1621. Number of Sets of K Non-Overlapping Line Segments Medium Math **Dynamic Programming**

This problem involves a set of points on a 1-D plane. Each point is designated by a unique integer position from 0 to n-1,

Problem Description

representing their placement on the line. We are interested in finding the number of ways to create k line segments, where each line segment covers at least two points and has endpoints at integral coordinates. It's important to note that these line segments must be non-overlapping, though they can share endpoints. The goal is to calculate the number of distinct configurations you can draw with these requirements. A crucial detail is that not all points need to be covered by the line segments; thus, some points can remain unused. Also, since the

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problems to keep the numbers manageable. Intuition To approach this problem, consider dynamic programming, which is a method used to solve complex problems by breaking them

down into simpler subproblems. The idea behind this approach is to create a table, with f and g to record the number of ways to

number of ways can be enormous, the solution should be given modulo 10^9 + 7, a common practice in combinatorics-related

satisfy the condition up to the i-th point with j segments used so far.

segments up to the i-th point, where the i-th point is not shared with the next segment (an endpoint). In contrast, g[i][j] counts the number of configurations where the i-th point is shared (not the end of the last segment), allowing a segment to extend to include more points. The key to solving the problem is to correctly update the values in f and g through iteration. Initially, we set f[1][0] to 1, as there is

The 2D arrays f and g are critical in our dynamic programming approach. f[i][j] represents the number of ways we can draw j line

only one way to have 0 segments with 1 point. Then we iterate through all points and all possible numbers of segments, updating f based on previous values and g based on both its previous value and the value of f. If we're allowed to add a new segment (j is greater than 0), we can look back at previous points and either start a new segment (f[i - 1][j - 1]) or extend an existing one (g[i-1][j-1]).In short, f and g are used to accumulate the number of valid configurations as we progress through the points and segments, with

and g[n][k], which are the number of configurations after considering all points and exactly k segments, modulo 10^9 + 7. By setting up these relations and carefully calculating the iterations, we consider all possible segment configurations and meet the constraints of non-overlapping with the option of shared endpoints, efficiently arriving at the correct count. **Solution Approach**

the updates accounting for starting, ending, and extending segments at each step. The final answer consists of the sum of f[n][k]

combinations of segments up to each point i for 0 to n-1 and each possible number of segments j from 0 to k. We start with the initialization: $1 \mod = 10**9 + 7$

Here, mod is the number we'll use to perform modulus operations to keep our solution within the required range to avoid integer

overflow. The tables f and g are initialized with zeros, since we start with no configurations, and set 1 for f[1] [0] since there's

The solution involves dynamic programming, which iteratively updates our two tables, f and g, to account for the various

Following the initialization, the main iteration begins: 1 for i in range(2, n + 1):

for j in range(k + 1):

g[i][j] += f[i - 1][j - 1]

g[i][j] += g[i - 1][j - 1] g[i][j] %= mod

4 f[1][0] = 1

2 $f = [[0] * (k + 1) for _ in range(n + 1)]$

3 $g = [[0] * (k + 1) for _ in range(n + 1)]$

exactly 1 way to have 0 segments among 1 point.

f[i][j] = (f[i - 1][j] + g[i - 1][j]) % mod

g[i][j] = g[i - 1][j]For each point i and each number of segments j, we update f[i][j] by the sum of the number of ways we can draw j segments up

Additionally, if it's allowed to add a new segment (j > 0), we have the following update rules:

to the previous point without it being an endpoint (f[i-1][j]) and the number of ways with it being a shared point (g[i-1][j]).

Then, we carry over the value of g[i - 1][j] to g[i][j] because we can always extend a line segment to point i without adding a new segment.

g[i][j] %= mod

configurations.

Example Walkthrough

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1 return (f[-1][-1] + g[-1][-1]) % mod
We use -1 as an index for convenience to refer to the last element of the list, which represents all points and exactly k segments. The
modulus operation ensures the result complies with the required number range.
```

Through this iterative and layered approach, the algorithm efficiently calculates the required configurations, respecting the rules of

non-overlapping segments which can optionally share endpoints. This method leverages dynamic programming to handle numerous

potential combinations, significantly reducing the time complexity compared to naively enumerating all possible segment

We increment g[i][j] by the number of ways to draw j-1 segments up to the previous point, and add either a new segment end

(f[i-1][j-1]) or extend an existing segment (g[i-1][j-1]). This accounts for all the possibilities at point i with j segments.

Consider a set with n = 4 points and we want to find the number of ways to create k = 1 line segment. The points on the 1-D plane can be represented as [0, 1, 2, 3].

There's no way to have a segment with just point [0] (f[1][1] = 0), so f[2][1] remains 0.

Finally, the return statement sums the number of configurations for n points and k segments from both tables:

1 f = [[0, 0],[1, 0], [0, 0], [0, 0], [0, 0]]

[0, 0],

[0, 0],

[0, 0],

[0, 0]]

1. For i = 2, j = 1:

At this point, our tables are:

[1, 0],

[0, 0],

[0, 0],

[0, 0]]

[0, 0],

[0, 1],

[0, 0],

[0, 0]]

2. For i = 3, j = 1:

[0, 1],

[0, 0]]

[0, 0],

[0, 1],

[0, 1],

[0, 0]]

3. For i = 4, j = 1:

[0, 1],

[0, 2]]

[0, 0],

[0, 1],

[0, 1],

[0, 1]]

Python Solution

class Solution:

MOD = 10**9 + 7

dp[1][0] = 1

g = [[0, 0],

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39 }

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g = [[0, 0],

Tables now:

g = [[0, 0],

11

1 f = [[0, 0],

g = [[0, 0],

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 $1 \mod = 10**9 + 7$

6 f[1][0] = 1

```
\circ Since j > 0, we add ways from f[i-1][j-1] to g[i][j], resulting in g[2][1] becoming 1 (one way to start a new segment at
 [1,2]).
```

Now f[3][1] becomes the sum of f[2][1] and g[2][1], which is 1.

Again, f[4][1] becomes the sum of f[3][1] and g[3][1], totaling to 2.

g[4][1] carries over g[3][1] and adds f[3][0].

without overlap, and with the possibility of shared endpoints.

def numberOfSets(self, n: int, k: int) -> int:

 $dp = [[0] * (k + 1) for _ in range(n + 1)]$

Fill the dynamic programming tables.

Define the modulus for large numbers to prevent overflow.

'cumulative' tracks the cumulative count of sets including previous points.

Initialize the base case: 1 way to form 0 segments using 1 point.

for j in range(k + 1): # For each number of segments from 0 to k

dp[i][j] = (dp[i - 1][j] + cumulative[i - 1][j]) % MOD

The number of ways to form 'j' segments considering 'i' as the last point

is equal to the sum of the previous point without the ith point and the cumulative count.

Add the number of ways to form 'j-1' segments not including the ith point.

Add the number of ways to form 'j-1' segments including previous points.

private static final int MOD = (int) 1e9 + 7; // Defining the modulus value for large numbers to prevent overflow

Create two 2D lists (tables) for dynamic programming.

for i in range(2, n + 1): # For each point from 2 to n

cumulative[i][j] += dp[i - 1][j - 1]

cumulative[i][j] += cumulative[i - 1][j - 1]

dpEnd[1][0] = 1; // Base case: There is 1 way to form 0 segments with 1 point

cumulative[i][j] %= MOD

cumulative[i][j] %= MOD

return (dp[-1][-1] + cumulative[-1][-1]) % MOD

public int numberOfSets(int n, int k) {

// Iterate over the number of points

for (int i = 2; i <= n; ++i) {

int numberOfSets(int n, int k) {

// Dynamic programming iterations

if (j > 0) {

2 function numberOfSets(n: number, k: number): number {

// possibly ending with a segment.

memset(dpSegmentEnd, 0, sizeof(dpSegmentEnd));

memset(dpNoSegmentEnd, 0, sizeof(dpNoSegmentEnd));

for (int i = 2; $i \le n$; ++i) { // Iterate over points

// Initialize dp arrays

cumulative = $[[0] * (k + 1) for _ in range(n + 1)]$

Now we start iterating over each point and potential number of segments:

Let's walk through a small example to illustrate the solution approach.

Using dynamic programming, we initialize our tables f and g like this:

 $= [[0] * (k + 1) for _ in range(n + 1)]$

 $5 g = [[0] * (k + 1) for _ in range(n + 1)]$

After initialization, f and g look like this:

1 f = [[0, 0],[1, 0], [0, 1],

∘ We carry over g[2][1] to g[3][1]. Since j > 0, g[3][1] will also include f[2][0], but since f[2][0] is 0, it doesn't change.

Tables now: 1 f = [[0, 0],[1, 0], [0, 1],

Finally, we return the sum of f[n][k] and g[n][k], which is f[4][1] + g[4][1] = 2 + 1 = 3 modulo $10^9 + 7$, so the result is 3.

So, for n = 4 and k = 1, there are 3 distinct ways we can draw 1 line segment that covers at least two points on the 1-D plane,

'dp' tracks the number of sets that can be formed using the first 'i' points with 'j' segments including the ith point.

The final answer is the sum of the ways to form 'k' segments including the nth point and not including the nth point.

int[][] dpEnd = new int[n + 1][k + 1]; // dpEnd[i][j] represents the number of ways to form j segments with i points, with

int[][] dpNotEnd = new int[n + 1][k + 1]; // dpNotEnd[i][j] represents the number of ways to form j segments with i points

22 # The cumulative count up to 'i' for 'j' segments is the same as the previous cumulative count. 23 cumulative[i][j] = cumulative[i - 1][j] 24 25 # If there is at least one segment to place, 26 if j: 27

Java Solution

1 class Solution {

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                 // Iterate over the number of segments
                 for (int j = 0; j \le k; ++j) {
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                     // The number of ways with a segment ending at the current point i is equal to the sum of the number of ways
                     // with and without a segment ending at the previous point (i-1), for the same number of segments
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                     dpEnd[i][j] = (dpEnd[i - 1][j] + dpNotEnd[i - 1][j]) % MOD;
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 18
                     // For the number of ways without a segment ending at the current point i carry over the number
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                     // from the previous point i-1 without a segment ending
                     dpNotEnd[i][j] = dpNotEnd[i - 1][j];
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                     // If we can form at least one segment
 23
                     if (j > 0) {
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                         // There are two ways to extend the number of ways to form j segments without a segment ending at point i:
                             1) Use the number of ways to form j-1 segments with a segment ending at point i-1, and
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 26
                                 start a new segment at point i-1 extending to i.
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                             2) Extend the number of ways to form j segments without a segment ending at point i-1 by adding a point i.
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                         dpNotEnd[i][j] += (dpEnd[i - 1][j - 1] + dpNotEnd[i - 1][j - 1]) % MOD;
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                         // Modulo operation to prevent overflow
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                         dpNotEnd[i][j] %= MOD;
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             // The total number of ways to form k segments with n points is the sum of the ways with and without a segment ending at po
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             return (dpEnd[n][k] + dpNotEnd[n][k]) % MOD;
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C++ Solution
  1 class Solution {
  2 public:
         int dpSegmentEnd[1010][1010]; // dp array for cases where a segment ends at a point
         int dpNoSegmentEnd[1010][1010]; // dp array for cases with no segment ending at a point
         const int MOD = 1e9 + 7;
                                       // Define the modulo operator value
```

// Method that calculates the number of ways to divide a line into k non-overlapping line segments

dpSegmentEnd[1][0] = 1; // Base case: only one way when there is one point and no segments

dpSegmentEnd[i][j] = (dpSegmentEnd[i - 1][j] + dpNoSegmentEnd[i - 1][j]) % MOD;

dpNoSegmentEnd[i][j] = (dpNoSegmentEnd[i][j] + dpSegmentEnd[i - 1][j - 1]) % MOD;

// Total number of ways is the sum of ways where last segment ends on point n and where it doesn't

// Create a 2D array `f` where `f[i][j]` represents the number of ways to draw `j` segments with `i` points,

// Create a 2D array `g` where `g[i][j]` represents the number of ways to draw `j` segments with `i` points,

1 // A function that calculates the number of ways to divide a line into `k` non-overlapping segments.

dpNoSegmentEnd[i][j] = (dpNoSegmentEnd[i][j] + dpNoSegmentEnd[i - 1][j - 1]) % MOD;

// If it is possible to add a new segment, update the ways count for segments ending at current point

for (int j = 0; j <= k; ++j) { // Iterate over number of segments</pre>

// Ways where a segment does not end at the current point

// Ways where a segment ends at the current point

dpNoSegmentEnd[i][j] = dpNoSegmentEnd[i - 1][j];

return (dpSegmentEnd[n][k] + dpNoSegmentEnd[n][k]) % MOD;

const f = Array.from({ length: n + 1 }, () => new Array(k + 1).fill(0));

const $g = Array.from(\{ length: n + 1 \}, () => new Array(k + 1).fill(0));$

// Base case: there's one way to draw 0 segments with 1 point.

// The modulo value for preventing integer overflow.

// Loop through the number of segments.

// If we have at least one segment.

g[i][j] += f[i - 1][j - 1];

g[i][j] += g[i - 1][j - 1];

// Number of ways to draw `j` segments up to point `i`, ending with a point. 18 f[i][j] = (f[i-1][j] + g[i-1][j]) % mod;19 20 21 // Number of ways to draw `j` segments up to point `i`, possibly ending with a segment (continuing). 22 g[i][j] = g[i - 1][j];23

Typescript Solution

f[1][0] = 1;

// ending with a point.

const mod = 10 ** 9 + 7;

if (j) {

// Loop through the points.

for (let i = 2; i <= n; ++i) {

for (let j = 0; $j \le k$; ++j) {

g[i][j] %= mod;

g[i][j] %= mod;

return (f[n][k] + g[n][k]) % mod;

```
Time and Space Complexity
The given Python code defines a class Solution with a method numberOfSets that calculates the number of ways to draw k non-
overlapping line segments from n distinct points. The dynamic programming approach is used with two 2D lists (or matrices if you
prefer), f and g. Here's an analysis of the time and space complexity of the code:
Time Complexity
The code consists of nested loops: the outer loop runs for n-1 iterations (starting from 2 up to n), and the inner loop runs for k+1
iterations (starting from 0 up to k). Since each iteration performs a constant number of operations including assignments and
```

// Return the number of ways to draw `k` segments with `n` points, which could either end with a point or a segment.

// Add ways to draw j-1 segments up to point i-1, then add a new segment ending at point i.

// Add ways to continue the segment from `j-1` segments up to point `i-1`.

Therefore, the time complexity is 0(n * k). **Space Complexity**

modular additions, the time complexity is a direct product of the number of iterations of these loops.

```
Two matrices f and g are created, each of size (n + 1) \times (k + 1). No additional space that grows with n or k is used in the
algorithm, so the space complexity is determined by the size of these matrices.
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Thus, the space complexity is O(n * k) since it's proportional to the amount of space used to store the matrices.