2845. Count of Interesting Subarrays

Medium | Array | Hash Table | Prefix Sum | Leetcode Link

Problem Description

The problem provides an integer array called `nums`, indexed from 0. Additionally, two integers `modulo` and `k` are given. The task is to count the number of subarrays considered "interesting." A subarray is defined as a contiguous non-empty sequence of elements within the array. For a subarray `nums[l..r]` to be interesting, it must satisfy the condition that among its elements, the number of indices `i` (where `l` ≤ `i` ≤ `r`) such that `nums[i] % modulo == k` must itself be congruent to `k` when taken modulo `modulo`, i.e., `cnt % modulo == k`.

To understand better, consider these points:

- You go through the array and find all possible contiguous subarrays.
- For each subarray, you count the elements that, when divided by `modulo`, leave a remainder of `k`.
- If the count of such elements in the subarray also, when divided by `modulo`, leaves a remainder of `k`, that subarray is called interesting, and you need to increase the interesting subarray count by one.
- The output is the total number of interesting subarrays found this way.

Intuition

To approach this problem efficiently, without checking every possible subarray individually (which would be too time-consuming), we can use a technique from combinatorics that involves keeping track of the cumulative sums of certain conditions.

Here's the intuitive step-by-step breakdown:

1. Transform the original array `nums` such that each element becomes `1` if it satisfies `nums[i] % modulo == k` or `0` otherwise. Let's call this array `arr`.
2. Create a prefix sum array `s` such that `s[i]` represents the total count of '1's from the start of `arr` to the current index `i`. This helps us to quickly calculate the total number of '1's in any subarray.
3. Use a hash-based data structure, like Counter in Python, to keep track of how many times each possible prefix sum modulo `modulo` has occurred. The key idea is that if the difference between the prefix sums of two indices is congruent to `k` modulo `modulo`, it implies the subarray between those two indices is interesting.
4. As we iterate through the `arr`, we add to a running sum (`s`) the value of the current element. We then look up in our Counter how many times we've seen prefix sums that are `k` less than the current sum `mod modulo`. These contribute to our answer.
5. We update our Counter with the new running sum `mod modulo` at each index, incrementing the count since we've now encountered another subarray with that sum.

Applying these ideas, we achieve a solution that is linear with respect to the size of `nums`, hence much more efficient than examining all possible subarrays individually.

Solution Approach

The provided solution utilizes an array transformation, prefix sums, modular arithmetic, and a hash map for efficient lookups to tackle the subarray counting challenge.

1. **Array Transformation**: First, the code transforms the original `nums` array into a binary array `arr` with the same length. Each element in `arr` is set to `1` if `nums[i] % modulo` equals `k`, otherwise, it is set to `0`. This transformation simplifies the problem by converting it into a problem of counting the number of subarrays whose sum is congruent to `k` modulo `modulo`.

   ```python
   arr = [int(x % modulo == k) for x in nums]
   ```

2. **Using a HashMap (Counter) for Prefix Sum Lookup**: The Counter is used to store the frequency of the occurrence of prefix sums modulo `modulo`. Initially, Counter is set to `{0: 1}` because we start with a sum of `0` and there is one way to have a sum of `0` (no elements).

   ```python
   cnt = Counter()
   cnt[0] = 1
   ```

3. **Calculating Prefix Sums and Counting Interesting Subarrays**: As we iterate through each element in the transformed array `arr`, we maintain a running sum `s`. For each new element `x`, the running sum `s` is incremented by `x`, representing the sum of a prefix ending at this index.

   ```python
   s += x
   ```

   We then determine the number of interesting subarrays that end at the current index by looking up how many times we've seen prefix sums that would make the sum of the current subarray equal to `k` modulo `modulo`. This is done by checking `cnt[(s - k) % modulo]`.

   ```python
   ans += cnt[(s - k) % modulo]
   ```

   After checking for interesting subarrays, we update the Counter with the new running sum modulo `modulo` to account for the new subarray ending at this index.

   ```python
   cnt[s % modulo] += 1
   ```

4. **Returning the Result**: The variable `ans` is used to accumulate the count of interesting subarrays. After iterating over the array `arr`, `ans` holds the final count of all interesting subarrays, which is returned as the result.

   ```python
   return ans
   ```

The overall time complexity of the solution is O(n), as it requires a single pass through the array, and space complexity is also O(n) due to the additional array `arr` and the Counter which might store up to `modulo` distinct prefix sums.

Example Walkthrough

Let's work through an example to illustrate the solution approach.

Consider the integer array `nums = [1, 2, 3, 4, 5]`, with `modulo = 2`, and `k = 1`. The task is to count the number of interesting subarrays based on the given criteria.

1. **Array Transformation**: We transform `nums` into `arr` by setting each `arr[i]` to `1` if `nums[i] % 2 == 1`, otherwise `0`. Thus, `arr` becomes `[1, 0, 1, 0, 1]`, since `1`, `3`, and `5` are odd numbers and give a remainder of `1` when divided by `2`.

2. **Using a HashMap (Counter) for Prefix Sum Lookup**: Initialize a Counter with `{0: 1}`, representing that the sum of zero occurs once at the beginning (no subarray).

   ```python
   cnt = Counter({0: 1})
   ```

3. **Calculating Prefix Sums and Counting Interesting Subarrays**: Now, we start iterating through `arr` and sketch out the process dynamically:

   - For `arr[0]`, which equals `1`, we increment our running sum `s = 0 + 1 = 1`. We then look in the Counter for `cnt[(1 - 1) % 2] = cnt[0]`, which is `1`, as we have seen a prefix sum (that sums to `0`) exactly once before adding `arr[0]`. We add `1` to our answer and update the Counter to `cnt[1] += 1`.

   - For `arr[1]` with a value of `0`, our running sum does not change (`s = 1`). We look up `cnt[(1 - 1) % 2] = cnt[0]`, again finding a `1`. We add it to our answer (now `ans = 2`) and leave the Counter unchanged since `arr[1]` is `0`.

   - For `arr[2] = 1`, `s` is updated to `2`. We check `cnt[(2 - 1) % 2] = cnt[1]` in the Counter, which is `1`, so our answer increments to `3`. We then increment `cnt[2 % 2] = cnt[0]` by `1`.

   By iterating over the entire array `arr`, we keep a running sum `s`, check the Counter, and update the counts of encountered modulated prefix sums, accumulating the number of interesting subarrays into `ans`.

4. **Returning the Result**: After iterating over the entire array `arr`, we will have the final count of all interesting subarrays stored in the variable `ans`. Assuming we were incrementing `ans` along with the iterations as described, the final result would be returned.

This approach simplifies the process, ensuring we only need to traverse the array once, giving an O(n) time complexity, which is efficient for large arrays.

Python Solution

```python
from collections import Counter

class Solution:
    def countInterestingSubarrays(self, nums: List[int], modulo: int, k: int) -> int:
        # This array will contain 1s at the positions where the element
        # in the original nums array satisfies the condition (x % modulo == k).
        interesting_elements = [int(num % modulo == k) for num in nums]

        # Counter to store the frequency of cumulative sums mod modulo
        cumulative_sum_frequency = Counter()
        # Initialize with 0 sum having 1 frequency as this represents empty subarray
        cumulative_sum_frequency[0] = 1

        # Initialize answer and cumulative sum
        count_interesting_subarrays = 0
        cumulative_sum = 0

        # Iterate through the boolean array to count interesting subarrays
        for element in interesting_elements:
            cumulative_sum += element
            # The current sum minus the target sum (k) mod modulo will tell us
            # if there is a subarray ending at the current index which is interesting
            count_interesting_subarrays += cumulative_sum_frequency[(cumulative_sum - k) % modulo]
            # Update cumulative frequency counter
            cumulative_sum_frequency[cumulative_sum % modulo] += 1

        # Return the total count of interesting subarrays
        return count_interesting_subarrays

```

Java Solution

```java
import java.util.List;
import java.util.Map;
import java.util.HashMap;

class Solution {
    // Method that counts the number of subarrays where the number of elements equal to k modulo is also k.
    public long countInterestingSubarrays(List<Integer> nums, int modulo, int k) {
        int totalCount = nums.size(); // Total number of elements in nums
        int[] remainders = new int[totalCount]; // Array to store the remainders

        // Populate the remainders array with 1 if nums[i] % modulo == k or with 0 otherwise
        for (int i = 0; i < totalCount; i++) {
            remainders[i] = nums.get(i) % modulo;
            if (remainders[i] == k) {
                remainders[i] = 1;
            } else {
                remainders[i] = 0;
            }
        }

        Map<Integer, Integer> remainderCounts = new HashMap<>(); // Map to store the remainder frequencies
        remainderCounts.put(0, 1); // Initialize with 0 remainder seen once
        long interestingSubarraysCount = 0; // Variable to hold the final count of interesting subarrays
        int sum = 0; // Variable to accumulate the sum of remainders

        // Iterate over the remainders array
        for (int remainder : remainders) {
            sum += remainder; // Increase the sum with the current remainder
            // Increase the count by the number of occurrences where the adjusted sum matches the expected remainder
            interestingSubarraysCount += remainderCounts.getOrDefault((sum - k + modulo) % modulo, 0);
            // Update the map with the current modulus of the sum and increase the count by 1 or set it to 1 if not present
            remainderCounts.merge(sum % modulo, 1, Integer::sum);
        }

        return interestingSubarraysCount; // Return the count of interesting subarrays
    }
}

```

C++ Solution

```cpp
#include <vector>
#include <unordered_map>

class Solution {
public:
    // Function that counts the number of "interesting" subarrays
    // An "interesting" subarray is one where the sum of its elements, modulo 'modulo', equals 'k'
    long long countInterestingSubarrays(std::vector<int>& nums, int modulo, int k) {
        int n = nums.size(); // Get the size of the 'nums' vector
        std::vector<int> modArray(n); // Array to store modulo transformations

        // Preprocessing: Populate 'modArray' with 1 if nums[i] % modulo == k, otherwise 0
        for (int i = 0; i < n; ++i) {
            modArray[i] = (nums[i] % modulo == k) ? 1 : 0;
        }

        // Hash map to keep track of the count of prefix sums modulo 'modulo'
        std::unordered_map<int, int> prefixCount;
        prefixCount[0] = 1; // Initialize for the case where subarray begins at index 0

        long long interestingSubarraysCount = 0; // Variable to store the count of interesting subarrays
        int currentSumModulo = 0; // Variable to store current prefix sum modulo 'modulo'

        // Iterate over the modified array to count "interesting" subarrays
        for (int element : modArray) {
            currentSumModulo += element; // Update current prefix sum
            // Calculate adjusted sum for negative cases and find count in 'prefixCount'
            interestingSubarraysCount += prefixCount[(currentSumModulo - k + modulo) % modulo];
            // Increase the count for this sum modulo 'modulo'
            prefixCount[currentSumModulo % modulo]++;
        }

        // Return the final count of "interesting" subarrays
        return interestingSubarraysCount;
    }
};

```

Typescript Solution

```typescript
function countInterestingSubarrays(nums: number[], modulo: number, targetRemainder: number): number {
    // Initialize an array to store binary values, 1 for integers that have a remainder equal to targetRemainder when divided by modu
    const binaryRemainderArray: number[] = [];
    for (const num of nums) {
        binaryRemainderArray.push(num % modulo === targetRemainder ? 1 : 0);
    }

    // Create a map to count the occurrences of cumulative sums modulo 'modulo'.
    const cumulativeSumCounts: Map<number, number> = new Map();
    cumulativeSumCounts.set(0, 1); // Initialize with a zero sum to account for subarrays that start from index 0.

    let interestingSubarraysCount = 0; // Initialize the count of interesting subarrays.
    let cumulativeSum = 0; // Initialize the cumulative sum of binary values.

    for (const binaryValue of binaryRemainderArray) {
        // Accumulate the binary values to keep track of the number of elements equal to targetRemainder modulo 'modulo'
        cumulativeSum += binaryValue;

        // Calculate the adjusted cumulative sum for the interesting subarray.
        const adjustedSum = (cumulativeSum - targetRemainder + modulo) % modulo;

        // Add the number of occurrences where the adjusted cumulative sum has been seen before.
        interestingSubarraysCount += cumulativeSumCounts.get(adjustedSum) || 0;

        // Increment the count for the current cumulative sum.
        cumulativeSumCounts.set(cumulativeSum % modulo, (cumulativeSumCounts.get(cumulativeSum % modulo) || 0) + 1);
    }

    return interestingSubarraysCount; // Return the total count of interesting subarrays found.
}

```

Time and Space Complexity

The time complexity of the code is `O(n)`, where `n` is the length of the input list `nums`. This is because the code iterates through the `nums` list once, performing a constant amount of work for each element by computing the modulo, updating the sum `s`, looking up and updating the count in the `cnt` dictionary, and incrementing the answer `ans`.

The space complexity of the code is also `O(n)` due to the use of the `cnt` dictionary, which stores up to `n` unique sums modulo the value of `modulo`, and the list `arr` which stores `n` elements.