835. Image Overlap

Medium Array Matrix

Leetcode Link

Problem Description

The problem presents us with two binary square matrices, `img1` and `img2`, each containing only 0's and 1's. Our task is to determine the largest overlap between these two images when one image is translated (slid in any direction) over the other. The overlap is quantified by counting the number of positions where both images have a 1. A key constraint is that rotation is not allowed, and any 1's that move outside the matrix bounds are discarded. Our goal is to find the maximum overlap after trying all possible translations.

Intuition

To solve this problem, we should think about how to track the overlap between the two images effectively. One way to do this is to consider each `1` in `img1` and match it against each `1` in `img2`. For each pairing, we can calculate the translation vector (the difference in x and y coordinates) required to move the `1` from `img1` directly onto the `1` in `img2`.

We approach the solution by iterating over all elements of `img1` and `img2`. For each `1` we find in `img1`, we iterate through `img2`, and for every `1` in `img2`, we calculate the translation vector. This vector is represented as a tuple (`delta_row`, `delta_col`), where `delta_row` is the difference in the row indices and `delta_col` is the difference in the column indices between a `1` in `img1` and a `1` in `img2`.

All these vectors are stored in a counter (a dictionary with tuples as keys and their counts as values), which is used to count how many times each vector occurs. The highest count in this counter, after considering all pairs of `1` bits, would represent the most overlapping translation, giving us the largest possible overlap between `img1` and `img2`.

The solution makes use of the `Counter` class from Python's collections module to conveniently keep track of the number of overlaps for each translation vector.

Solution Approach

The provided solution employs a brute-force approach, utilizing nested loops to iterate through each element of both `img1` and `img2`. Here's a step-by-step explanation:

1. We start by obtaining the size of the given images `n`, assuming both images are of size `n x n`.
2. We create a `Counter` from the `collections` module to keep track of all translation vectors and how frequently they result in an overlap. A translation vector is represented as a tuple `(delta_i, delta_j)`, which corresponds to the difference in rows (`delta_i`) and columns (`delta_j`) needed to align a `1` from `img1` with a `1` from `img2`.
3. We then proceed with four nested loops:
   - The outer two loops iterate over all the cells of `img1`:
     - For each position `(i, j)` in `img1` that contains a `1`, we then iterate over every position in `img2` using two more loops.
   - The inner two loops iterate over all the cells of `img2`:
     - For each position `(h, k)` in `img2` that contains a `1`, we calculate the translation vector needed to move the `1` from `(i, j)` in `img1` to `(h, k)` in `img2`.
4. The calculated translation vector `(i - h, j - k)` is then used to increment the count in our `Counter`. This effectively counts how many `1` bits from `img1` would overlap with `1` bits in `img2` when `img1` is translated according to this vector.
5. After evaluating all possible pairs of `1`s in both images, we find the maximum count from our `Counter`. This count represents the largest number of overlapping `1`s for the best translation. We return this maximum count as the solution.

In the worst case, every element of `img1` (which could be a `1`) has to be compared with every element of `img2`, leading us to a time complexity of $O(n^4)$, where `n` is the size of one dimension of the image.

Finally, the last line of the function accounts for the possibility of no overlap at all. If `cnt` (our `Counter`) remains empty because there are no `1` bits overlapping in any translation, we return `0`. Otherwise, we return the maximum value found in `cnt`, which is the result of `max(cnt.values())`.

Example Walkthrough

Let's illustrate the solution approach with an example where `img1` and `img2` are both 3×3 binary matrices:

```
img1 = [[1,0,0],
        [0,1,0],
        [0,0,0]]

img2 = [[0,0,0],
        [0,1,0],
        [0,0,1]]
```

Now, let's walk through the solution approach:

1. We first identify the size `n` of the square matrices, which in this case is `3`.
2. We create a `Counter` to keep track of the translation vectors - `delta_i` and `delta_j`, which denote the row and column differences, respectively.
3. Then, we iterate over each cell in `img1`:
   - For cell `(0,0)` in `img1`, which contains a `1`, we check every cell in `img2`. Only cells `(1,1)` and `(2,2)` contain a `1` in `img2`.
4. For each `1` in `img2`, we calculate the translation vectors necessary to overlap this `1` with the `1` at `(0,0)` in `img1`:
   - To overlap `(0,0)` from `img1` onto `(1,1)` in `img2`, we calculate the translation vector as `(1 - 0, 1 - 0)` which is `(1, 1)`.
   - To overlap `(0,0)` from `img1` onto `(2,2)` in `img2`, the translation vector is `(2 - 0, 2 - 0)` which is `(2, 2)`.
5. We add and/or increment the count of these vectors in our `Counter`.
6. We repeat steps 3-5 for the other `1` at position `(1,1)` in `img1`:
   - There is one `1` at `(1,1)` in `img1` which corresponds directly to the `1` at `(1,1)` in `img2`. The translation vector in this case is `(1 - 1, 1 - 1)` which is `(0, 0)`.
7. We increment the count of this vector in our `Counter`. Now our `Counter` has counted translation vectors with counts as follows:
   - `(1, 1)`: 1 count
   - `(2, 2)`: 1 count
   - `(0, 0)`: 1 count
8. Finally, we scan through our `Counter` to determine the maximum overlap, which is 1 in this case since each translation only overlaps one `1` in both images. The result, therefore, is 1.

So, the largest overlap by translating `img1` over `img2` or vice versa is a single `1`. The steps above demonstrate the brute-force approach systematically considering all translation vectors and assessing the overlaps.

Python Solution

```python
from typing import List
from collections import Counter

class Solution:
    def largestOverlap(self, A: List[List[int]], B: List[List[int]]) -> int:
        # Obtain the size of the square images
        size = len(A)

        # Initialize a counter to keep track of overlaps
        overlap_count = Counter()

        # Go through every pixel in first image
        for i in range(size):
            for j in range(size):
                # If we have a 1 (active pixel) in the first image
                if A[i][j] == 1:
                    # Check against every pixel in the second image
                    for k in range(size):
                        for l in range(size):
                            # If there's a 1 in the same position of the second image
                            if B[k][l] == 1:
                                # Count overlaps by storing the delta (difference) of positions
                                overlap_count[i - k, j - l] += 1

        # Find the maximum number of overlaps (if there are any), otherwise return 0
        return max(overlap_count.values()) if overlap_count else 0

```

Java Solution

```java
class Solution {
    public int largestOverlap(int[][] img1, int[][] img2) {
        int n = img1.length; // dimension of the images
        // Using a map to store the count of overlaps with particular vector shifts
        Map<List<Integer>, Integer> shiftCount = new HashMap<>();
        int maxOverlap = 0; // To track the maximum overlap

        // Iterate through the first image
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < n; ++j) {
                // Only interested in cells that are 1 (part of the image)
                if (img1[i][j] == 1) {
                    // Iterate through the second image
                    for (int h = 0; h < n; ++h) {
                        for (int k = 0; k < n; ++k) {
                            // Only interested in cells that are 1 (part of the image)
                            if (img2[h][k] == 1) {
                                // Calculate the vector shift needed to overlay img1's 1 over img2's 1
                                List<Integer> shift = List.of(i - h, j - k);
                                // Increase the count for this vector shift
                                shiftCount.merge(shift, 1, Integer::sum);
                                // Keeping track of the max overlap by comparing and choosing the larger count
                                maxOverlap = Math.max(maxOverlap, count);
                            }
                        }
                    }
                }
            }
        }
        // Return the max overlap found during iteration
        return maxOverlap;
    }
}
```

C++ Solution

```cpp
#include <vector>
#include <map>
#include <utility>
#include <algorithm>

class Solution {
public:
    int largestOverlap(std::vector<std::vector<int>>& image1, std::vector<std::vector<int>>& image2) {
        int size = image1.size();
        std::map<std::pair<int, int>, int> overlapCount; // Holds the count of overlaps per translation vector
        int maxOverlap = 0; // Keeps track of the maximum overlap count

        // Iterate over all the pixels in image1
        for (int i = 0; i < size; ++i) {
            for (int j = 0; j < size; ++j) {
                // Check if there is an image part at the current pixel in image1
                if (image1[i][j] == 1) {
                    // Iterate over all the pixels in image2
                    for (int h = 0; h < size; ++h) {
                        for (int k = 0; k < size; ++k) {
                            // Check if there is an image part at the current pixel in image2
                            if (image2[h][k] == 1) {
                                // Calculate the translation vector from the current pixel in image2 to the current pixel in image1
                                std::pair<int, int> translationVector = std::make_pair(i - h, j - k);
                                // Increment the overlap count for this translation vector
                                overlapCount[translationVector]++;
                                // Update the maxOverlap if the current overlap count is greater
                                maxOverlap = std::max(maxOverlap, overlapCount[translationVector]);
                            }
                        }
                    }
                }
            }
        }

        // Return the maximum overlap count found
        return maxOverlap;
    }
};
```

Typescript Solution

```typescript
function largestOverlap(image1: number[][], image2: number[][]): number {
    // Determine the dimension of the square images
    const size = image1.length;

    // Create a map to hold the counts of the overlaps
    const overlapCount: Map<number, number> = new Map();

    // Initialize the maximum number of overlaps to 0
    let maxOverlap = 0;

    // Iterate through each cell of the first image
    for (let row1 = 0; row1 < size; ++row1) {
        for (let col1 = 0; col1 < size; ++col1) {
            // Proceed if this cell of the first image has a '1'
            if (image1[row1][col1] === 1) {
                // Now, iterate through each cell of the second image
                for (let row2 = 0; row2 < size; ++row2) {
                    for (let col2 = 0; col2 < size; ++col2) {
                        // Check if the second image's cell is also a '1'
                        if (image2[row2][col2] === 1) {
                            // Calculate the translation vector as a unique key (assuming size < 200)
                            const translationKey = (row1 - row2) * 200 + (col1 - col2);

                            // Use the translation key to update and get the count of this overlap
                            const currentCount = (overlapCount.get(translationKey) ?? 0) + 1;
                            overlapCount.set(translationKey, currentCount);

                            // Keep track of the max number of overlaps
                            maxOverlap = Math.max(maxOverlap, currentCount);
                        }
                    }
                }
            }
        }
    }

    // Return the maximum number of overlapping ones
    return maxOverlap;
}
```

Time and Space Complexity

The given code snippet is designed to find the largest overlap of two given binary matrices `img1` and `img2` by shifting `img2` in all possible directions. Here is the analysis of its time complexity and space complexity:

Time Complexity

The time complexity of the function is governed by the four nested loops:

- The two outer loops iterate over each cell of the `img1` matrix, resulting in n^2 iterations, where n is the length of the matrix side.
- The two inner loops iterate over each cell of the `img2` matrix, also resulting in n^2 iterations.

For each '1' in `img1`, the inner loops iterate through the entire `img2` matrix. In the worst case, this results in n^2 iterations for the '1's in `img1`. Combining these factors, the worst-case time complexity is $O(n^2 * n^2)$, which simplifies to $O(n^4)$.

Space Complexity

The space complexity is determined by the additional space used by the program which is mainly due to the `Counter` object used to count the number of overlaps for each shift. The number of different shifts is bounded by the size of the matrix ($n * n$), as the possible shifts range from $(-n+1, -n+1)$ to $(n-1, n-1)$.

Therefore, in the worst case, the `Counter` could have up to n^2 entries if every possible shift occurs at least once. Thus, the space complexity is $O(n^2)$.

In summary, the time complexity of the code is $O(n^4)$ and the space complexity is $O(n^2)$.