

and

$$0 < r(x, y) < 1$$

Total Absorption

Total Reflection

Typical values of i(x, y)

- On a sunny day, illumination on earth's surface is 90,000 lm/m^2
- On a cloudy day it is 10,000 lm/m^2
- Full moon yields 0.01 lm/m^2
- Commercial office yields 1000 lm/m^2

Typical values of r(x, y)

- for black velvet – 0.01
- Stainless steel – 0.65
- Flat white wall paint – 0.90
- Snow – 0.93

Image Digitization

- Why do we need digitization?
- What is digitization?
- How to digitize an image?

Why Digitization?

- **Theory of Real numbers -** between any two given points there are infinite number of points
  - An image can be represented by infinite number of points
  - Each such image point may contain one of the infinitely many possible intensity/color values needing infinite number of bits
- **Obviously such a representation is not possible in any digital computer**

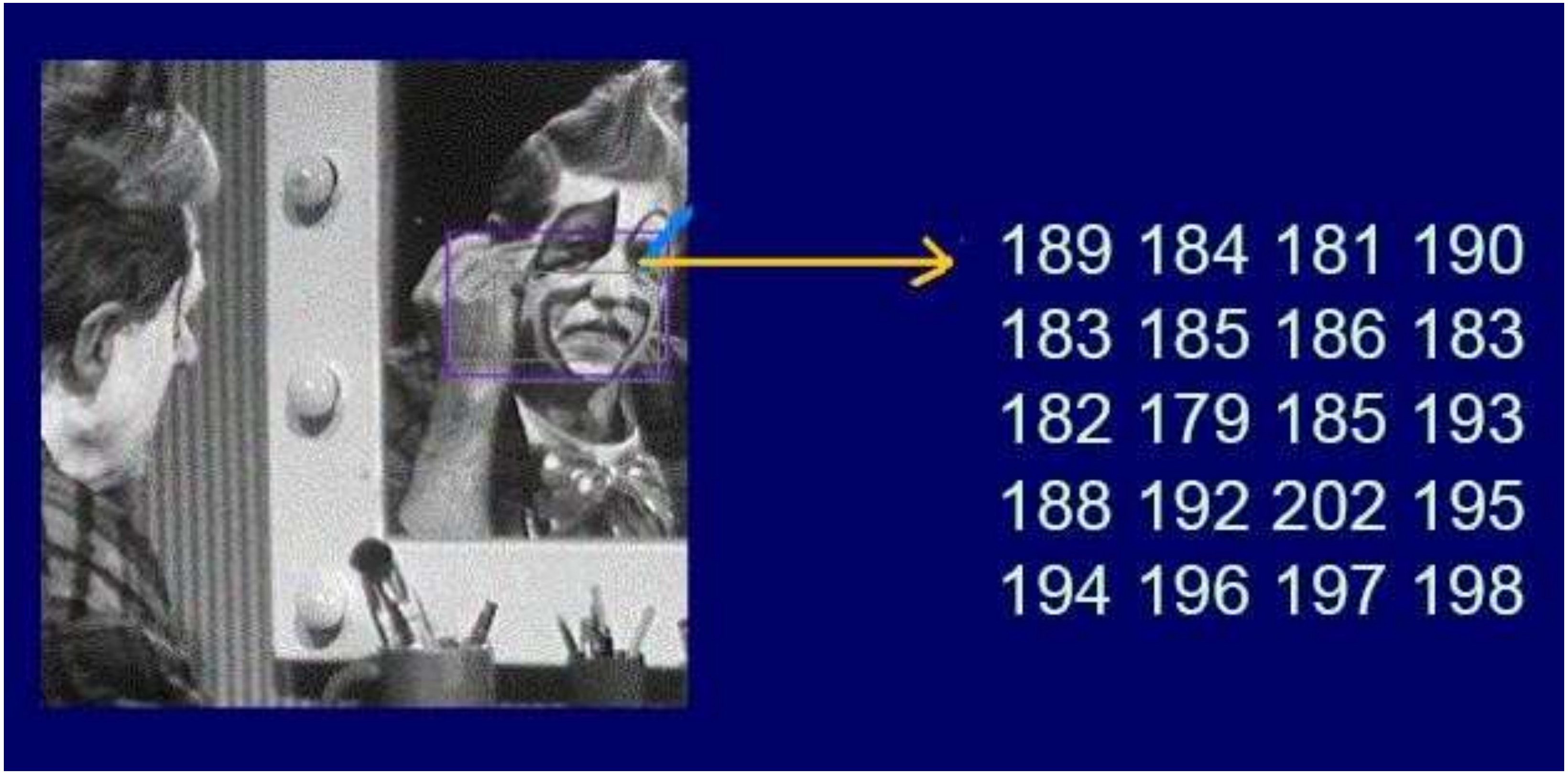
What is desired?

- An image to be represented in the form of a finite 2-D matrix

$$
I = \begin{bmatrix}
f(0,0) & f(0,1) & f(0,2) \ldots\ldots\ldots & f(0,N-1) \\
f(1,0) & f(1,1) & f(1,2) \ldots\ldots\ldots & f(1,N-1) \\
f(2,0) & f(2,1) & f(2,2) \ldots\ldots\ldots & f(2,N-1) \\
\vdots & \vdots & \vdots & \vdots \\
f(M-1,0) & f(M,1) & f(M,2) \ldots & f(M-1,N-1)
\end{bmatrix}
$$

Each of the matrix elements should assume one of finite discrete values

Image as a Matrix of Numbers



What is Digitization?

- **Image representation by 2-D finite matrix.** **It is** related to coordinates values

Sampling

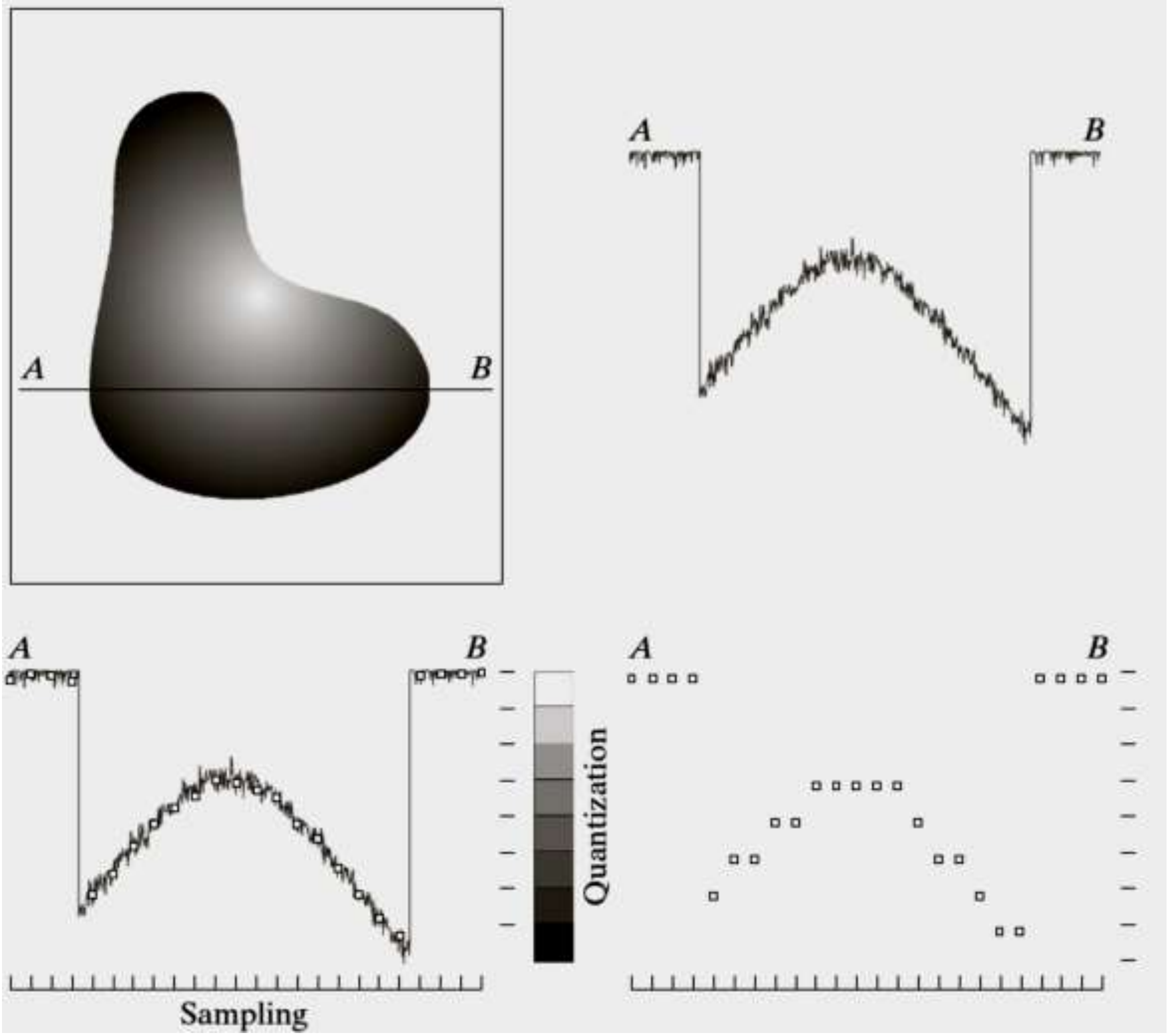
- **Each Matrix element represented by one of the finite set of discrete values. It is related to intensity values**

Quantization

Image Sampling and Quantization

- To convert an Image to digital form, we have to sample the Image in both coordinates (spatial domain) and in amplitude.
- Digitizing the coordinate (spatial domain) values is called ***sampling***. ***(Nyquist frequency)***
- Digitizing the amplitude values is called ***quantization***.

Sampling and Quantization



a b
c d

Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Sampling and Quantization

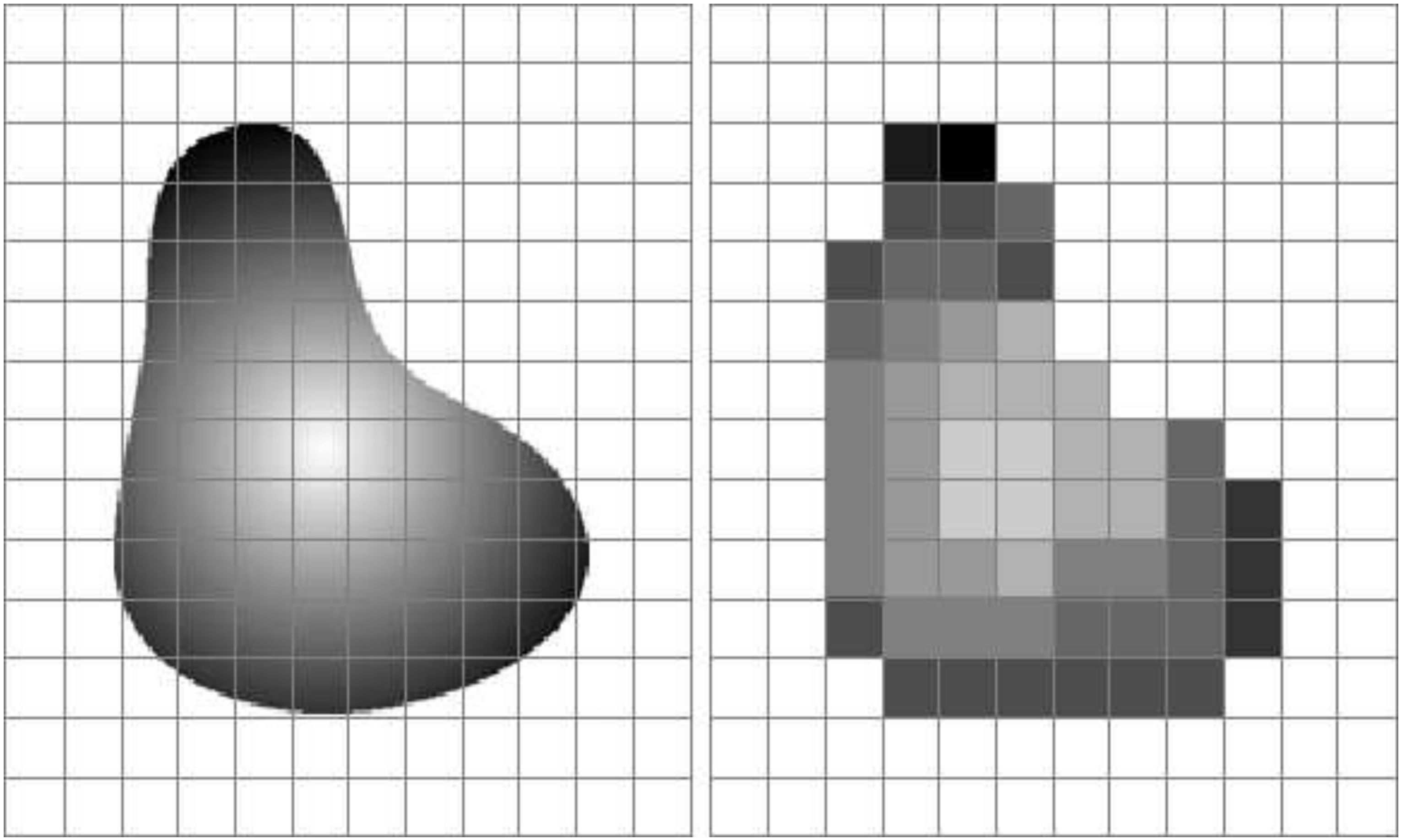


Image before sampling and quantization   Result of sampling and quantization

Aliasing

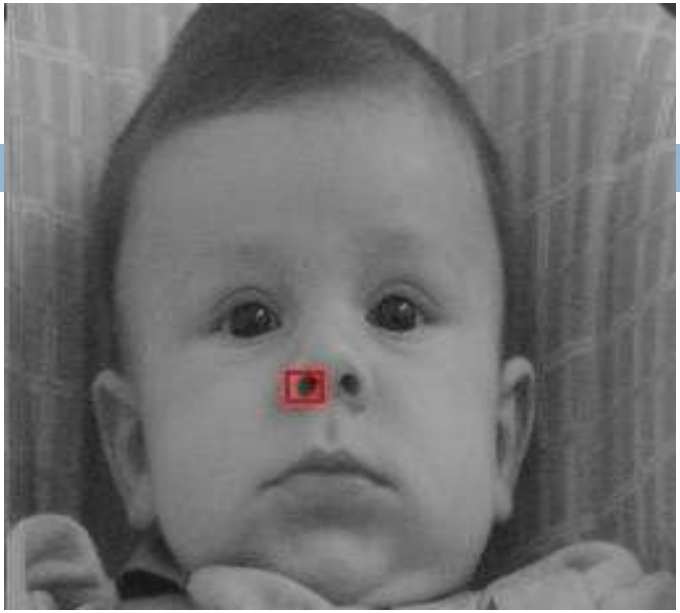
Spatial Aliasing: is insufficient sampling of data along the space axis, which occurs because of the insufficient spatial resolution of the acquired image.

Critical Sampling Distance: is the Sampling Distance to the corresponding Nyquist Rate.

Aliasing problem of a undersampled image

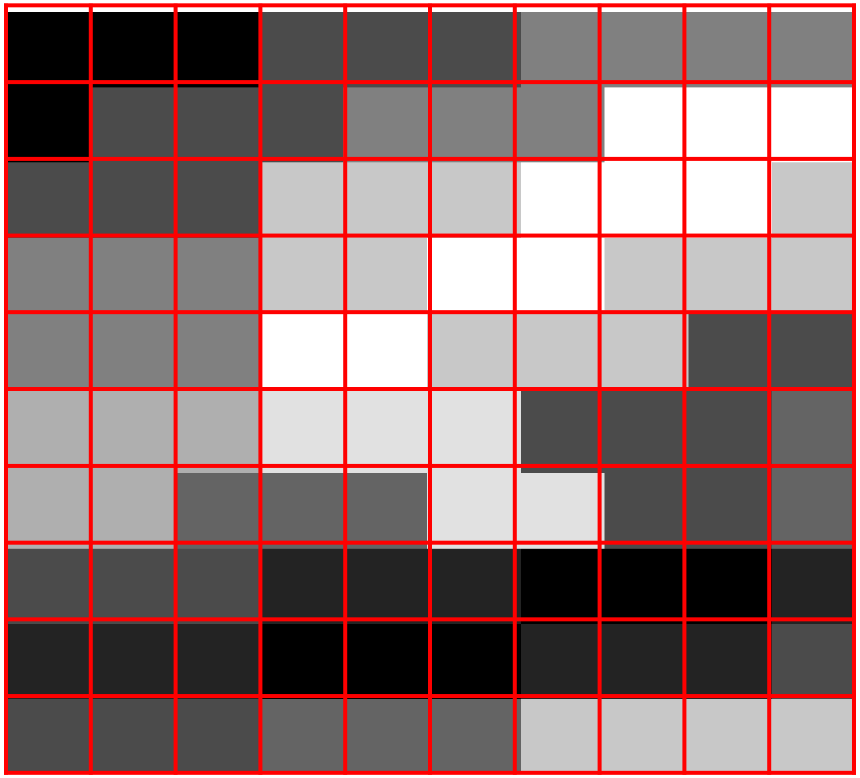
Digital Image?

- When x, y and the amplitude values of f are finite, discrete quantities, the image is called digital image
- A digital image is composed of a finite number of elements, each of which has a particular location and value. These elements are referred to as picture elements, image elements, pels and pixels.



99	71	61	51	49	40	35	53	86	99
93	74	53	56	48	46	48	72	85	102
101	69	57	53	54	52	64	82	88	101
107	82	64	63	59	60	81	90	93	100
114	93	76	69	72	85	94	99	95	99
117	108	94	92	97	101	100	108	105	99
116	114	109	106	105	108	108	102	107	110
115	113	109	114	111	111	113	108	111	115
110	113	111	109	106	108	110	115	120	122
103	107	106	108	109	114	120	124	124	132

Digital Image?



0	0	0	75	75	75	128	128	128	128
0	75	75	75	128	128	128	255	255	255
75	75	75	200	200	200	255	255	255	200
128	128	128	200	200	255	255	200	200	200
128	128	128	255	255	200	200	200	75	75
175	175	175	225	225	225	75	75	75	100
175	175	100	100	100	225	225	75	75	100
75	75	75	35	35	35	0	0	0	35
35	35	35	0	0	0	35	35	35	75
75	75	75	100	100	100	200	200	200	200

Image Size

- Requires decisions about values for M, N, and for the number, L, the number of gray levels typically is an integer power of 2:

  $$L = 2^K$$

  Where k is number of bits require to represent a grey value
- The discrete levels should be equally spaced and that they are integers in the interval [0, L-1].

Image Size

- The number, b, of bits required to store a digitized image is

$$b=M*N*k.$$

- For an image of 512 by 512 pixels, with 8 bits per pixel:
  - Memory required = 256K bytes= 0.25 megabytes

Coordinate Convention used

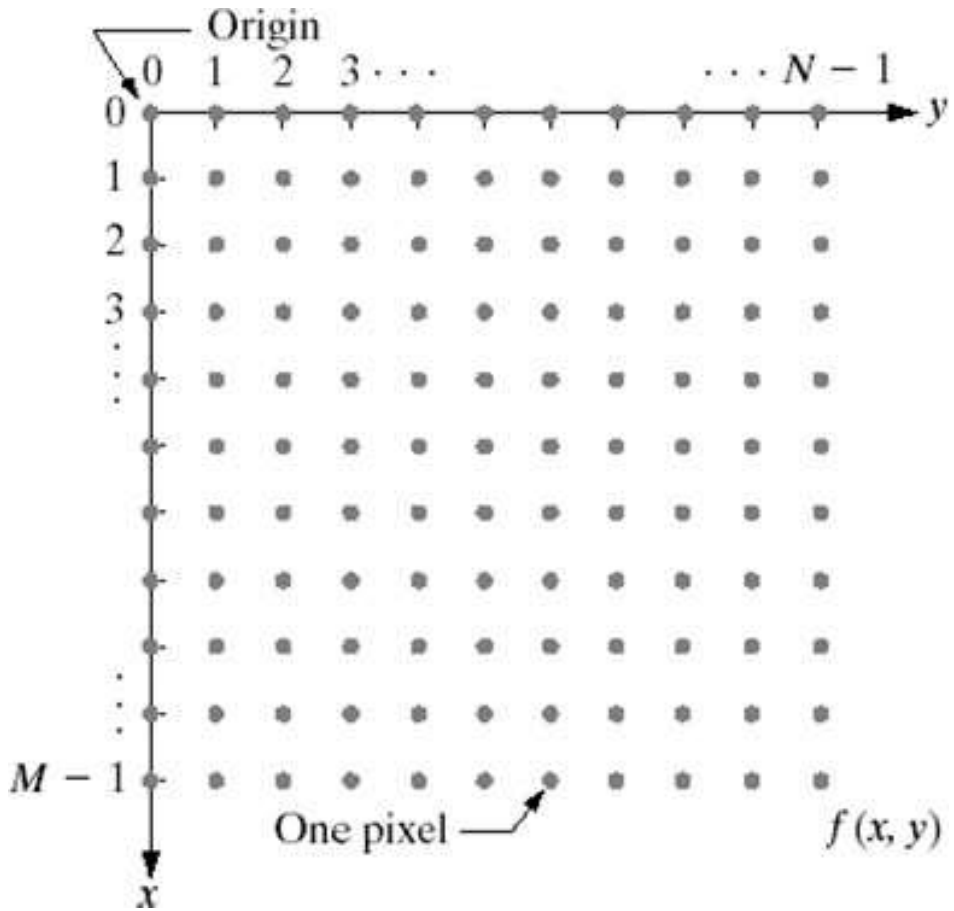


Image Resolution

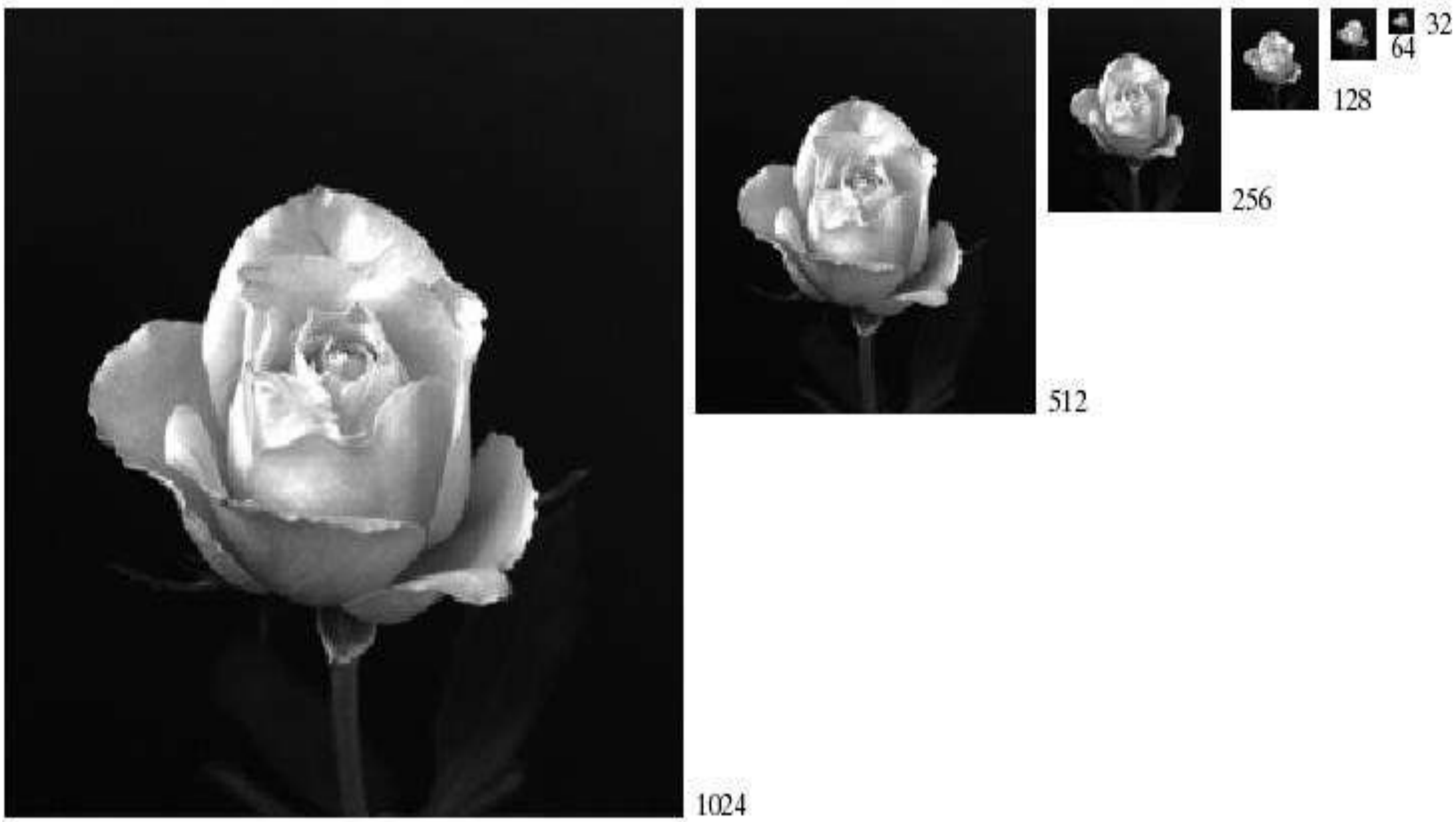
- **How many samples and gray levels are required for a good approximation?**
  - Resolution (the degree of discernible detail) of an image depends on sample number and gray level number.
  - i.e. the more these parameters are increased, the closer the digitized array approximates the original image.
  - **But: storage & processing requirements increase rapidly as a function of N, M, and k**

Image Resolution

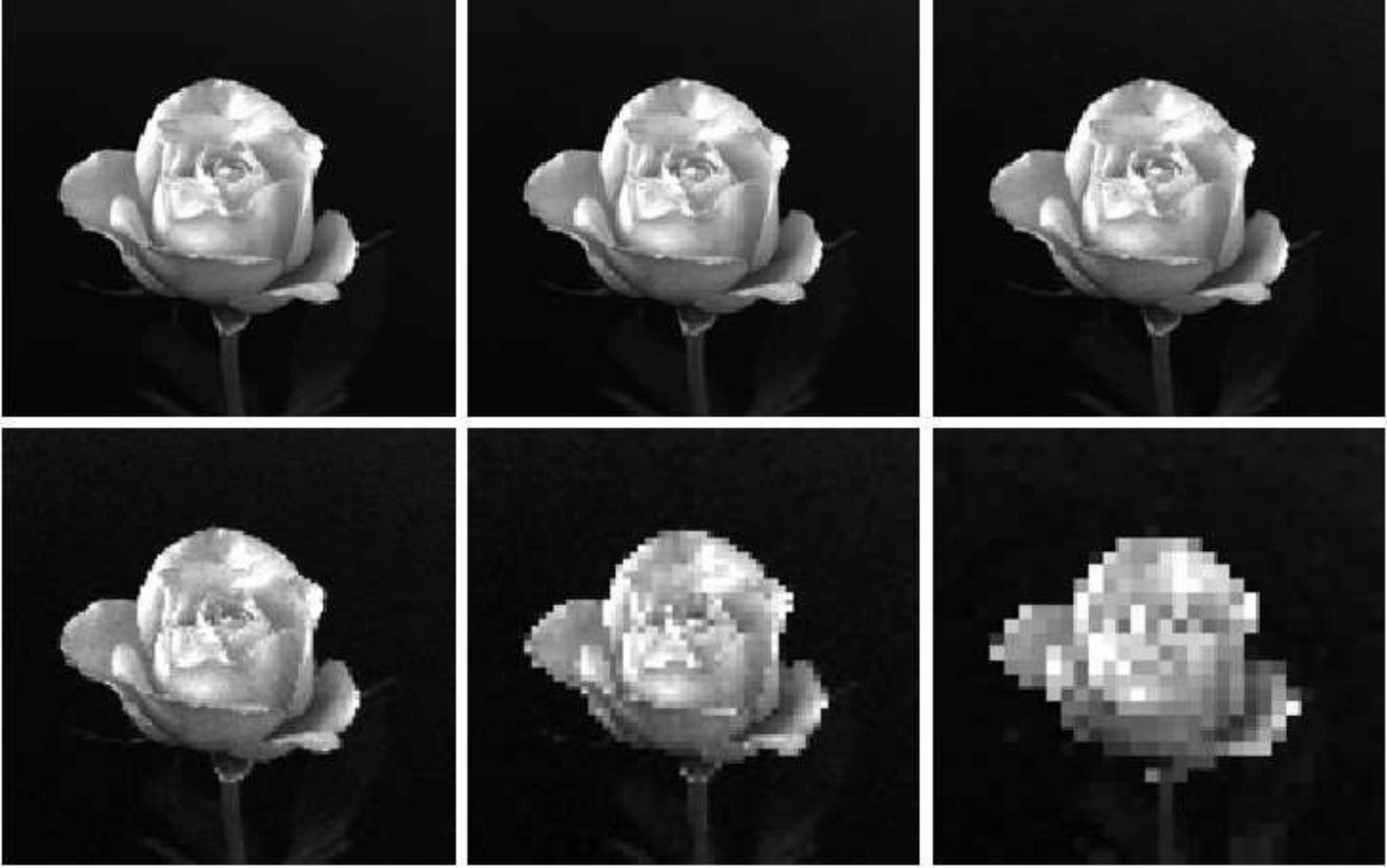
- **Spatial Resolution:** Spatial resolution is the smallest detectable detail in an image.
  - Dots/pixels per unit distance
    - dots per inch - dpi
- **Gray level (Intensity) Resolution:** Gray-level resolution similarly refers to the smallest detectable change in gray level.

- **The more samples in a fixed range, the higher the resolution**
- **The more bits, the higher the resolution**

Spatial Resolution



- A 1024*1024, 8-bit image subsampled down to size 32*32 pixels. The number of allowable gray levels was kept at 256



- (a) 1024*1024, 8-bit image. (b) 512*512 image resampled into 1024*1024 pixels by row and column duplication. (c) through (f) 256*256, 128*128, 64*64, and 32*32 images resampled into 1024*1024 pixels.

Checkerboard Effect

- When the no. of pixels in an image is reduced keeping the no. of gray levels in the image constant, fine checkerboard patterns are found at the edges of the image. This effect is called the checker board effect.

Gray level (Intensity) Resolution

Quantization

8-bit

7-bit

6-bit

5-bit

4-bit

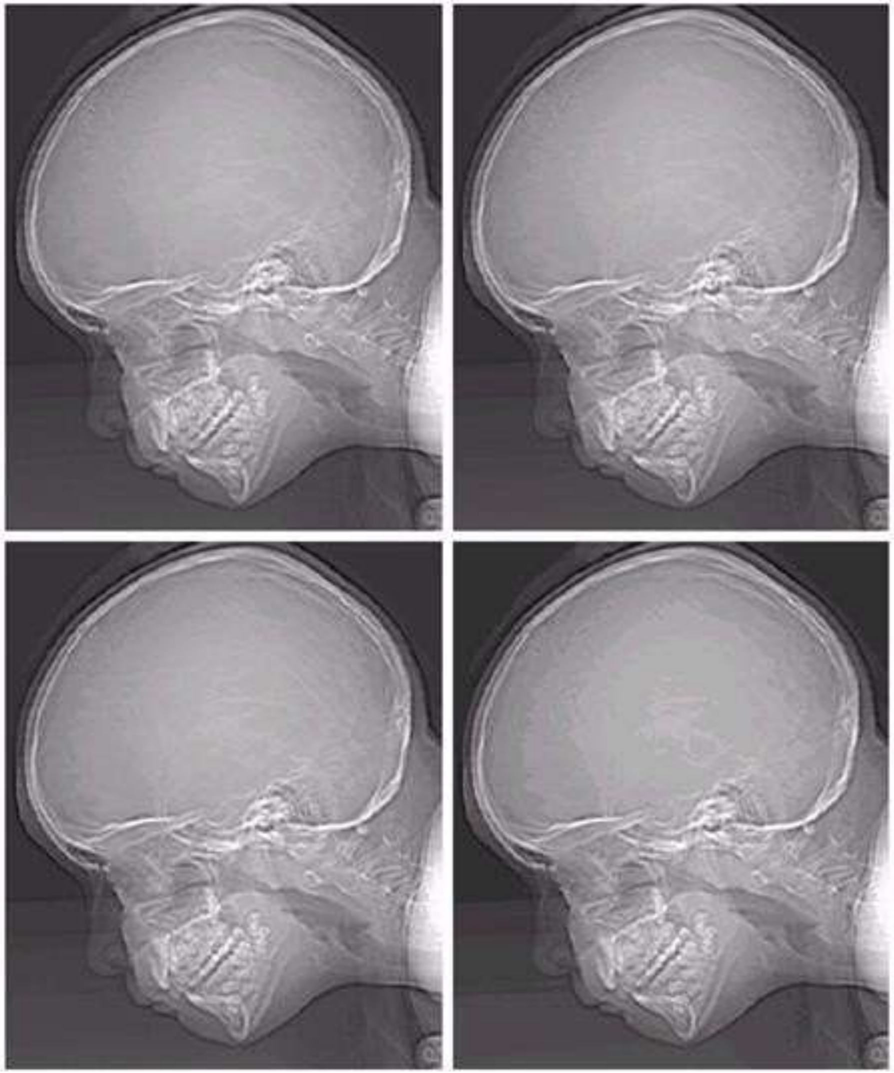
3-bit

2-bit

1-bit

Gray level (Intensity) Resolution …

Varying the number of gray levels



(a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

False Contouring

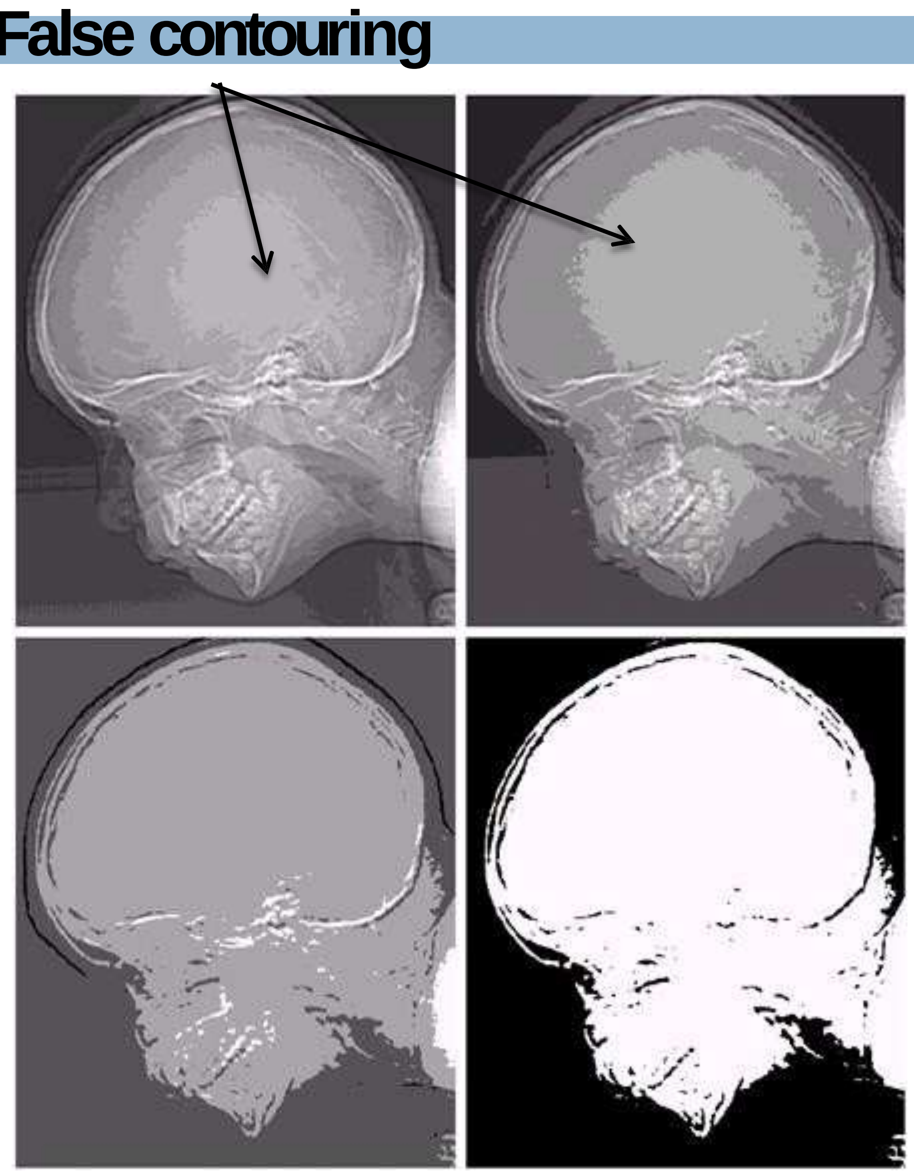
- When the no. of gray-levels in the image is low, the foreground details of the image merge with the background details of the image, causing ridge like structures. This degradation phenomenon is known as false contouring.

Gray level (Intensity) Resolution …

Varying the number of gray levels

e f
g h

(Continued) (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)



Resolution: How Much Is Enough?

- The big question with resolution is always *how much is enough*?
  - This all depends on what is in the image and what you would like to do with it
  - Key questions include
    - Does the image look pleasing?
    - Can you see what you need to see within the image?

Resolution: How Much Is Enough? ...

- The picture on the right is fine for counting the number of cars, but not for reading the number plate

Question

- **Q1:** If we want to resize a 1024x768 image to one that is 600 pixels wide with the same aspect ratio as the original image, what should be the height of the resized image?

- **Sol:**

$$\text{Aspect Ratio} = \frac{\text{width}}{\text{height}}$$

Question …

- For the original image the Aspect ratio is: 1027/768=1.33
- Now for the resized image, we want the same aspect ratio but a width of 600 pixels.

$$height = \frac{width}{Aspect\ ratio} = \frac{600}{1.33} = 451$$

- Hence the resized image will be 600x451

Question …

- **Question:** A common measure of transmission for digital data is the **baud rate**, defined as the number of bits transmitted per second. Transmission is accomplished in packets consisting of a start bit, a byte(8 bits) of information and a stop bit.

  a)How many minutes would it take transmit a 1024x1024 image with 256 gray levels if we use a 56 k baud modem?

  b)What would be the time required if we use a 750 k band transmission line?

Question …

- **Sol. :**

- Since we have 256 gray levels, we need 8-bits for representing each pixel.
- Along with these 8-bits, we also have start bit and a stop bit.
- Hence we have (8+2) bits per pixel.
- So total number of bits for transmission are

$$N=1024x1024x10=10485760 \text{ bits}$$

These bits are transmitted at 56 k baud .

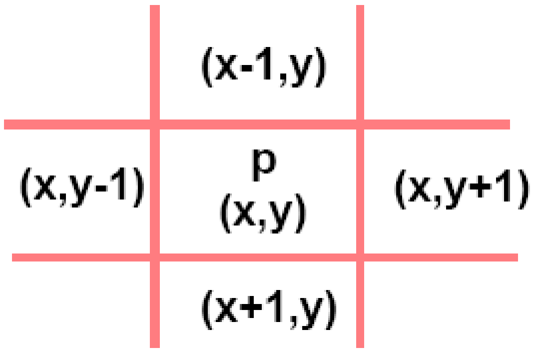
$$\text{So time taken}=N/56x10^3=187.25 \text{ sec}=3.1 \text{ minutes}$$

Basic Relationship between Pixels

- An image is denoted by a function f(x,y).
- Each element f(x,y) at location (x,y) is called a pixel.
- There exist some basic but important relationships between pixels.

Basic Relationship between Pixels …

- A pixel p at location (x,y) has two horizontal and two vertical neighbors.



- This set of four pixels is called 4-neighbors of $p = N_4(p)$.
- Each of these neighbors is at a unit distance from p.
- If p is a boundary pixel then it will have less number of neighbors.

Basic Relationship between Pixels …

- A pixel p has four diagonal neighbors=$N_D(p)$

(x-1,y-1)		(x-1,y+1)
	p (x,y)	
(x+1,y-1)		(x+1,y+1)

- The points of $N_4(p)$ and $N_D(p)$ together are called 8-neighbors of p.
- $N_8(p) = N_4(p) \cup N_D(p)$
- If p is a boundary pixel then both $N_D(p)$ and and $N_8(p)$ will have less number of pixels.

Basic Relationship between Pixels …

- Two pixels are said to be connected if they are adjacent in some sense
  - They are neighbors(N_4,N_D or N_8) and
  - Their intensity values (gray levels) are similar

- For a binary image B, two points p and q will be connected if q€N(p) or p€N(q) and B(p) = B(q).

Adjacency

- Let V be the set of intensity values
- **4-adjacency**: Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.
- **8-adjacency:** Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.
- **m-adjacency:** Two pixels p and q with values from V are m-adjacent if

  (i) q is in the set $N_4(p)$, or

  (ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ **is empty**

  (has no pixels whose values are from V).

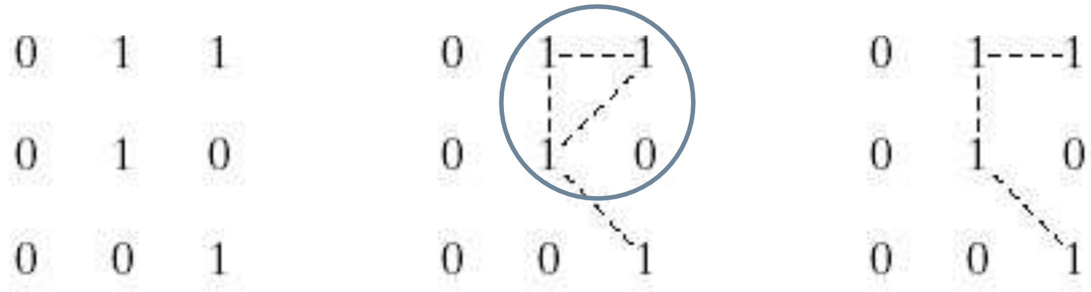
Examples: Adjacency and Path

0	1	1
0	1	0
0	0	1

Find 8-adjacency & m-adjacency of the pixel in the centre.

Note: V = {1}

Examples: Adjacency and Path



(a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

V = {1}

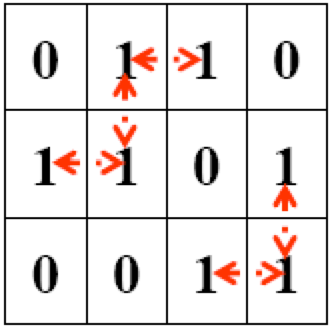
Fig (b) shows the ambiguity in 8-adjacency

Examples: Adjacency and Path …

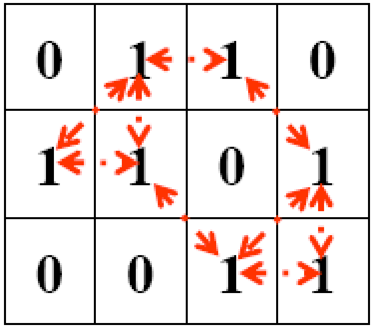
Mixed connectivity is a modification of 8-connectivity

-- Eliminates multiple path connections that often arise with 8-connectivity.

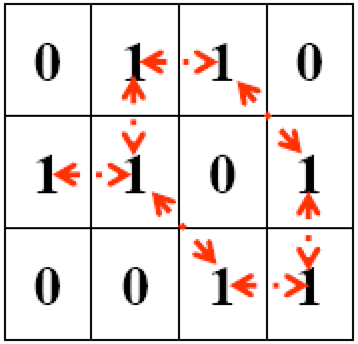
Ex: V = {1}



4 - connected



8 - connected



m - connected

Path

- A (digital) path (or curve) from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Where (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$.

- Here n is the *length* of the path.
- If $(x_0, y_0) = (x_n, y_n)$, the path is ***closed*** path.
- We can define 4-, 8-, and m-paths based on the type of adjacency used.

Connectivity

- **Connected in S**

  Let S represent a subset of pixels in an image. Two pixels p with coordinates (x_0, y_0) and q with coordinates (x_n, y_n) are said to be **connected in S** if there exists a path

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Where

Distance Measures

- Given pixels *p, q* and *z* with coordinates (x, y), (s, t), (u, v) respectively, the distance function D has following properties:

1. $D(p, q) \geq 0$ $[D(p, q) = 0, \text{ iff } p = q]$ **Identity**

2. $D(p, q) = D(q, p)$ **Symmetry**

3. $D(p, z) \leq D(p, q) + D(q, z)$ **Triangular inequality**

Distance Measures …

The following are the different Distance measures:

Euclidean Distance :

$$D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}$$

City Block Distance:

$$D_4(p, q) = |x-s| + |y-t|$$

Chess Board Distance:

$$D_8(p, q) = \max(|x-s|, |y-t|)$$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

City Block Distance

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Chess Board Distance

Assignment -1

1. When you enter a dark theater on a bright day, it takes an appreciable interval of time before you can see well enough to find an empty seat. Which of the visual process is at play in this situation?
2. Consider the two image subsets, S1 and S2, shown in the following figure. For V={1}, determine whether these two subsets are a) 4-adjacent, b) 8-adjacent, or c) m-adjacent.

		S1					S2		
0	0	0	0	0	0	0	1	1	0
1	0	0	1	0	0	1	0	0	1
1	0	0	1	0	1	1	0	0	0
0	0	1	1	1	0	0	0	0	0
0	0	1	1	1	0	0	1	1	1

Assignment -1 …

3. Consider the image segment shown

a)Let V={0,1} and compute the length of the shortest 4-,8-, and m-path between p and q. if a particular path does not exist between these two points, explain why.

b) Repeat for V={1,2}

	3	1	2	1	(q)
	2	2	0	2	
	1	2	1	1	
(p)	1	0	1	2	

Any Questions ?

Image Processing And Analysis

Mathematical Tools used in Digital Image Processing

Array versus Matrix Operations

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} ap + br & aq + bs \\ cp + dr & cq + ds \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} ap & bq \\ cr & ds \end{bmatrix}$$

When we multiply two images, we usually carry out array multiplication.

Linear versus Nonlinear Operations

- An operator is said to be linear if it satisfies the properties of *additivity* and *homogeneity.*

Additivity property:

$$H[f_1(x,y) + f_2(x,y)] = H[f_1(x,y)] + H[f_2(x,y)]$$

Homogeneity property:

$$H[K_1\, f_1(x,y)] = K_1\, H[f1(x,y)]$$

$$H[a_1\, f_1(x,y) + a_2\, f_2(x,y)] =$$

$$a_1\, H[f_1(x,y)] + a_2 H[f_2(x,y)]$$

Is the sum operator, Σ, linear?

$$\Sigma\,[a_1 f_1(x,y) + a_2 f_2(x,y)] =$$

$$\Sigma\, a_1 [f_1(x,y)] + \Sigma a_2 [f_2(x,y)]$$

{*Distributive Property*}

$$\Sigma\,[a_1 f_1(x,y) + a_2 f_2(x,y)] =$$

$$a_1 \Sigma\,[f_1(x,y)] + a_2\, \Sigma\,[f_2(x,y)]$$

Is the max operator, whose function is to find the maximum value of the pixels in an image, linear?

$$f_1 = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} \quad f_2 = \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}$$

$$a_1 = 1 \quad a_2 = -1$$

$H[a_1 f_1(x,y) + a_2 f_2(x,y)] =$

$a_1 H[f_1(x,y)] + a_2 H[f_2(x,y)]$

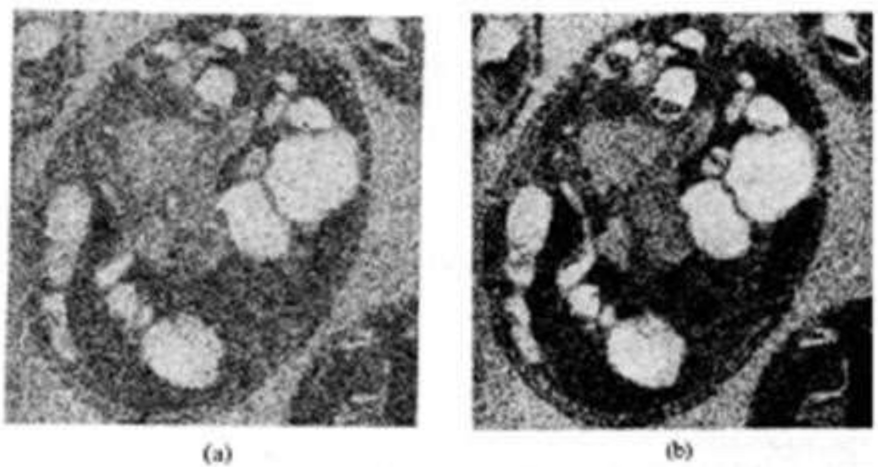
Arithmetic Operations

- **Arithmetic operations are performed on the pixels of two or more images**

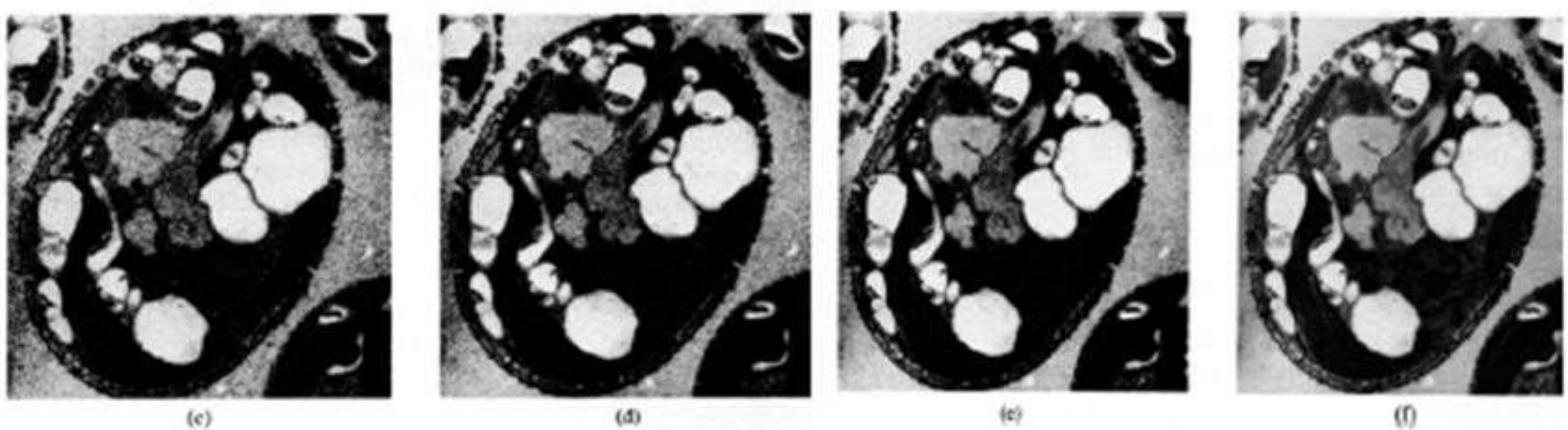
- Let p and q be the pixel values at location (x,y) in first and second images respectively

  - Addition: p+q
  - Subtraction: p-q
  - Multiplication: p.q
  - Division: p/q

Image Averaging: Example

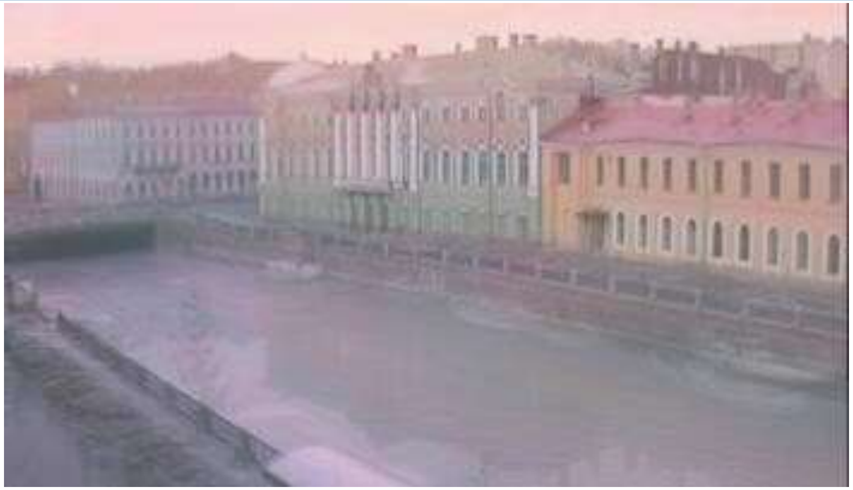


Example of noise reduction by averaging
(a) a typical noisy image,
(b)-(f) results of averaging 2, 8, 16, 32, and 128 noisy image.



Example

Example …



Average image

Median Image

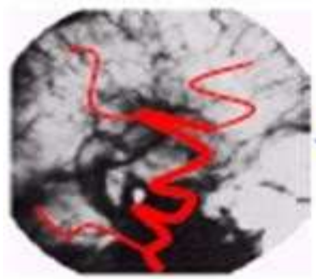
Image Subtraction

- Obtained by computing the difference between all pairs of corresponding pixels from images f & h

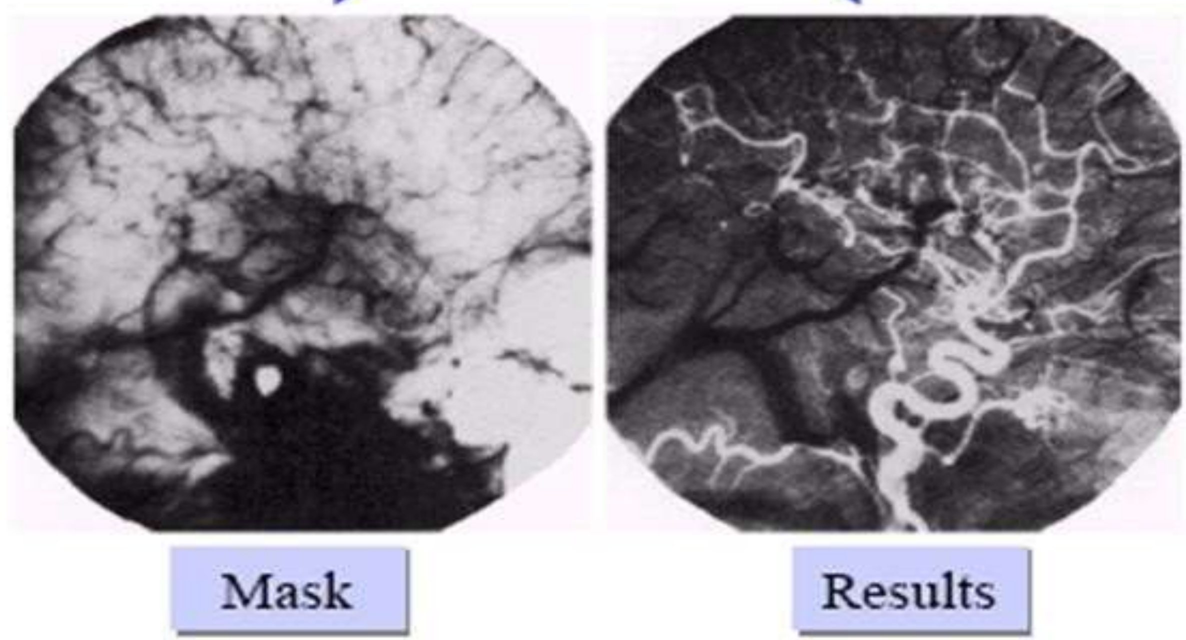
  $$g(x,y)=f(x,y)-h(x,y)$$

- Example (Mask mode radiography): imaging blood vessels and arteries in a body. Blood stream is injected with a dye and X-ray images are taken before and after the injection
  - f(x,y): image after injecting a dye
  - h(x,y): image before injecting the dye
- The difference of the 2 images yields a clear display of the blood flow paths.

Image Subtraction: Example



$$f(x,y) - h(x,y) = g(x,y)$$



Enhancement by image subtraction

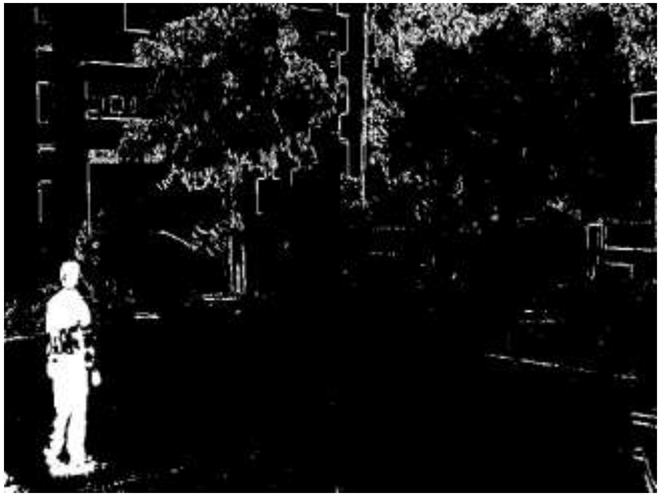
Image Subtraction: Example …

- **Frame differencing** is the most simple and easy to implement method.
- The video frame at time t-1 is used as the Reference frame for the frame at time t.
- The pixel is considered part of the foreground if the difference in pixel values for a given pixel is greater than a threshold T_s,

$$|frame_i - frame_{i-1}| > T_s$$

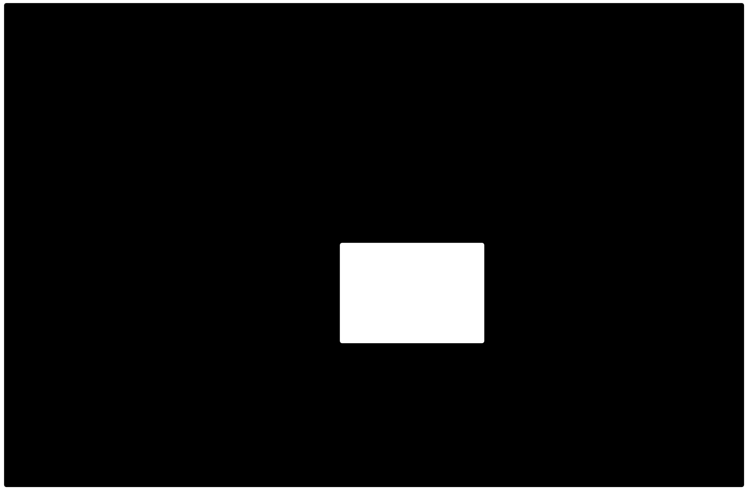
Image Subtraction: Example …

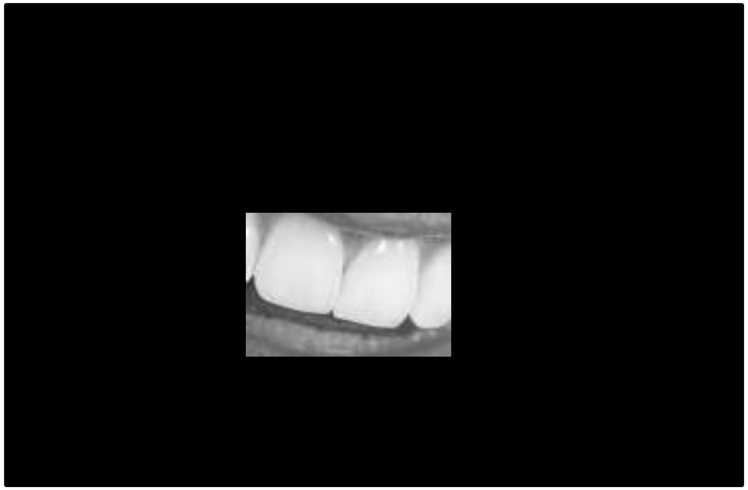
Image Subtraction: Example …



Region of Interest

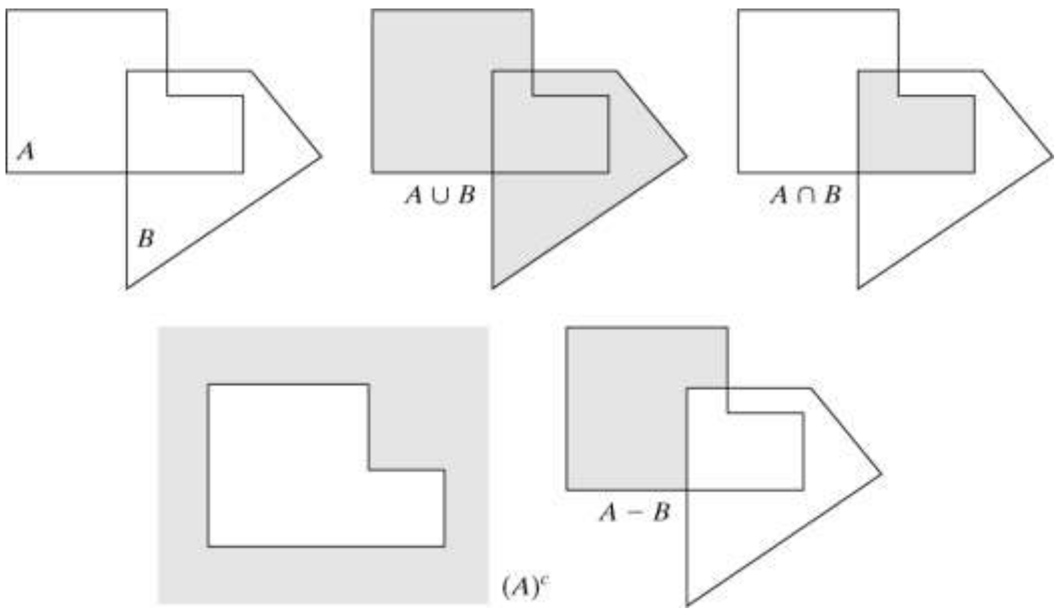
An important applications of image multiplication

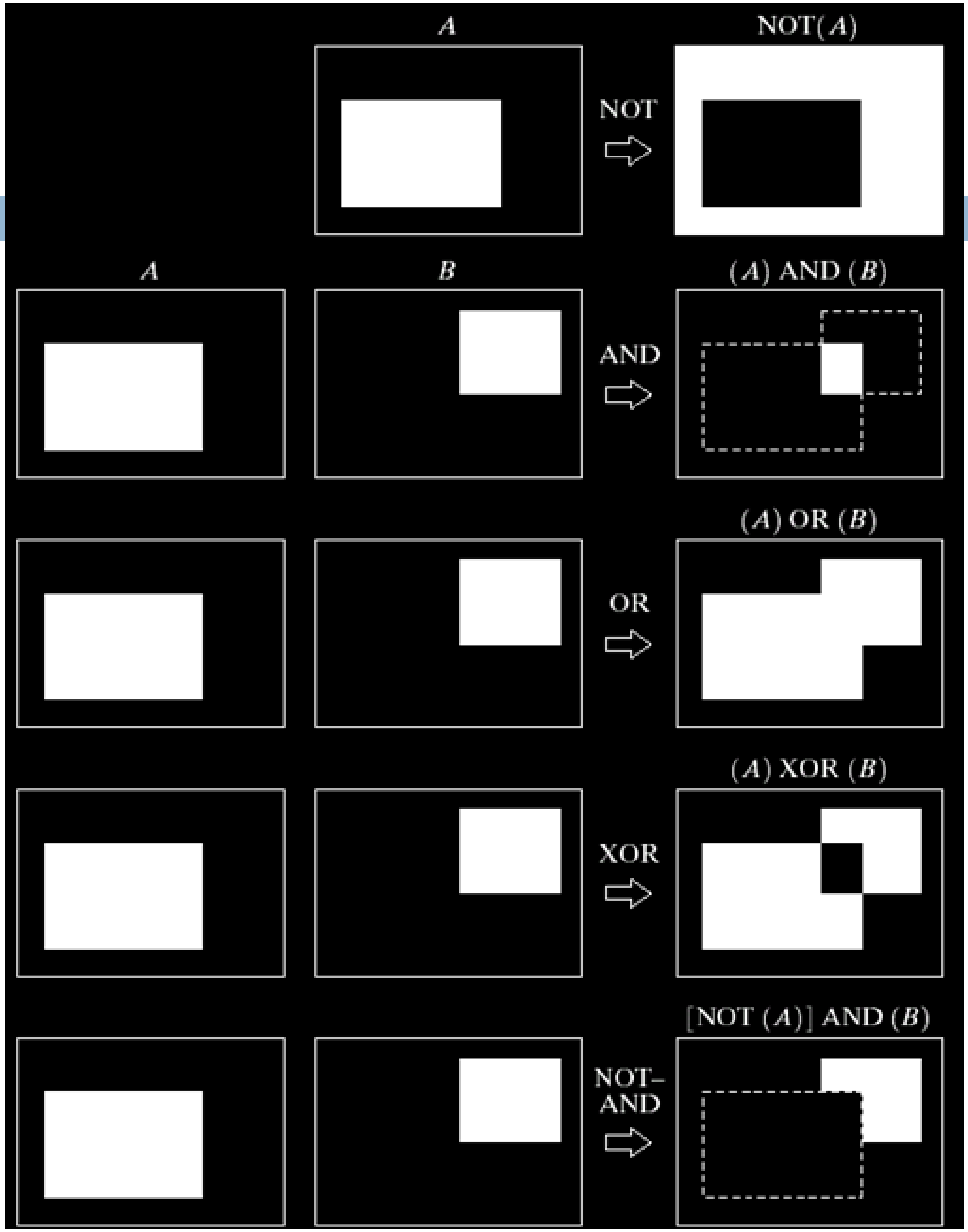




Logic Operations

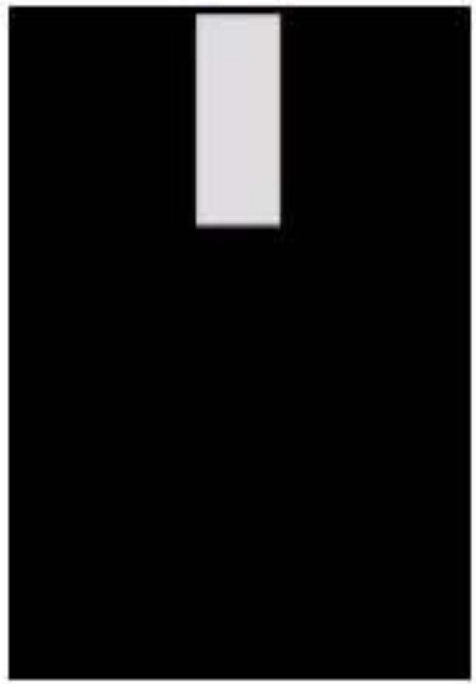
- When dealing with logic operations on gray-level images, pixel values are processed as strings of binary numbers.
- AND, OR, COMPLEMENT (NOT)




A
NOT(A)
NOT
A
B
(A) AND (B)
AND
(A) OR (B)
OR
(A) XOR (B)
XOR
[NOT (A)] AND (B)
NOT–
AND

Example of AND Operation

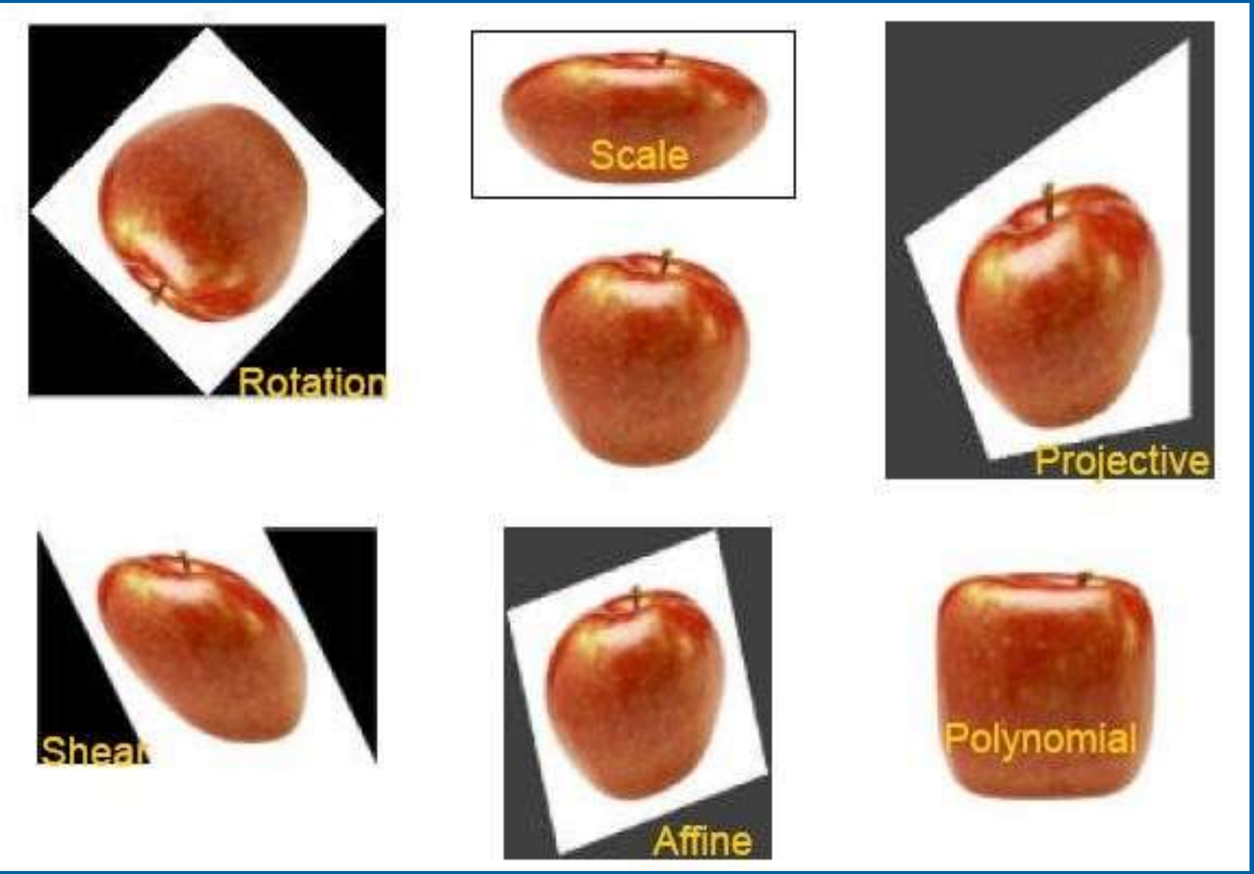
original image



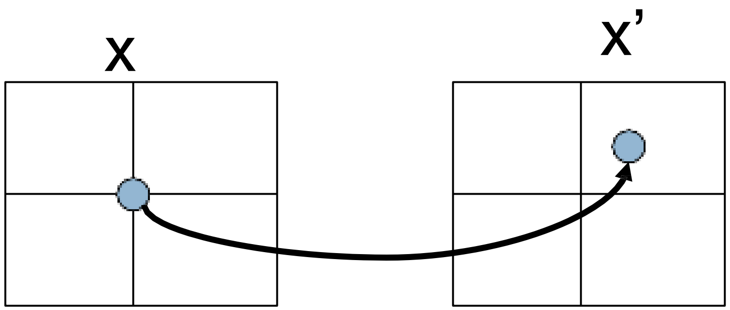
AND image mask

result of AND operation

Geometric Transformation



Geometric Spatial Transformations



Suppose that the true position of a pixel is (x,y) and the distorted position is (x',y')

A geometric transformation is a vector *T* that maps the pixel (*x*,*y*) to a new position (*x'*, *y'*)

$$x' = T_x(x,y) : y' = T_y(x,y)$$

The transformation equations *Tx* and *Ty* are either know in advance - in the case of rotation, translation, scaling or can be determined from known original and transformed images.

Geometric Spatial Transformations …

- Modify the spatial relationship between pixels in an image.
- Also known as *rubber-sheet* transformations

A geometric transform consists of two basis steps:

1.Pixel co-ordinate transformation, which maps the co-ordinates of the input image pixel to the point in the output image. The output point co-ordinates should be computed as continuous values (real numbers) as the position does not necessarily match the digital grid after the transform.

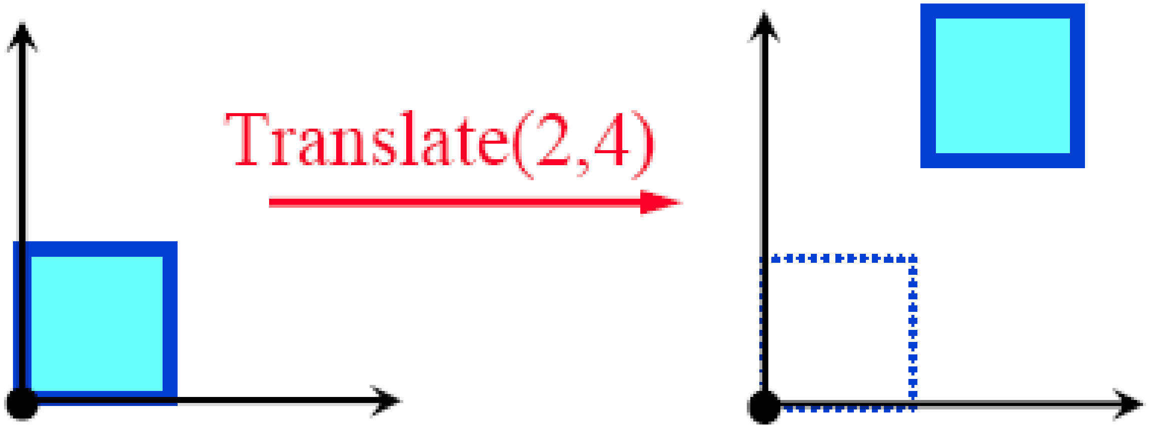
2.The second step is to determine its brightness value. It brightness is usually computed as an interpolation of the brightness of several points in the neighborhood .

Affine transformations

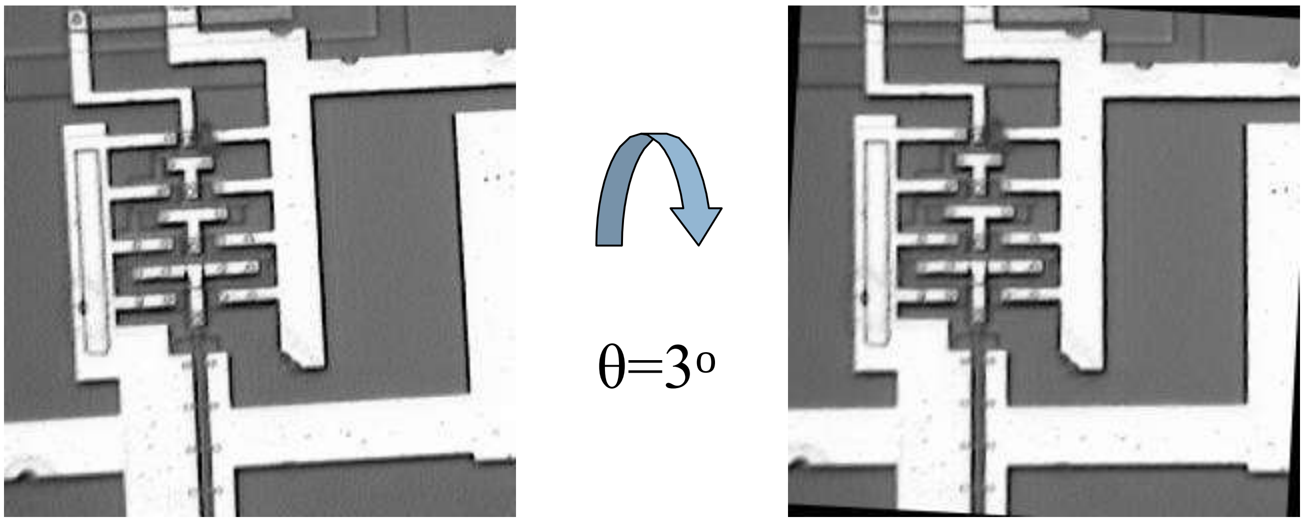
- An affine transformation is an important class of linear 2-D geometric transformations which maps variables (*e.g.* pixel intensity values located at position (x,y) in an input image) into new variables (*e.g.* in an output image (x',y') by applying a linear combination of translation, rotation, scaling and/or shearing (*i.e.* non-uniform scaling in some directions) operations.

Translation

- Translate (a,b): $(x,y) \longrightarrow (x+a,y+b)$



Rotation Example



Scale

- Scale (a,b): $(x,y) \longrightarrow (ax,by)$

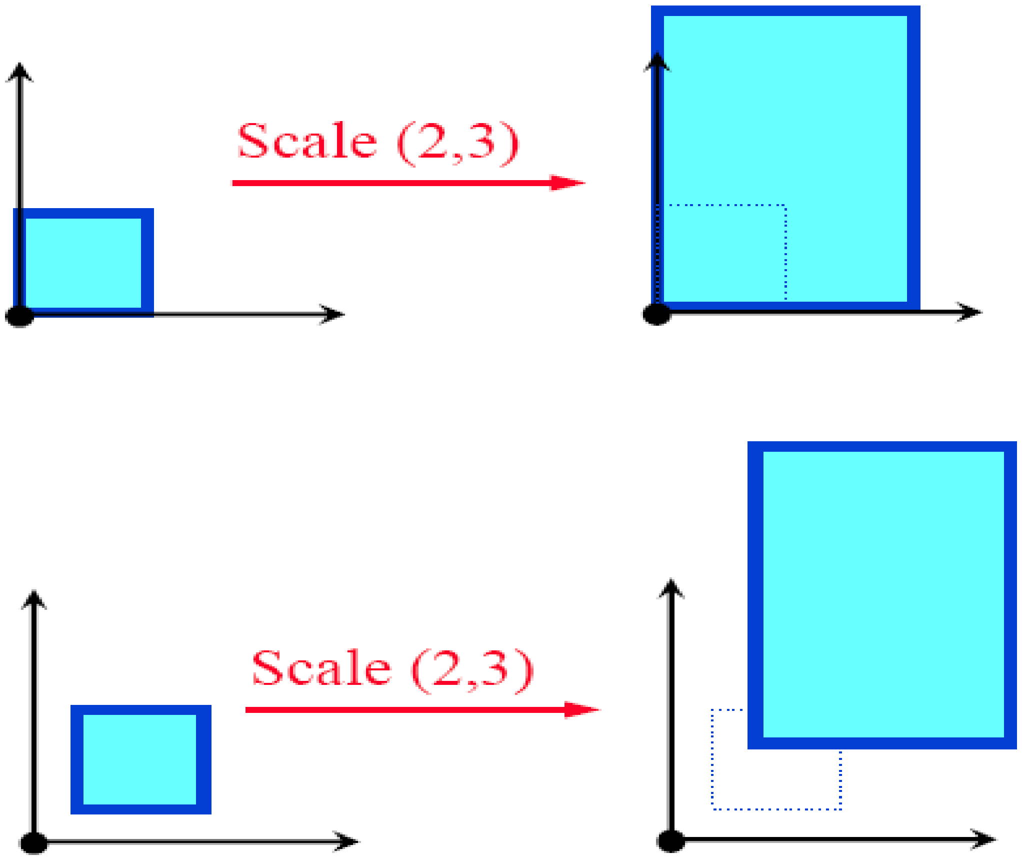


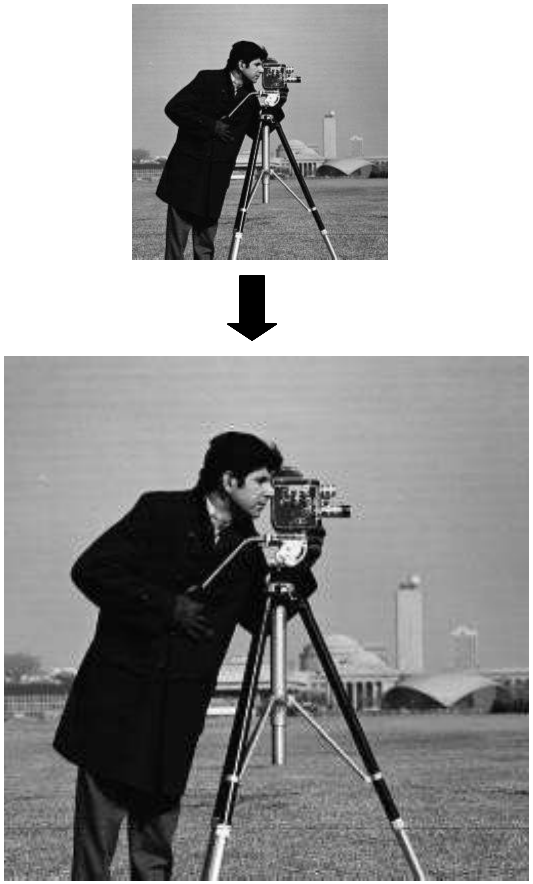
Image Interpolation

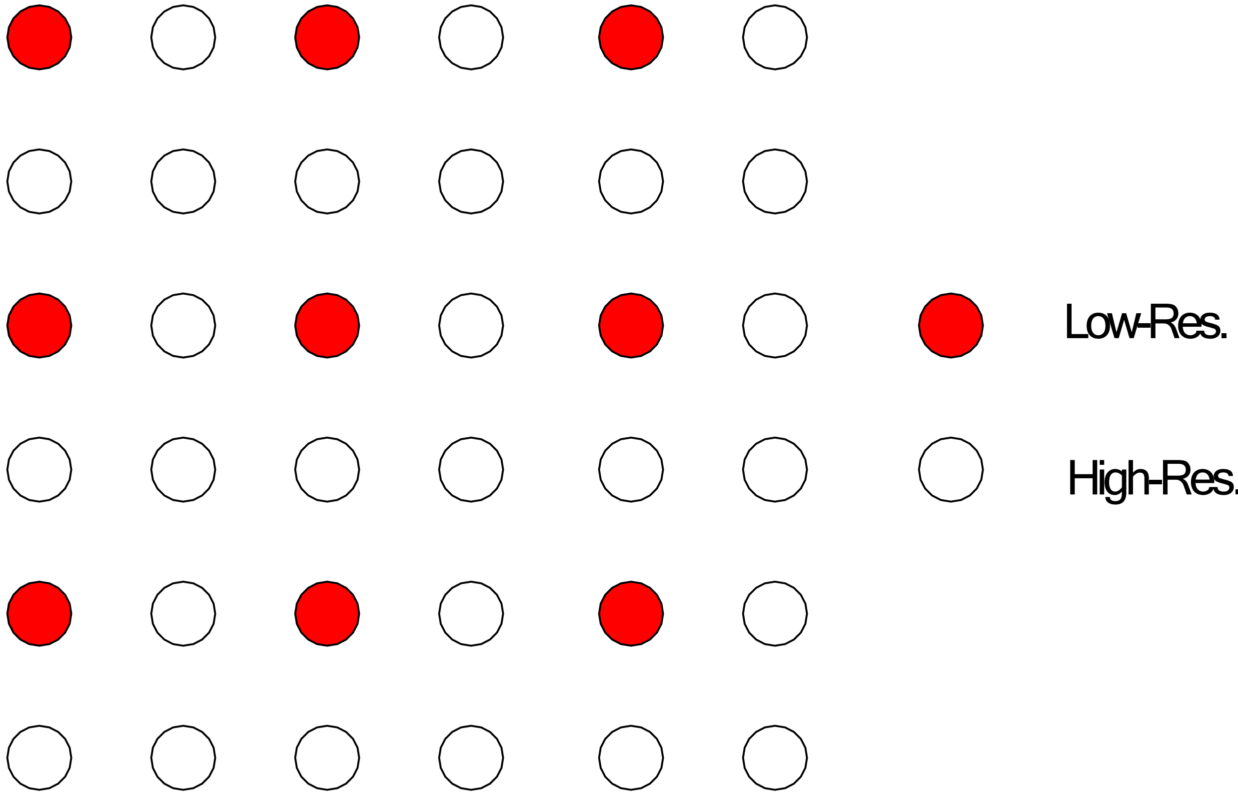
- What is image interpolation?
  - An image f(x,y) tells us the intensity values at the integral lattice locations, i.e., when x and y are both integers
  - Image interpolation refers to the "guess" of intensity values at missing locations, i.e., x and y can be arbitrary
  - Note that it is just a guess (Note that all sensors have finite sampling distance

Image Interpolation …

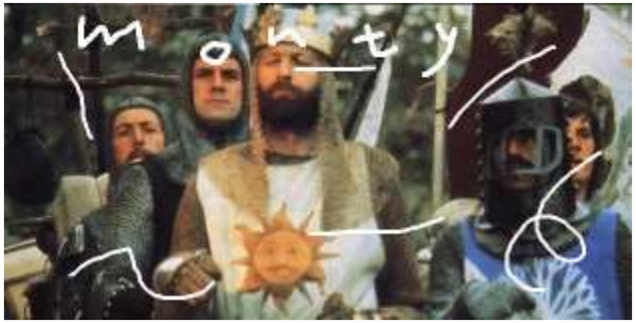
- Engineering Motivations
- Why do we need image interpolation?
  - We want BIG images
    - When we see a video clip on a PC, we like to see it in the full screen mode
  - We want GOOD images
    - If some block of an image gets damaged during the transmission, we want to repair it
  - We want COOL images
    - Manipulate images digitally can render fancy artistic effects as we often see in movies

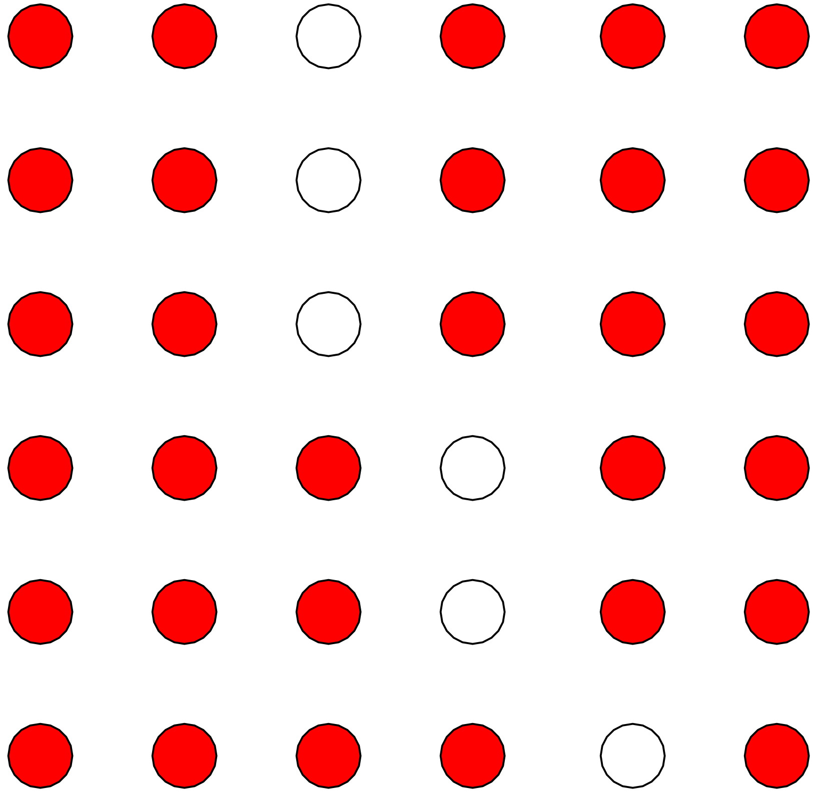
Scenario I: Resolution Enhancement





Scenario II: Image Inpainting





Non-damaged Damaged

Image Interpolation …

3 main type of 2D Interpolations :

- Nearest neighbor interpolation
- Bilinear interpolation
- Bicubic interpolation

Nearest Neighbour Interpolation

Nearest neighbor is the most basic and requires the least processing time of all the interpolation algorithms because it only considers one pixel – the closest one to the interpolated point. This has the effect of simply making each pixel bigger.

If you enlarge an image 200%, then one pixel in the input image will be represented by how many pixels in the output image?

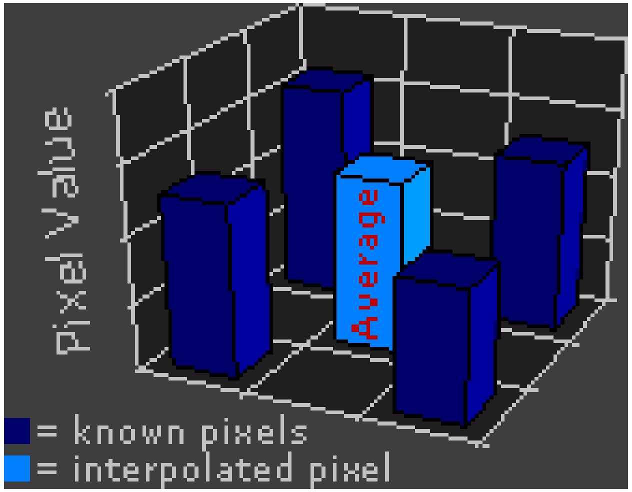
Nearest Neighbour Interpolation …

450% increases in size of this 106 x 40 crop from an image

Nearest Neighbour Interpolation

Bilinear Interpolation

- Bilinear interpolation considers the closest 2x2 neighborhood of known pixel values surrounding the unknown pixel. It then takes a weighted average of these 4 pixels to arrive at its final interpolated value.
- This results in much smoother looking images than nearest neighbor.



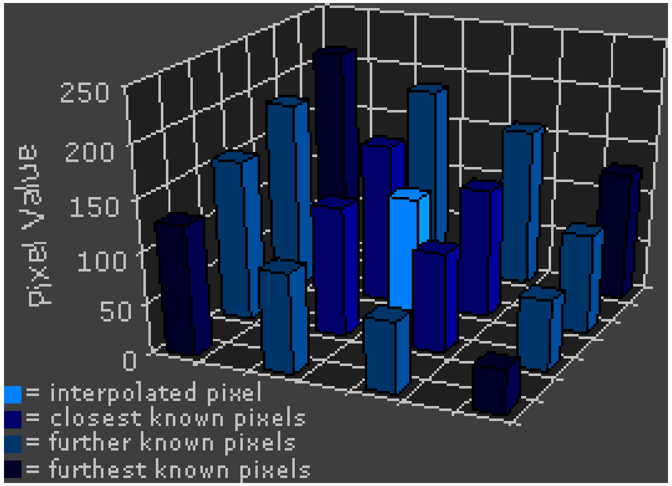
Bilinear Interpolation …

The averaging has an anti-aliasing effect and therefore produces relatively smooth edges with hardly any jaggies.

Bilinear Interpolation

Bicubic Interpolation

- Bicubic considers the closest 4x4 neighborhood of known pixels – for a total of 16 pixels.
- Since these are at various distances from the unknown pixel, closer pixels are given a higher weighting in the calculation.
- It is perhaps the ideal combination of processing time and output quality.
- For this reason it is a standard in many image editing programs (including Adobe Photoshop), printer drivers and in-camera interpolation.



Bicubic Interpolation …

Bicubic produces noticeably sharper images than the previous two methods. Notice the smoother eyelashes.

Lab Assignment

- Write MATLAB Program for following output:

A

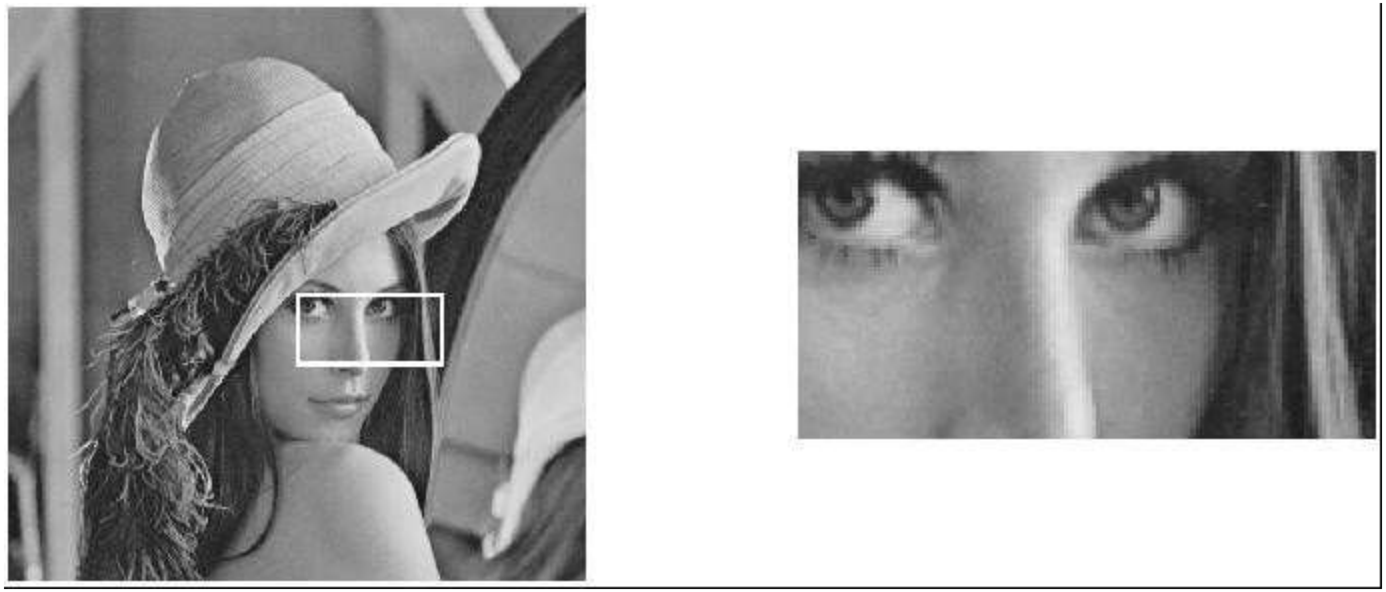
B

Lab Assignment …

- Write MATLAB Program for following output:

Lab Assignment …

- Write MATLAB Program for following output:



Lab Assignment …

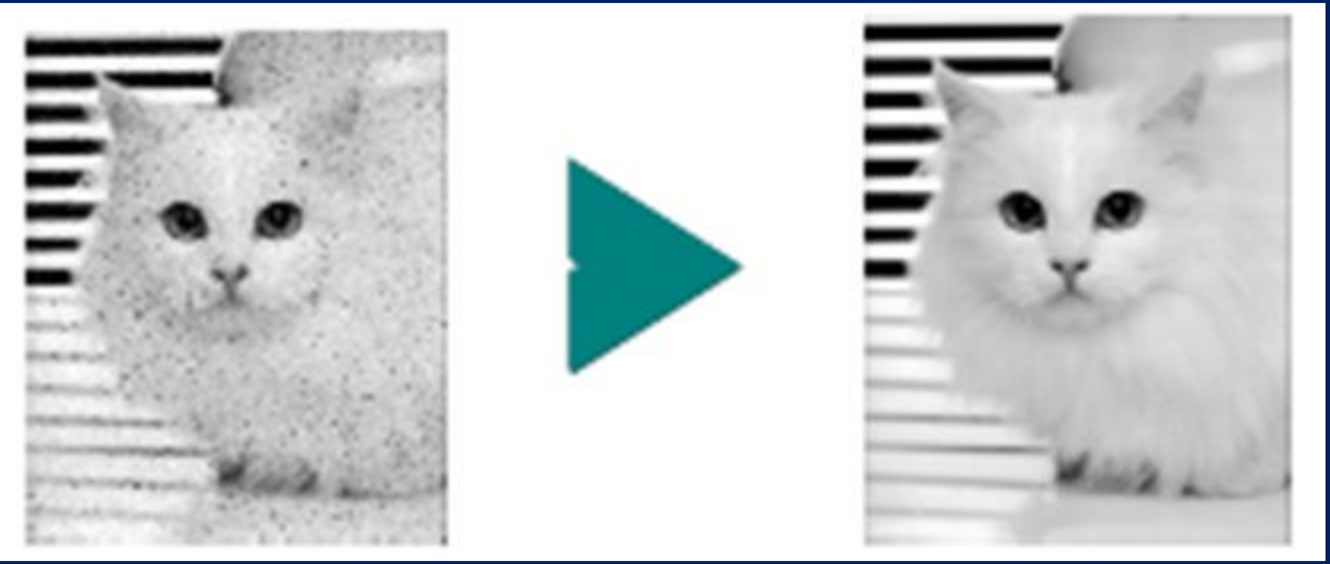
Implement the following on MATLAB.

1. Take two images & perform arithmetic operations on them.

2 Take your passport size image & zoom it 200% using the 3 interpolation techniques discussed.

Any Questions ?

Image Enhancement in the Spatial Domain



Dr A S Jalal

Image is NOT Perfect Sometimes

What Is Image Enhancement?

Image enhancement is the process of making images more useful

The reasons for doing this include:

- Highlighting interesting detail in images
- Removing noise from images
- Making images more visually appealing

Why do we need Image Enhancement?

- The goal of image enhancement is to process a digital image and make it more “suitable” than the original image for a specific application
- Images may suffer from the following degradations:
  - Poor contrast due to poor illumination or finite sensitivity of the imaging device
  - Electronic sensor noise or atmospheric disturbances leading to broad band noise
  - Aliasing effects due to inadequate sampling
  - Finite aperture effects or motion

After Image Enhancement

Image Enhancement

- There are Two broad categories of Image Enhancement Approaches
  - **Spatial Domain methods**
    - Based on direct manipulation of pixels in an image
  - **Frequency Domain methods**
    - Based on modifying the Fourier transform of an image

Spatial Domain

- **Intensity Transformations (Point processing)**
  - Operate on single pixels of an image
    - e.g., image averaging; logic operation; contrast stretching …
- **Spatial Filtering (Mask processing)**
  - Working in a neighbourhood of every pixel in an image
    - e.g., blurring, median

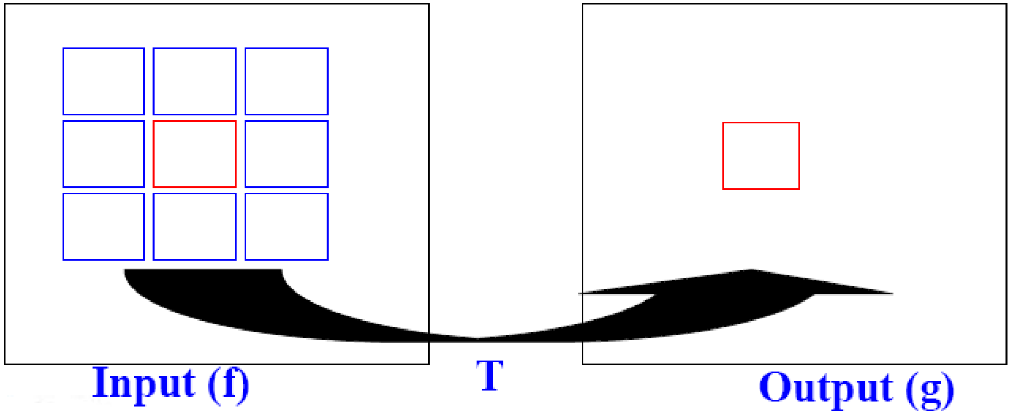
Spatial Domain

- Spatial domain can be denoted by:

$$g(x,y) = T[f(x,y)]$$

where

- $f(x,y)$ is the input image
- $g(x,y)$ is the processed image
- T is an operator on f defined over some neighborhood of (x,y)



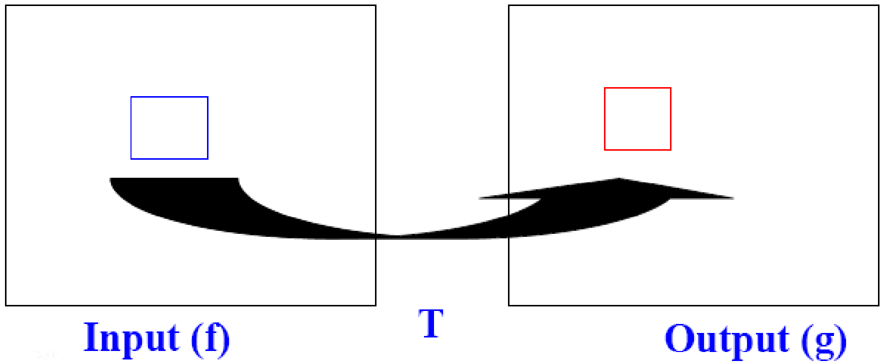
Intensity Transformations (Point processing)

Intensity Transformation

- **Simplest T** : operates on neighborhood of 1x1
- Then, it becomes **intensity transformation**
- g depends on only the value of f at (x,y)
- T = gray level (or intensity or mapping) transformation function

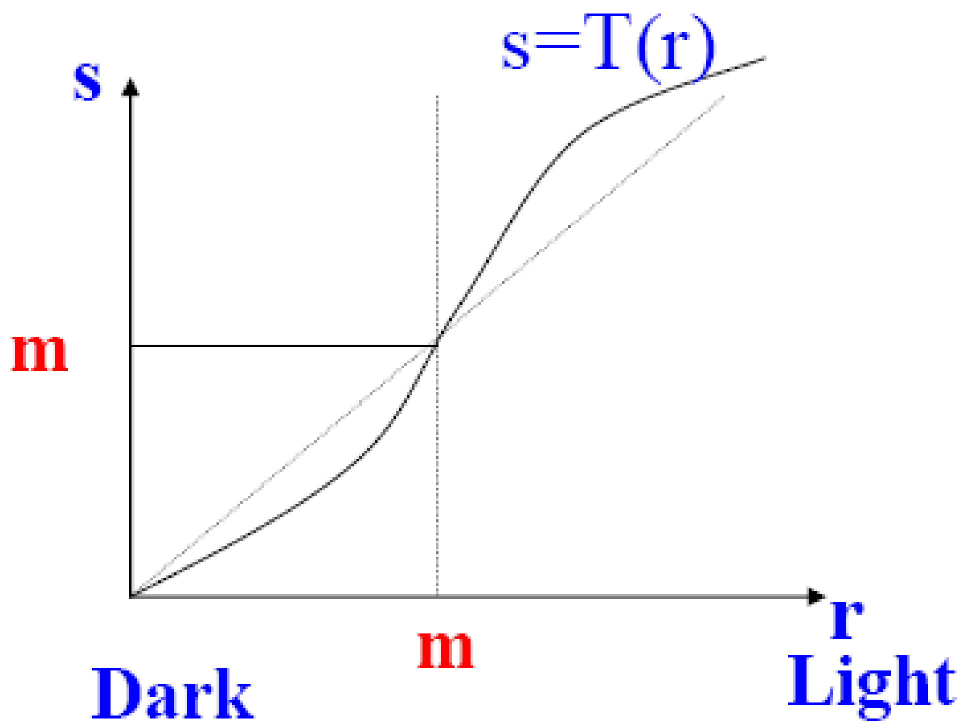
$$s = T(r)$$

- Where
  - r = gray level of $f(x,y)$
  - s = gray level of $g(x,y)$



Intensity Transformation …

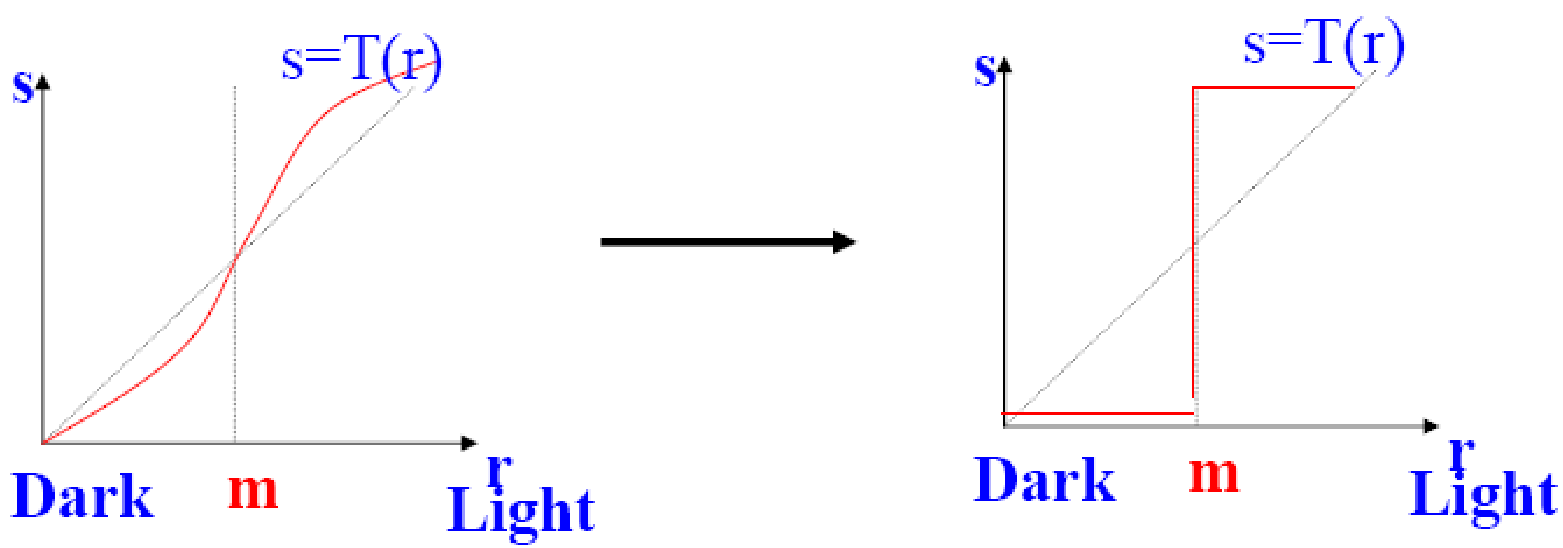
- The gray level below **m** are darkened and the levels above **m** are brightened.



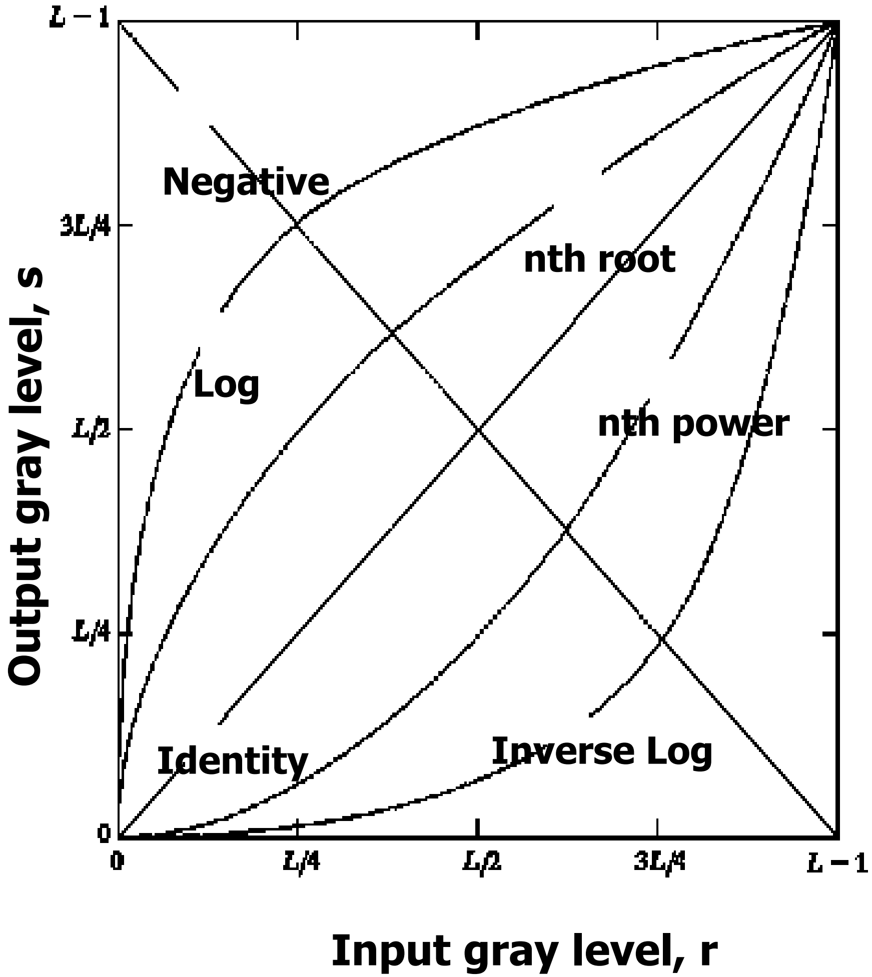
Intensity Transformation …

- **Limiting case:** Produces a binary image (two level) from input image and the function is known as Thresholding function

$$g(x,y) = \begin{cases} L & \text{if } f(x,y) > m, \\ 0 & \text{otherwise} \end{cases}$$



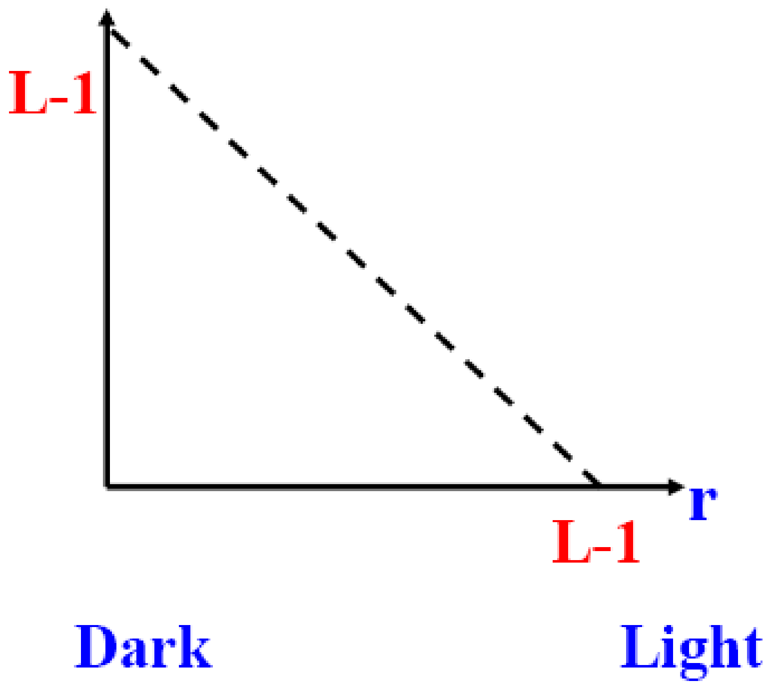
Basic Intensity Transformation Functions



- **Linear function**
  - Negative and identity transformations
- **Logarithm function**
  - Log and inverse-log transformation
- **Power-law function**
  - n^{th} power and n^{th} root transformations

Identity function: Output intensities are identical to input intensities.

Image Negatives



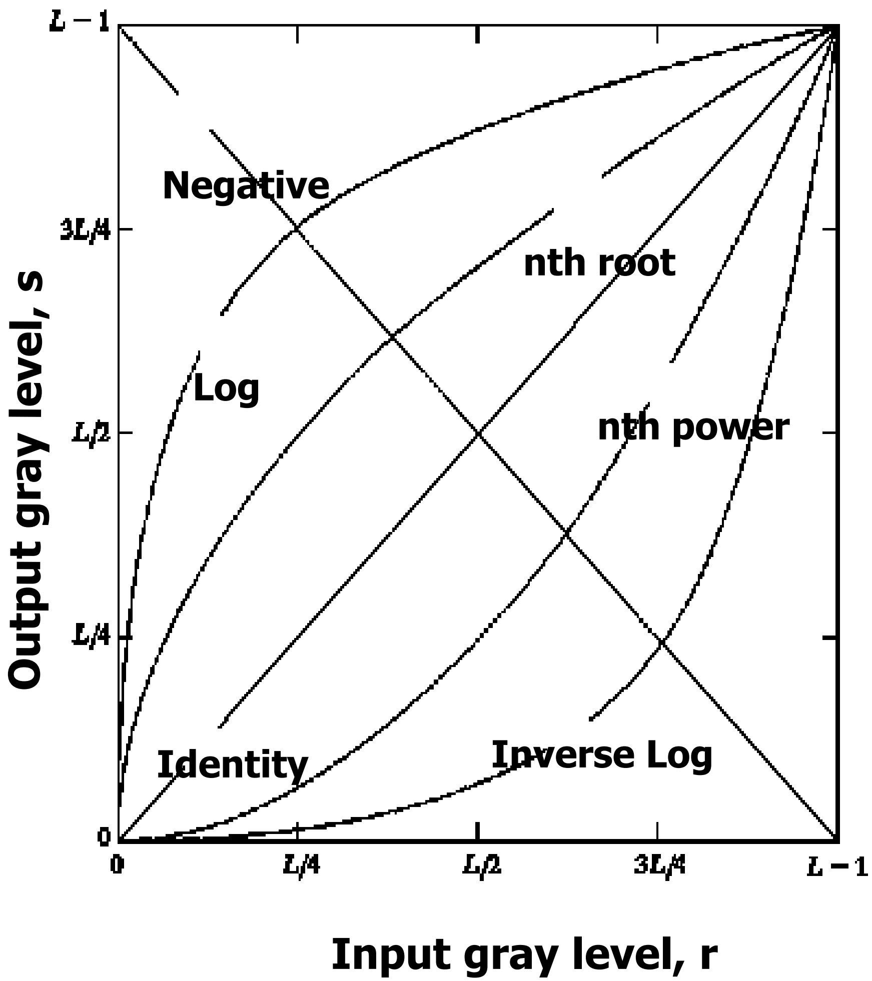
- An image with gray level in the range $[0, L-1]$ where $L = 2^n$; n = 1, 2…
- Negative transformation :

$$s = L - 1 - r$$

- Reversing the intensity levels of an image.
- Suitable for enhancing white or gray detail embedded in dark regions of an image, especially when the black area dominant in size.

Image Negatives

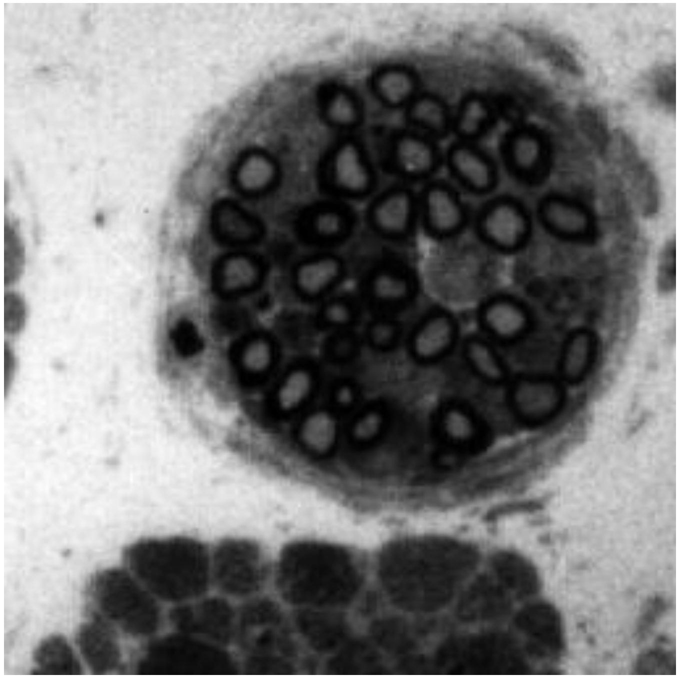
Log Transformations



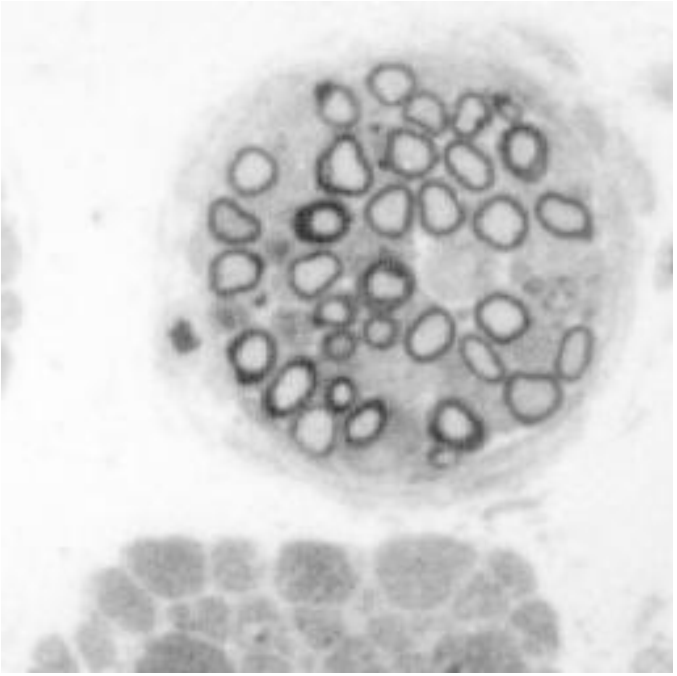
$$s = c \log (1+r)$$

- c is a constant and $r \geq 0$
- Log curve maps a narrow range of low gray-level values in the input image into a wider range of output levels.
- Used to expand the values of dark pixels in an image while compressing the higher-level values.

Log Transformations: Example

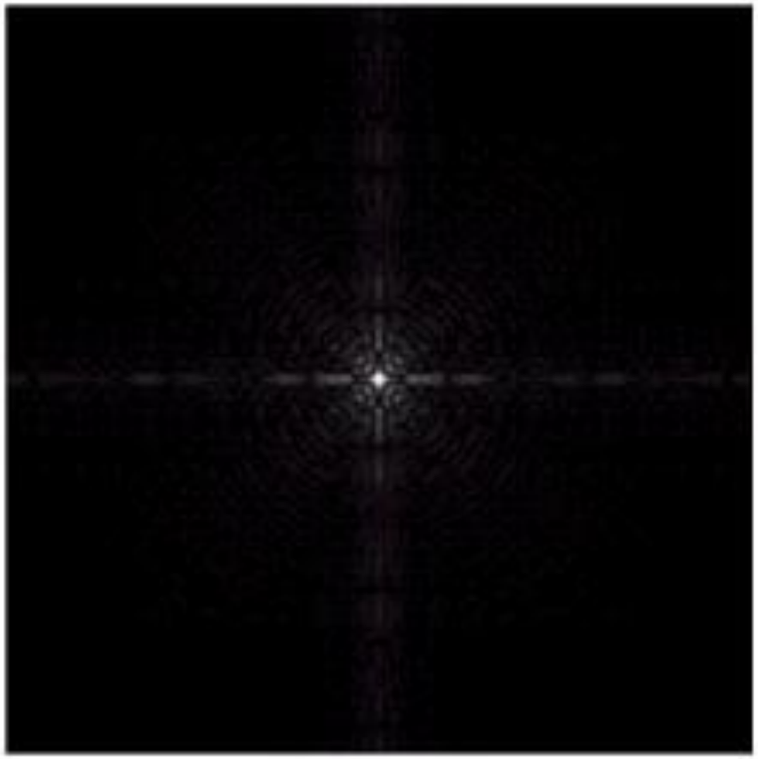


Original Image

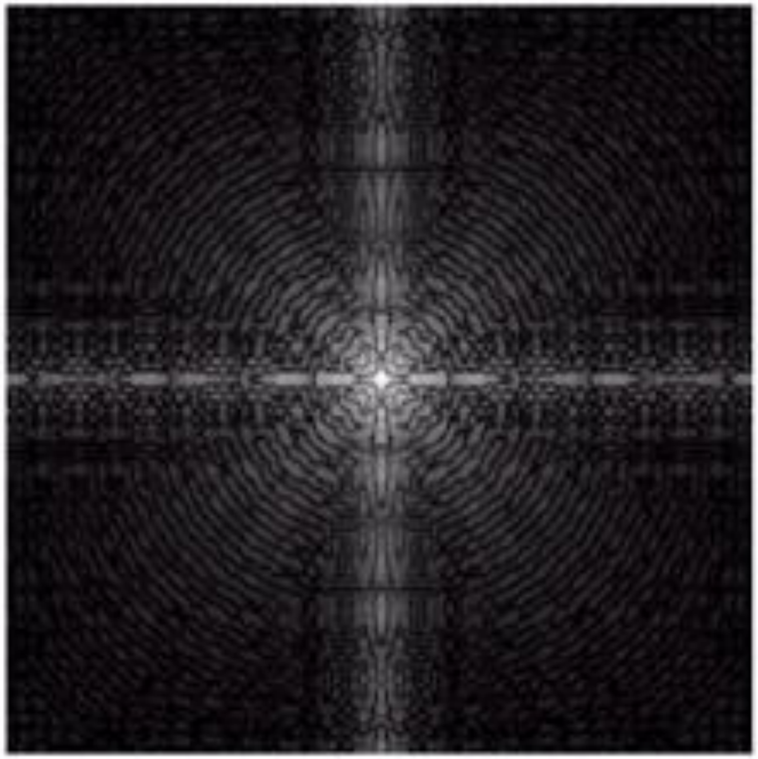


Mapped image

Example of Logarithm Image

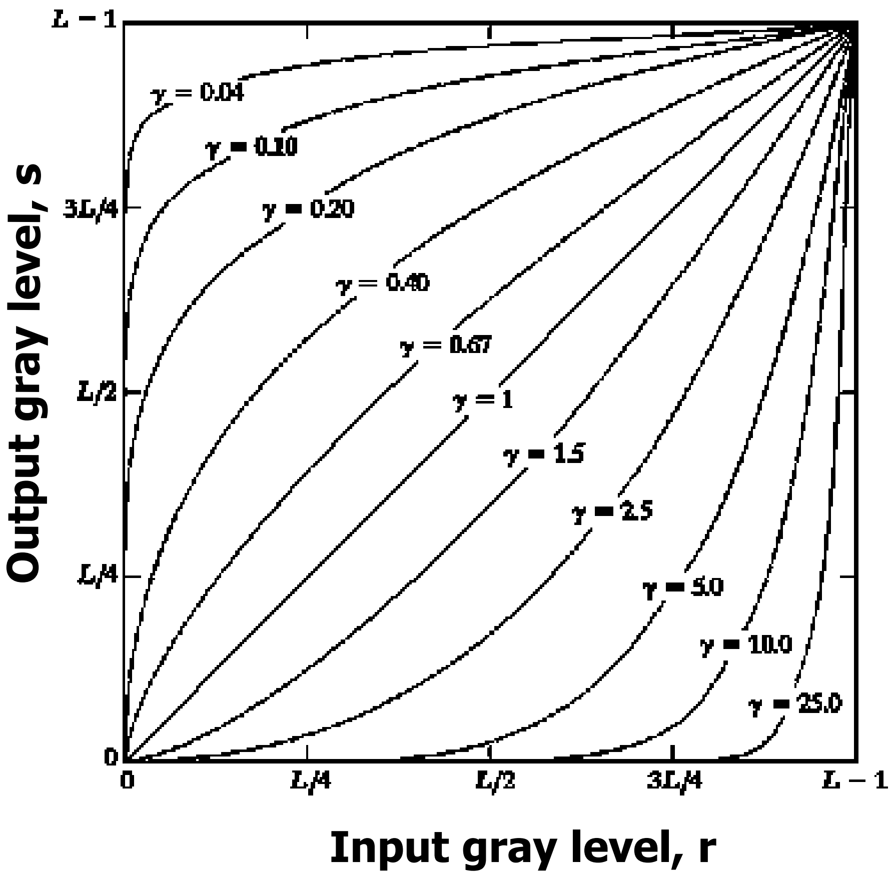


Fourier Spectrum with range = 0 to 1.5 x 10^6



Result after apply the log transformation with c = 1, range = 0 to 6.2

Power-Law Transformations

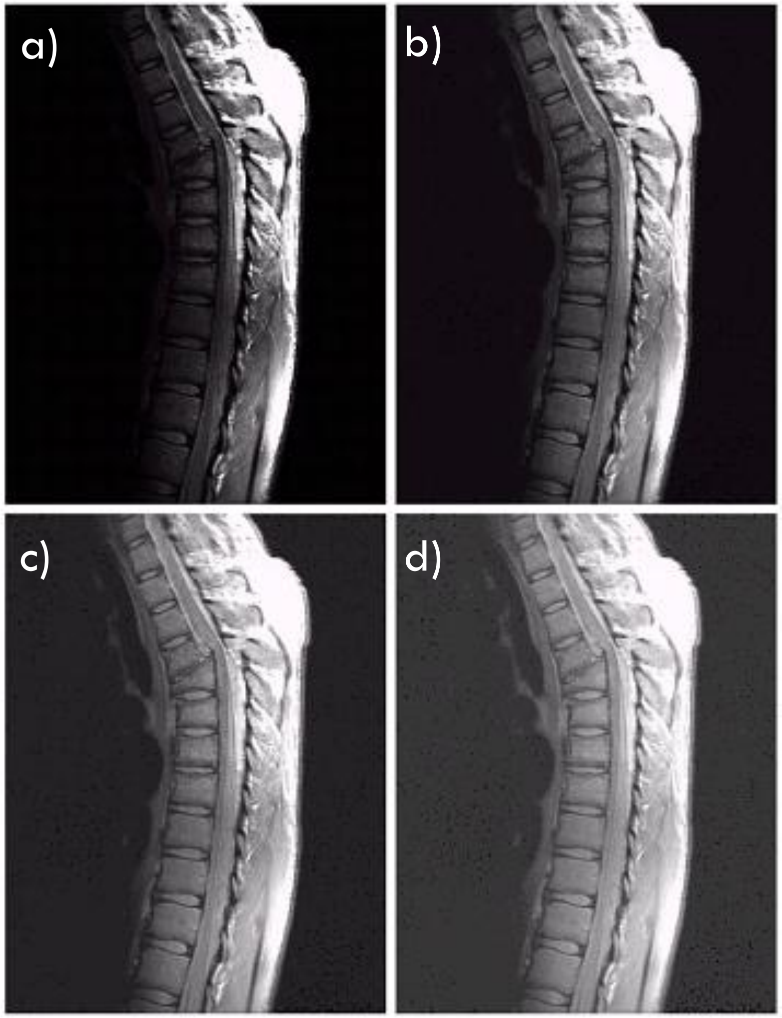


Plots of $s = cr^{\gamma}$ for various values of γ (c = 1 in all cases)

$$s = cr^{\gamma}$$

- c and γ are positive constants
- Power-law curves with fractional values of γ map a narrow range of dark input values into a wider range of output values, with the opposite being true for higher values of input levels.
- $c = \gamma = 1$ ⇨ Identity function

Example : MRI



(a) a magnetic resonance image of an upper human spine with a fracture dislocation and spinal cord impingement

- The picture is predominately dark
- An expansion of gray levels are desirable ⇨ needs $\gamma < 1$

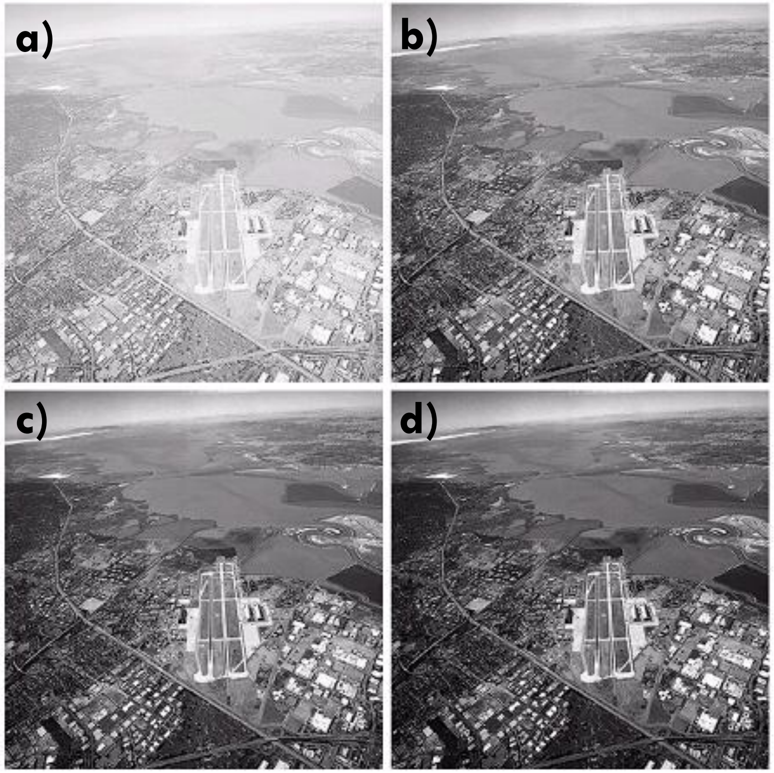
(b) result after power-law transformation with $\gamma = 0.6$, c=1

(c) transformation with $\gamma = 0.4$ (best result)

(d) transformation with $\gamma = 0.3$ (under acceptable level)

Effect of decreasing gamma

- **When the γ is reduced too much, the image begins to reduce contrast to the point where the image started to have very slight "wash-out" look, especially in the background**



(a) image has a washed-out appearance, it needs a compression of gray levels ⇨ needs $\gamma > 1$
(b) result after power-law transformation with $\gamma = 3.0$ (suitable)
(c) transformation with $\gamma = 4.0$ (suitable)
(d) transformation with $\gamma = 5.0$ (high contrast, the image has areas that are too dark, some detail is lost)

Piecewise-Linear Transformation Functions

- A complementary approach to the previous methods
- The form of piecewise functions can be arbitrarily complex
- Practical implementation of some important transformations can be formulated only as piecewise functions
- Their specification requires considerably more user input

Contrast Stretching

- The Simplest of piecewise linear functions.
- **Basic idea** – to increase the dynamic range of the gray levels in the image being processed.
- During image acquisition, low contrast images may result due to
  - Poor illumination
  - Lack of dynamic range in image sensor
  - Wrong setting of the lens aperture

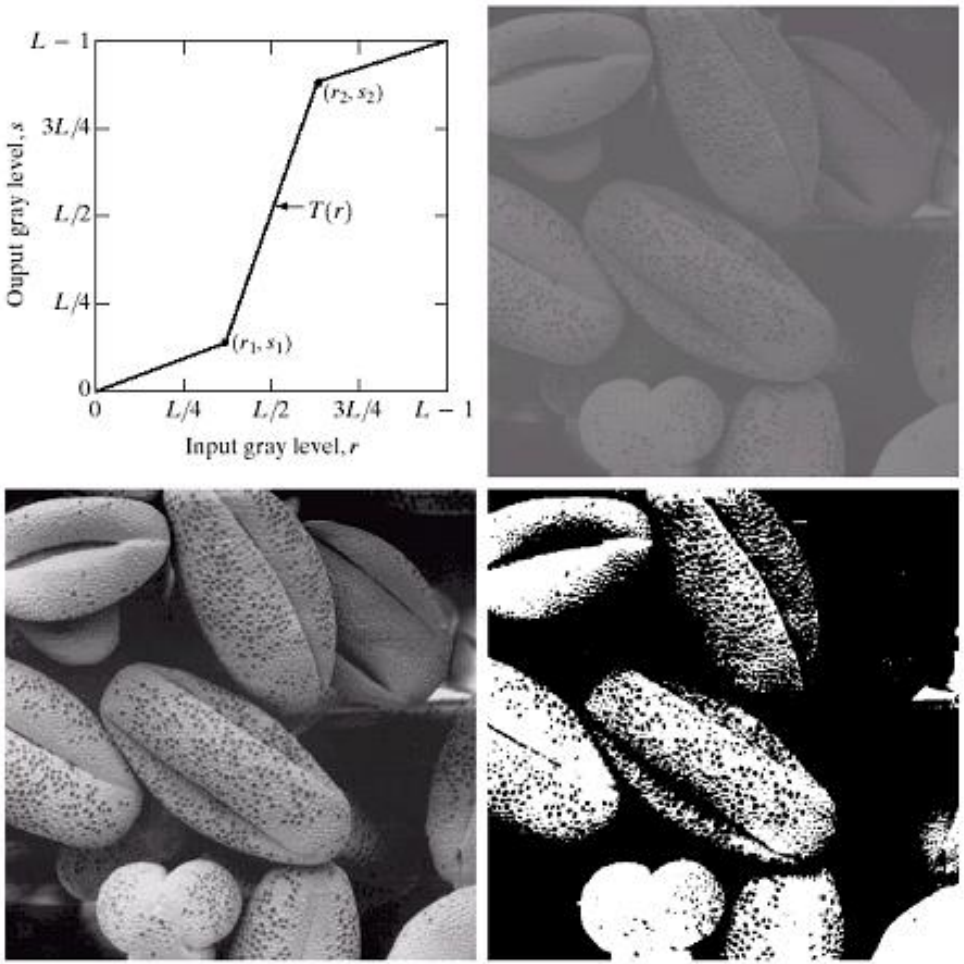
Contrast Stretching

- Contrast transform result

Original

Enhanced

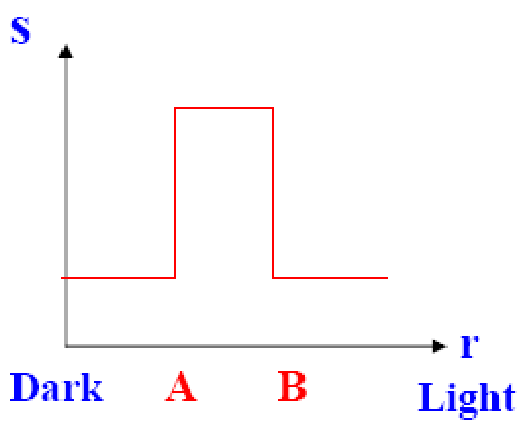
Contrast Stretching: Example

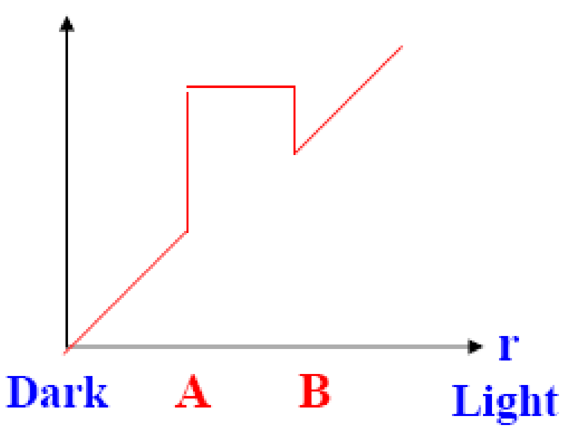


- increase the dynamic range of the gray levels in the image
- (b) a low-contrast image
- (c) result of contrast stretching: $(r_1,s_1) = (r_{min},0)$ and $(r_2,s_2) = (r_{max},L\text{-}1)$
- (d) result of thresholding

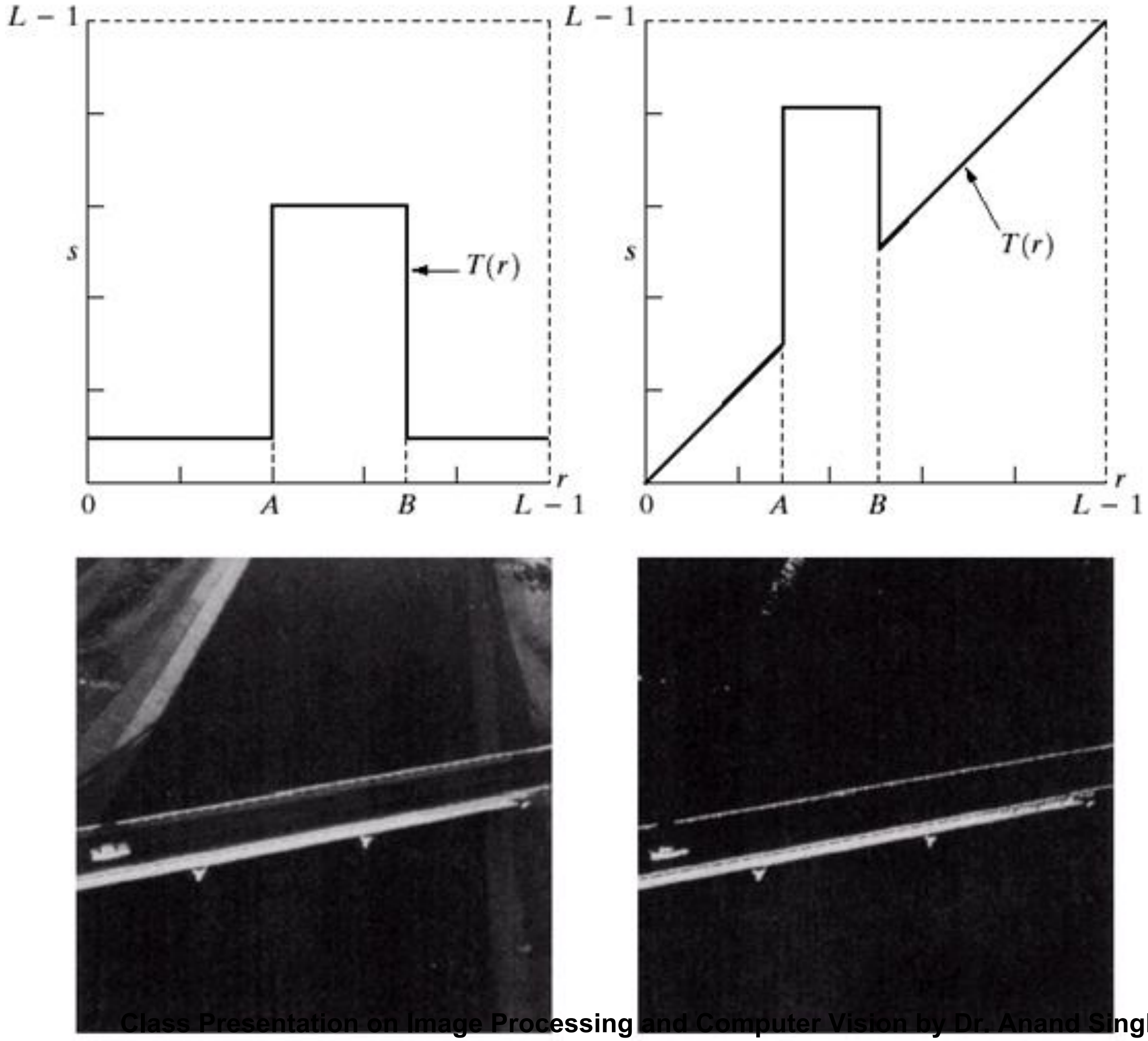
Gray-level slicing

- Highlight a specific range of gray values
- Two basic Methods
  - Display a high value for all gray levels in the range of interest and a low value for all other
  - Brighten the desired range of gray levels but preserve all other levels





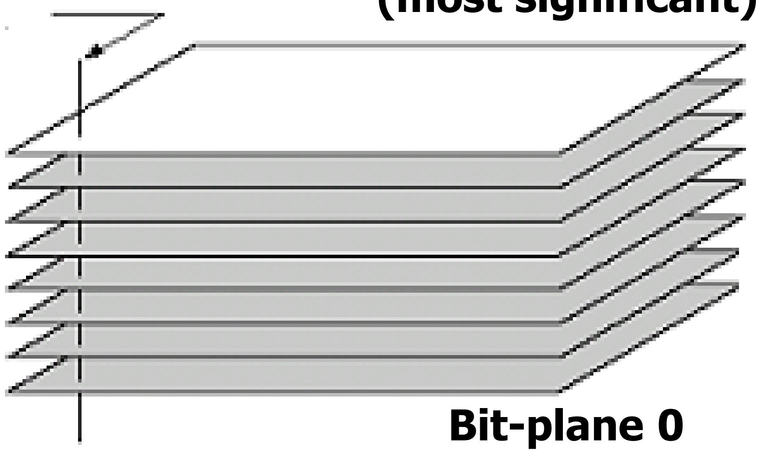
Gray-level slicing: Example



a b
c d

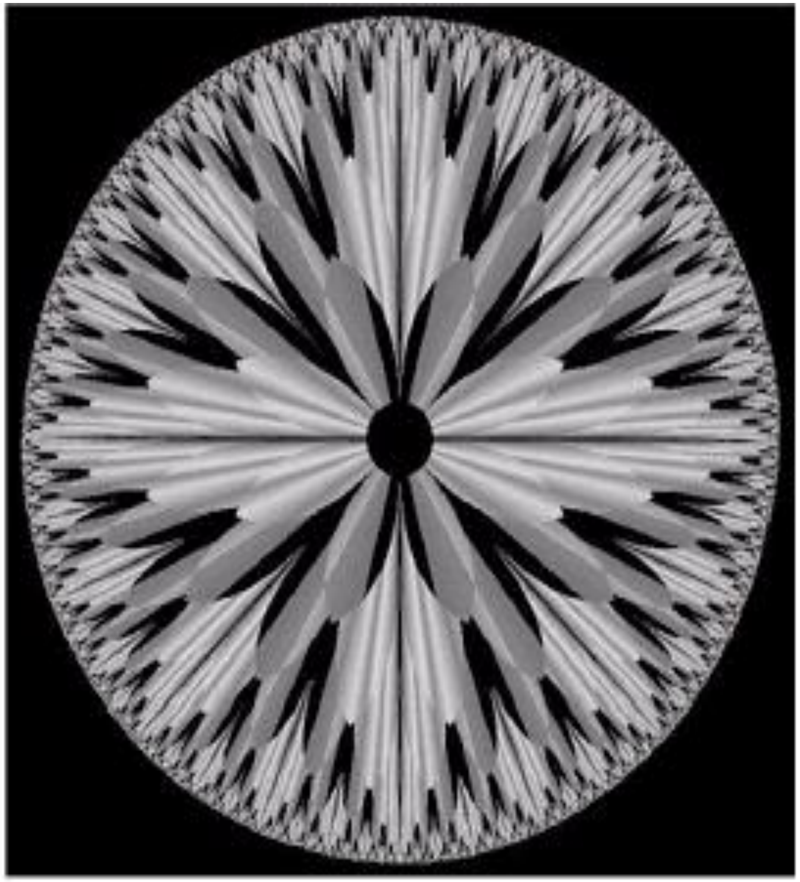
(a) This transformation highlights range [A, B] of gray levels and reduces all others to a constant level.
(b) This transformation highlights range [A, B] but preserves all other levels.
(c) An image.
(d) Result of using the transformation in (a).

Bit-plane slicing



- Highlighting the contribution made to total image appearance by specific bits
- Suppose each pixel is represented by 8 bits
- Higher-order bits contain the majority of the visually significant data
- Useful for analyzing the relative importance played by each bit of the image

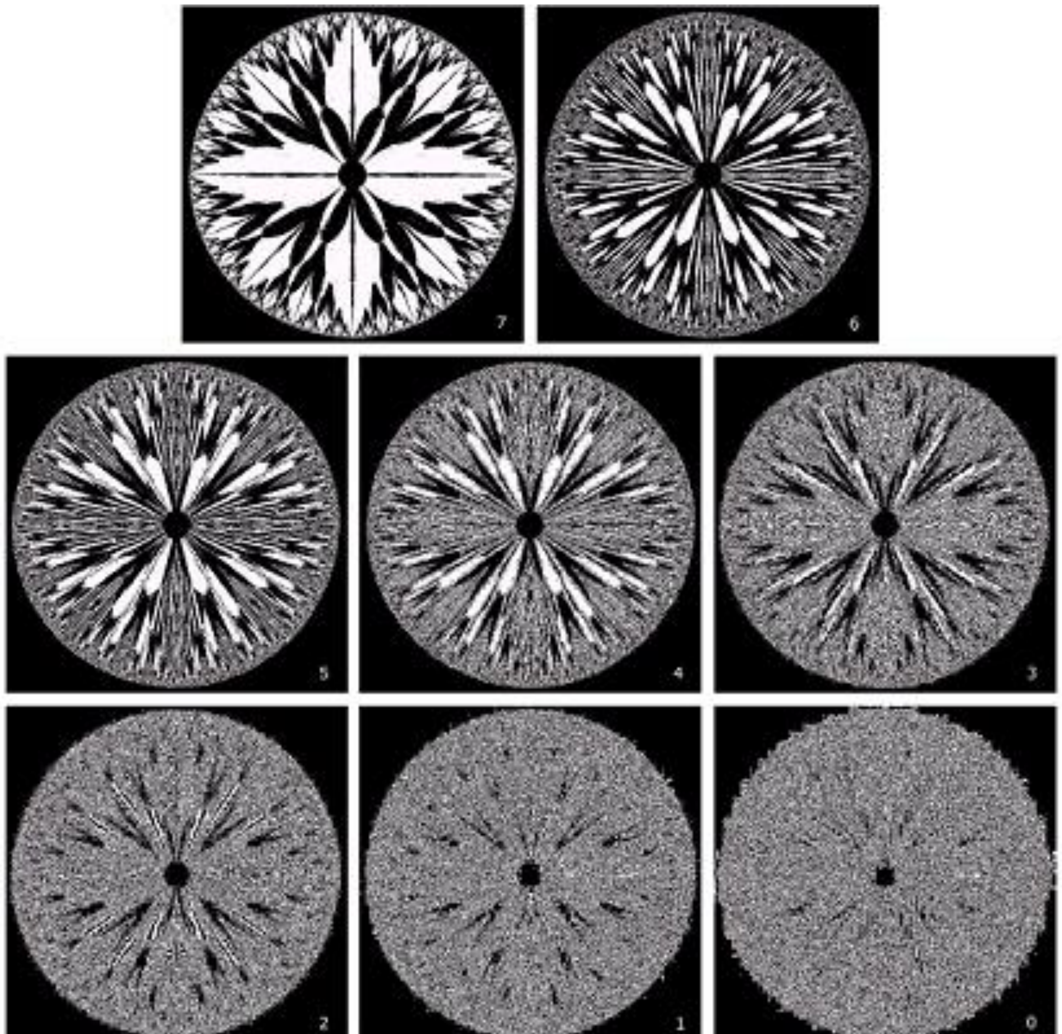
Bit-plane slicing: Example



An 8-bit fractal image

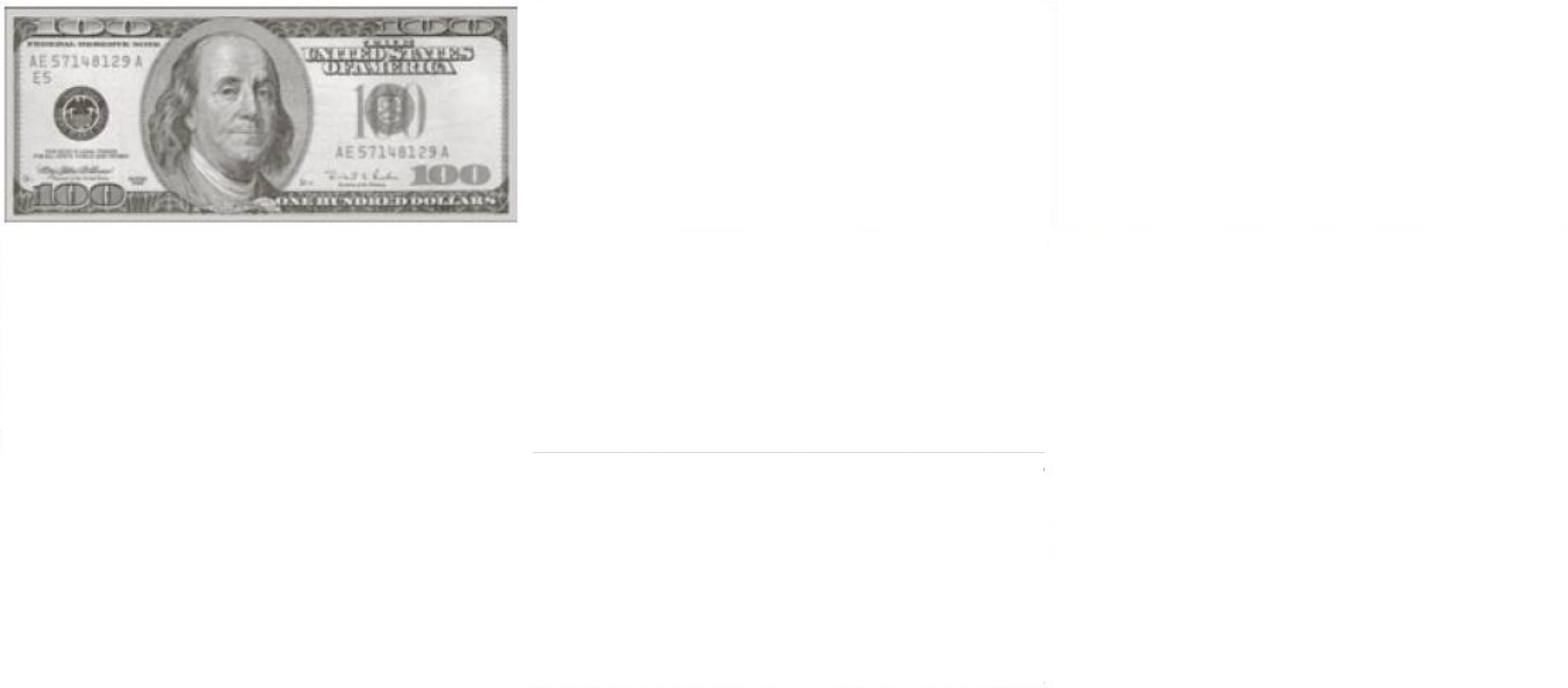
- Higher order bit planes of an image carry a significant amount of visually relevant details.
- Lower order planes contribute more to fine (often imperceptible) details.
- The (binary) image for bit-plane 7 can be obtained by processing the input image with a thresholding gray-level transformation.
  - Map all levels between 0 and 127 to 0
  - Map all levels between 129 and 255 to 255

Bit-plane slicing: Example (8 bit planes)



Bit-plane 7	Bit-plane 6	
Bit-plane 5	Bit-plane 4	Bit-plane 3
Bit-plane 2	Bit-plane 1	Bit-plane 0

Bit-plane slicing: Example



a	b	c
d	e	f
g	h	i

(a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

Bit-plane slicing: Example

a b c

Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).

Histogram Processing

- Histogram is a discrete function formed by counting the number of pixels that have a certain gray level in the image .
- Histogram of a digital image with gray levels in the range [0,L-1] is a discrete function

$$h(r_k) = n_k$$

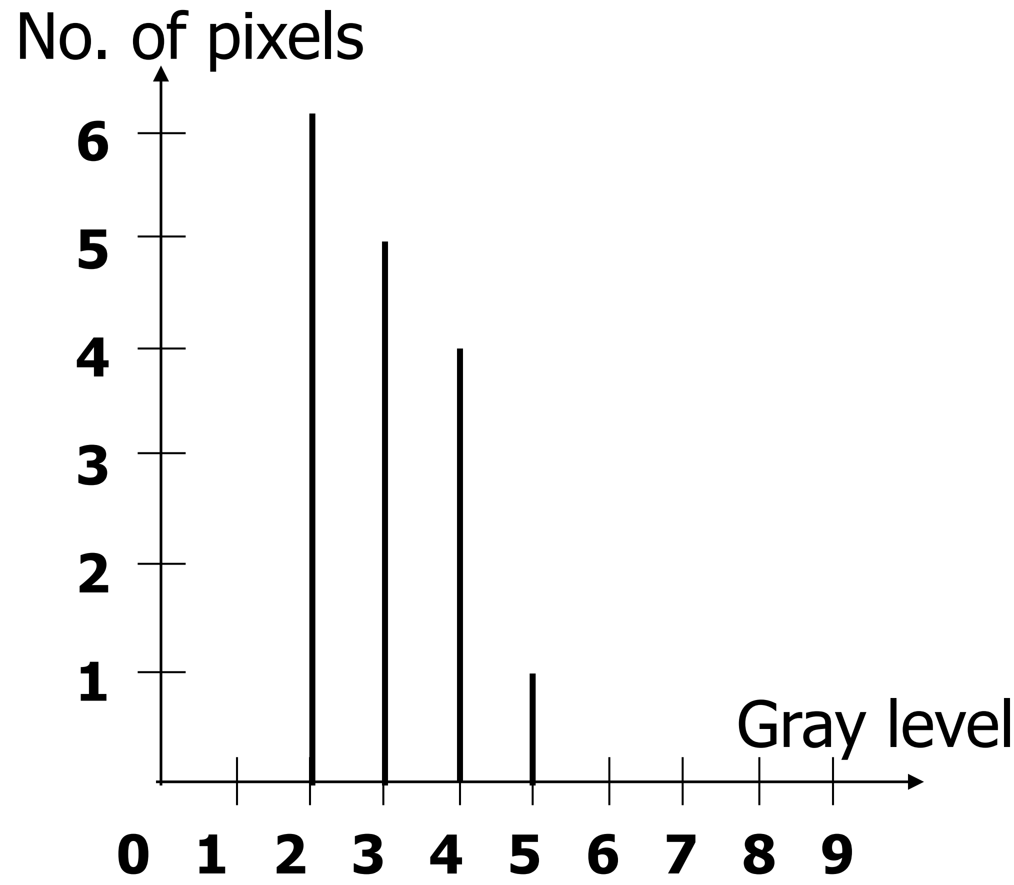
- Where
  - r_k : the k^{th} gray level
  - n_k : the number of pixels in the image having gray level r_k
  - $h(r_k)$: histogram of a digital image with gray levels r_k

Histogram Processing: Example

2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4

4x4 image

Gray scale = [0,9]



histogram

Normalized Histogram

- Dividing each of histogram at gray level r_k by the total number of pixels in the image, n

$$p(r_k) = n_k / n$$

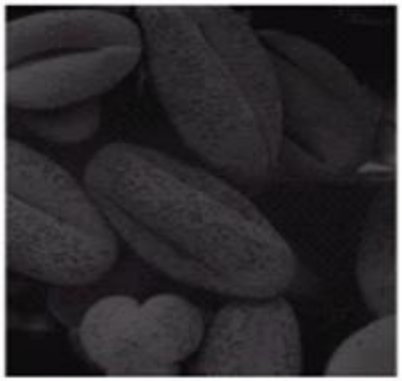
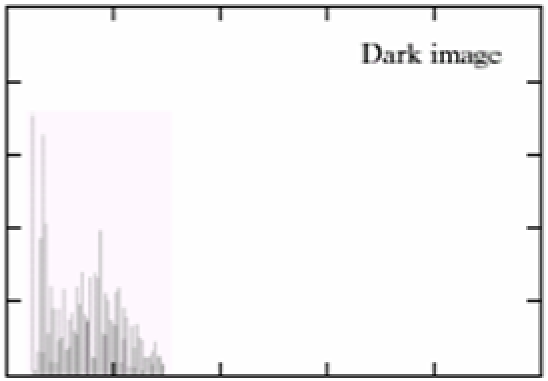
- For k = 0,1,...,L-1
- $p(r_k)$ gives an estimate of the probability of occurrence of gray level r_k
- The sum of all components of a normalized histogram is equal to 1

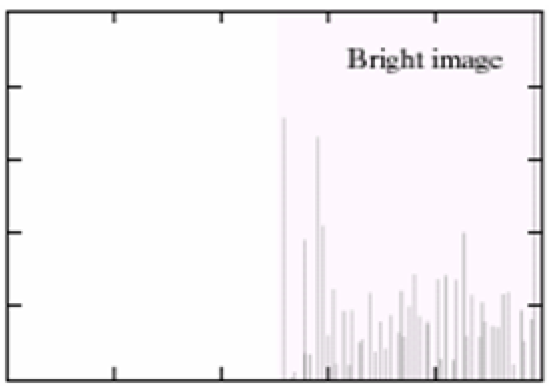
Histogram Processing: Example

- Problem: an image with gray levels between 0 and 7 is given below. Find the histogram of the image

1	6	2	2
1	3	3	3
4	6	4	0
1	6	4	7

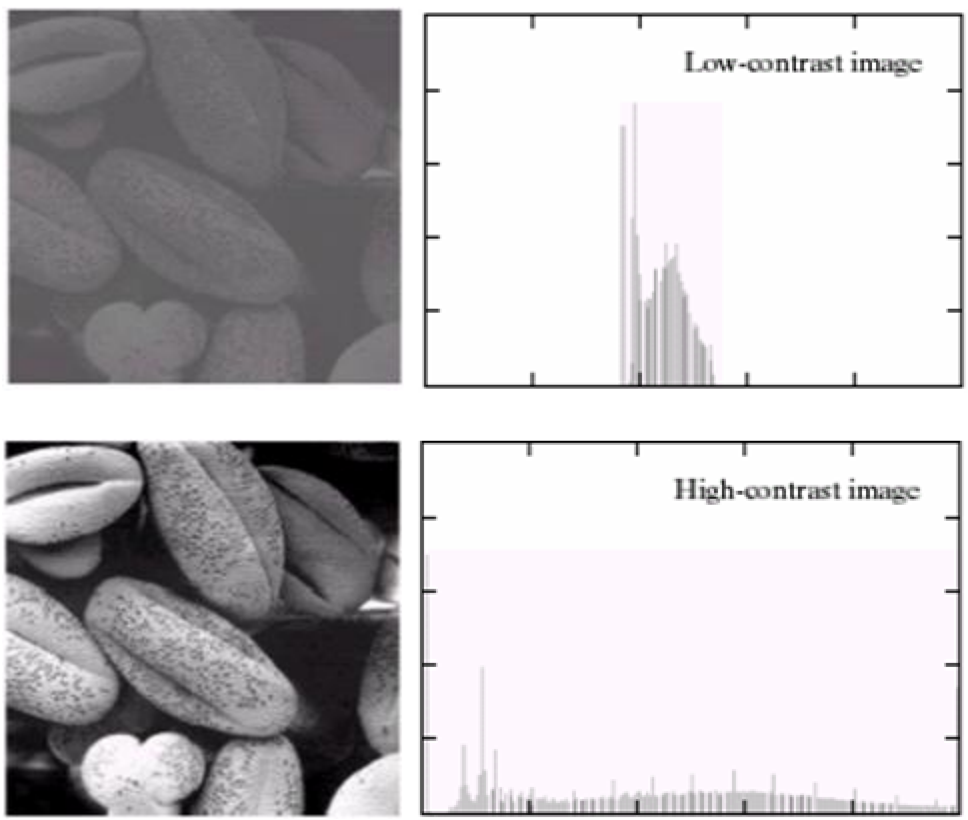
Histogram Processing: Example

- In the dark images, components of the histogram are concentrated on the low (dark) side of the gray scale.
- In bright images, the histogram is biased towards the high side of the gray scale.

Histogram Processing: Example



- In image with low contrast, the histogram will be narrow & centred towards the middle of the gray scale.
- An image with high contrast & a large variety of gray tones will have pixels that occupy the entire range of possible gray levels & are uniformly distributed.

Histogram Equalization OR Histogram Linearization

- It is the process that transforms the intensity values so that the histogram of the output image approximately matches the flat (uniform) histogram.
- The aim is to create an image with equally distributed brightness levels over the whole brightness scale.

Histogram Equalization OR Histogram Linearization

- As the low-contrast image's histogram is narrow and centered toward the middle of the gray scale, if we distribute the histogram to a wider range the quality of the image will be improved.
- We can do it by adjusting the probability density function of the original histogram of the image so that the probability spread equally.
- Histogram equalization (HE) results are similar to contrast stretching but offer the **advantage of full automation**, since HE automatically determines a transformation function to produce a new image with a uniform histogram.

Histogram Equalization

- Goal: find a transform $s=T(r)$ such that the transformed image has a flat (equalized) histogram.
- Where $T(r)$ satisfies the following conditions:
  - $T(r)$ is single-valued and monotonically increasing in interval [0,1];
  - $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$.

Histogram Equalization …

For discrete values, the probability of occurrence of gray level r_k in an image is given by:

$$P_r(r_k) = n_k / n$$

where k = 0, 1, …, L-1,

- n is the total no. of pixels in the image
- n_k is the no. of pixels with gray level r_k
- L is the total no. of possible gray levels in the image

Histogram Equalization …

The discrete transformation function is given by:

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_k)$$

$$= \sum_{j=0}^{k} \frac{n_j}{n} \quad k = 0, 1, \ldots, L-1$$

The o/p image is obtained by mapping each pixel with level r_k in the i/p image into a corresponding pixel with level s_k in the o/p image with the help of the above equation.

Histogram Equalization …

The following equations bring back the gray levels in the range [0, L-1]

Discrete values:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= (L-1)\sum_{j=0}^{k} \frac{n_j}{MN} = \frac{L-1}{MN}\sum_{j=0}^{k} n_j \qquad k=0,1,\ldots,L-1$$

M – no. of rows

N – no. of columns

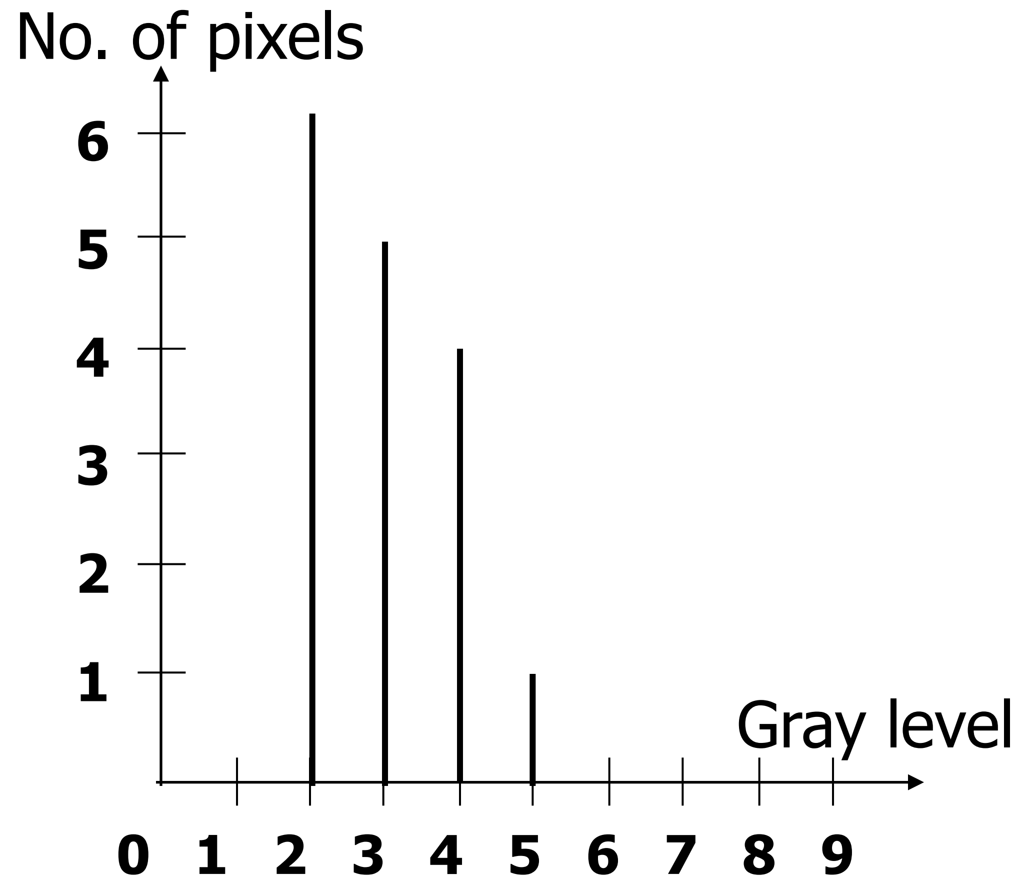
MN – total no. of pixels in the image

Histogram Equalization: Example 1

2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4

4x4 image

Gray scale = [0,9]



histogram

Histogram Equalization: Example 1…

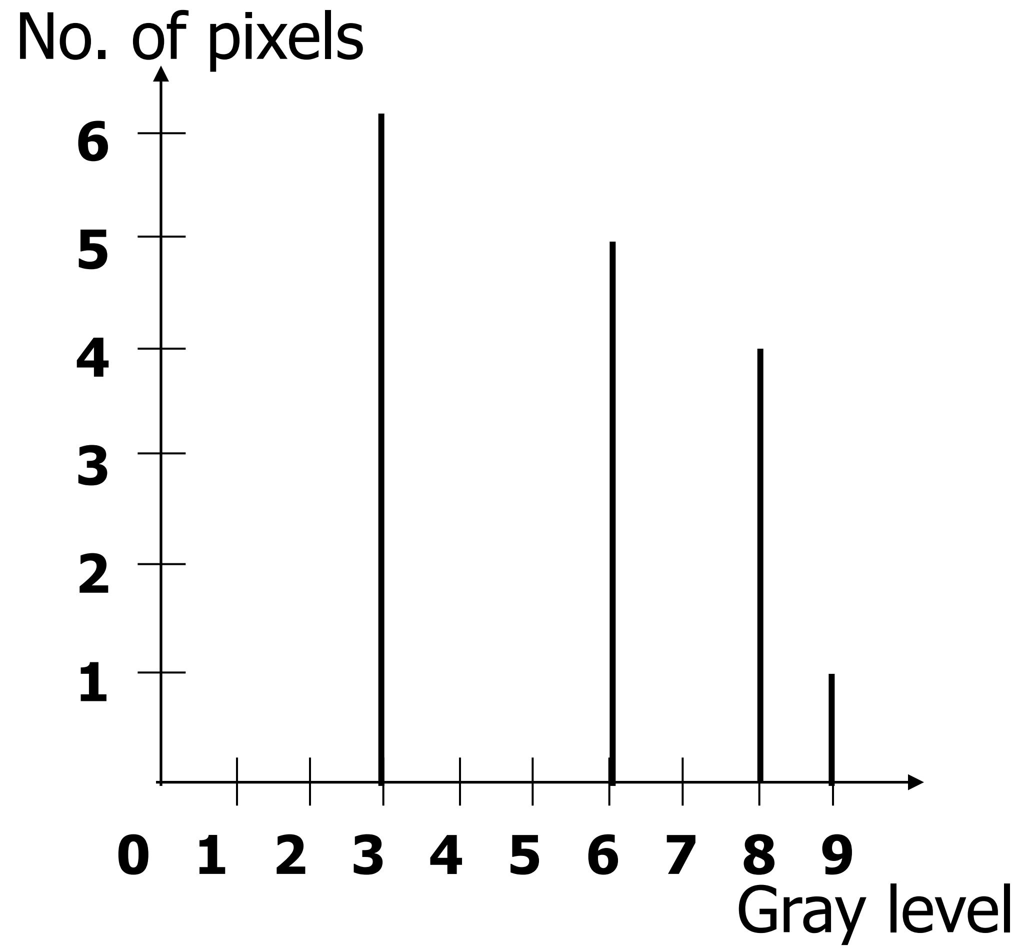
Gray Level(j)	0	1	2	3	4	5	6	7	8	9
No. of pixels	0	0	6	5	4	1	0	0	0	0
$\sum_{j=0}^{k} n_j$	0	0	6	11	15	16	16	16	16	16
$s = \sum_{j=0}^{k} \frac{n_j}{n}$	0	0	6 / 16	11 / 16	15 / 16	16 / 16	16 / 16	16 / 16	16 / 16	16 / 16
$s \times 9$	0	0	3.3 ≈3	6.1 ≈6	8.4 ≈8	9	9	9	9	9

Histogram Equalization: Example 1...

3	6	6	3
8	3	8	6
6	3	6	9
3	8	3	8

Output image

Gray scale = [0,9]



Histogram equalization

Histogram Equalization: Example 2

Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in following table.
Get the histogram equalization transformation function and give the $p_s(s_k)$ for each s_k.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

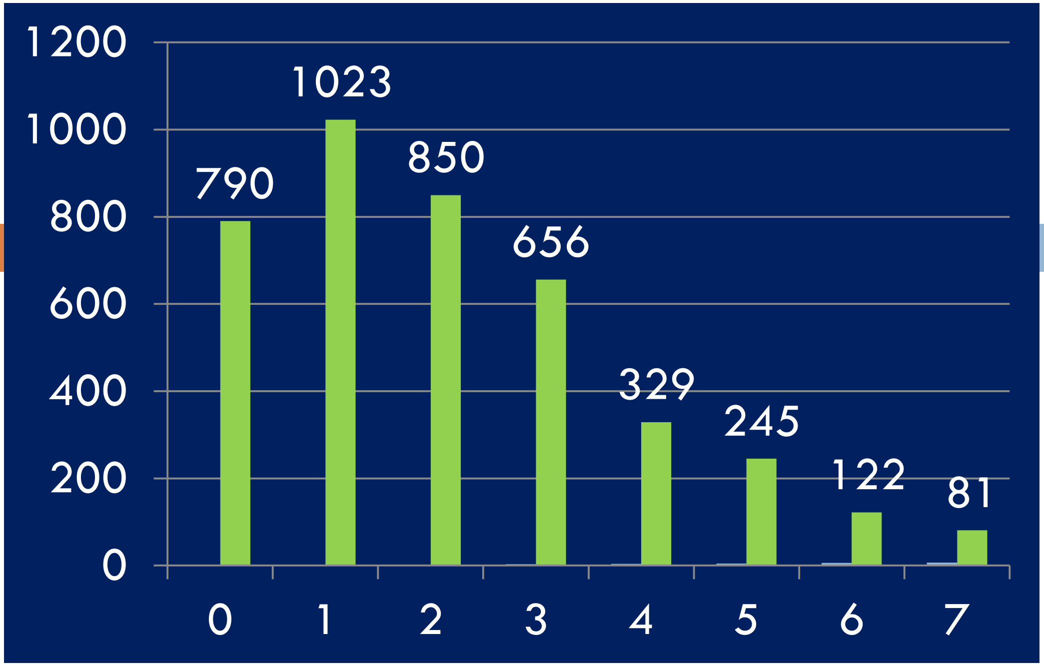
Histogram Equalization: Example 2 …

I/p Gray Level (r_k)	no. of pixels (n_k)	$p(r_k) = n_k/MN$	Σ	(L-1)Σ	O/p Gray Level (s)
0	790	0.19			
1	1023	0.25			
2	850	0.21			
3	656	0.16			
4	329	0.08			
5	245	0.06			
6	122	0.03			
7	81	0.02			

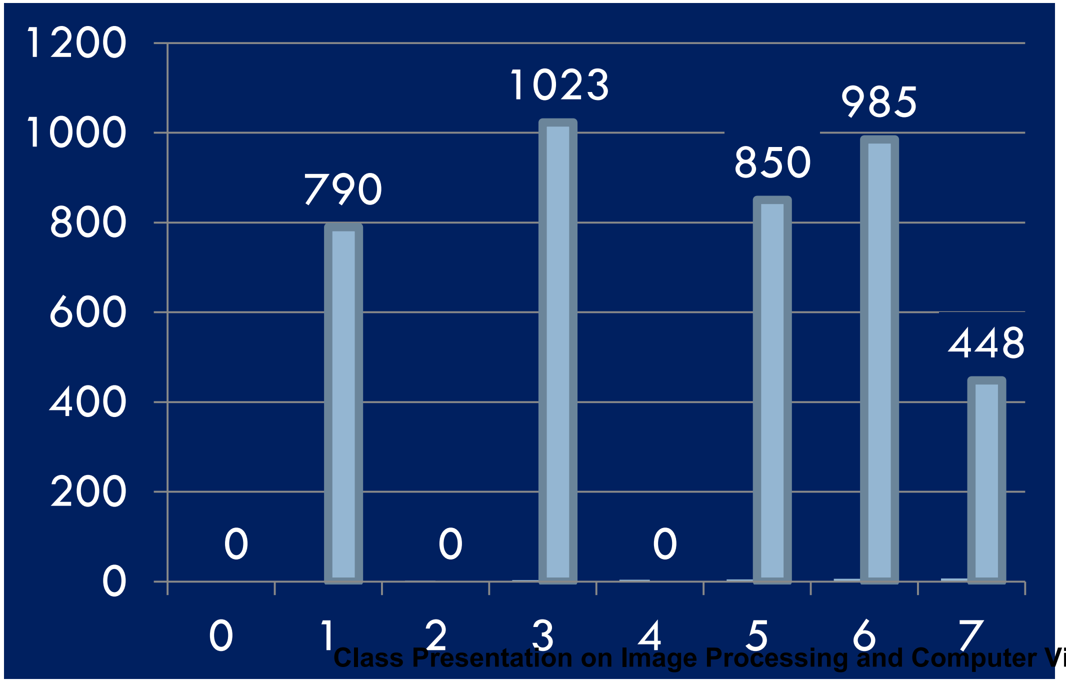
Grey level	n_k	$p_r(r_k)$	s_k
0	790	0.19	1
1	1023	0.25	3
2	850	0.21	5
3	656	0.16	6
4	329	0.08	6
5	245	0.06	7
6	122	0.03	7
7	81	0.02	7

What will be the histogram of the new image?

Equalized Grey level	n_k
0	0
1	790
2	0
3	1023
4	0
5	850
6	656 + 245 = 985
7	245 + 122 + 81 = 448

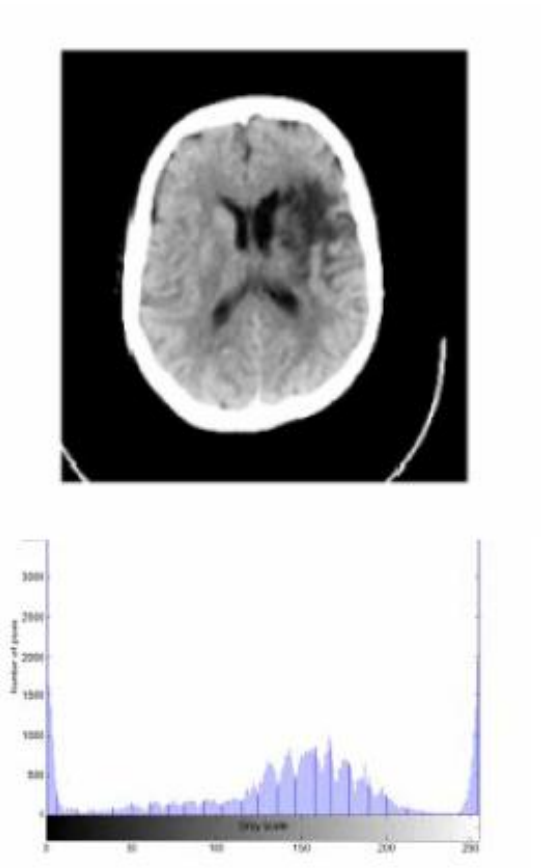


Histogram of the dark image

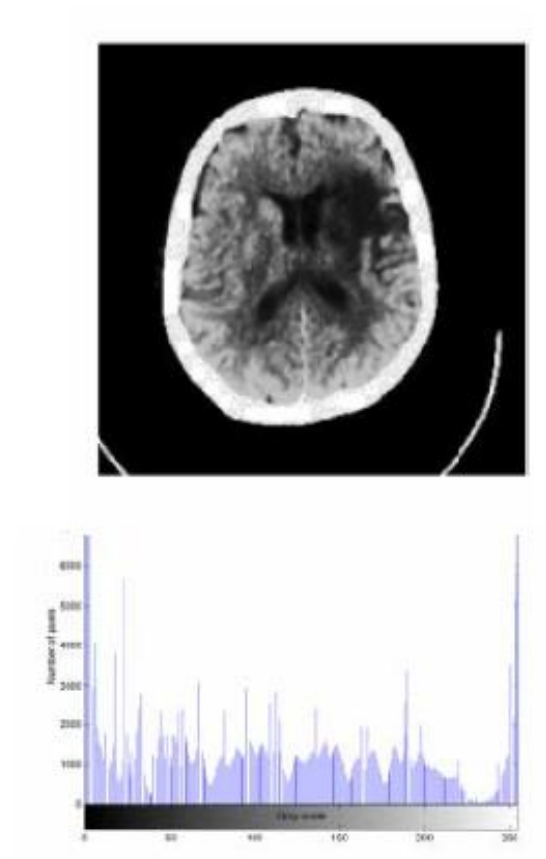


Equalized Histogram of the image

Histogram Equalization: Example

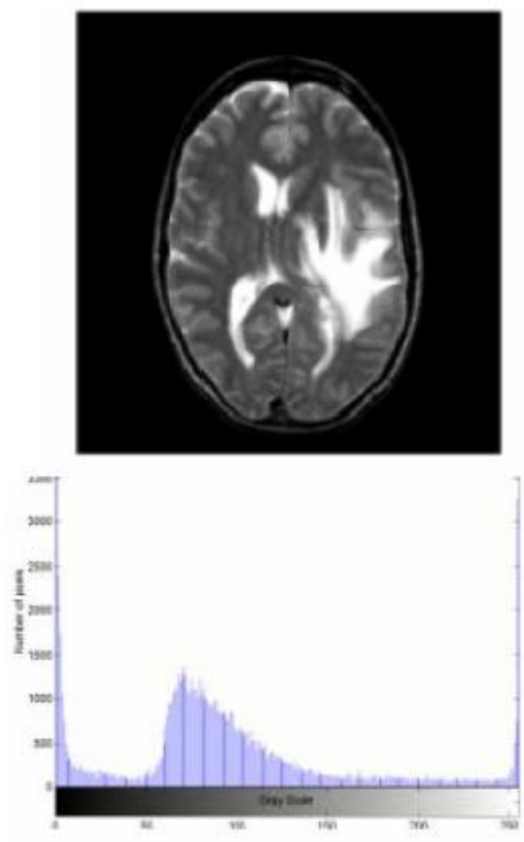


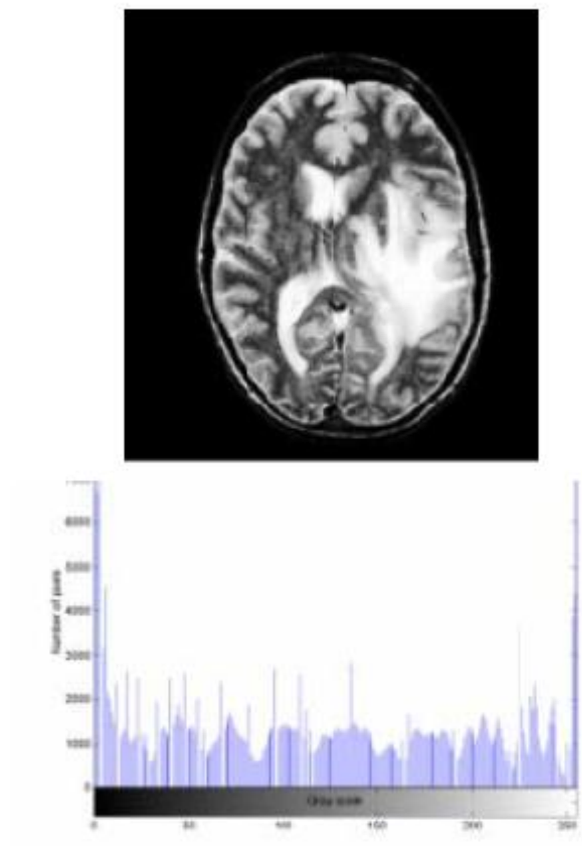
Before HE



After HE

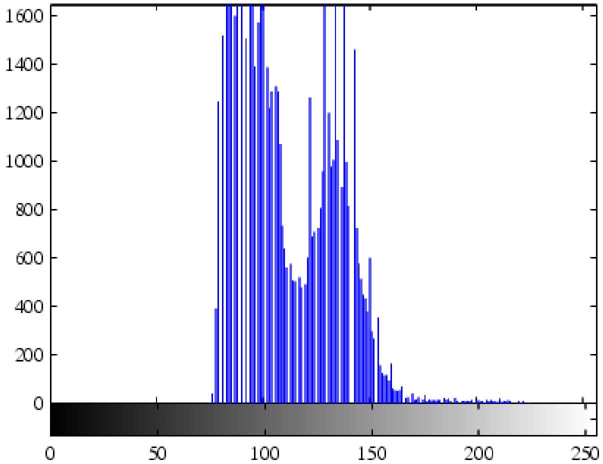
Histogram Equalization: Example

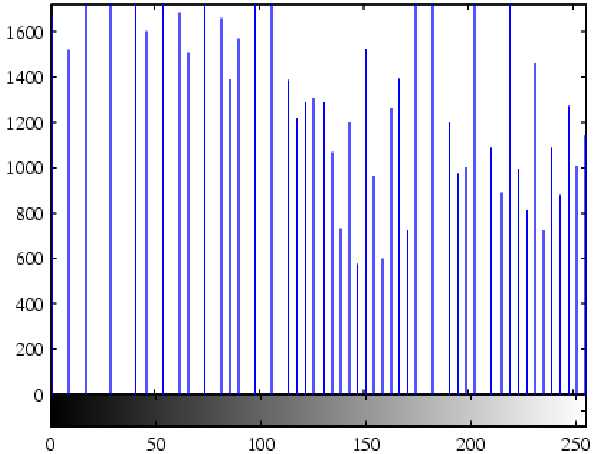




Before HE **After HE**

Histogram Equalization: Example





Histogram Matching (Specification)

- Histogram equalization has a disadvantage which is that it can generate only one type of output image.
- Sometimes it is useful to be able to specify the shape of the histogram that we wish the processed image to have.
- With Histogram Specification, we can specify the shape of the histogram that we wish the output image to have.
- It doesn’t have to be a uniform histogram

Histogram Matching (Specification) …

For an image, whose enhancement is to be done, we are given an histogram, $G(z_k)$, that actually shows how the processed image's histogram should look after applying the transformation function to the i/p image.

$$s_k = T(r_k)$$

$$G(z_k) = \sum_{i=0}^{k} p_k(z_i) = s_k$$

$$z_k = G^{-1}(s_k) = G^{-1}[T(r_k)]$$

$k = 0, 1, \ldots, L-1$

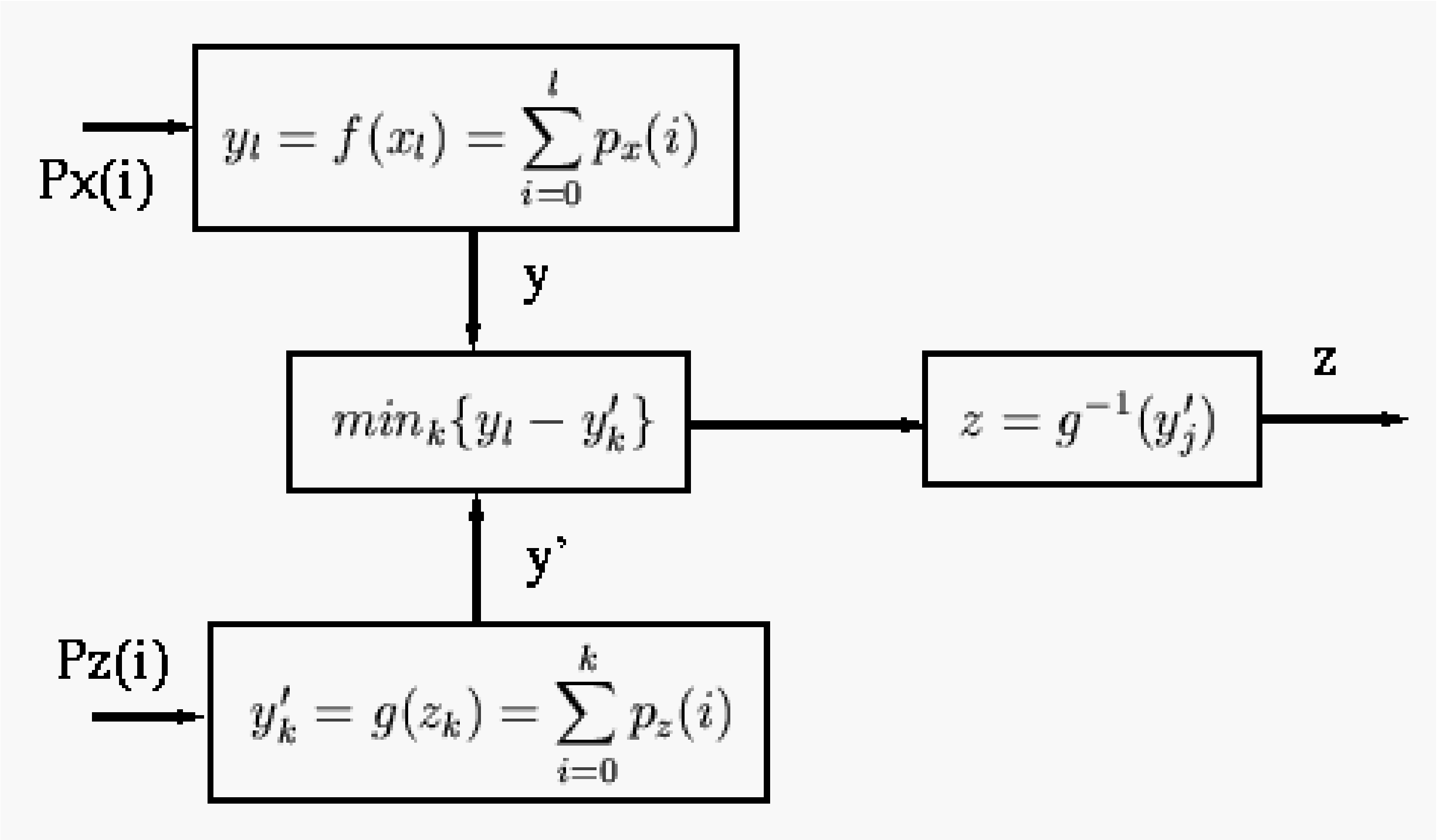
Histogram Matching (Specification) …

- Step 1: Find histogram of input image $p_r(rj)$, and find histogram equalization mapping.
- Step 2: Specify the desired histogram, and find histogram equalization mapping.
- Step 3: Build lookup table:
  - For each gray level k, find s_k and then find a 'l' level so that s_k best matches z_l:

$$\min|s_k - z_l|$$

  - setup a lookup entry lookup[k]=l.

Histogram Matching: Discrete Cases



Histogram Matching: Example

- Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in the following table (on the left). Get the histogram transformation function and make the output image with the specified histogram, listed in the table on the right.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

We have already equalized the first histogram.

I/p Gray Level (r_k)	no. of pixels (n_k)	$p(r_k) = n_k/MN$	Σ	(L-1)Σ	O/p Gray Level (s)
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1.00	7.00	7

Now equalize the second histogram.

Gray Level (z_k)	$p(z_k) = n_k/MN$	Σ	(L-1)Σ	O/p Gray Level $G(z_k)$
0	0.00	0	0	0
1	0.00	0	0	0
2	0.00	0	0	0
3	0.15	0.15	1.05	1
4	0.20	0.35	2.45	2
5	0.30	0.65	4.55	5
6	0.20	0.85	5.95	6
7	0.15	1	7	7

And now we do the matching

I/p Gray Level (r_k)	O/p Gray Level (s)
0	1
1	3
2	5
3	6
4	6
5	7
6	7
7	7

O/p Gray Level $G(z_k)$	Gray Level (z_k)
0	0
0	1
0	2
1	3
2	4
5	5
6	6
7	7

I/p Gray Level (r_k)		Gray Level (z_k)
0		0
1		1
2		2
3		3
4		4
5		5
6		6
7		7

Histogram Matching: Example

$$r_k \rightarrow z_q$$

$$0 \rightarrow 3$$

$$1 \rightarrow 4$$

$$2 \rightarrow 5$$

$$3 \rightarrow 6$$

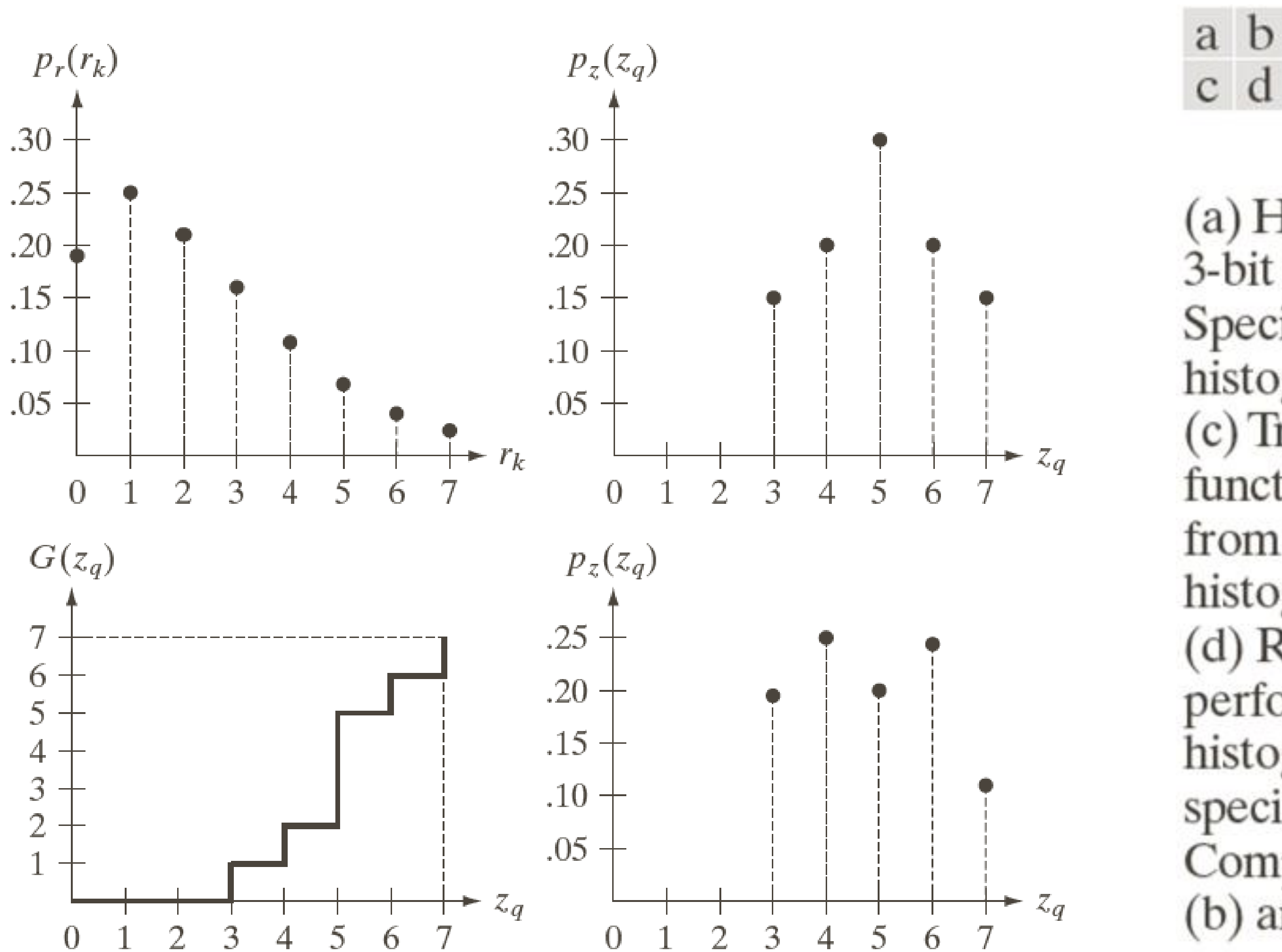
$$4 \rightarrow 6$$

$$5 \rightarrow 7$$

$$6 \rightarrow 7$$

$$7 \rightarrow 7$$

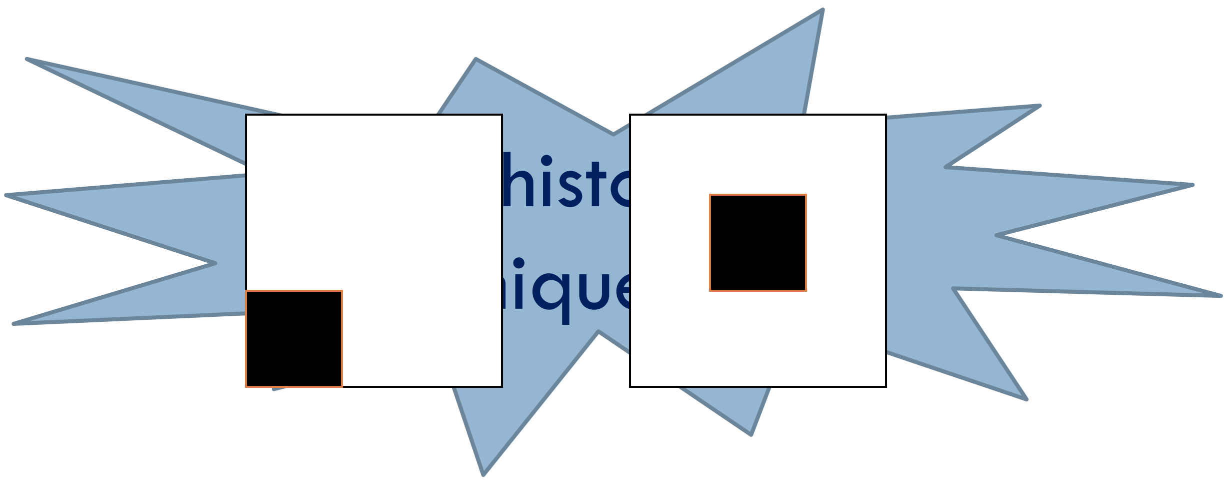
Histogram Matching: Example



(a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

Local Histogram Processing

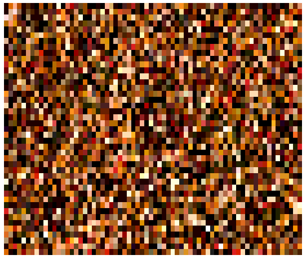
Histogram processing yields robust image processing results



Histograms of both the images are same !!

Local Histogram Processing …

- Q: What happens if I reshuffle all pixels within the image?



- A: It's histogram won't change. No point processing will be affected…
- Need spatial information to capture this.

Local Histogram Processing …

- Transformation should be based on gray-level distribution in the neighborhood of every pixel.
- Local histogram processing:
  - At each location the histogram of the points in the neighborhood is computed and a histogram equalization or histogram specification transformation function is obtained
  - The gray level of the pixel centered in the neighborhood is mapped
  - The center of the neighborhood is moved the next pixel and the procedure repeated

Assignment

- Construct the histogram of the following image.

0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7
8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15

- What will be the effect on the histogram if we set to zero the
  - Higher order bit plane?
  - Lower order bit plane ?
- Take your grey scale image & construct its histogram.

Assignment

Q. Equalize the following histogram and give the resultant histogram.

Gray levels (rk)	0	1	2	3	4	5	6	7
Number of pixels (nk)	50	100	100	300	200	200	50	0

Q. Perform histogram specification on the 8x8 image. The gray level distribution of the image is given below:

Gray levels (rk)	0	1	2	3	4	5	6	7
Number of pixels (nk)	8	10	10	2	12	16	4	2

Gray levels (rk)	0	1	2	3	4	5	6	7
Number of pixels (nk)	0	0	0	0	20	20	16	8

Assignment

Match the histogram A to the histogram B.

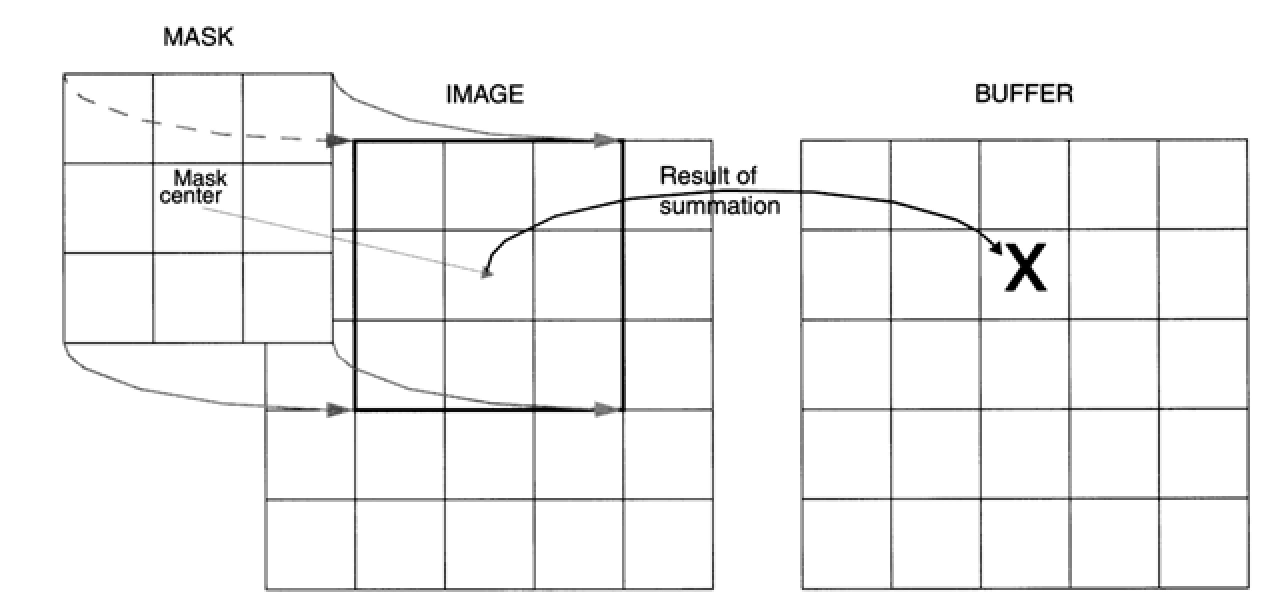
Histogram A

Gray Level	0	1	2	3	4	5	6	7
No. of Pixels	4	17	15	18	24	12	0	10

Histogram B

Gray Level	0	1	2	3	4	5	6	7
No. of Pixels	0	0	0	36	24	12	8	20

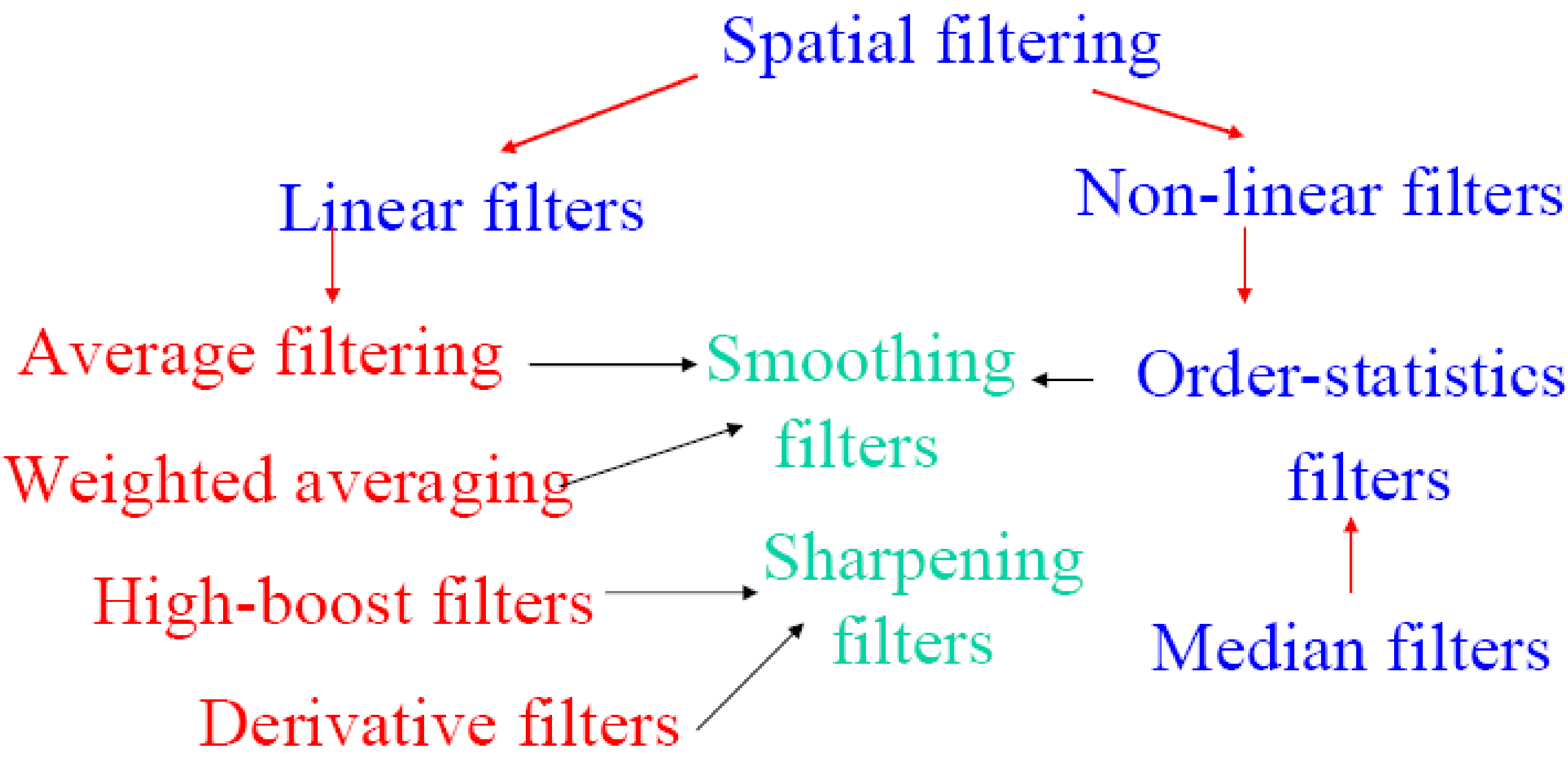
Spatial Filtering



Spatial Filtering

- Enhancement techniques based on the pixels of an image are often referred to as Spatial filtering
  - Working in a neighbourhood of every pixel in an image
- Filter term in "Digital image processing" is referred to the subimage
- There are others term to call subimage such as mask, kernel, template, or window
- The value in a filter subimage are referred as coefficients, rather than pixels.

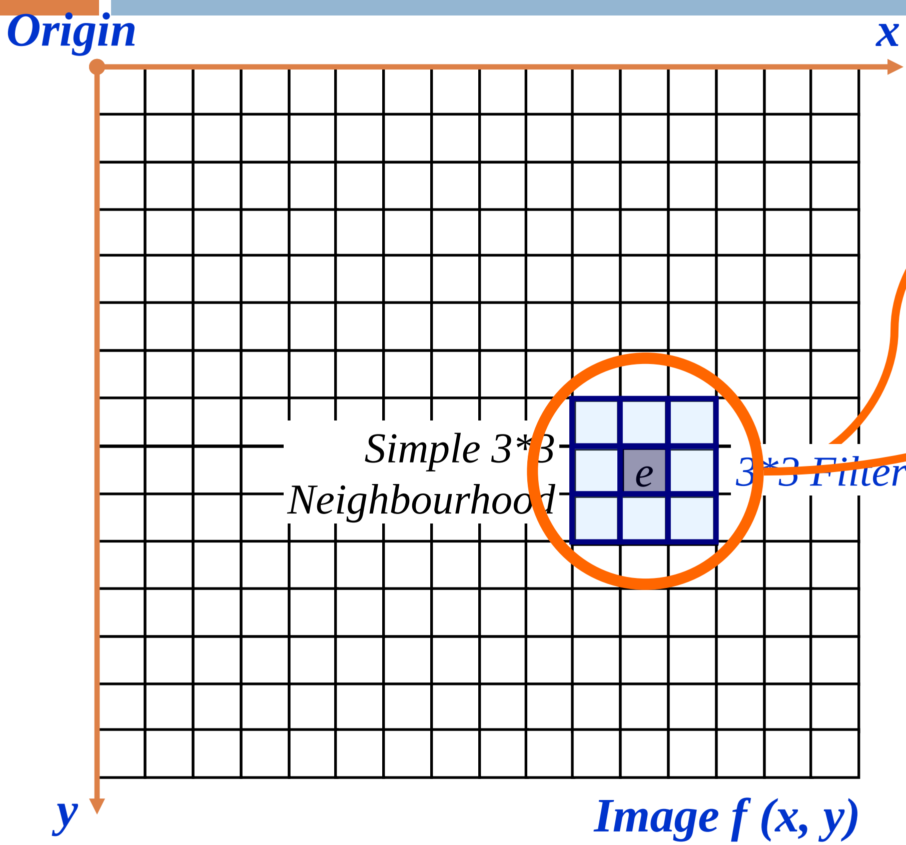
Spatial Filtering

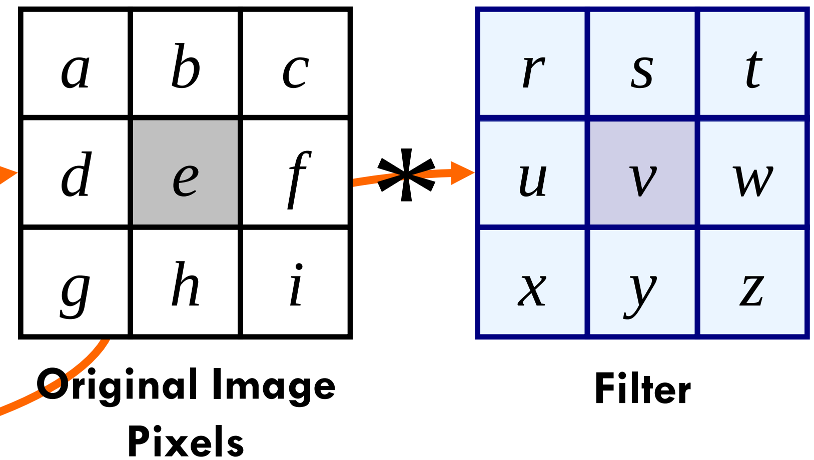


Mechanics of Spatial filtering

- The process consists simply of moving the filter mask from point to point in an image.
- At each point (x, y) the response of the filter at that point is calculated using a predefined relationship.
- If the operation performed on the pixel is linear, then the filter is called **linear spatial filter**.

Mechanics of Spatial filtering





$$e_{processed} = v*e + r*a + s*b + t*c + u*d + w*f + x*g + y*h + z*i$$

The above is repeated for every pixel in the original image to generate the filtered image

Vector Representation of Linear Filtering

- Simply move the filter mask from point to point in an image.
- At each point (x,y), the response of the filter at that point is calculated using a predefined relationship.

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn}$$
$$= \sum_{i=i}^{mn} w_i z_i$$

Spatial Filtering Process …

In general, linear filtering of an image f of size M x N with a filter mask of size m x n is given by the expression:

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\, f(x+s,y+t)$$

where (as per the previous discussion)

$a = (m - 1) / 2$ & $b = (n - 1) / 2$

To generate a complete filtered image, the above equation must be applied for

$x = 0, 1, \ldots, M-1$ & $y = 0, 1, \ldots, N-1$

Correlation Vs Convolution

Two closely related concepts

- Correlation

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

- Convolution

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x - s, y - t)$$

Find the result of Correlation & Convolution for the following image f and 1x5 mask w.

f

0	0	0	1	0	0	0	0

w

1	2	3	2	8

Origin

Note:

To allow every pixel in w to visit every pixel in f, pad f with enough 0s on either side.

If the filter is of size m, then the no. of 0s to be added are ???

m-1

Correlation

0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

$x = 0$

1	2	3	2	8

0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

$x = 1$

1	2	3	2	8

0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

$x = 3$

1	2	3	2	8

0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

$x = 11$

1	2	3	2	8

Correlation

Result of full correlation

0	0	0	8	2	3	2	1	0	0	0	0	0	0	0	0

Result of cropped correlation

0	8	2	3	2	1	0	0

Convolution

0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

x = 0

8	2	3	2	1

0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

x = 1

8	2	3	2	1

Result of full convolution

0	0	0	1	2	3	2	8	0	0	0	0	0	0	0	0

Result of cropped convolution

0	1	2	3	2	8	0	0

Smoothing Spatial Filters

- Used for blurring and for noise reduction
- Blurring is used in preprocessing steps, such as
  - removal of small details from an image prior to object extraction
  - bridging of small gaps in lines or curves
- noise reduction can be accomplished by blurring with a linear filter and also by a nonlinear filter
- There are 2 types of smoothing spatial filters
  - Smoothing Linear Filters
  - Order-Statistics Filters

Smoothing Linear Filters

- Output is simply the average of the pixels contained in the neighborhood of the filter mask.
- Also Called **Averaging filters** or **Lowpass filters**.
- Replacing the value of every pixel in an image by the average of the gray levels in the neighborhood will reduce the "sharp" transitions in gray levels.
- Sharp transitions
  - **random noise in the image**
  - **edges of objects in the image**

Thus, smoothing can reduce noises (desirable) and blur edges (undesirable)

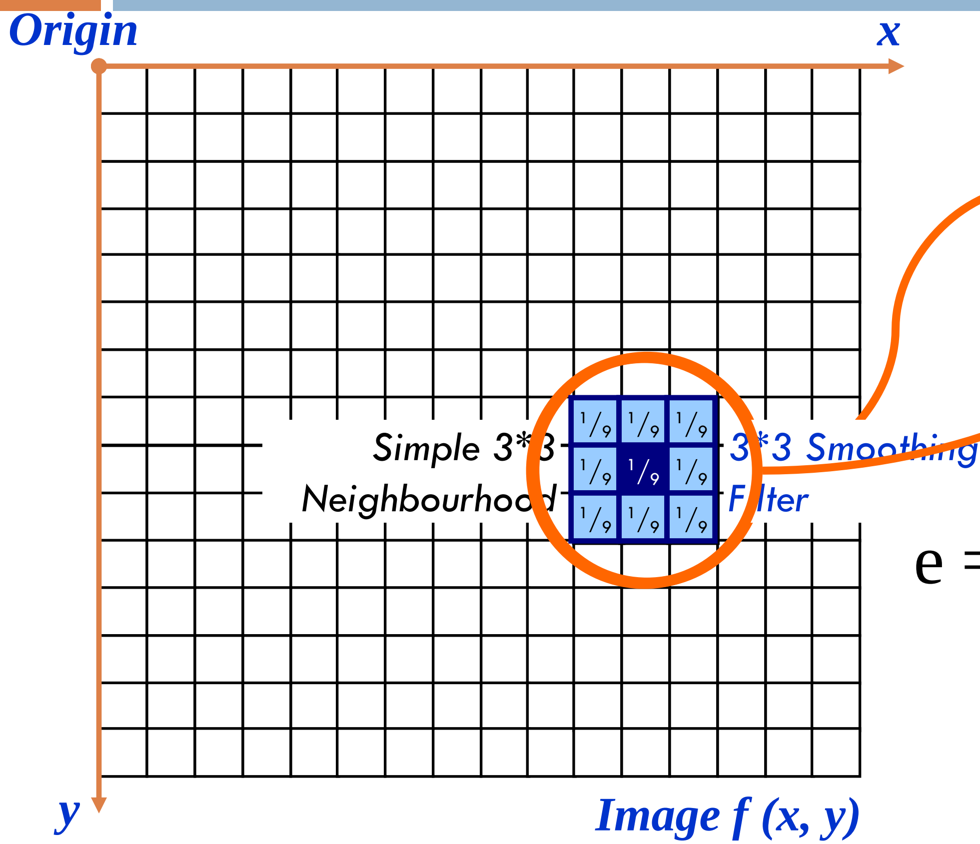
Smoothing Linear Filters …

- One of the simplest spatial filtering operations we can perform is a smoothing operation
  - Simply average all of the pixels in a neighbourhood around a central value
  - Especially useful in removing noise from images
  - Also useful for highlighting gross detail

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$

Simple averaging filter

Smoothing Linear Filters …



104	100	108
99	106	98
95	90	85

Original Image Pixels

$*$

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$

Filter

$$
\begin{aligned}
e = {} & {}^1/_9 * 106 + \\
& {}^1/_9 * 104 + {}^1/_9 * 100 + {}^1/_9 * 108 + \\
& {}^1/_9 * 99 + {}^1/_9 * 98 + \\
& {}^1/_9 * 95 + {}^1/_9 * 90 + {}^1/_9 * 85 \\
= {} & 98.3333
\end{aligned}
$$

The above is repeated for every pixel in the original image to generate the smoothed image

3x3 Smoothing Linear Filters

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Standard Average

$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

weighted average

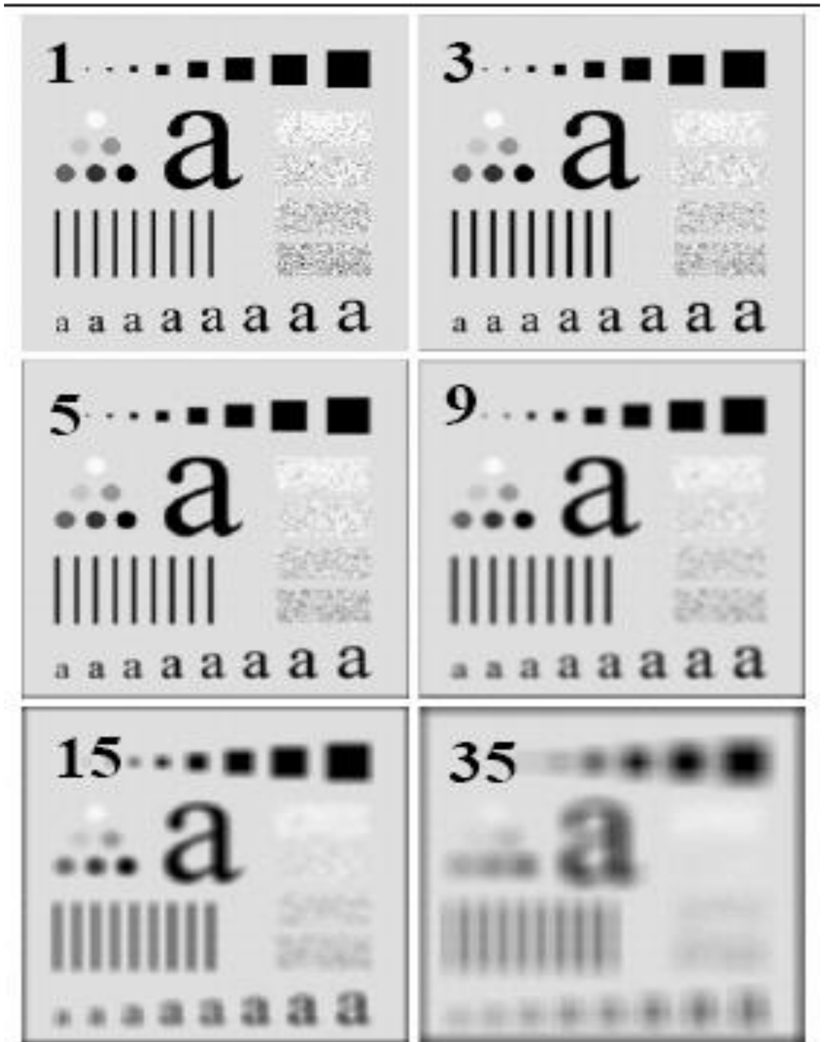
the center is the most important and other pixels are inversely weighted as a function of their distance from the center of the mask

Smoothing Linear Filters

- The general implementation for filtering an M x N image with a weighted averaging filter of size m x n is given by the expression

$$g(x,y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

Smoothing Linear Filters: Example



- Original image of 500 x 500 pixels
- Results of smoothing with square averaging filter masks of size m=3, 5, 9,15 and 35 respectively
- Note:
  - big mask is used to eliminate small objects from an image.
  - the size of the mask establishes the relative size of the objects that will be blended with the background.

Weighted Smoothing Filters

- More effective smoothing filters can be generated by allowing different pixels in the neighbourhood different weights in the averaging function
  - Pixels closer to the central pixel are more important
  - Often referred to as a *weighted averaging*

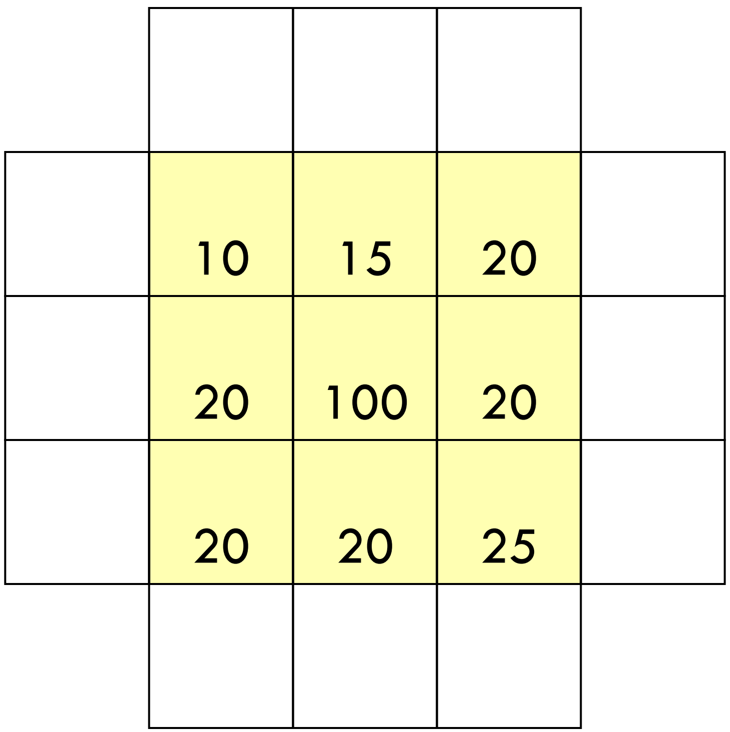
$^{1}/_{16}$	$^{2}/_{16}$	$^{1}/_{16}$
$^{2}/_{16}$	$^{4}/_{16}$	$^{2}/_{16}$
$^{1}/_{16}$	$^{2}/_{16}$	$^{1}/_{16}$

Weighted averaging filter

Order-Statistics Filters

- Order-statistics filters are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter,
- Example
  - median filter : $R = \text{median}\{z_k \mid k = 1,2,\ldots,n \times n\}$
  - max filter : $R = \max\{z_k \mid k = 1,2,\ldots,n \times n\}$
  - min filter : $R = \min\{z_k \mid k = 1,2,\ldots,n \times n\}$
- note: n x n is the size of the mask

Process of Median filter



10, 15, 20, 20, 20, 20, 20, 25, 100

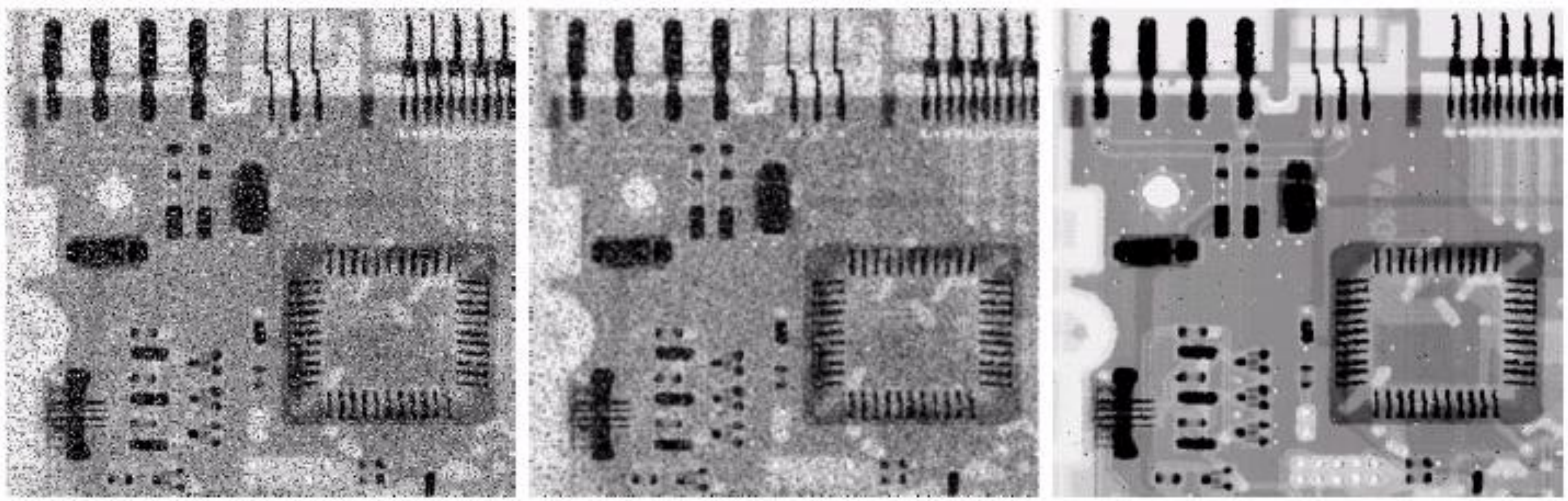
5^{th}

- Crop region of neighborhood
- Sort the values of the pixel in the region
- In the M x N mask the median is (M x N div 2) +1

Median Filtering

- Median filtering is particularly effective in the presence of impulse noise (salt and pepper noise).
- Unlike average filtering, median filtering does not blur too much image details.
- Advantages:
  - Removes impulsive noise
  - Preserves edges
- Disadvantages:
  - performance poor when # of noise pixels in the window is greater than 1/2 # in the window
  - performs poorly with Gaussian noise

Median Filtering: Example



a) X-ray image of circuit board corrupted by salt-pepper noise
b) Noise reduction with a 3 x 3 averaging mask
c) Noise reduction with a 3 x 3 median filter

Median Filtering: Example

Noise reduction with a 3 x 3 median filter

Sharpening Spatial Filters

- The principal objective of sharpening is to **highlight fine detail in an image** or **to enhance detail that has been blurred**, either in error or as a natural effect of a particular method of image acquisition.
- First and second order derivatives are commonly used for sharpening:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

Blurring vs. Sharpening

- As we know that blurring can be done in spatial domain by **pixel averaging** in a neighbors

- since **averaging is analogous to integration**

- Therefore, logically it can be concluded that the sharpening must be accomplished by **spatial differentiation.**

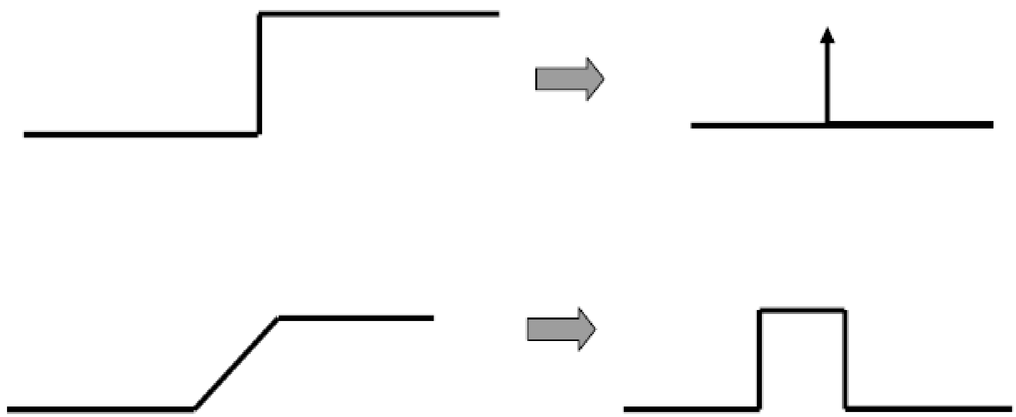
Derivative operator

- The strength of the response of a derivative operator is proportional to the degree of discontinuity of the image at the point at which the operator is applied.
- thus, image differentiation
  - enhances edges and other discontinuities (noise)
  - deemphasizes area with slowly varying gray-level values.

First-order derivative

- a basic definition of the first-order derivative of a one-dimensional function f(x) is the difference

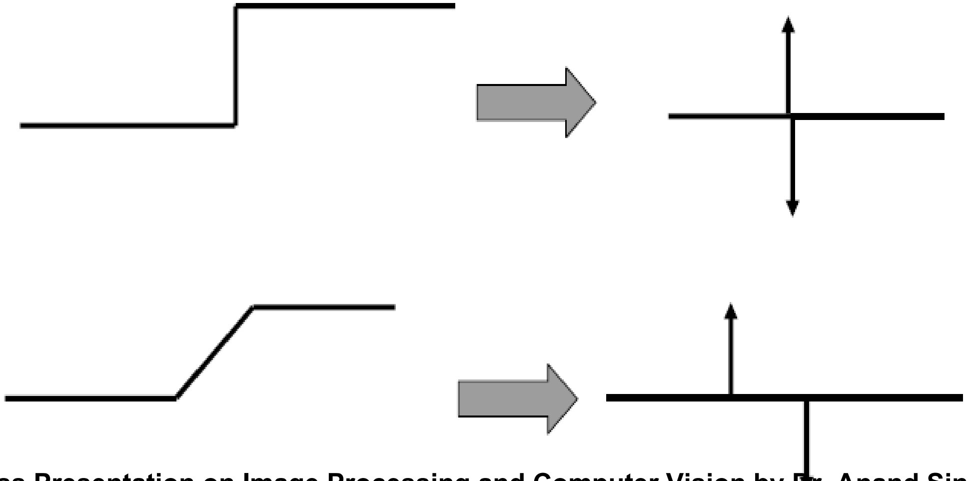
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$



Second-order derivative

- similarly, we define the second-order derivative of a one-dimensional function f(x) is the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$



Response of First and Second order derivatives

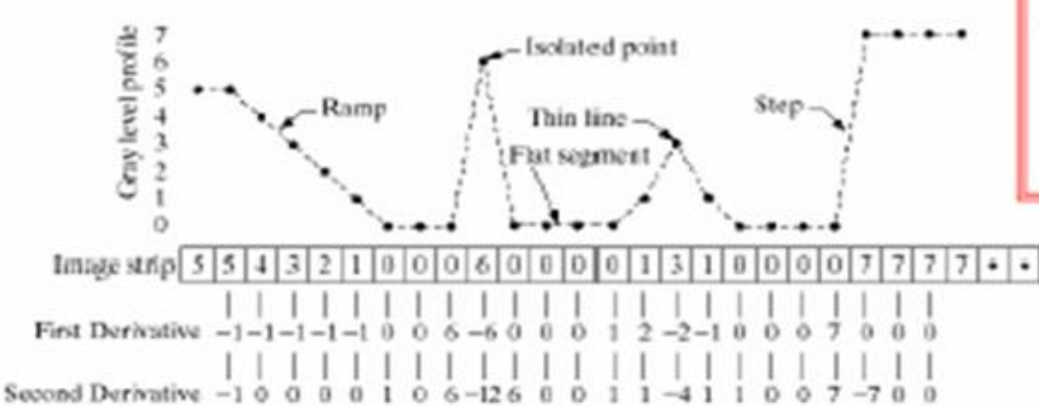
Response of first order derivative is:

- zero in flat segments (area of constant grey values)
- Non zero at the onset of a grey level step or ramp
- Non zero along ramps

Response of second order derivative is:

- Zero in flat areas
- Non zero at the onset of a grey level step or ramp
- Zero along ramps of constant slope

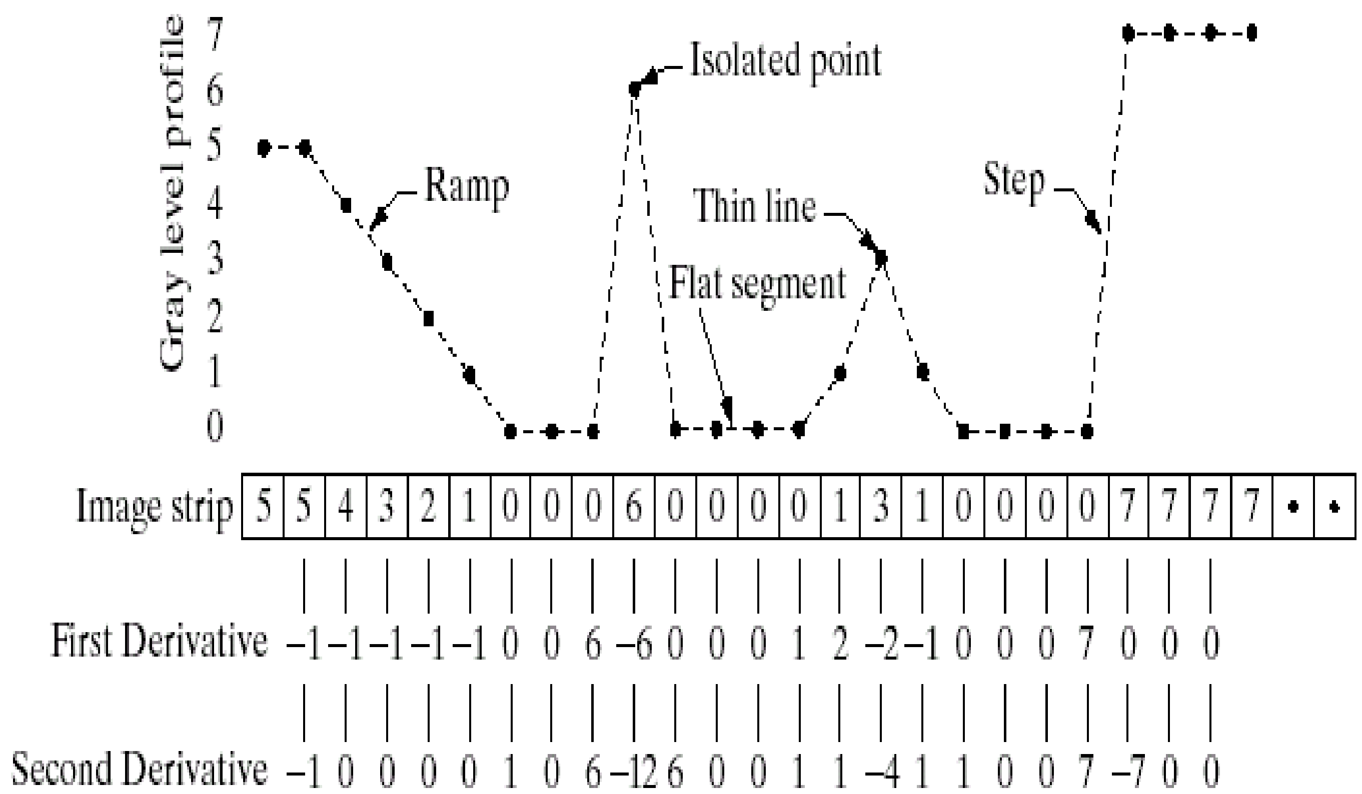
First and Second Order Derivatives: Example



$$\boxed{\frac{\partial f}{\partial x} = f_x(x) = f(x+1) - f(x)}$$

$$\boxed{\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= f_x(x) - f_x(x-1) \\ &= \big(f(x+1) - f(x)\big) - \big(f(x) - f(x-1)\big) \\ &= f(x+1) + f(x-1) - 2 \cdot f(x)\end{aligned}}$$

First and Second Order Derivatives: Example …



First and Second Order Derivatives …

- **Analysis**

1) The 1st-order derivative is nonzero along the entire ramp, while the 2nd-order derivative is nonzero only at the onset and end of the ramp.
2) Edges in an image represent this type (ramp) of transition. Therefore,
   1st make thick edge and 2nd make thin, much finer edges
3) The response at and around the point is much stronger for the 2nd- than for the 1st-order derivative.

1st and 2nd Derivative Comparison

- ✓ First-order derivatives generally produce thicker edges in an images.
- ✓ Second-order derivatives have a stronger response to fine detail (e.g. thin lines or isolated points).
- ✓ First-order derivatives generally have a stronger response to a gray-level step.
- ✓ Second-order derivatives produce a double response at step changes in gray level

Derivative Operator …

$f(x-1,y)$	$f(x,y)$	$f(x+1,y)$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$f(x,y-1)$
$f(x,y)$
$f(x,y+1)$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

The Laplacian
(Use of Second derivative for Enhancement)

- The filter is expected to be isotropic: response of the filter is independent of the direction of discontinuities in an image.
- Simplest 2-D isotropic second order derivative is the Laplacian:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

2-Dimensional Laplacian

The digital implementation of the 2-Dimensional Laplacian is obtained by summing 2 components

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial x^2}$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

0	1	0
1	-4	1
0	1	0

Laplacian Masks

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad 90^\circ \ isotropic$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad 45^\circ \ isotropic$$

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

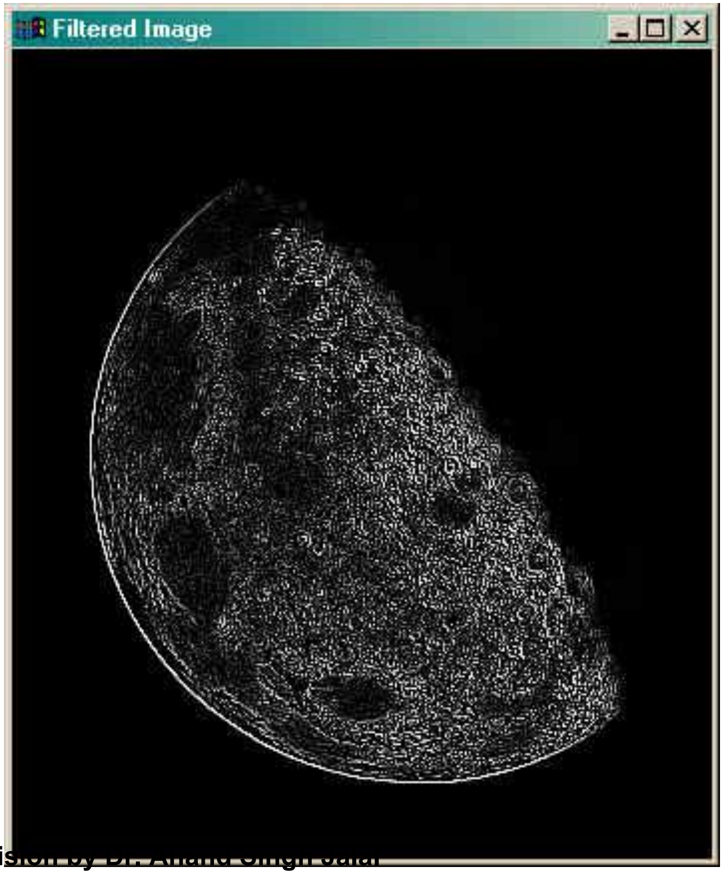
−1	−1	−1
−1	8	−1
−1	−1	−1

Effect of Laplacian Operator

- **as it is a derivative operator,**
  - it highlights gray-level discontinuities in an image
  - it deemphasizes regions with slowly varying gray levels
- **tends to produce images that have**
  - grayish edge lines and other discontinuities, all superimposed on a dark,
  - featureless background.

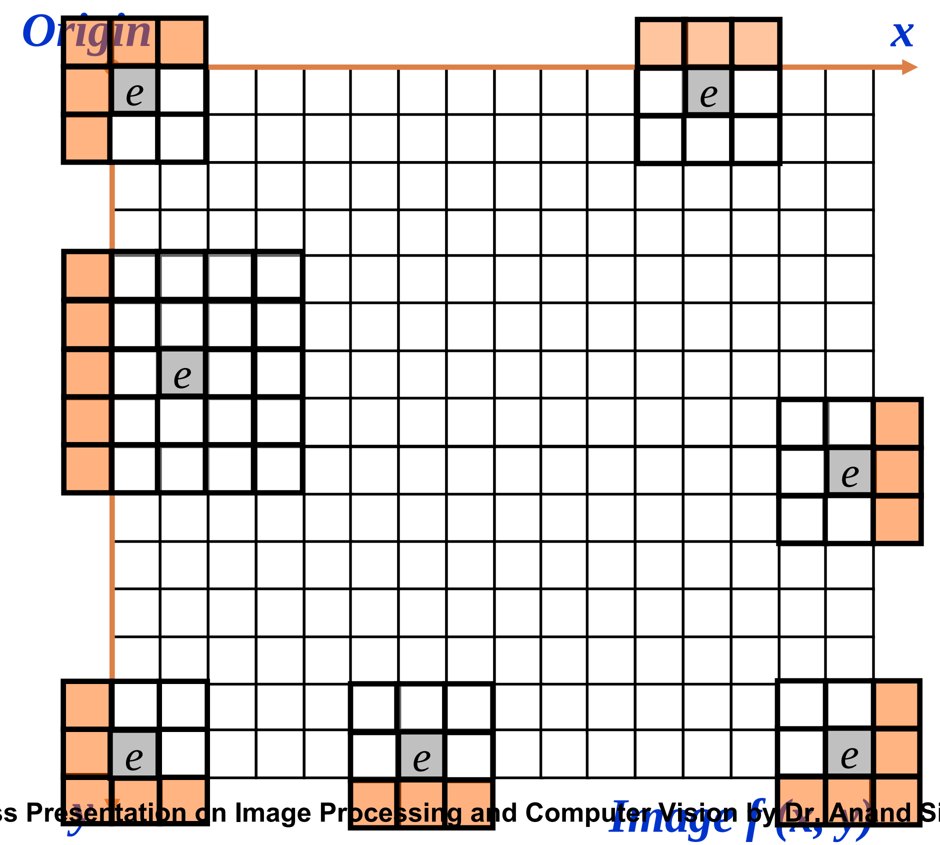
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Image of the north pole of the moon



Strange Things Happen At The Edges!

At the edges of an image we are missing pixels to form a neighbourhood



Strange Things Happen At The Edges …

- There are a few approaches to dealing with missing edge pixels:
  - Omit missing pixels
    - Only works with some filters
    - Can add extra code and slow down processing
  - Pad the image
    - Typically with either all white or all black pixels
  - Replicate border pixels

Using Fuzzy techniques for intensity transformations and spatial filtering

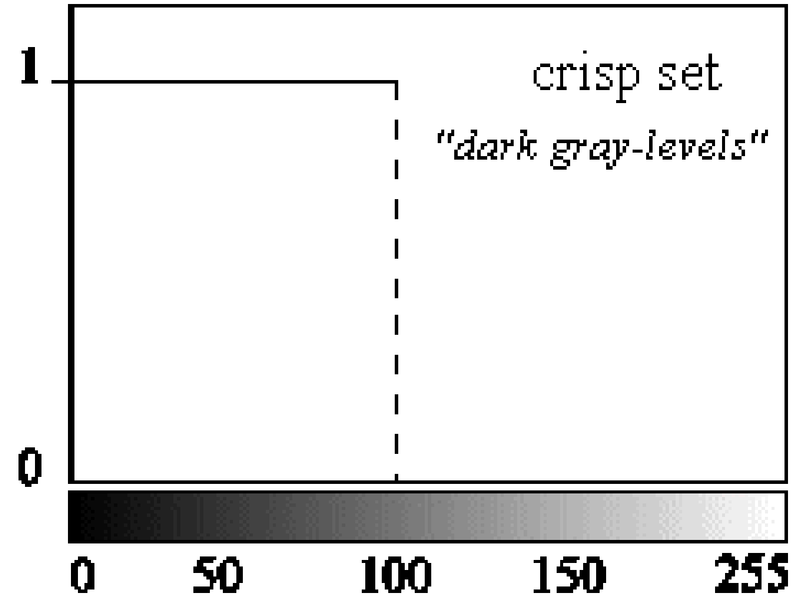
What is Fuzzy Set Theory?

- Fuzzy set theory is the extension of conventional (crisp) set theory.
- It handles the concept of partial truth (values between 1 (completely true) and 0 (completely false)).
- It was introduced by **Prof. Lotfi A. Zadeh** of UC/Berkeley in 1965 as a means to model the vagueness and ambiguity in complex systems.

Fuzzy Set …

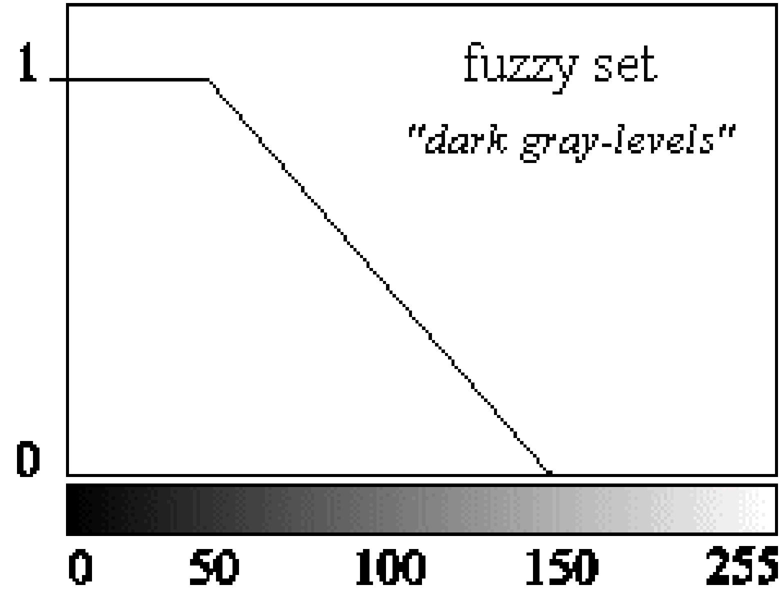
- The idea of fuzzy sets is simple and natural. For instance, we want to define a set of gray levels that share the property dark.

In classical set theory, we have to determine a threshold, say the gray level 100. All gray levels between 0 and 100 are element of this set, the others do not belong to the set



Fuzzy Set …

- But the darkness is a matter of degree.
- So, a fuzzy set can model this property much better. To define this set, we also need two thresholds, say gray levels 50 and 150.
- All gray levels that are less than 50 are the full member of the set,
- All gray levels that are greater than 150 are not the member of the set.
- The gray levels between 50 and 150, however, have a partial membership in the set

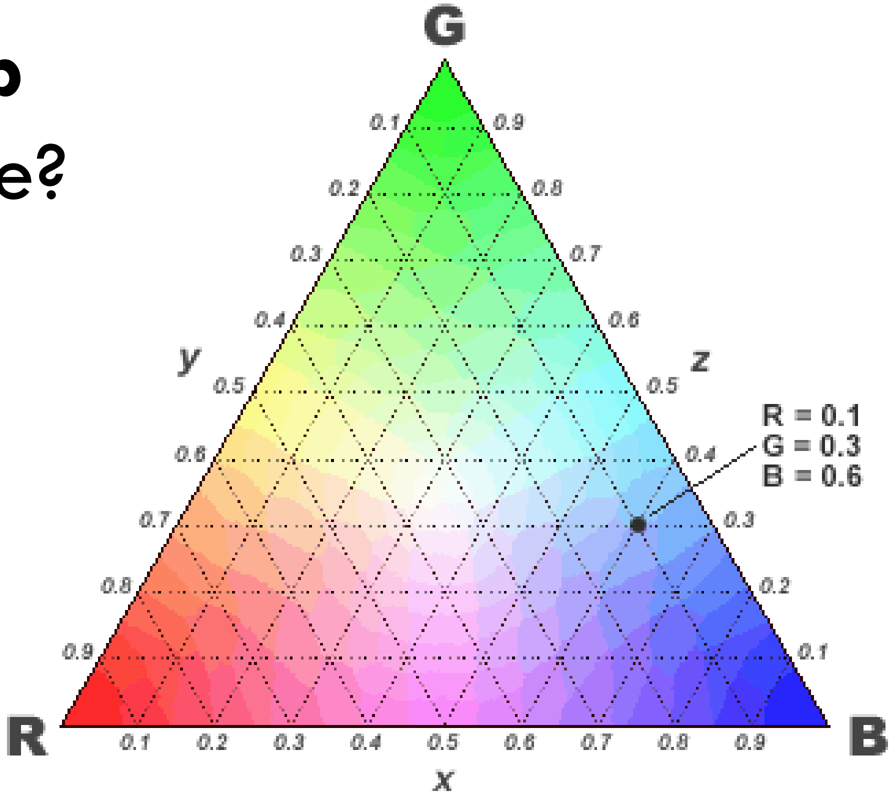


What is Fuzzy Logic?

- Definition of fuzzy
  - Fuzzy – “not clear, or imprecise or blurred”
- Definition of fuzzy logic
  - A form of knowledge representation suitable for notions that cannot be defined precisely, but which depend upon their contexts.

Fuzzy Logic …

Can we give a **crisp** definition to light blue?



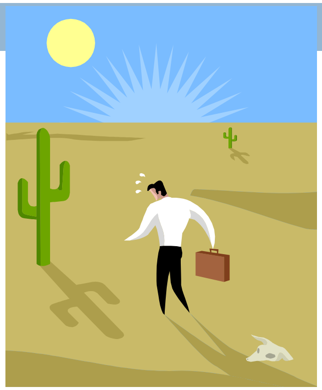
Probability vs Fuzzy logic

Probability, which ranges from 0.0 to 1.0, is used to gauge the likelihood of some particular, well-defined state, under conditions of ignorance or chance.

For instance: a fair coin has a 50% probability of coming up heads. Note:

1. we do not know the outcome ahead of time, due to chance and
2. there are only two, clearly-defined states: "heads" and "tails".

Suppose a person is dying of thirst in the desert. He finds two bottles of water.



Bottle's label says that it has a 0.9 membership in the class of fluids known as non-poisonous drinking water.

Bottle's label states that it has a 0.9 probability of being pure drinking water (i.e. only a 0.1 probability of being poison.)

Which bottle should he choose?

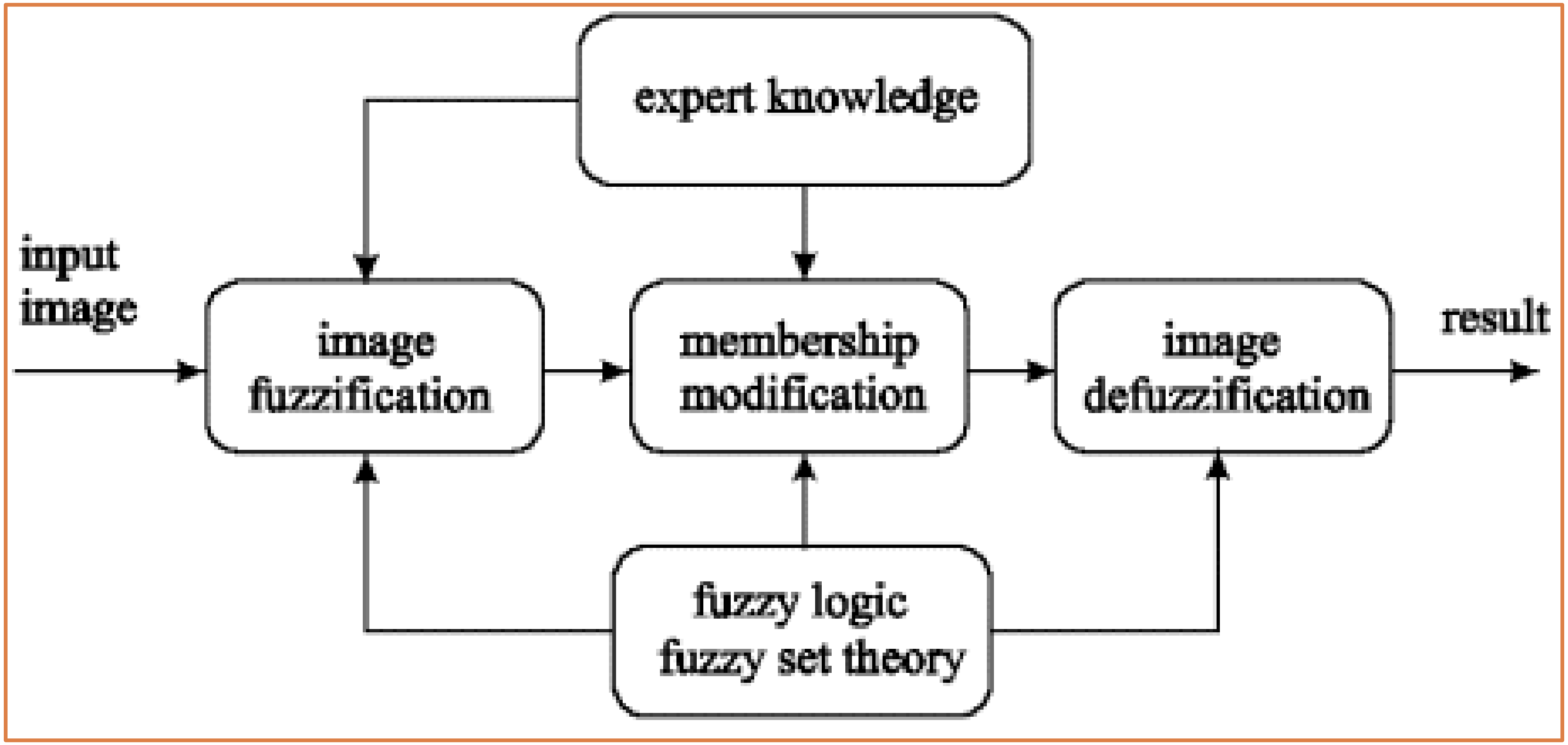
Fuzzy Image Processing

It is a collection of different fuzzy approaches to image processing

Fuzzy image processing is the collection of all approaches that understand, represent and process the images, their segments and features as fuzzy sets. The representation and processing depend on the selected fuzzy technique and on the problem to be solved.

(From: Tizhoosh, Fuzzy Image Processing, Springer, 1997)

Fuzzy image processing has three main stages: image fuzzification, modification of membership values, and, if necessary, image defuzzification

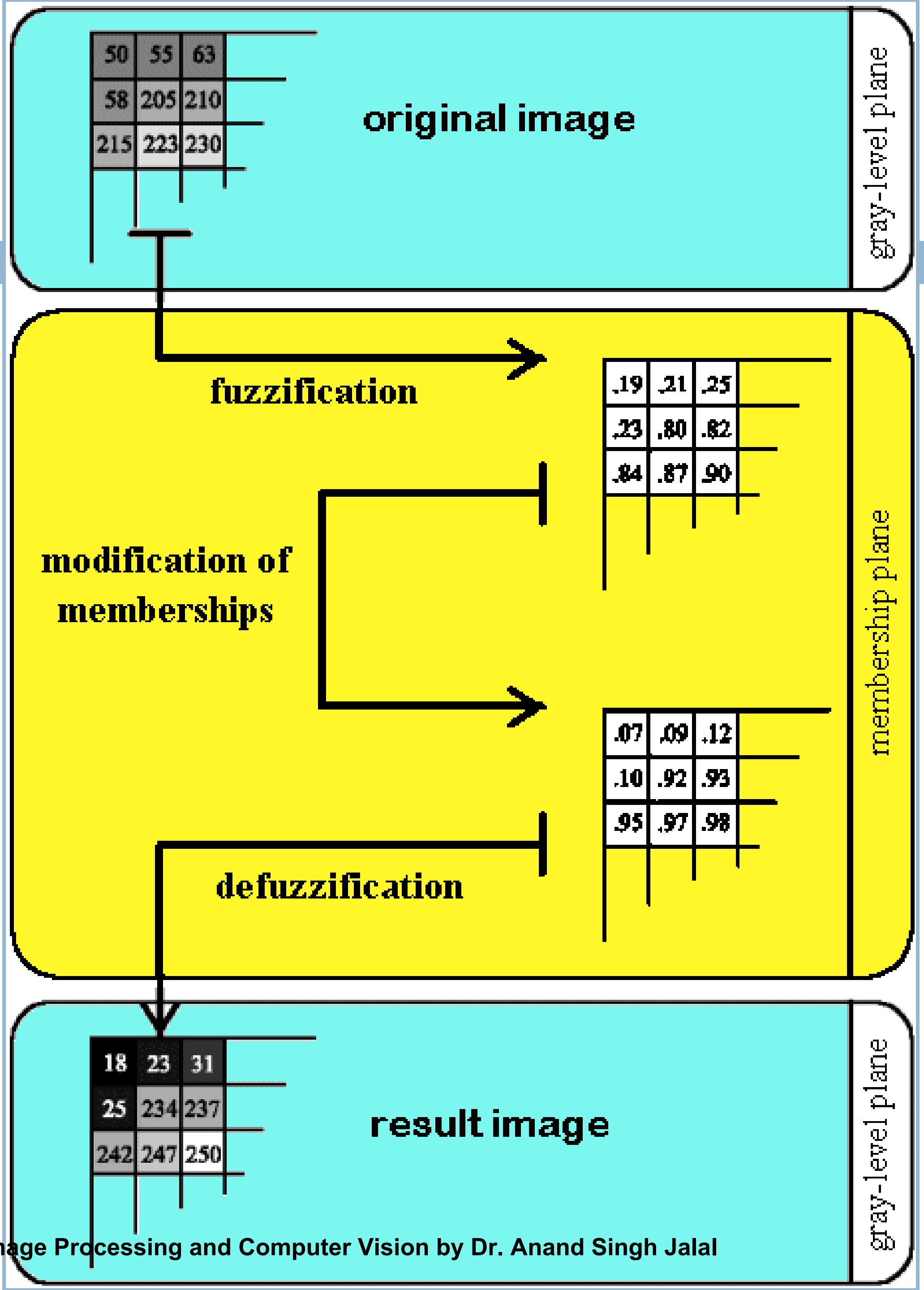


The general structure of fuzzy image processing

Steps of fuzzy image processing

After the image data are transformed from gray-level plane to the membership plane (fuzzification), appropriate fuzzy techniques modify the membership values.

This can be a fuzzy clustering, a fuzzy rule-based approach, a fuzzy integration approach and so on



Advantage of Fuzzy Image Processing

- Fuzzy techniques are powerful tools for knowledge representation and processing.
- Fuzzy techniques can manage the vagueness and ambiguity efficiently

In many image processing applications, we have to use expert knowledge to overcome the difficulties (e.g. object recognition, scene analysis).

Fuzzy set theory and fuzzy logic offer us powerful tools to represent and process human knowledge in form of fuzzy if-then rules.

Any Questions ?

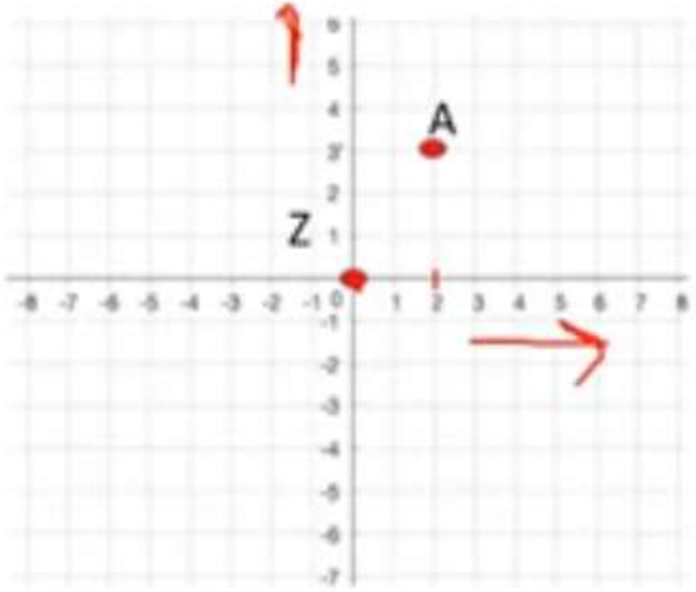
Fourier Transform

Jean Baptiste Joseph Fourier (1768-1830)

- **Had crazy idea (1807):**
- **Any** periodic function can be rewritten as a weighted sum of Sines and Cosines of different frequencies.
- **Don't believe it?**
  - Neither did Lagrange, Laplace, Poisson and other big wigs
  - Not translated into English until 1878!
- **But it's true!**
  - called Fourier Series
  - Possibly the greatest tool used in Engineering

What is Transform & Why is it Required ?

- Informally, a **transformation** is any way of changing something.



Plot A (2,3) i. e. x=2 & Y= 3

$Plot\ A(\infty,\infty)$

$$Z=\frac{1}{A}$$

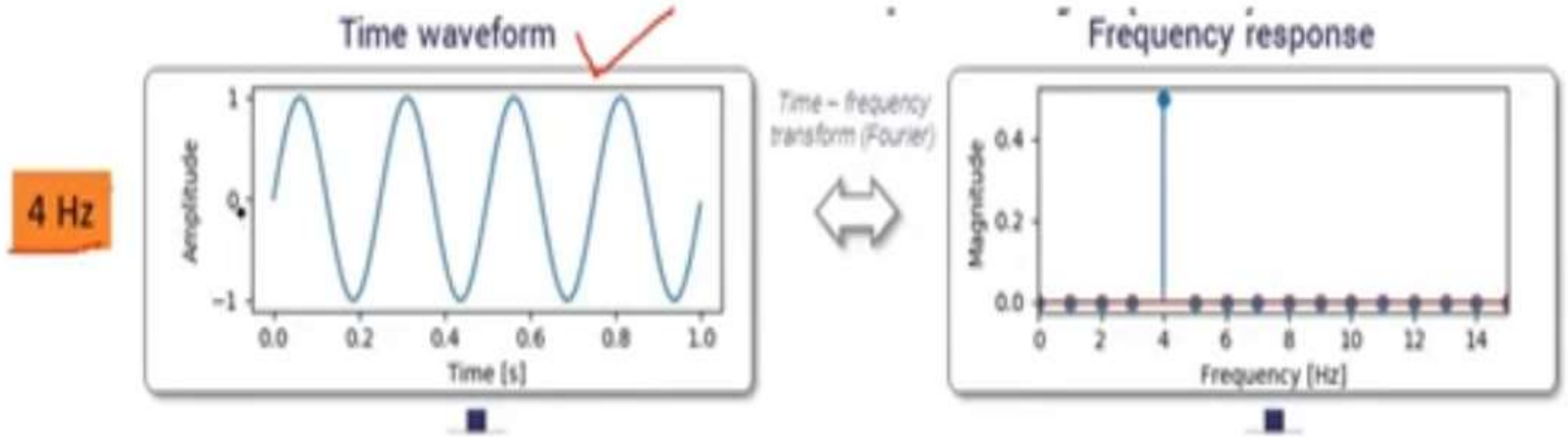
In mathematics, transformations are often used to move an object from a place where it is hard to work with it to a place where it is simpler.
For some people moving the object is equivalent to choosing a new way to view the object.

Why Do Transforms?

- **Fast computation**
  - E.g., convolution vs. multiplication for filter with wide support
- **Conceptual insights for various image processing**
  - E.g., spatial frequency info. (smooth, moderate change, fast change, etc.)
- **Obtain transformed data as measurement**
  - E.g., blurred images, radiology images (medical and astrophysics)
  - Often need inverse transform
  - May need to get assistance from other transforms
- **For efficient storage and transmission**
  - Pick a few "representatives" (basis)
  - Just store/send the "contribution" from each basis

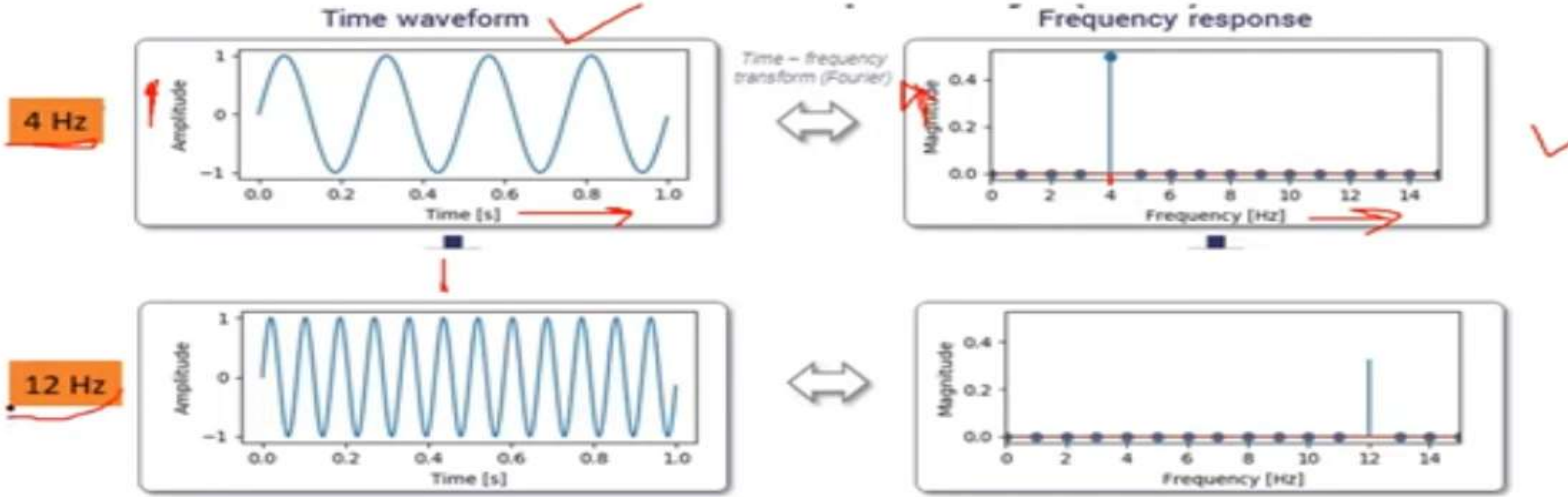
What is Fourier Transform ?

Convert time domain signal into frequency domain signal

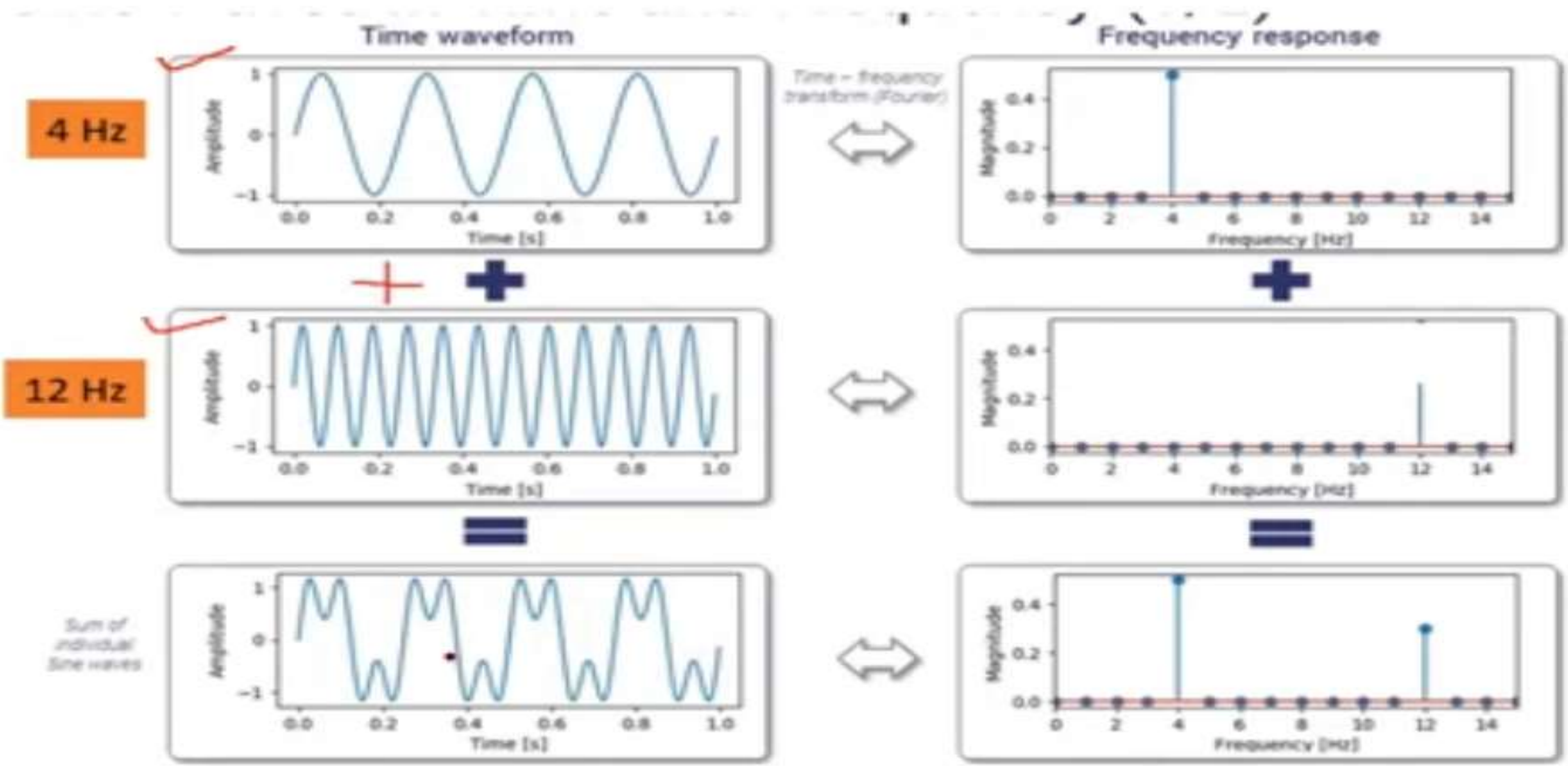


What is Fourier Transform ?

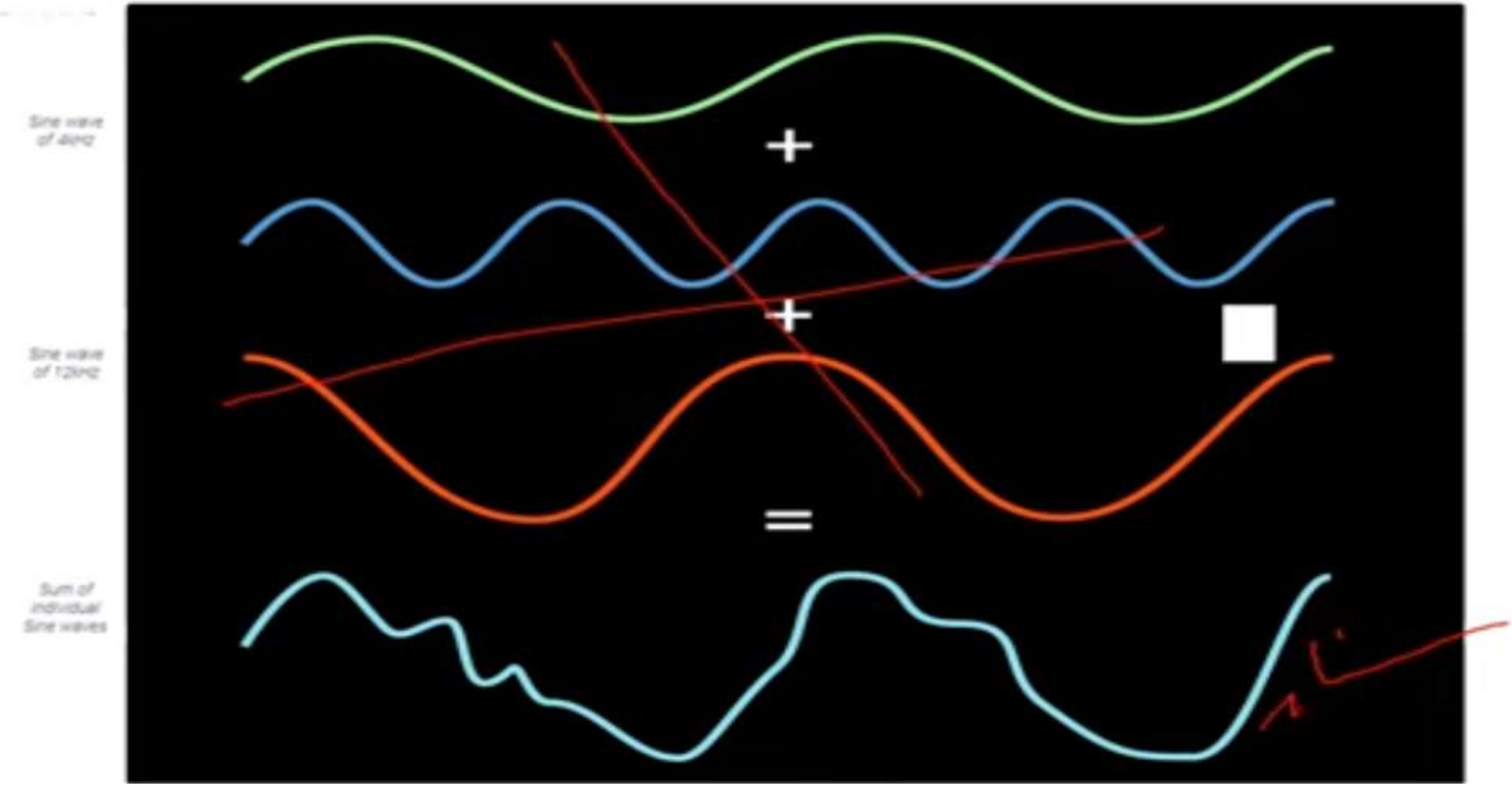
Convert time domain signal into frequency domain signal



Time Domain & Frequency Domain

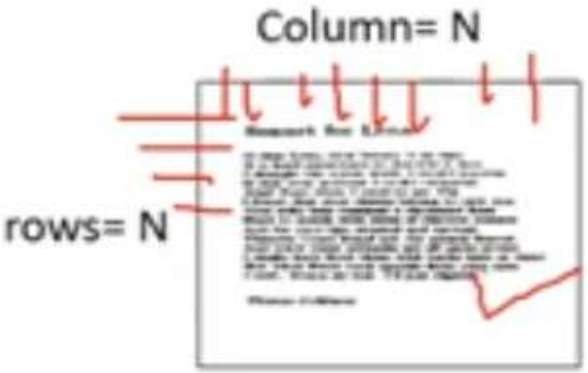


Time Domain & Frequency Domain



Fourier Transform of an Image

- The Fourier Transform is an important image processing tool which is used to decompose an image into its sine and cosine components.
- As we are only concerned with digital images, we will restrict this discussion to the Discrete Fourier Transform (DFT).
- For a square image of size N×N, the two-dimensional DFT is given by:



$$\boxed{F(u,v)} = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-i2\pi\left(\frac{xu}{N}+\frac{vy}{N}\right)}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

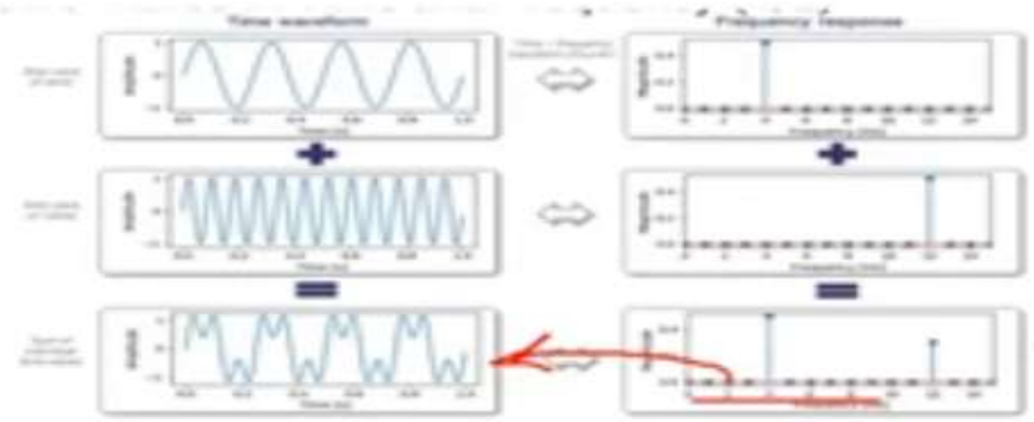
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

where f(x,y) is the image in the spatial domain

Exponential term is the basis function. The basis functions are sine and cosine waves with increasing frequencies,

Inverse Fourier Transform

Convert signal from frequency domain to time domain



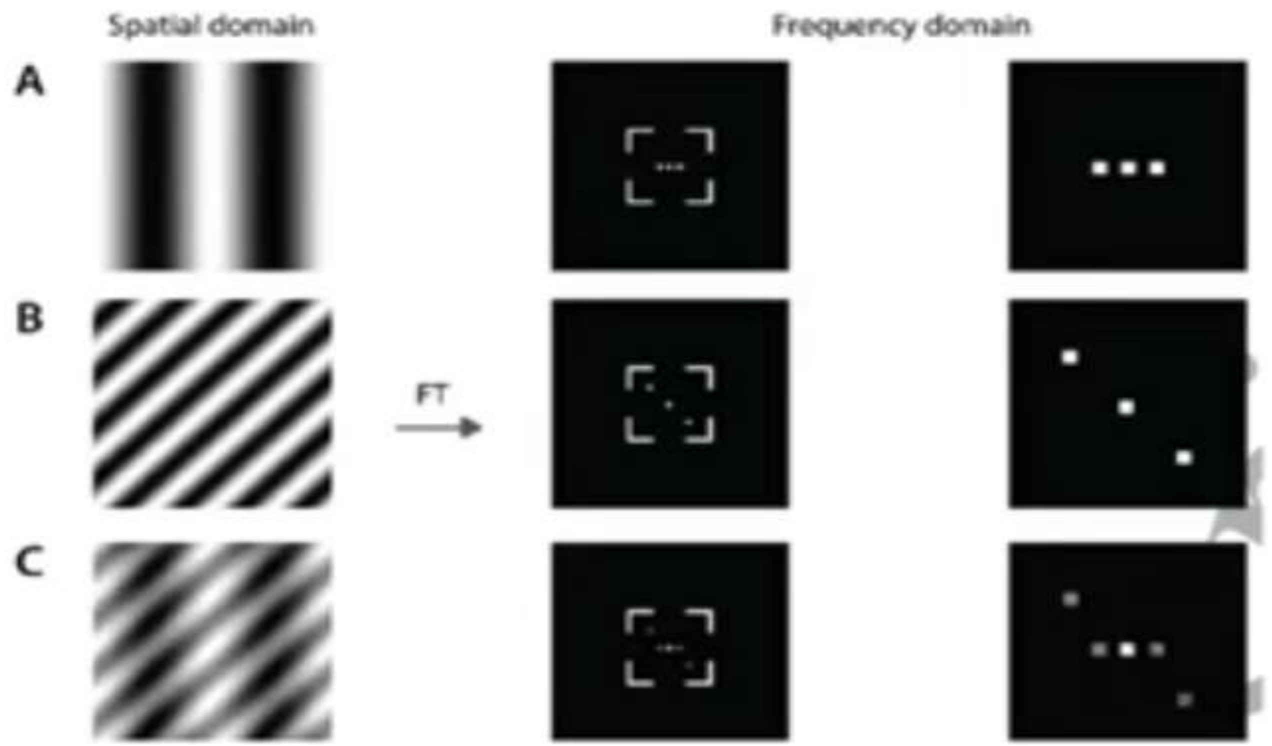
Fourier image can be re-transformed to the spatial domain.

- The inverse Fourier transform is given by

$$f(x,y) = \frac{1}{N^2} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi\left(\frac{xu}{N}+\frac{vy}{N}\right)}$$

What information Fourier Transform of an Image Give

The response of the Fourier Transform to periodic patterns in the spatial domain images can be seen very easily in the following artificial images



Solved Examples

Example1. Computer 2D DFT of 4 x 4 gray scale image

$$f(x, y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Example1. Computer 2D DFT of 4 x 4 gray scale image

$$f(x, y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Solution→ $\boxed{F(u, v)}$ = kernel $\cancel{x}$ f (x,y) x [kernel]^T

DFT basis function(kernel) for N = 4 is

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}.$$

F (u, v) =kernel x f (x,y) x [kernel]^T

$$F(u,v) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Solved Examples

Example1. A two dimensional DFT can be obtained using 1- dimensional –DFT algorithm twice, explain.
OR
Find 2–D DFT using separability property

Lets Understand this with following example

$$\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

Separability property

2D **DFT** can be obtained in two steps by successive application of 1D **DFT**.

F(u,v) can be obtained by applying 1D-**DFT** along the rows and then along the columns.

0	1	2	1
1	2	3	2
2	3	4	3
1	2	3	4

Calculate 1–D DFT along rows.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

→DFT of 1st Row

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

DFT of 2nd Row

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

DFT of 3rd Row

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

DFT of 4th Row

$$\begin{bmatrix} 4 & -2 & 0 & -2 \\ 8 & -2 & 0 & -2 \\ 12 & -2 & 0 & -2 \\ 8 & -2 & 0 & -2 \end{bmatrix}$$

Calculate 1–D DFT along columns.

$$DFT\ of\ 1st\ Column = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 12 \\ 8 \end{bmatrix} = \begin{bmatrix} 32 \\ -8 \\ 0 \\ -8 \end{bmatrix}$$

$$DFT\ of\ 2nd Column = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} -2 \\ -2 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$DFT\ of\ 3rd\ Column = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$DFT\ of 4th\ Column = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} -2 \\ -2 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution is

$$\begin{bmatrix} 32 & -8 & 0 & -8 \\ -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -8 & 0 & 0 & 0 \end{bmatrix}$$

Properties of DFT

- **The Separability Property**
- A 2-D DFT can be separated into two 1-D DFT

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

Properties of DFT …

The Separability Property

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

We can rearrange the terms as follows

$$F(u,v) = \frac{1}{M}\sum_{x=0}^{M-1} e^{-j2\pi\frac{ux}{M}}\ \frac{1}{N}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi\frac{vy}{N}}$$

$$F(u,v) = \frac{1}{M}\sum_{x=0}^{M-1} F(x,v)\, e^{-j2\pi\frac{ux}{M}}$$