763. Partition Labels

Medium Greedy Hash Table Two Pointers String

Problem Description

The problem presents us with a string `s`. Our goal is to partition `s` into the maximum number of parts such that no letter appears in more than one part. This means that once a letter appears in a part, it cannot appear in any other part. After partitioning, when we concatenate all parts back together in the original order, the string should remain unchanged; it should be `s`.

The resulting output should be a list of integers, where each integer represents the length of one of these partitions.

Intuition

The intuition behind this solution is to first understand that a partition can only end when all instances of a particular character included in the partition have been encountered. The last occurrence of each character defines the maximum possible extent of a partition.

To solve the problem, we track the last occurrence of each character using a dictionary. The keys are the characters from the string `s` and the values are the indices of their last occurrences.

As we iterate over the string `s`:

1. We continually update a `mx` variable to keep track of the farthest index that we must reach before we can end a partition. This is updated by looking at the last occurrence of the current character.
2. When the current index `i` matches `mx`, it means we've included all occurrences of the characters seen so far in the current partition. At this point, we can end the current partition and start a new one.
3. We append the size of the current partition to the answer list. The size is calculated as the difference between the current index `i` and the start index of the partition `j`, plus one.
4. We then set the start of the next partition to be one index after the current index `i`.

In the end, we get a list of partition sizes that fulfill the requirements.

Solution Approach

The solution uses a two-pass approach and utilizes a dictionary data structure to hold the last occurrence of each character in the string `s`.

1. **Building a dictionary of last occurrences**: The first pass is over the string `s`. As we iterate, we capture the last position or index of every unique character in the dictionary called `last`. This is constructed in the line `last = {c: i for i, c in enumerate(s)}`, making use of a dictionary comprehension.
2. **Partitioning based on last occurrences**: The next part of the solution is where the actual partitioning logic is implemented:
   - We initialize three variables: `mx`, `j`, and `ans`. The `mx` variable represents the furthest point a partition can reach before being cut. The `j` variable is the start index of the current partition, and `ans` is the list that will hold the sizes of the partitions.
   - We then iterate through the string `s` again, index by index, alongside the corresponding characters.
     - For each character at index `i`, we update `mx` to be the maximum value between the current `mx` and the last occurrence of the character `c`, like this: `mx = max(mx, last[c])`.
     - If the current index `i` equals `mx`, it means all characters in the current partition will not appear again in the following parts of the string. This is the right place to 'cut' this partition.
       - We calculate the size of the current partition by subtracting the start index `j` of this partition from the current index `i` and adding 1 (`i - j + 1`). This value is appended to the `ans` list.
       - Once we add the partition size to `ans`, we set the start `j` of the next partition to `i + 1`.
3. **Output the result**: After the iteration completes, we return the list `ans` which contains the sizes of all partitions.

The factors that make this algorithm work are:

- The dictionary captures the essential boundaries for our partitions by marking the last appearance of each character within the string.
- The two-pass mechanism ensures that we never 'cut' a partition too early, which ensures that each letter only appears in one part.
- The greedy approach - always updating `mx` to extend the partition as far as possible and only finalizing a partition when absolutely necessary - ensures that we maximize the number of partitions.

By following through with this approach, the function `partitionLabels` partitions the string optimally as per the problem statement.

Example Walkthrough

Let's consider a small example string `s = "abaca"`. We want to create partitions in such a way that no letters are repeated in any of the partitions and to maximize the number of partitions. The output should be a list of the lengths of these partitions.

1. **Building a dictionary of last occurrences**: The first pass over the string `s` will give us a dictionary `last` with the last position of every character.
   - For our example, `last` would be `{'a': 2, 'b': 1, 'c': 3}`.
2. **Partitioning based on last occurrences**:
   - We initialize `mx` to `-1`, `j` to `0`, and `ans` to an empty list.
   - We then loop through the string `s`, with the indices and characters.
     1. At index `0`, the character is `'a'`. We update `mx` to be the maximum of `mx` and the last occurrence of `'a'` (`mx = max(-1, 2)`), so `mx` becomes `2`.
     2. At index `1`, the character is `'b'`. We update `mx` to be `max(2, 1)` as the last occurrence of `'b'` is at index `1`. Since `2` is greater, `mx` remains `2`.
     3. At index `2`, the character is `'a'`. Since we have already accounted for the last occurrence of `'a'`, we don't need to update `mx` it is still the right end of our partition.
     4. At index `2` when `i` is equal to `mx`, we can 'cut' the partition here.
        - We append the size of this partition to `ans`, which is `i - j + 1` or `2 - 0 + 1`, so we append `3`.
        - We also set `j` to the next index `i + 1`, which is `3`, ready for the next potential partition.
     5. For the last character at index `3`, `'c'`, the last occurrence is at `3` itself. We update `mx` to `3`.
     6. Given that `i` equals `mx` at the last character, we have the end of another partition.
        - The size of the partition is again calculated as `i - j + 1`, yielding `3 - 3 + 1`, so we append `1` to `ans`.
3. **Output the result**: After the iteration completes, `ans` contains the sizes `[3, 1]`, denoting the lengths of the partitions `aba` and `c`. Hence, the final output is `[3, 1]`.

By following the steps above, the input string `abac` has been partitioned into `["aba", "c"]`, maximizing the number of parts and ensuring no letter appears in more than one part. The lengths of the partitions are `[3, 1]` as per the expected output.

Solution Implementation

Python

```python
class Solution:
    def partitionLabels(self, S: str) -> List[int]:
        # Create a dictionary to store the last occurrence of each character.
        last_occurrence = {char: index for index, char in enumerate(S)}

        # Initialize variables.
        # `max_last` represents the farthest index any character in the current partition has been seen.
        # `partition_start` represents the start of the current partition.
        max_last = partition_start = 0

        # This list will hold the sizes of the partitions.
        partition_sizes = []

        # Iterate through the characters of the string along with their indices.
        for index, char in enumerate(S):
            # Update `max_last` to be the max of itself and the last occurrence of the current character.
            max_last = max(max_last, last_occurrence[char])

            # If the current index is the last occurrence of all characters seen so far in this partition,
            # that means we can end the partition here.
            if max_last == index:
                # Append the size of the current partition to the list (`index - partition_start + 1`).
                partition_sizes.append(index - partition_start + 1)

                # Update `partition_start` to the next index to start a new partition.
                partition_start = index + 1

        # Return the list of partition sizes.
        return partition_sizes
```

Java

```java
import java.util.List;
import java.util.ArrayList;

class Solution {
    public List<Integer> partitionLabels(String s) {
        // Initialize an array to store the last appearance index of each character.
        int[] lastIndices = new int[26]; // Assuming 'a' to 'z' only appear in the string.
        int lengthOfString = s.length();
        // Fill the array with the index of the last occurrence of each character.
        for (int i = 0; i < lengthOfString; ++i) {
            lastIndices[s.charAt(i) - 'a'] = i;
        }

        // Create a list to store the lengths of the partitions.
        List<Integer> partitionLengths = new ArrayList<>();
        int maxIndexSoFar = 0; // To keep track of the farthest reach of the characters seen so far.
        int partitionStart = 0; // The index where the current partition starts.

        // Iterate through the characters in the string.
        for (int i = 0; i < lengthOfString; ++i) {
            // Update the maxIndexSoFar with the farthest last occurrence of the current character.
            maxIndexSoFar = Math.max(maxIndexSoFar, lastIndices[s.charAt(i) - 'a']);

            // If the current index matches the maxIndexSoFar, we've reached the end of a partition.
            if (maxIndexSoFar == i) {
                // Add the size of the partition to the list.
                partitionLengths.add(i - partitionStart + 1);
                // Update the start index for the next partition.
                partitionStart = i + 1;
            }
        }

        // Return the list containing the sizes of the partitions.
        return partitionLengths;
    }
}
```

C++

```cpp
#include <vector>
#include <string>
using namespace std;

class Solution {
public:
    // This function partitions the string such that each letter appears in at
    // most one part, and returns a vector of integers representing the size of these parts.
    vector<int> partitionLabels(string S) {
        int lastIndex[26] = {0}; // Array to store the last index of each letter
        int length = S.size(); // Get the size of the string

        // Fill the array with the index of the last occurrence of each letter in the string
        for (int i = 0; i < length; ++i) {
            lastIndex[S[i] - 'a'] = i;
        }

        vector<int> partitions; // This will store the sizes of the partitions
        int maxIndex = 0; // Maximum index of a character within the current partition
        int anchor = 0; // Starting index of the current partition

        for (int i = 0; i < length; ++i) {
            maxIndex = max(maxIndex, lastIndex[S[i] - 'a']); // Update maxIndex for the current partition

            // If the current index is the max index of the partition, it means we can make a cut here
            if (maxIndex == i) {
                partitions.push_back(i - anchor + 1); // Partition size is added to the result
                anchor = i + 1; // Update the starting index of the next partition
            }
        }

        return partitions; // Return the sizes of the partitions
    }
};
```

TypeScript

```typescript
function partitionLabels(inputString: string): number[] {
    // Initialize an array to hold the last index at which each letter appears.
    const lastIndex: number[] = Array(26).fill(0);
    // Helper function to convert a character to its relative position in the alphabet.
    const getCharacterIndex = (char: string) => char.charCodeAt(0) - 'a'.charCodeAt(0);
    // Get the length of the input string.
    const inputLength = inputString.length;

    // Update the lastIndex array with the last position for each letter in the input.
    for (let i = 0; i < inputLength; ++i) {
        lastIndex[getCharacterIndex(inputString[i])] = i;
    }

    // Prepare an array to hold the lengths of the partitions.
    const partitionLengths: number[] = [];
    // Initialize pointers for tracking the current partition.
    let start = 0, end = 0, maxIndex = 0;

    for (let i = 0; i < inputLength; ++i) {
        // Update the maximum index for the current partition.
        maxIndex = Math.max(maxIndex, lastIndex[getCharacterIndex(inputString[i])]);
        // If the current position matches the maxIndex, a partition is complete.
        if (maxIndex === i) {
            // Add the size of the current partition to the array.
            partitionLengths.push(i - start + 1);
            // Move the start to the next position after the current partition.
            start = i + 1;
        }
    }

    // Return the array of partition sizes.
    return partitionLengths;
}
```

```python
class Solution:
    def partitionLabels(self, S: str) -> List[int]:
        # Create a dictionary to store the last occurrence of each character.
        last_occurrence = {char: index for index, char in enumerate(S)}

        # Initialize variables.
        # `max_last` represents the farthest index any character in the current partition has been seen.
        # `partition_start` represents the start of the current partition.
        max_last = partition_start = 0

        # This list will hold the sizes of the partitions.
        partition_sizes = []

        # Iterate through the characters of the string along with their indices.
        for index, char in enumerate(S):
            # Update `max_last` to be the max of itself and the last occurrence of the current character.
            max_last = max(max_last, last_occurrence[char])

            # If the current index is the last occurrence of all characters seen so far in this partition,
            # that means we can end the partition here.
            if max_last == index:
                # Append the size of the current partition to the list (`index - partition_start + 1`).
                partition_sizes.append(index - partition_start + 1)

                # Update `partition_start` to the next index to start a new partition.
                partition_start = index + 1

        # Return the list of partition sizes.
        return partition_sizes
```

Time and Space Complexity

Time Complexity:

The time complexity for the provided code can be analyzed as follows:

- The first line within the `partitionLabels` function uses a dictionary comprehension to store the last appearance index of each character. This runs in $O(n)$ time, where n is the length of string `s`, because it iterates through the entire string once.
- The subsequent `for` loop also runs through the string once and performs a constant number of operations within the loop. Hence, the complexity of this part is $O(n)$.
- Overall, since both the creation of the dictionary and the for loop run in linear time relative to the size of the input `s`, and occur sequentially, the total time complexity of the function is $O(n)$.

Space Complexity:

- The space complexity is influenced by the additional storage used:
  - The `last` dictionary which stores the index of the last occurrence of each character has at most $O(\min(n, 26))$ space complexity because there are at most 26 letters in the alphabet. In the case where $n \geq 26$, the dictionary would have at most 26 elements.
  - The `ans` list where the sizes of the partitions are stored will have a space complexity of $O(n)$ in the worst case, where each character is a partition on its own.
- Since the `last` dictionary has a fixed upper limit due to a finite alphabet, it's typically considered $O(1)$ space, particularly in the context of large n.
- However, the list `ans` depends on the size of the input and can take up to $O(n)$ space in the worst case scenario.
- Therefore, the overall space complexity of the function is $O(n)$, reflecting the worst-case scenario where the output list could grow linearly with the input size.