Linear Differential Eqⁿ of nth order

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \text{-----} \; a_n y = Q$$

where $a_1, a_2 \text{---} a_n$ are constants.

$$(D^n + a_1 D^{n-1} + \text{----} \; a_n) y = Q$$

Nature of roots and corresponding C.F. →

Case I. when roots are real (rational) and distinct.

$$C.F. = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \text{---}$$

eg. $\frac{d^2 y}{dx^2} - 4y = 0$

$$(D^2 - 4) y = 0$$

A.E. $m^2 - 4 = 0$

$$m = +2, -2$$

$C.F. = c_1 e^{m_1 x} + c_2 e^{m_2 x}$, $P.I. = 0$

$\because y = C.F. + P.I.$

$$y = c_1 e^{2x} + c_2 e^{-2x}$$

eg. $(D^2 - D) y = 0$

A.E. $m^2 - m = 0$

$m(m-1) = 0 \Rightarrow m = 0, 1$

$C.F. = c_1 e^{m_1 x} + c_2 e^{m_2 x} \Rightarrow c_1 + c_2 e^x$

$y = C.F. + P.I.$, $P.I. = 0$

$$y = c_1 + c_2 e^x$$

Case II When roots are equal.

(i) when $m_1 = m_2$

$$C.F. = (C_1 + C_2 x) e^{m_1 x} + C_3 e^{m_3 x} + \cdots$$

(ii) when $m_1 = m_2 = m_3$

$$C.F. = (C_1 + C_2 x + C_3 x^2) e^{m_1 x} + C_4 e^{m_4 x} + \cdots$$

eg. $(D^2 - 4D + 4) y = 0$

A.E. $m^2 - 4m + 4 = 0$

$m = 2, 2$

$C.F. = (C_1 + C_2 x) e^{m_1 x}$, $P.I. = 0$

$y = (C_1 + C_2 x) e^{2x}$

eg. $(D^2 - 8D + 16) y = 0$

A.E. $m^2 - 8m + 16 = 0$

$m = 4, 4$

$C.F. = (C_1 + C_2 x) e^{4x}$

$y = C.F. + P.I.$

$= (C_1 + C_2 x) e^{4x}$

eg. $(D^2 - 6D + 9) y = 0$

eg. $(D^3 - D^2) y = 0$

Case III. when roots are imaginary →

$$m_1, m_2 = \alpha \pm i\beta$$

$$C.F. = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$$

eg. $(D^2 + 4)y = 0$

A.E. $m^2 + 4 = 0$

$m^2 = -4$

$m = \pm 2i$

$C.F. = (C_1 \cos 2x + C_2 \sin 2x)$

$y = C.F. + P.I.$

$= C_1 \cos 2x + C_2 \sin 2x$

eg. $(D^2 + 4D + 5)y = 0$

A.E. $m^2 + 4m + 5 = 0$

$$m = \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm i$$

$\alpha = -2, \quad \beta = 1$

$C.F. = e^{-2x}(C_1 \cos x + C_2 \sin x)$

$y = C.F. + P.I.$

eg. $(D^2 + 1)y = 0$

eg. $(D^2 + 9)y = 0$

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Case IV when roots are irrational

$m_1 = a + \sqrt{b}$, $m_2 = a - \sqrt{b}$

C.F. = $e^{ax}\left[c_1 \cosh\sqrt{b}\,x + c_2 \sinh\sqrt{b}\,x\right]$

eg. $(D^2 - 4D + 1)y = 0$

A.E. $m^2 - 4m + 1 = 0$

$m = 2 \pm \sqrt{3}$

C.F. = $e^{2x}\left[c_1 \cosh\sqrt{3}\,x + c_2 \sinh\sqrt{3}\,x\right]$

eg. $(D^3 - 5D^2 + 5D - 1)y = 0$

A.E. $m^3 - 5m^2 + 5m - 1 = 0$

$(m-1)(m^2 - 4m + 1) = 0$

$m = 1$, $m = \dfrac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

C.F. = $c_1 e^{x} + e^{2x}\left[c_2 \cosh\sqrt{3}\,x + c_3 \sinh\sqrt{3}\,x\right]$

eg. 1 $(D^2 + 6D + 9)y = 0$, $m = 3, 3$

2. $(D^3 + 2D^2 - D - 2)y = 0$

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eg.1 $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

$(D^2 - 3y + 2)y = 0$

A.E. $m^2 - 3m + 2 = 0$

$m = 1, 2$

C.F. $c_1 e^{x} + c_2 e^{2x}$

$y = c_1 e^{x} + c_2 e^{2x}$

eg.2 $(D^2 + \mu^2)y = 0$

A.E. $m^2 + \mu^2 = 0$

$m = \pm \mu i$

$y = c_1 \cos \mu x + c_2 \sin \mu x$

eg.3 $(D^2 - 8D + 16)y = 0$

A.E. $m^2 - 8m + 16 = 0$

$m = 4, 4$

$y = (c_1 + c_2 x)e^{4x}$

eg. $(D^2 + D - 30)y = 0$

A.E. $m^2 + m - 30 = 0$

$m = 5, -6$

$y = c_1 e^{5x} + c_2 e^{-6x}$

2.6 METHOD FOR FINDING THE COMPLEMENTARY FUNCTION

In finding the complementary function, R.H.S. of the given equation is replaced by zero.

Consider the differential equation

$$(D^n + a_1 D^{n-1} + a_2 D^{n-2} + \ldots + a_n D^0)\, y = 0 \qquad \ldots(1)$$

Where $a_1, a_2, \ldots a_n$ are all constants.

Let $\quad y = C_1 e^{mx}$ be its solution

$$Dy = C_1 m e^{mx}, \qquad D^2 y = C_1 m^2 e^{mx} \ldots \qquad D^n y = C_1 m^n e^{mx} \ldots$$

Putting these values in (1), we get

$$C_1\,(m^n + a_1 m^{n-1} + a_2 m^{n-2} + \ldots + a_n)\, e^{mx} = 0$$

Its auxiliary equation is $m^n + a_1 m^{n-1} + a_2 m^{n-2} + \ldots + a_n = 0 \qquad \ldots(2)$

This is Plynomial equation of degree n. So it has n roots i.e.

$$m_1,\quad m_2,\quad m_3 \ \ldots\ m_n$$

The complementary function of equation (1) depends upon the nature of the roots. The six nature of roots are as follows:

In brief Nature of roots and corresponding C.F.

Sl	*Nature of Roots of A.E.*	*Roots*	*C.F.*
1.	Real (ational) and Distinct roots	m_1, m_2, m_3	$C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{m_3 x}$
2.	Repeated roots	$m_1 = m_2$, $m_1 = m_2 = m_3$	$(C_1 + C_2 x)\ e^{m_1 x}$ $(C_1 + C_2 x + C_3 x^2)\ e^{m_1 x}$
3.	Complex roots	$m_1 = \alpha + i\beta$ $m_2 = \alpha - i\beta$	$e^{\alpha x}\,[C_1 \cos \beta x + C_2 \sin \beta x]$
4.	Repeated Complex roots	$m_1 = m_2 = \alpha + i\beta$ $m_3 = m_4 = \alpha - i\beta$	$e^{\alpha x}\,[(C_1 + C_2 x) \cos \beta x + (C_3 + C_4 x) \sin \beta x]$
5.	Irrational roots	$m_1 = a + \sqrt{b}$ $m_2 = a - \sqrt{b}$	$e^{ax}\left[C_1 \cosh \sqrt{b}\,x + C_2 \sinh \sqrt{b}x\right]$
6.	Repeated irrational roots	$m_1 = m_2 = a + \sqrt{b}$ $m_3 = m_4 = a - \sqrt{b}$	$e^{ax}\left[(C_1 + C_2 x)\cosh \sqrt{b}x + (C_3 + C_4 x)\sinh \sqrt{b}x\right]$

Case I. When the roots of auxiliary equation are (rational) and distinct.

The given equation (1) is equivalent to

$$(D - m_1)(D - m_2)(D - m_3) \ldots (D - m_n)\, y = 0$$

$$\Rightarrow \qquad (D - m_1)\, y = 0,\ (D - m_2)\, y = 0 \ldots (D - m_n)\, y = 0$$

Let us discuss $(D - m_1)\, y = 0 \Rightarrow \dfrac{dy}{dx} - m_1 y = 0$

I.F. = $e^{\cdots}$

Now its solution is $ye^{-m_1x} = \int 0.e^{-m_1x}\,dx$

$\Rightarrow \qquad ye^{-m_1x} = 0 + C_1$

$\Rightarrow \qquad ye^{-m_1x} = C_1$

$\Rightarrow \qquad y = C_1e^{m_1x}$

Similarly the solution of $(D-m_2)\,y = 0$ is $y = C_2e^{m_2x}$, the solution of $(D-m_n)\,y=0$ is $y = C_n e^{m_n x}$

C.F. of equation (1) is given by

$$\text{C.F.} = C_1e^{m_1x} + C_2e^{m_2x} + \ldots + C_ne^{m_nx}$$

Example 1. *Solve* $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0$

Solution. $(D^2 - 5D + 6)\,y = 0$

A.E. is $m^2 - 5m + 6 = 0 \quad \Rightarrow \quad (m-2)(m-3) = 0 \quad \Rightarrow \quad m = 2, 3$

$$\text{C.F.} = C_1e^{2x} + C_2e^{3x} \text{ and P.I.} = 0$$

$$y = \text{C.F.} + \text{P.I.} = C_1e^{2x} + C_2e^{3x}$$

Example 2. *Solve:* $\dfrac{d^3y}{dx^3} - 6\dfrac{d^2y}{dx^2} + 11\dfrac{dy}{dx} - 6y = 0$

Solution. The auxiliary equation is $m^3 - 6m^2 + 11\,m - 6 = 0$

$\Rightarrow (m-1)(m-2)(m-3) = 0 \quad \Rightarrow \quad m = 1, 2, 3$

The roots are real and distinct.

$\therefore$ Complementary function (C.F.) = $C_1e^x + C_2e^{2x} + C_3e^{3x}$

Particular Integral (P.I.) = 0

Hence, the complete solution is y = C.F. + P.I. = $C_1e^x + C_2e^{2x} + C_3e^{3x}$

where C_1, C_2 and C_3 are arbitrary constants of integration.

Case II When the roots of auxiliary equation are equal.

The given equation (1) is equivalent to

$$(D-m_1)(D-m_2)(D-m_3)\ldots(D-m_n)\,y = 0$$

If $m_1 = m_2$ then $(D-m_1)(D-m_1)(D-m_3)\ldots(D-m_n)\,y = 0$

Part of C.F. is the solution of the equation $(D-m_1)(D-m_1)\,y = 0$

Replacing $(D-m_1)\,y = v$ in (2), we get

$$(D-m_1)\,v = 0$$

$$\frac{dv}{dx} - m_1v = 0 \quad \Rightarrow \quad \frac{dv}{v} = m_1dx$$

$\Rightarrow \log v = m_1x + \log C_2 \Rightarrow \log v - \log C_2 = m_1x \Rightarrow \log\dfrac{v}{C_2} = m_1x \Rightarrow \dfrac{v}{C_2} = e^{m_1x}$

$$v = C_2e^{m_1x}$$

From (3)

This is the linear differential equation.

$$\text{I.F.} = e^{\int -m_3 dx} = e^{-m_3 x}$$

Solution is $y.e^{-m_3 x} = \int (C_2 e^{m_3 x})(e^{-m_3 x})\, dx + C_1 = \int C_2\, dx + C_1 = C_2 x + C_1$

$$y = (C_2 x + C_1)\, e^{m_3 x}$$

$$\text{C.F.} = (C_1 + C_2 x) e^{m_3 x} \qquad \ldots(4)$$

From (1) and (4) we have complete C.F. is

$$\text{C.F.} = (C_1 + C_2 x) e^{m_1 x} + C_3 e^{m_3 x} + C_4 e^{m_4 x} + \ldots + C_n e^{m_n x}$$

If $m_1 = m_2 = m_3$, then

$$\text{C.F.} = (C_1 + C_2 x + C_3 x^2) + C_4 e^{m_4 x} + \ldots + C_n e^{m_n x}$$

Example 3. *Solve:* $\dfrac{d^2 y}{dx^2} - 6\dfrac{dy}{dx} + 9y = 0$

Solution. Here, we have

$$\frac{d^2 y}{dx^2} - 6\frac{dy}{dx} + 9y = 0 \Rightarrow (D^2 - 6D + 9)\, y = 0, \text{ where } D \equiv \frac{d}{dx}$$

A.E. is $m^2 - 6m + 9 = 0 \Rightarrow (m - 3)^2 = 0 \Rightarrow m = 3, 3.$

$$\text{C.F.} = (C_1 + C_2 x)e^{3x} \text{ and P.I.} = 0$$

$$y = \text{C.F.} + \text{P.I.}$$

$$= (C_1 + C_2 x)e^{3x} \qquad \textbf{Ans.}$$

Example 4. *Solve:* $\dfrac{d^4 y}{dx^4} - 7\dfrac{d^3 y}{dx^3} + 15\dfrac{d^2 y}{dx^2} - 13\dfrac{dy}{dx} + 4y = 0$

Solution. $(D^4 - 7D^3 + 15D^2 - 13D + 4)\, y = 0$, where $D = \dfrac{d}{dx}$

The auxiliary equation is $m^4 - 7m^3 + 15\, m^2 - 13\, m + 4 = 0$

$$(m - 1)^3 (m - 4) = 0 \Rightarrow m = 1, 1, 1, 4$$

$$\text{C.F.} = (C_1 + C_2 x + C_3 x^2)\, e^x + C_4 e^{4x} \text{ and P.I.} = 0$$

$$y = \text{C.F.} + \text{P.I.} = (C_1 + C_2 x + C_3 x^2)e^x + C_4 e^{4x} \qquad \textbf{Ans.}$$

Example 5. *Find the general solution of the differential equation* (U.P., II Semester 2009)

$$\frac{d^5 y}{dx^5} - \frac{d^3 y}{dx^3} = 0$$

Solution. Here, we have

$$\Rightarrow \quad D^5 y - D^3 y = 0 \quad \Rightarrow \quad (D^5 - D^3)\, y = 0 \quad \Rightarrow \quad D^3 (D^2 - 1)\, y = 0$$

A.E. is $m^3 (m^2 - 1) = 0 \Rightarrow m = 0, 0, 0, 1, -1$

$$\ldots C_2 x + C_3 x^2) + C_4 e^{x} + C_5 e^{-x} \qquad \textbf{Ans.}$$

$$= e^{\alpha x}[C_1(\cos\beta x + i\sin\beta x) + C_2(\cos\beta x - i\sin\beta x)] + C_3e^{m_3x} + \ldots + C_ne^{m_nx}$$

$$= e^{\alpha x}[(C_1 + C_2)\cos\beta x + i(C_1 - C_2)\sin\beta x] + C_3e^{m_3x} + \ldots + C_ne^{m_nx}$$

$$\text{C.F.} = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x) + C_3e^{m_3x} + \ldots + C_ne^{m_nx}$$

where $c_1 = C_1 + C_2$, $c_2 = i(C_1 - C_2)$

This is the C.F. of differential equation,

Example 6. *Solve:* $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 5y = 0,$

Subject to $\quad y = 2$ *and* $\dfrac{dy}{dx} = \dfrac{d^2y}{dx^2}$ *when* $x = 0.$

Solution. Here the auxiliary equation is

$$m^2 + 4m + 5 = 0 \Rightarrow m = \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm i$$

Its roots are $-2 \pm i$

The complementary function is

$$y = e^{-2x}(C_1\cos x + C_2\sin x)$$

On putting $y = 2$ and $x = 0$ in (1), we get

$$2 = C_1$$

On putting $C_1 = 2$ in (1), we have

$$y = e^{-2x}[2\cos x + C_2\sin x]$$

On differentiating (2), we get

$$\frac{dy}{dx} = e^{-2x}[-2\sin x + C_2\cos x] - 2e^{-2x}[2\cos x + C_2\sin x]$$

$$= e^{-2x}[(-2C_2 - 2)\sin x + (C_2 - 4)\cos x]$$

$$\frac{d^2y}{dx^2} = e^{-2x}[(-2C_2 - 2)\cos x - (C_2 - 4)\sin x]$$

$$-2e^{-2x}[(-2C_2 - 2)\sin x + (C_2 - 4)\cos x]$$

$$= e^{-2x}[(-4C_2 + 6)\cos x + (3C_2 + 8)\sin x]$$

But $\quad \dfrac{dy}{dx} = \dfrac{d^2y}{dx^2}$

$$e^{-2x}[(-2C_2 - 2)\sin x + (C_2 - 4)\cos x] = e^{-2x}[(-4C_2 + 6)\cos x + (3C_2 + 8)\sin x]$$

On putting $x = 0$, we get

$$C_2 - 4 = -4C_2 + 6 \Rightarrow C_2 = 2$$

(2) becomes,

$$y = e^{-2x}[2\cos x + 2\sin x]$$

$$y = 2e^{-2x}[\sin x + \cos x]$$

Example 7. *Solve* $\dfrac{d^4y}{dx^4} + 81y = 0$

Solution. $(D^4 + 81)y = 0$

$$\Rightarrow \quad y = e^{3x}\,[(C_1 + C_2x)\cos 4x + (C_3 + C_4x)\sin 4x]$$

where C_1, C_2, C_3 and C_4 are arbitrary constant of integration.

Example 9. *Solve:* $(D^2 + 1)^2 (D - 1)\, y = 0$

Solution. The auxiliary equation is

$$(m^2 + 1)^2 (m - 1) = 0$$

Either $\quad m - 1 = 0 \Rightarrow m = 1$

$$m^2 + 1)^2 = 0 \Rightarrow m = \pm i, \pm i$$

$$\text{C.F.} = C_1e^x + [(C_2 + C_3x)\cos x + (C_4 + C_5x)\sin x]\, e^{0x}$$

$$\text{P.I.} = 0$$

The complete solution is

$$y = \text{C.F.} + \text{P.I.}$$
$$= C_1e^x + [(C_2 + C_3x)\cos x + (C_4 + C_5x)\sin x]$$

where C_1, C_2, C_3, C_4 and C_5 are arbitrary constants of integration.

Example 10. *Solve the differential equation:*

$$(D^2 + 1)^3 (D^2 + D + 1)^2 y = 0, \text{ where } D = \frac{d}{dx}.$$

Solution. Here, we have

$$(D^2 + 1)^3 (D^2 + D + 1)^2\, y = 0$$

Auxiliary equation is

$$(m^2 + 1)^3 (m^2 + m + 1)^2 = 0$$

$\Rightarrow$

and $\quad (m^2 + 1)^3 = 0$ gives $m = \pm i, \pm i, \pm i$

$$(m^2 + m + 1)^2 = 0$$

$$\Rightarrow \quad m = \frac{-1 \pm \sqrt{1-4}}{2}, \quad \frac{-1 \pm \sqrt{1-4}}{2}$$

$$\Rightarrow \quad m = \frac{-1 \pm \sqrt{3}i}{2}, \quad \frac{-1 \pm \sqrt{3}i}{2}$$

C.F. of differential equation is given by

$$\text{C.F.} = [(C_1 + C_2x + C_3x^2)\cos x + (C_4 + C_5x + C_6x^2)\sin x]\, e^{0x}$$
$$+ \left[(C_7 + C_8x)\cos\frac{\sqrt{3}}{2}x + (C_9 + C_{10}x)\sin\frac{\sqrt{3}}{2}x\right] e^{-\frac{x}{2}}, \text{ P.I.} = 0$$

$$y = \text{C.F.} + \text{P.I.}$$
$$= (C_1 + C_2x + C_3x^2)\cos x + (C_4 + C_5x + C_6x^2)\sin x + \Big[(C_7 + C \ldots \sqrt{3}$$

where $C_1, C_2, C_3, C \ldots$

EXERCISE 2.2

Solve the following equations :

1. $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 0$ **Ans.** $y = C_1 e^x + C_2 e^{2x}$

2. $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 30y = 0$ **Ans.** $y = C_1 e^{5x} + C_2 e^{-6x}$

3. $\dfrac{d^2y}{dx^2} - 8\dfrac{dy}{dx} + 16y = 0$ **Ans.** $y = (C_1 + C_2 x)\, e^{4x}$

4. $\dfrac{d^2y}{dx^2} + \mu^2 y = 0$ **Ans.** $y = C_1 \cos \mu x + C_2 \sin \mu x$

5. $(D^2 + 2D + 2)y = 0,\ y(0) = 0,\ y'(0) = 1$ **Ans.** $y = e^{x} \sin x$ *(A.M.I.E.T.E., June)*

6. $\dfrac{d^2y}{dt^2} - 3\dfrac{dy}{dt} + 4y = 0$ *(GBTU, II Sem., Jan. 2013)* **Ans.** $y = e^{\frac{3}{2}x}\left[C_1 \cos \dfrac{\sqrt{7}}{2}x + C_2 \sin \right.$

7. $\dfrac{d^3y}{dx^3} - 2\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} - 8y = 0$ **Ans.** $y = C_1 e^{2x} + C_2 \cos 2x + C_3 \sin 2x$

8. $\dfrac{d^4y}{dx^4} + 32\dfrac{d^2y}{dx^2} + 256 = 0$ *(A.M.I.E.T.E., Dec. 2004)* **Ans.** $y = (C_1 + C_2 x) \cos 4x + (C_3 + C_4$

9. $\dfrac{d^4y}{dx^4} - 4\dfrac{d^3y}{dx^3} + 8\dfrac{d^2y}{dx^2} - 8\dfrac{dy}{dx} + 4y = 0$ **Ans.** $y = e^x\,[(C_1 + C_2 x) \cos x + (C_3 + C_4$

10. $\dfrac{d^4y}{dx^4} + \dfrac{d^2y}{dx^2} = 0,\quad y(0) = y'(0) = y''(0) = 0,\ y'''(0) = 1$ **Ans.** $y = x -$

11. $E.I.\ \dfrac{d^2y}{dx^2} + Py = 0$

If $y = 0$ when $x = 0$, and $y = a$ when $x = \dfrac{1}{2}$, find y. **Ans.** $y = \dfrac{a \sin \sqrt{\dfrac{P}{EI}}\,x}{\sin \sqrt{\dfrac{P}{EI}}\,\dfrac{1}{2}}$

12. $\dfrac{d^3y}{dx^3} + 6\dfrac{d^2y}{dx^2} + 12\dfrac{dy}{dx} + 8y = 0,\ y(0) = 0$, and $y'(0) = 0$ and $y''(0) = 2$ *(A.M.I.E.T.E. Dec. 2008)*

13. $\dfrac{d^3y}{dx^3} + \dfrac{d^2y}{dx^2} + \dfrac{4dy}{dx} + 4y = 0,\ y(0) = 0,\ y'(0) = 0,\ y''(0) = -5,$ **Ans.** $y = x^2 e^{-2x}$

14. $(D^8 + 6D^6 - 32D^2)y = 0$ **Ans.** $y = -e^x + \cos 2x - \frac{1}{2}$

15. Show that non-trivial ...

Particular Integral →

(i) P.I. of L.D.E. $F(D)y = 0$ is given by

$$\frac{1}{F(D)} Q \quad , \quad (Q = e^{ax})$$

Thus $$P.I. = \frac{e^{ax}}{F(a)} \quad , \quad F(a) \neq 0$$

$$= x \cdot \frac{e^{ax}}{F'(a)} \quad \text{If } F(a) = 0$$

eg1. $(D^2 - 3D + 2)y = e^{3x}$

A.E. $m^2 - 3m + 2 = 0$

$m = 1, 2$

C.F. $c_1 e^{x} + c_2 e^{2x}$

P.I. $$\frac{e^{3x}}{(D^2 - 3D + 2)}$$

$$= \frac{1}{2} e^{3x}$$

$y = C.F. + P.I.$

$= c_1 e^{x} + c_2 e^{2x} + \frac{1}{2} e^{3x}$

eg2. $(D^3 + 2D^2 - D - 2)y = e^{x}$

Ans. $c_1 e^{x} + c_2 e^{-x} + c_3 e^{-2x} + \frac{x}{6} e^{x}$