2844. Minimum Operations to Make a Special Number

Medium Strategy Math String Enumeration

Problem Description

The problem requires us to transform a given string `num` that represents a non-negative integer into a special number using a series of operations. A number is deemed special if it's divisible by 25. Every operation involves choosing and deleting any single digit from `num`, and this process can be done as many times as needed. If we end up deleting all digits, the string `num` becomes `0`, which is a valid special number since 0 is divisible by 25. The ultimate goal is to find the minimum number of operations that make `num` a special number.

Intuition

To find the minimum number of operations needed, we need to think about the properties of numbers divisible by 25. Any number divisible by 25 ends with '00', '25', '50', or '75'. Therefore, our task is to rearrange the digits of `num` by deleting some of them to get a number ending with these two digits.

Let's focus on the two least significant digits in our operations because any digit(s) before them won't affect divisibility by 25 (e.g., 125, 225, 1025 - all end with 25 and thus are special). For `num` to be special, it must end with any of the pairs mentioned. With this in mind, we look for the occurrence of these pairs within `num`, and aim to remove as few digits as possible to form a special number.

We can achieve this using a depth-first search (DFS) strategy. We explore each position `i` in the string `num`, where we either:

1. Skip the current digit (which is equivalent to removing it) and recursively continue with the next digit.
2. Take the current digit into consideration, add it to our forming number and check the number's remainder when divided by 25.

Using this approach, we can recursively calculate the minimum operations needed and employ memoization to avoid recalculations (utilize previously solved sub-problems). Memoization is applied through the `@cache` decorator on the DFS function, which stores results of function calls with particular arguments for fast retrieval.

The recursion continues until we've checked every digit in `num` (`i == n`), at which point, if we've formed a number divisible by 25 (`k == 0`), we need 0 more operations, otherwise, we haven't formed a special number and return a high operation count (`n`). The minimum of these recursive calls at each step gives the least operations needed.

Solution Approach

The solution makes use of a recursive depth-first search (DFS) combined with memoization to optimize the computation. DFS is a powerful algorithmic technique for exploring all the options in a decision tree-like structure, one path at a time, until a goal state is reached.

The core function `dfs(i: int, k: int) -> int` plays a crucial role in this approach:

- `i` tracks the current digit's index within `num`.
- `k` is the current number formed by the selected digits from `num`, modulo 25. We use modulo 25 because we only care if the number formed is divisible by 25, not the actual number.

The base case for the recursion occurs when `i` equals the length of `num` (`n`). If `k` is 0 at this point, we've successfully formed a special number, so no more operations are needed, and 0 is returned. Otherwise, we return `n`, which is a large number signifying an unsuccessful attempt to form a special number (since we cannot perform more operations than the number of digits).

Within `dfs`, we consider two options at each digit:

1. Skip the current digit: We call `dfs(i + 1, k)` to continue to the next index while keeping the current value of `k`. We also add 1 to the result because skipping the digit is an operation.
2. Include the current digit: We call `dfs(i + 1, (k * 10 + int(num[i])) % 25)` to check if adding the current digit (`int(num[i])`) brings us closer to making the number special. We perform modulo 25 on the result to update `k`.

Then, we use the `min` function to choose the option with the fewer operations. Memoization is applied using the `@cache` decorator from Python's standard library, which effectively caches the results of the `dfs` function calls with specific `(i, k)` pairs.

The algorithm initiates with `dfs(0, 0)`, meaning we start with the first digit and a `k` value of 0, indicating no number formed yet. Finally, the outer `dfs` call returns the minimum number of operations required to achieve the task.

With this solution, we examine each possibility while ensuring we do not re-compute the same state (combination of `i` and `k`) multiple times, thereby significantly reducing the time complexity compared to a plain recursive approach without memoization.

The data structure used here is implicit within the function call stack for the recursive approach, and the caching mechanism uses some form of hash table to store the results.

Example Walkthrough

Let's consider an example where the input string `num` is "3527". Following the solution approach:

1. The goal is to minimize the operations to make "3527" a special number (divisible by 25). To do so, we need to find a way to end this number with '00', '25', '50', or '75'.
2. We start with the recursive depth-first search (DFS) function `dfs(i, k)`. Initially, we call `dfs(0, 0)` with `i = 0` (which is the first index of the string) and `k = 0` (no number formed yet).
3. At each step of the recursion, we have two choices:
   a. Skip the current digit: `dfs(i + 1, k)` and add 1 to the operations count.
   b. Include the current digit: `dfs(i + 1, (k * 10 + int(num[i])) % 25)`.
4. For the first digit '3', the initial call is `dfs(0, 0)`. Two recursive paths will be explored: `dfs(1, 0)` (skipping '3') and `dfs(1, 3 % 25)` (including '3').
5. Let's assume we first explore including the digit '3'. If we then include '5', the next steps are `dfs(2, (3 * 10 + 5) % 25)` → `dfs(3, 35 % 25)` → `dfs(3, 10)`. It looks like we won't end with '00', '25', '50', or '75' if we keep going this route. So we may backtrack and try skipping some digits instead.
6. By applying the same decision logic to all possibilities, we find that to end with '25', we can skip '3' and keep '527'. The operations would look like this:
   a. `dfs(1, 0)` by skipping '3' (operations = 1).
   b. `dfs(2, (0 * 10 + 5) % 25)` by including '5' (operations still = 1) → this is `dfs(2, 5)`.
   c. `dfs(3, (5 * 10 + 2) % 25)` by including '2' → this is `dfs(3, 2)`.
   d. `dfs(4, (2 * 10 + 7) % 25)` by including '7' → this is `dfs(4, 0)`. At `i = 4`, which is the length of `num`, we check if `k` is 0, which it is. Therefore, we've formed the number '527', which is divisible by 25, so no more operations are needed, and the minimum number of operations is the one obtained from this branch of the decision tree, which is 1.
7. If we had instead tried other combinations to end with '00', '50', or '75', we would have found that none of these paths gives a lower minimum operation count than ending with '25'.
8. The minimal number of operations is memorized for each state `(i, k)` to avoid redundant calculations.

Therefore, the answer for this particular `num` "3527" is 1. We perform one deletion operation (removing '3'), and then '527' is a special number.

Solution Implementation

Python

```python
from functools import lru_cache

class Solution:
    def minimum_operations(self, num: str) -> int:
        # Calculate the length of the input number string
        n = len(num)

        # Define a Depth First Search with memoization using the lru_cache decorator
        @lru_cache(maxsize=None)
        def dfs(index: int, remainder: int) -> int:
            # Base case: if we reached the end of the number string
            if index == n:
                # If current remainder is 0, it means we can form a number divisible by 25,
                # thus no more operations needed. Otherwise, we can't form such number hence return n
                return 0 if remainder == 0 else n

            # Case 1: Exclude the current digit, increment the operations counter
            exclude_current = dfs(index + 1, remainder) + 1

            # Case 2: Include the current digit, no increment to operations counter
            # Compute new remainder when the current digit is included
            include_current = dfs(index + 1, (remainder * 10 + int(num[index])) % 25)

            # Choose the minimum operations between including or excluding the current digit
            best_option = min(exclude_current, include_current)

            return best_option

        # Start DFS from the first digit, with remainder initialized to 0
        return dfs(0, 0)

# Example use case
solution = Solution()
print(solution.minimum_operations("1234"))  # Should output the minimum operations to get a multiple of 25
```

Java

```java
class Solution {
    private Integer[][] memoization;
    private String number;
    private int length;

    // This method initializes the problem and calls the depth-first search method to find the solution.
    public int minimumOperations(String num) {
        length = num.length();
        this.number = num;
        memoization = new Integer[length][25]; // 25 here because any two digits modulo 25 cover all possible remainders.
        return depthFirstSearch(0, 0);
    }

    // This is a recursive depth-first search method which tries to find the minimum operations to get a multiple of 25.
    private int depthFirstSearch(int index, int remainder) {
        // Check if we have reached the end of the string
        if (index == length) {
            // We return 0 if remainder is 0, meaning the current sequence is a multiple of 25. Otherwise, we return a high cost.
            return remainder == 0 ? 0 : length;
        }
        // Memoization to avoid recalculating subproblems
        if (memoization[index][remainder] != null) {
            return memoization[index][remainder];
        }
        // Option 1: Do not include the current index in our number and move to the next digit
        memoization[index][remainder] = depthFirstSearch(index + 1, remainder) + 1;
        // Option 2: Include the current index in the number, update the remainder, and see if it leads to a better solution
        int nextRemainder = (remainder * 10 + number.charAt(index) - '0') % 25;
        memoization[index][remainder] = Math.min(memoization[index][remainder], depthFirstSearch(index + 1, nextRemainder));

        // Return the computed minimum operations for the current subproblem
        return memoization[index][remainder];
    }
}
```

C++

```cpp
#include <vector>
#include <string>
#include <cstring>// For memset
#include <functional> // For std::function

class Solution {
public:
    int minimumOperations(std::string num) {
        int n = num.size(); // Get the size of the input string
        std::vector<std::vector<int>> memo(n, std::vector<int>(25, -1)); // Create a memoization table with -1 as default values

        // Define a recursive lambda function to find the minimum operations. It takes the current position and a remainder 'k'
        std::function<int(int, int)> dfs = [&](int index, int k) {
            if (index == n) { // Base case: if we have considered all digits
                return k == 0 ? 0 : n; // if remainder is 0, return 0 operations, else return n (max operations)
            }
            if (memo[index][k] != -1) { // If we have already calculated this state
                return memo[index][k]; // Return the calculated value
            }
            // If we skip the current digit, increment the operations count
            memo[index][k] = dfs(index + 1, k) + 1;
            // Try including the current digit and update the remainder, choose the minimum between skipping and taking this digit
            int newRemainder = (k * 10 + num[index] - '0') % 25;
            memo[index][k] = std::min(memo[index][k], dfs(index + 1, newRemainder)); // Return the minimum operations for this state
        };

        return dfs(0, 0); // Start the DFS from the first digit with a remainder of 0
    }
};
```

TypeScript

```typescript
function minimumOperations(num: string): number {
    const length = num.length;
    // Create a memoization table 'memo', initialized with -2, to store results of subproblems
    const memo: number[][] = Array.from({ length: length }, () => Array.from({ length: 25 }, () => -1));

    // Depth-First Search function to compute the minimum operations
    const dfs = (currentIndex: number, remainder: number): number => {
        // If we are at the end of the string and remainder is 0, no more operations are needed
        // Otherwise, the string cannot be made divisible by 25 using these digits
        if (currentIndex === length) {
            return remainder === 0 ? 0 : length;
        }
        // Check if we already calculated the result for this subproblem
        if (memo[currentIndex][remainder] !== -1) {
            return memo[currentIndex][remainder];
        }
        // Case 1: Skip the current digit, which costs us one operation
        memo[currentIndex][remainder] = dfs(currentIndex + 1, remainder) + 1;
        // Case 2: Take the current digit and update the remainder accordingly,
        // compare with the result from skipping and choose the minimum
        memo[currentIndex][remainder] = Math.min(
            memo[currentIndex][remainder],
            dfs(currentIndex + 1, (remainder * 10 + Number(num[currentIndex])) % 25)
        );
        // Return the computed minimum for this subproblem
        return memo[currentIndex][remainder];
    };

    // Start the DFS from the first index with a remainder of 0
    return dfs(0, 0);
}
```

```python
from functools import lru_cache

class Solution:
    def minimum_operations(self, num: str) -> int:
        # Calculate the length of the input number string
        n = len(num)

        # Define a Depth First Search with memoization using the lru_cache decorator
        @lru_cache(maxsize=None)
        def dfs(index: int, remainder: int) -> int:
            # Base case: if we reached the end of the number string
            if index == n:
                # If current remainder is 0, it means we can form a number divisible by 25,
                # thus no more operations needed. Otherwise, we can't form such number hence return n
                return 0 if remainder == 0 else n

            # Case 1: Exclude the current digit, increment the operations counter
            exclude_current = dfs(index + 1, remainder) + 1

            # Case 2: Include the current digit, no increment to operations counter
            # Compute new remainder when the current digit is included
            include_current = dfs(index + 1, (remainder * 10 + int(num[index])) % 25)

            # Choose the minimum operations between including or excluding the current digit
            best_option = min(exclude_current, include_current)

            return best_option

        # Start DFS from the first digit, with remainder initialized to 0
        return dfs(0, 0)

# Example use case
solution = Solution()
print(solution.minimum_operations("1234"))  # Should output the minimum operations to get a multiple of 25
```

Time and Space Complexity

Time Complexity

The given code defines a recursive function `dfs` with memoization to solve the problem. Let's analyze the time complexity:

- There are `n` digits in the input string `num`, so there are `n` levels of recursion.
- At each level `i`, there are two choices: either take the current digit into consideration by calculating `(k * 10 + int(num[i])) % 25` or skip it.
- Memoization is used to avoid recalculating the states by storing results for each unique `(i, k)`. The parameter `i` can take `n` different values (from 0 to `n - 1`) and `k` can take at most `25` different values (from 0 to 24) since we are only interested in the remainder when divided by 25.

Due to memoization, each state is computed only once. Therefore, the time complexity is $O(n * 25)$, which simplifies to $O(n)$ because the `25` is a constant factor.

Space Complexity

The space complexity is determined by the storage required for memoization and the depth of the recursion stack:

- Due to memoization, we store results for each unique `(i, k)`. Since `i` can take `n` different values and `k` can take `25` different values, the space required for memoization is $O(n * 25)$, which simplifies to $O(n)$.
- The recursion depth can go up to `n` levels, because we make a decision for each digit in the string `num`.

Taking these into consideration, the overall space complexity of the algorithm is $O(n)$ due to memoization and the recursion stack.