

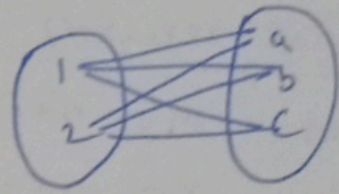
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## Relation

Relations are derived from Cartesian product.

$$A = \{1, 2\}$$

$$B = \{a, b, c\}$$



$$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$$

Definition  
Let  $A$  &  $B$  are two nonempty sets, then relation  $R$  from  $A$  to  $B$

$A$  to  $B$  is a subset of  $(A \times B)$

$$\boxed{R \subseteq A \times B}$$

Example of relation

$$\rightarrow \underline{\text{Ex}} \rightarrow A = \{1, 2\}$$

$$B = \{1, 2, 3\}$$

$$R_1 = \{(1,1), (1,3), (2,1), (2,2)\}$$

$\leftarrow R_1$  is a relation for  $A$  to  $B$  or not?

Find out all subset & check whether all the values of  $R_1$  is ~~present~~ present or not.

$$\rightarrow \text{all subset} \rightarrow \left\{ \begin{array}{l} (1,1), (1,2), (1,3) \\ (2,1), (2,2), (2,3) \end{array} \right\}$$

\* So we can called  $R_1$  is relation for  $A$  to  $B$



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$$\rightarrow R_{\max} = A \times B$$

$$\rightarrow R_{\min} = \{\emptyset\}, \text{ (empty set)}$$

$$\boxed{\text{Total no of relations} = 2^{m \cdot n}}$$

when  $m =$  no of element of set  $A$   
 $n =$  no of element of set  $B$ .

$$\text{exp } A = \{1, 2\}$$

$$B = \{2, 3, 4\}$$

$$2 \cdot 3$$

$$\text{The max}^{\text{m}} \text{ no of relations} \Rightarrow 2^{2 \cdot 3} = 2^6 = 64.$$

Types of relations  $\begin{cases} \text{Reflexive} \\ \text{Irreflexive} \end{cases}$

### ① Reflexive.

Let  $R$  be a relation in set  $A$ . Then  $R$  is called reflexive relation if  $(a, a) \in R$  for all  $a \in A$ .

$\hookrightarrow$  where  $a =$  elements of set  $A$ .

$$\text{exp } A = \{1, 2, 3, 4, 5\}$$

$$\left. \begin{array}{l} a = 1 \\ \quad 2 \\ \quad 3 \\ \quad 4 \\ \quad 5 \end{array} \right\}$$

$$R_1 = \{ \underbrace{(1,1)}, \underbrace{(3,3)}, \underbrace{(4,4)} \} \\ (2,1), (3,2)$$

(not reflexive) but not  $(5,5)$   $(2,2)$

Let say I have 2 sets

$$R_2 = \{ \underbrace{(1,1)}, \underbrace{(1,2)}, \underbrace{(2,2)} \} \\ \underbrace{(3,3)}, \underbrace{(4,4)}, \underbrace{(5,5)} \}$$

$$R_3 = \{\emptyset\} \quad \text{X (no)}$$

$$R_4 = \{A \times A\} \text{ (reflexive)}$$

$\hookrightarrow$  every elements are inside of  $A$ .

$\hookrightarrow$  Reflexive



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## (ii) Irreflexive Relation

Relation  $R$  on a set is said to be irreflexive if

$$a \in A, \& (a,a) \notin R.$$

→ It can also called as anti reflexive relation.

$$A = \{1, 2, 3, 4\}$$

$$R_1$$

↳

$$\{(1,2), (1,3), (2,2), (2,4)\}$$

X

as one (a,a) type value is here.

so it can not be irreflexive relation.

$$R_2$$

$$\{(3,4), (3,2), (2,1), (1,2)\}$$

no (a,a)

so it is a irreflexive relation

\* For this above example if any value like (a,a) is present i.e.  $\{(1,1), (2,2), (3,3), (4,4)\}$  then it is <sup>not</sup> called irreflexive relation.

$$R_1 = \{\emptyset\} \text{ is irreflexive relation.}$$

$$R_2 = \{A \times A\} \text{ X not an irreflexive relation}$$



part 4

## (a) Symmetric relation:

Let  $R$  be a relation on set  $A$ .

Then  $R$  is said to be symmetric relation.

$$\text{If } (a,b) \in R \Rightarrow (b,a) \in R.$$

$$\underline{a R b = b R a}$$

Exp  $A = \{1, 2, 3, 4\}$

$R_1 = \{(1,2), (2,1), (3,4), (4,3)\}$  ← symmetric

$R_2 = \{(1,3), (3,1), (3,4), (4,4)\}$  ←

→  $(A \times A)$  is a symmetric relation.

$R_3 = \{(1,1), (2,2), (3,3), (4,4)\}$

(symmetric)

$A \times A \rightarrow$   
 $(1,2,3) (1,2,3)$

$\left\{ \begin{array}{l} (1,1), (1,2), (1,3) \\ (2,1), (2,2), (2,3) \\ (3,1), (3,2), (3,3) \end{array} \right\}$

## Antisymmetric

A relation  $R$  on a set  $A$  is said to be anti-symmetric

if  $(a,b) \in R$  &  $(b,a) \in R$ , then  $a=b$ ,

Exp  $A = \{1, 2, 3, 4\}$

$R_2 = \{(1,2), (3,3), (4,3)\}$  (antisymmetric)

$\begin{array}{c} \text{a} \quad \text{b} \\ \text{a} = \text{b} \end{array}$



3) Can a relation be symmetric & anti symmetric both?

yes

$$A = \{a, b, c\}$$

$$R_1 = \{(a, a), (b, b), (c, c)\}$$

→ If  $(a, b) \in R, (b, a) \in R$  So symmetric.

→ If  $(a, b) \in R, (b, a) \in R$ , then  $a = b$  So also anti symmetric.

③ Asymmetric relation

A relation  $R$  on a set  $A$  is said to be Asymmetric relation

$$\text{If } (a, b) \in R \Rightarrow (b, a) \notin R$$

$$A = \{1, 2, 3, 4\}$$

$$R_1 = \{(1, 2), (2, 3), (3, 4), (4, 3)\} \leftarrow \text{Asymmetric.}$$

$$R_2 = \{(1, 2), (2, 1), (2, 3), (4, 3)\}$$

X → not an asymmetric.

exp

$$R = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$$

→ So it is not an asymmetric.

$$\begin{array}{c} a \quad b \\ a, b \notin R \\ b, a \in R \end{array}$$

NOTE 9  
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$R = A \times A$  ← no Asymmetric.

$R = \phi$  (Empty set). ← it is an Asymmetric.