$$= \frac{1}{13440}[112\,x^2y^6 - y^8]$$

The complete solution $= C.F. + P.I.$

$$= f_1(y+2ix) + f_2(y-2ix) + \frac{1}{13440}(112\,x^2y^6 - y^8) \qquad \textbf{Ans.}$$

EXERCISE 14.5

Solve the following equations :

1. $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x\,\partial y} + 2\frac{\partial^2 z}{\partial y^2} = 12xy$ *(A.M.I.E., Winter 2001)*

Ans. $z = f_1(y-x) + f_2(y-2x) + 2x^3y - \frac{3x^4}{2}$

2. $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x\partial y} - 6\frac{\partial^2 z}{dy^2} = xy$ **Ans.** $z = f_1(y-2x) + f_2(y+3x) + \frac{x^3y}{6} + \frac{x^4}{24}$

3. $(D^3 - 3D^2D')z = x^2y$ **Ans.** $z = \phi_1(y+x) + \phi_2(y-x) + \frac{1}{12}e^{2x-y} - xe^{x+y} - \frac{1}{3}\cos(x+2y)$

4. $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2\partial y} = 2e^{2x} + 3x^2y$ **Ans.** $z = f_1(y) + xf_2(y) + f_3(y+2x) + \frac{1}{60}(15e^{2x} + 3x^5y + x^6)$

5. $(D^2 - 6DD' + 9D'^2)z = 12x^2 + 36xy$ **Ans.** $z = f_1(y+3x) + xf_2(y+3x) + 6x^3y + 10x^4$

6. $(D^2 - D'^2 + D + 3D' - 2)z = e^{x-y} - x^2y$

Ans. $z = e^{-2x}f_1(y+x) + e^x f_2(y-x) - \frac{1}{4}e^{x-y} + \frac{1}{2}\left(x^2y + xy + \frac{3}{2}x^2 + \frac{3y}{2} + 3x + \frac{21}{4}\right)$

14.5 P.I. OF ANY FUNCTION

If the function on the R.H.S. of the P.D.E. is not of the form, given in previous cases. Then

$$\text{P.I.} = \frac{1}{F(D, D')}\phi(x, y)$$

$F(D, D')$ is factorized to get

$$F(D, D') = (D - m_1D')\,(D - m_2D') \;......\; (D - m_nD')$$

$$\text{P.I.} = \frac{1}{(D - m_1D)\,(D - m_2D')\ldots(D - m_nD')}\phi(x, y)$$

Let us consider

$$\text{P.I.} = \frac{1}{D - m_1D'}\phi\,(x, y) \qquad \text{(Taking only one term)}$$

$$\Rightarrow \qquad p - m_1q = \phi\,(x, y)$$

Subsidiary equations are (Lagrange's equations)

$$\frac{dx}{1} = \frac{dy}{-m_1} = \frac{dz}{\phi(x, y)}$$

From the first two

$$dy + m_1 dx = 0 \quad \Rightarrow \quad y + m_1 x = c$$

From the first and last equations, we get

$$dz = \phi(x, y)dx = \phi(x, c - m_1 x)\, dx$$

$$\Rightarrow \quad z = \int \phi(x, c - m_1 x)dx$$

$$\boxed{\text{P.I.} = \frac{1}{D - mD'}F(x, y) = \int \phi(x, c - mx)dx}$$

where c is replaced by $y + mx$ after integration.

Similarly we repeat the above method to get *P.I.*

Case V. When R.H.S. = Any function

Example 25. *Solve* $(D^2 - DD' - 2D'^2)z = (y-1)e^x$

Solution. $(D^2 - DD' - 2D'^2)z = (y-1)e^x$

A.E. is $(D^2 - DD' - 2D'^2) = 0$ or $m^2 - m - 2 = 0$

$$(m-2)(m+1) = 0, \quad \text{or} \quad m = 2, -1$$

$$\text{C.F.} = f_1(y+2x) + f_2(y-x)$$

$$\text{P.I.} = \frac{1}{D^2 - DD' - 2D'^2}(y-1)e^x$$

$$= \frac{1}{(D+D')(D-2D')}(y-1)e^x = \frac{1}{D+D'}\int[(c-2x-1)e^x dx] \qquad [\text{Put } y = c - 2x]$$

$$= \frac{1}{D+D'}[(c-2x-1)e^x + 2e^x]$$

$$[\text{Put } c = y + 2x]$$

$$= \frac{1}{D+D'}[ce^x - 2xe^x + e^x]$$

$$= \frac{1}{D+D'}[(y+2x)e^x - 2xe^x + e^x] = \frac{1}{D+D'}[ye^x + e^x]$$

$$[\text{Put } y = c + x]$$

$$[\text{Put } c = y - x]$$

$$= \int[(c+x)e^x + e^x]dx$$

$$= (c+x)e^x - e^x + e^x$$

$$= ce^x + xe^x = (y-x)e^x + xe^x$$

$$= ye^x \qquad \text{Ans.}$$

Hence, complete solution is

$$\therefore \quad z = f_1(y+2x) + f_2(y-x) + ye^x$$

Example 26. *Solve* $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial x \partial y} - 6\dfrac{\partial^2 z}{\partial y^2} = y\cos x.$

(R.G.P.V., Bhopal, June 2009, Feb. 2008, June 2006, 2004, Dec. 2002)

Solution. We have $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial x \partial y} - 6\dfrac{\partial^2 z}{\partial y^2} = y\cos x$

$$(D^2 + DD' - 6D'^2) = y\cos x$$

Its auxiliary equation is

$$m^2 + m - 6 = 0 \Rightarrow (m+3)(m-2) = 0 \Rightarrow m = 2, -3$$

$$\therefore \quad \text{C.F.} = f_1(y+2x) + f_2(y-3x)$$

$$\text{P.I.} = \frac{1}{D^2 + DD' - 6D'^2} y \cos x = \frac{1}{(D-2D')(D+3D')} y \cos x$$

$$= \frac{1}{D-2D'} \int (c+3x) \cos x \, dx \qquad \text{[Put } y = c + 3x]$$

$$= \frac{1}{D-2D'} [(c+3x)\sin x + 3\cos x] = \frac{1}{D-2D'} [y \sin x + 3 \cos x] \qquad \text{[Put } c + 3x = y]$$

$$= \int [(c-2x)\sin x + 3\cos x] \, dx \qquad \text{[Put } y = c - 2x]$$

$$= (c-2x)(-\cos x) - 2\sin x + 3\sin x = -y\cos x + \sin x \qquad \text{[Put } c - 2x = y]$$

Hence, the complete solution is

$$z = f_1(y+2x) + f_2(y-3x) + \sin x - y\cos x$$ **Ans.**

ample. 27. *Solve:* $(D^2 + D\,D' - 6\,D'^2)\,z = x^2 \sin(x+y)$

lution. Here, we have

$$(D^2 + D\,D' - 6\,D')\,z = x^2 \sin(x+y)$$

Putting $D = m$ and $D' = 1$, we have

A.E. is $m^2 + m - 6 = 0 \Rightarrow (m+3)(m-2) = 0 \Rightarrow m = 2, -3$

$$\text{C.F.} = f_1(y+2x) + f_2(y-3x)$$

$$\text{P.I.} = \frac{1}{D^2 + D\,D' - 6\,D'^2}\left[x^2 \sin(x+y)\right]$$

$$= \frac{1}{(D-2D')(D+3D')}\left[x^2 \sin(x+y)\right]$$

Let $\frac{1}{D+3D'}[x^2 \sin(x+y)] = u$

$$\Rightarrow \quad (D + 3D')\,u = x^2 \sin(x+y)$$

$$u = \int x^2 \sin(x + c + 3x)\,dx$$

$$= \int x^2 \sin(4x + c)\,dx \qquad [y = c + 3x]$$

$$= x^2\left(\frac{-\cos(4x+c)}{4}\right) - (2x)\left(\frac{-\sin(4x+c)}{16}\right) + 2\frac{\cos(4x+c)}{64}$$

$$= \left[\frac{-x^2}{4} + \frac{1}{32}\right]\cos(4x+c) + \frac{x}{8}\sin(4x+c) \qquad \text{...(1)}$$

On eliminating c, we put $c = y - 3x$ in (1) and get

$$u = \left(\frac{-x^2}{4} + \frac{1}{32}\right)\cos(4x + y - 3x) + \frac{x}{8}\sin(4x + y - 3x)$$

$$= \left(\frac{-x^2}{4} + \frac{1}{32}\right)\cos(x+y) + \frac{x}{8}\sin(x+y)$$

$$\text{P.I.} = \frac{1}{(D-2D')(D+3D')} x^2 \sin(x+y)$$

$$= \frac{1}{(D-2D')} u$$

$$= \frac{1}{D-2D'}\left[\left(\frac{-x^2}{4}+\frac{1}{32}\right)\cos(x+y)+\frac{x}{8}\sin(x+y)\right] \qquad (y = c-2x)$$

$$= \int\left[\left(\frac{-x^2}{4}+\frac{1}{32}\right)\cos(x+c-2x)+\frac{x}{8}\sin(x+c-2x)\right]dx$$

$$= \int\left[\left(\frac{-x^2}{4}+\frac{1}{32}\right)\cos(c-x)+\frac{x}{8}\sin(c-x)\right]dx$$

$$= \left(\frac{-x^2}{4}+\frac{1}{32}\right)\{-\sin(c-x)\}-\left(\frac{-x}{2}\right)\{-\cos(c-x)\}$$

$$+\left(\frac{-1}{2}\right)\sin(c-x)+\frac{x}{8}\cos(c-x)-\frac{1}{8}[-\sin(c-x)]$$

$$= \left(\frac{x^2}{4}-\frac{1}{32}-\frac{1}{2}+\frac{1}{8}\right)\sin(c-x)+\left(\frac{-x}{2}+\frac{x}{8}\right)\cos(c-x)$$

$$= \left(\frac{x^2}{4}-\frac{13}{32}\right)\sin(c-x)-\frac{3x}{8}\cos(c-x) \qquad \ldots(2)(c = 2x+y)$$

On eliminating c, we put $c = 2x + y$ in (2) and get

$$\text{P.I.} = \left(\frac{x^2}{4}-\frac{13}{32}\right)\sin(2x+y-x)-\frac{3x}{8}\cos(2x+y-x)$$

$$= \left(\frac{x^2}{4}-\frac{13}{32}\right)\sin(x+y)-\frac{3x}{8}\cos(x+y)$$

The complete solution = *C.F. + P.I.*

$$z = f_1(y+2x)+f_2(y-3x)+\left(\frac{x^2}{4}-\frac{13}{32}\right)\sin(x+y)-\frac{3x}{8}\cos(x+y) \quad \textbf{Ans.}$$

Example 28. *Solve* : $(r-4t) = \dfrac{4x}{y^2}-\dfrac{y}{x^2}$

Solution. Here, we have

$$(D^2-4D'^2)z = \frac{4x}{y^2}-\frac{y}{x^2}$$

A.E. is

$$m^2-4 = 0 \Rightarrow (m+2)(m-2) = 0 \Rightarrow m = 2, -2$$

$$\text{C.F.} = f_1(y+2x)+f_2(y-2x)$$

$$P.I. = \frac{1}{D^2-4D'^2}\left(\frac{4x}{y^2}-\frac{y}{x^2}\right)$$

$$= \frac{1}{(D+2D')(D-2D')}\left(\frac{4x}{y^2}-\frac{y}{x^2}\right)$$

$$y = c - 2x$$

Let $$u = \frac{1}{D-2D'}\left(\frac{4x}{y^2} - \frac{y}{x^2}\right)$$

$$\Rightarrow \quad (D-2D')\,u = \left(\frac{4x}{y^2} - \frac{y}{x^2}\right)$$

$$\Rightarrow \quad u = \int\left[\frac{4x}{(c-2x)^2} - \frac{c-2x}{x^2}\right]dx$$

$$= \int\left[\frac{-2(c-2x)+2c}{(c-2x)^2} - \frac{c}{x^2} + \frac{2}{x}\right]dx$$

$$= \int\left[\frac{-2}{(c-2x)} + \frac{2c}{(c-2x)^2} - \frac{c}{x^2} + \frac{2}{x}\right]dx$$

$$= \log(c-2x) + \frac{c}{c-2x} + \frac{c}{x} + 2\log x$$

On eliminating c, replace c by $2x + y$ and have

$$u = \log(2x+y-2x) + \frac{2x+y}{2x+y-2x} + \frac{2x+y}{x} + 2\log x$$

$$= \log y + \frac{2x+y}{y} + 2 + \frac{y}{x} + 2\log x$$

Now $$\text{P.I.} = \frac{1}{(D+2D')(D-2D')}\left(\frac{4x}{y^2} - \frac{y}{x^2}\right)$$

$$= \frac{1}{(D+2D')}\,u$$

$$= \frac{1}{D+2D'}\left[\log y + \frac{2x+y}{y} + 2 + \frac{y}{x} + 2\log x\right]$$

$$= \int\left[\log(c+2x) + \frac{2x+c+2x}{c+2x} + 2 + \frac{c+2x}{x} + 2\log x\right]dx \qquad [y = c+2x]$$

$$= \int\left[\log(c+2x) + \frac{2x}{c+2x} + 1 + 2 + \frac{c}{x} + 2 + 2\log x\right]dx$$

$$= \int\left[\log(c+2x) + \frac{2x+c-c}{2x+c} + 5 + \frac{c}{x} + 2\log x\right]dx$$

$$= \int\left[\log(c+2x) + 1 - \frac{c}{2x+c} + 5 + \frac{c}{x} + 1\cdot\log x^2\right]dx$$

$$= \left[x\log(c+2x) - \int x\,\frac{1}{c+2x}\,2\,dx + 6x - \frac{c}{2}\log(c+2x)\right.$$
$$\left. + c\log x + x\log x^2 - \int \frac{2x}{x^2}\cdot x\,dx\right]$$

$$= x\log(c+2x) - \int \frac{c+2x-c}{c+2x}\,dx + 6x - \frac{c}{2}\log(c+2x)$$
$$+ c\log x + x\log x^2 - \int 2\,dx$$

$$= x \log (c+2x) - x + \frac{c}{2} \log (c+2x) + 6x - \frac{c}{2} \log (c+2x) + c \log x + x \log x^2 - 2x$$

$$= x \log (c+2x) + 3x + c \log x + x \log x^2$$

On eliminating c, replacing c by $y - 2x$ and have

$$= x \log (y - 2x + 2x) + 3x + (y - 2x) \log x + x \log x^2$$

$$= x \log y + 3x + y \log x - 2x \log x + x \log x^2$$

$$= x \log y + 3x + y \log x - x \log x^2 + x \log x^2$$

$$= x \log y + 3x + y \log x$$

Hence, the complete solution is

$y = C.F + P.I. = f_1 (y+2x) + f_2 (y-2x) + x \log y + 3x + y \log x$ **Ans.**

Example 29. *Solve* : $[D^3 + D^2 D' - D D'^2 - D'^3] z = e^x \cos 2y$

Solution. We have

$$[D^3 + D^2 D' - D D'^2 - D'^3] z = e^x \cos 2y$$

A.E. is $m^3 + m^2 - m - 1 = 0 \Rightarrow (m+1)^2 (m-1) = 0, \quad m = 1, -1, -1$

$$\text{C.F.} = f_1 (y+x) + f_2 (y-x) + x f_3 (y-x)$$

$$\text{P. I.} = \frac{1}{D^3 + D^2 D' - D D'^2 - D'^3} e^x \cos 2y$$

$$= \frac{1}{(D+D')^2 (D-D')} e^x \cos 2y$$

Let $u = \frac{1}{D - D'} e^x \cos 2y \qquad (y = c - x)$

$$= \int e^x \cos 2(c-x)\, dx$$

$$= \frac{e^x}{1+4} [\cos 2(c-x) - 2 \sin (c-x)] \qquad \dots (1)$$

On eliminating c, replace c by $x + y$ in (1), and have

$$u = \frac{e^x}{5} [\cos 2(x+y-x) - 2 \sin (x+y-x)]$$

$$= \frac{e^x}{5} [\cos 2y - 2 \sin 2y]$$

Now $\left[\frac{1}{(D+D')} \frac{1}{(D-D')}\right] e^x \cos 2y = \frac{1}{(D+D')} u \qquad (y = c + x)$

$$= \frac{1}{D+D'} \left[\frac{e^x}{5} (\cos 2y - 2 \sin 2y)\right]$$

$$= \int \left[\frac{e^x}{5} \{\cos (2c + 2x) - 2 \sin (2c + 2x)\}\right] dx$$

$$= \int \frac{e^x}{5} \cos (2c + 2x)\, dx - 2 \int \frac{e^x}{5} \sin (2c + 2x)\, dx$$

$$= \frac{e^x}{5\,(1+4)}[\cos(2c+2x)+2\sin(2c+2x)] - \frac{2\,e^x}{5\,(1+4)}[\sin(2c+2x)-2\cos(2c+2x)]$$

$$= \frac{e^x}{25}\left[\cos(2c+2x)+2\sin(2c+2x)-2\sin(2c+2x)+4\cos(2c+2x)\right]$$

$$= \frac{e^x}{25}\left[5\cos(2c+2x)\right] = \frac{e^x}{5}\cos(2c+2x)$$

On eliminating c, replace c by $y-x$ and have

$$= \frac{e^x}{5}\cos(2y-2x+2x) = \frac{e^x}{5}\cos 2y$$

$$\text{P.I.} = \frac{1}{(D+D')}\left(\frac{1}{D+D'}\,\frac{1}{D-D'}\right)(e^x\cos 2y)$$

$$= \frac{1}{D+D'}\,\frac{e^x}{5}\cos 2y \qquad (y=c+x)$$

$$= \int \frac{e^x}{5}\cos 2\,(c+x)\,dx$$

$$= \frac{e^x}{5\,(1+4)}\left[\cos 2\,(c+x)+2\sin 2\,(c+x)\right]$$

On eliminating c, replace c by $(y-x)$ and get

$$= \frac{e^x}{25}\left[\cos 2\,(y-x+x)+2\sin 2\,(y-x+x)\right]$$

$$= \frac{e^x}{25}\left[\cos 2y+2\sin 2y\right]$$

The complete solution is $z = C.F. + P.I.$

$$= f_1(y+x)+f_2(y-x)+x\,f_3(y-x)+\frac{e^x}{25}(\cos 2y+2\sin 2y)] \textbf{ Ans.}$$

EXERCISE 14.6

Solve the following equations:

1. $(D-D')(D+2D')z=(y+1)e^x$ **Ans.** $z=f_1(y+x)+f_2(y-2x)+ye^x$
2. $\dfrac{\partial^2 z}{\partial x^2}-\dfrac{\partial^2 z}{\partial y^2}=\tan^3 x\tan y-\tan x\tan^3 y$ **Ans.** $z=f_1(y+x)+f_2(x-y)+\dfrac{1}{2}\tan x\tan y$
3. $(D^2-DD'-2D'^2)z=(2x^2+xy-y^2)\sin xy-\cos xy$ **Ans.** $z=f_1(y+2x)+f_2(y-x)+\sin xy$
4. $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial x\,\partial y}-2\dfrac{\partial^2 z}{\partial y^2}=(y-1)\,e^x$ (*Q. Bank, U.P. 2002*)
   Ans. $z=f_1(y+x)+f_2(y-2x)+(y-2)\,e^x$
5. $r-t=\tan^3 x\tan y-\tan x\tan^3 y$ **Ans.** $z= f_1(y+x)+f_2(y-x)+\dfrac{1}{2}\tan x\tan y$
6. $(D^2-6\,D\,D'+9\,D'^2)\,z=12x^2+36\,xy$ **Ans.** $z=f_1(y+3x)+x\,f_2(y+3x)+10x^4+6x^3y$

REDMI NOTE 8
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...6 NON-HOMOGENEOUS LINEAR EQUATIONS

The linear differential equations which are not homogeneous are called Non-homogeneous ...ear Equations.

For example, $3\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x\partial y}+4\frac{\partial^2 z}{\partial y^2}+5\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}+z=0$

$$f(D,D') = f_1(x,y)$$

Its solution, $z = \text{C.F.} + \text{P.I.}$

Complementary Function: Let the non-homogeneous equation be

$$(D-mD'-a)z=0 \quad \text{or} \quad \frac{\partial z}{\partial x}-m\frac{\partial z}{\partial y}-az=0$$

$$p-mq=az$$

The Lagrange's subsidiary equations are $\frac{dx}{1}=\frac{dy}{-m}=\frac{dz}{az}$

From first two relations, we have $-mdx=dy$

$$dy+mdx=0 \Rightarrow \quad y+mx=c_1 \quad \text{... (1)}$$

and from first and third relation, we have

$$dx=\frac{dz}{az} \Rightarrow x=\frac{1}{a}\log z+c_2 \Rightarrow z=c_3e^{ax} \quad \text{... (2)}$$

From (1) and (2), we have $z=e^{ax}\phi(y+mx)$

Similarly the solution of $(D-mD'-a)^2z=0$ is

$$z=e^{ax}\phi_1(y+mx)+xe^{ax}\phi_2(y+mx)$$

...7 IF THE EQUATION IS OF THE FORM

$$(\alpha D+\beta D'+\gamma)z=0$$

$$\Rightarrow \quad \alpha p+\beta q=-\gamma z$$

It is of Lagrange's form.

Lagrange's subsidiary equations are $\frac{dx}{\alpha}=\frac{dy}{\beta}=\frac{dz}{-\gamma z}$

From first two, we have $\alpha y-\beta x=C_1$

From first and last, we have $\frac{dz}{z}=-\frac{\gamma}{\alpha}dx$

$$\Rightarrow \quad \log z=-\frac{\gamma}{\alpha}x+\log c_2 \quad \Rightarrow \quad z=c_2e^{-\frac{\gamma}{\alpha}x}=\phi(C_1)e^{-\frac{\gamma}{\alpha}x}$$

$$\Rightarrow \quad z=C_2e^{-\frac{\gamma}{\alpha}x}=\varphi(\alpha y-\beta x)$$

where φ is an arbitrary function.

Example 30. *Solve* $(D+D'-2)(D+4D'-3)z=0$

Solution. The equation can be rewritten as $\{D-(-D')-2\}\{D-(-4D')-3\}z=0$

Hence the solution is

$$z=e^{2x}\phi_1(y-mx)+e^{3x}\phi_2(y-4mx) \quad \textbf{Ans.}$$

Example 31. *Solve* $(D+3D'+4)^2 z=0$

Solution. The equation is rewritten as

$$[D-(-3D')-(-4^2)]z=0$$

Hence the solution is $z=e^{-4x}\phi_1(y-3x)+xe^{-4x}\phi_2(y-3x)$ **Ans.**

Example 32. *Solve* $r+2s+t+2p+2q+z=0$

Solution. The equation is rewritten as

$$(D^2+2DD'+D'^2+2D+2D'+1)z=0$$

$$\Rightarrow \quad [(D+D')^2+2(D+D')+1]z=0$$

$$\Rightarrow \quad (D+D'+1)^2 z=0$$

$$\Rightarrow \quad [D-(-D')-(-1)]^2 z=0$$

Hence the solution is

$$z=e^{-x}\phi_1 y(y-x)+xe^{-x}\phi_2(y-x)$$ **Ans.**

Example 33. *Solve* $r-t+p-q=0$

Solution. The equation is rewritten as

$$(D^2-D'^2+D-D')z=0$$

$$\Rightarrow \quad [(D-D')(D+D')+1(D-D')]z=0$$

$$\Rightarrow \quad (D-D')(D+D'+1)z=0$$

Hence the solution is

$$z=\phi_1(y+x)+e^{-x}\phi_2(y-x)$$ **Ans.**

EXERCISE 14.7

Solve the following equations

1. $(D-D')(D+D'-3)z=0$ **Ans.** $z=\phi(y+x)+e^{3x}\phi_2(y-x)$
2. $(D-D'-1)(D-D'-2)z=0$ **Ans.** $z=e^{x}\phi_1(y+x)+e^{2x}\phi_2(y+x)$
3. $(D+D'-1)(D+2D'-2)z=0$ **Ans.** $z=e^{x}\phi_1(y-x)+e^{2x}\phi_2(y-2x)$
4. $(D^2+DD'+D'-1)z=0$ **Ans.** $z=e^{-x}\phi_1(y)+e^{x}\phi_2(y-x)$
5. $(D^2-DD'-2D'^2+2D+2D')z=0$ **Ans.** $z=\phi_1(y-x)+e^{-2x}\phi_2(y+2x)$
6. $[D^2-D'^2+D+3D'-2]z=0$ **Ans.** $z=e^{-2x}\phi_1(y+x)+e^{x}\phi_2(y-x)$
7. $(D^2-a^2D'^2+2abD+2abD')z=0$ **Ans.** $z=\phi_1(y-ax)+e^{-2abx}\phi_2(y+ax)$
8. $t+s+q=0$ **Ans.** $z=\phi_1(x)+e^{-x}\phi_2(y-x)$
9. $(D+D'-1)(D+2D'-3)z=0$ **Ans.** $z=e^{x}\phi_1(y-x)+e^{3x}\phi_2(y-2x)$
10. $(D-2D'+5)^2 z=0$ **Ans.** $z=e^{-5x}\phi_1(y+2x)+xe^{-5x}\phi_2(y+2x)$

Particular Integral

Case I. $\dfrac{1}{F(D,D')}e^{ax+by}=\dfrac{1}{F(a,b)}e^{ax+by}$