

2467. Most Profitable Path in a Tree

Description

There is an undirected tree with `n` nodes labeled from `0` to `n - 1`, rooted at node `0`. You are given a 2D integer array `edges` of length `n - 1` where `edges[i] = [ai, bi]` indicates that there is an edge between nodes `ai` and `bi` in the tree.

At every node `i`, there is a gate. You are also given an array of even integers `amount`, where `amount[i]` represents:

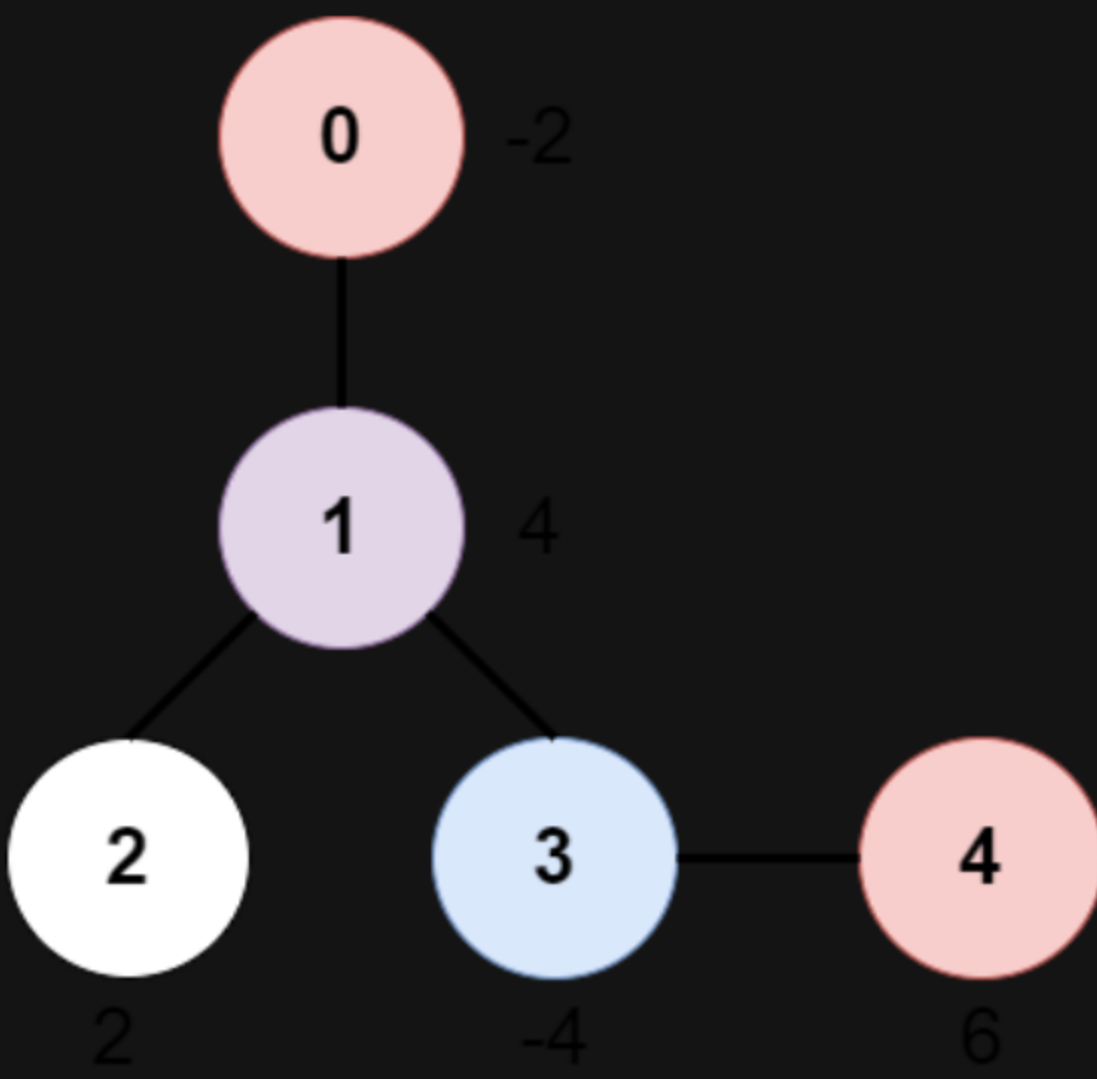
- the price needed to open the gate at node `i`, if `amount[i]` is negative, or,
- the cash reward obtained on opening the gate at node `i`, otherwise.

The game goes on as follows:

- Initially, Alice is at node `0` and Bob is at node `bob`.
- At every second, Alice and Bob **each** move to an adjacent node. Alice moves towards some **leaf node**, while Bob moves towards node `0`.
- For **every** node along their path, Alice and Bob either spend money to open the gate at that node, or accept the reward. Note that:
 - If the gate is **already open**, no price will be required, nor will there be any cash reward.
 - If Alice and Bob reach the node **simultaneously**, they share the price/reward for opening the gate there. In other words, if the price to open the gate is `c`, then both Alice and Bob pay `c / 2` each. Similarly, if the reward at the gate is `c`, both of them receive `c / 2` each.
- If Alice reaches a leaf node, she stops moving. Similarly, if Bob reaches node `0`, he stops moving. Note that these events are **independent** of each other.

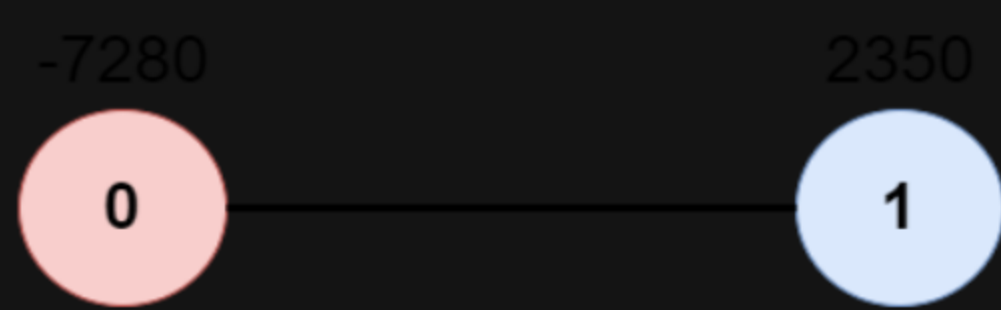
Return *the maximum net income Alice can have if she travels towards the optimal leaf node.*

Example 1:



```
Input: edges = [[0,1],[1,2],[1,3],[3,4]], bob = 3, amount = [-2,4,2,-4,6]
Output: 6
Explanation:
The above diagram represents the given tree. The game goes as follows:
- Alice is initially on node 0, Bob on node 3. They open the gates of their respective nodes.
  Alice's net income is now -2.
- Both Alice and Bob move to node 1.
  Since they reach here simultaneously, they open the gate together and share the reward.
  Alice's net income becomes -2 + (4 / 2) = 0.
- Alice moves on to node 3. Since Bob already opened its gate, Alice's income remains unchanged.
  Bob moves on to node 0, and stops moving.
- Alice moves on to node 4 and opens the gate there. Her net income becomes 0 + 6 = 6.
Now, neither Alice nor Bob can make any further moves, and the game ends.
It is not possible for Alice to get a higher net income.
```

Example 2:



```
Input: edges = [[0,1]], bob = 1, amount = [-7280,2350]
Output: -7280
Explanation:
Alice follows the path 0->1 whereas Bob follows the path 1->0.
Thus, Alice opens the gate at node 0 only. Hence, her net income is -7280.
```

Constraints:

- `2 <= n <= 105`
- `edges.length == n - 1`
- `edges[i].length == 2`
- `0 <= ai, bi < n`
- `ai != bi`
- `edges` represents a valid tree.
- `1 <= bob < n`
- `amount.length == n`
- `amount[i]` is an **even** integer in the range `[-104, 104]`.

