1954. Minimum Garden Perimeter to Collect Enough Apples

Medium Math Binary Search

Problem Description

In this problem, we are given an infinite 2D grid where an apple tree is planted at every integer coordinate point. The amount of apples growing on a tree is determined by the sum of the absolute values of its x and y coordinates (i.e., `|i| + |j|`). Our goal is to find the smallest axis-aligned square plot centered at the origin (`0, 0`) that contains at least a certain number of apples, `neededApples`.

The challenge lies in minimizing the perimeter of this square plot while ensuring it includes or exceeds the specified amount of apples. The problem requires us to return the minimum perimeter of this plot.

Intuition

To solve this problem, we can observe a few things about the distribution of apple trees and their yield:

1. Since the trees are symmetrically placed with respect to both axes, we can focus on any one of the four quadrants and extrapolate for the others.
2. The number of apples grows linearly as we move away from the origin along any straight line, but since we're considering a square area, the growth rate of total apples is quadratic.

The key insight here is that the number of apples within a square plot with side length `2 * L` (centered at the origin) can be computed in a formulaic way. This formula must take into account the count of apples for trees along the perimeter and inside the square. Due to the quadratic nature of the problem, the number of apples within a plot is related to the sum of a series of integers.

Given these insights, we can calculate the number of apples within a plot of side length `2 * L` without explicitly adding apples from each tree.

Our solution uses a binary search approach within a reasonable range (`1` to `r`) to find the minimum side length such that the square plot defined by `2 * L` has at least `neededApples`. Binary search is effective here as it avoids a linear scan of all possible side lengths and instead narrows down by halves, significantly reducing the number of checks needed.

The width of the plot is `2 * L`, and since the plot is a square, the perimeter will be `4` times the side length, which is `8 * L`, giving us our final answer. The formula `2 * mid * (mid + 1) * (2 * mid + 1)` inside the binary search loop represents a mathematical formula that relates to the number of apples within a square plot.

By conducting a binary search on the side lengths, it's possible to identify the side length where the number of apples meets or exceeds the `neededApples`, resulting in the smallest square plot that satisfies the problem constraints.

Solution Approach

In the provided code solution, a binary search algorithm is used to efficiently find the smallest integer length 'L' such that a square plot with this length satisfies the condition of containing at least `neededApples`. Here's how the solution works, step by step:

1. Initialize the search space with variables `l` (for left) and `r` (for right) representing the possible range of lengths for the sides of the plot. Note that the range is set between `1` and `100000` which is an assumed limit that ensures our search space includes the solution.
2. Perform a binary search within this range. A binary search starts with the midpoint of the current `l` and `r` and systematically reduces the search space in half based on whether the condition is satisfied (`neededApples` are within or on the plot).
3. Calculate the midpoint between `l` and `r` using `(l + r) >> 1`. The `>>` operator is a bitwise shift that effectively divides by 2, finding the midpoint for the next iteration of our search.
4. For the current midpoint, compute the number of apples in the square plot using the formula `2 * mid * (mid + 1) * (2 * mid + 1)`. This formula is derived from the problem's premise that calculates the number of apples based on the coordinates of the trees, taking advantage of the symmetry and quadratic nature of apple growth in the grid.
5. Compare the number of apples calculated with the `neededApples`. If the computed number is greater than or equal to `neededApples`, it means that a square plot with the current midpoint length is sufficient, and thus we adjust the `r` variable to `mid`, effectively reducing our search space to the left half.
6. Otherwise, if the computed number is less than `neededApples`, we need a larger plot. Thus, we adjust the `l` variable to `mid + 1`, focusing on the right half of our current search space for the next iteration.
7. The binary search continues until `l` and `r` converge, meaning we have found the smallest `l` that meets the condition.
8. The result is then multiplied by 8 to give the perimeter of the square plot (`l * 8`), as there are four sides to the square and each side is `2 * L`.

The use of binary search is crucial in this solution as it provides a logarithmic time complexity, much more efficient than a linear scan, especially when the range of possible lengths is large.

Note: In the code, `l` eventually holds the value of the minimum required length of the square plot's side that houses the `neededApples`. It's important to remember that the binary search relies on the observation that as the side length increases, the number of apples within or on the plot increases as well, allowing us to find the precise length needed efficiently.

Example Walkthrough

Let's say we need to find a square plot with at least `neededApples = 10` apples. Here's how we would apply the solution approach described above:

1. We initialize our search space with `l = 1` and `r = 100000`, though in practice, the right bound `r` could be anything sufficiently large.
2. To start our binary search, we calculate the midpoint `mid = (l + r) >> 1`. Initially, it's `((1 + 100000) >> 1)`, so `mid` will be `50000`. This is an exaggerated mid value for our small example, but for the purposes of our walk-through, we'll proceed with more reasonable numbers in the following steps.
3. Let's assume a more practical midpoint `mid = 2`. We'd calculate the number of apples using the formula `2 * mid * (mid + 1) * (2 * mid + 1)` which simplifies to `2 * 2 * 3 * 5`, giving us 60 apples.
4. Since `60` is greater than `10`, our needed number of apples, we adjust our `r` to be equal to `mid`, bringing `r` down from `100000` to `2`.
5. We won't update `l` because the number of apples at `mid` is already more than `10`, so we need to find if there's an even smaller square that still satisfies the requirement.
6. Now we recalculate the `mid` since we brought down `r` to `2`, the new `mid` is `((1 + 2) >> 1)`, which is `1.5`, but since we are dealing with integer values, we take `mid = 1`.
7. Calculate the number of apples with `mid = 1` using the formula, which gives us `2 * 1 * 2 * 3` or 12 apples.
8. Once again, we see that 12 apples are greater than the 10 needed, so we again adjust `r` to `mid`, but since `mid` is already `1`, both `l` and `r` are now `1`.
9. Since `l` and `r` have converged, we have found our minimum `L = 1`. The final step is to calculate the perimeter of the square plot `1 * 8`, which gives us `8`.

By condensing the range step by step, we were able to identify that a square plot of side length `2` centered at the origin (`0, 0`) results in a square that covers enough apple trees to meet our required number of apples. The perimeter of this plot is `8`, which is the solution to our problem.

Solution Implementation

Python

```python
class Solution:
    def minimumPerimeter(self, neededApples: int) -> int:
        # Initialize the left and right boundaries for binary search.
        left, right = 1, 100000

        # Perform binary search to find the minimum distance from the tree to collect the needed apples.
        while left < right:
            # Calculate mid-point to partition the search range.
            mid = (left + right) // 2

            # The formula calculates the total number of apples within a square with side length 'mid'.
            if 2 * mid * (mid + 1) * (2 * mid + 1) >= neededApples:
                # If we have enough apples, try to find a smaller square.
                # hence we move the right boundary to the current midpoint.
                right = mid
            else:
                # If we don't have enough apples, we need a larger square,
                # so we move the left boundary past the midpoint.
                left = mid + 1

        # Once binary search is complete, left will hold the minimum distance,
        # We multiply by 8 to get the perimeter of the square (distance from the tree is 'left').
        return left * 8
```

Java

```java
class Solution {
    public long minimumPerimeter(long neededApples) {
        // Initialize low and high bounds for binary search
        long low = 1, high = 100000;

        // Apply binary search to find the minimum distance from the tree
        // at which the number of apples will meet or exceed the 'neededApples'
        while (low < high) {
            // Find the middle point between low and high
            long mid = (low + high) >> 1;

            // Check if the number of apples within this perimeter is sufficient
            // Calculate the number of apples by the formula explained.
            // The formula is for the sum of the first 'mid' odd numbers
            // and then multiplied by 4, because the garden has 4 quadrants.
            // 2 * mid * (mid + 1) * (2 * mid + 1) directly gives the sum of
            // apples in one quadrant, so we compare that sum with 'neededApples'
            if (2 * mid * (mid + 1) * (2 * mid + 1) >= neededApples) {
                high = mid; // Adjust the high bound of the search to 'mid'
            } else {
                low = mid + 1; // Adjust the low bound of the search to 'mid' + 1
            }
        }

        // Once we find the closest perimeter that meets the requirement,
        // we multiply it by 8 to get the minimum perimeter of the square.
        // This is because the distance from the tree to the edge of the square
        // on one side forms the perimeter of the square after being multiplied by 4
        // (imagine extending the distance to all four sides of the square),
        // and the condition is checking for each half hence the multiplication by 2.
        return low * 8;
    }
}
```

C++

```cpp
class Solution {
public:
    // This function calculates the minimum perimeter of a square garden
    // from which one can collect at least 'neededApples' apples.
    long long minimumPerimeter(long long neededApples) {
        // Initialize the binary search boundaries
        long long left = 1, right = 100000;

        // Perform binary search to find the smallest side length of the square
        while (left < right) {
            // Find the middle point between current left and right
            long long mid = (left + right) >> 1;

            // Calculate the number of apples that can be collected from a square garden:
            // The formula is based on an arithmetic series that derives from the pattern
            // in which apples are distributed around the perimeter of the garden.
            long long apples = 2 * mid * (mid + 1) * (2 * mid + 1);

            // Check if the current garden size can meet or exceed the needed apples
            if (apples >= neededApples) {
                // If enough apples can be acquired, bring the right bound inwards
                right = mid;
            } else {
                // Otherwise, increase the left bound to enlarge the garden size
                left = mid + 1;
            }
        }

        // The final result is the side length times 8, which is the perimeter of the square.
        // left now holds the value of the minimum required side length
        return left * 8;
    }
};
```

TypeScript

```typescript
function minimumPerimeter(neededApples: number): number {
    // Initialize left and right bounds for binary search
    let left = 1;
    let right = 100000;

    // Use binary search to find the smallest perimeter
    while (left < right) {
        // Calculate the middle point
        const mid = Math.floor((left + right) / 2);

        // Calculate the number of apples within a (mid x mid) square
        // Formula: 2 * mid * (mid + 1) * (2 * mid + 1)
        const applesInSquare = 2 * mid * (mid + 1) * (2 * mid + 1);

        // Check if the current square has enough apples
        if (applesInSquare >= neededApples) {
            // If it does, move the right bound to mid
            right = mid;
        } else {
            // If it doesn't, move the left bound to mid + 1
            left = mid + 1;
        }
    }

    // Return the perimeter, which is 8 times the side of the square
    return 8 * left;
}
```

```python
class Solution:
    def minimumPerimeter(self, neededApples: int) -> int:
        # Initialize the left and right boundaries for binary search.
        left, right = 1, 100000

        # Perform binary search to find the minimum distance from the tree to collect the needed apples.
        while left < right:
            # Calculate mid-point to partition the search range.
            mid = (left + right) // 2

            # The formula calculates the total number of apples within a square with side length 'mid'.
            if 2 * mid * (mid + 1) * (2 * mid + 1) >= neededApples:
                # If we have enough apples, try to find a smaller square,
                # hence we move the right boundary to the current midpoint.
                right = mid
            else:
                # If we don't have enough apples, we need a larger square,
                # so we move the left boundary past the midpoint.
                left = mid + 1

        # Once binary search is complete, left will hold the minimum distance,
        # We multiply by 8 to get the perimeter of the square (distance from the tree is 'left').
        return left * 8
```

Time and Space Complexity

The given Python code is performing a binary search to find the minimum perimeter of a square that encloses at least `neededApples` apples. Here's the analysis of time and space complexity:

- **Time Complexity:**

  The binary search algorithm divides the search interval in half each time, which creates a logarithmic time complexity. Considering that the initial search range is defined between `1` and `100000`, the maximum number of iterations this search will perform is log2(100000) because it's effectively cutting the range in half with each iteration until it finds the perimeter that satisfies the condition. Therefore, the time complexity is $O(\log n)$ where n is the upper limit of the search range.

  The condition checked in the binary search involves a calculation based on `mid`: `2 * mid * (mid + 1) * (2 * mid + 1)`. The evaluation of this arithmetic expression is done in constant time since it only depends on the current value of `mid` and doesn't grow with the size of the input.

  Consequently, the overall time complexity of the function is $O(\log n)$, with n being the maximum number `100000`.

- **Space Complexity:**

  Space complexity refers to the amount of space or memory taken by an algorithm to run as a function of the length of the input.

  In this code, there are only a fixed number of integer variables (`l`, `r`, `mid`) used, and there's no use of any additional data structures that grow with the size of the input. Thus, the space used by the algorithm does not depend on the input size. Therefore, the space complexity is constant, $O(1)$.