1533. Find the Index of the Large Integer

Medium Array Binary Search Interactive Leetcode Link

Problem Description

In this problem, we are given a unique integer array where all elements are equal except for one that is larger than the others. Instead of direct access to the array, we are provided an API `ArrayReader` with two functions: `compareSub` and `length`. Our objective is to find the index of the element that is larger than the rest using the `ArrayReader` API, with the constraint of calling `compareSub` a maximum of 20 times.

The `compareSub` function compares the sum of two sub-arrays within given indices and returns `1` if the first is larger, `-1` if the second is larger, and `0` if they are equal. `length` returns the size of the array. It's important to note that both API functions are considered to operate in constant time, `O(1)`.

Intuition

Given that all elements of the array are equal except for one, we can use a divide and conquer strategy to narrow down the search space efficiently. To do this, we can partition the array into three equal parts and use the `compareSub` method to compare the sums of these parts.

The intuition is that the unique element, which is larger, will cause the sum of the part containing it to be greater than the others. Based on the comparison results, we can eliminate two-thirds of the array in each step, as follows:

- If the sum of the first and the second third is the same (`compareSub` returns `0`), the element must be in the last third.
- If the sum of the first third is greater than the sum of the second (`compareSub` returns `1`), the element must be in the first third.
- If the sum of the second third is greater than the sum of the first (`compareSub` returns `-1`), the element must be in the second third.

By repeating this process and shrinking the search interval, we eventually isolate the unique element's index. Our approach guarantees that we will find the index with a minimal number of calls to the `compareSub` function, well within the 20 call limit.

Solution Approach

The implementation of the solution follows a ternary search approach to identify the segment containing the unique element that is larger than the others.

The algorithm initializes two pointers, `left` and `right`, representing the search boundaries, which at the beginning correspond to the start and end indices of the array, respectively.

In each iteration of the while loop, which continues as long as `left` is less than `right`, the search space is divided into thirds by calculating two indices, `t2` and `t3`:

1. `t1` is set to `left`, the beginning of the current search space.
2. `t2` is calculated as `left + (right - left) // 3`, which is one-third into the current search space.
3. `t3` is `left + 2 * ((right - left) // 3) + 1`, which is two-thirds into the current search space.

The solution then uses these indices to call `compareSub(t1, t2, t2 + 1, t3)`. This comparison effectively evaluates the sum of the first third against the sum of the second third of the search space. Based on the return value of `compareSub`, the algorithm adjusts the `left` and `right` pointers:

- If `compareSub` returns `0`, this means the sums of the first and second thirds are equal, implying that the unique element is not in either of the first two thirds. Thus, the algorithm eliminates these two segments from consideration by setting `left` to `t3 + 1`.
- If `compareSub` returns `1`, this indicates that the first third contains the unique element, so the `right` pointer is adjusted to `t2` to discard the latter two-thirds of the search space.
- If `compareSub` returns `-1`, this shows that the unique element is in the second third, and the search space is updated by setting `left` to `t2 + 1` and `right` to `t3`.

The loop continues until `left` equals `right`, which means the search space has been narrowed down to a single element—the unique element that is larger than the others. At this point, the algorithm returns `left`, the index of the unique element.

This approach ensures that the search space is halved at each step, making it possible to find the element within a logarithmic number of comparisons, specifically `log3(n)`, where `n` is the length of the array. Using ternary search instead of binary search allows us to reduce the number of necessary comparisons, capitalizing on the specific nature of the problem.

Example Walkthrough

Let's consider that we have an `ArrayReader` that represents the following array: `[1, 1, 1, 2, 1, 1, 1, 1]`. We need to find the index of the element that is larger than the rest using the solution approach described above.

We have a total of 8 elements in the array, so our initial `left` is `0` and our `right` is `7`.

- Initially, `t1 = left = 0`.
- `t2 = left + (right - left) // 3 = 0 + (7 - 0) // 3 = 2`.
- `t3 = left + 2 * ((right - left) // 3) + 1 = 0 + 2 * ((7 - 0) // 3) + 1 = 5`.

We call `compareSub(0, 2, 3, 5)` and suppose it returns `0`, meaning the sum of elements from index 0 to 2 is the same as the sum of elements from index 3 to 5. Therefore, our unique element must be in the last third of the array.

So now, we update `left` to `t3 + 1 = 6`.

On the next iteration:

- `t1 = left = 6`.
- Since `right` remains the same, we calculate `t2` and `t3` within the new bounds.
- `t2 = left + (right - left) // 3 = 6 + (7 - 6) // 3 = 6`.
- `t3 = left + 2 * ((right - left) // 3) + 1 = 6 + 2 * ((7 - 6) // 3) + 1 = 7`.

We call `compareSub(6, 6, 7, 7)` and it can only return `1`, because the unique larger element must be at index `7` (since `6` and `6` are the same index, and the only remaining index is `7`).

Now that `left` and `right` have converged (both are `7`), we found our unique larger value at the index `7`. We return `7`.

This example demonstrates that by narrowing down the search space with each comparison and adjusting the pointers accordingly, we can find the index of the unique larger element with a minimal number of calls to the API, satisfying our constraints.

Python Solution

```python
class Solution:
    def getIndex(self, reader: 'ArrayReader') -> int:
        # Initialize two pointers for binary search
        left, right = 0, reader.length() - 1

        while left < right:
            # Divide the range into three equal parts
            # and determine their endpoints
            one_third = (right - left) // 3
            two_thirds = 2 * one_third

            first_part_end = left + one_third
            second_part_start = first_part_end + 1
            second_part_end = left + two_thirds
            third_part_start = second_part_end + 1

            # Compare the sum of the first third with the sum of the second third
            comparison_result = reader.compareSub(left, first_part_end, second_part_start, second_part_end)

            # If both sums are equal, the pivot index must be in the third part
            if comparison_result == 0:
                left = third_part_start
            # If the sum of the first third is greater, the pivot index is in the first part
            elif comparison_result == 1:
                right = first_part_end
            # If the sum of the second third is greater, the pivot index is in the second part
            else:
                left, right = second_part_start, second_part_end

        # When 'left' is equal to 'right', we have found the pivot index
        return left

```

Java Solution

```java
class Solution {
    public int getIndex(ArrayReader reader) {
        // Initialize pointers for left and right boundaries
        int left = 0;
        int right = reader.length() - 1;

        // Use a modified version of binary search to find the specific index
        while (left < right) {
            // Divide the range into three equal parts
            int mid1 = left + (right - left) / 3;
            int mid2 = left + (right - left) / 3 * 2 + 1;

            // Compare the sum of the first two-thirds with the sum of the last two-thirds
            int comparisonResult = reader.compareSub(left, mid1, mid1 + 1, mid2);
            // If the sums are equal, the desired index is in the last third
            if (comparisonResult == 0) {
                left = mid2 + 1;
            // If the sum of the first third is greater, the desired index is in the first third
            } else if (comparisonResult == 1) {
                right = mid1;
            // Otherwise, the desired index is in the middle third
            } else {
                left = mid1 + 1;
                right = mid2;
            }
        }
        // Left should now be the desired index as the range has been narrowed down to a single element
        return left;
    }
}

```

C++ Solution

```cpp
class Solution {
public:
    // Assuming the array contains a peak element (an element that is greater
    // than its neighbours), this function finds the index of one such peak element.
    int getIndex(ArrayReader& reader) {
        int left = 0; // Begin search at the start of the array
        int right = reader.length() - 1; // End search at the last element of the array

        // Continue searching while the range is not narrowed down to one element
        while (left < right) {
            // Divide the current range into three equal parts
            int part1End = left + (right - left) / 3;
            int part2Start = part1End + 1;
            int part2End = left + (right - left) / 3 * 2;
            int part3Start = part2End + 1;

            // Compare sum of elements in the first and second parts
            int comparisonResult = reader.compareSub(left, part1End, part2Start, part2End);

            if (comparisonResult == 0) {
                // If sums are equal, the peak must be in the third part, discard first two parts
                left = part3Start;
            } else if (comparisonResult == 1) {
                // If sum of first part is greater, the peak is in the first part, discard the rest
                right = part1End;
            } else {
                // If sum of second part is greater, the peak is between part2Start and part2End,
                // discard the parts outside of this range
                left = part2Start;
                right = part2End;
            }
        }

        // The narrowed down range will eventually converge to a single element.
        // This element is a peak (a candidate index where the element is not smaller than its neighbors).
        return left;
    }
};

```

Typescript Solution

```typescript
// Function to find the index of one peak element.
// A peak element is greater than its neighbors.
function getIndex(reader: { length: () => number, compareSub: (start1: number, end1: number, start2: number, end2: number) => number
    let left = 0; // Start of the search range
    let right = reader.length() - 1; // End of the search range

    // Continue search while there is more than one element in the range
    while (left < right) {
        // Divide the current range into three equal parts
        const part1End = left + Math.floor((right - left) / 3);
        const part2Start = part1End + 1;
        const part2End = left + Math.floor((right - left) / 3 * 2);
        const part3Start = part2End + 1;

        // Compare the sum of elements in the first and second parts
        const comparisonResult = reader.compareSub(left, part1End, part2Start, part2End);

        if (comparisonResult === 0) {
            // Sums are equal, peak must be in the third part
            left = part3Start;
        } else if (comparisonResult === 1) {
            // Sum of first part is greater, peak is in the first part
            right = part1End;
        } else {
            // Sum of second part is greater, peak is in between part2Start and part2End
            left = part2Start;
            right = part2End;
        }
    }

    // Once the range is narrowed down to one element, return it
    // This element is not guaranteed to be a peak, but based on the problem
    // statement it is considered as a correct answer.
    return left;
}

```

Time and Space Complexity

The given code uses a ternary search approach to find an index in a simulated array using the `ArrayReader` API. The while loop repeatedly narrows down the search range by dividing it into thirds and comparing the sums of different segments of the array.

Time Complexity:

The time complexity is `O(log3(n))`, where `n` is the length of the array. In each iteration of the loop, the size of the current search range is reduced by 1/3. Since we are making two calls to `compareSub` per iteration, these calls do not significantly increase the time complexity, and the dominating factor remains the division by three in each step. The complexity is determined by how many times we can divide the range length by 3 until we reach a range of length 1.

Space Complexity:

The space complexity is `O(1)` because no additional space is allocated proportional to the input size. The algorithm only uses a fixed number of variables to store the indices (`left`, `right`, `t1`, `t2`, and `t3`) and the comparison result `cmp` regardless of the size of the array.