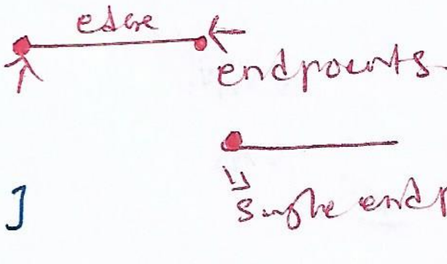
graph theory

→ A **graph** is a set of ~~vertices~~ nodes & edges.

→ **endpoints**

↳ each edge has one or two vertices ; that associated with it called Endpoints.

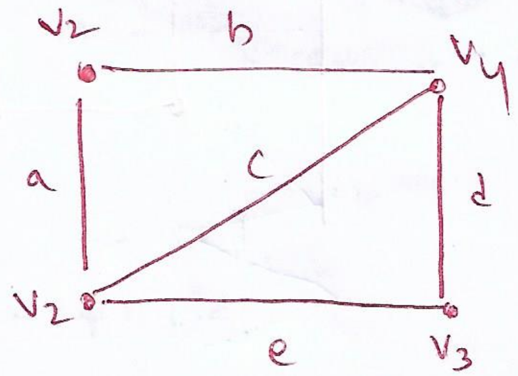


node or vertices → denoted by V[ ]

edges ————→ E[ ]

edge

↳ it is a line that connected to two nodes / vertices.



$V = \{v_1, v_2, v_3, v_4\}$

$E = \{a, b, c, d, e\}$

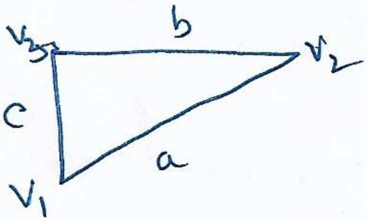
$G = (V, E)$

Types

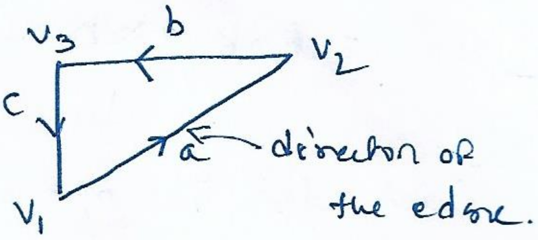
graphs are of 2 types according to direction of edges

- ~~unoriented~~ unoriented / undirectioned graph
- oriented / Directed graph.

unoriented | oriented.



$a \begin{cases} (V_1, V_2) \\ (V_2, V_1) \end{cases}$ } unordered pair.



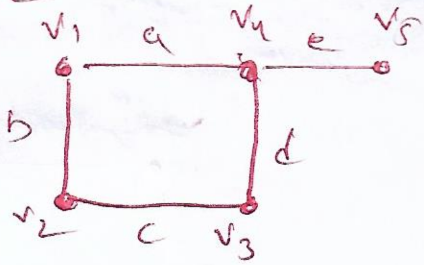
$a = (V_1, V_2)$
$b = (V_2, V_3)$
$c = (V_3, V_1)$
} ordered paired of edges.

common types of graphs

① **simple graph**

it is a graph in which each edge connects two different vertices & where no two edge connects the same pair of vertices.

Exp

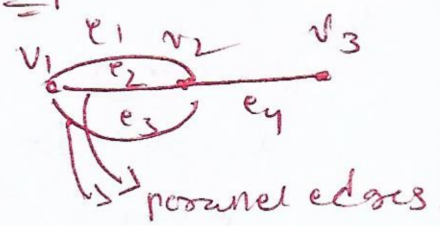


[it always without self loop & parallel edges.

② **Multigraph**

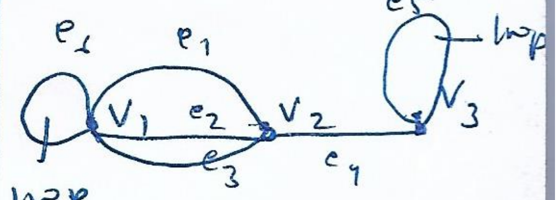
These graphs may have multiple edges that connected with same vertices.
(they may have some parallel edges)

Exp



③ **pseudographs**

These are the graphs that include loops, & possibly multiple edges connecting same pair of vertices.



loop

It is a edge that connect a vertex to itself.

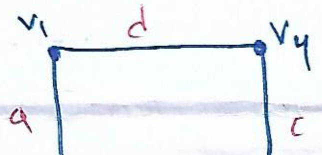
(4) null graph

a graph that do not have any edges called null graph

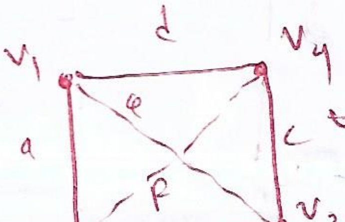
(5) Complete graph

It has an edge between every pair of vertex

Exp



→



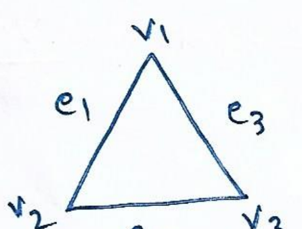
complete graph.

(not complete graph)

no edge, between $(v_1\ v_3)$ & (v_2, v_4)

(6) Regular graph

In this graph degree of each vertex ~~with~~ is same.



$$\left.\begin{array}{l} v_1 \to (e_1, e_3) \\ v_2 \to (e_1, e_2) \\ v_3 \to (e_2, e_3) \end{array}\right\} \text{degree} = 2$$

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degree of vertex in linear graph.

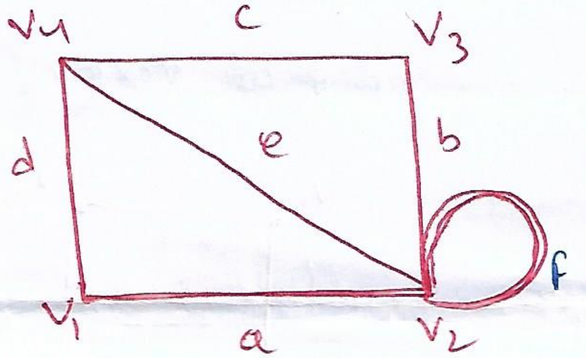
we can represent for the degree of a vertex.

$$\boxed{deg(V) = n_e + 2n_L}$$

n_e = no of edges incident at vertex 'V'.

n_L = no of selfloop incident.

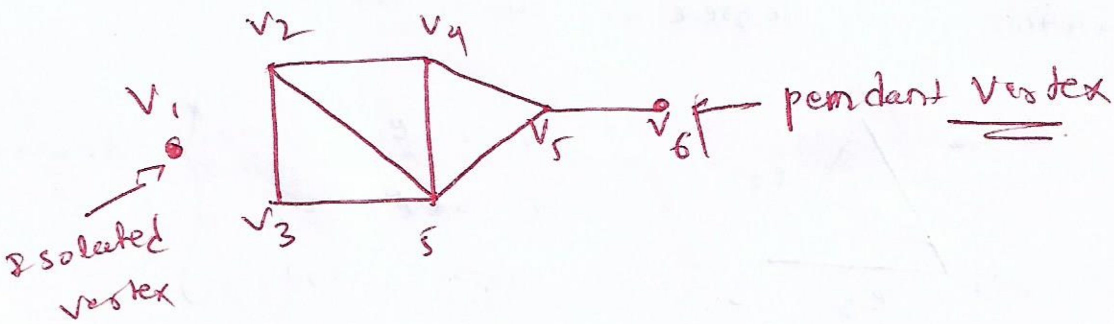
questions



$d(V_1) = (a, d) = 2$
$d(V_2) = 5 (a, b, e), f, = 3 + 2 \times 1$
$d(V_3) = 2 \; (b, c)$
$d(V_4) = 3 \; (c, e, d)$

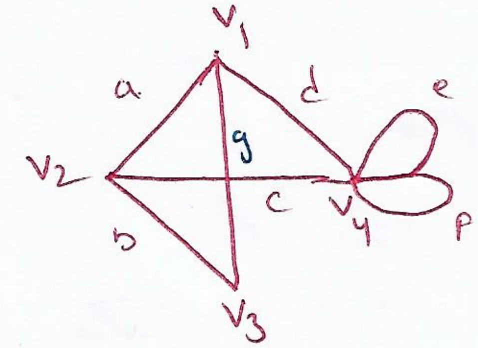
Isolated → the vertex which has degree '0' is known as Isolated vertex.

(2) vertex of degree '1' is called as pandant Vertex.



eg.

(eg)



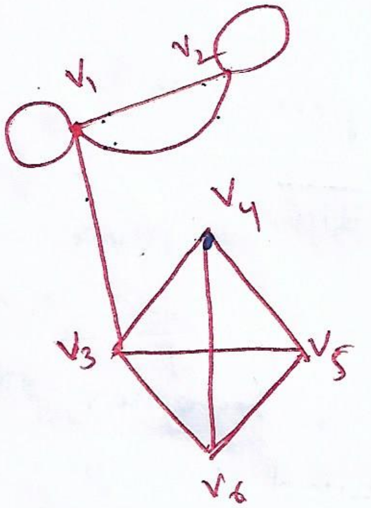
$d(v_1) = (a, d, c) = 3$

$d(V_2) = (a, b, c) = 3$

$d(V_3) = (b, g) = 2$

$d(V_4) = (c, d), 2, 2 \times 2 = 4 = 6$

(eg)



$d(v_1) = 2 + 3 = 5$

$d(v_2) = 2 + 2 = 4$

$d(v_3) = 4$

$d(v_4) = 3$

$d(v_5) = 3$

$d(v_6) = 3$

walk on graph.

A walk is nothing but sequence of edges/vertices of a given graph.

it is of 2 types

- open walk. (PATH)
- closed walk (circuit).

open walk

→ Here the initial & end vertex are different.

EXP

path → $v_1 - v_2 - v_3 - v_4 - v_5 - v_6 - v_7$

start

end

both diff

or no vertex comes twice.

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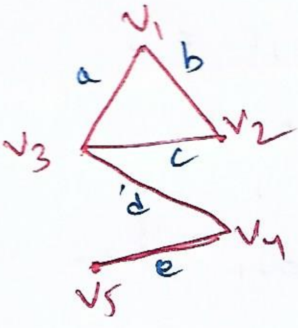


two path are there -> (a-b-c-d-e)

↳ v_3 apper twice.

so it is not a open path

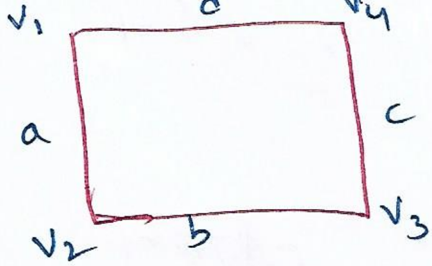
another path

(b-c-d-e)

no vertices comes together

~~is ca~~ is called open path.

Closed walk / Circuit



-> Here the initial & end vertex are same.

-> or, repetation of vertex may be there.

->(a-b-c-d)

↳ v_1-> v_2 - v_3 - v_4 - v_1)

end

initial

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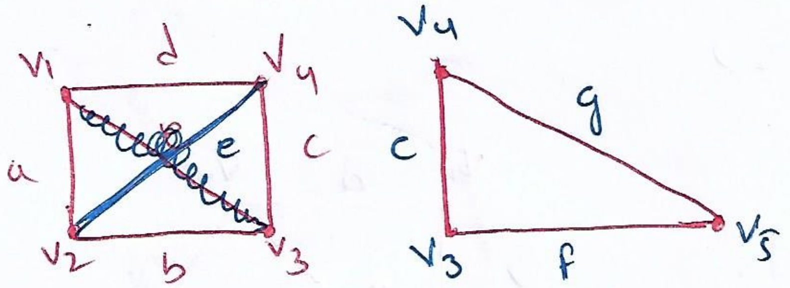
operations on graph.

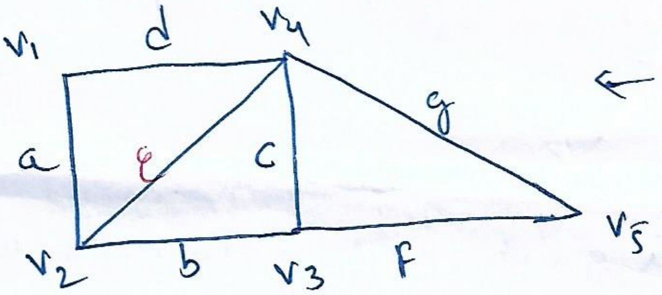


(a) (b)

① union of two graphs.

$G = G_1 \cup G_2$ } ← it contain edges either in G_1 or G_2 or in both

union represent.



← $G_1 \cup G_2$

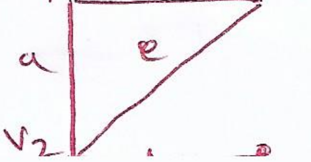
(b) intersection of 2 graphs.

↳ it contains all the edges which are in both G_1 & G_2

for the above example.

(c) difference of 2 graphs.

$G = G_1 - G_2$ } it contain all the edges that are in G_1 but not G_2.



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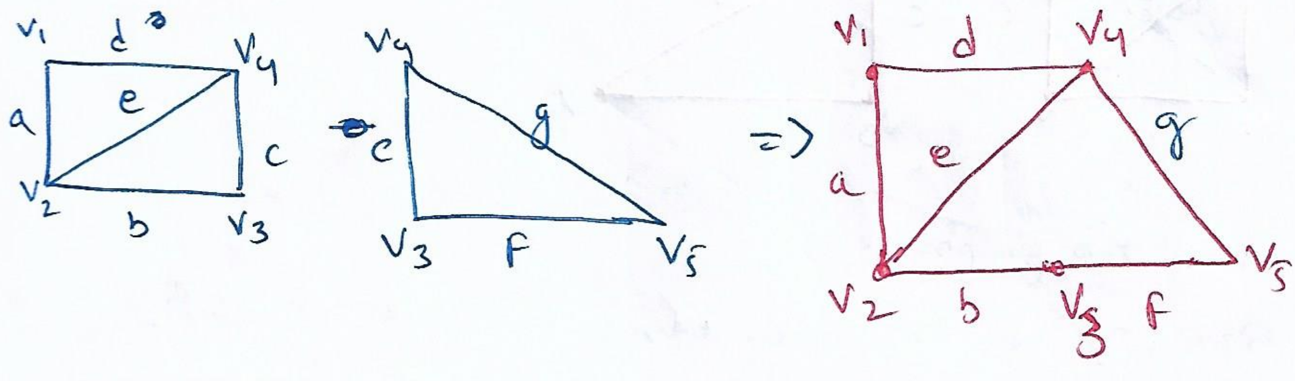
(d) addition of a graph.

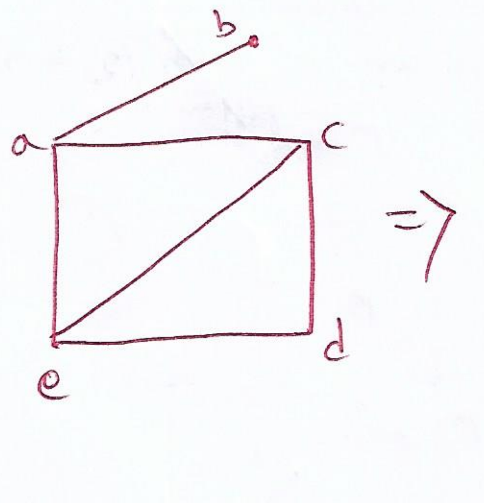
$G_1 \oplus G_2 = (G_1 \cup G_2) - (G_1 \cap G_2)$



GRAPH REPRESENTATION.

(1) Adjacency list:

It is a way to represent a graph, without multiple edges. It also specify all the vertices that are adjacent to each vertex of a graph.



=>

vertex	~~Adjescent~~ Adjacent vertices.
a	b, c, e
b	a
c	a, d, e
d	c, e
e	a, c, d

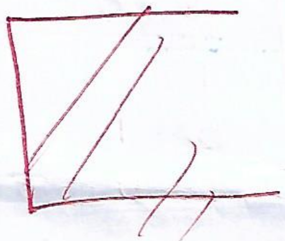
adjacency list.

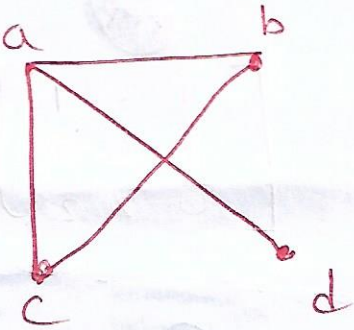
③ Adjacency Matrices

this is a $n \times n$ ~~matrix~~, ~~zero~~ combination of '0' & '1' matrix, where we can put '1' represent the connection between two edges & '0' represent there is no connection between them.

$$A[i,j] = \begin{cases} 1 & \text{if } i \,\&\, j \text{ edges are connected.} \\ 0 & \text{else.} \end{cases}$$

(Q5) Represent the following graph with use of adjacency matrix.

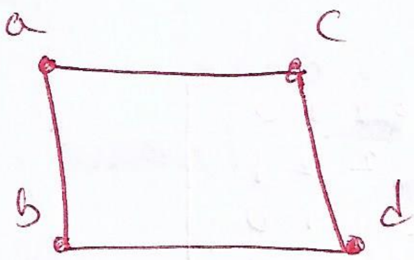


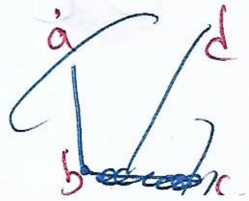


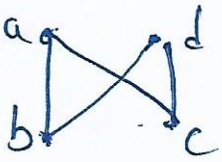
$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \hline \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(Q6) Draw a graph from the given ~~adjacency~~ adjacency matrix.

$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \hline \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$







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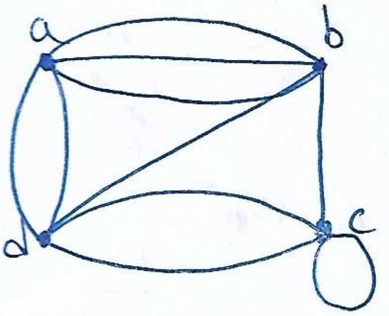
(43) use/draw an adjacency Matrix to represent the below pseudograph a.



$\longrightarrow$

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & & & & \\ \end{array}$$

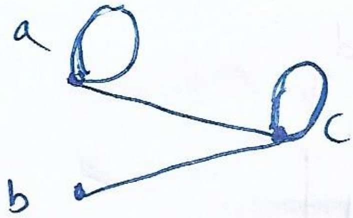
$$\begin{array}{cc} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

(44) draw the graph represented by the given adjacency Matrix.

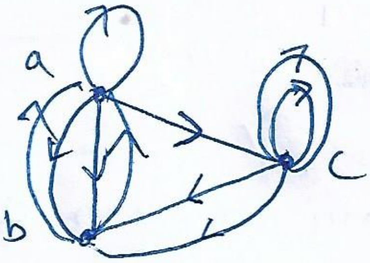
(IMP)

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix}, \quad \begin{bmatrix} 0 & 2 & 3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

$$\begin{array}{cc} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

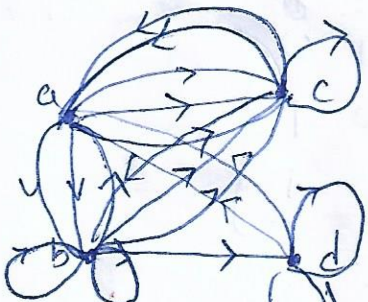


$$\begin{array}{cc} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix} \end{array} \longrightarrow$$



(IMP) direction

$$\begin{array}{cc} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 2 & 3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix} \end{array}$$



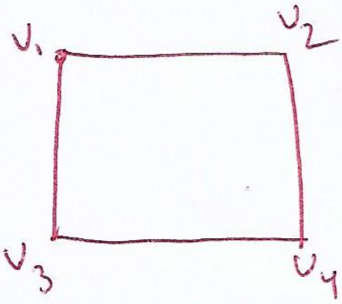
re-11

Isomorphic

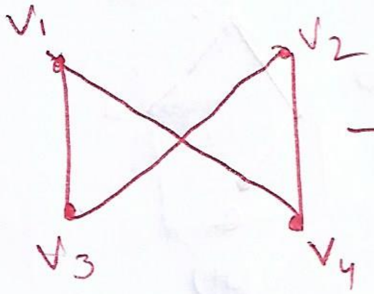
properties

1. Same no of vertices
2. Same no of edges
3. equal no of vertices with given degree.

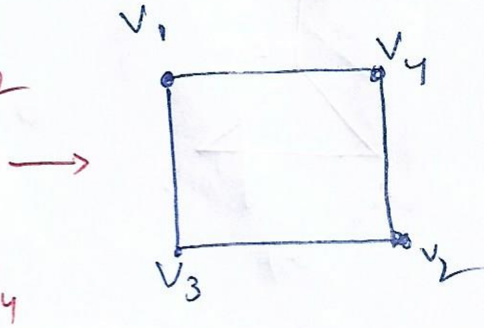
Exp



(G)



(H)



step 1

one to one fun mapping

(from this graph, show that G & H are Isomorphic)

$f(v_1) \to V_1$ — vertex v_1 mapped to V_1

$f(v_2) \to V_4$

$f(v_3) \to V_3$

$f(v_4) \to V_2$

← one to one fun mapping.

step 2 check for edges through adjacent vertex.

$(v_1, v_2) \to$ (adjacent) $\to (V_1, V_4)$ ✓ (adjacent)

$(v_2, v_4) \longrightarrow (V_4, V_2)$ ✓

$(v_3, v_4) \longrightarrow (V_3, V_2)$ ✓

$(v_1, v_3) \longrightarrow (V_1, V_3)$ ✓

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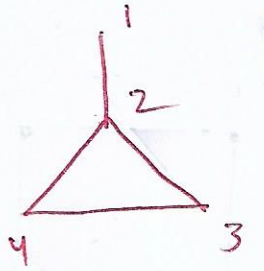
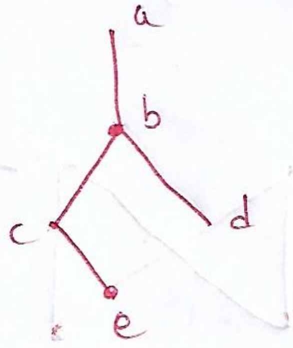
diff → Two graphs which contain the same no of graph vertices connected in the same way are said to be isomorphic.

check G & H are isomorphic or not ?



(G)

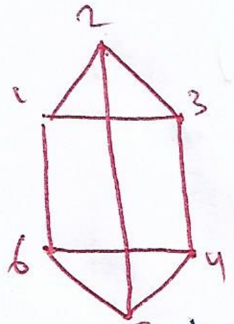


(H)

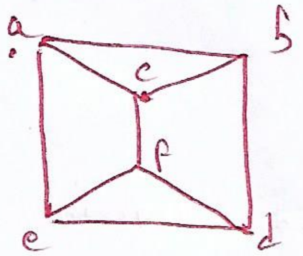
$N_G \to 4$
$N_H \to 4$ } ✓

vertices
$V_G = 4$
$V_H = 5$ } X

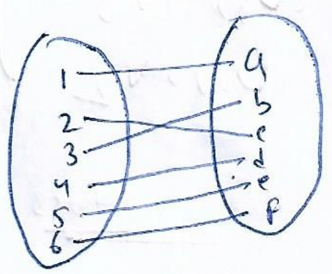
So that G & H are not isomorphic.

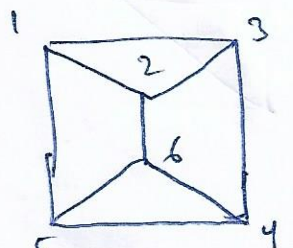


G



(H)





Mapping

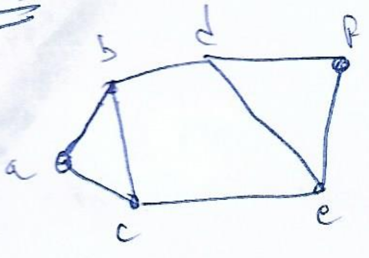
G (1) → H (a)
G (2) → H (c)
G (3) → H (b)
G (4) → H (d)
G (5) → H (e)
G (6) → H (f)

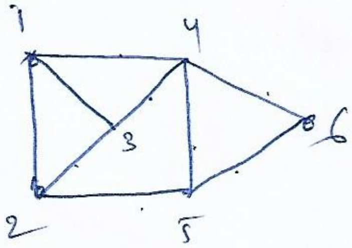
adjacent edge

(1,2) ≅ (a,c)
(2,3) ≅ (c,b)
(1,3) ≅ (a,b)
(2,6) ≅ (c,f)
(5,6) ≅ (e,f)
(6,4) ≅ (f,d)

(1,5) ≅ (
(3,4) ≅ (

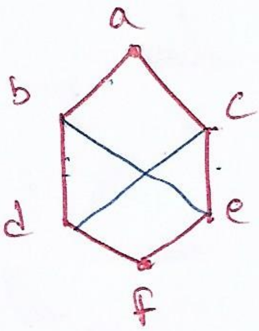
ge-13

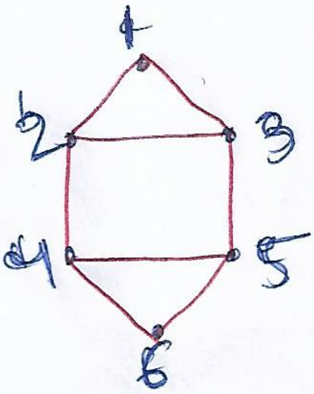




Samed node, but edge diff. (X)

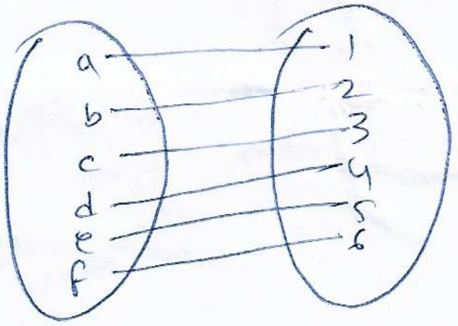
(as)





1. node are same (6)
2. ~~vertixes~~ edges are same (8)
3. degree -> a-2, b-3, c-3, e-3, d-3, f-2 ✓

when ~~it~~ you mapped.

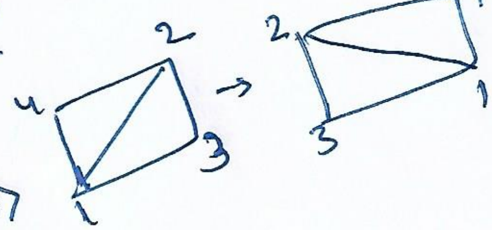


$(b,e) \to (2,5)$ X

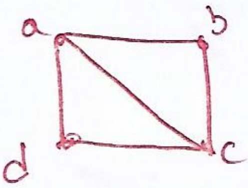
$(c,d) \to (3,4)$ X

when

(not Isomorphic)

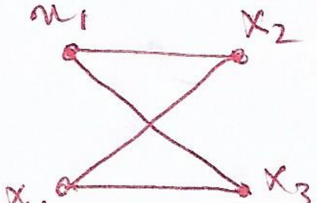


Check for Isomorphic.



G_1 ✓

G_2 ✓



G_3 (X)

G_1 & G_2 are Isomorphic.