1055. Shortest Way to Form String

Medium Prefix Two Pointers String

Leetcode Link

Problem Description

The problem involves finding how to form a given `target` string by concatenating subsequences of a `source` string. A subsequence can be obtained by deleting zero or more characters from a string without changing the order of the remaining characters. We need to determine the minimum number of these subsequences from `source` needed to create the `target` string. If it's not possible to form the `target` from subsequences of `source`, the function should return `-1`.

To visualize the problem, think about how you can create the word "cat" using letters from the word "concentrate". You can select `c`, omit `o`, `n`, pick `c`, skip `e`, `n`, `t`, and then pick `a` and skip `r`, `a`, `t`. That forms one subsequence "cac". Moreover, if you had a target like "catat", you'd need two subsequences from "concentrate" to form it—"cat" and "at".

Intuition

The intuition for the solution is built on the idea that we can iterate over the `source` and `target` strings simultaneously, matching characters from `target` and moving through `source`. We do this in a loop that continues until we've either created the `target` string or determined it's impossible.

For each iteration (which corresponds to constructing a single subsequence), we do the following:

1. Start from the beginning of the `source` string and look for a match for the current character in `target`.
2. Each time we find a match, we move to the next character in `target` but continue searching from the current position in `source`.
3. If we reach the end of the `source` without finding the next character in `target`, we start over from the beginning of `source` and increment our subsequence count.
4. If when restarting the `source` string we don't make any progress in `target` (meaning we didn't find even the next character in the target), we conclude that the `target` cannot be formed and return -1.

The concept is similar to using multiple copies of the `source` string side by side, and crossing out characters as we match them to `target`. Whenever we reach the end of a `source` string copy and still have characters left to match in `target`, we move on to the next copy of `source`, symbolizing this with an increment in our subsequence counter. This continues until we've matched the entire `target` string or have verified that it's impossible.

Solution Approach

The solution uses a two-pointer technique to iterate through both the `source` and `target` strings. One pointer (`i`) traverses the `source` string, while the other pointer (`j`) iterates over the `target` string. Here's a step-by-step breakdown of the key components of the algorithm:

1. **Function `f(i, j)`:** This is a helper function that takes two indices, `i` for the `source` and `j` for the `target`. The purpose of `f` is to try to match as many characters of `target` starting from index `j` with the `source` starting from index `i` until we reach the end of `source`. The function runs a `while` loop until either `i` or `j` reaches the end of their respective strings. Inside this loop:
   - If the characters at `source[i]` and `target[j]` match, increment `j` to check for the next character in `target`.
   - Whether or not there is a match, always increment `i` because we can skip characters in `source`.
   - The function returns the updated index `j` after traversing through `source`.
2. **Main Algorithm:** Once we have our helper function, the main algorithm proceeds as follows:
   - We initialize two variables, `n` and `m` as the lengths of `source` and `target` respectively, and `ans` and `j` to keep track of the number of subsequences needed and the current index in `target`.
   - We use a `while` loop that continues as long as `j < n`, meaning there are still characters in `target` that have not been matched.
   - Inside the `while` loop, we call our helper function `f(0, j)` which tries to match `target` starting from the current `j` index with `source` starting from `0`. If the returned index `k` is the same as `j`, it means no further characters from `target` could be matched and we return `-1` as it's impossible to form `target`.
   - If `k` is different from `j`, this means we've managed to match some part of `target`, and we update `j` to `k` and increment `ans` to signify the creation of another subsequence from `source`.
   - The process repeats until all characters of `target` are matched.
3. **Return Value:** The loop ends with two possibilities; either we were able to form `target` successfully, hence we return the `ans` which is the count of subsequences needed, or we determined that `target` cannot be formed from `source` and returned `-1`.

Complexity Analysis

- **Time Complexity:** $O(n * m)$, where `n` is the length of `source` and `m` is the length of `target`. In the worst case, we iterate through the entire `source` for every character in `target`.
- **Space Complexity:** $O(1)$, we only use a fixed amount of extra space for the pointers and the `ans` variable regardless of the input size.

By thoroughly understanding the definition of a subsequence and carefully managing the iteration through both strings, this solution efficiently determines the minimum number of subsequences of `source` required to form `target` or establishes that it's not possible.

Example Walkthrough

Let's walk through a small example to illustrate the solution approach.

Example:

- `source`: "abcab"
- `target`: "abccba"

Walkthrough:

1. Initialize the count of subsequences (`ans`) needed to 0 and the index `j` in the `target` to 0.
2. Since `j < n` (where n is the length of `target`), start the iteration and call the helper function `f` with `f(0, 0)`.
3. Inside `f(0, 0)`, iterate over `source` and `target`. For each character in `source`, check if it matches the current `target[j]`.
   - For `source[0]` = 'a' and `target[0]` = 'a', there's a match, increment `j` to 1 (next character in `target`).
   - Continue to `source[1]` = 'b', which matches `target[1]` = 'b', increment `j` to 2.
   - Continue to `source[2]` = 'c', which matches `target[2]` = 'c', increment `j` to 3.
   - The character `source[3]` = 'a' does not match `target[3]` = 'c', so just increment `i`.
   - The character `source[4]` = 'b' does not match `target[3]` either, increment `i` again and now `i` reaches the end of `source`.
4. The `f` function returns `j` which is now 3. Since `j` has increased from 0 to 3, one subsequence "abc" has been matched from `source`.
   - Increment `ans` to 1 and start matching the next subsequence with `f(0, 3)`.
5. In the second call to `f(0, 3)`, we iterate from the start of `source` again:
   - Skip `source[0]` = 'a', since it doesn't match `target[3]` = 'c'.
   - Skip `source[1]` = 'b', since it doesn't match `target[3]` either.
   - `source[2]` matches `target[3]`, so increment `j` to 4.
   - `source[3]` matches `target[4]`, increment `j` to 5.
   - No more characters in `source` match `target[5]` = 'a', but once we reach the end of `source`, `f` returns `j` which is now 5.
6. Increment `ans` to 2 and start matching the last character with `f(0, 5)`.
7. In the third call to `f(0, 5)`, the first character `source[0]` = 'a' matches the last character `target[5]` = 'a'. The `j` is incremented to 6, which is the length of `target`, so the entire target string has been matched.
8. Increment `ans` to 3 which is our final answer.

Return:

The function would return `3` as it takes three subsequences of `source` to form the `target` string "abccba".

This example collapses the entire iteration into a concise explanation, demonstrating how the algorithm works in practice and matches subsequences in the source to form the target string.

Python Solution

```python
class Solution:
    def shortestWay(self, source: str, target: str) -> int:
        # Helper function to find the first unmatched character in 'target'
        # starting from 'target_index' in the 'source' string.
        def find_unmatched_index(source_index, target_index):
            # Iterate over both 'source' and 'target' strings.
            while source_index < len_source and target_index < len_target:
                # If the current characters match, move to the next character in 'target'.
                if source[source_index] == target[target_index]:
                    target_index += 1
                # Move to the next character in 'source'.
                source_index += 1
            # Return the index in 'target' where the characters stop matching.
            return target_index

        # Initialize the length variables of 'source' and 'target'.
        len_source, len_target = len(source), len(target)
        # Initialize the number of subsequences needed.
        subsequences_count = 0
        # Initialize 'target_index' to keep track of progress in the 'target' string.
        target_index = 0

        # Main loop to iterate until the entire 'target' string is checked.
        while target_index < len_target:
            # Find the index of the first unmatched character after 'target_index'.
            unmatched_index = find_unmatched_index(0, target_index)

            # Check if 'target_index' did not move forward; if so, 'target' cannot be constructed.
            if unmatched_index == target_index:
                return -1

            # Update 'target_index' to the index of the first unmatched character.
            target_index = unmatched_index

            # Increment the count of subsequences used.
            subsequences_count += 1

        # Return the total number of subsequences from 'source' needed to form 'target'.
        return subsequences_count
```

Java Solution

```java
class Solution {
    // Method to find the minimum number of subsequences of 'source' which concatenate to form 'target'
    public int shortestWay(String source, String target) {
        // 'sourceLength' is the length of 'source', 'targetLength' is the length of 'target'
        int sourceLength = source.length(), targetLength = target.length();
        // 'numSubsequences' will track the number of subsequences used
        int numSubsequences = 0;
        // 'targetIndex' is used to iterate through the characters of 'target'
        int targetIndex = 0;

        // Continue until the whole 'target' string is covered
        while (targetIndex < targetLength) {
            // 'sourceIndex' is used to iterate through characters of 'source'
            int sourceIndex = 0;
            // 'subsequenceFound' flags if a matching character was found in the current subsequence iteration
            boolean subsequenceFound = false;

            // Loop both 'source' and 'target' strings to find subsequence matches
            while (sourceIndex < sourceLength && targetIndex < targetLength) {
                // If the characters match, move to the next character in 'target'
                if (source.charAt(sourceIndex) == target.charAt(targetIndex)) {
                    subsequenceFound = true; // A match in the subsequence was found
                    targetIndex++;
                }
                // Always move to the next character in 'source'
                sourceIndex++;
            }

            // If no matching subsequence has been found, it's not possible to form 'target'
            if (!subsequenceFound) {
                return -1;
            }
            // A subsequence that contributes to 'target' was used, so increment the count
            numSubsequences++;
        }

        // Return the minimum number of subsequences needed to form 'target'
        return numSubsequences;
    }
}
```

C++ Solution

```cpp
class Solution {
public:
    // Function to find the minimum number of subsequences of 'source' required to form 'target'.
    int shortestWay(string source, string target) {
        int sourceLength = source.size(), targetLength = target.size(); // Source and target lengths
        int subsequencesCount = 0; // Initialize the count of subsequences needed
        int targetIndex = 0; // Pointer for traversing the target string

        // Loop until the entire target string is covered
        while (targetIndex < targetLength) {
            int sourceIndex = 0; // Reset source pointer for each subsequence iteration
            bool subsequenceFound = false; // Flag to check if at least one matching character is found in this iteration

            // Traverse both source and target to find the subsequence
            while (sourceIndex < sourceLength && targetIndex < targetLength) {
                // If the characters match, move the pointer in the target string to find the next character
                if (source[sourceIndex] == target[targetIndex]) {
                    subsequenceFound = true;
                    ++targetIndex;
                }
                ++sourceIndex; // Always move to the next character in the source string
            }

            // If no matching character was found, it's impossible to form target from source
            if (!subsequenceFound) {
                return -1;
            }

            ++subsequencesCount; // A new subsequence is found for this iteration
        }

        // Return the total count of subsequences required
        return subsequencesCount;
    }
};
```

Typescript Solution

```typescript
// Function to find the minimum number of subsequences of 'source' required to form 'target'.
function shortestWay(source: string, target: string): number {
    let sourceLength: number = source.length; // Source length
    let targetLength: number = target.length; // Target length
    let subsequencesCount: number = 0; // Initialize the count of subsequences needed
    let targetIndex: number = 0; // Pointer for traversing the target string

    // Loop until the entire target string is covered
    while (targetIndex < targetLength) {
        let sourceIndex: number = 0; // Reset source pointer for each subsequence iteration
        let subsequenceFound: boolean = false; // Flag to check if at least one matching character is found in this iteration

        // Traverse both source and target to find the subsequence
        while (sourceIndex < sourceLength && targetIndex < targetLength) {
            // If the characters match, move the pointer in the target string to find the next character
            if (source.charAt(sourceIndex) === target.charAt(targetIndex)) {
                subsequenceFound = true;
                targetIndex++; // Move to the next character in target
            }
            sourceIndex++; // Always move to the next character in the source string
        }

        // If no matching character was found, it's impossible to form target from source
        if (!subsequenceFound) {
            return -1;
        }

        subsequencesCount++; // A new subsequence is found for this iteration
    }

    // Return the total count of subsequences required
    return subsequencesCount;
};
```

Time and Space Complexity

Time Complexity

The primary function of the algorithm, `shortestWay`, iterates over the `target` string while repeatedly scanning the `source` string to find subsequences that match the `target`. The function `f(i, j)` is called for each subsequence found and runs in a while loop that continues until either the end of the `source` or `target` string is reached. The worst-case scenario occurs when every character in the `source` has to be visited for every character in the `target`.

Given:

- `n` is the length of `source`
- `m` is the length of `target`

The worst-case time complexity can be roughly bounded by $O(n * m)$ since, in the worst case, the substring search could traverse the entire source string for each character in the target string.

Space Complexity

The space complexity of the algorithm is $O(1)$ as it only uses a fixed number of integer variables `n`, `m`, `ans`, `j`, and `k`, and does not allocate any additional space proportional to the input size.