Introduction to DIP

Image: An Image is a 2D function that represents a measure of some characteristic such as brightness or colour of a viewed scene.

Pixel is the smallest controllable element of a picture represented T.V Screen on the screen.

DIP - Refers to processing of digital images by means of digital computer.

Low Level - I/P & O/P are images
Mid level - outputs are extracted from I/P img.
High Level - An ensemble of recognition of individual objects.

Applications of DIP →

(a) Gamma Ray Imaging (Nuclear Medicine)
(b) X-Ray Imaging (Med. diagnostic)
(c) Imaging in U.V Band (lasers)
(d) Imaging in Radio Band (MRI) (Magnetic Resonance Imaging)
(e) Imaging in Visible & Infrared Band (Remote Sensing, Micro Inspection)
(f) Imaging In ~~Radio Band~~ Microwave - (Radar)

Steps in DIP -
— Image Acquisition
(a) Image Enhancement
(b) Image Restoration
(c) Color Image Processing
(d) Wavelets (To convert image into diff resolutions)
(e) Image Segmentation (To divide image into segments)

(f) Image Compression (To compress image or reduce size)

(g) Image Recognition (To identify that this is my this particular image)

(h) Morphological Processing.
(To know various things about image so that we can describe the image{ }structure of the image)

Components of DIP →

(a) Image Sensors
(b) Specialized Image processing Hardware.
(c) Computer
(d) Image Processing Software
(e) Mass Storage → Image → process → O/p
(f) Image Display → □
(g) Hardcopy Devices
(h) Networking

Sampling & Quantization

★ Need for Sampling & Quantization →

Mostly the output of image sensors is in the form of Analog Signal.

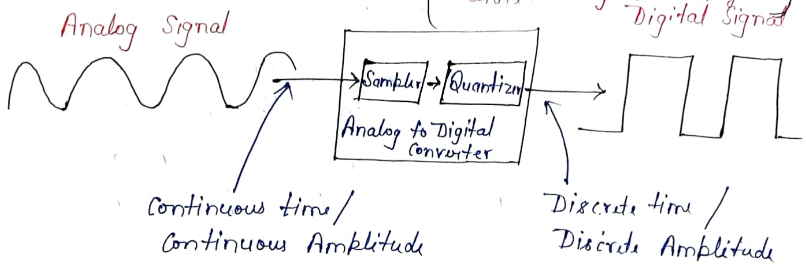
Now the problem is that we can't apply digital image processing & its techniques on analog signals. This is due to the fact that cannot store the output of image sensors which are in the form of analog signals because it requires Infinite memory to store a signal that can have infinite values.

So we have to convert this analog signal into digital signal.

To create a digital image we need to convert the continuous data into digital form. This conversion from analog to digital involves two process:

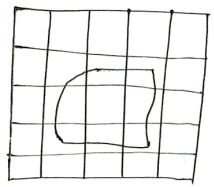
(i) Sampling
(ii) Quantization

black - low Intensity
gray - slightly higher Intensity
Blue - Medium intensity
light blue - high intensity
White - very high intensity

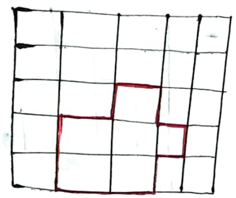


Sampling: Digitization of co-ordinate values

Quantization: Digitization of amplitude values



Continuous Image



Result of Image after Sampling & Quantization

Unit - 2 Image Enhancement in Spatial Domain

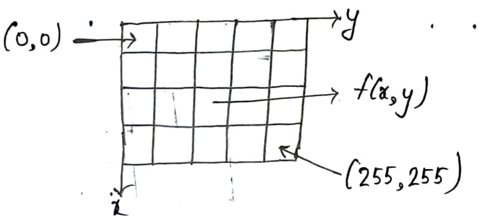
Spatial Domain → The term spatial domain means working in the given space, in the case, the image.

* It implies working with the pixel values or in other words, working directly with the raw data.

$f(x,y)$ → be the original image (It tells us where the image color is dark or light)
where f → gray level value, and
(x,y) → are image co-ordinates

* for a 8-bit image, f can take values from 0-255 where 0 represents black, 255 represents white & all the intermediate values represent shades of gray.

* In an image of size 256 X 256 x and y can take values from (0,0) to (255,255)



* The modified image can be expressed as

$$g(x,y) = T[f(x,y)]$$

where $f(x,y)$ → is the original image
T → is the transformation applied to get a new modified image $g(x,y)$

* Spatial domain enhancement can be carried out in 2 ways:

(a) Point Processing
(b) Neighbourhood processing

Image Enhancement

Spatial Domain Technique	Frequency Domain Technique
(1) The SD methods are those which operate directly on the individual pixels, composing image.	(1) In FD method, we process an image by changing the frequency components in an image.
(2) SD processing method includes point processing, mask process & global process.	(2) It includes only global process.
(3) SD image enhancement techniques are gray scale manipulation, histogram equalization, image smoothing & image sharpening.	(3) FD enhancement techniques are filtering & homomorphic filtering.
(4) In SD image enhancement, some convolution process/operation is involved, hence computation cost is higher & processes are complex.	(4) It does not involve any convolution operation, so computation cost becomes lesser & processes become simpler.

Point Pro. T.

Digital Negative

(Gray level ...

* As the name suggests, negative simply means inverting the gray levels i.e, black in the original image will now look white and vice versa.

* If we consider a 8-bit image, then the digital negative image can be obtained by using a simple transformation given by,

$$\boxed{S = 255 - r}$$

∵ In general,

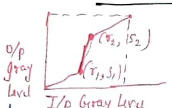
$$\boxed{S = (L-1) - r} \quad ; \ L \rightarrow \text{no of gray levels}$$

* Example → displaying of an X-ray Image.

Point Processing Technique of Spatial Domain Enhancement

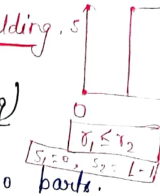
↓

Contrast Stretching



* Many times we obtain low contrast images due to poor illuminations or due to wrong setting of the lens aperture.
* The idea behind contrast stretching is to increase the contrast of the images by making the dark portions darker & the bright portions brighter.
* The contrast stretching transformation increases the dynamic range of the modified image.

* Extreme contrast stretching yields thresholding.



Gray Level Slicing (Intensity Slicing)

* Thresholding splits the gray level into two parts.
* At times, we need to highlight a specific range of gray values, for example enhancing the flaws in an X-ray or a CT image.

* In such circumstances, we use a transformation known as gray level slicing.

* It looks similiar to the thresholding function except that there we select a band of gray level values.

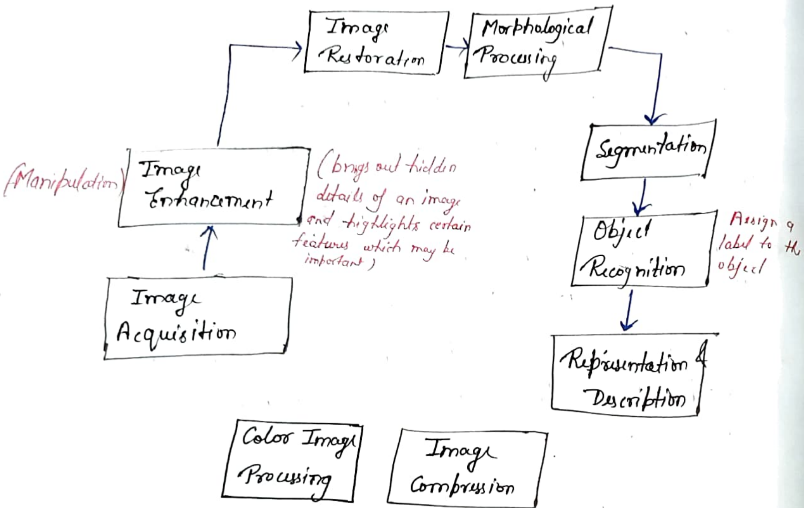
Bit Plane Slicing

* In this technique, we find out the contribution made by each bit to the final image.

* Let consider an image as 256 x 256 x 8
In this 256 x 256 → no of pixels present in the image and 8 → no of bits required to represent each pixel.
where 8-bits simply mean 2^8 or 256 gray levels
* Now each pixel will be represented by 8-bits for ex → black → 00000000 , white - 11111111 and b/w them, 254 gray levels are accomodated.
* In bit plane slicing, we see the importance of each bit in the final image.

Key Stages in DIP

(improving the appearance of an image)



Power Law Transformation

* Non-linearities encountered during image capturing, printing & displaying can be corrected using gamma correction.

* Hence Gamma correction is important if the image needs to be displayed on the computer.

* formula for Power Law Transformation is

$$\boxed{g(x,y) = c \times f(x,y)^{\gamma}}$$

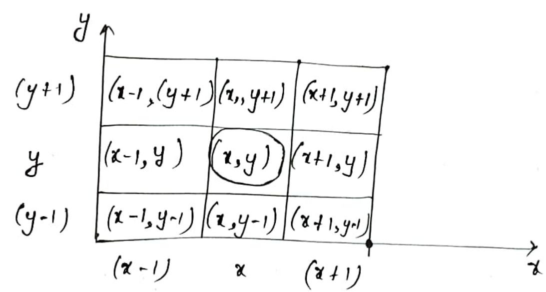
where c and γ are positive constants

"Simply it makes all the non-linear pt to the linear pt in terms of curve. Connects the pt to remove the non-linearity and it increases the dynamic range of the image."

* Power Law Transformation can also be used to improve the dynamic range of an image.

Relationship b/w Pixels Neighbourhood and Adjacency of Pixels

(1) Neighbourhood of Pixels -



(a) $N_4(P)$ 4 - Neighbours : the set of horizontal & Vertical neighbours.

	(x, y+1)	
(x-1, y)	(x, y)	(x+1, y)
	(x, y-1)	

(b) $N_D(P)$ diagonal neighbours - the set of 4 diagonal neighbours

$(x-1, y+1)$		$(x+1, y+1)$
	(x, y)	
$(x-1, y-1)$		$(x+1, y-1)$

(c) $N_8(P)$ - 8 neighbours

Union of 4-neighbour and diagonal neighbours.

$$\boxed{N_8(P) = N_4(P) + N_D(P)}$$

(2) Connectivity / Adjacency ⇝

A two pixels that are neighbours and have the same gray level are adjacent.

(a) 4-adjacency

Binary Image

8-adjacent

$$\begin{matrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{matrix}$$

4-adjacent

$V = \{1\}$

(b) 8-adjacency.

Gray Scale Image.

$$\begin{matrix} 54 & 10 & 100 & 8 \\ 81 & 150 & 2 & 34 \\ 201 & 200 & 3 & 45 \\ 7 & 70 & 147 & 56 \end{matrix}$$

8-adj

4-adj.

$V = \{1, 2, 3, \ldots 10\}$

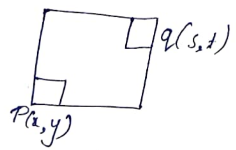
(c) m-adjacency (mixed adjacency)

1. Both pixels should be D-adjacent
2. Common 4-adjacent value should not be included in 'V'

* If two pts are 4-adjacent then they are m-adjacent

Distance Measures b/w Pixels.

Let P and q be pixels with co-ordinates (x, y) & (s, t) respectively.



(1) Euclidean Distance ⇝

$$D_e(P, q) = [(x-s)^2 + (y-t)^2]^{1/2}$$

(2) City Block Distance.

$$D_4(P, q) = |x-s| + |y-t|$$

Ex - The pixels with D_4 distance ≤ 2 from (x, y) form the following contours of constant distance.

$$\begin{matrix} & & 2 & & \\ & 2 & 1 & 2 & \\ 2 & 1 & 0 & 1 & 2 \\ & 2 & 1 & 2 & \\ & & 2 & & \end{matrix}$$

(3) Chessboard Distance →

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

$D_8(p,q) = m$

Ex- The pixels with D_8 distance ≤ 2 from (x,y) from the following contours of constant distance.

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

(mixed distance)

(4) D_m Distance – It is defined as the shortest m-path between the points. This distance between 2 pixels depends on the values of the pixels along the path as well as the values of their neighbours.

$P(x,y)$ – – – – $q(s,t)$

★ Note The D_4 and D_8 distances b/w p & q are independent of any paths that might exist b/w the pts because these distances involve only the co-ordinates of the points.

Ques An img segment is given. Let V be the set of gray level values used to define connectivity in the img. Compute D_4, D_8 and D_m distances b/w pixels 'p' & 'q' for

(i) $V = \{2, 3\}$

(ii) $V = \{2, 6\}$

(0,0)

2(p)	3	2	6	1
6	2	3	6	2
5	3	2	3	5
2	4	3	5	2
4	5	2	3	6 (q)

(4,4)

Coordinates of $P(x,y) = (0,0)$

coordinates of $q(s,t) = (4,4)$

$$D_4(p,q) = |x-s| + |y-t|$$
$$= |0-4| + |0-4|$$
$$= 4 + 4$$
$$= 8$$

$$D_8(p,q) = \max(|x-s|, |y-t|)$$
$$= \max(|0-4|, |0-4|)$$
$$= \max(4, 4)$$
$$= 4$$

(i) $V = \{2, 3\}$ There is no path b/w p & q as (q)6 is not included in the set V

(ii) $V = \{2, 6\}$ There is no path b/w p & q

Ques Consider the following img segment. Compute D_m, D_4 and D_8 distances b/w pixels 'p' and 'q' for $r = \{0, 1\}$ where r is the set of gray level values used to define connectivity. Repeat for $r = \{1, 2\}$

$$\begin{array}{cccc} 3 & 1 & 2 & 1\,(q) \\ 0 & 2 & 0 & 2 \\ 1 & 2 & 1 & 1 \\ 1\,(p) & 0 & 1 & 2 \end{array}$$

Coordinates of $p(x,y) = (0,0)$

coordinates of $q(s,t) = (3,3)$

$$D_4(p,q) = |x-s| + |y-t|$$
$$= |0-3| + |0-3|$$
$$= 3+3$$
$$= 6$$

$$D_8(p,q) = \max(|x-s|, |y-t|)$$
$$= \max(|-3|, |-3|)$$
$$= 3$$

for D_m (i) $r = \{0, 1\}$

$\boxed{5 \text{ units}}$

$$\begin{array}{cccc} 3 & 1 & 2 & 1\ q \\ 0 & 2 & 0 & 2 \\ 1 & 2 & 1 & 1 \\ p\ 1 & 0 & 1 & 2 \end{array}$$

(ii) $r = \{1, 2\}$

$$\begin{array}{cccc} 3 & 1 & 2 & 1\,(q) \\ 0 & 2 & 0 & 2 \\ 1 & 2 & 1 & 1 \\ (p)\,1 & 0 & 1 & 2 \end{array}$$

= 6 units.

shortest path = 6 units

Arithmatic Operations Between Images

These are array operations which are carried out b/w corresponding pixel pairs. The four arithmetic operations are denoted as.

Pixel pairs.

$$s(x,y) = f(x,y) + g(x,y)$$
$$d(x,y) = f(x,y) - g(x,y)$$
$$p(x,y) = f(x,y) \times g(x,y)$$
$$v(x,y) = f(x,y) \div g(x,y)$$

Few Important Points -

→ If the result is a floating pt no, round off its value.

→ If the result is above the pixel range, select the max range value.

→ If the result is below the pixel range, select the min range value.

→ If the result is infinity, write it as zero.

Addition

$$\begin{bmatrix} 0 & 100 & 10 \\ 4 & 0 & 10 \\ 8 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 10 & 100 & 5 \\ 2 & 0 & 0 \\ 0 & 10 & 10 \end{bmatrix} = \begin{bmatrix} 100 & 200 & 15 \\ 6 & 0 & 10 \\ 8 & 10 & 15 \end{bmatrix}$$

Uses -

- Addition of noisy images for noise reduction
- Image averanging in the field of astronomy.

Subtraction -

$$\begin{bmatrix} 0 & 100 & 10 \\ 4 & 0 & 10 \\ 8 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 10 & 100 & 5 \\ 2 & 0 & 0 \\ 0 & 10 & 10 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 5 \\ 2 & 0 & 10 \\ 8 & 0 & 0 \end{bmatrix}$$

Uses -

→ Enhancement of differences b/w images.
→ Mask mode radiography in medical imaging.

Multiplication -

$$\begin{bmatrix} 0 & 100 & 10 \\ 4 & 0 & 10 \\ 8 & 0 & 5 \end{bmatrix} * \begin{bmatrix} 10 & 100 & 5 \\ 2 & 0 & 0 \\ 0 & 10 & 10 \end{bmatrix} = \begin{bmatrix} 0 & 255 & 50 \\ 8 & 0 & 0 \\ 0 & 0 & 50 \end{bmatrix}$$

Uses → Shading Correction.

Masking or Region of interest (ROI) operations.

Division -

$$\begin{bmatrix} 0 & 100 & 10 \\ 4 & 0 & 10 \\ 8 & 0 & 5 \end{bmatrix} \quad \begin{bmatrix} 10 & 100 & 5 \\ 2 & 0 & 0 \\ 0 & 10 & 10 \end{bmatrix} = \begin{bmatrix} 0 & 255 & 50 \\ 8 & & \\ & & \end{bmatrix}$$

Uses - Shading correction.

Logical Operations on Images -

(a) AND Operation (Both input 1 then O/p 1 else 0)
(b) OR Operation (If atleast any I/p 1 then O/p 1 else 0)
(c) NOT Operation (If I/p = 0 then O/p = 1 vice versa)

Point Operations

PTO

Bit Plane Slicing Numericals

Ques Given the following 3x3 image find its bit planes.

Ans Bit plane slicing divides the image into bit planes on the basis of contribution of each bit in the image.

1	2	3
4	5	0
7	2	1

001	010	011
100	101	000
111	010	001

How many bit planes it will make?

The no of bit represents the single pixel of my image.

MSB

0	0	0
1	1	0
1	0	0

middle

0	1	1
0	0	0
1	1	0

LSB

1	0	1
0	1	0
1	0	1

Point Operations

They are a method of image processing in which each pixel in the o/p image is only dependent upon the corresponding pixel in the input image and is independent of its location or neighbouring pixels.

$$\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & - \\ - & - \end{bmatrix}$$

Let r be the gray value at a pt (x,y) of the input image $f(x,y)$ and s be the gray value at a pt (x,y) of the o/p image $g(x,y)$, then the point operation can be defined as:

$$s = T(r)$$

Where T is the pt. operation of a certain gray-level mapping relationship b/w the original img and o/p img.

(1) Digital Negative

(2) $s = (L-1) - r$

$= 8 - 4 - 1$

$= 3$

$7 - 3 = 4$

$7 - 5 = 2$

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

$\boxed{2^3 = 8}$

3 bits req'd to represent a pixel

3	4	2	5
4	1	3	1
5	5	1	2
0	1	3	6

(2) Thresholding with $T = 4$

$L = 8$ $\boxed{2^3 = 8}$

$L - 1 = 7$

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

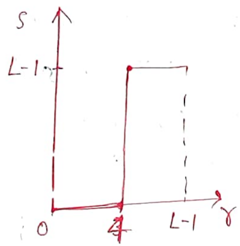
$$S = \begin{cases} L-1 = 7 ; & r \geq 4 \\ 0 ; & r < 4 \end{cases}$$

$r = 0\ 1\ 2\ 3 \longrightarrow s = 0$

$r = 4\ 5\ 6\ 7 \longrightarrow s = 7$

7	0	7	0
0	7	7	7
0	0	7	7
7	7	7	0

O/p image



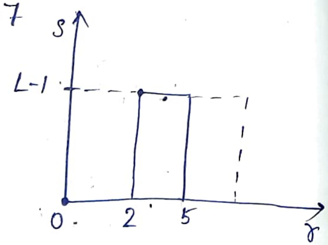
(3) Clipping with $r_1 = 2$ & $r_2 = 5$

7	7	7	7
7	0	7	0
7	7	0	7
0	0	7	0

$$S = \begin{cases} L-1 = 7 ; & 2 \leq r \leq 5 \\ 0 , & \text{otherwise} \end{cases}$$

$r = 0, 1, 6, 7 \longrightarrow s = 0$

$r = 2, 3, 4, 5 \longrightarrow s = 7$



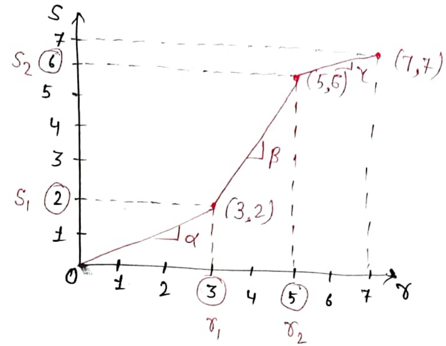
(4) Bit Plane Slicing - Done.

(5) Contrast Stretching →

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

Given $r_1 = 3$ & $r_2 = 5$

$s_1 = 2$ & $s_2 = 6$



$$M = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\alpha = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{3 - 0} = \frac{2}{3} = 0.66$$

$$\beta = \frac{6 - 2}{5 - 3} = \frac{4}{2} = 2$$

$$\gamma = \frac{7 - 6}{7 - 5} = \frac{1}{2} = 0.5$$

$$S = \begin{cases} \alpha . r & 0 \leq r < 3 & 0, 1, 2 \\ \beta . (r - r_1) + s_1 & 3 \leq r < 5 & 3, 4 \\ \gamma (r - r_2) + s_2 & 5 \leq r \leq 7. & 5, 6, 7 \end{cases}$$

r	S
0	$S = \alpha \cdot r = 0.66 \times 0 = 0 = 0$
1	$S = \alpha \cdot r = 0.66 \times 1 = 0.66 \simeq 1$
2	$S = \alpha \cdot r = 0.66 \times 2 = 1.32 \simeq 1$
3	$S = \beta(r - r_1) + S_1 = 2(3-3) + 2 = 2$
4	$S = \beta(r - r_1) + S_1 = 2(4-3) + 2 = 4$
5	$S = \gamma(r - r_2) + S_2 = 0.5(5-5) + 6 = 6$
6	$S = \gamma(r - r_2) + S_2 = 0.5(6-5) + 6 = 6.5 \simeq 7$
7	$S = \gamma(r - r_2) + S_2 = 0.5(7-5) + 6 = 7$

O/p Image

4	2	6	1
2	7	4	7
1	1	7	6
7	7	4	1

(6) Intensity level Slicing

Que Intensity level slicing with $r_1 = 3$ & $r_2 = 5$

(i) Without Background

(ii) With Background

(i)

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

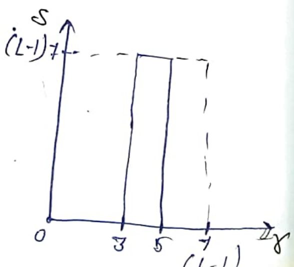
$$S = \begin{cases} L-1 = 7 & ; 3 \le r \le 5 \\ 0 & ; \text{otherwise} \end{cases}$$

$r = 0, 1, 2, 6, 7 \Rightarrow S = 0$

$r = 3, 4, 5 \Rightarrow S = 7$

7	7	7	0
7	0	7	0
0	0	0	7
0	0	7	0

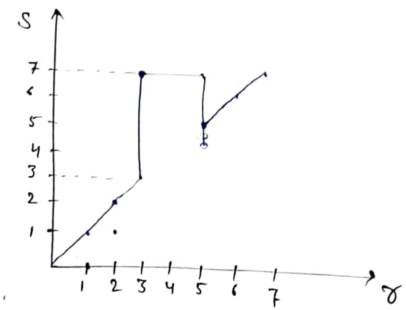
O/p Image.



(ii) With Background

$$S = \begin{cases} L-1 = 7 & ; 3 \le r \le 5 \\ r & ; \text{otherwise} \end{cases}$$

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1



7	7	7	2
7	6	7	6
2	2	6	7
7	6	7	1

O/p Image with background

Logarithmic Transformation

$$\boxed{S = c \log(1+r)}$$

S = pixel value of the O/p Image

r = pixel value of the I/p Image

Where c is a constant & it is assumed that $r \geq 0$

110	120	90
91	94	98
90	91	99

$2^7 = 128$

$\boxed{L = 2^n = 128}$

$\boxed{n = 7}$

(i) $c = 1$

(ii) $c = \dfrac{L}{\log_{10}(1+L)}$

(i) $c = 1$

r	s
110	$S = 1 \cdot \log_{10}(1+110) = \log(111) = 2.04 \approx 2$
120	$s = 1 \cdot \log(1+120) = \log(121) = 2.08 \approx 2$
90	$s = 1 \cdot \log(1+90) = \log(91) = 1.95 \approx 2$
91	$s = 1 \cdot \log(1+91) = \log(92) = 1.96 \approx 2$
94	$s = 1 \cdot \log(1+94) = \log(95) = 1.97 \approx 2$
98	$s = 1 \cdot \log(1+98) = \log(99) = 1.99 \approx 2$
99	$s = 1 \cdot \log(1+99) = \log(100) = 2 \approx 2$

$$\boxed{S = c(\log_{10}(1+r))}$$

$$\begin{bmatrix} 110 & 120 & 90 \\ 91 & 94 & 98 \\ 90 & 91 & 99 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

(ii) $c = \dfrac{L}{\log(1+L)} = \dfrac{128}{\log(129)} = 60.66 \approx 61$

$61 \times 2.04 = 124.44$

$61 \times 2.08 = 126.88$

$61 \times 1.95 = 118.95$

$61 \times 1.96 = 119.56$

$61 \times 1.97 = 120.17$

$61 \times 1.99 = 121.39$

$61 \times 2 = 122$

$$\begin{bmatrix} 124 & 127 & 119 \\ 120 & 120 & 121 \\ 119 & 120 & 122 \end{bmatrix}$$

O/P Image.

(8) Power Law Transformation.

$$\boxed{S = cr^{\gamma}}$$

Where c & γ are positive constants

Given: $c = 1$ & $\gamma = 0.2$ calculate for:

$$\begin{bmatrix} 110 & 120 & 90 \\ 91 & 94 & 98 \\ 90 & 91 & 99 \end{bmatrix}$$

$$\boxed{S = c\, r^{\gamma}}$$
$$= 1 \cdot r^{0.2}$$
$$= (r)^{0.2}$$

r	S
110	$1 \cdot (110)^{0.2} = 2.56 \approx 3$
120	$1 \cdot (120)^{0.2} = 2.60 \approx 3$
90	$1 \cdot (90)^{0.2} = 2.45 \approx 2$
91	$1 \cdot (91)^{0.2} = 2.46 \approx 2$
94	$1 \cdot (94)^{0.2} = 2.48 \approx 2$
98	$1 \cdot (98)^{0.2} = 2.50 \approx 3$
90	$1 \cdot (90)^{0.2} = 2.45 \approx 2$
91	$1 \cdot (91)^{0.2} = 2.46 \approx 2$
99	$1 \cdot (99)^{0.2} = 2.50 \approx 3$

$$\begin{bmatrix} 3 & 3 & 2 \\ 2 & 2 & 3 \\ 2 & 2 & 3 \end{bmatrix}$$

O/P Image

Image Enhancement →

The process of manipulating the image so that the result is more suitable than the original for specific applications.

Technique of Image Enhancement

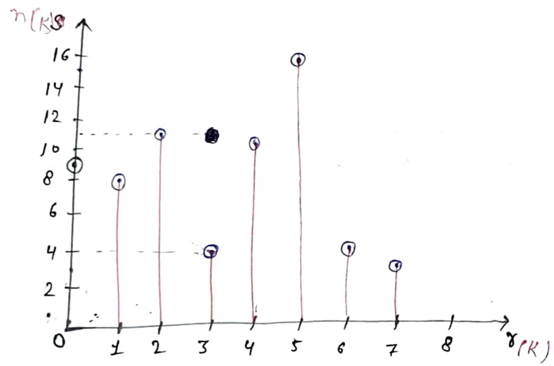
Spatial Domain Frequency Domain Combination

Spatial Domain

(a) Histogram Equilization

Ques Perform the histogram equilization for an 8 x 8 image shown below

Gray Level	0	1	2	3	4	5	6	7
Pixels (No)	9	8	11	4	10	15	4	3

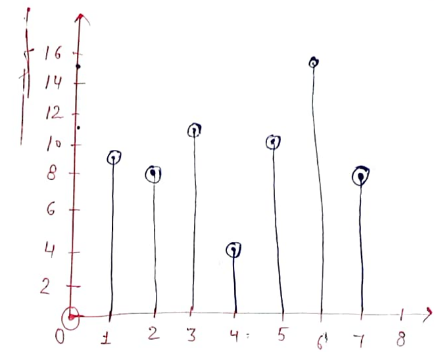


Histogram of I/p Image

Gray Levels	No of pixels	$P(r_K) = n_K/n$ (PDF)	S_K (CDF)	$S_K \times 7$	Histogram Equilization level
0	9	0.141	0.141	0.987	1
1	8	0.125	0.266	1.862	2
2	11	0.172	0.438	3.066	3
3	4	0.0625	0.5005	3.5035	4
4	10	0.156	0.6565	4.5955	5
5	15	0.234	0.8905	6.2335	6
6	4	0.0625	0.953	6.671	7
7	3	0.047	1	7	7
	n = 64				

PDF - Probability distribution Function.
CDF - Cummulative distribution Function.

Gray Levels	1	2	3	4	5	6	7
No of Pixels	9	8	11	4	10	15	7



(Histogram of Processed Image)

(2) Histogram Equalization

Gray level	Pixels	$P(r_K)$ PDF	S(K) CDF	$S_K \times 7$	Histogram Equalization level
0	0	0	0	0	0
1	8	0.32	0.32	2.24	2
2	8	0.32	0.64	4.48	4
3	2	0.08	0.72	5.04	5
4	0	0	0.72	5.04	5
5	7	0.28	1.00	7	7
6	0	0	01	7	7
7	0	0	1	7	7

n = 25

I/p Image.

1	2	1	1	1
2	5	3	5	2
2	5	5	5	2
2	5	3	5	2
1	1	1	2	1

max val = 5

$\boxed{2^3 = 8}$

L = 8

$\boxed{L - 1 = 7}$

Gray levels	0	2	4	5	7
No of Pixels	0	8	8	2	7

2	4	2	2	2
4	7	5	7	4
4	7	7	7	4
4	7	5	7	4
2	2	2	4	2

(2) Histogram Matching (Specification)

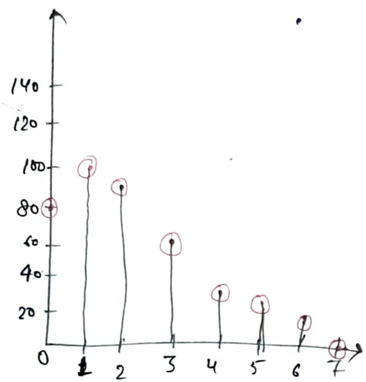
(1) Given below two Histograms

(i)

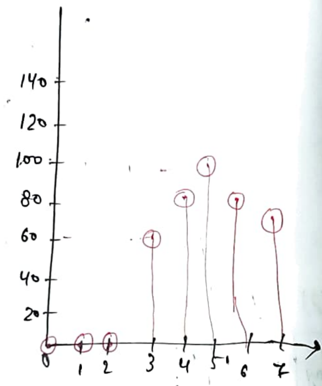
GL	0	1	2	3	4	5	6	7
NP	80	100	90	60	30	20	10	0

(ii)

GL(r)	0	1	2	3	4	5	6	7
NP	0	0	0	60	80	100	80	70



(i)



(ii)

Equalize Histogram:

Gray Level	nk	$P(r_k) = n_k/n$ (PDF)	SK (CDF)	SK×7	Histogram Equalization	New
0	80	0.20	0.20	1.4	1	80
1	100	0.25	0.45	3.15	3	100
2	90	0.23	0.68	4.76	5	90
3	60	0.15	0.83	5.81	6	90 (60+30)
4	30	0.07	0.9	6.3	6	
5	20	0.05	0.95	6.65	7	
6	10	0.02	0.97	6.79	7	30
7	0	0	0.97	6.79	7	

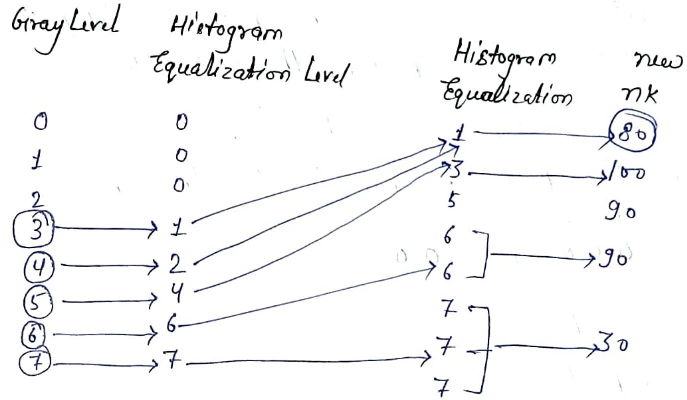
$n = 390$

Gray Level	nk	$P(r_k) = n_k/n$ (PDF)	SK (CDF)	SK×7	Histogram Equalization Level
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	60	0.15	0.15	1.05	1
4	80	0.20	0.35	2.45	2
5	100	0.25	0.60	4.2	4
6	80	0.20	0.8	5.6	6
7	70	0.17	0.97	6.79	7

Mapping

→ Take first & last column of Histogram (ii)

→ Take the last 2 columns of Histogram (i)



GL	0	1	2	3	4	5	6	7
NP	0	0	0	80	80	100	90	30

Fundamentals of Spatial Filtering →

The name 'filter' is borrowed from frequency domain processing. It basically means (refers to) (accepting) or rejecting certain frequencies is called lowpass filter.

Ex - A filter that passes low frequencies is called lowpass filter.

We can accomplish a similar smoothing directly on the image itself using spatial filtering filters (Also called masks, kernals, templates and windows)

(1) Convolution

Ques 1 Let $I = \{0, 0, 1, 0, 0\}$ be an image. Using the mask $K = \{3, 2, 8\}$, perform the convolution.

$I = \{0, 0, 1, 0, 0\}$ $K = \{3, 2, 8\}$

In convolution process we have to rotate the kernal by 180°

8 2 3 8 2 3
0 0 0 0 1 0 0 0 0

(ii) Initial posn →

Template

8 2 3
0 ~~0~~ 0 0 1 0 0 0 0
0

(Now multiply respective pair only)

$(8\times0)+(2\times0)+(3\times0)$
$=0$ → just below the center bit.

Output is 0 located at the center pixel.

(iii) Position After One Shift →

Template is shifted by 1 bit.

8 2 3
0 0 0 0 1 0 0 0 0
0 0

from last step ← 0 ; 0 → from this step

$(8\times0)+(2\times0)+(3\times0)$

0 → at center

Output is 0.

(iv) Position after 2 Shifts →

Template is shifted again.

8 2 1
0 0 0 0 1 0 0 0 0
0 0 3

$(8\times0)+(2\times0)+(3\times1)$
$=3$

Output produced is 3.

(v) Position after 3 shift.

Template is shifted again

8 2 1
0 0 0 0 1 0 0 0 0
0 0 3 2

$(8\times0)+(2\times1)+(1\times0)$
$=2$

Output produced is 2.

(vi) Position after 4 shifts

Template is shifted again.

8 2 3
0 0 0 0 1 0 0 0 0
0 0 3 2 8

Output produced is 8

(vii) Position after 5 shifts

Template is shifted again

8 2 3
0 0 0 0 1 0 0 0 0
0 0 3 2 8 0

Output produced is 0.

(2) Correlation →

Ques Let $I = \{0, 0, 1, 0, 0\}$ be an image using the mask $K = \{3, 2, 8\}$, perform the correlation.

$I = \{0, 0, 1, 0, 0\}$, $K = \{3, 2, 8\}$

(i) Zero padding process for correlation

```
3 2 8             3 2 8
0 0 0 0 1 0 0 0 0
```

(ii) Initial position →

Template

```
3 2 8
0 0 0 0 1 0 0 0 0
0
```

Output produced is 0.

(iii) Position after one shift →

```
  3 2 8
0 0 0 0 1 0 0 0 0
  0 0
```

Output produced is 0.

(iv) Position after 2 shift →

```
    3 2 8
0 0 0 0 1 0 0 0 0
  0 0 8
```

Output produced is 8.

(v) Position after 3 shifts

Template

```
      3 2 8
0 0 0 0 1 0 0 0 0
  0 0 8 2
```

Output produced is 2

(vi) Position after 4 shifts.

Template

```
        3 2 8
0 0 0 0 1 0 0 0 0
  0 0 8 2 3
```

Output produced is 3.

(vii) Position after 5 shifts

Template

```
          3 2 8
0 0 0 0 1 0 0 0 0
  0 0 8 2 3 0
```

Output produced is 0.

(viii) final state (Position)

Template

```
              3 2 8
0 0 0 0 1 0 0 0 0
  0 0 8 2 3 0 0
```

Output produced is 0, Further shifting exceeds the range.

So in the final position, the output produced is $\{0, 0, 8, 2, 3, 0, 0\}$.

Ques Let $I = \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$ be an image and $K = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ be a Kernal (mask). Perform convolution & correlation.

(i) **Convolution -**

Rotate the Kernal by 180°

$K' = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$

$\begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}$

3—4
1—2
↓
$\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$

placing mask here

(a)
$$\begin{matrix} 0_4 & 0_3 & 0 & 0 \\ 0_2 & 3_1 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$$K' = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} \longrightarrow \begin{matrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

(b)
$$\begin{matrix} 3 & 0_4 & 0_3 & 0 \\ 0 & 3_2 & 3_1 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \longrightarrow \begin{matrix} 3 & 9 & 0 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$(4 \times 0) + (3 \times 0) + (2 \times 3) + (3 \times 1)$
$= 0 + 0 + 6 + 3 = 9$

(c)
$$\begin{matrix} 3 & 9 & 0_4 & 0_3 \\ 0 & 3 & 3_2 & 0_1 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \longrightarrow \begin{matrix} 3 & 9 & 6 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$(4 \times 0) + (3 \times 0) + (2 \times 3) + (1 \times 0)$
$= 6$

(d)
$$\begin{matrix} 3 & 9 & 6 & 0 \\ 0_4 & 3_3 & 3 & 0 \\ 0_2 & 3_1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \longrightarrow \begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 3 & 3 & 0 \\ 0 & 9 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

(e)
$$\begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 3_4 & 3_3 & 0 \\ 0 & 3_2 & 3_1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \longrightarrow \begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 30 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

(f)
$$\begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 30 & 3_4 & 0_3 \\ 0 & 3 & 3_2 & 0_1 \\ 0 & 0 & 0 & 0 \end{matrix} \longrightarrow \begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$120 + 9 + 6 + 3 = 138$

(g)
$$\begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 9_4 & 3_3 & 3 & 0 \\ 0_2 & 0_1 & 0 & 0 \end{matrix} \longrightarrow \begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 9 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$(9 \times 4) + (3 \times 3) + (0 \times 2) + (0 \times 1)$

(h)
$$\begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 9 & 3_4 & 3_3 & 0 \\ 0 & 0_2 & 0_1 & 0 \end{matrix} \longrightarrow \begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 9 & 21 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

(i)

$$\begin{array}{cccc} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 9 & 21 & 3_{4} & 0_{3} \\ 0 & 0 & 0_{2} & 0_{1} \end{array} \longrightarrow \begin{array}{cccc} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 9 & 21 & 12 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

$(3\times 4) + (0\times 3) + (0\times 2) + (0\times 1)$

$= 12$

for correlation perform the same steps without rotating the Kernal by 180°

Translation Property

Sampling Quantization

Image size based Questions

Logical & Arithmatic Operation.

Masking, filter, Laplacian,

Histogram & Matching

Fourier Transform

Intensity Transformation – Contrast Stretching

Ques How many diff. 100 x 100 binary image can exist.

$= 2^{100\times 100}$

$$\frac{8\times 4\times \cancel{100}\times \cancel{100}}{\cancel{10000}}$$

$= 32$ sec.

Ques Find DFT of 1D $f(x) = \{0, 1, 2, 1\}$

$$= \sum_{x=0}^{N-1} f(x) \times e^{-j2\pi kx/N}$$

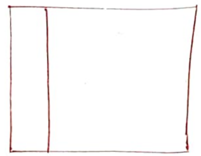
$$= f(0)e^{-0} + f(1)e^{-j\pi/2} + f(2)e^{-j\pi} + f(3)e^{-j3\pi/2}$$

$$= 0 + 1\left[\cos\tfrac{\pi}{2} - j\sin\tfrac{\pi}{2}\right] + 2\left[\cos\pi - j\sin\pi\right] + 1\left[\cos\tfrac{3\pi}{2} - j\sin\tfrac{3\pi}{2}\right]$$

$$= 0 + 1(0-j) + 2(-1) + 1(0+j)$$

$$\cancel{-j} - 2 + \cancel{j}$$

$$= -2$$



* The discretization of image data in spatial co-ordinate – sampling.

* The no of bits required to store a 1024 x 512 image with 512 gray level shades is.

$2^{10}\times 2^{9}\times 9$

$= 9\times 2^{19}$

* Suppose there is a multi spectral image of size 100 x 100 this image has 4 bands and each having 256 gray levels this image needs to transformated at the rate of 10k bits per sec Calculate the time req^d to transmit the image in sec.

Ques Compute 2D DFT of 4 x 4 gray scale image

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

A $$\boxed{F(u,v) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi\left(\frac{ux}{N} + \frac{vy}{N}\right)}}$$

basis function.

$$F(u,v) = \text{Kernal} \times f(x,y) \times [\text{Kernal}]^T$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Ques What is the smallest discernible details in an image?

Ans Pixel (Spatial Resolution)

Ques What is the smallest discermined change in an image?

Ans Gray level

A When the no of pixels in an img is reduced keeping the no of gray levels in the image constant, fine checker board patterns are found at the edges of the image. Effect is called checker board effect.

A When the no of gray levels in the image is low the foreground details of image merged with background of the image causing rich like structure this degradation phenomenon is called — false contouring.