2002. Maximum Product of the Length of Two Palindromic Subsequences

Dynamic Programming

The problem is about finding two non-overlapping palindromic subsequences in a given string s. A subsequence is a sequence that

Problem Description

Bit Manipulation

String

Medium

can be derived from another sequence by deleting some or no elements without changing the order of the remaining elements. The goal is to maximize the product of the lengths of these two palindromic subsequences. A string is palindromic if it reads the same forward and backward. The main challenge is to ensure that the subsequences are disjoint, meaning they do not share characters at Intuition

Backtracking

Bitmask

Leetcode Link

the same index positions.

3. Attempt to pair each palindromic subsequence with another, ensuring they do not overlap (disjoint).

2. Check which of these subsequences are palindromic.

1. Generate all possible subsequences of the string.

The intuition behind the solution involves the following steps:

- 4. Calculate the product of the lengths of each pair and keep track of the maximum product found.
- The solution uses bitwise operations to efficiently represent and iterate over all subsequences. The bitmask representing each
- subsequence is used to check if two subsequences are disjoint by using XOR and AND operations. It also counts the number of set bits (1s) in the bitmask using .bit_count() to determine the length of the subsequence without actually constructing the subsequence string, which saves time and memory. Finally, it uses the precomputed palindromic status of each bitmask to quickly
- check if a subsequence is palindromic, avoiding repeated calculations. **Solution Approach**

The solution provided leverages bit manipulation and dynamic programming to tackle the problem. The approach can be broken down into the following steps: 1. Precompute Palindromic Subsequences:

• We iterate over all possible subsequences represented by bitmasks where each bit corresponds to an index in the string s. A

bit set to 1 represents the inclusion of that character in the subsequence, while 0 represents exclusion. • The variable p is an array where each index corresponds to a bitmask of a subsequence, p is used to store whether the

represented subsequence is palindromic or not. Initially, all values in p are set to True. • The first for loop goes through all possible bitmasks, using k. Nested while loops are then used to iterate from the outermost characters toward the center, checking if the characters are equal. If a pair of characters is not equal, p[k] is set to False

- and the loop breaks, indicating that this subsequence is not palindromic. 2. Find Maximized Product of Lengths:
 - With all palindromic subsequences identified, we loop through them with the bitmask i. The bitmask mx is computed as the XOR of i with the bitmask representing all characters in the string (i.e., (1 << n) - 1). This essentially inverts i, marking all indices not included in i. Then, we initialize j with mx and enter a nested loop. Inside this loop, we only consider bitmasks j that are palindromic (p[j]
 - == True). For each such bitmask, we use .bit_count() to calculate the length of the palindromic subsequences corresponding to the bitmasks i and j (stored in variables a and b, respectively). • The product of a and b is calculated and checked against the current maximum product ans. If it is larger, it becomes the

• The critical optimization here is to iterate through all smaller bitmasks of mx that are still palindromic. This is done by

• After all combinations are checked, the maximum product calculated is stored in ans, which is returned as the result.

The algorithm makes use of bit manipulation to efficiently enumerate subsequences and dynamically checks for palindromic

properties to reduce redundant calculations. By exploiting bit counts and clever looping, it is able to quickly find the maximum

decrementing j at each step using j = (j - 1) & mx. By ANDing with mx, we ensure we get smaller bitmasks that represent

4. Return Result:

Example Walkthrough

new maximum.

3. Iterate Over All Combinations:

subsequences disjoint from subsequence i.

product of the lengths of two disjoint palindromic subsequences.

have $2^5 - 1 = 31$ possible non-empty subsequences.

• p[5] = True since the bitmask 5 (00101) is palindromic.

p[20] = True for 20 (10100) since it's palindromic.

p[10] = False for 10 (01010) because it's not palindromic.

For simplicity, let's consider a few bitmasks and their corresponding subsequences:

yield the maximum product of their lengths. 1. Precompute Palindromic Subsequences:

• First, we generate all possible subsequences using bit masks. For the string s = "ababa" which has a length of 5, we will

Let's consider a small example with the string s = "ababa". We want to find two non-overlapping palindromic subsequences that

 00101 which represents the subsequence "aa" (palindromic) 01010 which represents the subsequence "bbb" (not palindromic) ■ 10100 which represents the subsequence "aba" (palindromic) • The array p would reflect if these subsequences are palindromic (True or False). For our example:

2. Find Maximized Product of Lengths: Now, we look for pairs of palindromic subsequences that do not overlap and calculate the product of their lengths.

∘ If we take the bitmask 20 (10100) which corresponds to the subsequence "aba", we would then find the inverted bitmask mx = 31 XOR 20 = 11 (01111) representing 'b', 'ab', 'bb' or 'abb'. Within the bitmask 11 (01111), we look for palindromic subsequences.

• We check the lengths using .bit_count(): the length of 20 (10100) is 3 and the length of 3 (00011) is 2. The product is 3 *

• Let's say we find the bitmask 3 (00011) which represents the subsequence "ab" and is also palindromic.

 \circ We keep decrementing j using j = (j - 1) & mx to find all smaller, non-overlapping palindromic subsequences. • For instance, if j was initially 11 (01111), the next j would be 7 (00111), representing the subsequence "aab", which is not

find the maximum product of lengths of two non-overlapping palindromic subsequences.

Precompute all palindromic substrings using bit representation

while left < right and (bitmask >> left & 1) == 0:

Find the next '1' bit from the left

Find the next '1' bit from the right

- product. In this example, the maximum product is 6, given by the subsequences "aba" (length 3) and "ab" (length 2).
- def maxProduct(self, s: str) -> int: # Length of the string n = len(s)# is_palindrome will denote if the binary representation of a number corresponds to a palindromic substring

After examining all possible palindrome subsequence combinations, we determine the ones that give us the maximum

This walkthrough provides a conceptual understanding of how the solution uses bit masks and dynamic programming to efficiently

25 26 # Initialize the result for maximum product of the lengths 27 max_product = 0 28 29 # Iterate over all possible bitmasks

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2 = 6.
3. Iterate Over All Combinations:
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4. Return Result:

Python Solution

class Solution:

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C++ Solution

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p.

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1 class Solution {

int maxProduct(string s) {

is_palindrome = [True] * (1 << n)</pre>

for bitmask in range(1, 1 << n):</pre>

left, right = 0, n - 1

break

left += 1

right -= 1

return max_product

while left < right:</pre>

palindromic.

- 17 while left < right and (bitmask >> right & 1) == 0: 18 right -= 1 # If the corresponding characters do not match, this is not a palindrome 19 20 if left < right and s[left] != s[right]:</pre> 21 is_palindrome[bitmask] = False
- for i in range(1, 1 << n): 30 # Proceed only if the bitmask represents a palindrome 31 32 if is_palindrome[i]: 33 # Inverse bitmask: set bits become unset and vice versa 34 inverse mask = $((1 << n) - 1) ^ i$
- # Iterate over all submasks of the inverse bitmask 36 37 = inverse mask 38 len_a = i.bit_count() # Count of set bits, giving the length of palindrome A 39 while j: 40 # If j represents a palindrome, calculate the product of the lengths
- 41 if is_palindrome[j]: 42 len_b = j.bit_count() # Length of palindrome B 43 max_product = max(max_product, len_a * len_b) 44 # Move to the next submask 45 $j = (j - 1) \& inverse_mask$
- 48 Java Solution
 - 1 import java.util.Arrays; class Solution { public int maxProduct(String s) {
- // Get the length of the string int stringLength = s.length(); 6 // Initialize a boolean array for palindrome checks with size as all possible subsets boolean[] isPalindrome = new boolean[1 << stringLength];</pre> 8 // Default all entries to true 9 Arrays.fill(isPalindrome, true); 10 11 12 // Check each subset to see if it forms a palindrome 13 for (int subset = 1; subset < 1 << stringLength; ++subset) {</pre> 14 for (int left = 0, right = stringLength - 1; left < stringLength; ++left, --right) {</pre> 15 // Find the next index 'left' where subset has a bit set while (left < right && (subset >> left & 1) == 0) { 16
- 17 ++left; 18 19 // Find the next index 'right' where subset has a bit set 20 while (left < right && (subset >> right & 1) == 0) { 21 --right; 22 // If the characters at 'left' and 'right' don't match, it's not a palindrome 23 24 if (left < right && s.charAt(left) != s.charAt(right)) {</pre> 25 isPalindrome[subset] = false; 26 break; 27 28
- // Iterate through all subsets of complement 41 for (int j = complement; j > 0; j = (j - 1) & complement) { 42 43 if (isPalindrome[j]) { // If the subset at index j forms a palindrome 44 int countB = Integer.bitCount(j); // Count the number of set bits 45 // Update the maximum product if the current pair product is larger maximumProduct = Math.max(maximumProduct, countA * countB); 46 47 48 49 50 51 52 return maximumProduct; // Return the maximum product found 53 54

// Check each subsequence represented by a bitmask to see if it is a palindrome

for (int left = 0, right = n - 1; left < right; ++left, --right) {</pre>

// Function to compute the maximum product of the lengths of two non-overlapping palindromic subsequences

// Advance the left index until it points to a character included in the subsequence

// Move the right index back until it points to a character included in the subsequence

// Iterate over all bitmasks to find the maximum product of palindromic subsequence pairs

if (isPalindrome[i]) { // Only consider the bitmask if it represents a palindrome

for (int j = complementMask; j; j = (j - 1) & complementMask) {

if (isPalindrome[i]) { // Only consider the bitmask if it represents a palindrome

// Function to count the number of set bits in a bitmask (equivalent to __builtin_popcount in C++)

const lengthB = bitCount(j); // Compute the length of palindrome B

const lengthA = bitCount(i); // Compute the length of palindrome A

for (let $j = complementMask; j; j = (j - 1) & complementMask) {$

int lengthA = __builtin_popcount(i); // Compute the length of palindrome A

// If the characters at the current left and right indices do not match, this is not a palindrome

int complementMask = $((1 << n) - 1) ^ i; // Generate a bitmask for the complementary subsequence$

int lengthB = __builtin_popcount(j); // Compute the length of palindrome B

maxProduct = max(maxProduct, lengthA * lengthB); // Update the maximum product

// Find the other palindromic subsequence with the maximum length that can pair with the current one

vector<bool> isPalindrome(1 << n, true); // Initialize a vector to track if a subsequence represented by bitmask is a palin

int maximumProduct = 0; // Initialize the maximum product of palindrome lengths

if (isPalindrome[i]) { // If the subset at index i forms a palindrome

int countA = Integer.bitCount(i); // Count the number of set bits

// Calculate the product of lengths for all pairs of palindromic subsets

for (int i = 1; i < 1 << stringLength; ++i) {</pre>

// Calculate the complement of subset i

int n = s.size(); // Get the size of the input string

while (left < right && !(mask >> left & 1)) {

while (left < right && !(mask >> right & 1)) {

if (left < right && s[left] != s[right]) {</pre>

int maxProduct = 0; // Initialize the maximum product to 0

isPalindrome[mask] = false;

for (int mask = 1; mask < (1 << n); ++mask) {</pre>

++left;

--right;

break;

for (int i = 1; i < (1 << n); ++i) {

if (isPalindrome[j]) {

int complement = ((1 << stringLength) - 1) ^ i;</pre>

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             return maxProduct; // Return the final maximum product of palindromic subsequence lengths
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    };
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Typescript Solution
  1 // Function to compute the maximum product of the lengths of two non-overlapping palindromic subsequences
  2 function maxProduct(s: string): number {
         const n: number = s.length; // Get the length of the input string
         const isPalindrome: boolean[] = new Array(1 << n).fill(true); // Initialize an array to track if a subsequence is a palindrome</pre>
         // Check each subsequence represented by a bitmask to see if it is a palindrome
         for (let mask = 1; mask < (1 << n); ++mask) {</pre>
             let left = 0, right = n - 1;
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             while (left < right) {</pre>
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                 // Advance the left index until it points to a character included in the subsequence
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                 while (left < right && !((mask >> left) & 1)) {
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                     ++left;
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 14
                 // Move the right index back until it points to a character included in the subsequence
 15
                 while (left < right && !((mask >> right) & 1)) {
 16
                     --right;
 17
 18
                 // If the characters at the current left and right indices do not match, this is not a palindrome
                 if (left < right && s[left] !== s[right]) {</pre>
 19
 20
                     isPalindrome[mask] = false;
 21
                     break;
 22
                 left++;
 23
 24
                 right--;
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         let maxProduct = 0; // Initialize the maximum product to 0
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 30
         // Iterate over all bitmasks to find the maximum product of palindromic subsequence pairs
 31
         for (let i = 1; i < (1 << n); ++i) {
```

const complementMask = $((1 << n) - 1) ^ i; // Generate a bitmask for the complementary subsequence$

// Find the other palindromic subsequence with the maximum length that can pair with the current one

maxProduct = Math.max(maxProduct, lengthA * lengthB); // Update the maximum product

The time complexity of the code above can be analyzed as follows: 1. The first for loop, running from k in range(1, 1 \ll n), enumerates all possible subsets of the string s where n is the length of s.

if (isPalindrome[j]) {

function bitCount(mask: number): number {

count += mask & 1;

Time and Space Complexity

The space complexity is determined by:

let count = 0;

while (mask) {

return count;

mask >>= 1;

return maxProduct; // Return the final maximum product

is 2^n , so this part of the code runs in $0(n * 2^n)$ time. 2. The second part of the code contains two nested loops. The outer loop runs for 2ⁿ - 1 iterations, and for each iteration, the inner while loop potentially runs multiple times. The maximal number of times the inner loop can run can be approximated by 2^n

again because it starts at mx and decreases until it reaches 0. However, the average number of iterations is less due to the

For each subset, the while loop checks if it forms a palindrome, which takes 0(n) time in the worst case. The number of subsets

- bitwise AND operation with mx. Since the exact number of iterations is hard to determine without a deeper analysis of the distribution of palindromes, we can approximate the time complexity of the inner loop with the upper bound of 0(2ⁿ) as well. The calculation within the loop includes bit count $(0(\log n))$ and a max operation (0(1)), which are considerably less than $0(2^n)$, so they don't affect the overall time complexity. Thus, the second part of the code runs in $0(2^n * 2^n)$ time.
- 2. The variables and constant space usage inside the loops do not contribute to the space complexity significantly as compared to Therefore, the total time complexity of the algorithm can be estimated as $0(n * 2^n + 2^n * 2^n)$ which simplifies to $0(2^n * 2^n)$

because $2^n * 2^n$ dominates $n * 2^n$. The space complexity is $0(2^n)$.

1. The boolean array p of size 2ⁿ, which results in a space complexity of 0(2ⁿ).