1769. Minimum Number of Operations to Move All Balls to Each Box

Medium Array String Leetcode Link

Problem Description

In this problem, we're working with a simulated array of `n` boxes, each represented by `0` if the box is empty, or by `1` if the box contains one ball. We are tasked with calculating the minimum number of operations required to move all balls into each individual box one by one. In each operation, we are allowed to move one ball to an adjacent box (to the left or right). The goal is to figure out for every box `i` the number of moves needed if all balls were to be moved to box `i`. The answer should be returned as an array where each index `i` contains the minimum operations for box `i`.

The problem tests one's ability to efficiently compute cumulative operations while taking into account the both directions—left to right and right to left.

Intuition

To approach this problem, we need to come up with a way to count the operations for all positions without simulating each individual move, as that would be too slow. We do so in two sweeps, first from left to right, and then from right to left.

In the first sweep, we start from the leftmost box and move right, counting how many balls we've seen so far and adding that count to our operations because for each ball we pass, it would take one more operation to move it to the current box. We keep track of the total operations we'd need if we were moving all the balls to the right.

In the second pass, we do the opposite. We start from the rightmost box and move left, also counting balls and accumulating the operations needed to move all balls to the current position from the right. We then add this value to the corresponding position in our result from the first pass.

The accumulation of these movements gives us the minimum operation count for each box because it folds in the moves needed from both sides.

This approach allows us to calculate the minimum number of operations for each box in a single iteration from each direction, optimizing the solution to run with linear time complexity.

Solution Approach

The given solution takes a two-pass approach, which can be thought of as dynamic programming to some extent where we are building up the answer based on previous computations.

First Pass: Left to Right

- We initialize a counter `cnt` to keep track of the number of balls seen so far and an array `ans` of zeros with the same length as the `boxes` string to store the operations required for each box.
- Starting with the second element (since the first box would always require zero moves to get to itself), we iterate through the boxes string from left to right.
  - If the previous box (at index `i-1`) contains a ball (`'1'`), we increment `cnt` as we have encountered another ball.
  - The operations to move all balls encountered so far to the current box `i` is the sum of operations needed for the previous box `ans[i-1]` plus the number of balls encountered so far `cnt`. We set this value in the `ans` array at index `i`.

Second Pass: Right to Left

- We then initialize another counter `s` that will serve a similar purpose as `cnt` but for the right-to-left pass. We also reset `cnt` back to zero.
- Now, we iterate through the string from right to left starting from the second-to-last box.
  - If the next box (at index `i+1`) contains a ball, we increment `cnt` by one because we've encountered another ball.
  - We then add `cnt` to `s`. The reasoning behind this is the same as the left to right pass but in reverse; `s` keeps a running total of the number of operations needed to bring the balls from the right to the current box.
  - Finally, we add `s` to the existing value in `ans[i]` to account for the operations needed from the right side.

Final Answer

- After completing both passes, the `ans` array now contains the sum of the operations needed from both sides (left and right) for each box and thus gives us the correct minimum number of operations for each box.
- The function returns the array `ans` as the final result.

This algorithm has a time complexity of O(n) because it passes through the boxes string only twice, irrespective of the number of balls. It uses additional space for the array `ans`, giving it a space complexity of O(n) as well.

Example Walkthrough

Let's consider a small example to illustrate the solution approach. Suppose we have an array of boxes: `boxes = "001011"`. Let's walk through the algorithm.

First Pass: Left to Right

1. We start with `cnt = 0` and `ans = [0, 0, 0, 0, 0, 0]` representing the initial minimum operations for each box.
2. At `i = 1` (`boxes[1] = "0"`), the previous box is empty, so `cnt` remains `0` and `ans[1] = ans[0] + cnt = 0`.
3. At `i = 2` (`boxes[2] = "1"`), we find the first ball. We increment `cnt` to `1` since we've encountered a ball, and `ans[2] = ans[1] + cnt = 0 + 1 = 1`.
4. At `i = 3` (`boxes[3] = "0"`), we've seen one ball so far, so `ans[3] = ans[2] + cnt = 1 + 1 = 2`.
5. At `i = 4` (`boxes[4] = "1"`), we encounter another ball, increment `cnt` to `2`, and `ans[4] = ans[3] + cnt = 2 + 2 = 4`.
6. At `i = 5` (`boxes[5] = "1"`), we once again increment `cnt` to `3` as we've found another ball, and `ans[5] = ans[4] + cnt = 4 + 3 = 7`.

Now, `ans` represents the operations to move the balls from left to right, and it looks like this: `[0, 0, 1, 2, 4, 7]`.

Second Pass: Right to Left

1. We reset `cnt` to `0` and introduce `s = 0`. We start from the rightmost box and move leftward.
2. At `i = 4` (`boxes[4] = "1"`), since the next box contains a ball, we increment `cnt` to `1`. We add `cnt` to `s`, which makes `s = 1`, and add `s` to `ans[4]`: `ans[4] = ans[4] + s = 4 + 1 = 5`.
3. At `i = 3` (`boxes[3] = "0"`), we have seen one ball on the right, so `s` becomes `1 + 1 = 2`, and `ans[3] = ans[3] + s = 2 + 2 = 4`.
4. At `i = 2` (`boxes[2] = "1"`), we encounter another ball, and hence increment `cnt` to `2`. Now, `s = s + cnt = 2 + 2 = 4`, and `ans[2] = ans[2] + s = 1 + 4 = 5`.
5. At `i = 1` (`boxes[1] = "0"`), `s = s + cnt = 4 + 2 = 6`, and `ans[1] = ans[1] + s = 0 + 6 = 6`.
6. At `i = 0` (`boxes[0] = "0"`), we continue and `s = s + cnt = 6 + 2 = 8`, but since it's the leftmost box, we don't need to add it to `ans[0]`, which remains `0`.

The final `ans` array after accumulating operations from both sides is: `[0, 6, 5, 4, 5, 7]`.

This array represents the minimum number of moves to get all balls in front of each respective box. For example, `ans[1] = 6` means if we want to move all balls to box 1, it will take us a total of 6 moves.

Thus, our function would return `[0, 6, 5, 4, 5, 7]` as the final output for the input string `"001011"`. This example demonstrates the efficiency of the algorithm by avoiding the brute force approach of simulating each move, which would be impractical for large strings.

Python Solution

```python
from typing import List

class Solution:
    def minOperations(self, boxes: str) -> List[int]:
        # Calculate the length of the boxes string
        num_boxes = len(boxes)
        # Initialize an array to hold the answer (number of operations for each box)
        operations = [0] * num_boxes
        # Initialize a counter for the number of operations moving from left to right
        left_to_right_count = 0

        # Traverse the boxes from left to right to calculate the number of operations needed
        for i in range(1, num_boxes):
            # If the previous box contains a ball, increment the count
            if boxes[i - 1] == '1':
                left_to_right_count += 1
            # The current box operation count is the previous box's count plus
            # the number of operations carried over (cumulative)
            operations[i] = operations[i - 1] + left_to_right_count

        # Reset the counter for the number of operations when moving from right to left
        right_to_left_count = 0
        # Reset a variable to store the sum of operations when moving right to left
        sum_operations = 0

        # Traverse the boxes from right to left to add remaining operations
        for i in range(num_boxes - 2, -1, -1):
            # If the next box contains a ball, increment the count
            if boxes[i + 1] == '1':
                right_to_left_count += 1
            # Accumulate the sum of operations from right to left
            sum_operations += right_to_left_count
            # Add the number of operations from the right to the current answer
            operations[i] += sum_operations

        return operations

```

Java Solution

```java
class Solution {
    public int[] minOperations(String boxes) {
        // Get the length of the input string
        int length = boxes.length();
        // Initialize the answer array with the same length
        int[] operations = new int[length];

        // Forward pass to calculate the operations required for each box from the left
        for (int i = 1, count = 0; i < length; ++i) {
            // If the previous box contains a ball, increment the count
            if (boxes.charAt(i - 1) == '1') {
                ++count;
            }
            // Accumulate the number of operations required for the current box
            operations[i] = operations[i - 1] + count;
        }

        // Backward pass to calculate the operations required for each box from the right
        for (int i = length - 2, count = 0, sum = 0; i >= 0; --i) {
            // If the next box contains a ball, increment the count
            if (boxes.charAt(i + 1) == '1') {
                ++count;
            }
            // Accumulate the number of operations required from the right side
            sum += count;
            // Add the right side operations to the current box's operations
            operations[i] += sum;
        }

        // Return the final array with minimum operations required for each box
        return operations;
    }
}

```

C++ Solution

```cpp
#include <vector>
#include <string>

class Solution {
public:
    // Function to calculate the minimum number of operations required to move all balls to each box.
    std::vector<int> minOperations(std::string boxes) {
        int numberOfBoxes = boxes.size();
        std::vector<int> operations(numberOfBoxes); // This vector will store the result.

        // Forward pass to calculate the operations from the left.
        // Start from the second box (index 1), accumulate the count of balls (1's)
        // and the operations needed to bring them to each box.
        for (int i = 1, ballCount = 0; i < numberOfBoxes; ++i) {
            if (boxes[i - 1] == '1') {
                ballCount++; // Increment ball count if the previous box has a ball.
            }
            // Accumulate the number of operations needed to bring all balls from left up to the current box.
            operations[i] = operations[i - 1] + ballCount;
        }

        // Backward pass to calculate the operations from the right.
        // Start from the second-to-last box, accumulating the ball count going backward.
        // To operate both passes separately we use 'operationSum' to accumulate operations in this pass.
        for (int i = numberOfBoxes - 2, ballCount = 0, operationSum = 0; i >= 0; --i) {
            if (boxes[i + 1] == '1') {
                ballCount++; // Increment ball count if the next box has a ball.
            }
            operationSum += ballCount; // Accumulate the number of operations needed from right side up to the current box.
            // Add the current right pass operations to the forward pass operations for each box.
            operations[i] += operationSum;
        }

        return operations; // Return the final operations required for each box.
    }
};

```

Typescript Solution

```typescript
/**
 * Calculate the minimum number of operations to move all balls to each box.
 *
 * @param {string} boxes - A string where each character represents a box, '1' contains a ball, '0' does not.
 * @returns {number[]} - An array containing the minimum number of operations needed for each box
 */
function minOperations(boxes: string): number[] {
    const totalBoxes = boxes.length;
    const operations = new Array(totalBoxes).fill(0);

    // Forward pass: Count operations from left to right
    let countLeft = 0; // Count of balls to left of current box
    for (let i = 1; i < totalBoxes; i++) {
        if (boxes[i - 1] === '1') {
            countLeft++; // Increment if a ball is found
        }
        operations[i] = operations[i - 1] + countLeft; // Add count to operations
    }

    // Backward pass: Count operations from right to left
    let countRight = 0; // Count of balls to right of current box
    let totalOperations = 0; // Sum of operations needed
    for (let i = totalBoxes - 2; i >= 0; i--) {
        if (boxes[i + 1] === '1') {
            countRight++; // Increment if a ball is found
        }
        totalOperations += countRight; // Accumulate total operations
        operations[i] += totalOperations; // Add to previous count of operations
    }

    return operations;
}

```

Time and Space Complexity

Time Complexity

The given code consists of two separate for loops that each iterate through the array `boxes`. Each loop runs in linear time relative to the number of elements `n` in the input string `boxes`.

The first loop runs from index 1 to `n`, with constant-time operations within the loop, leading to an `O(n)` complexity.

The second loop similarly runs with constant-time operations but in reverse order, from `n-2` down to 0. This loop also results in an `O(n)` complexity.

Since these loops do not nest and run in sequence, the overall time complexity of the function is `O(n) + O(n)`, which simplifies to `O(n)`.

Space Complexity

The space complexity is determined by the additional memory used by the algorithm besides the input. In this case, the algorithm only allocates space for one additional array, `ans`, which stores the result and has the same length as the input string `boxes`.

Thus, the space complexity is directly proportional to the length of the input, resulting in a space complexity of `O(n)`.