Now, $\quad (y)_{m=-1} = x^{-1}\left[a_0\left(1-\frac{x^2}{1.2}+\frac{x^4}{1.2.3.4}-\ldots\right)+a_1\left(x-\frac{x^3}{2.3}+\frac{x^5}{2.3.4.5}-\ldots\right)\right]$

$= x^{-1}\,[a_0 \cos x + a_1 \sin x]$

Hence complete solution is given by

$$y = (y)_{m=-1}$$

$$\Rightarrow \qquad y = \frac{1}{x}(a_0 \cos x + a_1 \sin x).$$

Note. All those problems, in which $x = 0$, was an ordinary point of $y'' + P(x)y' + Q\ldots$ can also be solved by Frobenius method as given in Art. 7.9 and explained in above illustrati...

EXERCISE 7.5

Solve the following differential equations:

1. $(1-x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+y=0$ **Ans.** $y = a_0\left(1-\frac{1}{2}x^2+\frac{1}{8}x^4+\ldots\right)+b\left(x-\frac{1}{2}x^3+\frac{1}{4!}\ldots\right.$

2. $(2+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(1+x)\,y=0$

   Ans. $y = a_0\left(1-\frac{1}{4}x^2-\frac{1}{12}x^3+\frac{5}{56}x^4+\ldots\right)+b\left(x-\frac{1}{6}x^3-\frac{1}{24}\ldots\right.$

3. $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0$ **Ans.** $y = a_0(1-2x^2)+a_1\cdot\left(x-\frac{x^3}{2}-\frac{x^5}{8}+\ldots\right.$

OBJECTIVE TYPE QUESTIONS

Choose the correct alternative:

1. The singular point of $x(x-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+x^2y=0$ is

   (*i*) 0 (*ii*) 1 (*iii*) 2 (*iv*) 3

2. The singular point of $x^2(x-4)\frac{d^2y}{dx^2}+5(x-4)\frac{dy}{dx}+6y=0$ is

   (*i*) 2 (*ii*) 3 (*iii*) 4 (*iv*) 0

3. The singular point of $x(x-1)(x-2)\dfrac{d^2y}{dx^2}+x(x-1)\dfrac{dy}{dx}+2x(x-1)y=0$ is

(*i*) 1 (*ii*) 2 (*iii*) 3 (*iv*) 4

4. The irregular singular point of $(x-1)(x-2)^3\dfrac{d^2y}{dx^2}+(x-1)^2\dfrac{dy}{dx}+3(x-1)y=0$ is

(*i*) 0 (*ii*) 1 (*iii*) 2 (*iv*) 3

5. The regular singular point of $\dfrac{d^2y}{dx^2}+\dfrac{1}{x}\dfrac{dy}{dx}+\dfrac{1}{x^2}y=0$ is

(*i*) 0 (*ii*) 1 (*iii*) 2 (*iv*) 3

6. At $x=0$ the differential equation $\dfrac{d^2y}{dx^2}+\dfrac{1}{x^2}\dfrac{dy}{dx}+\dfrac{1}{x^3}y=0$ has

(*i*) Ordinary point (*ii*) Regular singular point

(*iii*) irregular singular point (*iv*) None of these point

7. If $x=0$ is an ordinary point of a differential equation, then for its solution we take

(*i*) $\sum a^{m+k}x^k$ (*ii*) $\sum a_2 x^1$

(*iii*) $\sum a_k x^{m+k}$ (*iv*) $\sum\limits_{k=0}^{\infty} a_k x^k$

8. If $x=0$ is a regular singular point of a differential equation, then for its solution w

(*i*) $\sum a_k x^{m+k}$ (*ii*) $\sum u_k x^m$

(*iii*) $\sum a_k x^k$ (*iv*) None of these

9. If $x=0$ is an irregular singular point of a differential equation, then for its solutic

(*i*) $\sum a_k x^{m+k}$ (*ii*) $\sum a_k x^k$

(*iii*) $\sum a^k x^m$ (*iv*) None of these

Indicate True or False for the following :

10. $x=0$ is an ordinary point of $(x-1)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+5=0$.

11. $x=0$ is a singular point of $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+4y=0$.

12. $x=1$ is a singular point of $(x-1)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+2y=0$.

13. $x=1$ is a not an ordinary point of $(x-1)^2\dfrac{d^2y}{dx^2}+4(x-1)\dfrac{dy}{dx}+4y=0$.

14. $x=0$ and $x=3$ are regular singular points of $x(x-3)^2\dfrac{d^2y}{dx^2}+2(x-3)\dfrac{dy}{dx}$