2545. Sort the Students by Their Kth Score

`Medium` `Array` `Matrix` `Sorting`

Problem Description

In this problem, we have a class containing m students and n exams. We are presented with a matrix `score` of size `m x n`, where `score[i][j]` represents the score of the `i`th student in the `j`th exam. It's important to note that the scores are distinct, meaning no two students have received the exact same score in an exam.

We are also given an integer `k`, which refers to a specific exam. The task is to sort the students based on their scores in this `k`th exam. The sorting should be done in descending order - the student with the highest score on the `k`th exam should come first, and so on. What we aim to achieve is a new matrix with the same rows as the original `score` matrix, but ordered such that when looking at column `k`, the scores are in decreasing order.

Ultimately, we need to return the matrix once it has been sorted by these criteria.

Intuition

The solution to this problem is relatively straightforward due to Python's powerful sorting functions. We can use the built-in `sorted` function to sort the rows (students) based on their score in the `k`th exam (`k` is a 0-indexed exam number).

To specify the sorting criteria, we provide a lambda function to the `key` argument of `sorted`, which tells it to use the score at index `k` of each row as the key for sorting. Since we want to sort the scores in descending order, we negate the scores by using `-x[k]`. This means that higher scores will be treated as "smaller" by the sorting algorithm because of the negative sign, thereby placing them at the front of the sorted list.

This approach leverages Python's negative value ordering where it treats smaller values (negative values in this case) as having higher priority in the sort order, hence sorting the array in descending order based on the `k`th index.

So, the solution, in essence, is simply a one-liner that returns a new matrix `score` sorted by the score of the `k`th exam for each student, in descending order.

Solution Approach

The implementation is concise because it relies on Python's sorting algorithm, TimSort, an optimized hybrid sorting algorithm derived from merge sort and insertion sort. Here is a step-by-step breakdown:

1. The `sorted` function is called on the matrix `score`. In this case, `score` is a list of lists, with each inner list representing a student's scores across all exams.

2. The `key` parameter of the `sorted` function takes a lambda function. This lambda function `lambda x: -x[k]` is used to determine the sorting order. The `key` parameter defines the basis on which the elements of `score` are sorted. In this instance, the lambda returns the score of the `k`th exam (0-indexed) but negated, so high scores become lower numbers, and because Python sort is ascending by default, this negative transformation ensures that we get a descending order.

3. The inner workings of the `sorted` function then automatically handle the comparison of these keys (i.e., the negated `k`th exam scores) for each row in the matrix and arrange them in ascending order (which, due to the negation, corresponds to descending order of the actual scores).

4. As Python lists are 0-indexed, `k` refers directly to the correct column in the list of lists without requiring any adjustment.

5. The sorted list of lists, i.e., the sorted `score` matrix, is then returned as output. This new matrix has rows ordered by descending scores of the `k`th exam, which is exactly what the problem statement asked for.

Thus, the algorithm makes efficient use of Python's built-in sorting facilities and does not require any additional complex data structures or patterns. It's a simple, elegant solution that takes advantage of the way Python's sort works with keys and negation to achieve the desired descending order.

The full implementation in Python, as provided in the original solution, is as follows:

```
1  class Solution:
2      def sortTheStudents(self, score: List[List[int]], k: int) -> List[List[int]]:
3          return sorted(score, key=lambda x: -x[k])
```

Example Walkthrough

Let's consider a small example to better understand the solution approach described above.

Suppose we have the following matrix `score` where m=3 (students) and n=4 (exams), and each cell `score[i][j]` represents the score of student `i` in exam `j`:

```
1  score = [
2      [60, 85, 75, 90],   # Student 1
3      [88, 92, 67, 75],   # Student 2
4      [85, 85, 85, 88]    # Student 3
5  ]
```

We are also given k=2, which refers to the third exam (remember, indices start at 0).

Our goal is to sort the students based on their scores in the `k`th exam. So we need to look at the scores in exam 3:

```
1  Exam 3 scores: [75, 67, 85]
```

We need to sort these scores in descending order and then rearrange the `score` matrix accordingly.

Now, using the solution approach:

1. We call the `sorted` function on the `score` matrix.
2. The `key` parameter is provided with a lambda function `lambda x: -x[k]`. Since k=2, the lambda function becomes `lambda x: -x[2]`.
3. The `sorted` function applies this key (sorting criteria) to each student's list of exam scores.
4. Negative values are created for the scores of the third exam, which results in the following transformed scores: `[-75, -67, -85]`.
5. These negative values are sorted in ascending order automatically by the `sorted` function, leading to the following order (while preserving the original scores): `[-85, -75, -67]`.
6. This order corresponds to sorting the original scores of the third exam in descending order: `[85, 75, 67]`.
7. Finally, the `score` matrix is reordered to match this sorting, giving us the following result:

```
1  sorted_score = [
2      [85, 85, 85, 88],   # Student 3
3      [60, 85, 75, 90],   # Student 1
4      [88, 92, 67, 75]    # Student 2
5  ]
```

Here, if you look at the third column, which corresponds to exam 3, you'll see the scores are sorted in descending order: 85, 75, 67.

The code for this in Python, using the provided solution approach, would look like this:

```
1  score = [
2      [60, 85, 75, 90],
3      [88, 92, 67, 75],
4      [85, 85, 85, 88]
5  ]
6  k = 2
7
8  sorted_score = sorted(score, key=lambda x: -x[k])
9  print(sorted_score)
```

The output of this code snippet will be the sorted matrix:

```
1  [[85, 85, 85, 88],
2   [60, 85, 75, 90],
3   [88, 92, 67, 75]]
```

As you can see, the students are now sorted by their scores in the third exam, in descending order, as required.

Python Solution

```
1  from typing import List
2
3  class Solution:
4      def sortTheStudents(self, scores: List[List[int]], k: int) -> List[List[int]]:
5          # Sort the students by the k-th score in descending order.
6          # The lambda function extracts the k-th element from each sub-list for comparison.
7          # The minus sign '-' is used to sort in descending order.
8          sorted_scores = sorted(scores, key=lambda student_scores: -student_scores[k])
9          return sorted_scores
10
```

Java Solution

```
1  import java.util.Arrays; // Import Arrays class for sorting functionality
2
3  class Solution {
4
5      /**
6       * Sorts the students based on the scores in a specific column.
7       *
8       * @param scores A 2D array representing the scores of the students. Each row is a student, and each column is a score for a different
9       * @param k The index of the score based on which the sorting should be performed.
10      * @return The sorted 2D array of students' scores.
11      */
12      public int[][] sortTheStudents(int[][] scores, int k) {
13          // Sort the 2D array 'scores' using a custom comparator
14          // that compares the elements in column 'k' in descending order.
15          Arrays.sort(scores, (a, b) -> b[k] - a[k]);
16          return scores; // Return the sorted array.
17      }
18  }
19
```

C++ Solution

```
1  #include <vector> // Include the necessary header for using vectors
2  #include <algorithm> // Include the algorithm header for the sort function
3
4  class Solution {
5  public:
6      // Function to sort the students based on their scores at index 'k'
7      // scores: A 2D vector containing students and their scores
8      // k: The index of the score to sort by
9      // Returns: The sorted 2D vector of students
10     vector<vector<int>> sortTheStudents(vector<vector<int>>& scores, int k) {
11
12         // Use the standard sort function to reorder the 'scores' vector
13         // Sort is done in descending order based on the value at index 'k'
14         sort(scores.begin(), scores.end(), [k](const auto& a, const auto& b) {
15             return a[k] > b[k]; // Lambda function to compare scores at index 'k'
16         });
17
18         return scores; // Return the sorted vector
19     }
20 };
21
```

Typescript Solution

```
1  /**
2   * Sorts a matrix of student scores based on a particular score index.
3   * @param {number[][]} scores - A matrix where each row represents a student's scores.
4   * @param {number} scoreIndex - The index of the score to sort the students by.
5   * @returns {number[][]} - The sorted matrix with students ordered by the score at the given index.
6   */
7  function sortTheStudents(scores: number[][], scoreIndex: number): number[][] {
8      // Sorts the array of scores in descending order based on the score at scoreIndex within each student's array.
9      return scores.sort((studentA, studentB) => studentB[scoreIndex] - studentA[scoreIndex]);
10 }
11
```

Time and Space Complexity

The time complexity of the provided sorting algorithm is $O(n \log n)$, where n is the number of items in the `score` list. This is because the built-in `sorted` function in Python uses an algorithm called Timsort, which has this time complexity on average.

The space complexity of the provided code is $O(n)$, due to the space required to hold the new sorted list that is returned by the `sorted` function. The original list is not modified in place; instead, a new list is created to hold the sorted result.