

75. Sort Colors

Medium Array Two Pointers Sorting

Problem Description

In this problem, you are given an array `nums` which contains `n` elements. Each element represents a color coded as an integer: `0` for red, `1` for white, and `2` for blue. Your task is to sort this array in a way that the colors are grouped together and in the order of red, white, and blue. The [sorting](#) has to be done in-place, without using any extra space for another array, and you cannot use the library's sort function.

Intuition

To solve this problem, the solution approach uses a variant of the famous Dutch National Flag algorithm proposed by Edsger Dijkstra. The crux of this algorithm is a three-way partitioning technique that segments an array into three parts to sort the elements of three different types.

In this case, we will maintain three pointers:

- `i` - All elements before index `i` are `0`s (reds).
- `j` - All elements from index `j` onwards are `2`s (blues).
- `k` - Current element that is being inspected.

Initially, `i` is set to `-1`, indicating there are no `0`s in the beginning, and `j` is set to the length of `nums`, indicating there are no `2`s at the end. The `k` pointer will start from `0` and move towards `j`.

We iterate through the array with `k`, and when we find a `0`, we increment `i` and swap the values at `i` and `k`. If we find a `2`, we decrement `j` and swap the values at `k` and `j` but we don't move the pointer `k` because the new element we swapped from position `j` might be `0`, so it needs to be rechecked. If the element is `1`, it's already in its correct place since we are ensuring all `0`s and `2`s are moved to their correct places. So, for `1`, we just move `k` forward.

We continue this process until `k` meets `j`, at which point all elements to the left of `i` are `0`s, elements between `i` and `j` are `1`s, and all elements from `j` onwards are `2`s, resulting in a sorted array.

Solution Approach

The solution implements the Dutch National Flag algorithm, which is a partitioning strategy.

Here's a step-by-step explanation using the Reference Solution Approach:

- Initialize three pointers (`i`, `j`, and `k`):
 - `i` starts just before the array at `-1`. This will eventually track the position up to which `0`s have been sorted.
 - `j` starts after the end of the array at `len(nums)`. This will eventually track the position from which `2`s have been sorted.
 - `k` starts at `0` and is used to iterate through the array.
- Perform iterations while `k < j`:
 - If `nums[k] == 0`, this element needs to be moved to the front.
 - Increment `i` to move it to the next unsorted position.
 - Swap the elements at `i` and `k` (`nums[i], nums[k] = nums[k], nums[i]`), effectively moving the `0` to its correct place.
 - Increment `k` to move on to the next element.
 - Else, if `nums[k] == 2`, this element needs to be moved to the end.
 - Decrement `j` to move it towards the first unsorted position from the end.
 - Swap the elements at `k` and `j` (`nums[j], nums[k] = nums[k], nums[j]`), moving the `2` closer to its correct place. Here we don't increment `k` because the newly swapped element could be `0` or `1` and it has not been evaluated yet.
 - If `nums[k] == 1`, no action is needed as `1`s are automatically sorted when `0`s and `2`s are moved to their correct places.
 - Simply increment `k` to continue to the next element.

By following this approach, we continue to partition the array into three parts: `0`s before `i`, `1`s between `i` and `j`, and `2`s after `j`. The loop continues until `k` becomes equal to `j`, meaning all elements have been examined and placed in their correct position. Therefore, the array is now sorted in-place with red (`0`), white (`1`), and blue (`2`) colors in the correct order without using any additional space or the library sort function.

Example Walkthrough

Let's say we have an array `nums` as `[2, 0, 1, 2, 1, 0]`. We need to sort this array using the Dutch National Flag algorithm so that all `0`s (reds) come first, followed by `1`s (whites), and then `2`s (blues).

Here's a step-by-step process of how the algorithm would sort this array:

- Initialize the pointers `i`, `j`, and `k`:
 - `i` is set to `-1`
 - `j` is set to `6` (since the array length is `6`)
 - `k` is set to `0`
- Start iterating with `k` while `k < j` (while `k` is less than `6`):
 - Iteration 1:
 - `nums[k]` is `2`. Since `k==0`, we need to move this `2` to the end.
 - We decrement `j` to `5`.
 - We swap `nums[k]` with `nums[j]`. So the array becomes `[0, 0, 1, 2, 1, 2]`.
 - We don't increment `k` as we need to evaluate the swapped element.
 - Iteration 2:
 - Now `nums[k]` is `0`. This needs to go at the beginning.
 - We increment `i` to `0`.
 - We swap `nums[i]` with `nums[k]`. The array is still `[0, 0, 1, 2, 1, 2]` since both `nums[i]` and `nums[k]` are `0`.
 - We increment `k` to `1`.
 - Iteration 3:
 - `nums[k]` is another `0`.
 - We increment `i` to `1`.
 - We swap `nums[i]` with `nums[k]`, but the array remains unchanged `[0, 0, 1, 2, 1, 2]` as they are the same value.
 - Increment `k` to `2`.
 - Iteration 4:
 - `nums[k]` is `1`. This is already in the correct position.
 - We simply increment `k` to `3`.
 - Iteration 5:
 - `nums[k]` is `2`, needs to move to the end.
 - We decrement `j` to `4`.
 - We swap `nums[k]` with `nums[j]`. Now the array looks like `[0, 0, 1, 1, 2, 2]`.
 - Do not increment `k` as we need to evaluate the swapped element.
 - Iteration 6:
 - `nums[k]` is now `1`. It should stay in place.
 - Increment `k` to `4`. Now `k == j`, so we stop.

The final sorted array is `[0, 0, 1, 1, 2, 2]`, with all the colors grouped together in the correct order without using any extra space or sorting functions.

Python Solution

```
1 class Solution:
2     def sortColors(self, nums: List[int]) -> None:
3         # Initialize pointers for the next position of 0, the next position of 2, and the current element
4         next_zero_index, next_two_index, current_index = -1, len(nums), 0
5
6         # Process elements until the current_index reaches the next_two_index
7         while current_index < next_two_index:
8             if nums[current_index] == 0:
9                 # Move the 0 to the next position for 0
10                next_zero_index += 1
11                nums[next_zero_index], nums[current_index] = nums[current_index], nums[next_zero_index]
12                # Move to the next element
13                current_index += 1
14            elif nums[current_index] == 2:
15                # Move the 2 to the next position for 2
16                next_two_index -= 1
17                nums[next_two_index], nums[current_index] = nums[current_index], nums[next_two_index]
18                # Do not increment current_index because we need to check the newly swapped element
19            else:
20                # If the current element is a 1, just move to the next element
21                current_index += 1
22        # The function modifies the list in place, so there is no return value
23
```

Java Solution

```
1 class Solution {
2
3     // Method to sort the array containing 0s, 1s, and 2s
4     public void sortColors(int[] nums) {
5         // Initialize pointers for the current element (currIndex),
6         // the last position of 0 (lastZeroIndex) and the first position of 2 (firstTwoIndex)
7         int lastZeroIndex = -1;
8         int firstTwoIndex = nums.length;
9         int currIndex = 0;
10
11        // Process elements until currIndex reaches firstTwoIndex
12        while (currIndex < firstTwoIndex) {
13            if (nums[currIndex] == 0) {
14                // If the current element is 0, swap it to the position after the last 0 we found
15                swap(nums, ++lastZeroIndex, currIndex++);
16            } else if (nums[currIndex] == 2) {
17                // If the current element is 2, swap it with the element at the position
18                // just before the first 2 we found
19                swap(nums, --firstTwoIndex, currIndex);
20            } else {
21                // If the current element is 1, just move to the next element
22                ++currIndex;
23            }
24        }
25    }
26
27    // Helper method to swap two elements in an array
28    private void swap(int[] nums, int i, int j) {
29        int temp = nums[i];
30        nums[i] = nums[j];
31        nums[j] = temp;
32    }
33 }
34
```

C++ Solution

```
1 #include <vector>
2 using namespace std;
3
4 class Solution {
5 public:
6     // This function is used to sort the colors, represented by numbers 0, 1, and 2.
7     // It uses the Dutch National Flag algorithm to sort in place with O(n) complexity.
8     void sortColors(vector<int>& nums) {
9         // Initialize pointers:
10        // 'left' is the position where the next 0 should go,
11        // 'right' is the position one more than where the next 2 should go,
12        // 'current' is the current index being considered.
13        int left = -1, right = nums.size(), current = 0;
14
15        while (current < right) { // Process elements until 'current' reaches 'right'
16            if (nums[current] == 0) {
17                // When a 0 is found, swap it with the element at 'left' position,
18                // then move both 'left' and 'current' one step right.
19                swap(nums[++left], nums[current++]);
20            } else if (nums[current] == 2) {
21                // When a 2 is found, swap it with the element just before 'right' position,
22                // then decrement 'right' to move it leftward.
23                // Note 'current' is not incremented because the swapped element needs to be checked.
24                swap(nums[--right], nums[current]);
25            } else {
26                // If the element is 1, just move 'current' one step to the right.
27                ++current;
28            }
29        }
30    }
31 };
32
```

Typescript Solution

```
1 /**
2  * Sorts an array of numbers in-place, so that all 0s come first,
3  * followed by all 1s, and then all 2s. This pattern is known as the Dutch national flag problem.
4  *
5  * @param {number[]} nums - The input array containing 0s, 1s, and 2s.
6  */
7 function sortColors(nums: number[]): void {
8     let zeroIndex = -1; // Initialize the index where 0s will be placed.
9     let twoIndex = nums.length; // Initialize the index where 2s will be placed.
10    let currentIndex = 0; // The current index we're scanning from the array.
11
12    while (currentIndex < twoIndex) {
13        if (nums[currentIndex] === 0) {
14            // When the current element is 0, swap it with the element at zeroIndex,
15            // then increment zeroIndex and currentIndex.
16            zeroIndex++;
17            [nums[zeroIndex], nums[currentIndex]] = [nums[currentIndex], nums[zeroIndex]];
18            currentIndex++;
19        } else if (nums[currentIndex] === 2) {
20            // When the current element is 2, decrement twoIndex and swap the current element
21            // with the element at twoIndex.
22            twoIndex--;
23            [nums[twoIndex], nums[currentIndex]] = [nums[currentIndex], nums[twoIndex]];
24            // Do not increment currentIndex here because the element swapped from twoIndex
25            // may be 0, which will need to be moved to zeroIndex in the next iteration.
26        } else {
27            // If the element is 1, just move on to the next element.
28            currentIndex++;
29        }
30    }
31 }
32
```

Time and Space Complexity

The time complexity of the code is $O(n)$, where `n` is the length of the input list `nums`. This is because the while loop iterates through each element of the list at most once. The variables `i`, `j`, and `k` are used to traverse the array without the need to revisit elements. The increment and decrement operations on `i`, `j`, and `k`, as well as the swaps, all occur in constant time, and the loop runs until `k` is no longer less than `j`.

The space complexity of the code is $O(1)$ because the sorting is done in place. No additional storage is needed that scales with the input size `n`. The only extra space used is for the three pointers `i`, `j`, and `k`, which use a constant amount of space regardless of the size of the input list.