124. Binary Tree Maximum Path Sum

<u>Depth-First Search</u> <u>Dynamic Programming</u> <u>Binary Tree</u>

Problem Description

Hard

tree. Here, a path is defined as a sequence of nodes where consecutive nodes are connected by an edge, and a node can only be used once within a single path. The path sum is the total of the values of all nodes in that path. Notably, the path does not have to go through the root of the tree. Our goal is to calculate the maximum sum from all such possible paths in the tree.

In this problem, we are given the root of a binary tree and need to find the maximum path sum from any sequence of nodes in the

Intuition

When dealing with trees, recursion is often a natural approach. Since we are looking for the maximum path sum, at every node we have a choice: include that node on a path extending left or right, or start a new path through that node. We can recursively find the maximum path sums going through the left child and the right child. However, when combining these sums at a node, we must realize that we cannot include both child paths since that would create a loop, not a valid path.

positive, since any path would only include a subtree if it contributed positively to the total sum. We define the dfs function that does the following: • When the node is None, return 0 because an empty node contributes nothing to the path sum.

The solution's intuition hinges on realizing that for any node, the maximum sum wherein that node is the highest point (i.e., the

'root' of that path) is its value plus the maximum sums of its left and right subtrees. We only add the subtree sums if they are

• Recursively calculate the maximum path sum for the left and right subtrees. If the sum is negative, we reset it to 0, as described earlier. • Calculate the potential maximum path sum at the current node by adding the node's value to the maximum path sums from both subtrees. This represents the largest value that could be achieved when passing through that node and potentially including both left and right paths (but not combining them).

• Update a global variable ans that tracks the overall maximum path sum found anywhere in the tree with this new potential maximum. • Finally, for the recursion to continue upwards, return the node's value plus the greater of the two maximum sums from the left or right subtree.

This represents the best contribution that node can make towards a higher path.

- The maxPathSum function initiates this recursive process, starting from the root, and returns the maximum path sum found. Solution Approach
 - The solution uses a Depth-First Search (DFS) algorithm to recursively traverse the binary tree and calculate the path sums. This approach ensures that every node is visited, and that the maximum path sums for the subtrees of each node are considered.
- Here are the steps and ideas involved:

The core function is dfs, which is a recursive function that takes a node of the tree as an argument and returns the maximum path sum obtained by including the current node and extending to either its left or right child (not both).

negative path sums since including a negative path would decrease the overall sum, which is not optimal.

In the base case, if the current node is None, the function returns 0, meaning there is no contribution to the path sum.

When dfs is called on a non-null node, it first recursively calculates the maximum path sums of the left and right subtrees

with left = max(0, dfs(root.left)) and right = max(0, dfs(root.right)). The max(0, ...) pattern is used to ignore

The global variable ans is updated with the maximum of its current value and the newly calculated sum (ans = max(ans,

Finally, dfs returns root.val + max(left, right), taking into account the current node's value and the maximum

contribution from either the left or right child. This allows the function to relay upward through the recursion the best path

tree.

subtree for which root is the highest point.

sum contribution of the current node to its parent.

3. At 11, dfs goes to 7 (left child), which returns 7 because it is a leaf node.

4. Then dfs goes to the 2 (right child), which returns 2.

returned is 5 + max(left, right) to propagate up the tree.

After obtaining the maximum non-negative path sums from the left and right subtrees, it calculates the maximum path sum that includes the current node and both children: root.val + left + right. This is not the value returned by the function because a path should only extend in one direction; however, this value is crucial because it is the highest path sum for the

- root.val + left + right)). This step is essential because ans keeps track of the maximum path sum found in the entire tree, including paths that do not extend to the root of the entire tree.
- The overall maximum path sum is found by setting the initial value of ans to -inf to ensure that any path sum in the tree will be larger (since the tree is non-empty), and then calling dfs(root) which triggers the recursive process. The final value of ans after

the recursion completes gives us the result we are seeking – the maximum path sum of any non-empty path in the given binary

Example Walkthrough Let's illustrate the solution approach with a small binary tree example: Consider the following binary tree:

We want to find the maximum path sum in this tree using the solution approach described. 1. Start with the function dfs at the root node with the value 5. 2. The dfs function is called recursively on the left child 4 and the function call sequence continues down to 11.

5. At node 11, the function now has left and right values: the dfs result from the left is 7 and the right is 2. Node 11 itself has a value of 11, so

the local maximum we can generate at this node (including its value and both children) is 11 + 7 + 2 = 20. However, for the purposes of

9. The overall maximum path that can be formed including node 5 and its children is compared against the global maximum ans. The value

12. The final value of ans after recursion completes represents the maximum path sum from any sequence of nodes in the given binary tree.

For our example, let's assume that the left side (with nodes 5, 4, and 11) produces a path sum of 18, and the right side (with

nodes 5, 8, 4, and 1) produces a path sum of 18 as well. The highest path sum that includes the root would be 5 + 18 + 18 =

41. But suppose there's an even higher sum path that doesn't include the root, say 13 + 8 + 4, which equals 25 (node 13 is the

7. The recursion unfolds, and a similar process occurs for node 4 on the left and for 8, 13, 4, and 1 on the right side of the tree.

8. At node 5, the maximum path sums from left and right sides are collected (already calculated by dfs for child nodes).

11. The process continues until the topside of the recursion, where the dfs function was initially called with the root of the tree.

10. In each step, whenever a higher path sum is found at a node, the global maximum ans is updated.

propagation up the tree, we return $11 + \max(7, 2) = 11 + 7 = 18$. 6. The global maximum ans is updated if 20 is greater than the current value of ans.

Solution Implementation

self.val = val

self.left = left

self.right = right

if not node:

return 0

max_path_sum = float('-inf')

Start DFS with the root of the tree

def maxPathSum(self, root: TreeNode) -> int:

def dfs(node: TreeNode) -> int:

Helper function to perform depth-first search

left_max = max(0, dfs(node.left))

Base case: if the current node is None, return 0

To account for potentially all negative-valued trees

Recursively calculate the maximum path sum on the left subtree

If negative, we take 0 to avoid decreasing the overall path sum

After DFS is done, max_path_sum holds the maximum path sum for the tree

Python

class Solution:

root of this path). Hence, the final answer for this example would depend on which sum is larger, and ans would be updated with the maximum sum found.

Definition for a binary tree node. class TreeNode: def init (self. val=0, left=None, right=None):

Similarly, do the same for the right subtree right_max = max(0, dfs(node.right)) # Update the overall maximum path sum # This includes the node value and the maximum paths from both subtrees nonlocal max path sum max_path_sum = max(max_path_sum, node.val + left_max + right_max) # Return the maximum path sum without splitting # The current node's value plus the greater of its left or right subtree paths return node.val + max(left_max, right_max) # Initialize the overall maximum path sum to negative infinity

class Solution { private int maxSum = Integer.MIN_VALUE; // Initialize maxSum with the smallest possible integer value. // Returns the maximum path sum of any path that goes through the nodes of the given binary tree. public int maxPathSum(TreeNode root) {

return max_path_sum

dfs(root)

Java

```
calculateMaxPathFromNode(root);
        return maxSum;
   // A helper method that computes the maximum path sum from a given node and updates the overall max{\sf Sum} .
   private int calculateMaxPathFromNode(TreeNode node) {
        if (node == null) {
           // If the current node is null, we return 0 since null contributes nothing to the path sum.
           return 0;
       // Compute and get the maximum sum of paths from the left child;
       // if the value is negative, we ignore the left child's contribution by taking 0.
       int leftMaxSum = Math.max(0, calculateMaxPathFromNode(node.left));
       // Compute and get the maximum sum of paths from the right child;
       // if the value is negative, we ignore the right child's contribution by taking 0.
        int rightMaxSum = Math.max(0, calculateMaxPathFromNode(node.right));
       // Update maxSum with the greater of the current maxSum or the sum of the current node value plus leftMaxSum and rightMaxSum.
       // This accounts for the scenario where the path involving the current node and its left and right children yields the max pa
       maxSum = Math.max(maxSum, node.val + leftMaxSum + rightMaxSum);
       // This call must return the maximum path sum including the currently evaluated node and one of its subtrees
       // since a path cannot have branches and must be straight through the parents or children nodes.
        return node.val + Math.max(leftMaxSum, rightMaxSum);
/**
* Definition for a binary tree node.
*/
class TreeNode {
   int val;
   TreeNode left;
   TreeNode right;
   TreeNode() {}
   TreeNode(int val) {
       this.val = val;
   TreeNode(int val, TreeNode left, TreeNode right) {
       this.val = val;
       this.left = left;
        this.right = right;
```

class TreeNode { val: number; left: TreeNode | null; right: TreeNode | null: constructor(val?: number, left?: TreeNode | null, right?: TreeNode | null) {

};

TypeScript

C++

struct TreeNode {

TreeNode *left;

TreeNode *right;

// Constructors

int val:

class Solution {

};

public:

// Definition for a binary tree node.

int maxPathSum(TreeNode* root) {

if (!node) {

// Start DFS from root.

depthFirstSearch(root);

// Definition for a binary tree node.

this.val = (val === undefined ? 0 : val);

this.left = (left === undefined ? null : left);

// Node's value

TreeNode() : val(0), left(nullptr), right(nullptr) {}

TreeNode(int x) : val(x), left(nullptr), right(nullptr) {}

// Pointer to the left child

// Pointer to the right child

TreeNode(int x, TreeNode *left, TreeNode *right) : val(x), left(left), right(right) {}

// Depth-first search function to explore nodes and calculate max path sum.

std::function<int(TreeNode*)> depthFirstSearch = [&](TreeNode* node) {

int leftMax = std::max(0, depthFirstSearch(node->left));

return node->val + std::max(leftMax, rightMax);

return maxValue; // Return the global maximum path sum found.

int rightMax = std::max(0, depthFirstSearch(node->right));

maxValue = std::max(maxValue, leftMax + rightMax + node->val);

int maxValue = INT_MIN; // Initialize the maximum path sum as the smallest possible integer.

return 0; // Base case: If the node is null, return 0 as it adds nothing to the path sum.

// Recursively calculate the max path sum for the left and right children, ignoring negative sums.

// Return the maximum path sum of either the left or right subtree plus the current node value.

// Update the global max path sum by considering the current node value and its maximum left and right path sums.

```
this.right = (right === undefined ? null : right);
// Initialize a global variable to hold the maximum path sum
let maxPathSumValue = -1001;
/**
 * Helper function to calculate the maximum path sum
 * @param {TreeNode | null} node - The current node of the binary tree
 * @return {number} The maximum sum from the current node to any leaf
function maxPathSumDfs(node: TreeNode | null): number {
    // Base case: If the node is null, return 0
    if (!node) {
        return 0;
    // Calculate maximum sum starting from left child
    const leftMax = Math.max(0, maxPathSumDfs(node.left));
    // Calculate maximum sum starting from right child
    const rightMax = Math.max(0, maxPathSumDfs(node.right));
    // Update global maxPathSumValue with the maximum sum of the current subtree
    // which includes the current node and both left and right maximum sums
    maxPathSumValue = Math.max(maxPathSumValue, leftMax + rightMax + node.val);
    // Return the maximum sum from current node to any leaf
    return Math.max(leftMax, rightMax) + node.val;
/**
* Function to find the maximum path sum of a binary tree
* @param {TreeNode | null} root - The root of the binary tree
 * @return {number} The maximum path sum in the binary tree
 */
function maxPathSum(root: TreeNode | null): number {
    maxPathSumDfs(root): // Call the helper function starting from the root
    return maxPathSumValue; // Return the global maximum path sum
# Definition for a binary tree node.
class TreeNode:
   def init (self, val=0, left=None, right=None):
       self.val = val
       self.left = left
       self.right = right
class Solution:
    def maxPathSum(self, root: TreeNode) -> int:
       # Helper function to perform depth-first search
       def dfs(node: TreeNode) -> int:
           # Base case: if the current node is None, return 0
            if not node:
               return 0
           # Recursively calculate the maximum path sum on the left subtree
           # If negative, we take 0 to avoid decreasing the overall path sum
```

Time and Space Complexity

Start DFS with the root of the tree

left_max = max(0, dfs(node.left))

right_max = max(0, dfs(node.right))

nonlocal max path sum

max_path_sum = float('-inf')

dfs(root)

return max_path_sum

Update the overall maximum path sum

Similarly, do the same for the right subtree

Return the maximum path sum without splitting

To account for potentially all negative-valued trees

Initialize the overall maximum path sum to negative infinity

return node.val + max(left_max, right_max)

This includes the node value and the maximum paths from both subtrees

The current node's value plus the greater of its left or right subtree paths

max_path_sum = max(max_path_sum, node.val + left_max + right_max)

After DFS is done, max_path_sum holds the maximum path sum for the tree

the number of nodes in the binary tree. // The space complexity of the code is O(h), where h is the height of the tree. This is due to the recursion stack during the

depth-first search (DFS). In the worst case of a skewed tree, the space complexity becomes O(n), where n is the number of nodes, because the height of the tree can be n in the case of a completely unbalanced tree. For a balanced tree, the space complexity would be $O(\log n)$, because the height h would be proportional to $\log n$.

// The time complexity of the code is O(n) because the dfs function visits each node in the binary tree exactly once, where n is