

# 481. Magical String

MediumTwo PointersString

Leetcode Link

## Problem Description

The problem presents a unique sequence called a "magical string" that is built based on its own sequence of numbers. We start with a string `s` that only contains the characters `'1'` and `'2'`. The magic is that when we group the characters in `s` by the count of consecutive `'1'`s and `'2'`s, these grouped counts form the same string `s`.

As an example, we start with the known sequence: "12211221221121122...". Grouping the consecutive characters yields "1 22 11 2 1 22 1 22 11 2 11 22 ...", and then the counts of `'1'`s and `'2'`s in these groups are "1 2 2 1 1 2 1 2 2 1 2 2 ...", which is the same as the original sequence `s`.

The task is to determine the count of `'1'`s in the first `n` characters of the magical string `s`. Specifically, given an integer `n`, return how many `'1'`s appear in the initial segment of length `n` in the string `s`.

## Intuition

To solve this problem, we don't need to generate the entire string, which could be very long. Instead, we can build the string `s` only as far as necessary to count the number of `'1'`s within the first `n` characters.

The solution uses a list `s` to simulate the magical string and a pointer `i` to keep track of where in `s` we are currently looking to determine how many times to repeat the next character. We begin with the known start of the magical string as `[1, 2, 2]`.

The core idea is to iteratively extend the magical string based on its definition. For each step, we look at the value of `s[i]`, which tells us how many times to repeat the next character. We alternate the characters to add based on the last character in the string: if it's a `'1'`, we add `'2'`s; if it's a `'2'`, we add `'1'`s. The number of characters to add is equal to the current value of `s[i]`.

The use of `pre` helps us know what the last character was (either `'1'` or `'2'`), and `cur` is used to determine what the next character should be, by switching between `'1'` and `'2'`.

We continue extending the string until its length is at least `n`. Once the length of `s` reaches `n`, we simply take the first `n` elements of `s` and count the number of `'1'`s to get our answer.

## Solution Approach

The implementation relies on a simple Python list `s` to simulate the construction of the magical string. An integer `i` serves as a pointer that moves through the list, indicating how many times the next character should be repeated. Here's how the implementation unfolds:

- We initialize the representation of the magical string as `s = [1, 2, 2]`. This is because the sequence always starts with "122".
- We set the pointer `i = 2`. This is because `s[2]` points to the second `'2'` in the initial list, which indicates that the next sequence to be added should consist of two numbers. The pointer `i` will indicate which number in `s` we should look at to determine the next sequence to append to `s`.
- We then enter a while loop which will run as long as the length of `s` is smaller than `n`, ensuring that we build enough of the magical string to count the number of `'1'`s in the first `n` characters.
- Inside the loop, we first store the last element of the current string in the variable `pre`. This is either a 1 or a 2, and it represents the last character in the string.
- Next, we calculate `cur` as `3 - pre`. Since we only have `'1'` and `'2'` in our sequence, if the last character (`pre`) is `'1'`, `cur` will be `'2'` (since `3 - 1 = 2`), and if it's `'2'`, `cur` will be `'1'`.
- The next step is to extend the list `s` by adding `cur` repeated `s[i]` times. This is akin to saying "if we have a `'2'` in `s[i]`, and `cur` is `'1'`, then append `'1'` to `s` two times." We append `[cur] * s[i]` to `s`.
- We then increment `i` by 1 to move the pointer to the next character in `s`.
- Once we exit the while loop, it means we've built a section of the magical string that is at least as long as `n`. The last step is to return the count of `'1'`s in the first `n` characters of `s` by using `s[:n].count(1)`.

This approach is efficient because it constructs only as much of the magical string as is necessary to determine the number of `'1'`s within the first `n` elements. It uses simple list operations and arithmetic to achieve this task.

## Example Walkthrough

Let's walk through a small example to illustrate the solution approach. Suppose we are asked to find the number of `'1'`s in the first 10 characters of the magical string `s`.

- We start off by initializing our list `s` with the known beginning of the magical string: `[1, 2, 2]`.
- The pointer `i` is initially set to 2, since `s[2]` points to `'2'`, which will determine what we append next to `s`.
- Our `while` loop continues as long as the length of `s` is less than 10 (the `n` value in our example). At the start, the length of `s` is 3, so we enter the loop.
- The last element, `pre`, of the current string `s` is 2.
- We calculate `cur` as `3 - pre`. Since `pre` is 2, `cur` becomes 1.
- We extend `s` by adding `cur` repeated `s[i]` times. In this case, since `s[i]` is 2 and `cur` is 1, we append two `'1'`s to `s`, resulting in `s` becoming `[1, 2, 2, 1, 1]`.
- We then increment `i` by 1, moving the pointer to the next character in `s`, so `i` is now 3.
- Now, `pre` is 1 (the last element in `s`), so now `cur` will be `3 - pre`, which is 2. `s[i]` is 1, so we append one `'2'` to `s`, changing `s` to `[1, 2, 2, 1, 1, 2]`.
- We repeat steps 4 through 7, with `i` now at 4. `s[i]` is 1, the current `pre` is 2, so `cur` is 1. We add one `'1'` to `s` to get `[1, 2, 2, 1, 1, 2, 1]`.
- Continuing this process, we increment `i`, calculate `cur`, and extend `s` until `s` is at least 10 characters long. After a few iterations, `s` is `[1, 2, 2, 1, 1, 2, 1, 2, 1, 2, 1, 2, 1]`, and the length of `s` is 10.
- The loop stops since `s` now has a length of 10. We count the number of `'1'`s in `s[:10]`, which is 5.

Thus, for `n = 10`, our function would return 5, as there are five `'1'`s in the first 10 characters of the magical string `s`.

## Python Solution

```
1 class Solution:
2     def magicalString(self, n: int) -> int:
3         # Initialize the magical string with the known starting sequence
4         magical_str = [1, 2, 2]
5
6         # Use index to track the position in the string for generating the next elements
7         index = 2
8
9         # Generate the magical string until its length is at least 'n'
10        while len(magical_str) < n:
11            # Get the last value in the magical string
12            last_value = magical_str[-1]
13
14            # The current value to be appended is the "opposite" of the last value (1 switches to 2, and 2 switches to 1)
15            current_value = 3 - last_value
16
17            # Append 'current_value' to the list as many times as the value at the current 'index'
18            magical_str.extend([current_value] * magical_str[index])
19
20            # Move to the next index
21            index += 1
22
23        # Slice the magical string until 'n' and count the occurrences of '1'
24        return magical_str[:n].count(1)
25
```

## Java Solution

```
1 class Solution {
2     public int magicalString(int n) {
3         // Initialize the magical string as a list with the first three numbers
4         List<Integer> magicalStr = new ArrayList<>(Arrays.asList(1, 2, 2));
5
6         // Use a pointer to iterate through the magical string to generate the next numbers
7         int i = 2; // Starting from index 2 because the first three numbers are already in the list
8         while (magicalStr.size() < n) {
9             int lastNum = magicalStr.get(magicalStr.size() - 1); // Get the last number in the current magical string
10            int nextNum = 3 - lastNum; // Calculate the next number (if lastNum is 1, then nextNum is 2; if lastNum is 2, then nextNum is 1)
11
12            // Add 'nextNum' to the magical string 's.get(i)' times as per the current number's frequency
13            for (int j = 0; j < magicalStr.get(i); ++j) {
14                magicalStr.add(nextNum);
15            }
16
17            i++; // Move to the next number
18        }
19
20        // Count the number of occurrences of 1 in the first 'n' elements of the magical string
21        int countOnes = 0;
22        for (int idx = 0; idx < n; ++idx) {
23            if (magicalStr.get(idx) == 1) {
24                countOnes++;
25            }
26        }
27
28        // Return the count of 1's in the first 'n' elements of the magical string
29        return countOnes;
30    }
31 }
32
```

## C++ Solution

```
1 #include <vector>
2 #include <algorithm>
3
4 class Solution {
5 public:
6     int magicalString(int n) {
7         std::vector<int> magical_seq = {1, 2, 2}; // Initialize the magical sequence with its first three elements
8
9         // Use 'index' to iterate through the magical sequence
10        // The loop continues until the size of magical_seq is at least n
11        for (int index = 2; magical_seq.size() < n; ++index) {
12            int last_number = magical_seq.back(); // Get the last number in the current sequence
13            int next_number = 3 - last_number; // Determine the next number to add, which will be 1 if the last is 2, and 2 if the last is 1
14
15            // Append the next number to the sequence s[i] times where s[i] is the current element at position i
16            // This loop controls the count of the next number to be appended
17            for (int count = 0; count < magical_seq[index]; ++count) {
18                magical_seq.emplace_back(next_number); // Append the number to the end of the sequence
19            }
20        }
21
22        // Count the number of 1's up to the nth element and return that count
23        return std::count(magical_seq.begin(), magical_seq.begin() + n, 1);
24    }
25 };
26
```

## Typescript Solution

```
1 function magicalString(n: number): number {
2     // Initialize the magical string with the known beginning sequence
3     let magicalStr = [...'1221121'];
4     // Initialize the index for counting group occurrences
5     let readIndex = 5;
6
7     // Generate the magical string up to the required length 'n'
8     while (magicalStr.length < n) {
9         // Get the last character of the current magical string
10        const lastChar = magicalStr[magicalStr.length - 1];
11        // Append the opposite character to the string ('1' becomes '2', and '2' becomes '1')
12        magicalStr.push(lastChar === '1' ? '2' : '1');
13        // If the current read index character is '2', repeat the action once more
14        if (magicalStr[readIndex] !== '1') {
15            magicalStr.push(lastChar === '1' ? '2' : '1');
16        }
17        // Move to the next index
18        readIndex++;
19    }
20
21    // Calculate the number of '1's in the first 'n' characters of the magical string
22    return magicalStr.slice(0, n).reduce((count, char) => count + (char === '1' ? 1 : 0), 0);
23 }
24
```

## Time and Space Complexity

### Time Complexity

The time complexity of the function primarily comes from the while loop that generates the magical string until its length is at least `n`. In each iteration, the loop appends up to `s[i]` elements to the list `s` where `s[i]` could be either `1` or `2`. This results in a maximum of `2` additions per iteration. However, since each iteration of the loop adds at least one element, and we iterate until we have `n` elements, the overall time complexity can be approximated as  $O(n)$ .

### Space Complexity

The space complexity of this function is also defined by the size of the list `s`, which grows to match the input `n` in the worst-case scenario. Since we need to store each element of the magical string up to the `n`th position, the space complexity is  $O(n)$ .