

# 509. Fibonacci Number

## Problem Description

The problem is to find the  $n$ th number in the Fibonacci sequence. The Fibonacci sequence is a series of numbers where each number is the sum of the two preceding ones. The sequence starts with 0 and 1. Mathematically, it can be defined as:

```
1 F(0) = 0, F(1) = 1
2 F(n) = F(n - 1) + F(n - 2), for n > 1.
```

The task is to write a function that, given a non-negative integer  $n$ , returns the  $n$ th Fibonacci number.

## Intuition

To solve this problem, we think about the properties of the Fibonacci sequence. Since  $F(n)$  depends only on the two previous numbers,  $F(n - 1)$  and  $F(n - 2)$ , we can compute it using an iterative approach. We start with the first two numbers, 0 and 1, and repeat the process of adding the last two numbers to get the next number until we reach the  $n$ th term. This removes the need for recursion, which can be inefficient and possibly lead to a stack overflow for large values of  $n$ .

The intuition behind the iterative solution is based on the observation that we don't need to retain all the previous numbers computed - just the last two numbers to calculate the next number in the sequence. This leads to a time and space efficient algorithm.

## Solution Approach

The solution implements an iterative approach to calculate Fibonacci numbers. The main algorithm used here doesn't require complex data structures or patterns - it simply relies on variable swapping and updating values in each iteration.

Here's a step-by-step explanation of the code:

- We initialize two variables  $a$  and  $b$  to the first two Fibonacci numbers, 0 and 1, respectively. These two variables will keep track of the last two numbers in the sequence at each step.
- We then enter a loop that will iterate  $n$  times. On each iteration, we simulate the progression of the sequence.
- Inside the loop, we have a single line of code that performs the update:

```
1 a, b = b, a + b
```

- This line is a tuple unpacking feature in Python, which allows for the simultaneous update of  $a$  and  $b$ . Here's the breakdown:
- $b$  is assigned to  $a$ , which moves the sequence one step forward.
  - $a + b$  is the sum of the current last two numbers, producing the next number in the sequence, and it is assigned to  $b$ .

This operation is repeated until we have looped  $n$  times, by which point  $a$  will contain the  $n$ th Fibonacci number, and the function returns  $a$ .

One important note is that the looping starts from 0 and goes to  $n-1$ , thus iterating  $n$  times. The reason for this is that we start counting from 0 in the Fibonacci sequence, so after  $n$  iterations, we've already achieved the  $n$ th term.

There are no recursive calls, which makes this algorithm run in  $O(n)$  time complexity, which is the number of iterations equal to  $n$ , and  $O(1)$  space complexity, because we're only ever storing two values, regardless of the size of  $n$ .

## Example Walkthrough

Let's illustrate the solution approach with an example where we want to find the 5th number in the Fibonacci sequence.

- We start by initializing two variables  $a$  and  $b$  with the first two Fibonacci numbers, 0 and 1, respectively. So  $a = 0$  and  $b = 1$ .
- We need to loop from 0 to  $n-1$ , where  $n$  is 5 in this example, since we want to find the 5th number in the sequence.
- The loop starts, and at each iteration, we will perform the following operation:

```
1 a, b = b, a + b
```

- Let's see how the values of  $a$  and  $b$  change with each iteration:
    - Iteration 1:  $a = 1, b = 1$  ( $0 + 1$ )
    - Iteration 2:  $a = 1, b = 2$  ( $1 + 1$ )
    - Iteration 3:  $a = 2, b = 3$  ( $1 + 2$ )
    - Iteration 4:  $a = 3, b = 5$  ( $2 + 3$ )
- After 4 iterations (which is  $n-1$  for  $n=5$ ), we can stop since  $a$  now holds the value of the 5th Fibonacci number.

- The function will then return  $a$ , which is 3. So, the 5th number in the Fibonacci sequence is 3.

Now implementing this approach with actual Python code would look like this:

```
1 def fibonacci(n):
2     a, b = 0, 1
3     for _ in range(n):
4         a, b = b, a + b
5     return a
6
7 # Example usage:
8 print(fibonacci(5)) # Output will be 3
```

This walkthrough demonstrates how the algorithm works with a small example and assures us that the result of `fibonacci(5)` is indeed the 5th number in the Fibonacci sequence according to our zero-indexing in the sequence definition.

## Python Solution

```
1 class Solution:
2     def fib(self, N: int) -> int:
3         # Initialize the first two Fibonacci numbers
4         previous, current = 0, 1
5
6         # Iterate N times to calculate the N-th Fibonacci number
7         for _ in range(N):
8             # Update the previous and current values to move one step forward in the Fibonacci sequence
9             previous, current = current, previous + current
10
11        # After N iterations, previous holds the value of the N-th Fibonacci number
12        return previous
13
```

## Java Solution

```
1 class Solution {
2     public int fib(int n) {
3         // Initializing the first two numbers of the Fibonacci sequence.
4         int previousNumber = 0; // Previously known as 'a'.
5         int currentNumber = 1; // Previously known as 'b'.
6
7         // Looping to calculate Fibonacci sequence until the nth number.
8         while (n-- > 0) {
9             // Calculate the next number in the Fibonacci sequence.
10            int nextNumber = previousNumber + currentNumber;
11
12            // Update the previous number to be the current number.
13            previousNumber = currentNumber;
14
15            // Update the current number to be the next number.
16            currentNumber = nextNumber;
17        }
18
19        // After completing the loop, 'previousNumber' holds the nth Fibonacci number.
20        return previousNumber;
21    }
22 }
23
```

## C++ Solution

```
1 class Solution {
2 public:
3     int fib(int n) {
4         // Initialize the first two fibonacci numbers
5         int previous = 0, current = 1;
6
7         // Loop to calculate the nth fibonacci number
8         while (n--) {
9             // Calculate the next fibonacci number
10            int next = previous + current;
11
12            // Update the previous and current values for the next iteration
13            previous = current;
14            current = next;
15        }
16
17        // Return the nth fibonacci number which is now stored in 'previous'
18        return previous;
19    }
20 };
21
```

## Typescript Solution

```
1 // Function to calculate the nth Fibonacci number
2 function fib(n: number): number {
3     // Initialize the first two Fibonacci numbers
4     let currentFib = 0; // The first Fibonacci number, F(0)
5     let nextFib = 1; // The second Fibonacci number, F(1)
6
7     // Iterate until the nth number
8     for (let i = 0; i < n; i++) {
9         // Update the current and next numbers using tuple assignment
10        // currentFib becomes the nextFib, and nextFib becomes the sum of currentFib and nextFib
11        [currentFib, nextFib] = [nextFib, currentFib + nextFib];
12    }
13
14    // Return the nth Fibonacci number
15    return currentFib;
16 }
17
```

## Time and Space Complexity

### Time Complexity

The provided code consists of a single loop that iterates  $n$  times, where  $n$  is the input number to calculate the Fibonacci sequence. In each iteration of the loop, a constant number of operations are performed, specifically the assignments  $a, b = b, a + b$ . Therefore, this loop runs in linear time with respect to the input  $n$ . This gives us a time complexity of  $O(n)$ .

### Space Complexity

The space complexity of the code is constant, as it only uses a fixed number of variables ( $a$  and  $b$ ) regardless of the input size. This means no additional space is used that scales with the input size  $n$ , leading to a space complexity of  $O(1)$ .