# 2050. Parallel Courses III

## Description

You are given an integer `n`, which indicates that there are `n` courses labeled from `1` to `n`. You are also given a 2D integer array `relations` where `relations[j] = [prevCourse_j, nextCourse_j]` denotes that course `prevCourse_j` has to be completed **before** course `nextCourse_j` (prerequisite relationship). Furthermore, you are given a **0-indexed** integer array `time` where `time[i]` denotes how many **months** it takes to complete the `(i+1)^th` course.

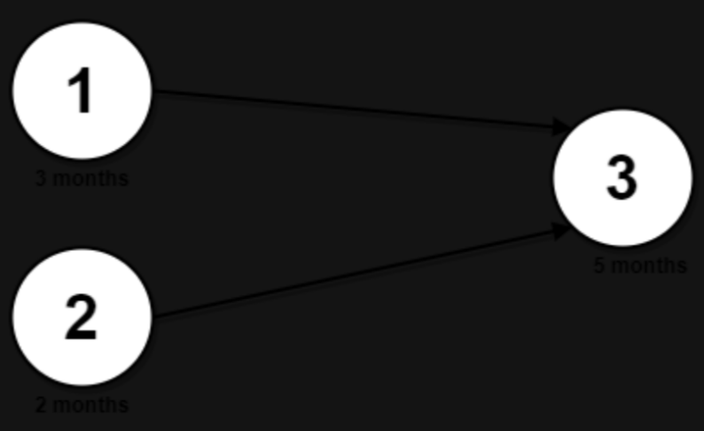
You must find the **minimum** number of months needed to complete all the courses following these rules:

- You may start taking a course at **any time** if the prerequisites are met.
- **Any number of courses** can be taken at the **same time**.

Return *the **minimum** number of months needed to complete all the courses*.

**Note:** The test cases are generated such that it is possible to complete every course (i.e., the graph is a directed acyclic graph).

**Example 1:**




```
Input: n = 3, relations = [[1,3],[2,3]], time = [3,2,5]
Output: 8
Explanation: The figure above represents the given graph and the time required to complete each course.
We start course 1 and course 2 simultaneously at month 0.
Course 1 takes 3 months and course 2 takes 2 months to complete respectively.
Thus, the earliest time we can start course 3 is at month 3, and the total time required is 3 + 5 = 8 months.
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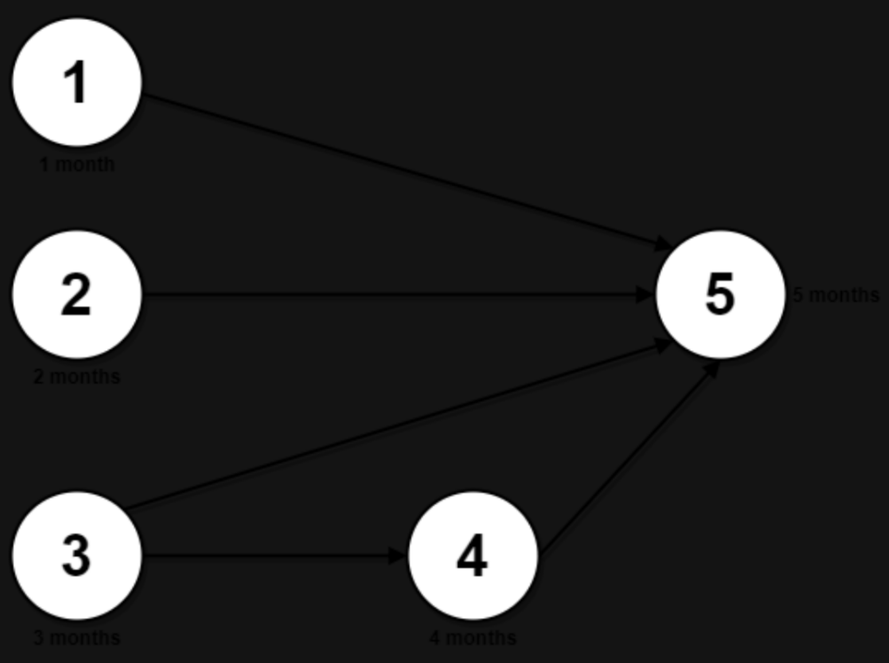
**Example 2:**




```
Input: n = 5, relations = [[1,5],[2,5],[3,5],[3,4],[4,5]], time = [1,2,3,4,5]
Output: 12
Explanation: The figure above represents the given graph and the time required to complete each course.
You can start courses 1, 2, and 3 at month 0.
You can complete them after 1, 2, and 3 months respectively.
Course 4 can be taken only after course 3 is completed, i.e., after 3 months. It is completed after 3 + 4 = 7 months.
Course 5 can be taken only after courses 1, 2, 3, and 4 have been completed, i.e., after max(1,2,3,7) = 7 months.
Thus, the minimum time needed to complete all the courses is 7 + 5 = 12 months.
```

**Constraints:**

- `1 <= n <= 5 * 10^4`
- `0 <= relations.length <= min(n * (n - 1) / 2, 5 * 10^4)`
- `relations[j].length == 2`
- `1 <= prevCourse_j, nextCourse_j <= n`
- `prevCourse_j != nextCourse_j`
- All the pairs `[prevCourse_j, nextCourse_j]` are **unique**.
- `time.length == n`
- `1 <= time[i] <= 10^4`
- The given graph is a directed acyclic graph.