2656. Maximum Sum With Exactly K Elements

Easy Greedy Array

Leetcode Link

Problem Description

You are provided with an array `nums`, which contains integer elements and is indexed starting from 0. Along with this, you are given an integer `k`. The goal is to maximize your score through a specific operation that you can perform exactly `k` times.

The operation consists of the following steps:

1. Pick an element `m` from the array `nums`.
2. Remove this element from the array.
3. Insert a new element into the array that has a value of `m + 1`.
4. Increase your score by `m`.

After performing this operation exactly `k` times, you need to determine the maximum score you can achieve.

Intuition

To maximize the score from each operation, it's logical to always select the largest element available in the array. If you always choose the largest element, which we will denote as `x`, you ensure that your score increments by the maximum possible value each time.

Here's the thinking process step by step:

- In the first operation, choose the maximum element in the array, `x`. Your score increases by `x` and now the array has a new element `x + 1`.
- For the second operation, since you now have `x + 1` in the array (which is greater than the original `x`), you select and remove `x + 1`, and then add `x + 2`. Your score increases by `x + 1` this time.
- You continue this process, each time selecting the current maximum, which continues to increase by 1 with each operation.

After `k` operations, your total score will be an accumulated sum of all selected elements: `x` from the first operation, `x + 1` from the second, and so on, up to `x + k - 1` for the `k`th operation.

So, you can calculate the total score by taking the sum of the arithmetic sequence that starts with `x`, has `k` terms, and a common difference of 1.

Thus, the formula to calculate the final score is:

```
Score = k * x + (1 + 2 + ... + (k - 1))
```

We can simplify the summation using the formula for the sum of the first `n` natural numbers:

```
Sum = n * (n + 1) / 2
```

Therefore, you can express the score as:

```
Score = k * x + (k - 1) * k / 2
```

Applying this formula grants us the maximum score after `k` operations without the need to simulate the operations step by step.

Solution Approach

The solution outlined above hinges on understanding and applying a mathematical concept rather than running complex algorithms or relying on particular data structures. Here's how we translate the mathematical solution into a concrete implementation:

- First, we need to determine the maximum element from the array `nums`, which is denoted as `x`. In Python, this can be easily done by using the built-in `max()` function.

  ```
  x = max(nums)
  ```

- Now that we have the maximum number (`x`), we must calculate the score based on the formula derived earlier:

  ```
  Score = k * x + (k - 1) * k / 2
  ```

  In the formula, `k * x` accounts for selecting the maximum element `x` `k` times, while the second part of the formula `(k - 1) * k / 2` is the sum of the arithmetic series contributing to the incrementality of the score with each operation.

- The implementation of this calculation is straightforward and directly derived from the formula. It can be written in a single line of code in Python:

  ```
  return k * x + k * (k - 1) // 2
  ```

  Here, the `//` operator is used to perform integer division to avoid any floating-point result, which is suitable since we are dealing with integers only.

By abstracting the problem into a formula, the time complexity of the solution is `O(n)`, where `n` is the length of the array, coming from the time complexity of finding the maximum element. The space complexity is `O(1)`, as we only store the maximum element and use no additional data structures to hold intermediate values.

Example Walkthrough

Let's illustrate the solution approach with a small example:

Suppose our input array `nums` is `[3, 5, 2]` and we are allowed to perform the operation `k = 2` times.

- We first find the maximum element in the array, which is `x = 5`.
- Then, we calculate the maximum score by using the earlier derived formula:

  ```
  Score = k * x + (k - 1) * k / 2
  ```

  Plugging in the values:

  ```
  Score = 2 * 5 + (2 - 1) * 2 / 2 Score = 10 + 1 Score = 11
  ```

So, without physically removing elements and adding them back to the array, we can directly compute that the maximum score after performing the operation twice is `11`.

To break it down further, after the first operation, we would have selected element `5`, removed it, added `6` to the array, and increased our score by `5`. After the second operation, we would select the new maximum element, which is `6`, remove it, add `7` to the array, and increase our score by another `6`. Therefore, the total score would be `5 (initial score) + 6 (second operation) = 11`.

This hypothetical set of operations confirms the correctness of our direct calculation using the formula. The formula saves time by avoiding multiple iterations for selecting and replacing elements with their increments.

Implementing this step-by-step:

1. From array `nums = [3, 5, 2]`, find the maximum element `x`, which is `5`.
2. Calculate the maximum score using the given values `k = 2` and `x = 5`.
3. Substitute into the formula to get the result: `Score = 2 * 5 + 1 * (2 - 1) = 11`.

A Python implementation of this example would be:

```python
nums = [3, 5, 2]
k = 2

x = max(nums)
score = k * x + k * (k - 1) // 2

print(score)  # Output should be 11
```

This program would output `11`, which represents the maximum score achieved after performing the operation `k` times.

Python Solution

```python
from typing import List

class Solution:
    def maximizeSum(self, nums: List[int], k: int) -> int:
        # Find the maximum number in the given list of numbers
        max_num = max(nums)

        # The strategy to maximize the sum is to repeatedly increment the maximum number.
        # We will do this k times, each time adding the maximum number to the sum.
        # And for each iteration, we also add the i-th increment (0 to k-1).
        # This is equivalent to max_num * k (adding max_num k times), plus the sum
        # of the first k natural numbers, which is k * (k - 1) // 2 using the formula for the sum
        # of the first 'n' natural numbers.
        maximized_sum = k * max_num + k * (k - 1) // 2

        # Return the calculated maximized sum
        return maximized_sum

```

Java Solution

```java
class Solution {
    public int maximizeSum(int[] nums, int k) {
        // Initialize a variable to store the maximum value found in the array
        int maxVal = 0;

        // Iterate through all elements in the array to find the maximum value
        for (int value : nums) {
            maxVal = Math.max(maxVal, value);
        }

        // Calculate and return the sum of the largest k numbers in an arithmetic progression
        // starting from maxVal and with a common difference of 1.
        return k * maxVal + k * (k - 1) / 2;
    }
}

```

C++ Solution

```cpp
#include <vector>
#include <algorithm> // Include algorithm for std::max_element

class Solution {
public:
    // Function to maximize sum by performing k increments on the maximum element
    int maximizeSum(std::vector<int>& nums, int k) {
        // Find the maximum element in the vector
        int maxElement = *std::max_element(nums.begin(), nums.end());

        // Calculate the total increment on the max element after k operations
        // This is done by multiplying the maximum element with k
        // And then adding the sum of first k natural numbers (k*(k-1)/2)
        int totalIncrement = k * maxElement + k * (k - 1) / 2;

        // Return the sum after all increments are done
        return totalIncrement;
    }
};

```

Typescript Solution

```typescript
/**
 * Maximizes the sum by adding the top k numbers where the
 * first number is the maximum in the array and every subsequent
 * number is one less than the previous.
 *
 * @param {number[]} nums - The array of numbers to search through.
 * @param {number} k - The number of top elements to consider for maximizing the sum.
 * @returns {number} The maximized sum according to the described formula.
 */
function maximizeSum(nums: number[], k: number): number {
    // Find the maximum value in the nums array.
    const maxValue = Math.max(...nums);
    // Calculate the sum of the top k numbers starting from the maxValue.
    // The formula used is the sum of the first k elements of an arithmetic series
    // starting with maxValue and decrementing by 1 each time.
    const sum = k * maxValue + (k * (k - 1)) / 2;

    // Return the calculated sum.
    return sum;
}

```

Time and Space Complexity

The given Python code consists of two main operations: finding the maximum value in the list `nums`, and performing arithmetic calculations to obtain the result.

Time Complexity

The time complexity of finding the maximum element in a list of `n` elements using `max(nums)` is `O(n)`, as it requires checking each element exactly once. The arithmetic operations `k * x + k * (k - 1) // 2` involve constant time calculations, hence their complexity is `O(1)`.

Overall, the time complexity of the function is `O(n) + O(1)`, which simplifies to `O(n)`.

Space Complexity

In terms of space complexity, there are no additional data structures used that grow with the input size. Only a constant amount of additional space is used for variables like `x`. Therefore, the space complexity of the function is `O(1)`.

In conclusion, the time complexity of the provided code is `O(n)` and the space complexity is `O(1)`.