2860. Happy Students

Medium Array Enumeration Sorting

Problem Description

In this problem, we are dealing with a situation that involves making a group of students happy based on selection criteria. The array `nums` represents the number of students in a class, and each element `nums[i]` implies a certain requirement that the `i`th student has to be happy. There are two conditions under which a student will be happy:

1. The student is selected, and the total number of selected students is more than their own specified number (`nums[i]`).
2. The student is not selected, and the total number of selected students is less than their own specified number (`nums[i]`).

We need to find out how many different ways we can select groups of students so that all students are happy according to their individual requirements.

Intuition

The solution to this problem involves sorting the `nums` array first. This action will group students by their happiness criteria, allowing us to consider students in sorted order, which is easier to handle.

After sorting the array, we iterate through potential group sizes—from 0 students selected up to all students being selected (hence the range from 0 to `n` inclusive). During this iteration, we are looking for the positions where the following is true:

- If the current position is `i`, then all students up to `i-1` must require a group size smaller than `i` (or be included themselves) to be happy, and all students from `i` onwards must require a group size greater than `i` to be happy (or not be included).

The provided code snippet, however, seems incomplete as it sets up the logic and loop for counting valid ways to select students but lacks the actual implementation for updating the `ans` variable, indicating the number of ways to select students. Therefore, the reference solution approach is necessary to understand the complete logic for properly incrementing `ans` based on valid selections. Without this crucial part of the logic, we cannot deduce the correct number of ways to fulfill the problem statement requirements.

The correct approach will evaluate each position to check whether it satisfies the conditions mentioned above. If a valid position is found, it should update the count of happiness-satisfying selections (`ans`). We also need to handle edge cases, for example, when groups can't be formed because all students have a happiness criteria that's higher or lower than possible group sizes.

Solution Approach

To implement the solution for this problem, we need a way to count scenarios where a selection of students makes all students happy. Since the original solution provided is incomplete, let's infer the approach that would lead to a complete solution.

After sorting the `nums` array in non-decreasing order, we perform a sweep across potential group sizes. This requires comparing the group size `i` at each step to the numbers in the sorted array and deciding if students are happy. Below is the pattern we can follow for a complete solution:

1. Initialize a count variable (`ans`) to store the total number of ways to select students, starting at 0.
2. Sort the array `nums` to make a linear sweep feasible.
3. Iterate through the array, checking each possible group size from 0 to `n`, inclusive.
4. To analyze each group size `i`, determine if selecting `i` students will satisfy both happy conditions for students before and after the `i`th) student:
   - For students at index less than `i` (selected students), they should all have a happiness criterion `nums[i]` that is less than or equal to `i`.
   - For students at index greater than or equal to `i` (not selected), their happiness criterion `nums[i]` should be strictly greater than `i`.
5. Only if the current group size satisfies both criteria, it implies a valid configuration. Increment the `ans` counter for each such case.
6. Return the `ans` as the count of all valid selections.

In terms of algorithms and data structures:

- **Sorting Algorithm**: We use the built-in sort function, which is likely implemented as a variant of quicksort, heapsort, or mergesort in most programming languages, giving O(n log n) complexity for sorting the array.
- **Linear Sweep pattern**: After sorting, we iterate over each element once, giving us an O(n) complexity for the sweep.

The complete solution, therefore, will have an overall time complexity of O(n log n) due to the sorting with an additional O(n) sweep, resulting in O(n log n) total time complexity.

Example Walkthrough

Let's consider a small example to illustrate the solution approach. Assume we have the following array `nums` that indicates the students' happiness criteria:

```
nums = [1, 2, 3, 4]
```

According to the problem, there are 4 students in the class, and they will be happy under the following conditions:

- Student 0 wants more than 1 student to be selected.
- Student 1 wants more than 2 students to be selected.
- Student 2 wants more than 3 students to be selected.
- Student 3 wants more than 4 students to be selected.

Following the solution approach:

1. Initialize `ans` to 0.
2. Sort the array `nums` in non-decreasing order. However, in our example, the array is already sorted, so no action is required.
3. Iterate through the possible group sizes:
   - With a group size of 0 (`i = 0`):
     - No student is selected, which means all the students' conditions are checked.
     - All students have a happiness criterion greater than 0, so this configuration makes all students unhappy.
   - With a group size of 1 (`i = 1`):
     - Student 0 is selected. This doesn't satisfy Student 0's happiness criteria (`nums[0] = 1`) since 1 is not more than 1.
     - This configuration does not make all students happy.
   - With a group size of 2 (`i = 2`):
     - Students 0 and 1 are selected. This satisfies both of their criteria since 2 is more than 1 (`nums[0]`) and 2 is more than 2 (`nums[1]`).
     - Students 2 and 3 are not selected. This satisfies their criteria since 2 is less than 3 (`nums[2]`) and 4 (`nums[3]`).
     - This configuration makes all students happy. Increment `ans` to 1.
   - With a group size of 3 (`i = 3`):
     - Students 0, 1, and 2 are selected. This satisfies their criteria since 3 is more than 1 (`nums[0]`), 2 (`nums[1]`), and 3 (`nums[2]`).
     - Student 3 is not selected. This satisfies their criterion since 3 is less than 4 (`nums[3]`).
     - This configuration makes all students happy. Increment `ans` to 2.
   - With a group size of 4 (`i = 4`):
     - All students are selected. Student 3's criteria are not satisfied since 4 is not more than 4 (`nums[3]`).
     - This configuration does not make all students happy.
4. After iterating through all possible group sizes, we find that there are 2 valid selections that make all students happy.

Thus, the final `ans` is 2.

Solution Implementation

Python

```python
from typing import List

class Solution:
    def count_ways(self, nums: List[int]) -> int:
        # Sort the input list to make it easier to process
        nums.sort()

        # Get the length of the sorted list
        length = len(nums)

        # Initialize the answer (the number of ways) to zero
        answer = 0

        # Iterate over all elements in the sorted list
        for i in range(length + 1):
            # If not the first element, and the previous number is greater than or equal to the index, skip
            if i > 0 and nums[i - 1] >= i:
                continue

            # If not the last element, and the current number is less than or equal to the index, skip
            if i < length and nums[i] <= i:
                continue

        # At this point the answer is not changed by the loop, this loop has no effect.
        # The function currently will always return 0 as it stands.
        return answer
```

Java

```java
import java.util.Collections;
import java.util.List;

class Solution {

    /**
     * Counts the ways in which numbers can be assigned to their indices
     * in the list such that each number is greater than its index.
     *
     * @param nums List of Integer values presumably between 0 and list size
     * @return Count of the possible ways numbers can be arranged fulfilling the condition
     */
    public int countWays(List<Integer> nums) {
        // Sort the list in non-decreasing order
        Collections.sort(nums);

        // Get the size of the list
        int n = nums.size();

        // Initialize answer count to 0
        int answer = 0;

        // Iterate through all possible positions in the list
        for (int i = 0; i <= n; i++) {
            // Check if the current number is greater than it's index (after considering zero-based index adjustment)
            // Also, consider the cases when the index is at the start or the end of the list
            if ((i == 0 || nums.get(i - 1) < i) && (i == n || nums.get(i) > i)) {
                // Increment the answer if the condition is satisfied
                answer++;
            }
        }

        // Return the total count of valid ways
        return answer;
    }
}
```

C++

```cpp
#include <vector>
#include <algorithm> // For sort()

class Solution {
public:
    // Function to count the number of distinct ways to form index-value pairs.
    int countWays(vector<int>& nums) {
        // Sort the input vector of numbers in non-decreasing order.
        sort(nums.begin(), nums.end());

        // Initialize the answer (count of distinct ways) to 0.
        int count_distinct_ways = 0;

        // Get the number of elements in nums.
        int n = nums.size();

        // Loop through the numbers from 0 to n (inclusive at the beginning and exclusive at the end).
        for (int i = 0; i <= n; ++i) {
            // Skip the current number if:
            // - It's not the first number and the previous number is greater or equal to the current index.
            // OR
            // - It's not the last number and the current number is less or equal to the current index.
            if ((i > 0 && nums[i - 1] >= i) || (i < n && nums[i] <= i)) {
                continue;
            }
            // If none of the above conditions are met, increment the count of distinct ways.
            ++count_distinct_ways;
        }

        // Return the total count of distinct ways.
        return count_distinct_ways;
    }
};
```

TypeScript

```typescript
function countWays(nums: number[]): number {
    // Sort the input array in ascending order.
    nums.sort((a, b) => a - b);

    // Initialize the count of ways to 0.
    let waysCount = 0;

    // Get the length of the nums array.
    const arrayLength = nums.length;

    // Iterate through the array including a position at the end.
    for (let i = 0; i <= arrayLength; ++i) {
        // Skip the current iteration if the value at the previous index is greater than or equal to the current index
        // OR if the value at the current index is less than or equal to the current index.
        if (i > 0 && nums[i - 1] >= i) || (i < arrayLength && nums[i] <= i)) {
            continue;
        }

        // If conditions are met, increment the count of ways.
        ++waysCount;
    }

    // Return the total count of valid ways.
    return waysCount;
}
```

```python
from typing import List

class Solution:
    def count_ways(self, nums: List[int]) -> int:
        # Sort the input list to make it easier to process
        nums.sort()

        # Get the length of the sorted list
        length = len(nums)

        # Initialize the answer (the number of ways) to zero
        answer = 0

        # Iterate over all elements in the sorted list
        for i in range(length + 1):
            # If not the first element, and the previous number is greater than or equal to the index, skip
            if i > 0 and nums[i - 1] >= i:
                continue

            # If not the last element, and the current number is less than or equal to the index, skip
            if i < length and nums[i] <= i:
                continue

        # At this point the answer is not changed by the loop, this loop has no effect.
        # The function currently will always return 0 as it stands.
        return answer
```

Time and Space Complexity

The given Python code snippet is meant to count the number of ways or a particular quantity relating to a list of integers. However, the function `countWays` does not include logic to increment the `ans` variable, thus the actual purpose of the function is not clear from the provided code, and its time and space complexity are discussed as it is.

Time Complexity

The overall time complexity of the `countWays` function is determined by the sorting operation and the for loop.

1. The `sort()` method applied on `nums` list is the most costly operation in this snippet. The sort function in Python uses Timsort, which has a worst-case time complexity of `O(n log n)` where 'n' is the length of the list.
2. The for loop runs from 0 to `n + 1` where 'n' is the length of the `nums` list. However, the loops body has `continue` statements which may terminate the iterations early without performing any additional operations. In the worst case, the loop runs `n + 1` times.

Considering the above points, the dominant part in terms of time complexity is the sorting operation. Therefore, the overall time complexity of the function is `O(n log n)` due to the sort, irrespective of the for loop, which has a best case of `O(1)` and a worst case of `O(n)`.

Space Complexity

The space complexity of the function is determined by the storage requirements that are not directly dependent on the input size. In the snippet provided:

1. The `nums` list is sorted in place, so no additional space is necessary for sorting beyond the space already used to store `nums`.
2. The variable `ans` and the loop variable `i` each take constant space.

As there are no additional data structures used that grow with the size of the input, the space complexity of the function is `O(1)`, which is constant space.