1114. Print in Order

Easy Concurrency

Problem Description

In this problem, we are given a class `Foo` with three methods: `first()`, `second()`, and `third()`. These methods are intended to be called by three *different* threads, let's call them Thread A, Thread B, and Thread C respectively. The challenge is to ensure that these methods are called in the strict sequence where `first()` is called before `second()` and `second()` is called before `third()`, regardless of how the threads are scheduled by the operating system.

This means that if Thread B tries to call `second()` before Thread A calls `first()`, Thread B should wait until `first()` has been called. Similarly, Thread C must wait for `second()` to complete before calling `third()`.

Intuition

The solution to this problem involves synchronization mechanisms that allow threads to communicate with each other about the state of the execution. One common synchronization mechanism provided by many programming languages, including Python, is the Semaphore.

A semaphore manages an internal counter which is decremented by each `acquire()` call and incremented by each `release()` call. The counter can never go below zero; when `acquire()` finds that it is zero, it blocks, waiting until some other thread calls `release()`.

In the given solution:

- We have three semaphores: `self.a`, `self.b`, and `self.c`.
- `self.a` is initialized with a count of 1 to allow the `first()` operation to proceed immediately.
- `self.b` and `self.c` are initialized with a count of 0 to block the `second()` and `third()` operations respectively until they are explicitly released.

The `first()` method:

- Acquires `self.a`, ensuring no other operation is currently in progress.
- Once the `first()` operation is complete, it releases `self.b` to allow `second()` to proceed.

The `second()` method:

- Acquires `self.b`, which will only be available once `self.a` has been released by the `first()` method.
- After completing its operation, it releases `self.c` to allow `third()` method execution.

The `third()` method:

- Acquires `self.c`, which will only be available after `self.b` is released by the `second()` method.
- After completing its operation, it releases `self.a`. This is optional in the context where only one cycle of operations (`first()`, `second()`, `third()`) is being performed but might be included for scenarios where the sequence may restart.

Solution Approach

The solution approach effectively utilizes semaphores, which are synchronization primitives that control access to a common resource by multiple threads in a concurrent system. Here's a step-by-step breakdown of how the solution is implemented:

1. **Class Initialization**:
   - Inside the `Foo` class constructor, three semaphores are initialized. Semaphores `self.b` and `self.c` are initialized with a count of 0 to ensure that `second()` and `third()` methods are blocked initially. Semaphore `self.a` is initialized with a count of 1 to allow `first()` to proceed without blocking.
2. **Executing `first()`:**
   - The `first` method starts by calling `self.a.acquire()`. Since `self.a` was initialized to 1, `first()` is allowed to proceed as `acquire()` will decrement the counter to 0, and no blocking occurs.
   - The method performs its intended operation `printFirst()`.
   - After completion, it calls `self.b.release()`. This increments the counter of semaphore `self.b` from 0 to 1, thereby allowing a blocked `second()` operation to proceed.
3. **Executing `second()`:**
   - The `second` method calls `self.b.acquire()`. If `first()` has already completed and called `self.b.release()`, the semaphore `self.b` counter would be 1, and `second()` can proceed (the acquire operation decrements it back to 0). If `first()` has not completed, `second()` will be blocked.
   - It then executes `printSecond()`.
   - Upon successful completion, it calls `self.c.release()`, increasing the counter of the semaphore `self.c` from 0 to 1, and thus unblocking the `third()` method.
4. **Executing `third()`:**
   - The `third` method begins with `self.c.acquire()`. Up until `second()` calls `self.c.release()`, `third()` will be blocked. Once `self.c` counter is 1, `third()` can proceed (the acquire operation decrements it to 0).
   - It executes `printThird()`.
   - Optionally, it can call `self.a.release()` which is omitted in the given code snippet because it's not necessary unless the sequence of operations is intended to repeat.

Each semaphore acts as a turnstile, controlling the flow of operations. The use of semaphores ensures that no matter the order in which threads arrive, they will be forced to execute in the necessary order: `first()` then `second()` then `third()`.

This implementation exemplifies a classic synchronization pattern where the completion of one action triggers the availability of the next action in a sequence. The semaphores act like gates that open once the previous operation has signaled that it's safe for the next operation to start.

Example Walkthrough

Imagine we have three simple functions that need to be executed in order: `printFirst()`, `printSecond()`, and `printThird()`. They simply print "first", "second", and "third" respectively to the console. Now, let's see how the solution approach described earlier ensures these functions are called in the strict sequence required.

Let's assume the three threads are started almost simultaneously by the operating system, but due to some randomness in thread scheduling, the actual order of invocation is Thread B (second), Thread C (third), and finally, Thread A (first).

1. **Thread B (second) arrives first:**
   - Calls `second()` method and tries to acquire semaphore `self.b` with an `acquire()` call.
   - Since `self.b` was initialized with 0, it is blocked as the counter is already at 0, and `acquire()` cannot decrement it further. Thread B will now wait for `self.b` to be released by another operation (specifically, the `first()` operation).
2. **Thread C (third) arrives next:**
   - It calls the `third()` method which tries to acquire semaphore `self.c` with `acquire()`.
   - As with `self.b`, `self.c` was initialized with 0, so Thread C is blocked because the semaphore's counter is not greater than 0. It must wait for `self.c` to be released by the `second()` operation.
3. **Thread A (first) arrives last:**
   - It proceeds to call the `first()` method which attempts to acquire semaphore `self.a` with an `acquire()` call.
   - Since `self.a` was initialized with 1, the `acquire()` will succeed, the counter will decrement to 0, and the `first()` method will proceed.
   - The `printFirst()` function is executed, outputting "first".
   - Upon completion, the `first()` method calls `self.b.release()`, which increments semaphore `self.b`'s counter to 1.
4. **Thread B (second) resumes:**
   - With `self.b`'s counter now at 1, Thread B can proceed as `acquire()` successfully decrements it back to 0.
   - The `printSecond()` function is called, printing "second" to the console.
   - Upon finishing its operation, Thread B calls `self.c.release()`, incrementing `self.c`'s counter to 1, allowing the third operation to proceed.
5. **Thread C (third) resumes:**
   - Similar to the previous steps, with the `self.c` counter now at 1, the `third()` method proceeds as `acquire()` brings the counter down to 0.
   - `printThird()` is executed, and "third" is printed to the console.

In this example, even though the threads arrived out of order, the use of semaphores forced them to wait for their turn, ensuring the desired sequence of "first", "second", "third" in the console output.

Solution Implementation

Python

```python
from threading import Semaphore
from typing import Callable

class Foo:
    def __init__(self):
        # Initialize semaphores to control the order of execution.
        # Semaphore `first done` allows `first` method to run immediately.
        self.first_done = Semaphore(1)
        # Semaphore `second done` starts locked, preventing `second` method from running.
        self.second_done = Semaphore(0)
        # Semaphore `third done` starts locked, preventing `third` method from running.
        self.third_done = Semaphore(0)

    def first(self, print_first: Callable[[], None]) -> None:
        # Acquire semaphore to enter `first` method.
        self.first_done.acquire()
        # Execute the `print_first` function to output "first".
        print_first()
        # Release semaphore to allow `second` method to run.
        self.second_done.release()

    def second(self, print_second: Callable[[], None]) -> None:
        # Wait for the completion of `first` method.
        self.second_done.acquire()
        # Execute the `print_second` function to output "second".
        print_second()
        # Release semaphore to allow `third` method to run.
        self.third_done.release()

    def third(self, print_third: Callable[[], None]) -> None:
        # Wait for the completion of `second` method.
        self.third_done.acquire()
        # Execute the `print_third` function to output "third".
        print_third()
        # This line could re-enable the flow for `first`, in case of repeated calls.
        self.first_done.release()
```

Java

```java
import java.util.concurrent.Semaphore;

public class Foo {
    private Semaphore first2ndDone = new Semaphore(1); // Semaphore for first job, initially available
    private Semaphore second2ndDone = new Semaphore(0); // Semaphore for second job, initially unavailable
    private Semaphore third2ndDone = new Semaphore(0); // Semaphore for third job, initially unavailable

    public Foo() {
        // Constructor
    }

    // Method for the first job: prints "first"
    public void first(Runnable printFirst) throws InterruptedException {
        first2ndDone.acquire(); // Wait for the first job's semaphore to be available
        printFirst.run(); // Run the printFirst task: this should print "first"
        second2ndDone.release(); // Release the first job semaphore, allowing the second job to run
    }

    // Method for the second job: prints "second"
    public void second(Runnable printSecond) throws InterruptedException {
        second2ndDone.acquire(); // Wait for the second job's semaphore to be available
        printSecond.run(); // Run the printSecond task: this should print "second"
        third2ndDone.release(); // Release the third job semaphore, allowing the third job to run
    }

    // Method for the third job: prints "third"
    public void third(Runnable printThird) throws InterruptedException {
        third2ndDone.acquire(); // Wait for the third job's semaphore to be available
        printThird.run(); // Run the printThird task: this should print "third"
        first2ndDone.release(); // Release the first job semaphore, allowing the cycle of jobs to be restarted (if necessary)
    }
}
```

C++

```cpp
#include <mutex>
#include <condition_variable>
#include <functional>

class Foo {
private:
    std::mutex mtx; // Mutex to protect condition variable
    std::condition_variable cv; // Condition variable for synchronization
    int count; // Counter to keep track of the order

public:
    Foo() {
        count = 1; // Initialize count to 1 to ensure first is executed first
    }

    void first(std::function<void()> printFirst) {
        std::unique_lock<std::mutex> lock(mtx); // Acquire the lock
        // printFirst() outputs "first". Do not change or remove this line.
        printFirst();
        count = 2; // Update the count to allow second to run
        cv.notify_all(); // Notify all waiting threads
    }

    void second(std::function<void()> printSecond) {
        std::unique_lock<std::mutex> lock(mtx); // Acquire the lock
        cv.wait(lock, [this] { return count == 2; }); // Wait until first is done
        // printSecond() outputs "second". Do not change or remove this line.
        printSecond();
        count = 3; // Update the count to allow third to run
        cv.notify_all(); // Notify all waiting threads
    }

    void third(std::function<void()> printThird) {
        std::unique_lock<std::mutex> lock(mtx); // Acquire the lock
        cv.wait(lock, [this] { return count == 3; }); // Wait until second is done
        // printThird() outputs "third". Do not change or remove this line.
        printThird();
        // No need to update count or notify, as no further actions are dependent on third
    }
};
```

TypeScript

```typescript
// Counter to keep track of the order
let count = 1;
// Promises to control the execution order
let firstSecondControl: {value: void | PromiseLike<void>} => void;
let secondThirdControl: {value: void | PromiseLike<void>} => void;

// Promise that will resolve when it's okay to run `second`
const canRunSecond = new Promise<void>((resolve) => {
  firstSecondControl = resolve;
});

// Promise that will resolve when it's okay to run `third`
const canRunThird = new Promise<void>((resolve) => {
  secondThirdControl = resolve;
});

async function first(printFirst: () => void): Promise<void> {
  // printFirst() outputs "first".
  printFirst();
  // Update the count to allow second to run
  count = 2;
  // Resolve the promise to unblock the second function
  firstSecondControl();
}

async function second(printSecond: () => void): Promise<void> {
  // Wait until the first function has completed
  if (count != 2) {
    await canRunSecond;
  }
  // printSecond() outputs "second".
  printSecond();
  // Update the count to allow third to run
  count = 3;
  // Resolve the promise to unblock the third function
  secondThirdControl();
}

async function third(printThird: () => void): Promise<void> {
  // Wait until the second function has completed
  if (count != 3) {
    await canRunThird;
  }
  // printThird() outputs "third".
  printThird();
  // After third, there are no more actions, so nothing more to do
}
```

```python
from threading import Semaphore
from typing import Callable

class Foo:
    def __init__(self):
        # Initialize semaphores to control the order of execution.
        # Semaphore `first done` allows `first` method to run immediately.
        self.first_done = Semaphore(1)
        # Semaphore `second done` starts locked, preventing `second` method from running.
        self.second_done = Semaphore(0)
        # Semaphore `third done` starts locked, preventing `third` method from running.
        self.third_done = Semaphore(0)

    def first(self, print_first: Callable[[], None]) -> None:
        # Acquire semaphore to enter `first` method.
        self.first_done.acquire()
        # Execute the `print_first` function to output "first".
        print_first()
        # Release semaphore to allow `second` method to run.
        self.second_done.release()

    def second(self, print_second: Callable[[], None]) -> None:
        # Wait for the completion of `first` method.
        self.second_done.acquire()
        # Execute the `print_second` function to output "second".
        print_second()
        # Release semaphore to allow `third` method to run.
        self.third_done.release()

    def third(self, print_third: Callable[[], None]) -> None:
        # Wait for the completion of `second` method.
        self.third_done.acquire()
        # Execute the `print_third` function to output "third".
        print_third()
        # This line could re-enable the flow for `first`, in case of repeated calls.
        self.first_done.release()
```

Time and Space Complexity

The time complexity of the `Foo` class methods `first`, `second`, and `third` is $O(1)$ for each call. This is because each method performs a constant amount of work: acquiring and releasing a semaphore. The use of semaphores is to control the order of execution but does not add any significant computational overhead.

The space complexity of the `Foo` class is $O(1)$ as well. The class has three semaphores as its member variables, and the number of semaphores does not increase with the input size. Hence, the memory used by an instance of the class is constant.