1558. Minimum Numbers of Function Calls to Make Target Array

Medium | Greedy | Bit Manipulation | Array | Leetcode Link

Problem Description

You are provided with two integer arrays. The first one is called `nums`, and it contains some integers. The second one is called `arr`, which is the same length as `nums` but initially filled with `0`s. Your goal is to make `arr` look exactly like `nums`.

To achieve your goal, you are allowed to use a special `modify` function, which can perform one of two operations:

1. Increment by 1: Choose any index `i` and increase the value of `arr[i]` by 1.
2. Double all: Double the value of each element in the array `arr`.

Your task is to figure out the minimum number of calls to the `modify` function required to transform `arr` into `nums`. The end goal is to achieve this transformation with the smallest possible number of operations.

Intuition

The solution strategy revolves around working backwards from `nums` to `arr`, as decreasing or halving elements is easier to track than incrementing or doubling. The insight is that any number in `nums` is formed by starting with 0, potentially doubling several times, and then incrementing.

To minimize calls to `modify`, we should maximize the use of the doubling operation, because each doubling operation is equivalent to many increment operations. Thus, the strategy is to:

1. Find the total number of increment operations needed for each number in `nums` by counting the number of set bits (since each set bit represents an increment operation that has occurred).
2. Find the maximum number of doubling operations needed. This is achieved by determining the highest power of 2 required to reach the maximum number in `nums`, which is equivalent to the bit length of the maximum number minus 1 (since starting from 0 and doubling does not count as an operation).

By combining the count of set bits across all numbers with the maximum bit length, we arrive at the minimum number of operations needed to transform `arr` into `nums`.

Solution Approach

The solution uses simple bitwise operations to find the answer. Here's how the implementation goes by breaking it down into logical steps:

- Iterate through each number in the given `nums` array to calculate the total number of increment operations for each number and the highest number of doubling operations for the whole array.

- For each number, we use the bitwise operation `bit_count()`, which returns the count of set bits (1-bits) in the number. The count of set bits actually represents how many times the increment operation has been performed to reach this number from 0 because each increment operation can only set a bit from 0 to 1.

- To get the maximum number of doubling operations across all numbers in the array, we look for the maximum value in `nums`. We call `bit_length()` on this number, which gives us the position of the highest set bit, i.e., the number of times we can potentially divide the number by 2 until it reaches 0. This represents the doubling operations needed to reach this number from 1. Since we start with 0, not 1, we subtract 1 from the bit length as the initial state does not count as a doubling operation.

- The total minimum number of function calls is the sum of all increment operations (`sum(v.bit_count() for v in nums)`) and the maximum number of doubling operations (`max(0, max(nums).bit_length() - 1)`). We also ensure that the number of doubling operations is not negative (which can happen if `nums` contains all zeros), by taking `max(0, ...)`.

The Python implementation provided uses a list comprehension to calculate the total count of bits for all numbers in `nums` and finds the maximum bit length among all numbers. Then it adds these two values to get the final answer.

Here is what the code does, broken down step by step:

1. `sum(v.bit_count() for v in nums)` - This part goes through every number in `nums` and accumulates the total increment operations required by summing up the number of set bits in each of the numbers.

2. `max(nums).bit_length()` - This part finds the maximum number in `nums` and then calculates the length of the bits needed to represent that number, which corresponds to the number of doubling operations needed plus one (since index starts at zero).

3. Subtracting 1 from the bit length - We subtract 1 because the sequence starts from 0 and doubling from 0 does not change the number, so the first doubling operation effectively starts when going from 1 to 2.

Therefore, by combining these steps, the implementation efficiently computes the minimum number of operations with a time complexity of O(n) where n is the length of the `nums` array.

Example Walkthrough

Let's assume we have the following input:

`nums` array: `[3, 8, 1]`

We want to convert the `arr` array: `[0, 0, 0]` to look exactly like `nums`.

Now, let's walk through the solution approach for this example:

1. **Increment Operations**:
   - For the number 3, it has a binary representation of `11`, which has 2 set bits.
   - For the number 8, it has a binary representation of `1000`, which has 1 set bit.
   - For the number 1, it has a binary representation of `1`, which has 1 set bit.

   So the total number of increment operations needed is `2 (for 3) + 1 (for 8) + 1 (for 1) = 4`.

2. **Doubling Operations**:
   - The maximum number in `nums` is 8, which has a binary representation of `1000`. The bit length is 4. Therefore, we would need 3 doubling operations to get from `1` to `8`. Since we start from `0`, we subtract one, leaving us with `4 - 1 = 3` doubling operations.

Now, combining both the increment and doubling operations:

- Total increment operations = 4 (from step 1)
- Total doubling operations = 3 (from step 2)

Therefore, the minimum number of function calls required to transform `arr` into `nums` is `4 + 3 = 7`.

This walk through shows that we need 7 `modify` operations to make `arr` look exactly like `nums`, which are `[3, 8, 1]`, by incrementing and doubling appropriately.

Python Solution

```python
class Solution:
    def minOperations(self, nums: List[int]) -> int:
        # Calculate the total number of set bits (1s) in all the numbers.
        # This corresponds to the number of increment operations needed.
        num_of_increment_ops = sum(bin(num).count('1') for num in nums)

        # Find the maximum number in the list to determine the number of
        # double operations needed.
        # We need to find the position of the highest set bit (most significant bit),
        # which determines the number of times we need to double from 1 to reach
        # that value. This is equivalent to the bit length of the maximum number minus 1
        # since starting from 1 (2^0) requires no doubling operations.
        highest_num = max(nums) if nums else 0
        num_of_double_ops = highest_num.bit_length() - 1 if highest_num > 0 else 0

        # The total number of operations is the sum of the increment and double operations.
        return num_of_increment_ops + num_of_double_ops

```

Java Solution

```java
class Solution {

    /**
     * Method to calculate minimum operations to make all elements of nums equal to zero.
     * An operation is defined as either:
     *   1) Subtract 1 from an element if it's not 0 or,
     *   2) Divide all elements by 2 if all elements are even
     *
     * @param nums array of integers
     * @return the minimum number of operations required
     */
    public int minOperations(int[] nums) {
        int totalOperations = 0; // to track the total minimum operations
        int maxNumber = 0; // to track the maximum number in the array

        // Calculate the sum of bit counts (number of 1's) for all numbers
        // and find the maximum number simultaneously
        for (int number : nums) {
            maxNumber = Math.max(maxNumber, number);
            totalOperations += Integer.bitCount(number); // add number of 1's in binary representation
        }

        // Add the length of the binary string of the maximum number minus 1
        // which accounts for the division by 2 operations
        totalOperations += Integer.toBinaryString(maxNumber).length() - 1;

        return totalOperations;
    }
}

```

C++ Solution

```cpp
#include <vector>
#include <algorithm> // For std::max

class Solution {
public:
    int minOperations(std::vector<int>& nums) {
        int operationsCount = 0; // Initialize counter for minimum operations
        int maxNum = 0; // Variable to store the maximum value in nums

        // Loop through all numbers in the vector
        for (int value : nums) {
            // Update maxNum to the maximum value encountered so far
            maxNum = std::max(maxNum, value);
            // Add the number of set bits (1s) in the current value to operationsCount
            operationsCount += __builtin_popcount(value);
        }

        // If maxNum is greater than 0, add the number of bits to reach the most significant bit
        // of the largest number in nums (the number of left shifts needed for the largest number)
        if (maxNum) {
            operationsCount += 31 - __builtin_clz(maxNum);
        }

        // Return the total count of operations needed
        return operationsCount;
    }
};

```

Typescript Solution

```typescript
// Importing Array object
import { max } from 'lodash';

let operationsCount: number = 0; // Initialize counter for minimum operations
let maxNum: number = 0; // Variable to store the maximum value in nums

/**
 * Calculate the minimum number of operations to make all element of nums equal to zero.
 * @param {number[]} nums - The array of numbers to be processed.
 * @return {number} The minimum number of operations required.
 */
function minOperations(nums: number[]): number {
    operationsCount = 0; // Reset the counter for each function call
    maxNum = 0;

    // Loop through all numbers in the array
    for (let value of nums) {
        // Update maxNum to the maximum value encountered so far
        maxNum = max([maxNum, value]) || 0;
        // Add the number of set bits (1s) in the current value to operationsCount
        operationsCount += countSetBits(value);
    }

    // If maxNum is greater than 0, add the number of bits to reach the most significant bit
    // (the number of left shifts needed for the largest number)
    if (maxNum) {
        operationsCount += 31 - clz(maxNum);
    }

    // Return the total count of operations needed
    return operationsCount;
}

/**
 * Count the number of set bits in an integer (Naive method).
 * @param {number} n - The number in which set bits are to be counted.
 * @return {number} The count of set bits in n.
 */
function countSetBits(n: number): number {
    let count: number = 0;
    while (n) {
        count += n & 1; // Increment count if the least significant bit is set
        n = n >>> 1; // Right shift n by 1 bit to process the next bit
    }
    return count;
}

/**
 * Compute the number of leading zero bits in the integer's binary representation.
 * @param {number} num - The number whose leading zeros are to be counted.
 * @return {number} The count of leading zero bits in num.
 */
function clz(num: number): number {
    if (num === 0) return 32;
    let leadingZeros = 0;
    for (let i = 31; i >= 0; i--) {
        if ((num & (1 << i)) === 0) {
            leadingZeros++;
        } else {
            break;
        }
    }
    return leadingZeros;
}

```

Time and Space Complexity

The time complexity of the code can be calculated based on two operations: the calculation of bit counts for all numbers and finding the maximum number to calculate its bit length. Calculating the bit count for a single number is `O(1)`, and we do this for every number in the list, resulting in `O(n)` complexity where `n` is the length of the list. Finding the maximum value in the list also takes `O(n)` time. The `bit_length` function is also `O(1)` as it typically involves calculating the position of the highest bit set in the integer representation, which does not depend on the size of the number itself but on the number of bits.

Thus, the time complexity of the function is `O(n)` where `n` is the size of the `nums` list.

As for the space complexity, since no additional significant space is used apart from a few variables to store intermediate results, the space complexity is `O(1)`.

In summary:

- Time Complexity: `O(n)`
- Space Complexity: `O(1)`