2857. Count Pairs of Points With Distance k

Problem Description

In this problem, we are given a list of points on a 2D plane, where each point is represented as a pair of integers (x, y). These pairs are the coordinates of the points. We are also given an integer `k`. The task is to count how many unique pairs of points `(i, j)` have a specific distance between them, where the distance is defined as `(x1 XOR x2) + (y1 XOR y2)` and XOR stands for the bitwise XOR operation. A key point to note is that a valid pair `(i, j)` must have the indices `i` less than `j` to ensure pairs are not counted multiple times and to maintain uniqueness.

Intuition

The intuition behind the solution emerges from understanding how XOR operation works and then optimizing the brute force approach of checking each possible pair of points to see if they meet our distance criteria.

Here are the steps to understand and arrive at the solution:

1. **Understanding XOR**: The XOR operation has a crucial property: if `a XOR b = c`, then `a = b XOR c` and `b = a XOR c`. This property will help us quickly find potential pairs that can have the distance `k` between them.
2. **Iterating Efficiently**: A naive approach would involve checking every possible pair of points, which would be inefficient with large datasets. We can optimize this by smartly iterating over possible values of one coordinate, reducing the search space.
3. **Counter Dictionary**: We use a Counter (dictionary subclass in Python that helps keep track of element counts) to keep track of how many points with certain coordinates we already have seen. The key of the counter will be the coordinate pair, and the value is the number of times we have seen that specific pair.
4. **Calculating Potential Matches**: For each new point `(x2, y2)` we're inspecting, we iterate through possible values of `a` (from `0` to `k`) and calculate a corresponding `b` such that `a + b = k`. With these values of `a` and `b`, we can determine what the coordinate `(x1, y1)` of a potential matching point would need to be using the XOR property mentioned earlier.
5. **Counting Valid Pairs**: For each of these potential matching coordinates `(x1, y1)`, we use our counter to see if we have previously come across such a point. If we have, we know that the current point `(x2, y2)` and the counted `(x1, y1)` would form a valid pair, so we add the count from the counter to our answer.
6. **Updating the Counter**: After running through all possible `a` and corresponding `b`, we add the current point `(x2, y2)` to our counter, indicating that we have now seen this particular point.
7. **Return the Answer**: Once we have iterated over all points and calculated the valid pairs, we return the total count.

Through this method, we can efficiently and systematically count all the pairs that have the XOR-based distance of `k`, without having to evaluate each pair of points individually.

Solution Approach

The solution uses a blend of counting methodology with smart iteration based on the properties of the XOR operation to count the number of valid point pairs that are at a distance `k` from each other.

Here is a step-by-step explanation of the algorithm:

1. **Initialization**:
   - A `Counter` object `cnt` is initialized to keep track of how many times a particular point `(x, y)` is seen. In Python, a `Counter` is a dictionary that stores elements as keys and their counts as values.
   - An integer `ans` is initialized to `0`, which will accumulate the number of valid pairs that meet the distance criteria.
2. **Iterate Over Points**:
   - We iterate over each point `(x2, y2)` in the `coordinates` array using a for-loop. This loop is responsible for looking at each point and determining how many points we've seen so far can form a valid pair with it at distance `k`.
3. **Determining Potential Pairs Based on Distance `k`**:
   - Inside the first loop, another for-loop runs through integer values from `0` to `k` inclusive. With each iteration, we take `a` as the current value of the loop, and we compute `b` such that `b = k - a`.
   - We then use the `XOR` property to find what `x1` and `y1` would be if `x2 XOR x1 = a` and `y2 XOR y1 = b`. This gives us the coordinates of a point that could be at the required distance `k` from `(x2, y2)`.
4. **Counting and Summing Valid Pairs**:
   - Using the calculated `(x1, y1)`, we check the `cnt` dictionary to see if we already came across this point in our previous iterations. If so, the value from the dictionary for the key `(x1, y1)` is added to `ans`. This step counts how many times we have a point that can pair with our current point to create a distance of `k`.
5. **Updating the Counter**:
   - After inspecting all possible matching points for `(x2, y2)`, we update the `cnt` dictionary by incrementing the count of the current point `(x2, y2)`. This means that now `(x2, y2)` can be considered as a potential match for future points.
6. **Returning the Result**:
   - Once all points have been accounted for, and all valid pairs have been counted, the final value of `ans`, which represents the number of valid pairs with the specified distance, is returned.

The algorithm efficiently uses a single pass through the points while leveraging the constant-time lookup feature of dictionaries in Python to keep the overall performance manageable. Moreover, the approach takes full advantage of the XOR operation's properties, avoiding an exhaustive enumeration of all possible point pairs, which could otherwise become computationally expensive.

Example Walkthrough

Let's take an example with the following points and `k = 5`:

```
points = [(0, 0), (1, 4), (4, 1), (5, 0)]
```

Now, let's walk through the solution approach step by step for `k = 5`:

1. **Initialization**:
   - `cnt = Counter()` is initialized to keep track of how many times a particular point is seen.
   - `ans = 0` is initialized to accumulate the number of valid pairs.
2. **Iterate Over Points**:
   - First we consider the point `(0, 0)`. There are no previous points to compare it with, so we simply move on after adding it to the `cnt`.
3. **Determining Potential Pairs Based on Distance `k`**:
   - Examining the next point `(1, 4)`. We iterate for `a` from `0` to `5` (our `k` value).
     - When `a = 0`: Then `b = 5`, we look for a point with coordinates such that `0 XOR x1 = 0` and `4 XOR y1 = 5`. This gives us `x1 = 0` and `y1 = 1`. But we have not seen `(0, 1)` before, so there is no match.
     - When `a = 1`: Then `b = 4`, we seek a point where `1 XOR x1 = 1` and `4 XOR y1 = 4`. We get `x1 = 0` and `y1 = 0` which we have seen, so we add `1` to `ans`.
     - We continue this process for other values of `a` until `5` but find no more matches.
   - `cnt[(1,4)] += 1`
4. **Next Points**:
   - For `(4, 1)`, repeating the process for values of `a`:
     - When `a = 0`: Then `b = 5`, and we look for `4 XOR x1 = 0` & `1 XOR y1 = 5` which yields `x1 = 4`, `y1 = 4`. No such point exists yet.
     - When `a = 1`: Then `b = 4`, and `4 XOR x1 = 1` & `1 XOR y1 = 4` which yields `x1 = 5`, `y1 = 5`. No match.
     - And so on, until a match is found for `a = 4`: Then `b = 1`. Here, `x1 = 5` & `y1 = 0` is a match, so we add `1` to `ans`.
   - `cnt[(4, 1)] += 1`
5. **Last Point**:
   - For `(5, 0)`, following the same method for different values of `a`, we find a match for `a = 1` & `b = 4`, which corresponds to the point `(4, 4)`. It's not in `cnt`, so no match is added to `ans`.
   - `cnt[(5, 0)] += 1`
6. **Returning the Result**:
   - Having processed all points, we find that `ans = 2`. There are two unique pairs of points with XOR-based distance `5`: `(0, 0)` & `(1, 4)`, `(0, 0)` & `(4, 1)`.

The algorithm allowed us to figure out the number of valid point pairs with the desired property by efficiently iterating through the list of points and using the Counter to avoid unnecessary pair checks.

Python Solution

```python
from collections import Counter

class Solution:
    def count_pairs(self, coordinates: List[List[int]], k: int) -> int:
        # Create a counter to keep track of the number of occurrences of each coordinate
        cnt = Counter()
        # Initialize the answer to zero
        ans = 0
        # Iterate over all coordinates
        for x2, y2 in coordinates:
            # Check all possible combinations of (a, b) where a + b = k
            for a in range(k + 1):
                b = k - a

                # Calculate the original coordinates (x1, y1) before being XOR'ed
                # by using the current coordinate (x2, y2) and the calculated values of a and b
                x1, y1 = a ^ x2, b ^ y2

                # Add the number of times the original coordinate (x1, y1) has been seen so far
                ans += cnt[(x1, y1)]

            # Increment the count of the current coordinate (x2, y2)
            cnt[(x2, y2)] += 1

        # Return the total count of pairs found
        return ans
```

Java Solution

```java
import java.util.*;

class Solution {

    // Method to count the number of pairs whose bitwise XOR meets specified conditions
    public int countPairs(List<List<Integer>> coordinates, int k) {
        // Create a HashMap to store the frequency of occurrences of each coordinate pair
        HashMap<Long, Integer> frequencyCount = new HashMap<>();
        int answer = 0; // Initialize count of valid pairs to 0

        // Iterating through each coordinate pair in the list
        for (List<Integer> coordinate : coordinates) {
            // Extract x2 and y2 from the current coordinate
            int x2 = coordinate.get(0);
            int y2 = coordinate.get(1);

            // Calculate all possible pairs (x1, y1) within the range 0 to k
            for (int a = 0; a <= k; ++a) {
                int b = k - a; // Since a + b should be equal to k

                // Compute x1 and y1 using XOR operation on a, b with x2, y2 respectively
                int x1 = a ^ x2;
                int y1 = b ^ y2;

                // Increment count for this pair if it's already in the hashmap
                answer += frequencyCount.getOrDefault(list.of(x1, y1), 0);
            }

            // Update the frequencyMap with the current coordinate,
            // incrementing its count or adding it if doesn't exist
            frequencyCount.merge(coordinate, 1, Integer::sum);
        }

        // Return the final count of valid pairs
        return answer;
    }
}
```

C++ Solution

```cpp
#include <vector>
#include <unordered_map>

class Solution {
public:
    // Method to count the number of distinct pairs of points from the input coordinates that have a Manhattan distance of exactly k.
    int countPairs(vector<vector<int>>& coordinates, int k) {
        // Create a hash map to count occurrences of points
        unordered_map<long long, int> pointCount;

        // Helper function to convert a 2D point into a unique long long value
        auto getUniqueKey = [](int x, int y) -> long long {
            return static_cast<long long>(x) * 1000000LL + y;
        };

        int pairCount = 0; // Initialize the count of pairs to zero

        // Iterate through each point in coordinates
        for (const auto& point : coordinates) {
            int x2 = point[0], y2 = point[1];

            // Check all possible points (x1, y1) that could form a pair with (x2, y2) having Manhattan distance k
            for (int a = 0; a <= k; ++a) {
                int b = k - a;
                // Use the XOR operation to find the corresponding x1 and y1
                int x1 = a ^ x2;
                int y1 = b ^ y2;

                // Increase the count of pairs by the occurrences of the (x1, y1) point
                pairCount += pointCount[getUniqueKey(x1, y1)];
            }
            // Register the occurrence of the current point (x2, y2)
            ++pointCount[getUniqueKey(x2, y2)];
        }

        // Return the total count of pairs found
        return pairCount;
    }
};
```

Typescript Solution

```typescript
function countPairs(coordinates: number[][], k: number): number {
    // Initialize a map to keep track of the count of each coordinate pair
    const countMap: Map<number, number> = new Map();

    // Define a helper function to create a unique hash for a pair of coordinates
    const hashCoordinates = (x: number, y: number): number => x * 1000000 + y;

    // Initialize the count of valid pairs
    let pairCount = 0;

    // Loop through each coordinate in the array
    for (const [coordinateX2, coordinateY2] of coordinates) {
        // Check all possible pairs with Manhattan distance = k
        for (let a = 0; a <= k; ++a) {
            const b = k - a;
            // Find the counterpart for the current coordinate that will form a valid pair
            const partnerX = a ^ coordinateX2;
            const partnerY = b ^ coordinateY2;
            // Increase the count by the number of times the counterpart has been seen
            pairCount += countMap.get(hashCoordinates(partnerX, partnerY)) ?? 0; // Nullish coalescing to handle undefined values
        }
        // Increment the count for the current coordinate pair in the map
        const currentHash = hashCoordinates(coordinateX2, coordinateY2);
        countMap.set(currentHash, ((countMap.get(currentHash) ?? 0) + 1);
    }

    // Return the final count of valid pairs
    return pairCount;
}
```

Time and Space Complexity

Time Complexity

The given Python code has a nested loop where the outer loop goes through each coordinate in the list `coordinates`, and for each of these coordinates, the inner loop iterates `k + 1` times. If `n` is the number of coordinates, the total number of iterations of the inner loop across all executions of the outer loop is `n * (k + 1)`. Since the other operations inside the inner loop, including dictionary access and updates, take constant time, the time complexity is $O(n * k)$.

Space Complexity

The space complexity of the code is determined by the additional space required for the `Counter` object `cnt`. In the worst-case scenario, if all coordinates are unique, the counter will have an entry for each coordinate in the list. Therefore, if there are `n` coordinates, the space complexity will be $O(n)$ for storing all the unique coordinates in `cnt`.