309. Best Time to Buy and Sell Stock with Cooldown

Medium Array Dynamic Programming Leetcode Link

Problem Description

In this challenge, you are given an array `prices` where `prices[i]` represents the price of a given stock on the `i`-th day. Your task is to calculate the maximum profit that can be achieved through making as many buy-and-sell transactions as you wish, with a specific restriction: after selling a stock, you must wait one day before buying again.

To be more explicit, you can buy one share and then sell it. However, after you sell, you need to skip a day before you can initiate another purchase. This is called a cooldown period. Additionally, it is important to note that you cannot hold more than one share at a time; before you can buy another share, you must have already sold the previous one. Your goal is to find out the best strategy to maximize your profit under these terms.

Intuition

The intuition behind the solution involves dynamic programming. Since we cannot engage in a new transaction on the day following a sale, we can make a decision for each day - to buy, sell, or rest (do nothing). The key is to track the profits of these decisions while considering the cooldown.

We can maintain two states for each day that represent the decisions and conditions:

1. `f0`: The maximum profit we can have if we rest (do not buy or sell) on the current day or if we sell on the current day.
2. `f1`: The maximum profit we can have if we buy on the current day.

For day 0, our states would be initialized as `f0 = 0` (we start with no stock and no cooldown) and `f1 = -prices[0]` (since buying the stock would cost us the price of the stock).

We need a third temporary variable `f` to store the previous value of `f0` (from the day before), which will be used to update `f1` since we cannot buy the stock on the same day we sell.

The dynamic programming transitions can be described as follows:

- `f0` will be the maximum of itself (choose to rest, and profit doesn't change) or `f1 + x` (choosing to sell the stock bought at a cost of `f1`, which means adding the current price `x`).
- `f1` will be the maximum of itself (choosing to do nothing with the previously bought stock) or `f - x` (buying a new stock after cooldown, which means subtracting the current price `x` from the profit made before cooldown indicated by `f`).

In each iteration, we go through the prices starting with the second day, because the first day's decisions are predetermined as `f0 = 0` and `f1 = -prices[0]`. The result returned will be `f0`, which indicates the maximum profit that could be made with the last action being a rest or a sale.

The implementation of this solution successfully combines the conditions of the problem into a dynamic iterative process that captures the essence of buy-and-sell strategies while complying with the cooldown constraint.

Solution Approach

The solution is implemented using a dynamic programming approach, which utilizes iteration and optimal substructure properties common in such problems.

- **Initialization**: Before the loop begins, we prepare our initial state variables:
  - `f0` is initialized to `0`, representing the profit with no stock on hand at the start.
  - `f1` is set to `-prices[0]`, accounting for the cost of buying stock on the first day.
  - A temporary variable `f` will hold the previous value of `f0` during iteration, serving as a memory for the state of profits two days ago.
- **Iteration**: We begin iterating through the prices array from the second day onward (since the first day's decisions have already been set up). During each iteration, we perform two key updates to represent possible actions and their outcomes:
  - `f0` gets updated to `max(f0, f1 + x)`, where `x` is the current day's stock price. This represents the maximum profit between not selling/buying anything on the current day (`f0`) and selling the stock bought at `f1` price (therefore, `f1 + x`).
  - `f1` gets updated to `max(f1, f - x)`, which represents the maximum profit between keeping the stock obtained before (`f1`) or buying today (which requires the previous day's profit `f` minus the current price `x`; note this action is only permitted after a cooldown).
- **Data Structures**: We use three integer variables `f0`, `f1`, and `f`, which change with each iteration to keep track of our decision impacts. No additional data structures are used, making the space complexity linear.
- **Pattern**: The pattern applied here is reminiscent of state machine logic used in dynamic programming. Multiple states (`f0` and `f1`) are considered, with transitions based on conditions specified by the problem statement. The decision on each day depends on the state of the previous days (either one or two days back).
- **Algorithm Completion**: The loop continues to make decisions and update states based on the stock prices for each day. The algorithm completes after the last day in the input array. The return value is `f0` because it represents the maximum profit obtainable with the last action being rest or sale, aligned with the problem requirements.

The algorithm's overall complexity is **O(n)**, as it involves a single pass through the `prices` array, and the operations within each iteration are constant time. This approach effectively balances the problem constraints and the need to make optimized decisions at each step, leading to an efficient and intuitive solution.

Example Walkthrough

Let's consider a small example using the solution approach outlined above. Suppose we have the following stock prices over a series of days: `prices = [1, 2, 3, 0, 2]`. Using the provided algorithm and dynamic programming approach, we'll calculate the maximum profit possible.

- **Day 0**: Before the loop starts, let's initialize our variables.
  - `f0` is set to `0`, because we haven't made any transaction yet.
  - `f1` is set to `-prices[0]` which is `-1`, because we bought one share of stock at the price of `1`.
- **Day 1**:
  - Update `f` to the previous value of `f0` which is `0`.
  - Calculate `f0` as `max(f0, f1 + prices[1])` = `max(0, -1 + 2)` = `1`.
  - Update `f1` as `max(f1, f - prices[1])` = `max(-1, 0 - 2)` = `-1`.
  - At the end of Day 1, `f0` is `1` and `f1` is `-1`.
- **Day 2**:
  - Update `f` to the previous value of `f0` which is `1`.
  - Calculate `f0` as `max(f0, f1 + prices[2])` = `max(1, -1 + 3)` = `2`.
  - Update `f1` as `max(f1, f - prices[2])` = `max(-1, 1 - 3)` = `-1`.
  - At the end of Day 2, `f0` is `2` and `f1` is `-1`.
- **Day 3**:
  - Update `f` to the previous value of `f0` which is `2`.
  - Calculate `f0` as `max(f0, f1 + prices[3])` = `max(2, -1 + 0)` = `2`.
  - Update `f1` as `max(f1, f - prices[3])` = `max(-1, 2 - 0)` = `2`.
  - At the end of Day 3, `f0` is `2` and `f1` is `2`.
- **Day 4**:
  - Update `f` to the previous value of `f0` which is `2`.
  - Calculate `f0` as `max(f0, f1 + prices[4])` = `max(2, 2 + 2)` = `4`.
  - Update `f1` as `max(f1, f - prices[4])` = `max(2, 2 - 2)` = `2`.
  - At the end of Day 4, `f0` is `4` and `f1` is `2`.

After iterating through all the days, our algorithm concludes and the maximum profit that we can yield from the given prices is stored in `f0`, which is `4`. This means the best strategy would have resulted in a total profit of `4`, considering all the buy-and-sell transactions and the compulsory cooldown period after each sale.

Python Solution

```python
from typing import List

class Solution:
    def maxProfit(self, prices: List[int]) -> int:
        # Initialize variables:
        # freeze_profit (f) - profit of the day before cooldown
        # sell_profit (f0) - profit after selling the stock
        # hold_profit (f1) - profit after buying the stock or holding onto the stock bought previously
        freeze_profit, sell_profit, hold_profit = 0, 0, -prices[0]

        # Iterate through the stock prices, starting from the second day
        for current_price in prices[1:]:
            # Update profits for the current day
            # freeze_profit remains as the sell_profit from the previous day
            # sell_profit is the maximum of either keeping the previous sell_profit or selling stock today (hold_profit + current_pri
            # hold_profit is the max of either keeping the stock bought previously or buying new stock after cooldown (freeze_profit
            freeze_profit, sell_profit, hold_profit = (
                sell_profit,
                max(sell_profit, hold_profit + current_price),
                max(hold_profit, freeze_profit - current_price)
            )

        # The maximum profit will be after all trades are done, which means no stock is being held, hence sell_profit
        return sell_profit

# Example usage:
# sol = Solution()
# profit = sol.maxProfit([7,1,5,3,6,4])
# print(profit) # Output: 5

```

Java Solution

```java
class Solution {
    public int maxProfit(int[] prices) {
        // Initialize the placeholders for the maximum profits
        int previousNoStock = 0;          // f0 represents the max profit till previous day with no stock in hand
        int previousWithStock = -prices[0]; // f1 represents the max profit till previous day with stock in hand
        int tempPreviousNoStock;          // Used to store previous no stock state temporarily

        // Parser through the price list starting from day 1 as we have initial state for day 0 already considered
        for (int i = 1; i < prices.length; i++) {
            int currentNoStock = Math.max(previousNoStock, previousWithStock + prices[i]); // Either keep no stock or sell the stock
            previousWithStock = Math.max(previousWithStock, previousNoStock - prices[i]);  // Either keep the stock we have or buy ne
            tempPreviousNoStock = previousNoStock; // Temporarily store the previous no stock state
            previousNoStock = currentNoStock; // Update the previous no stock state with the current state
        }
        return previousNoStock; // At the end, the profit with no stock in hand will be the maximum profit
    }
}

```

C++ Solution

```cpp
class Solution {
public:
    int maxProfit(vector<int>& prices) {
        // Initialize the profit states:
        // f0: the max profit we can have at this state if we don't hold a stock
        // f1: the max profit we can have at this state if we hold one stock
        // holdProfit: stores the previous f0 state to calculate the new f1 state
        int noStockProfit = 0;
        int holdProfit = 0;
        int oneStockProfit = -prices[0]; // Assume we bought the first stock

        // Loop through the list of prices starting from the second price
        for (int i = 1; i < prices.size(); ++i) {
            // Calculate the max profit if we don't hold a stock today
            // either we did not hold a stock yesterday (noStockProfit) or
            // we sold the stock we were holding (oneStockProfit + prices[i]).
            int newNoStockProfit = max(noStockProfit, oneStockProfit + prices[i]);

            // Calculate the max profit if we hold one stock today
            // either we were already holding a stock (oneStockProfit) or
            // we buy a new stock today (holdProfit - prices[i]).
            oneStockProfit = max(oneStockProfit, holdProfit - prices[i]);

            // Update holdProfit to the previous noStockProfit at the end of the day
            holdProfit = noStockProfit;
            // Update noStockProfit to the new calculated noStockProfit
            noStockProfit = newNoStockProfit;
        }
        // Since we want to maximize profit, we should not hold any stock at the end
        // hence we return noStockProfit which represents the max profit with no stock in hand
        return noStockProfit;
    }
};

```

Typescript Solution

```typescript
function maxProfit(prices: number[]): number {
    // Initialize the first day's profit status variables;
    // f0 represents the profit having no stock at day's end,
    // f1 represents the profit having stock at day's end,
    // prevProfitWithStock represents the profit having stock at the previous day's end.
    let noStockProfit: number = 0;
    let prevNoStockProfit: number = 0;
    let profitWithStock: number = -prices[0];

    // Iterate over the prices from the second day onward.
    for (const price of prices.slice(1)) {
        // Update the profit status by choosing the best profit strategy for the day:
        // Keep no stock or sell the stock (prevProfitWithStock + price),
        // Keep the stock or buy new stock (prevNoStockProfit - price).
        const tempNoStockProfit = prevNoStockProfit;
        prevNoStockProfit = Math.max(prevNoStockProfit, profitWithStock + price);
        profitWithStock = Math.max(profitWithStock, noStockProfit - price);
        noStockProfit = tempNoStockProfit;
    }

    // Return the maximum profit status when ending with no stock.
    return prevNoStockProfit;
}

```

Time and Space Complexity

The given Python code defines a method `maxProfit` designed to find the maximum profit that can be achieved from a sequence of stock prices, where `prices` is a list of stock prices.

Time Complexity

The time complexity of the code is driven by a single loop that iterates through the list of stock prices once (excluding the first price which is used for initial setup). Inside the loop, the code performs a fixed number of comparisons and arithmetic operations for each price. Because these operations occur within the loop and their number does not depend on the size of `prices`, they constitute constant-time operations.

Therefore, the time complexity is determined solely by the number of iterations, which is `n - 1`, where `n` is the length of `prices`. Considering big O notation, which focuses on the upper limit of performance as the input size grows, the time complexity of this algorithm is `O(n)`.

Space Complexity

The space complexity is assessed based on the additional memory used by the algorithm as a function of the input size. In this code, only a fixed number of variables `f`, `f0`, and `f1` are used, regardless of the size of the input list `prices`. There is no use of any additional data structures that would grow with the input size.

Thus, the space complexity of the algorithm is `O(1)`, denoting constant space usage.