2145. Count the Hidden Sequences

Medium Array Prefix Sum Leetcode Link

Problem Description

You are presented with an array `differences` which is used to calculate the values of a hidden sequence `hidden`. The `differences` array gives you the difference between each pair of consecutive elements in the `hidden` sequence. That is, for each `i` in the `differences` array, `differences[i] = hidden[i + 1] - hidden[i]`.

The `hidden` sequence itself is not known, but it does have length `n + 1`, where `n` is the length of the `differences` array. The problem also stipulates that any number in the `hidden` sequence must be within an inclusive range denoted by two given integers, `lower` and `upper`.

Your task is to find out how many different `hidden` sequences can be possibly made that adhere to the differences provided and also stay within the boundaries given by `lower` and `upper`. If no such sequences can be made, your answer should be `0`.

Intuition

To solve this problem, we think about the constraints that the `lower` and `upper` bounds put on the possible values of the `hidden` sequence. Since we know the differences between each pair of consecutive values, we can simulate the sequence to find the minimum and maximum values that occur if we start the sequence at zero. These minimum and maximum values are offset versions of what the real sequence could look like.

Knowing the most extreme (minimum and maximum) values that our sequence reaches, we can calculate how many different starting points for the `hidden` sequence are possible that would keep the sequence within the bounds of `lower` and `upper`. Essentially, we are sliding the window of the extreme values of our simulated sequence along the scale of `lower` to `upper` and checking for overlaps.

Since we are given that the hidden sequence values must be in the range `[lower, upper]` (inclusive), we subtract the maximum value we found from `upper` and the result is the span of numbers that could potentially be the start of a `hidden` sequence. We also subtract the range span of the hidden sequence (max - min), so we get the number of sequences that can be formed. Adding `1` accounts for the inclusive boundaries. If the span is negative, that means there are no possible sequences, so we use `max` function to set the count to `0` in such cases.

The one-liner calculation in the provided solution performs this sliding window computation to find the count of valid starting numbers, and thus, by extension, the count of valid sequences.

Solution Approach

To implement the solution, we make use of a simple linear scan algorithm to iterate through the `differences` array and accumulate the changes to calculate the possible minimum and maximum values that the `hidden` sequence can take.

1. We start by initializing a variable `num` to `0`, which represents the current value of the `hidden` sequence, assuming it starts at `0`. Alongside, we initialize two other variables, `mi` and `mx`, which stand for the minimum and the maximum values that we encounter as we construct the `hidden` sequence. Both are initially set to `0`.

2. Then, we iterate over the `differences` array, and for each difference `d`, we add `d` to `num`. As we simulate the sequence, we update `mi` and `mx` with these two lines:

   ```
   mi = min(mi, num)
   mx = max(mx, num)
   ```

   If `num` is lesser than our current minimum, we update `mi` to reflect `num`, and similarly, if `num` is greater than our current maximum, we update `mx` to be `num`.

3. After we finish iterating through all the elements in `differences`, we will have the most extreme values that the `hidden` sequence could attain if it started at 0. Now, we must consider the given bounds `lower` and `upper`.

4. The formula to calculate the number of valid starting points for the `hidden` sequence, and hence the number of possible sequences, is:

   ```
   return max(0, upper - lower - (mx - mi) + 1)
   ```

   We subtract `mx - mi` from `upper - lower` to get the number of positions between `lower` and `upper` that could be the start of the `hidden` sequence, while still ensuring all of its values do not breach the bounds. We add 1 because both `lower` and `upper` are inclusive.

5. If the result of the subtraction is less than `0`, it implies that there's no possible starting point that keeps the sequence within the bounds, therefore there are `0` possible sequences. We use the `max` function to handle this scenario, which helps to ensure that we do not return a negative number of possible sequences.

This approach is efficient, with a time complexity of O(n) where n is the length of the `differences` array, since we only need to scan through the `differences` array once to arrive at the solution.

Example Walkthrough

Let's consider the `differences` array as `[-2, -1, 2, 1]`, with the bounds `lower` equal to `1`, and `upper` equal to `6`.

1. We start by initializing our variables `num`, `mi`, and `mx` to `0`. These will keep track of our current sequence value (assuming we start at 0), and the minimum and maximum values we encounter.

2. We iterate through the `differences` array:
   - For the first difference, `-2`, we update `num` to `0 - 2 = -2`. We also update `mi` to `-2` (since `-2` is less than the old minimum `0`) and `mx` remains as `0`.
   - For the second difference, `-1`, `num` becomes `-2 - 1 = -3`. `mi` is updated to `-3` and `mx` remains as `0`.
   - For the third difference, `2`, `num` becomes `-3 + 2 = -1`. `mi` remains as `-3` and `mx` remains as `0`.
   - For the last difference, `1`, `num` is now `-1 + 1 = 0`. Again `mi` and `mx` remain as `-3` and `0`, respectively.
3. After iterating through the array, we have `mi = -3` and `mx = 0`. Our hidden sequence, if starting at `0`, would range in values between `-3` and `0`.

4. We now compare the span of our possible hidden sequence values with the given bounds:
   - We first calculate the span of numbers between `lower` and `upper`: `upper - lower` which is `6 - 1 = 5`.
   - Next, we find the span of our hidden sequence by computing `mx - mi`: `0 - (-3) = 3`.
   - To find how many sequences can fit, we subtract the hidden sequence span from the bounds span and add `1`: `(5 - 3) + 1 = 3`.
5. The final result is `3`. Meaning we can have `3` different possible starting points for the `hidden` sequence that would keep the sequence within the bounds given by `lower` and `upper`. The valid sequences would start at `1`, `2`, and `3`, leading to the following possible sequences within the bounds `[1, 6]`:
   - Starting at `1`: [1, -1, -2, 0, 1]
   - Starting at `2`: [2, 0, -1, 1, 2]
   - Starting at `3`: [3, 1, 0, 2, 3]

Each starting value leads to a sequence that, when applying the differences, remains within the bounds of `1` to `6`. Hence, there are 3 valid `hidden` sequences.

Python Solution

```python
from typing import List

class Solution:
    def numberOfArrays(self, differences: List[int], lower: int, upper: int) -> int:
        # Initialize the variables: current_sum to track the running sum,
        # min_value to keep the minimum value encountered, and max_value
        # for the maximum value encountered.
        current_sum = min_value = max_value = 0

        # Iterate through each difference in the array
        for diff in differences:
            # Add the current difference to the running sum
            current_sum += diff
            # Update the minimum value if the new current_sum is lower
            min_value = min(min_value, current_sum)
            # Update the maximum value if the new current_sum is higher
            max_value = max(max_value, current_sum)

        # Calculate the width of the range spanned by the differences
        range_width = max_value - min_value
        # Calculate the total number of distinct arrays that can be formed
        # within the given upper and lower bounds. Here we also include the
        # '+ 1' offset to account for inclusive bounds.
        # If the resulting number is negative, we use max(0, ...) to default to 0.
        # This represents cases where no valid arrays can be formulated.
        num_of_arrays = max(0, (upper - lower) - range_width + 1)

        return num_of_arrays

```

Java Solution

```java
class Solution {

    /**
     * Calculate the number of valid arrays that can be constructed with the given conditions.
     *
     * @param differences An array of integers representing the difference between consecutive elements in the target array.
     * @param lower The lower bound for the elements of the target array.
     * @param upper The upper bound for the elements of the target array.
     * @return The number of valid arrays that can be constructed.
     */
    public int numberOfArrays(int[] differences, int lower, int upper) {
        // Initialize running sum, minimum and maximum values observed while simulating the array creation
        long runningSum = 0;
        long minObserved = 0;
        long maxObserved = 0;

        // Iterate over the array of differences
        for (int difference : differences) {
            // Update the running sum with the current difference
            runningSum += difference;

            // Update the minimum observed sum, if necessary
            minObserved = Math.min(minObserved, runningSum);

            // Update the maximum observed sum, if necessary
            maxObserved = Math.max(maxObserved, runningSum);
        }

        // Compute the number of possible starting values that satisfy the bounds
        int totalValidArrays = Math.max(0, (int) (upper - lower - (maxObserved - minObserved) + 1));

        // Return the computed total number of valid arrays
        return totalValidArrays;
    }
}

```

C++ Solution

```cpp
#include <vector> // Include necessary header for using vectors
#include <algorithm> // Include necessary header for using min and max functions

class Solution {
public:
    int numberOfArrays(vector<int>& differences, int lower, int upper) {
        long long runningSum = 0; // This will keep track of the accumulated sum of differences
        long long minSum = 0; // This will keep the minimum sum encountered
        long long maxSum = 0; // This will keep the maximum sum encountered

        // Iterate over the differences array
        for (int &difference : differences) {
            runningSum += difference; // Accumulate the sum of differences
            minSum = std::min(minSum, runningSum); // Update the minimum sum if necessary
            maxSum = std::max(maxSum, runningSum); // Update the maximum sum if necessary
        }

        // Calculate the range of the final array values
        long long validRange = upper - lower - (maxSum - minSum) + 1;

        // If the range is negative, set it to zero
        return std::max(0LL, validRange);
    }
};

```

Typescript Solution

```typescript
// TypeScript does not have a standard header system like C++,
// so you don't 'include' modules. Instead, you import them if necessary.
// In this case, no import is needed since arrays and Math functions are built-in.

// A global variable to keep track of the accumulated sum of differences
let runningSum: number = 0;

// A global variable to keep track of the minimum sum encountered
let minSum: number = 0;

// A global variable to keep track of the maximum sum encountered
let maxSum: number = 0;

// Function to calculate the number of valid arrays from the given differences
// and the bounds provided by lower and upper limits
function numberOfArrays(differences: number[], lower: number, upper: number): number {
    // Reset the global variables for a new function call
    runningSum = 0;
    minSum = 0;
    maxSum = 0;

    // Iterate over the differences array
    for (let difference of differences) {
        // Accumulate the sum of differences
        runningSum += difference;

        // Update the minimum and maximum sums if necessary
        minSum = Math.min(minSum, runningSum);
        maxSum = Math.max(maxSum, runningSum);
    }

    // Calculate the range of the final array values
    let validRange: number = upper - lower - (maxSum - minSum) + 1;

    // Return the number of valid arrays, ensuring the number is not negative
    return Math.max(0, validRange);
}

```

Time and Space Complexity

The given Python function `numberOfArrays` calculates how many valid arrays can be generated from a list of differences within the given upper and lower bounds.

Time Complexity: The time complexity of the `numberOfArrays` function is `O(n)`, where `n` is the length of the `differences` list. This is because we iterate through each element of `differences` exactly once, performing constant-time operations (addition, minimum, maximum) at each step.

Space Complexity: The space complexity of the function is `O(1)` as the function uses a fixed number of integer variables (`num`, `mi`, `mx`) and does not allocate any additional space that grows with the input size. The space used for the input `differences` list does not count towards the space complexity of the function itself as it is provided as input.