2827. Number of Beautiful Integers in the Range Dynamic Programming Math Hard

Leetcode Link

The problem requires us to find out how many integers within a given range [low, high] can be considered "beautiful". An integer is

Problem Description

"beautiful" if it meets two criteria: The count of even digits in the number is equal to the count of odd digits.

- 2. The number must be divisible by k. We're tasked with returning the count of such beautiful integers.

Intuition

lead: a flag to denote if we're still at leading zeroes.

memoization for efficiency is a hallmark of the Digit DP approach.

The solution uses the following algorithms, data structures, and patterns:

To solve this problem, the solution employs a technique known as "Digit Dynamic Programming" (Digit DP). The idea behind Digit DP is to build numbers digit by digit, keeping track of various conditions that must be met. It allows us to calculate the number of valid numbers below a certain threshold.

The solution uses a depth-first search (DFS) function with memoization to incrementally construct possible integers digit by digit, and at each step, checks if the conditions of beauty are satisfied. 1. The dfs function is recursively called for each position pos in the number while tracking:

limit: a flag to indicate if we should be bounded by the number formed by the integer's digits up to this point or be free to

mod: the current remainder when the number constructed so far is divided by k.

diff: a measure to determine if the count of even digits is equal to the count of odd digits.

- consider all digits from 0 to 9. 2. The diff parameter is maintained as an offset of 10 and adjusted by adding 1 for odd digits and subtracting 1 for even digits with
- the goal of reaching a diff of exactly 10 to ensure there's an equal count of odd and even digits. 3. The recursive calls construct the number by exploring all the possibilities for each digit's place, respecting the current

constraints (like if we are limited by the maximum range or if we are considering leading zeroes).

limit and separately up to the low - 1 limit. 5. The actual count of beautiful integers within the [low, high] range is determined by subtracting the count obtained for low - 1

4. The function returns the count of valid consecutive integer sequences that fit the defined beauty requirements up to the high

- from the count obtained for high. The solution captures the subtleties of the problem by using smart state transitions that ensure no possibility is left unexamined
- **Solution Approach**

1. Depth-First Search (DFS): DFS allows us to explore all possible integer combinations by traversing through each digit from most

while at the same time preventing wasteful repetitions through memoization. The combination of DFS for enumeration and

2. Memoization: The @cache decorator in Python is used for memoization, effectively storing the results of the DFS function calls with a particular set of parameters to avoid recalculations. 3. Dynamic Programming (DP): By using memoization and the DFS pattern, the solution utilizes dynamic programming to build

digit by digit.

or low - 1).

significant to least significant.

upon previously computed states.

The implementation specifics are as follows:

digits add 1, even digits subtract 1).

Iterating over all digits 0-9 (0-up):

outside the given range.

(the leftmost digit).

computed.

function is then reset by clearing its cache.

• The key function in the solution is dfs(pos, mod, diff, lead, limit), which is used to recursively explore all possible numbers

 pos keeps track of the current digit index starting from the leftmost digit. mod represents the current remainder when the partially constructed number is divided by k.

diff is managed such that its final value should be 10, representing an equal number of odd and even digits (initialized to 10, odd)

• The DFS function is implemented to stop when pos exceeds the length of the string representation of the current boundary (high

If a nonzero digit is placed, the function updates mod, diff, and sets lead to false, as we are now creating a nonzero number.

The answer for the upper bound high is obtained by converting high to a string and passing it through the dfs function. The DFS

- Lead is a boolean flag indicating whether the current series of recursions are still in the leading zeros part of the number. limit is a flag to ensure we don't exceed the upper bound of the high-end of the range during the DFS.
- If the current digit is a leading zero, the recursive call adjusts only pos and limit. It continues leading zero considerations by keeping lead as true.
- Another call to dfs is made using low 1 to count the valid numbers up to the lower limit, ensuring that we don't count numbers

Finally, the difference between these two values gives us the count of beautiful integers within the [low, high] range.

The algorithm effectively breaks down a complicated counting problem into manageable states by using DFS and DP, while

memoization guarantees that the time complexity remains controlled by caching the results of states that have already been

Example Walkthrough

range [20, 23] where k = 2, meaning each beautiful number must be divisible by 2.

• The number 20 is divisible by 2, which satisfies the second condition (k = 2).

For 22, we adjust diff to 8, and since it is divisible by 2, it meets the conditions.

For 23, diff is back to 9, but since 23 isn't divisible by 2, it isn't considered.

def numberOfBeautifulIntegers(self, low: int, high: int, k: int) -> int:

check if it's divisible by k and has 10 distinct digits

upper_limit = int(num_str[position]) if is_limited else 9

If we've constructed a number of the same length,

return modulo == 0 and distinct_count == 10

Find the count of beautiful numbers up to 'high'

dfs.cache_clear() # Clear the cache to reuse the function

51 # and 'is_limited' indicates if we have a digit limit based on the target number.

// If we are not in leading zero or limit, update our cache

int numberOfBeautifulIntegers(int low, int high, int k) {

// Convert the high limit number into a string.

std::string high_str = std::to_string(high);

if (position >= high_str.size()) {

// Initialize the memoization table for dynamic programming.

// Base case - if we've looked at all digits in high_str.

return memo[position][remainder][even_odd_diff];

int upper_bound = limit ? high_str[position] - '0' : 9;

// Calculate the next even/odd differential.

memo[position][remainder][even_odd_diff] = count;

// First, find the count of beautiful integers up to the high limit.

// Return running count of beautiful integers.

for (int digit = 0; digit <= upper_bound; ++digit) {</pre>

// Determine the limit for the current digit.

if (digit == 0 && leading_zeros) {

if (!leading_zeros && !limit) {

int count_high = dfs(0, 0, 10, true, true);

memset(memo, -1, sizeof(memo));

// Reset the memoization table for the next call.

return (remainder == 0 && even_odd_diff == 10) ? 1 : 0;

// Declare the depth first search (dfs) function that we'll use for our digit DP.

// Check if the number has equal odds and evens and if it's divisible by k.

if (!leading_zeros && !limit && memo[position][remainder][even_odd_diff] != -1) {

// Go one level deeper while preserving the leading zeros.

int next_diff = even_odd_diff + (digit % 2 == 1 ? 1 : -1);

// Go one level deeper taking current digit into account.

// Store the count for the current state into the memoization table.

// Check if we can use memoized data (We memoize on non-leading zeros and when we are not at the

count += dfs(position + 1, remainder, even_odd_diff, true, limit && digit == upper_bound);

count += dfs(position + 1, (remainder * 10 + digit) % k, next_diff, false, limit && digit == upper_bound);

std::function<int(int, int, int, bool, bool)> dfs = [&](int position, int remainder, int even_odd_diff, bool leading_zeros,

return ans; // Return the cumulative count of beautiful numbers

if (!isLeadingZero && !isLimit) {

cache[pos][mod][diff] = ans;

4. Continuing With Numbers 22 and 23: We continue this process for 22 and 23:

(dfs(high)) and the lower bound adjusted by one (dfs(low - 1)).

from functools import lru_cache

if position == len(num_str):

@lru_cache(None)

else:

return ans

num_str = str(high)

44 # parity (even or odd) alternates.

 $count_high = dfs(0, 0, 10, 1, 1)$

Let's illustrate the solution approach with a small example. Suppose we want to find the number of beautiful integers within the

2. Exploring Number 20: For the first number, 20: The leftmost digit is 2, which is even. So, we decrease diff by 1 (diff = 9), signifying one more even than odd digits so far.

The next digit is 0, which is even, and since it's a leading zero, we only change our position and maintain lead as true.

1. Initialization: We start off by initializing diff to 10. We'll use the dfs function to explore possible numbers starting at position 0

3. Moving to the Next Number: We cannot increment beyond the number 23 as it is our high limit, so we look at the number 21, in which we add 1 to diff for the odd last digit, making it diff = 10 again. The number 21, however, is not divisible by 2 and hence isn't beautiful.

which in this case would be [20, 19]. We perform the same operation for this range but expect to find 0 beautiful integers since

1 class Solution:

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54 }

C++ Solution

1 #include <functional>

int memo[11][21][21];

memset(memo, -1, sizeof(memo));

// numerical limit).

int count = 0;

} else {

return count;

};

2 #include <cstring>

class Solution {

public:

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memo = Array(11)

.fill(0)

for mod, and 10 options for diff.

#include <string>

Java Solution

there are 2 beautiful integers.

it's below our range starting point. 7. That yields our final answer: After subtracting the count from low - 1, we still have 2 beautiful integers as our count.

This example demonstrates how the dfs function would explore all possible numbers for the given range, adjusting mod and diff

from the cache instead of recomputing. The final answer is obtained by the arithmetic difference between the upper bound

def dfs(position: int, modulo: int, distinct_count: int, is_leading: int, is_limited: int) -> int:

Adjust the distinct digit count depending on the parity of the digit

next_distinct = distinct_count + (1 if digit % 2 == 1 else -1)

accordingly to identify valid integers. Memoization ensures that if we were to calculate the same state again, we retrieve the count

6. Adjusting for Lower Limit: We also need to subtract the number of valid sequences up to one less than the lower limit (low - 1),

5. Counting Beautiful Numbers: Out of the numbers 20 through 23, the ones that satisfy all the conditions are 20 and 22. Hence,

- Python Solution
- 13 ans = 014 15 # Try all possible digits for current position 16 for digit in range(upper_limit + 1): # Leading zeros are treated differently since they don't affect the distinct digit count 17 18 if digit == 0 and is_leading:

ans += dfs(position + 1, modulo, distinct_count, 1, is_limited and digit == upper_limit)

ans += dfs(position + 1, (modulo * 10 + digit) % k, next_distinct, 0, is_limited and digit == upper_limit)

33 $num_str = str(low - 1)$ 34 $count_{low} = dfs(0, 0, 10, 1, 1)$ 35 36 # The difference will give the count for the inclusive range [low, high] 37 return count_high - count_low 38 39 # Explanation: 40 # The above class Solution contains a method named 'numberOfBeautifulIntegers' which calculates the

41 # count of beautiful integers within the closed interval [low, high] that are also divisible by 'k'.

43 # A 'beautiful integer' is defined as an integer that contains exactly 10 distinct digits and their

46 # The internal 'dfs' (depth first search) function is a recursive function that explores all the valid

47 # combinations of digits from the current 'position' up to the length of the number in string form 'num_str'.

49 # The 'modulo' parameter represents the current value modulo 'k', 'distinct_count' keeps track of the count

50 # of distinct digits encountered so far, 'is_leading' is a flag to check if we're still at leading zeroes,

Find the count of beautiful numbers below 'low' (since we're excluding the lower boundary)

```
1 class Solution {
       private String numberString; // The string representation of the number we are working with
       private int k;
                                     // The given k value
       private Integer[][][] cache = new Integer[11][21][21]; // Cache to store intermediate results for dynamic programming
 5
       // Main method to find the number of 'beautiful' integers between low and high inclusive
 6
       public int numberOfBeautifulIntegers(int low, int high, int k) {
 7
           this.k = k; // Set the global k value
 8
 9
           numberString = String.valueOf(high); // Convert the upper limit to string
10
           int countUpToHigh = dfs(0, 0, 10, true, true); // Count beautiful numbers up to high
           numberString = String.valueOf(low - 1); // Convert the lower limit to string after subtracting one
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           cache = new Integer[11][21][21]; // Reset the cache for the new range
13
           int countUpToLowMinusOne = dfs(0, 0, 10, true, true); // Count beautiful numbers up to low-1
14
           return countUpToHigh - countUpToLowMinusOne; // Return the difference as the result
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       // Helper method to perform depth-first search and count beautiful numbers
18
       private int dfs(int pos, int mod, int diff, boolean isLeadingZero, boolean isLimit) {
19
           // Termination condition: if we have reached the end of the number string
           if (pos >= numberString.length()) {
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21
               // If at the end the mod is 0 and diff is 10 (no number has been used),
22
               // we have found a valid number, otherwise return 0
23
               return mod == 0 && diff == 10 ? 1 : 0;
24
25
26
           // Check our cache to save time if the result is already computed
27
           if (!isLeadingZero && !isLimit && cache[pos][mod][diff] != null) {
               return cache[pos][mod][diff];
28
29
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31
           int ans = 0;  // Initialize the answer for the current position to 0
32
           int upperBound = isLimit ? numberString.charAt(pos) - '0' : 9; // Set the maximum digit we can place here
33
34
           // Iterate through all possible digits we can place
35
           for (int digit = 0; digit <= upperBound; ++digit) {</pre>
36
               if (digit == 0 && isLeadingZero) {
37
                   // If the current digit is 0 and we are still in the leading zeroes part
38
                   ans += dfs(pos + 1, mod, diff, true, isLimit && digit == upperBound);
               } else {
39
40
                   // Decide the next diff value based on the current digit
41
                   int nextDiff = diff + (digit % 2 == 1 ? 1 : -1);
42
                   // Add the count of beautiful numbers starting with the current digit
43
                   ans += dfs(pos + 1, (mod * 10 + digit) % k, nextDiff, false, isLimit && digit == upperBound);
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```

55 56 57 58 59 };

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             // Convert the low limit number into a string.
 53
             std::string low_str = std::to_string(low - 1);
 54
            // Next, find the count of beautiful integers up to the low limit minus one.
             int count_low = dfs(0, 0, 10, true, true);
             // The final result is the difference between the two counts.
             return count_high - count_low;
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Typescript Solution
    function numberOfBeautifulIntegers(low: number, high: number, k: number): number {
         let highAsString = String(high);
         let memo: number[][][] = Array(11)
             .fill(0)
             .map(() =>
                 Array(21)
                     .fill(0)
  8
                     .map(() \Rightarrow Array(21).fill(-1)),
            );
  9
 10
         // Depth-first search function to find the number of beautiful integers
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 12
         const depthFirstSearch = (position: number, remainder: number, digitDifference: number, leadingZero: boolean, isLimit: boolean)
 13
             // Base case: If the current position is at the end of the string length,
 14
             // check if the number is divisible by k and the digit difference is 10 (beautiful).
 15
             if (position === highAsString.length) {
 16
                 return remainder === 0 && digitDifference === 10 ? 1 : 0;
 17
 18
            // If there's no leading zero, not at limit, and value is already computed in memo array, return the value.
 19
             if (!leadingZero && !isLimit && memo[position][remainder][digitDifference] !== -1) {
 20
                 return memo[position][remainder][digitDifference];
 21
 22
             let count = 0;
 23
             const upperBound = isLimit ? Number(highAsString[position]) : 9;
 24
 25
             // Explore all possible next digits from 0 to upper bound
 26
             for (let digit = 0; digit <= upperBound; ++digit) {</pre>
 27
                 if (digit === 0 && leadingZero) {
                     // If the digit is 0 and it's a leading zero, we keep the leadingZero flag true.
 28
 29
                     count += depthFirstSearch(position + 1, remainder, digitDifference, true, isLimit && digit === upperBound);
 30
                 } else {
 31
                     // Update the digit difference and the remainder when adding the current digit
 32
                     const nextDigitDifference = digitDifference + (digit % 2 === 0 ? -1 : 1);
 33
                     count += depthFirstSearch(position + 1, (remainder * 10 + digit) % k, nextDigitDifference, false, isLimit && digit
 34
 35
 36
             // Memoize the result if there's no leading zero and not at the digit limit
 37
 38
             if (!leadingZero && !isLimit) {
                 memo[position][remainder][digitDifference] = count;
 39
 40
 41
            return count;
        };
 42
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 44
        // First, calculate the count of beautiful numbers less than or equal to high
 45
        const countHigh = depthFirstSearch(0, 0, 10, true, true);
 46
        // Update highAsString to the value of 'low - 1'
        highAsString = String(low - 1);
```

to k - 1, diff can theoretically range from -L to L, lead can be either 0 or 1, and limit can also be either 0 or 1, their multiplications define the number of states. However, note that diff values are actually going from 0 to 9 when counting unique differences, since

The time complexity can be roughly estimated as 0(10 * k * L * 2 * 2).

// Reset memo array for the new range calculation

51 .map(() =>52 Array(21) 53 .fill(0) 54 .map(() => Array(21).fill(-1)),55); 56 // Calculate the count of beautiful numbers less than 'low' 57 const countLow = depthFirstSearch(0, 0, 10, true, true); 58 // Return the difference to get the count of beautiful integers in the range [low, high] 59 return countHigh - countLow; 60 61 Time and Space Complexity The time complexity of the DFS function primarily depends on the number of possible states. The state is defined by the parameters (pos, mod, diff, lead, limit). Since pos can take values from 0 to L where L is the number of digits in high, mod can range from 0

the maximum depth of the DFS. For memoization, we store a unique result for each of the possible states, giving us a space

complexity similar to time complexity, which is 0(10 * k * L) because we don't need to consider the space for precomputing limit and lead, which are just passed along in the recursive calls without consuming additional space. Thus, the overall space complexity is 0(10 * k * L).

diff == 10 implies a valid count of unique differences. Hence, we have 2 options for lead and limit each, L options for pos, k options

The space complexity is affected by the depth of the recursion and the memoization used. The recursion depth is O(L) since that is