264. Ugly Number II

Hash Table

Problem Description

Medium

is divisible by any prime factor other than 2, 3, or 5 is not an ugly number. The sequence of ugly numbers starts with 1 since it has no prime factors. The task is to find and return the nth ugly number in the sequence of all ugly numbers. Intuition

The straightforward way to find the nth ugly number would be to loop through all the integers, factorize each, and check if their

factors are limited to 2, 3, and 5. However, this method would be highly inefficient as it requires factorization of each number up

An ugly number is defined as a positive integer that only has prime factors of 2, 3, and 5. In other words, any positive integer that

Math Dynamic Programming Heap (Priority Queue)

to the nth ugly number. An efficient approach exploits the fact that every new ugly number must be built by multiplying a smaller ugly number by 2, 3, or 5. The idea is to maintain a list of ugly numbers found so far. For generating the next ugly number, the algorithm selects the

smallest number that can be obtained by multiplying any ugly number by 2, 3, or 5. This ensures that we are building upon the sequence of ugly numbers we have already generated and not missing any in between. The given code uses <u>Dynamic Programming</u> to keep track of the ugly numbers generated so far. It initializes an array dp where

dp[i] will represent the i+1th ugly number. It uses three pointers (p2, p3, p5) to track the position for the next multiplication with 2, 3, and 5 respectively. To find the next ugly number in the sequence, the algorithm will do the following:

• Multiply the ugly numbers at the respective pointers p2, p3, and p5 by 2, 3, and 5 to get next2, next3, and next5. • The minimum value of these three is the next ugly number.

corresponding to that pointer. This step simultaneously allows multiple increments if there are ties (i.e., two or three of the products have the

• Each pointer is incremented if and only if the minimum value is equal to the product of the respective ugly number and the prime number

same minimum value), ensuring that the same ugly number is not generated more than once.

- Continuing in this manner fills the dp array with the first n ugly numbers. The nth ugly number, or dp[n-1] since the array index
- is 0-based, is then returned.
- The solution uses dynamic programming as its main algorithmic pattern to build up a series of ugly numbers, ensuring that each new ugly number is the smallest possible one that follows the previous ugly numbers.

Let's look at the major steps of the implementation in detail:

Initialization: The solution starts by initializing a list dp containing n elements, all set to 1. The first element of the list, dp [0],

is set to 1 because 1 is the first ugly number by definition. It also sets up three pointers p2, p3, and p5, each starting at 0.

Building the Ugly Numbers List: The algorithm goes into a loop starting from i = 1 (since 1 is already in the list) up to i = 1.

is equal to the current minimum (the new ugly number just calculated), that pointer is incremented. This means that pointer is

now pointing at the next candidate that needs to be multiplied by its respective prime to compete for a place in the ugly

Returning the Result: After filling up the list with n ugly numbers, the algorithm returns the nth ugly number, which is dp[n -

These pointers will keep track of which ugly number should next be multiplied by 2, 3, and 5, respectively.

Within the loop:

number sequence.

progresses in the correct order, and no ugly numbers are missed.

programming method described. We will start with n = 7.

dp[i]. So let's illustrate the iterations:

- next2 = 2 * dp[p2] = 2 * dp[0] = 2

- next3 = 3 * dp[p3] = 3 * dp[0] = 3

- next5 = 5 * dp[p5] = 5 * dp[0] = 5

- next2 = 2 * dp[p2] = 2 * dp[1] = 4

- next3 = 3 * dp[p3] = 3 * dp[0] = 3

p3 is incremented because next3 was used.

dp[3] = 4 $p2 \longrightarrow 2$, $p3 \longrightarrow 1$, $p5 \longrightarrow 0$

dp[4] = 5 $p2 \longrightarrow 2$, $p3 \longrightarrow 1$, $p5 \longrightarrow 1$

dp[5] = 6 $p2 \longrightarrow 3$, $p3 \longrightarrow 2$, $p5 \longrightarrow 1$

dp[6] = 8 $p2 \longrightarrow 4$, $p3 \longrightarrow 2$, $p5 \longrightarrow 1$

entries and correct sequence progression.

def nth ugly number(self, n: int) -> int:

index_2, index_3, index_5 = 0, 0, 0

Generate ugly numbers up to the nth one

next 2 = uqly numbers[index 2] * 2

next 3 = uqly numbers[index 3] * 3

next_5 = ugly_numbers[index_5] * 5

Initialize pointers for multiples of 2, 3, and 5

Calculate the next multiples for 2, 3, and 5

ugly_numbers[i] = min(next_2, next_3, next_5)

// Dynamic programming array to store the ugly numbers

int nextMultipleOf2 = uglyNumbers[index2] * 2;

int nextMultipleOf3 = uglvNumbers[index3] * 3;

int nextMultipleOf5 = uglyNumbers[index5] * 5;

dp[i] = std::min(next2, std::min(next3, next5));

* Ugly numbers are numbers whose only prime factors are 2, 3, or 5.

dp.push(nextUglyNumber); // Append it to the dp array

// Initialize pointers for the multiples of 2, 3, and 5

// Initialize dp (dvnamic programming) array with the first ugly number

// Determine the next ugly number as the minimum of the next multiples

// Increment the corresponding pointer if it matches the ugly number

const nextUglyNumber = Math.min(nextMultipleOf2, nextMultipleOf3, nextMultipleOf5);

if (dp[i] == next2) ++pointer2;

if (dp[i] == next3) ++pointer3;

if (dp[i] == next5) ++pointer5;

* @param n The position of the ugly number to find.

let pointer2 = 0, pointer3 = 0, pointer5 = 0;

// Find the next multiples of 2, 3, and 5

const nextMultipleOf2 = dp[pointer2] * 2;

const nextMultipleOf3 = dp[pointer3] * 3;

const nextMultipleOf5 = dp[pointer5] * 5;

if (nextUqlyNumber === nextMultipleOf2) {

function nthUglyNumber(n: number): number {

// Return the nth ugly number.

return dp[n - 1];

* Calculates the nth ugly number.

* @return The nth ugly number.

let dp: number[] = [1];

for (let i = 1; i < n; ++i) {

};

/**

*/

TypeScript

// Increase the respective pointer if it was used.

if (uqlyNumbers[i] == nextMultiple0f2) {

// Next ugly number is the minimum of the next multiples of 2, 3 and 5

uglyNumbers[i] = Math.min(nextMultipleOf2, Math.min(nextMultipleOf3, nextMultipleOf5));

// Increment indices depending on which multiple the current ugly number is based on

// This ensures that for each prime factor, we only count multiples in ascending order

 $ugly_numbers = [1] * n$

for i in range(1, n):

index 3 += 1

 $index_5 += 1$

Return the nth ugly number

// Function to find the nth ugly number

int[] uqlyNumbers = new int[n];

// Populate the uglyNumbers array

for (int i = 1; i < n; ++i) {

// The first ualy number is always 1

// Pointers for multiples of 2, 3, and 5

int index2 = 0, index3 = 0, index5 = 0;

// Next multiples of 2, 3, and 5

public int nthUglyNumber(int n) {

uglyNumbers[0] = 1;

if ugly numbers[i] == next_5:

Initialize a dynamic programming array with 1 as the first ugly number

Select the minimum of these multiples to be the next ugly number

Python

class Solution:

First Iteration (i = 1):

Here is a visualization of how the algorithm processes the sequence for n = 10:

Solution Approach

- ∘ It calculates next2, next3, and next5 by multiplying dp[p2] by 2, dp[p3] by 3, and dp[p5] by 5, respectively. It then finds the minimum of these three values. This minimum value is the next ugly number, which is then added to the list at dp[i]. Pointer Incrementation: For each pointer (p2, p3, and p5), if the value they pointed to times its respective factor (2, 3, or 5)
- 1]. The use of dynamic programming in this problem helps to drastically reduce the time complexity. Instead of recalculating each possibility for each new number, the algorithm builds on previously calculated values. Pointers ensure that the sequence
- Ugly Numbers [1]: p2 --> 2, p3 --> 3, p5 --> 5Add min to list [1, 2]: p2 --> 4, p3 --> 3, p5 --> 5 Add min to list [1, 2, 3]: p2 --> 4, p3 --> 6, p5 --> 5

As the algorithm finds each new ugly number, it updates the list and the pointers appropriately until it reaches the nth element.

Example Walkthrough Let's walk through an example to illustrate the solution approach for finding the 7th ugly number using the efficient dynamic

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Initialization: We begin by initializing our dp array with a size of n. Initially, dp = [1, _, _, _, _, _, _, _] where 1 is the first
ugly number and represents uninitialized values. The pointers p2, p3, and p5 are all set to 0, which points to dp[0].
```

Building the Ugly Numbers List: We will iterate and calculate next2, next3, next5, pick the minimum, and then store it in

min(next2, next3, next5) = 2 - dp[1] = 2

p2 is incremented because next2 was used. Second Iteration (i = 2):

- next5 = 5 * dp[p5] = 5 * dp[0] = 5 min(next2, next3, next5) = 3 - dp[2] = 3
- give the 7th ugly number): dp[1] = 2 $p2 \longrightarrow 1$, $p3 \longrightarrow 0$, $p5 \longrightarrow 0$ dp[2] = 3 $p2 \longrightarrow 1$, $p3 \longrightarrow 1$, $p5 \longrightarrow 0$

And here's the sequence of the array and pointer updates for each step until dp[6] is filled (as dp is 0-indexed, dp[6] will

Pointer Incrementation: At each iteration, the pointer for which the next ugly number was found (either p2, p3, or p5) is

incremented. If there is a tie (i.e., two or three next values are the same), all tied pointers are incremented, ensuring unique

Returning the Result: After looping until i = 6 (which is when we have our 7th ugly number), the array looks like [1, 2, 3, 1]

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4, 5, 6, 8]. The loop stops and we have generated our n ugly numbers. The 7th ugly number is dp[6] which is 8.
  The algorithm efficiently calculates the sequence of ugly numbers, keeping track of the next potential candidates for ugly
  numbers using pointers and only iterates n times. Thus, for n = 7, the 7th ugly number is 8.
Solution Implementation
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- # Increment the corresponding indices when their multiples are used if ugly numbers[i] == next_2: index 2 += 1if uqly numbers[i] == next_3:
- return ugly_numbers[n 1] Java class Solution {

```
index2++;
            if (uglyNumbers[i] == nextMultiple0f3) {
                index3++;
            if (uqlvNumbers[i] == nextMultiple0f5) {
                index5++;
        // Return the nth ugly number
        return uglyNumbers[n - 1];
C++
#include <vector>
#include <algorithm>
class Solution {
public:
    // Function to find the nth ugly number.
    int nthUglyNumber(int n) {
        // Create a dynamic programming table to store ugly numbers.
        std::vector<int> dp(n);
        // The first ugly number is 1.
        dp[0] = 1;
        // Initialize pointers for multiples of 2, 3, and 5.
        int pointer2 = 0, pointer3 = 0, pointer5 = 0;
        // Populate the table with the next ugly numbers.
        for (int i = 1; i < n; ++i) {
            // Find the next multiples for 2, 3 and 5.
            int next2 = dp[pointer2] * 2;
            int next3 = dp[pointer3] * 3;
            int next5 = dp[pointer5] * 5;
            // Select the minimum of these multiples as the next ugly number.
```

```
pointer2++;
       if (nextUglyNumber === nextMultipleOf3) {
           pointer3++;
       if (nextUglvNumber === nextMultipleOf5) {
           pointer5++;
   // Return the nth ugly number, at index n-1
   return dp[n - 1];
class Solution:
   def nth ugly number(self, n: int) -> int:
       # Initialize a dynamic programming array with 1 as the first ugly number
       ugly_numbers = [1] * n
       # Initialize pointers for multiples of 2, 3, and 5
       index_2, index_3, index_5 = 0, 0, 0
       # Generate ugly numbers up to the nth one
       for i in range(1, n):
           # Calculate the next multiples for 2, 3, and 5
           next 2 = uqly numbers[index 2] * 2
           next 3 = uqlv numbers[index 3] * 3
           next_5 = ugly_numbers[index_5] * 5
           # Select the minimum of these multiples to be the next ugly number
           ugly_numbers[i] = min(next_2, next_3, next_5)
           # Increment the corresponding indices when their multiples are used
           if ualy numbers[i] == next_2:
               index 2 += 1
           if ugly numbers[i] == next_3:
```

The given Python code implements the Ugly Number II solution to find the nth ugly number using dynamic programming. An ugly number is a number whose prime factors are limited to 2, 3, and 5.

Time and Space Complexity

index 3 += 1

index 5 += 1

Return the nth ugly number

return ugly_numbers[n - 1]

if uqly numbers[i] == next_5:

Time Complexity

The time complexity of the code is O(n). This is because there's a single loop that runs n-1 times (since the loop starts at 1 and

runs until n), and within each iteration of the loop, the operations performed are constant time operations such as multiplication

and comparison.

Space Complexity The space complexity of the code is also O(n). The dp array is the primary space-consuming data structure, which stores the

ugly numbers up to the nth ugly number. Since we have n elements in the dp array, the space consumed is proportional to n.