2021. Brightest Position on Street

Medium | Array | Ordered Set | Prefix Sum

Leetcode Link

Problem Description

In this problem, we are provided with a representation of a street and its street lamps. The street is visualized as a number line, and street lamps are given as a 2D integer array `lights`. Each entry in this array, `lights[i]`, contains two integers: `position_i` and `range_i`. The first integer `position_i` indicates the location of the street lamp on the number line, while the second integer `range_i` tells us how far to the left and right the lamp illuminates from its position. Consequently, the area that each street lamp lights up is from `position_i - range_i` to `position_i + range_i` inclusive.

The problem defines the **brightness** of a position on the street as the number of street lamps that illuminate that particular position. Our task is to determine the **brightest position** on the street, which is the position that is lit by the highest number of street lamps. If there happens to be more than one position with the same maximum brightness, the problem asks us to return the smallest of these brightest positions.

Intuition

The intuition behind solving this problem involves treating the street as a range of positions and each lamp as contributing a unit of brightness to a specific range. We can use sweep line algorithm concepts to efficiently determine the brightness of each position. Here's how we can approach the problem:

1. Consider each lamp as triggering an 'event' at the start and end of its range.
2. When a lamp starts at `position_i - range_i`, it adds to the brightness (+1), and when its influence ends at `position_i + range_i + 1`, it subtracts from the brightness (-1). This is because the range is inclusive, so the end of the influence is technically one position beyond the range.
3. We can sweep over all the positions influenced by any lamp, and at each point, we can add or subtract the brightness accordingly.
4. We use a dictionary to keep track of all these 'events' of starting and ending of a lamp's influence, then sort this dictionary by the key (which represents positions on the street).
5. We sweep the sorted positions and maintain a running sum (`s`) of the brightness, updating the maximum brightness (`mx`) seen so far and the position (`ans`) where this maximum brightness occurs.
6. At the end of the sweep, `ans` will hold the smallest position with the maximum brightness because we sorted the positions and the calculation respects the order.

The implementation translates this intuition into a working solution, utilizing Python's `defaultdict` to facilitate the management of the brightness changes and keeping track of the running sum and maximum brightness dynamically as we sweep through the positions.

Solution Approach

The implementation of the solution uses a variety of data structures and an intelligent algorithm to achieve an efficient approach.

- **Data Structures Used:**
  - **defaultdict(int):** A Python `defaultdict` with `int` type is used to track the starting and ending influence points of each lamp's light on the street. If an index is not already in the dictionary, it defaults to `0` which is convenient because it represents the initial state of brightness for any position on the street.
- **Algorithms and Patterns:**
  - **Sweep Line Algorithm:** This algorithm is used to simulate the process of 'sweeping' through the street from left to right. As we 'sweep' through the street, we encounter events that start or end a lamp's range.
  - **Event Processing:** Each lamp generates two events: one where its light begins (`position_i - range_i`) and one where it ends (`position_i + range_i + 1`). In the `defaultdict`, we increase the brightness by `1` at the start and decrease it by `1` at the end of the range.
  - **Sorting the Events:** Sorting the keys of the `defaultdict` ensures that we process these events in the order of their occurrence along the street.
  - **Running Sum and Maximization:** In the sweep process, a running sum `s` is maintained which updates at each event, increasing or decreasing according to the lamp's influence. We then compare this running sum to the maximum brightness `mx` encountered so far. If the current sum is greater, we update the `mx` and record the position `ans` at which this new maximum occurs.

The code reflects these concepts as follows:

- We iterate through each lamp in `lights` and calculate the starting (`l`) and ending (`r`) points of its range, updating the defaultdict accordingly.
- We then sort the keys of the defaultdict (the positions where brightness changes occur) and perform the 'sweep' by iteratively adding or subtracting from the running sum `s`.
- Whenever the running sum `s` exceeds the previously tracked maximum brightness `mx`, we update `mx` with `s` and set `ans` to the current position `k`.

As a result, the code does not require us to calculate the brightness for every single position on the street. Instead, it efficiently tracks changes in brightness at specific points, which allows us to find the brightest position without redundant calculations.

Example Walkthrough

Let's go through the solution approach with a small example. Suppose we are given the following `lights` array representing the street lamps:

```
lights = [[1, 1], [4, 1]]
```

Each of the subarrays represents a street lamp with `[position_i, range_i]`. Now, we will walk through the algorithm step by step.

1. **Initialization of Events:**
   - For the first lamp `[1, 1]`, it influences the street in the range `[0, 2]` (from `position_i - range_i` to `position_i + range_i`).
   - For the second lamp `[4, 1]`, it influences the street in the range `[3, 5]`.
2. **Creating and Populating Event Points:**
   - We treat the start and end of each range as events. We use a defaultdict to store the brightness increase and decrease at specific points:

```python
events = defaultdict(int)
events[0] += 1
events[3] += 1
events[2+1] -= 1  # We add 1 because the range end is inclusive
events[5+1] -= 1  # Same as above
```

3. **Sorting Events:**
   - We now have the events `{0: 1, 3: 1, 3: -1, 6: -1}`.
   - Sorting them by key gives us the order `[0, 3, 3, 6]`.
4. **Sweeping Through Sorted Event Points:**
   - We begin with a running sum `s = 0`, maximum brightness `mx = 0`, and the answer `ans = 0`.
   - At position `0`, we encounter the start of the first lamp's influence, so `s += events[0]` (now `s = 1`).
   - At position `3`, we have two events. The start of the second lamp's influence (`s += events[3]`, now `s = 2`) and the end of the first lamp's influence (`s += events[3]`, now `s = 1`).
   - At position `6`, we encounter the end of the second lamp's influence, so `s -= events[6]` (now `s = 0`).
   - After processing each event, we update `mx` and `ans` whenever we find a new maximum brightness.
5. **Finding the Brightest Position:**
   - During the process, we found the maximum brightness to be `mx = 2` at positions `3`, but since there are two events at `3`, this number decreases back to `1` by the end of processing all events at `3`.
   - `ans` during the occurrence of `mx` is set to the position at which this happened, which in this case, initially, is `3`.

Hence, by the end of the sweep, we determined that the brightest position on the street is `3` with a brightness of `2`, and that is our final answer.

By following this approach, we only calculate the brightness at specific event points instead of for every position on the street, which makes the algorithm efficient and effective for solving this kind of problem.

Python Solution

```python
from collections import defaultdict

class Solution:
    def brightestPosition(self, lights: List[List[int]]) -> int:
        # Initialize a dictionary to store the changes in brightness
        brightness_changes = defaultdict(int)

        # Iterate over each light
        for position, radius in lights:
            # Determine the range of positions affected by the light's brightness
            left_border = position - radius
            right_border = position + radius

            # Increment the brightness at the start of the range
            brightness_changes[left_border] += 1
            # Decrement the brightness just after the end of the range
            brightness_changes[right_border + 1] -= 1

        # Initialize variables to keep track of the current brightness,
        # the maximum brightness observed, and the position of the maximum brightness
        current_brightness = max_brightness = 0
        brightest_position = 0

        # Iterate over the positions in the sorted order of the keys
        for position in sorted(brightness_changes):
            # Update the current brightness
            current_brightness += brightness_changes[position]
            # If the current brightness is greater than the maximum recorded brightness
            if max_brightness < current_brightness:
                # Update the maximum brightness and brightest position
                max_brightness = current_brightness
                brightest_position = position

        # Return the position with the maximum brightness
        return brightest_position

```

Java Solution

```java
class Solution {
    public int brightestPosition(int[][] lights) {
        // Use a TreeMap to easily manage the range of light contributions on the positions
        TreeMap<Integer, Integer> deltaBrightness = new TreeMap<>();

        // Iterate over each light array to calculate the influence ranges and store them
        for (int[] light : lights) {
            int leftBoundary = light[0] - light[1]; // Calculate left boundary of the light
            int rightBoundary = light[0] + light[1]; // Calculate right boundary of the light

            // Increase brightness at the start of the range
            deltaBrightness.merge(leftBoundary, 1, Integer::sum);
            // Decrease brightness right after the end of the range
            deltaBrightness.merge(rightBoundary + 1, -1, Integer::sum);
        }

        int brightestPosition = 0; // To hold the result position with the brightest light
        int currentBrightness = 0; // Current accumulated brightness
        int maxBrightness = 0; // Max brightness observed at any point

        // Iterate over the entries in the TreeMap
        for (var entry : deltaBrightness.entrySet()) {
            int changeInBrightness = entry.getValue();
            currentBrightness += changeInBrightness; // Apply the change on the current brightness

            // Check if the current brightness is the maximum observed so far
            if (maxBrightness < currentBrightness) {
                maxBrightness = currentBrightness; // Update the maximum brightness
                brightestPosition = entry.getKey(); // Update the position of the brightest light
            }
        }

        return brightestPosition; // Return the position with the maximum brightness
    }
}

```

C++ Solution

```cpp
class Solution {
public:
    int brightestPosition(vector<vector<int>>& lights) {
        // Create a map to store the changes in brightness at different positions
        map<int, int> brightnessDeltas;

        // Loop through each light and update the map with the range it illuminates
        for (auto& light : lights) {
            // Calculate the left and right bounds of the light's effect
            int left = light[0] - light[1];
            int right = light[0] + light[1];

            // Increase brightness at the starting position of the light's effect
            ++brightnessDeltas[left];
            // Decrease brightness just after the end of the light's effect
            --brightnessDeltas[right + 1];
        }

        // Variable to store the brightest position found so far
        int brightestPosition = 0;
        // Variable to keep track of the current sum of brightness as we iterate
        int currentBrightness = 0;
        // Variable to store the maximum brightness encountered
        int maxBrightness = 0;

        // Iterate through brightnessDeltas to find the brightest position
        for (auto& [position, delta] : brightnessDeltas) {
            // Update the current brightness based on the delta
            currentBrightness += delta;

            // If we find a brighter position, update maxBrightness and brightestPosition
            if (maxBrightness < currentBrightness) {
                maxBrightness = currentBrightness;
                brightestPosition = position;
            }
        }

        // Return the position with the maximum brightness
        return brightestPosition;
    }
};

```

Typescript Solution

```typescript
// Define the type structure for the light positions array
type LightPosition = [number, number];

/**
 * Returns the brightest position on a street given the array of lights.
 * @param lights - An array of tuples representing lights, where each tuple consists of the position and range.
 * @return The position of the brightest point.
 */
const brightestPosition = (lights: LightPosition[]): number => {
    // Create a map where the key is the position on the street and the value is the change in brightness at that point.
    const deltaBrightness = new Map<number, number>();

    // Populate the map with brightness changes, accounting for the lights turning on at their start position
    // and off immediately after their end position.
    for (const [position, range] of lights) {
        const start = position - range;
        const end = position + range;
        deltaBrightness.set(start, (deltaBrightness.get(start) ?? 0) + 1);
        deltaBrightness.set(end + 1, (deltaBrightness.get(end + 1) ?? 0) - 1);
    }

    // Extract and sort the keys from the map to iterate over positions in ascending order.
    const sortedPositions = Array.from(deltaBrightness.keys()).sort((a, b) => a - b);

    let currentBrightness = 0; // Accumulated brightness at the current position.
    let maxBrightness = 0;     // Maximum brightness encountered so far.
    let brightestPos = 0;      // Position with the maximum brightness.

    // Iterate over all positions to find the maximum accumulated brightness.
    for (const position of sortedPositions) {
        currentBrightness += deltaBrightness.get(position) || 0;

        // Update maximum brightness and position if the current brightness is greater.
        if (maxBrightness < currentBrightness) {
            maxBrightness = currentBrightness;
            brightestPos = position;
        }
    }

    // Return the position with the maximum brightness.
    return brightestPos;
};

// Example usage (Optional):
// const lights: LightPosition[] = [[1, 2], [3, 6], [5, 5]];
// console.log(brightestPosition(lights)); // Should output the position of the brightest point

```

Time and Space Complexity

The time complexity of the code is `O(N log N)` where `N` is the number of light ranges in the `lights` list. This complexity arises because we sort the keys of our dictionary `d`, which contains at most `2N` keys (each light contributes two keys: the start and end of its illumination range). Sorting these keys dominates the runtime complexity.

The space complexity of the code is `O(N)` since we use a dictionary to store the changes to brightness at each key point. In the worst case, if every light has a unique range, the dictionary could have as many as `2N` keys, where `N` is the number of light ranges in the `lights` list.