2748. Number of Beautiful Pairs

Easy Array Math Number Theory

Problem Description

The LeetCode problem provides an array `nums`, indexed from 0. We need to find all the "beautiful pairs" in this array. A pair of indices `(i, j)` is considered beautiful if the first digit of `nums[i]` and the last digit of `nums[j]` are coprime. Two numbers are coprime if their greatest common divisor (gcd) is 1, meaning they do not have any prime factors in common besides 1.

To solve the problem, we have to count each such pair where `i < j`.

Intuition

To solve this problem efficiently, we recognize that there are only 10 possible digits (0 through 9), so it's possible to count occurrences of leading and trailing digits without having to compare every possible pair (which would be inefficient). The solution provided keeps track of the count of leading digits encountered so far in a count array `cnt`, as we iterate through the array.

For each number `x` in `nums`, we check if the last digit of `x` is coprime with each digit we've seen as a first digit, using the gcd function. If they are coprime, we add the count recorded for that first digit to our answer `ans`, which accumulates the total number of beautiful pairs. After checking against all first digits we've seen, we then record the first digit of x in our `cnt` count array, incrementing the count for that digit, which will be used for subsequent numbers in `nums`.

This approach ensures that we are only iterating through the array once and are maintaining a constant-size array to track the counts of first digits, leading to a time-efficient solution.

Solution Approach

The solution uses a count array `cnt` of size 10 (since we have 10 digits from 0 to 9) to keep a tally on the number of times a digit appears as the first digit of a number in `nums`. Furthermore, it leverages the Greatest Common Divisor (gcd) to check for coprimality.

Here's a step-by-step breakdown of the algorithm:

1. Initialize a count array `cnt` with 10 zeros, corresponding to the digits from 0 to 9.
2. Initialize a variable `ans` to 0, which will hold the count of the beautiful pairs.

For every number `x` in `nums`:

3. Extract the **last digit** of `x` by calculating `x % 10`.
4. For every possible **first digit** `y` ranging from 0 to 9:
   - Check if we have previously encountered `y` as a first digit (`cnt[y] > 0`).
   - Calculate `gcd(x % 10, y)`. If it is 1, `x`'s last digit and `y` are coprime.
   - If they are coprime, increment `ans` by the count of numbers (`cnt[y]`) that had `y` as their first digit.
5. Convert `x` to a string and take the first character, convert it back to an integer, and increment the respective count in `cnt`.

By using this calculation method, only a single pass through the `nums` array is needed, and we avoid comparing every pair of numbers. Complexity is reduced from potentially O(n^2) to O(n) because we're only performing a constant amount of work for each of the `n` elements in `nums`.

Example Walkthrough

Let's walk through a small example to illustrate the solution approach. Suppose our input array `nums` is `[12, 35, 46, 57, 23]`.

1. We initialize our count array `cnt` with 10 zeros: `cnt = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]`.
2. We initialize our variable `ans` to 0.

Now for each number in `nums`:

- For num `12`:
  - The last digit is `2` (`12 % 10`).
  - No other numbers have been processed yet, so we add `1` to `cnt[1]` because `1` is the first digit.
  - The `cnt` array becomes `[0, 1, 0, 0, 0, 0, 0, 0, 0, 0]`.
- For num `35`:
  - The last digit is `5` (`35 % 10`).
  - We check against all first digits we've seen so far, which is only `1`.
  - `gcd(5, 1)` is `1`, they are coprime, so we increment `ans` by the count of numbers with the first digit `1`, which is `1`.
  - `ans` becomes `1`.
  - We add `1` to `cnt[3]` for the first digit of `35`.
  - The `cnt` array becomes `[0, 1, 0, 1, 0, 0, 0, 0, 0, 0]`.
- For num `46`:
  - The last digit is `6`.
  - We check against first digits `1` and `3` (from `12` and `35`).
  - `gcd(6, 1)` is `1`, so they are coprime, increment `ans` by `cnt[1]` which is `1`.
  - `gcd(6, 3)` is `3`, so `6` and `3` are not coprime, do nothing for `cnt[3]`.
  - `ans` becomes `2`.
  - We add `1` to `cnt[4]` for the first digit of `46`.
  - The `cnt` array becomes `[0, 1, 0, 1, 1, 0, 0, 0, 0, 0]`.
- For num `57`:
  - The last digit is `7`.
  - We check against first digits `1`, `3`, and `4`.
  - `gcd(7, 1)`, `gcd(7, 3)`, `gcd(7, 4)` are **all** `1`, so `7` is coprime with `1`, `3`, and `4`.
  - Increment `ans` by `cnt[1] + cnt[3] + cnt[4]` which is `1 + 1 + 1 = 3`.
  - `ans` becomes `5`.
  - We add `1` to `cnt[5]` for the first digit of `57`.
  - The `cnt` array becomes `[0, 1, 0, 1, 1, 1, 0, 0, 0, 0]`.
- For num `23`:
  - The last digit is `3`.
  - We check against first digits `1`, `3`, `4`, and `5`.
  - `gcd(3, 1)` is `1`, so they are coprime, increment `ans` by `cnt[1]` which is `1`.
  - `gcd(3, 3)` is `3`, so not coprime with itself, do nothing for `cnt[3]`.
  - `gcd(3, 4)` is `1`, so they are coprime, increment `ans` by `cnt[4]` which is `1`.
  - `gcd(3, 5)` is `1`, so they are coprime, increment `ans` by `cnt[5]` which is `1`.
  - `ans` becomes `5 + 1 + 1 + 1 = 8`.
  - We add `1` to `cnt[2]` for the first digit of `23`.
  - The `cnt` array becomes `[0, 1, 1, 1, 1, 1, 0, 0, 0, 0]`.

After processing all the numbers, we have gone through the array once, and the total count of beautiful pairs `ans` is `8`.

Solution Implementation

Python

```python
from typing import List
from math import gcd

class Solution:
    def count_beautiful_pairs(self, nums: List[int]) -> int:
        # Initialize count array with zeroes for each digit from 0 to 9
        count = [0] * 10

        # Initialize the answer to 0, which will hold the number of beautiful pairs
        answer = 0

        # Iterate over each number in the input list
        for number in nums:
            # Check against each digit from 0 to 9
            for digit in range(10):
                # If there is a previously encountered number whose last digit has GCD=1 with current last digit of 'number'
                if count[digit] and gcd(number % 10, digit) == 1:
                    # Increment 'answer' by the count of that digit since it forms a beautiful pair
                    answer += count[digit]

            # Increment the count of the first digit of 'number'
            count[int(str(number)[0])] += 1

        # Return the total count of beautiful pairs
        return answer
```

Java

```java
class Solution {
    public int countBeautifulPairs(int[] nums) {
        // An array to count the occurrences of the last digits encountered.
        int[] countLastDigits = new int[10];

        // Initialize a variable to keep track of the number of beautiful pairs.
        int beautifulPairs = 0;

        // Iterate over each number in the input array.
        for (int number : nums) {
            // For each digit from 0 to 9, check if we have seen it before as a last digit.
            for (int y = 0; y < 10; ++y) {
                // If we have seen this last digit and the gcd of current number's last digit
                // and y is 1, increment the count of beautiful pairs by the number of times we've seen y.
                if (countLastDigits[y] > 0 && gcd(number % 10, y) == 1) {
                    beautifulPairs += countLastDigits[y];
                }
            }
            // Reduce the current number to its last digit.
            while (number > 9) {
                number /= 10;
            }
            // Increment the count of the last digit of the current number.
            ++countLastDigits[number];
        }

        // Return the total count of beautiful pairs found.
        return beautifulPairs;
    }

    // A helper method to calculate the greatest common divisor of two numbers.
    private int gcd(int a, int b) {
        // If b is zero, then a is the gcd. Otherwise, recursively call gcd with b and a % b.
        return b == 0 ? a : gcd(b, a % b);
    }
}
```

C++

```cpp
#include <vector>
#include <numeric> // For std::gcd

class Solution {
public:
    // Function to count beautiful pairs
    // A beautiful pair is defined such that the greatest common divisor (gcd) of the
    // least significant digit of one number and any digit of another number is 1
    int countBeautifulPairs(std::vector<int>& nums) {
        // Count array to keep track of the least significant digits of the numbers
        std::vector<int> countDigits(10, 0); // Initialize all elements to 0
        int beautifulPairs = 0; // Initialize beautiful pairs count

        // Loop through all numbers in the vector
        for (int number : nums) {
            // Check against all digits from 0 to 9
            for (int digit = 0; digit < 10; ++digit) {
                // If the count of digits is not zero and
                // the gcd of the number's least significant digit and current digit is 1,
                // increment the beautifulPairs by the count of that digit
                if (countDigits[digit] && std::gcd(number % 10, digit) == 1) {
                    beautifulPairs += countDigits[digit];
                }
            }
            // Reduce the number to its least significant digit
            while (number > 9) {
                number /= 10;
            }
            // Increment the count of the least significant digit
            ++countDigits[number];
        }

        // Return the total count of beautiful pairs
        return beautifulPairs;
    }
};
```

TypeScript

```typescript
/**
 * Calculates the count of beautiful pairs in the array.
 * A pair (i, j) is considered beautiful if the GCD of the last digit of nums[i] and nums[j] is 1,
 * and i < j.
 * @param nums - array of numbers
 * @returns the count of beautiful pairs
 */
function countBeautifulPairs(nums: number[]): number {
    // Initialize an array to keep count of the last digit frequency
    const lastDigitCount: number[] = Array(10).fill(0);
    let beautifulPairsCount = 0;

    // Loop through each number in the nums array
    for (let num of nums) {
        // Check against all possible last digits
        for (let digit = 0; digit < 10; ++digit) {
            // If there's a number with this last digit and their GCD of last digits is 1, count it
            if (lastDigitCount[digit] > 0 && gcd(num % 10, digit) === 1) {
                beautifulPairsCount += lastDigitCount[digit];
            }
        }
        // Reduce the number to its last digit
        while (num > 9) {
            num = Math.floor(num / 10);
        }
        // Increment the count for this last digit
        ++lastDigitCount[num];
    }
    // Return the total count of beautiful pairs
    return beautifulPairsCount;
}

/**
 * Recursively calculates the Greatest Common Divisor (GCD) of two numbers using Euclid's algorithm.
 * @param a - first number
 * @param b - second number
 * @returns the GCD of a and b
 */
function gcd(a: number, b: number): number {
    // Base case: if second number is 0, return the first number
    if (b === 0) {
        return a;
    }
    // Recursive case: return the GCD of b and the remainder of a divided by b
    return gcd(b, a % b);
}
```

```python
from typing import List
from math import gcd

class Solution:
    def count_beautiful_pairs(self, nums: List[int]) -> int:
        # Initialize count array with zeroes for each digit from 0 to 9
        count = [0] * 10

        # Initialize the answer to 0, which will hold the number of beautiful pairs
        answer = 0

        # Iterate over each number in the input list
        for number in nums:
            # Check each digit from 0 to 9
            for digit in range(10):
                # If there is a previously encountered number whose last digit has GCD=1 with current last digit of 'number'
                if count[digit] and gcd(number % 10, digit) == 1:
                    # Increment 'answer' by the count of that digit since it forms a beautiful pair
                    answer += count[digit]

            # Increment the count of the first digit of 'number'
            count[int(str(number)[0])] += 1

        # Return the total count of beautiful pairs
        return answer
```

Time and Space Complexity

Time Complexity

The time complexity of the given code is `O(n)` where `n` is the length of the input list `nums`. The analysis is as follows:

- We have a single loop that iterates over each element of `nums`, which contributes a factor of `O(n)` to the complexity.
- Inside the loop, we execute a fixed number of iterations (10, for the range of `j` from `0` to `9`), checking the greatest common divisor (`gcd`) of a pair of single digits. Since both the number of iterations and the `gcd` operation on single-digit numbers are constant-time operations, the loop inside does not depend on `n` and thus contributes a constant factor, `O(1)`, per each outer loop iteration.
- Updating the count array `cnt` also operates in constant time, `O(1)`, because it accesses a predetermined index determined by the first digit of `x`.

Hence, combining these, the time complexity is `O(n) = O(1) = O(n)`.

Space Complexity

The space complexity of the code is `O(1)` which is a constant space overhead, regardless of the input size. This is explained as follows:

- The `cnt` array has a fixed length of `10`, which does not depend on the size of the input list `nums`.
- The `ans` variable is just a single integer counter.

As neither of these two grows with the size of the input `nums`, the space complexity of the algorithm is constant.