2367. Number of Arithmetic Triplets

Problem Description

You are given an array of distinct integers which is in increasing order and a positive integer representing the difference value, diff. The goal is to find out how many unique triplets of indices (i, j, k) within the array satisfy two conditions: first, nums[j] - nums[i] is equal to diff, and second, nums[k] - nums[j] is equal to diff as well, with the indices following the relationship of i < j < k. This is essentially looking for sequences of three numbers that form an arithmetic progression with a common difference of diff.

Intuition

The intuition behind the solution involves understanding that for an arithmetic triplet to exist for a number x (nums[i]), there must be two other numbers x + diff (nums[j]) and x + 2 (nums[k]) present in the array. The solution utilizes a set to store the numbers for constant-time lookups, which makes checking the existence of x + diff and x + diff * 2 efficient.

The approach is as follows:

1. Convert the list of numbers into a set for faster lookup. This is beneficial because we want to be able to quickly check if a number + diff exists in the original array.
2. Iterate over each number in the array (which is already in increasing order) and for each number x, check if both x + diff and x + diff * 2 are present in the set.
3. Sum up the results of the checks for each element. If for a given number x, both x + diff and x + diff * 2 are found, this indicates the existence of an arithmetic triplet, so we count it.
4. The sum gives the total count of such unique triplets in the array.

This method is efficient because we capitalize on the properties of sets and the given ordered nature of the array to avoid unnecessary computations. By doing a single pass through the array and checking for the presence of the other two elements in the triplet, we achieve a solution that is simple and effective.

Solution Approach

The solution is implemented using a set data structure and a single for-loop iteration through the array. Here is the breakdown of how the solution works step by step:

1. Convert the nums array into a set called vis which stands for visited or seen. The conversion to a set is critical because it allows for O(1) time complexity lookups to check if an element exists within the array.

   ```
   1  vis = set(nums)
   ```

2. Use list comprehension combined with the sum function to iterate over each number x in nums. The iteration results in a boolean expression for each number which checks if both the next two numbers in the arithmetic sequence are present in the set vis. For each x in nums, if x + diff and x + diff == 2 exist in vis, the condition is True and contributes 1 to the sum, otherwise, it contributes 0.

   ```
   1  sum(x + diff in vis and x + diff * 2 in vis for x in nums)
   ```

3. For each iteration, the algorithm checks for the two required conditions:

   ○ x + diff is in vis: This checks if there is another number ahead in the array that is diff greater than the current number x. This is looking for the j index such that nums[j] - nums[i] == diff.

   ○ x + diff * 2 is in vis: This checks if there is a number that is twice the diff greater than the current number x. This is looking for the k index such that nums[k] - nums[j] == diff.

4. The sum function then adds up the results from the list comprehension. Each True represents a valid arithmetic triplet, and the sum is therefore the total count of unique arithmetic triplets in the array.

The pattern used here is effectively checking each possible starting point of an arithmetic triplet and rightly assuming that due to the strictly increasing nature of the inputs, if an x + diff and x + diff * 2 exist, they will be part of a valid arithmetic triplet. The use of set for constant-time lookups and list comprehension for concise code makes the implementation both efficient and readable.

Example Walkthrough

Let's illustrate the solution approach with a small example. Suppose our array of distinct integers is nums = [1, 3, 5, 7, 9], and the given difference value is diff = 2.

Following the steps outlined in the solution approach:

1. We first convert the nums array into a set vis to make element lookups more efficient:

   ```
   1  nums = [1, 3, 5, 7, 9]
   2  vis = set(nums)  # vis is now {1, 3, 5, 7, 9}
   ```

2. Next, we use list comprehension and the sum function to iterate over each number in nums. For each number x, we check if x + diff and x + diff * 2 are present in the set vis.

   Here's what happens for each element of nums:

   ○ For x = 1: Check if 1 + 2 and 1 + 4 are in vis. This is True because both 3 and 5 are in vis.

   ○ For x = 3: Check if 3 + 2 and 3 + 4 are in vis. This is True because both 5 and 7 are in vis.

   ○ For x = 5: Check if 5 + 2 and 5 + 4 are in vis. This is True because both 7 and 9 are in vis.

   ○ For x = 7: Check if 7 + 2 and 7 + 4 are in vis. This is False since 9 is in vis but 11 is not.

   ○ For x = 9: Check if 9 + 2 and 9 + 4 are in vis. This is False since neither 11 nor 13 is in vis.

   The above checks can be summed up with the following line of code:

   ```
   1  sum(x + diff in vis and x + diff * 2 in vis for x in nums)
   ```

3. As we iterate over the array, we find that x = 1, x = 3, and x = 5 satisfy both conditions of being a valid starting point for an arithmetic triplet with a common difference of diff. Therefore, for these elements, the corresponding boolean will be True.

4. The sum function will sum these True values. In our example, we have three True values corresponding to starting points 1, 3, and 5, resulting in a sum of 3.

Therefore, there are three unique triplets that form an arithmetic progression with a common difference of 2 in the array nums = [1, 3, 5, 7, 9].

This walkthrough demonstrates the utility of the set data structure for lookups and the effectiveness of iterating through the ordered list to identify valid arithmetic triplets.

Python Solution

```python
1  from typing import List
2
3  class Solution:
4      def arithmeticTriplets(self, nums: List[int], diff: int) -> int:
5          # Create a set from the list for constant time look-up.
6          seen_numbers = set(nums)
7
8          # Count the number of valid arithmetic triplets
9          # An arithmetic triplet is a sequence of three numbers where
10         # the difference between consecutive numbers is equal to diff.
11         # Here it checks if for a given number x in nums, both x + diff and
12         # x + 2 x diff are present in nums. If so, that contributes to one valid triplet.
13         triplet_count = sum(x + diff in seen_numbers and x + 2 * diff in seen_numbers for x in nums)
14
15         # Return the total count of triplets found.
16         return triplet_count
```

Java Solution

```java
1  class Solution {
2      // Function to find the number of arithmetic triplets in an array
3      public int arithmeticTriplets(int[] nums, int diff) {
4          // An array to keep track of the presence of elements up to the maximum possible value
5          boolean[] seen = new boolean[301];
6
7          // Mark the presence of each number in the 'seen' array
8          for (int num : nums) {
9              seen[num] = true;
10         }
11
12         // Initialize the count for arithmetic triplets
13         int count = 0;
14
15         // Iterate through the array to find arithmetic triplets
16         for (int num : nums) {
17             // Check if the two subsequent numbers (with the given difference 'diff') are present
18             if (seen[num + diff] && seen[num + 2 * diff]) {
19                 // If both are present, we found an arithmetic triplet, increment the count
20                 ++count;
21             }
22         }
23
24         // Return the total count of arithmetic triplets found
25         return count;
26     }
27 }
```

C++ Solution

```cpp
1  #include <vector>
2  #include <bitset>
3
4  class Solution {
5  public:
6      // This function counts the number of arithmetic triplets in the array.
7      // An arithmetic triplet is a sequence of three numbers such that
8      // the difference between consecutive numbers is the same as 'diff'.
9      int arithmeticTriplets(vector<int>& nums, int diff) {
10         bitset<301> visitedNumbers; // Initialize a bitset where we will mark the numbers present in 'nums'.
11
12         // Mark all the numbers present in the 'nums' vector within the bitset 'visitedNumbers'.
13         for (int number : nums) {
14             visitedNumbers[number] = 1;
15         }
16
17         int countTriplets = 0; // Initialize counter for arithmetic triplets.
18
19         // Iterate through all the numbers in 'nums'.
20         for (int number : nums) {
21             // Check if there are two other numbers in the sequence that make an arithmetic triplet
22             // with the current 'number' and the given 'diff' (difference).
23             // Increases countTriplets when the two other numbers forming the triplet are present, i.e., when
24             // both 'number + diff' and 'number + 2 * diff' are set in the 'visitedNumbers' bitset.
25             countTriplets += visitedNumbers[number + diff] && visitedNumbers[number + 2 * diff];
26         }
27
28         return countTriplets; // Return the total number of arithmetic triplets found in 'nums'.
29     }
30 };
```

Typescript Solution

```typescript
1  function arithmeticTriplets(nums: number[], diff: number): number {
2      // Initialize an array that will keep track of the presence of numbers within 'nums'
3      const visited: boolean[] = new Array(301).fill(false);
4
5      // Populate the 'visited' array with true at indexes that exist in the 'nums' array
6      for (const num of nums) {
7          visited[num] = true;
8      }
9
10     // Initialize a counter for the number of arithmetic triplets found
11     let tripletCount = 0;
12
13     // Iterate through the 'nums' array to find arithmetic triplets
14     for (const num of nums) {
15         // Check if the two successors (num+diff and num+2*diff) are present in 'nums'
16         if (visited[num + diff] && visited[num + 2 * diff]) {
17             // Increment the counter if both successors are found, signifying an arithmetic triplet
18             tripletCount++;
19         }
20     }
21
22     // Return the total count of arithmetic triplets
23     return tripletCount;
24 }
```

Time and Space Complexity

Time Complexity

The given code traverses through all the elements in the nums list once to construct the vis set, and again it iterates through all elements of nums in the return statement.

For each element x in nums, the code checks if x + diff and x + diff * 2 are present in the vis set. Looking up an element in a set has an average time complexity of O(1) because sets in Python are implemented as hash tables.

Therefore, the overall time complexity is O(n) where n is the number of elements in nums, since the set lookup operation is constant time on average, and we are doing this operation twice for each element in nums.

Space Complexity

The space complexity of the code is determined by the additional data structures that are used. Here, a set vis is created based on the elements of nums. In the worst case, if all elements in nums are unique, the vis set will contain the same number of elements as nums.

So, the space complexity would be O(n), where n is the number of elements in nums.