part-1

Transitive Relation.

A relation 'R' is said to be transitive, if $(a,b) \in R$, $(b,c) \in R$, then $(a,c) \in R$.

here we have some example to check, wheather these are transitive or not.

(i) $\{(1,2), (2,1), (1,1), (2,2)\}$

$1 \to 2 \to 1$ $(1,1)$

$(2,1)$ $(1,2)$

$2 - 1 - 2$ $(2,2)$

(so it is transitive).

(ii) $\{(1,1), (2,2), (3,3)\}$

it is not in the form of $\left\{ \begin{matrix} (a,b), (b,c) \\ \Rightarrow \\ (a,c) \end{matrix} \right\}$

so it is transitive.

(iii) $\{(1,2)\}$ —— (transitive)

(iv) $\{(1,2), (1,3)\}$

1-2. (not form '2')
reverse (1,3), (1,2)
$1 \to 3$ (not)

} so transitive

(v) $\{(1,2), (2,3)\}$

$1 \to 2 - 3$ $(1,3) \to$ as it is not in the relation

So it is not transitive

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(vi) $\{(1,2),(3,2)\}$

$1 \to 2 \to$ (no)

(3,2), (1,2)

$3 \to 2 \to$ but no value from '2'.

so that it is transitive.

(vii) $\{(1,2),(2,3),(1,3),(1,1)\}$

1 - 2 - 3

so (transitive)

(viii) $\{(3,1),(2,3)\}$

3 — 1 (no value from '1')

(2,3) (3,1)

2 - 3 - 1 (2,1) should be there.

as, (2,1) is not in the relation, so it is not a transitive.

For checking a relation is transitive or not.

If $\boxed{(a,b) \; \& \; (b,c) \to (a,c)}$

then

(this condition always true)

else, any (a,b), or (b,c), or (a,a), (b,b) are there

then, it is transitive.

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(b) Inverse Relation

Let 'R' be a relation from set 'A' to set 'B', then the inverse relation R^{-1} from set 'B' to set 'A' is defend as by

$\{(b,a) : (a,b) \in R\}$.

Exp Let $A = \{1,2,3\}$

$B = \{a,b\}$

$R = \{(1,a),(1,b),(3,a),(2,b)\}$ be a relation from 'A' to 'B'

then find the inverse relation.

$R^{-1} = \{(a,1),(b,1),(a,3),(b,2)\}$

(c) Identity relation.

→ A relation 'R' in a set 'A' is said to be identity relation (I_A), if $I_A = \{(x,x) : x \in A\}$.

Exp $A = \{2,4,5\}$

$I_A = \{(2,2),(4,4),(5,5)\}$

Complement of Relation.

→ it is denoted by $\bar{R}$ or R^c.

$\bar{R} = \{(a,b) / (a,b) \in A \times B$ and $(a,b) \notin R\}$

$\bar{R} = (A \times B) - R$

ag-3

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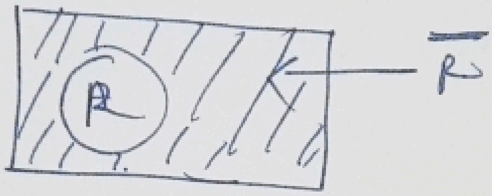
Case-4

Exp $A = \{a, b\}$ & $B = \{1, 2, 3\}$.

If $R = \{(a, 2), (b, 1), (b, 3)\}$.

Find $\overline{R}$?

Ans



$A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

$$\boxed{\overline{R} = \{(a, 1), (a, 3), (b, 2)\}}$$

Equivalance Relation.

A relation 'R' is equivalence relation with set 'A' if & only if

(i) 'R' is ~~reflexive~~ reflexive i.e for all $a \in R$, $(a, a) \in R$

(ii) 'R' is symmetric i.e for $(a, b) \in R$, then $(b, a) \in R$

(iii) 'R' is Transitive i.e for $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$.

Question

$A = \{a, b, c\}$

Find the below examples are equivalence Relation or not.

Exp

① $X_1 = \{\ \}$

↳ as it follow, R, S, T ← Transitive ↳ Reflexive ↳ Symmetric

as it belongs to set (A)

for reflexive it need, $\{(a,a), (b,b), (c,c)\}$ ←

as it is not there, X_1 is not equivalence relation

② $X_2 = \{(a,a), (b,b), (c,c)\}$

Reflexive = true.
Symmetric = true.
Transitive = true.

} ← ~~thus~~ X_2 is an equivalence relation.

③ $X_3 = \{(a,a), (b,b), (c,c), (b,a)\}$.

→ Reflexive = yes, as (a,a), (b,b) & (c,c) are present.

→ Symmetric => there is no issue for (a,a), (b,b), (c,c)

now issue for (b,a),

As (b,a) is there, it should be a term (a,b) which is missing

(not symmetric.)

Transitive ↳ ~~(a,a)~~ (b,a) (a,a) (b,b) (b,a)

(✓)

*④ $x_4 = \{(a,a), (a,c), (b,a), (c,a)\}$

Reflexive ⇒ not (not (b,b) & (c,c) is present

Symmetric ⇒ not (~~(b,a) is present but a~~

(c,a) is present but not (a,b).

Transitivity ⇒

b − a − c ⇐ (b,c) not

(not transitive).

⑤ $x_5 = \{(a,a), (b,b), (c,c), (a,b), (a,c), (b,a), (c,a)\}$

Reflexive = yes, (a,a) (b,b), (c,c)

Symmetric = yes (a,b) → (b,a)

(a,c) → (c,a)

Transitive ⇒ a − b a − (a,a) (present)

c − a c − (c,c) (present)

b − a − b − (b,b) (present)

(yes)

So it is an equivalent Relation.

⑥ $x_6 = \{A \times A\}$

← it follow all, so that equivalence relation.