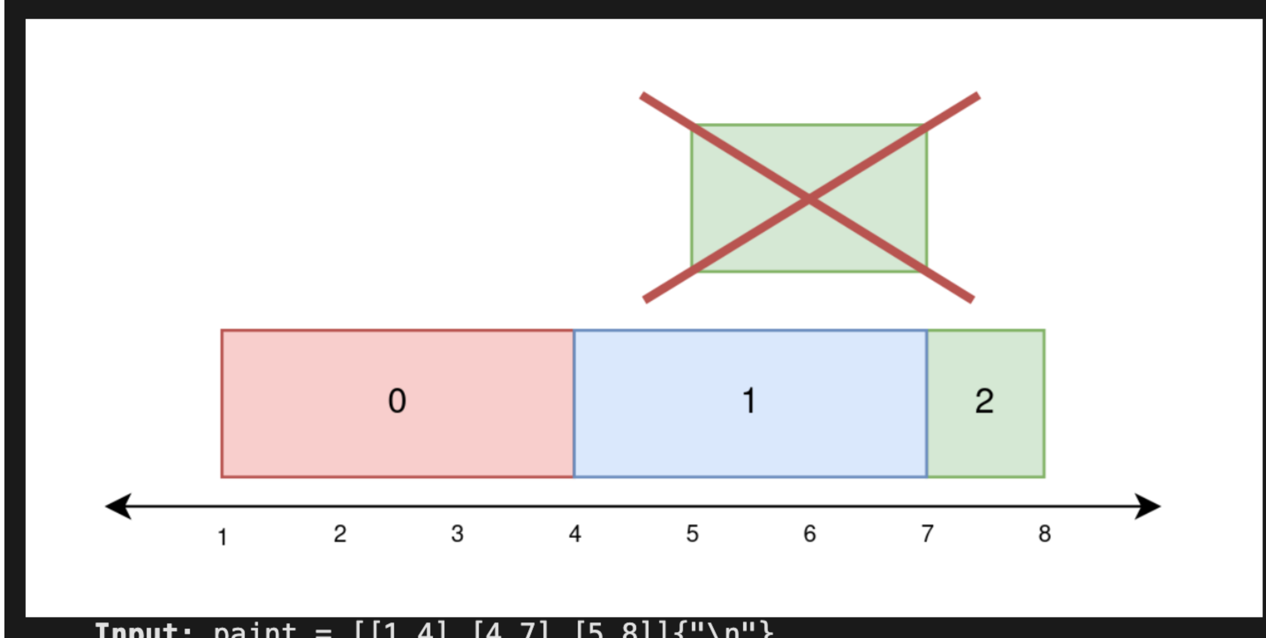
2158. Amount of New Area Painted Each Day

There is a long and thin painting that can be represented by a number line. You are given a **0-indexed** 2D integer array{" "} `paint` of length `n`, where{" "} `paint[i] = [start_i, end_i]` . This means that on the{" "} `i^th` {" "} day you need to paint the area **between**{" "} `start_i` {" "} and{" "} `end_i` .

Painting the same area multiple times will create an uneven painting so you only want to paint each area of the painting at most **once**.

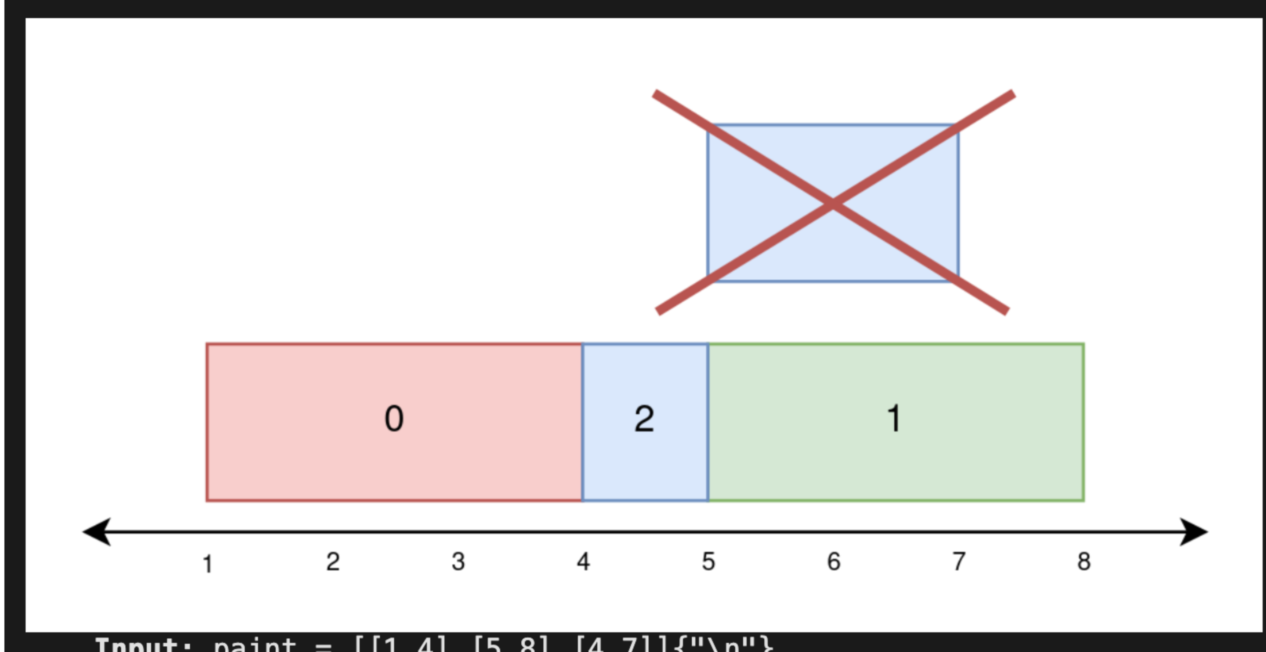
Return *an integer array* `worklog` *of length* `n` *, where* `worklog[i]` *{" "} is the amount of **new** area that you painted on the{" "}* `i^th` *day.*

Example 1:



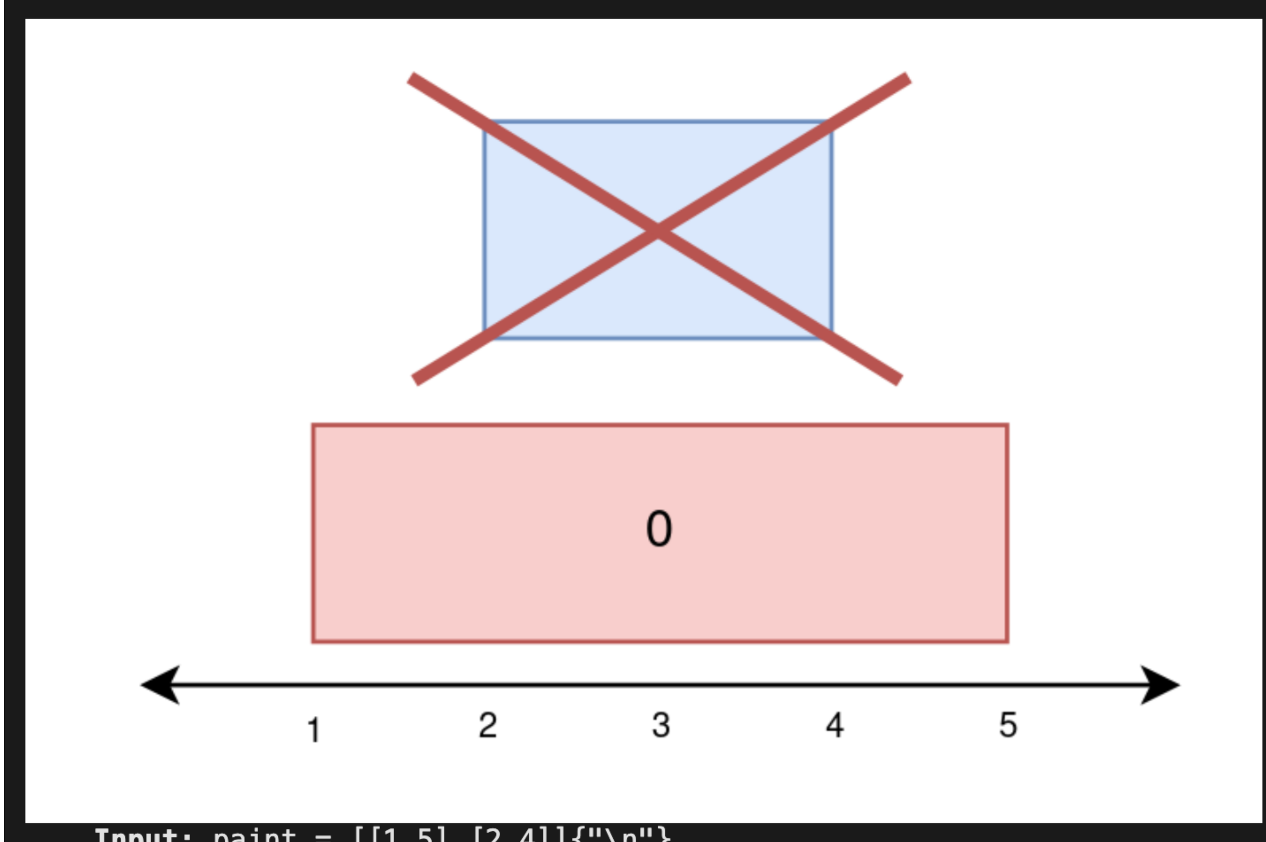

```
Input: paint = [[1,4],[4,7],[5,8]]{"\n"}
Output: [3,3,1]{"\n"}
Explanation:
{"\n"}On day 0, paint everything between 1 and 4.{"\n"}The amount of new
area painted on day 0 is 4 - 1 = 3.{"\n"}On day 1, paint everything between
4 and 7.{"\n"}The amount of new area painted on day 1 is 7 - 4 = 3.{"\n"}On
day 2, paint everything between 7 and 8.{"\n"}Everything between 5 and 7 was
already painted on day 1.{"\n"}The amount of new area painted on day 2 is 8
- 7 = 1. {"\n"}
```

Example 2:




```
Input: paint = [[1,4],[5,8],[4,7]]{"\n"}
Output: [3,3,1]{"\n"}
Explanation:
{"\n"}On day 0, paint everything between 1 and 4.{"\n"}The amount of new
area painted on day 0 is 4 - 1 = 3.{"\n"}On day 1, paint everything between
5 and 8.{"\n"}The amount of new area painted on day 1 is 8 - 5 = 3.{"\n"}On
day 2, paint everything between 4 and 5.{"\n"}Everything between 5 and 7 was
already painted on day 1.{"\n"}The amount of new area painted on day 2 is 5
- 4 = 1. {"\n"}
```

Example 3:




```
Input: paint = [[1,5],[2,4]]{"\n"}
Output: [4,0]{"\n"}
Explanation:
{"\n"}On day 0, paint everything between 1 and 5.{"\n"}The amount of new
area painted on day 0 is 5 - 1 = 4.{"\n"}On day 1, paint nothing because
everything between 2 and 4 was already painted on day 0.{"\n"}The amount of
new area painted on day 1 is 0.{"\n"}
```

Constraints:

- `1 <= paint.length <= 10^5`
- `paint[i].length == 2`
- `0 <= start_i < end_i <= 5 * 10^4`

Solution

Let's split the number line into blocks such that for the ith block covers the interval $[i, i+1]$. Create a boolean array to store whether each block has been painted.

On day i, we are tasked with painting blocks start_i to $\text{end}_i - 1$. We can check each of these blocks, painting the unpainted ones (we also keep count of how many blocks we paint because that's what the question asks for). In the worst case, we have to check every block on every day. Let n be the number of days (up to 100000) and let m be the largest number that appears in the input (up to 50000). The time complexity is $\mathcal{O}(nm)$. This is not fast enough.

Instead of using a boolean array, we can use a BBST (balanced binary search tree) to store the indices of the unpainted blocks. At the start, we insert $0, 1, 2, \ldots, m-1, m$ into the BBST. When we paint a node, we delete its node from the BBST. In our time complexity analysis, it will become clear why we chose to use a BBST.

On each day, we search for the first node $\geq \text{left}_i$. If it's also $< \text{right}_i$, we delete it. We repeatedly do this until there are no more blocks between left_i and $\text{right}_i - 1$.

<mark style={{ backgroundColor: "lightblue" }}>The intuition behind this solution is that we don't want need to needlessly loop over painted blocks; as soon as a block is painted, it's no longer useful, so we delete it. Otherwise, in future days, we'd have to keep checking whether each block has been painted. A BBST can do what we need: find and delete single items quickly.

Time complexity

Inserting $0, 1, 2, \ldots, m-1, m$ into the BBST at the start takes $\mathcal{O}(m \log m)$ time.

Finding the first node $\geq \text{left}_i$ and deleting a node both take $\mathcal{O}(\log m)$, and we do them at most $n+m$ and m times, respectively.

In total, our algorithm takes $\mathcal{O}(m \log m + (n+m) \log m + m \log m) = \mathcal{O}((n+m) \log m)$.

Space complexity

A BBST of m elements takes $\mathcal{O}(m)$ space.

Built-in BBSTs

Most programming languages have built-in BBSTS so we don't have to code them ourselves. C++ has `set`, Java has `TreeSet`, Python has `SortedList`, and JavaScript has `SortedSet` (but it's not supported on LeetCode).

```cpp
class Solution {
public:
    vector<int> amountPainted(vector<vector<int>>& paint) {
        set<int> unpainted;
        vector<int> ans(paint.size());
        for (int i = 0; i <= 50000; i++) {
            unpainted.insert(i);
        }
        for (int i = 0; i < paint.size(); i++) {
            int left = paint[i][0], right = paint[i][1];
            // Repeatedly delete the first element >= left until it becomes >= right
            // This clears values in [left, right) from the set
            for (auto it = unpainted.lower_bound(left); *it < right; it = unpainted.erase(it), ans[i]++);
        }
        return ans;
    }
};
```

```java
class Solution {
    public int[] amountPainted(int[][] paint) {
        TreeSet<Integer> unpainted = new TreeSet<>();
        int[] ans = new int[paint.length];
        for (int i = 0; i <= 50000; i++) {
            unpainted.add(i);
        }
        for (int i = 0; i < paint.length; i++) {
            int left = paint[i][0], right = paint[i][1];
            // Repeatedly delete the first element >= left until it becomes >= right
            // This clears values in [left, right) from the TreeSet
            while (true) {
                int next = unpainted.ceiling(left);
                if (next >= right)
                    break;
                unpainted.remove(next);
                ans[i]++;
            }
        }
        return ans;
    }
}
```

```python
from sortedcontainers import SortedList

class Solution:
    def amountPainted(self, paint: List[List[int]]) -> List[int]:
        unpainted = SortedList([i for i in range(0, 50001)])
        ans = [0 for _ in range(len(paint))]
        for i in range(len(paint)):
            left, right = paint[i]
            # Repeatedly delete the first element >= left until it becomes >= right
            # This clears values in [left, right) from the SortedList
            while unpainted[ind := unpainted.bisect_left(left)] < right:
                unpainted.__delitem__(ind)
                ans[i] += 1
        return ans
```

```javascript
var SortedSet = require("collections/sorted-set");
/**
 * @param {number[][]} paint
 * @return {number[]}
 */
var amountPainted = function (paint) {
  const n = paint.length;
  const ans = new Array(n).fill(0);
  const unpainted = new SortedSet(Array.from(Array(50001).keys()));
  for (let i = 0; i < n; i++) {
    (left = paint[i][0]), (right = paint[i][1]);
    // Repeatedly delete the first element >= left until it becomes >= right
    // This clears values in [left, right) from the SortedSet
    while ((node = unpainted.findLeastGreaterThanOrEqual(left)).value < right) {
      unpainted.delete(node.value);
      ans[i]++;
    }
  }
  return ans;
};
```

Instead of storing the unpainted blocks, we can store the painted segments. We store them as `(left, right)` pairs in a BBST, where no segments intersect. Each day, we delete segments fully contained in `[left_i, right_i]`, then merge partially overlapping segments with it, all while keeping count of how many blocks we've painted this day. We create, delete, and check for overlaps in $\mathcal{O}(n)$ segments for a total time complexity of $\mathcal{O}(n \log n)$. This solution is trickier to implement—code will not be presented here.