PRAJJWAL SRIVASTAVA CLASS ROLL NO. 40 SEC → C
(20150048 9) DISCRETE ASSIGNMENT

A1. Let suppose n is odd,
and n is an integer & n^3+5 is odd (given).

$\therefore$ $n = 2K+1$; where K is any integer;

$\Rightarrow$ $n^3 = (2k+1)^3$.

$\Rightarrow$ $n^3 = 8K^3 + 1 + 6k(2k+1)$.

$n^3 \Rightarrow 8k^3 + 1 + 12k^2 + 6K$.

Now $n^3+5 \Rightarrow 8K^3 + 1 + 12K^2 + 6K + 5$

$n^3+5 \Rightarrow 8K^3 + 12K^2 + 6K + 6$.

$n^3+5 \Rightarrow 2(4K^3 + 6K^2 + 3K + 3)$.

$\therefore$ n^3+5 turns out to be even; but it is odd as it is a given statement;

$\therefore$ Our supposition is wrong

And n is even.

A2. Let $P(n): 3^{2n}$ when divided by 8, the remainder is 1.

or, $P(n) = 3^{2n} = 8\lambda + 1$ for some $\lambda \in M$

$\therefore$ $3^2 = 8 \times 1 + 1 = 8 + 1$ for some $\lambda \in M$

$\therefore$ P(1) is true.

Step-2: Let P(M) be true then:-

$3^{2m} = 8 + 1$;

Now; P(m+1):-

$3^{2(m+1)} = 3^{2m} \cdot 3^2 = 8(\lambda+1) \cdot 9$

$= 72\lambda + 9 = 72\lambda + 8 + 1 = 8(9\lambda+1) + 1$; where $r = 9\lambda + 1 \in N$

$\therefore$ P(m+1) is true wherever P(m) is true.

Hence it is true for all m.

A3. To prove: $2^n < n!$ for $n \geq 4$

$\therefore$ for $n = k$; $k \geq 4$.

Suppose. $P(k) := 2^k < k!$ is true

For :-

$\therefore P(k+1) \Rightarrow 2^{k+1} = 2 \cdot 2^k$. (by defn. of expo)

$2^{k+1} < 2 \cdot k!$ (by inductive hypo..)

$2^{k+1} < (k+1)k!$ (because $2 < k+1$)

$= (k+1)!$ (by defn. of fact.)

$\therefore$ P(k+1) is also true

$\therefore$ P(k) holds true always.

A4. $10^{2n-1} + 1$

for $n = 1$;

$10^{2-1} + 1 \Rightarrow 11(1)$

Hence divisible.

for $n = k$

$10^{2k-1} + 1 = 11(m)$ — ①

$\therefore$ for $n = k+1$,

$10^{2(k+1)-1} + 1 \Rightarrow 10^{2k+2-1} + 1 \Rightarrow 10^{2k+1} + 1$..

$\Rightarrow 10^{2k-1+2} + 1 \{ 10^{2k-1} = (11m - 1)$.

$\Rightarrow (11m - 1) \cdot 10^2 + 1$

$\Rightarrow 11m \times 100 + 11m - 100 + 1$

$\Rightarrow 11m \times 100 + 11m - 99$.

$\Rightarrow 11m \times 100 + 11m - 11 \times 9$

$\Rightarrow 11(m \times 100 + m - 9)$.

$\Rightarrow$ divisible by 9. Proved

$5^{2n+1} + 3^{n+2} \cdot 2^{n-1}$.

for $n=1$

$5^3 + 3^3 \cdot 2^0 = 152$

$= 19(8)$

when $n=K$

$5^{2K+1} + 3^{K+2} \cdot 2^{K-1} = 19m$ — ①.

when $n=K+1$

$\Rightarrow 5^{2K+2+1} + 3^{K+1+2} \times 2^{K+1-1}$

$\Rightarrow 5^{2K+3} + 3^{K+3} \times 2^{K}$

$\Rightarrow 5^{2K+1} \times 5^2 + 3^{K+2} \times 3 \times 2^{K-1} \times 2$

$\Rightarrow 5^{2K+1} \times 25 + 3^{K+2} \times 2^{K-1} \times 6$.

$\Rightarrow 5^{2K+1} \times (19+6) + 3^{K+2} \times 2^{K-1} \times 6$.

$\Rightarrow 5^{2K+1} \times 19 + 5^{2K+1} \times 6 + 3^{K+2} \times 2^{K-1} \times 6$.

$= 5^{2K+1} \times 19 + 6(5^{2K+1} + 3^{K+2} \times 2^{K-1})$

$\Rightarrow 19(5^{2K+1} + 6m)$

$\Rightarrow 19x$

Hence proved.

A6. $p, q \to T$ & $r, s \to F$.

a. $(\neg(p \wedge q) \vee \neg r) \vee ((q \leftrightarrow \neg p) \to (r \vee \neg s))$

$\Rightarrow (\neg(T \wedge T) \vee \neg F) \vee ((T \leftrightarrow \neg T) \to (F \vee \neg F))$

$\Rightarrow (\neg T \vee T) \vee ((T \leftrightarrow F) \to (F \vee T))$

$\Rightarrow (F \vee T) \vee ((F \to T))$

$\Rightarrow T \vee T$

$\Rightarrow T$

b. $(p \leftrightarrow r) \wedge (\neg q \rightarrow s)$.

$\Rightarrow (T \leftrightarrow F) \wedge (\neg T \rightarrow F)$.

$\Rightarrow F \wedge (F \rightarrow F)$.

$\Rightarrow F \wedge T$

$\Rightarrow F$.

c. $(p \vee (q \rightarrow (r \wedge \neg p))) \leftrightarrow (q \vee \neg s)$

$\Rightarrow (T \vee (T \rightarrow (F \wedge \neg T))) \leftrightarrow (T \vee \neg F)$

$\Rightarrow (T \vee (T \rightarrow (F \wedge F))) \leftrightarrow (T \vee T)$

$\Rightarrow (T \vee (T \rightarrow F)) \leftrightarrow (T)$.

$\Rightarrow (T \vee F) \leftrightarrow T$

$\Rightarrow T \leftrightarrow T$

$\Rightarrow T$

7. a. $(p \rightarrow q) \equiv (\sim p \vee q)$

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

b. $(p \rightarrow (q \rightarrow r)) \equiv p \rightarrow (\sim q \vee r) \equiv (p \vee q) \rightarrow r$

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$\sim q \vee r$	$p \rightarrow (\sim q \vee r)$	$p \vee q$	$\rightarrow r$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	F	F	T	F
T	F	T	T	T	T	T	F	T
T	F	F	T	T	T	T	F	T
F	T	T	T	T	T	T	F	T
F	T	F	F	T	F	T	F	T
F	F	T	T	T	T	T	F	T
F	F	F	T	T	T	T	F	T

c. $p \to (q \to p) \equiv \sim p \to (p \to q)$.

p	q	$q \to p$	$p \to q \to p$	$\sim p$	$p \to q$	$\sim p \to (p \to q)$
T	T	T	T	F	T	T
T	F	T	T	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T

d. $p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$.

p	q	r	$q \vee r$	$p \to (q \vee r)$	$p \to q$	$p \to r$	$(p \to q) \vee (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

e. $(p \to q) \wedge (r \to q) \equiv (p \vee r) \to q$

p	q	r	$p \to q$	$r \to q$	$(p \to q) \wedge (r \to q)$	$(p \vee r)$	$(p \vee r) \to q$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	F
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T
F	F	T	T	F	F	T	F
F	F	F	T	T	T	F	T

f. $(p \oplus q) \equiv (p \vee q) \wedge \sim (p \wedge q)$.

p	q	$p \leftrightarrow q$	$p \vee q$	$p \wedge q$	$\sim p \wedge q$	$(p \vee q) \wedge \sim (p \wedge q)$
T	T	T	T	T	F	T
T	F	F	T	F	F	F
F	T	F	T	F	T	F
F	F	T	F	F	T	T

A8. a. N: If it dosen't rain, then they will drive the car.
C: They will drive the car, if it doesn't rain.

b. N: If Mohan is not a singer, then he will not be poor.
C: If Mohan is not poor, then he is not a singer.

c. N: If he dosen't walk, then he will not be healthy.
C: If he will not be healthy, then he dosen't walk.

d. N: Only if Mohan doesn't work hard, then he will fail the test.
C: If he will not pass the test, then Mohan didn't work hard.

A9. a) $\sim p \vee \sim q$
b) $\sim p \wedge q$
c) $p \rightarrow \sim q$
d) $\sim p \rightarrow \sim q$

A10. a. $p \wedge (p \rightarrow q)$
$\Rightarrow p \wedge (\sim p \vee q)$.

b. $\sim (p \vee q) \leftrightarrow (p \wedge q)$.

p	q	$\sim(p \vee q)$	$p \wedge q)$	$\sim(p \vee q) \leftrightarrow (p \wedge q)$
T	T	F	T	F
T	F	F	F	T
F	T	F	F	T
F	F	T	F	F

CNF :- $(\sim p \vee \sim q) \wedge (p \vee q)$.

$(\sim p \to r) \land (q \leftrightarrow p)$

p	q	r	$(\sim p \to r)$	$(q \leftrightarrow p)$	$(\sim p \to r) \land (q \leftrightarrow p)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	F	F
F	T	T	T	F	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	F	T	F

PCNF $\Rightarrow (\sim p \lor \sim q \lor \sim r) \land (\sim p \lor q \lor r) \land (p \lor \sim q \lor r) \land$
$(p \lor \sim q \lor r) \land (p \lor q \lor r)$

A12. a) $\sim(p \land q)$

p q PNF:→ $(\sim p \lor \sim q)$

b) $\sim p \lor q$

p	q	$\sim p$	$\sim p \lor q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

PNF = $(p \land q) \lor (\sim p \land q) \lor (\sim p \land \sim q$

A13. $p \to \neg q$, $q \lor r$, $\neg s \to p$, $\neg r$.

i) $((p \cancel{\lor} p) \cancel{\to} \neg q) \to / p$.

i). $p \cancel{\to} \neg q / \Rightarrow (\neg p \cancel{\lor} \neg q)$ ∴.

On the next page.

A13.

p	q	r	s	¬q	¬r	¬s	p → ¬q	q ∨ r	¬s → p
T	T	T	T	F	F	F	F	T	T
T	T	T	F	F	F	T	F	T	T
T	T	F	T	F	T	F	F	T	T
T	T	F	F	F	T	T	F	T	T
T	F	T	T	T	F	F	T	T	T
T	F	T	F	T	F	T	T	T	T
T	F	F	T	T	T	F	T	F	T
T	F	F	F	T	T	T	T	F	T
F	T	T	T	F	F	F	T	T	T
F	T	T	F	F	F	T	T	T	F
F	T	F	T	F	T	F	T	T	T
F	T	F	F	F	T	T	T	T	F
F	F	T	T	T	F	F	T	T	T
F	F	T	F	T	F	T	T	T	F
F	F	F	T	T	T	F	T	F	T
F	F	F	F	T	T	T	T	F	F

s is Not a valid conclusion.

A14.

p	q	r	¬q	¬r	p → ¬q	r → p	q → ¬r
T	T	T	F	F	F	T	F
T	T	F	F	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	T	T	T	T	T
F	T	T	F	F	T	F	F
F	T	F	F	T	T	T	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	T	T

* q → ¬r is not a valid conclusion.

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14.

p	q	r	s	t	$p \to q$	$q \to r$	$r \to s$	$s \wedge t$
T	T	T	T	T	T	T	T	T
T	T	T	T	F	T	T	T	F
T	T	T	F	T	T	T	F	F
T	T	T	F	F	T	T	F	F
T	T	F	T	T	T	F	T	T
T	T	F	T	F	T	F	T	F
T	T	F	F	T	T	F	T	F
T	T	F	F	F	T	F	T	F
T	F	T	T	T	F	T	T	T
T	F	T	T	F	F	T	T	F
T	F	T	F	T	F	T	F	F
T	F	T	F	F	F	T	F	F
T	F	F	T	T	F	T	T	T
T	F	F	T	F	F	T	T	F
T	F	F	F	T	F	T	T	F
T	F	F	F	F	F	T	T	F
F	T	T	T	T	T	T	T	T
F	T	T	T	F	T	T	T	F
F	T	T	F	T	T	T	F	F
F	T	T	F	F	T	T	F	F
F	T	F	T	T	T	F	T	T
F	T	F	T	F	T	F	T	F
F	T	F	F	T	T	F	T	F
F	T	F	F	F	T	F	T	F
F	F	T	T	T	T	T	T	T
F	F	T	T	F	T	T	T	F
F	F	T	F	T	T	T	F	F
F	F	T	F	F	T	T	F	F
F	F	F	T	T	T	T	T	T
F	F	F	T	F	T	T	T	F
F	F	F	F	T	T	T	T	F
F	F	F	F	F	T	T	T	F

* S is not a valid conclusion. for $p \to q$, $q \to r$, $s \wedge t$.
but for $r \to s$ S is valid.

Ans.

p	q	r	s	t	$t \to s$	$p \wedge q$	$r \to s$	$(p \wedge q) \vee (r \to s)$	$t \to r$	$\neg(p \wedge q)$
T	T	T	T	T	T	T	T	T	T	F
T	T	T	T	F	T	T	T	T	T	F
T	T	T	F	T	F	T	F	T	T	F
T	T	T	F	F	T	T	F	T	T	F
T	T	F	T	T	T	T	T	T	F	F
T	T	F	T	F	T	T	T	T	T	F
T	T	F	F	T	F	T	T	T	F	F
T	T	F	F	F	T	T	T	T	T	F
T	F	T	T	T	T	F	T	T	T	T
T	F	T	T	F	T	F	T	T	T	T
T	F	T	F	T	F	F	F	F	T	T
T	F	T	F	F	T	F	F	F	T	T
T	F	F	T	T	T	F	T	T	F	T
T	F	F	T	F	T	F	T	T	T	T
T	F	F	F	T	F	F	T	T	F	T
T	F	F	F	F	T	F	T	T	T	T
F	T	T	T	T	T	F	T	T	T	T
F	T	T	T	F	T	F	T	T	T	T
F	T	T	F	T	F	F	F	F	T	T
F	T	T	F	F	T	F	F	F	T	T
F	T	F	T	T	T	F	T	T	F	T
F	T	F	T	F	T	F	T	T	T	T
F	T	F	F	T	F	F	T	T	F	T
F	T	F	F	F	T	F	T	T	T	T
F	F	T	T	T	T	F	T	T	T	T
F	F	T	T	F	T	F	T	T	T	T
F	F	T	F	T	F	F	F	F	T	T
F	F	T	F	F	T	F	F	F	T	T
F	F	F	T	T	T	F	T	T	F	T
F	F	F	T	F	T	F	T	T	T	T
F	F	F	F	T	F	F	T	T	F	T
F	F	F	F	F	T	F	T	T	T	T

$\therefore$ $t \to s$ is not a valid conclusion.

Q.17.

p	q	r	$\neg q$	$\neg r$	$p \to \neg q$	$r \to p$	$q \to \neg r$
T	T	T	F	F	F	T	F
T	T	F	F	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	T	T	T	T	T
F	T	T	F	F	T	F	F
F	T	F	F	T	T	T	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	T	T

$\therefore$ $q \to \neg r$ is not a valid conclusion.