704. Binary Search

Easy Array Binary Search

Problem Description

This problem asks us to create a function that takes two inputs: an array of integers `nums` sorted in ascending order, and an integer `target`. The goal is to search for the `target` value within the `nums` array. If the `target` is found, the function should return the index of the `target` in the array. If `target` is not in the array, the function should return `-1`.

The critical constraint is that the algorithm used to search for the `target` value must have a runtime complexity of `O(log n)`. This is a strong hint that a binary search algorithm should be implemented because binary search has a logarithmic time complexity and typically operates on sorted data.

Intuition

To satisfy the `O(log n)` runtime complexity requirement, we use a binary search algorithm, which effectively splits the array in half during each iteration to narrow down the possible location of the target value.

Here are the key steps of the intuition behind the binary search solution approach:

1. Initialize two pointers, `left` and `right`, which represent the start and end of the range within the array we're currently considering. Initially, `left` is `0` and `right` is `len(nums) - 1`.
2. Enter a `while` loop which runs as long as `left` is less than `right`. This loop will continue until we either find the target or the search space is empty.
3. Determine the `mid` point index by averaging `left` and `right`. The `>> 1` operation is a bitwise shift that divides the sum by 2, efficiently finding the midpoint.
4. Compare the middle element `nums[mid]` with the target. If `nums[mid]` is greater than or equal to the target, we move the `right` pointer to `mid`, as the target, if it exists, must be to the left of `mid`.
5. If `nums[mid]` is less than the target, the target, if it exists, must be to the right of `mid`, so we update `left` to `mid + 1`.
6. The loop continues, halving the search space each time, until `left` becomes equal to `right`.
7. After exiting the loop, we check if `nums[left]` is equal to `target`. If it is, we return `left`, which is the index of the target. If not, we return `-1` to indicate that the target is not present in the array.

This solution approaches the problem using classic binary search, ensuring the runtime complexity meets the required `O(log n)` by continuously halving the search space, which is characteristic of logarithmic runtime algorithms.

Solution Approach

The implementation of the solution uses a binary search algorithm. Binary search is a classic algorithm in computer science for efficiently searching for an element in a sorted list. The algorithm repeatedly divides the search interval in half, and because the list is sorted, it can quickly determine if the element can only be in either the left or the right half of the list.

Here's a step-by-step walkthrough of the implementation based on the provided code:

1. Initialize two pointers: `left` is set to `0`, which is the first index of `nums`, and `right` is set to `len(nums) - 1`, which is the last index of `nums`.
2. Enter a `while` loop that continues as long as `left` is less than `right`. This ensures that we don't stop searching until the search space has been fully narrowed down. We're searching for the exact placement of the `target` or concluding that it's not in the array.
3. Calculate the mid-point index `mid` by adding `left` and `right`, then shifting the result to the right by one bit (`>> 1`). This is equivalent to `mid = (left + right) // 2` but is more efficient. The `mid` variable represents the index of the element in the middle of the current search space.
4. The `if` statement `if nums[mid] >= target:` evaluates whether the middle element is greater than or equal to the `target`. If so, that means the `target`, if it exists, must be at `mid` or to the left of `mid` within the search space. In response, `right` is updated to `mid`, narrowing the search to the left half of the current search space (inclusive of the `mid` point).
5. If the middle element is less than the `target`, the `target` can only be to the right of `mid`. Therefore, we update `left` to be `mid + 1`, now searching to the right of the original `mid` point.
6. The loop continues until `left` is equal to `right`, which means we can no longer divide the search space, and we must check if we have found the target.
7. Once the loop exits, we have one final check to perform. We compare the element at the index `left` with the `target`. If they're equal, then `left` is the index where `target` is found in the array `nums`. Otherwise, if they're not equal, this means the `target` is not present in the array, and we return `-1`.

No other data structures are utilized in this implementation as binary search operates directly on the sorted input list, and no additional memory is required other than a few variables for indices.

This solution leverages the efficiency of binary search, taking advantage of the sorted nature of the input array `nums` to deliver the expected outcome within the required `O(log n)` time complexity.

Example Walkthrough

Let's consider an example to illustrate the solution approach described above.

Suppose we have the following array of integers `nums` and a target integer `target`:

```
nums = [1, 3, 5, 6, 9]
target = 5
```

Here's how the binary search algorithm would be applied step by step:

1. Initialize two pointers: `left` = 0 (beginning of the array) and `right` = 4 (index of the last element in the array).
2. Our `while` loop starts because `left` (0) is less than `right` (4).
3. Calculate `mid` index: `mid` = (0 + 4) >> 1 = 4 >> 1 = 2 (bitwise shift of 4 to the right by 1 is the same as integer division by 2). The element at index 2 in `nums` is 5.
4. Check if `nums[mid] >= target`. In this case, `nums[2]` is 5, which is equal to `target` (5). Therefore, we found our target, and `right` is updated to `mid`.
5. The array is not further divided since we already located the target. Therefore, the `while` loop does not continue further.
6. The loop concludes, and we no longer need to narrow down our search.
7. Finally, outside of the loop, we compare `nums[left]` with `target`. Here, `nums[left]` is `nums[2]` which is 5, and this is equal to the `target` value. We return `left`, which is `2`, the index where `target` is found.

By following this method, we efficiently find our target using a binary search approach, ensuring the search process conforms to the logarithmic runtime, `O(log n)`.

Solution Implementation

Python

```python
from typing import List

class Solution:
    def search(self, nums: List[int], target: int) -> int:
        # Initialize the left and right pointers
        left, right = 0, len(nums) - 1

        # Use a binary search algorithm
        while left < right:
            # Calculate the middle index using bit shifting (equivalent to floor division by 2)
            mid = (left + right) >> 1

            # If the middle element is greater than or equal to the target
            if nums[mid] >= target:
                # Narrow the search to the left half including the middle element
                right = mid
            else:
                # Narrow the search to the right half excluding the middle element
                left = mid + 1

        # After exiting the loop, left should be the smallest index of the target value
        # Check if the element at the left index is the target
        return left if nums[left] == target else -1
        # If the element is not found, return -1

# Example usage
# sol = Solution()
# result = sol.search([1, 2, 3, 4, 5], 3)
# print(result) # Output: 2
```

Java

```java
class Solution {
    public int search(int[] nums, int target) {
        // Initialize the starting index of the search range.
        int left = 0;
        // Initialize the ending index of the search range.
        int right = nums.length - 1;

        // Continue searching while the range has more than one element.
        while (left < right) {
            // Calculate the middle index of the current range.
            int mid = left + (right - left) / 2;

            // If the middle element is greater than or equal to the target,
            // narrow the search range to the left half (including the middle element).
            if (nums[mid] >= target) {
                right = mid;
            } else {
                // If the middle element is less than the target,
                // narrow the search range to the right half (excluding the middle element).
                left = mid + 1;
            }
        }

        // At this point, left is the index where the target may be if it exists.
        // Check if the element at the 'left' index is the target.
        // If it is, return the index. Otherwise, return -1 indicating the target is not found.
        return nums[left] == target ? left : -1;
    }
}
```

C++

```cpp
#include <vector> // Include the vector header for using the std::vector container

class Solution {
public:
    int search(std::vector<int>& nums, int target) {
        int left = 0; // Initialize left boundary of the search
        int right = nums.size() - 1; // Initialize right boundary of the search

        // Perform binary search
        while (left < right) {
            // Calculate the midpoint to avoid potential overflow
            int mid = left + (right - left) / 2;

            // If the middle element is greater or equal to the target,
            // we need to move the right boundary to the middle
            if (nums[mid] >= target) {
                right = mid;
            } else {
                // The target is greater than the middle element,
                // move left boundary one step to the right of mid
                left = mid + 1;
            }
        }

        // After the loop ends, left should point to the smallest
        // number not smaller than target. Check if it's the target,
        // otherwise return -1 as the target is not found
        return (nums[left] == target) ? left : -1;
    }
};
```

TypeScript

```typescript
/**
 * Searches for a target value in a sorted array of numbers using binary search.
 *
 * @param {number[]} nums - Sorted array of numbers where we search for the target.
 * @param {number} target - The value to search for.
 * @return {number} The index of the target if found; otherwise, -1.
 */
const search = (nums: number[], target: number): number => {
    // Initialize left and right pointers for the binary search.
    let left: number = 0;
    let right: number = nums.length - 1;

    // Continue to search while the left pointer has not surpassed the right.
    while (left < right) {
        // Calculate the middle index using bitwise right shift
        // equivalent to Math.floor((left + right) / 2)
        const mid: number = (left + right) >> 1;

        // If the middle element is greater or equal to target, move the right pointer to the middle.
        if (nums[mid] >= target) {
            right = mid;
        } else {
            // If the middle element is less than the target, move the left pointer past the middle.
            left = mid + 1;
        }
    }

    // Once the search space is narrowed down to a single element,
    // check if it's equal to the target and return the appropriate index or -1.
    return nums[left] === target ? left : -1;
};

// Example usage:
const nums: number[] = [1, 3, 5, 7, 9];
const target: number = 7;
const result: number = search(nums, target); // result should be 3 as nums[3] = 7
```

```python
from typing import List

class Solution:
    def search(self, nums: List[int], target: int) -> int:
        # Initialize the left and right pointers
        left, right = 0, len(nums) - 1

        # Use a binary search algorithm
        while left < right:
            # Calculate the middle index using bit shifting (equivalent to floor division by 2)
            mid = (left + right) >> 1

            # If the middle element is greater than or equal to the target
            if nums[mid] >= target:
                # Narrow the search to the left half including the middle element
                right = mid
            else:
                # Narrow the search to the right half excluding the middle element
                left = mid + 1

        # After exiting the loop, left should be the smallest index of the target value
        # Check if the element at the left index is the target
        return left if nums[left] == target else -1
        # If the element is not found, return -1

# Example usage
# sol = Solution()
# result = sol.search([1, 2, 3, 4, 5], 3)
# print(result) # Output: 2
```

Time and Space Complexity

Time Complexity:

The given Python code performs a binary search on the array `nums` by repeatedly dividing the search interval in half. The search ends when left and right indices converge, at which point it either finds the target or concludes that the target is not in the array.

The while loop will run until `left` is equal to `right`. On each iteration, the interval size is halved, which means the time complexity is logarithmic with regard to the size of the array `nums`. Hence, the time complexity is `O(log n)`, where `n` is the length of `nums`.

Space Complexity:

The space complexity of the code is `O(1)` since it uses a constant amount of extra space. The variables `left`, `right`, and `mid` only take up a fixed amount of space regardless of the input list size.