2303. Calculate Amount Paid in Taxes

Easy Array Simulation

LeetCode Link

Problem Description

The given LeetCode problem involves calculating the amount of taxes that need to be paid based on a set of tax brackets. Each bracket is defined by two parameters: `upper`, which is the upper bound of taxable income for that bracket, and `percent`, which is the tax rate for that bracket. The `brackets` array contains these pairs in ascending order of `upper`.

To calculate the total tax:

- For the first bracket, tax is applied on the entire income if the income is less than or equal to `upper[0]`, otherwise, tax is applied only up to `upper[0]`.
- For each subsequent bracket, tax is applied on the difference between the current bracket's `upper` and the previous bracket's `upper`, but only if the income is more than the previous `upper`. This continues until all the income is taxed or until all the brackets are exhausted.

The goal is to find out the total tax owed based on the income provided, where `income` is the total amount of money earned by an individual.

Intuition

The intuition behind the solution is to iterate through the tax brackets and calculate the taxes for each bracket incrementally. Since the income can span multiple brackets, the approach must consider partial amounts taxed at different brackets.

To implement this, we keep track of the previous bracket's upper bound as we iterate through the brackets so that we can calculate the taxable amount within the current bracket. The key steps in the solution are as follows:

1. Initialize `ans` as the accumulator for the total tax and `prev` to keep the upper bound of the previous bracket, starting with `0` for the initial condition.
2. Iterate over each bracket in the `brackets` list, which has `upper` and `percent` values.
3. For each bracket, calculate the amount of income that falls within this bracket. This is the minimum of the actual income and the current bracket's `upper` minus `prev`, which signifies the taxed amount in the previous brackets.
4. Calculate the tax for this bracket by multiplying the bracket's taxable income by the `percent`.
5. Update `prev` to be the current bracket's `upper` so that it can be used in the next iteration.
6. The tax rates are provided in percentages, so divide the final answer by `100` to get the actual tax amount.
7. Continue this process until all the brackets are covered or until the entire income is taxed.

The provided code implements this logic correctly and calculates the tax in an efficient manner.

Solution Approach

The algorithm for calculating taxes based on tax brackets is straightforward. The solution takes advantage of the sorted order of the tax brackets, following these steps:

1. Initialize two variables: `ans` to keep track of the total taxes paid so far, which starts at `0`, and `prev` to track the upper bound of the previous bracket, starting with `0` since there's no previous bracket at the beginning.

2. Loop through each bracket in the `brackets` array, extracting `upper`, the upper bound for the bracket, and `percent`, the tax rate for that bracket.

3. In each iteration, calculate the taxable amount for the current bracket. This is done by subtracting `prev` from the minimum of `income` and the current `upper`. The subtraction of `prev` ensures only the income falling within the current bracket is considered. This is represented by the following expression:

```
taxable_amount = max(0, min(income, upper) - prev)
```

4. Calculate the tax for the current bracket by multiplying the `taxable_amount` by the `percent` rate. Since `percent` represents a percentage, you'll need to multiply the tax amount by the rate and then divide by 100 to convert it into the actual tax value. The tax for the current bracket is added to the `ans` accumulator:

```
ans += taxable_amount * percent
```

5. Once the tax for this bracket is calculated, update `prev` to the current bracket's `upper`, which will be used in the next iteration to calculate the next bracket's taxable income.

```
prev = upper
```

6. After completing the loop over all brackets, the final tax value stored in `ans` is divided by `100` to adjust for the percentage calculation:

```
return ans / 100
```

The use of $\max(0, \min(\text{income}, \text{upper}) - \text{prev})$ ensures that the solution also handles cases where the `income` does not reach the current bracket's `upper`. It also handles cases where the `income` is exactly on the `upper` of the previous bracket, thus not bleeding into the current tax bracket's range.

No additional data structures are needed, as the algorithm only requires simple variable assignments and arithmetic operations. The pattern followed is iterative, straightforward, and efficient, as it only requires one pass through the brackets array, making the complexity of this algorithm proportional to the number of brackets ($O(n)$ time complexity, where n is the number of brackets).

This solution effectively calculates the total taxes for a given income according to the provided set of tax brackets.

Example Walkthrough

Let's walk through a small example to illustrate the solution approach. Suppose we have the following set of brackets:

```
[[10000, 10], [20000, 20], [30000, 30]]
```

This example states that:

- Tax is 10% on income up to $10,000,
- Tax is 20% on income between $10,000 and $20,000,
- Tax is 30% on income between $20,000 and $30,000.

Let's calculate the total tax for an income of $25,000 using the algorithm outlined in the solution approach:

1. Initialize `ans` to 0 and `prev` to 0.
2. Start looping through the brackets.

- **First bracket**: (10000, 10)

  The taxable income for the first bracket is $\min(25000, 10000) - 0 = 10000$.

  The tax for the first bracket is $10000 * 10\% = 1000$.

  Update `ans` by 1000 (now `ans` = 1000) and `prev` by the `upper` of the current bracket (now `prev` = 10000).

- **Second bracket**: (20000, 20)

  The taxable income for the second bracket is $\min(25000, 20000) - 10000 = 10000$.

  The tax for the second bracket is $10000 * 20\% = 2000$.

  Update `ans` by 2000 (now `ans` = 3000) and `prev` by the `upper` of the current bracket (now `prev` = 20000).

- **Third bracket**: (30000, 30)

  Since the income is 25,000, *which is less than the 'upper' of* 30,000, the calculation here is on the remaining income. The taxable income for the third bracket is $\min(25000, 30000) - 20000 = 5000$.

  The tax for the third bracket is $5000 * 30\% = 1500$.

  Update `ans` by 1500 (now `ans` = 4500). There's no need to update `prev` since we've already covered all the income.

3. After all the brackets are processed, divide `ans` by 100 to adjust the percentage calculation. Therefore, $\text{ans}/100 = 4500/100 = 45$.

So the total tax on an income of 25, 000 *with the given tax brackets is* 45. The algorithm correctly breaks down the income into portions that fall within each tax bracket, calculates the tax on each portion accordingly, and sums up the taxed amounts to determine the total tax liability.

Python Solution

```python
class Solution:
    def calculateTax(self, brackets: List[List[int]], income: int) -> float:
        # Initialize the total tax amount to 0
        total_tax = 0

        # Initialize 'previous_upper_bound' which will hold the upper bound of the previous bracket
        previous_upper_bound = 0

        # Loop through the collected tax brackets
        for upper_bound, tax_rate in brackets:
            # Calculate the taxable income for the current bracket.
            # `max(0, min(income, upper_bound) - previous_upper_bound)` ensures that the income does not exceed the current bracket's
            # and that income is not taxed twice for the lower brackets.
            taxable_income = max(0, min(income, upper_bound) - previous_upper_bound)

            # Calculate the tax for the current bracket and accumulate it into 'total_tax'
            total_tax += taxable_income * tax_rate

            # Update 'previous_upper_bound' to the current bracket's upper bound for the next iteration
            previous_upper_bound = upper_bound

        # Divide the 'total_tax' by 100 to convert the tax rate to a percentage and return the result
        return total_tax / 100
```

Java Solution

```java
class Solution {
    public double calculateTax(int[][] brackets, int income) {
        // `taxAmount` will store the calculated tax based on brackets.
        int taxAmount = 0;

        // `previousBracketUpperLimit` holds the upper limit of the previous tax bracket.
        int previousBracketUpperLimit = 0;

        for (int[] bracket : brackets) {
            // Each bracket contains an upper limit and the tax rate percent for the bracket range.
            int currentBracketUpperLimit = bracket[0];
            int taxRatePercent = bracket[1];

            // Calculate the taxed income at this bracket by taking the lesser of
            // income or the bracket's upper limit minus the previous bracket's upper limit.
            // This is the amount of income that falls within the current bracket's range.
            int taxedIncomeAtCurrentBracket = Math.max(0, Math.min(income, currentBracketUpperLimit)
                                              - previousBracketUpperLimit);

            // Update the total taxAmount with the tax from this bracket's range.
            taxAmount += taxedIncomeAtCurrentBracket * taxRatePercent;

            // Update the previousBracketUpperLimit for the next iteration.
            previousBracketUpperLimit = currentBracketUpperLimit;
        }

        // Convert the taxAmount to dollars and cents (as the percent was in whole number).
        return taxAmount / 100.0;
    }
}
```

C++ Solution

```cpp
class Solution {
public:
    // Function to calculate the tax based on given brackets and income
    double calculateTax(vector<vector<int>>& brackets, int income) {
        int tax = 0; // Variable to store the total tax
        int prevBracketUpperLimit = 0; // Variable to keep track of the previous bracket upper limit

        // Iterate over the tax brackets
        for (const auto& bracket : brackets) {
            int currentBracketUpperLimit = bracket[0]; // Upper limit for the current tax bracket
            int taxRate = bracket[1]; // Tax rate for the current bracket

            // Calculate the tax for the income falling within the current tax bracket range
            // max(0, ...) ensures we don't get negative values in case income is less than the previous bracket
            // min(income, currentBracketUpperLimit) ensures we don't calculate tax for income beyond the current bracket
            // Subtracting prevBracketUpperLimit gives us the taxable amount in the current bracket
            tax += max(0, min(income, currentBracketUpperLimit) - prevBracketUpperLimit) * taxRate;

            prevBracketUpperLimit = currentBracketUpperLimit; // Update the previous bracket upper limit for the next iteration

            // If income is less than the current bracket upper limit, no need to continue
            if (income < currentBracketUpperLimit) {
                break;
            }
        }

        // Tax rates are given in percentage, divide by 100 to get the actual tax amount
        return tax / 100.0;
    }
};
```

Typescript Solution

```typescript
/**
 * Calculates the total tax based on tax brackets.
 * Each tax bracket specifies the upper limit and the tax rate.
 * Income is taxed in a progressive manner according to these brackets.
 *
 * @param brackets - An array of arrays where each inner array contains 2 numbers:
 *                   the upper limit and the tax rate (as a percentage) for that bracket.
 * @param income - The total income to calculate tax for.
 * @returns The total tax calculated based on the brackets.
 */
function calculateTax(brackets: number[][], income: number): number {
    let totalTax = 0; // Stores the cumulative tax amount
    let previousBracketUpper = 0; // The upper limit of the previous bracket

    // Loop over each bracket
    for (const [currentBracketUpper, taxRatePercent] of brackets) {
        // Calculate the taxable income for the current bracket
        const taxableIncome = Math.max(0, Math.min(income, currentBracketUpper) - previousBracketUpper);

        // Calculate the tax for the current bracket and add it to the total tax
        totalTax += taxableIncome * taxRatePercent;

        // Update the previous bracket upper limit for the next iteration
        previousBracketUpper = currentBracketUpper;
    }

    return totalTax / 100; // Convert percentage to a decimal representation
}
```

Time and Space Complexity

The provided code snippet is designed to calculate taxes based on various tax brackets and the given income. Here is the analysis of both the time complexity and space complexity of the code:

Time Complexity

The time complexity of the `calculateTax` function is $O(n)$, where n is the number of tax brackets. This is because the function contains a loop that iterates through each bracket exactly once.

```
for upper, percent in brackets:
    ans += max(0, min(income, upper) - prev) * percent
    prev = upper
```

Inside the loop, operations are performed in constant time, including comparisons, arithmetic operations, and variable assignments.

Space Complexity

The space complexity of the `calculateTax` function is $O(1)$. The algorithm uses a fixed amount of extra space for variables `ans` and `prev`. No additional space which grows with the input size is utilized, as there are no data structures dependent on the size of the input.

```
ans = prev = 0
```

It's important to note that the input `brackets`, which is provided to the function, does not count towards the space complexity as it is considered input to the function and not extra space used by the function itself.