2598. Smallest Missing Non-negative Integer After Operations

Medium Greedy Array Hash Table Math Leetcode Link

Problem Description

This problem presents an optimization challenge with a combination of array manipulation and a novel concept called MEX (minimum excluded), which represents the smallest non-negative integer not present in the array. We are given an array `nums` and an integer `value`. The operation at our disposal allows us to add or subtract `value` from any element in `nums`. The goal is to apply this operation as many times as we want to maximize the MEX of the resulting array.

Understanding the MEX is crucial as it is the key evaluation metric for the outcome of array manipulations. For instance, in an array `[-1, 2, 3]`, since `0` and `1` are not present, the MEX is `0`, as it is the smallest non-negative integer not in the array. However, if we had `[1, 0, 3]`, the MEX would be `2`. What makes this problem interesting is the strategic decision of how to add or subtract 'value' in order to maximize the MEX.

Intuition

To solve the problem, we need a smart way to track the potential MEX without exhaustively enumerating all possible permutations of array changes, which would be computationally infeasible. The intuition here lies in realizing that adding or subtracting `value` doesn't affect the remainder when each element of `nums` is divided by `value`. Consequently, the remainders form certain 'buckets' that we can only fill up to a certain level - dictated by how many times a remainder appears in the original array.

This leads to the insight that we can use the frequencies of these remainders to determine the MEX without iterating through all possible array operations. Specifically, by initializing a counter for the frequency of each remainder (modulo `value`) present in the array, we can iterate from `0` and move up to find the smallest non-negative integer (our potential MEX) that doesn't collide with existing remainder frequencies.

When the counter for a remainder corresponding to `i % value` is `0`, it signals that we've hit a 'gap', or a value that can't be created through any combination of operations on the current array, since none of the array elements produce a remainder equal to `i % value` when divided by `value`. That's our desired maximum MEX. If not, we decrement the count and move to the next integer, repeating the process until we find our MEX.

Solution Approach

The solution strategy is built on a counting approach, which significantly simplifies the operation and determination of MEX. Let's expound on the implementation of the solution:

1. **Counting Modulo Frequencies**: We realize that the operation of adding or subtracting `value` from any element in `nums` retains its modulo `value`. Thus, we store the count of occurrences of each remainder after dividing the elements by `value` using a `Counter` data structure. This gives us the frequency distribution of remainders.

2. **Modulo Value as a Key Insight**: We use the modulo operation considering `value` as an invariant in our problem. This insight solves the optimization problem by allowing us to work with a limited number of remainders (from `0` to `value - 1`) and their counts instead of dealing with potentially large numbers after multiple addition or subtraction operations.

3. **Iterating for MEX**: We initialize an iteration from `0` upwards, continuing to `len(nums) + 1` to cover all potential MEX values. In each iteration:

   - We check if the count for the current number `i` modulo `value` is `0`. If it is, no number in `nums` can be changed through addition or subtraction operations to get a remainder that matches `i % value`. Hence, `i` represents a MEX that cannot be obtained through any variant of the `nums` array, and we immediately return it as the maximum MEX.

   - If the count is not `0`, it implies that `i % value` is a possible remainder and thereby not the MEX. We decrement the count for the remainder `i % value` by `1` because, conceptually, we have 'used up' one instance where a number in the array could achieve this remainder. We continue to the next iteration to find the MEX.

This approach avoids brute-force computation and provides an elegant, efficient path to determining the MEX. We rely on the cyclic nature of modulo operation and the Counters that track the possibility of numbers to be fashioned into a specific modulus bucket through the given operation.

The algorithm is optimized as it avoids iterating through all possible operations on the array and instead, operates on a simple linear search for the smallest non-present modulo within the range, thus optimizing it to a time complexity of O(n).

Example Walkthrough

Let's illustrate the solution with an example. Consider the array `nums = [1, 3, 4]` and let `value = 3`. We want to maximize the MEX of the array by adding or subtracting `3` to elements in `nums`.

Step 1: Counting Modulo Frequencies

First, we count the modulo frequencies. The remainders of numbers in `nums` when divided by `value = 3` are:

- `1 % 3 = 1`
- `3 % 3 = 0`
- `4 % 3 = 1`

So, our frequency distribution (remainder counts) is:

- Remainder `0`: 1 occurrence
- Remainder `1`: 2 occurrences
- Remainder `2`: 0 occurrences

Step 2: Utilizing Modulo Value as an Insight

We acknowledge that we can deal with numbers in `nums` only in terms of their remainders when divided by `value`.

Step 3: Iterating for MEX

Starting from `0`, we check the remainder counts:

- For `i = 0`: The remainder `0 % 3 = 0` has a count of `1` (from the number `3` in `nums`). This is not our MEX, so we decrement the count for remainder `0`.

- For `i = 1`: The remainder `1 % 3 = 1` has a count of `2`. Again, we decrement the count for remainder `1`.

- For `i = 2`: The remainder `2 % 3 = 2` has a count of `0`. This means that we've found a gap here; we can't get a remainder of `2` by adding or subtracting `3` from any element in `nums`. Hence, the MEX is `2`.

This concludes our example. Without performing any operations on the array `nums`, we have efficiently determined that the maximum MEX achievable is `2` by just iterating through remainders and their counts.

Python Solution

```python
from collections import Counter
from typing import List

class Solution:
    def findSmallestInteger(self, nums: List[int], value: int) -> int:
        # Create a counter to keep track of the frequency of each number modulo 'value'
        modulo_counter = Counter(num % value for num in nums)

        # Iterate over the numbers from 0 to length of nums (inclusive)
        for i in range(len(nums) + 1):
            # If there is no number i modulo 'value' in our collection, return i
            if modulo_counter[i % value] == 0:
                return i
            # Otherwise, decrease the count of i modulo 'value' by one
            modulo_counter[i % value] -= 1

```

Java Solution

```java
class Solution {

    // Method to find the smallest integer that is not present in the array when modulus with value
    public int findSmallestInteger(int[] nums, int value) {
        // Create an array to count occurrences of each modulus result
        int[] countModulo = new int[value];

        // Iterate over each number in nums and increment the count for the corresponding modulus
        for (int num : nums) {
            countModulo[(num % value + value) % value]++;
        }

        // Start looking for the smallest integer that is not in the array, by checking modulus occurrences
        for (int i = 0; ; ++i) { // no termination condition here since we are guaranteed to find a number eventually
            // Use the i % value to wrap around the countModulo array
            // Check if the current number has a count of zero, which means it's not present in the nums array when modulus with valu
            if (countModulo[i % value] == 0) {
                // If it's not present, this is the smallest number we are looking for
                return i;
            }
            // Otherwise, decrease the count and keep looking
            countModulo[i % value]--;
        }
    }
}

```

C++ Solution

```cpp
#include <vector>
#include <cstring>

class Solution {
public:
    int findSmallestInteger(vector<int>& nums, int value) {
        // Create an array to count occurrences of each modulus value
        int countArray[value];

        // Initialize the countArray with zeros
        memset(countArray, 0, sizeof(countArray));

        // Fill the countArray with the count of each modulus value
        for (int num : nums) {
            // Correct negative values using modulo operation, then increment the count
            ++countArray[(num % value + value) % value];
        }

        // Iterate to find the smallest integer not present in nums when modulo `value`
        for (int i = 0; ; ++i) {
            // Calculate the current index modulo value
            int modIndex = i % value;

            // If the count for the current index is zero,
            // it means the integer `i` is not present in nums as mod value
            if (countArray[modIndex] == 0) {
                // Return the smallest integer not present
                return i;
            }

            // Decrement the count for the current index as it is now being used
            --countArray[modIndex];
        }
        // The loop is designed to run indefinitely, will break internally.
    }
};

```

Typescript Solution

```typescript
function findSmallestInteger(nums: number[], value: number): number {
    // Initialize a counter array with size 'value' and fill it with 0s.
    const count: number[] = new Array(value).fill(0);

    // Increment the count for each number in nums based on its modulo 'value'
    for (const num of nums) {
        const index = ((num % value) + value) % value; // Handles negative numbers
        count[index]++;
    }

    // Iterate to find the smallest integer not in 'nums' after modulo 'value'.
    for (let i = 0; ; ++i) {
        const index = i % value; // Calculate the index in the count array

        // If the count at the current index is 0, this is the smallest integer not found in 'nums'
        if (count[index] === 0) {
            return i;
        }

        // Otherwise, decrement the count at the current index
        count[index]--;
    }
    // The loop is intended to run indefinitely, as it will always return inside the loop body.
}

```

Time and Space Complexity

The given Python code defines a method `findSmallestInteger` meant to find the smallest integer that is not present in the input list `nums` when considering each number modulo `value`. It manages this by creating a frequency counter for the modulo results and then iteratively checking for the smallest index that is not present in the frequency counter.

Time Complexity

The time complexity of the function is `O(n)`, where `n` is the length of the input list `nums`. This arises from the following operations:

1. Initializing the counter with `(x % value for x in nums)` has a linear runtime proportional to the size of `nums`.
2. The subsequent for loop runs for `n + 1` iterations in the worst case, as it stops as soon as it finds an integer that is not in the counter. Each operation within the for loop (accessing `cnt` and decrementing the count) is `O(1)` thanks to Python's `Counter` which is a hash map under the hood.

Therefore, as the initialization of the counter and the for loop are both linear in terms of `n` and are not nested, the overall time complexity remains linear or `O(n)`.

Space Complexity

The space complexity of the algorithm is `O(value)`, where `value` is the input parameter to the method. This complexity is attributed to the following points:

1. The counter `cnt` stores at most `value` different keys, since each number in `nums` is taken modulo `value`, which results in a possible range of `[0, value)`.
2. No other data structures are used that depend on the size of the input or `value`.

Thus, the space required by the counter is directly proportional to the `value`, leading to a space complexity of `O(value)`.