2028. Find Missing Observations

Medium Array Math Simulation

Leetcode Link

Problem Description

You are given a certain amount of 6-sided dice roll observations m, but unfortunately, n additional observations are missing. The average value of all $m + n$ dice rolls, including the missing ones, has been calculated and is known to you. It is your task to reconstruct the missing n observations such that when combined with the provided m observations, the overall average remains the same.

The known m observations are provided in an array called `rolls`, the overall average (which is an integer) is given by `mean`, and the number of missing observations is n.

Your goal is to return an array of length n that represents the missing observations. The constraints you must abide by are:

- The sum of the resulting n s observations must be equal to $\text{mean} \times (n + m)$ since that product gives the total sum implied by the average `mean`.
- Each element in the resulting array must be a valid dice roll number between 1 and 6.

If it's impossible to find such an array that satisfies these conditions, you are to return an empty array instead.

Intuition

Understanding this problem requires recognizing that it's a matter of distributing a specific sum (s) across n rolls such that each roll is between 1 and 6.

Since the problem states that the average is an integer, we know the total sum of all observations must be divisible by the number of observations, which is a key starting point. Moreover, we are dealing with average values, meaning the total sum must be spread over can be determined by the formula $(n + m) * \text{mean}$.

Here's how we arrive at the solution approach:

1. Calculate the total sum `s` needed for the missing observations by subtracting the sum of given rolls from the desired total sum. This gives us the amount that the missing rolls must sum up to.
2. Confirm that it's possible to construct a valid array with the missing observations:
   - The minimum sum we could get from n dice rolls is n, with each roll being 1.
   - The maximum sum we could get from n dice rolls is $6 * n$, with each roll being 6.
   - So, if s is outside of the range $[n, 6 * n]$, constructing an array is impossible, and we should return an empty list.
3. If the sum s falls within the valid range, we distribute it evenly across the n missing observations. Each missing observation will initially be assigned the integer part of the division $s // n$.
4. However, since s may not be exactly divisible by n, there could be a remainder. This remainder has to be distributed amongst the observations as well. We do this by adding 1 to each of the first $s \% n$ observations since the remainder can be at most $n-1$.
5. After these steps, we would have an array that contains the missing observations and satisfies the average value requirement.

The approach used in the Python code above implements these steps succinctly, ensuring that all constraints of the problem are met.

Solution Approach

The implementation follows a direct approach, which aligns with the intuition previously explained.

To outline the steps of the implementation:

1. Calculate the sum s that the missing rolls must add up to. This is done by multiplying the overall mean by the total number of observations (both missing and known), and then subtracting the sum of the known observations:

```
s = (n + m) * mean - sum(rolls)
```

   n is the number of known observations, so `len(rolls)` gives us this value.

2. Check if the sum s is within the bounds of what is achievable with n rolls. The conditions for s to be valid are that it can't be less than n (since every roll is at least 1) and it can't be greater than $6 * n$ (since every roll is at most 6):

```
if s > 6 * n or s < n:
    return []
```

   If s does not satisfy these conditions, an empty list is returned, signaling that it's impossible to have such a distribution.

3. Allocate the minimum guaranteed value to each of the n missing observations by performing integer division of s by n. This ensures that we are using up as much of s as possible while keeping the value of each roll within the dice's constraints ($1 - 6$ inclusive).

```
ans = [s // n] * n
```

   Here, `ans` is the list that will be returned, initially filled with the integer part of the division.

4. Distribute the remainder of s across the n missing observations. The modulus operator `%` gives the remainder of the division between s and n:

```
for i in range(s % n):
    ans[i] += 1
```

   We only need to iterate over the first $s \% n$ elements of the `ans` array. This ensures that the extra values are as evenly distributed as possible without exceeding the die's face value limit of 6.

This approach assumes that there are valid values for all missing observations such that their sum is s. By using integer division and modular arithmetic, the solution fills the `ans` array with valid die rolls that reach the sum s without violating any of the dice constraints. The algorithm has a constant time complexity with respect to the input array `rolls` since it's only dependent on the number of missing observations n. The space complexity is also linear with respect to the number of missing observations n.

Example Walkthrough

Let's walk through a small example to illustrate the solution approach.

Suppose we are given the following parameters:

- **rolls**: `[3, 4, 3]` representing the known dice rolls
- **mean**: 4 as the overall average of all the dice rolls (including the missing ones).
- **n**: 2 indicating that there are two missing observations.

We want to reconstruct the missing observations such that the resulting array, when combined with the provided observations, results in the average value 4.

Following the steps outlined in the solution approach:

1. **Calculate the total sum `s` needed for the missing observations:**

   First, we determine the total number of observations (missing + known), which is $3 + 2 = 5$.

   Then, we calculate the total sum implied by the average:

```
1 n = (n + m) * mean - sum(rolls)
2 n = (2 + 3) * 4 - sum([3, 4, 3])
3 n = 5 * 4 - 10
4 n = 20 - 10
5 n = 10
```

   So, the sum of the two missing observations must be 10.

2. **Check if the sum `s` is within the achievable bounds:**

   We must check if `s` is at least n (every roll is at least 1) and at most $6 * n$ (every roll is at most 6):

   Since `s` is 10, which is within the range of $[2, 12]$ ($[n, 6 * n]$), there is a possible solution.

3. **Allocate the minimum guaranteed value to each missing observation:**

   We divide the sum `s` by the number of missing observations n:

```
1 ans = [s // n] * n
2 ans = [10 // 2] * 2
3 ans = [5] * 2
4 ans = [5, 5]
```

   This provides us with an initial distribution for the missing observations.

4. **Distribute the remainder of `s` across the missing observations:**

   Since `s` is exactly divisible by `n` in this case, there is no remainder and this step is not necessary. The distribution `[5, 5]` already satisfies the conditions since both numbers are between 1 and 6 and the total is 10.

Thus, the reconstructed array `[5, 5]` when combined with the original rolls `[3, 4, 3]`, maintains the average value 4:

- Total value of all rolls: $3 + 4 + 3 + 5 + 5 = 20$
- Number of all rolls: 5
- Average (Total value / Number of rolls): $20 / 5 = 4$

Therefore, the final reconstructed array with the missing observations would be `[5, 5]`. This completes the example walkthrough using the given approach.

Python Solution

```python
class Solution:
    def missingRolls(self, rolls: List[int], mean: int, n: int) -> List[int]:
        # Calculate the total sum of all the dice rolls based on the given mean
        total_dice_rolls = (len(rolls) + n) * mean
        # Calculate the sum of the missing rolls by subtracting the sum of existing rolls
        sum_missing_rolls = total_dice_rolls - sum(rolls)

        # If the sum of the missing rolls is not between n and 6n, it's impossible to get such a sequence
        if sum_missing_rolls > n * 6 or sum_missing_rolls < n:
            return []

        # Start with an even distribution of the sum across all the missing rolls
        missing_rolls = [sum_missing_rolls // n] * n

        # Distribute the remainder among the first (remainder) rolls, adding 1 to each
        for i in range(sum_missing_rolls % n):
            missing_rolls[i] += 1

        # Return the final list of missing rolls
        return missing_rolls
```

Java Solution

```java
class Solution {

    // Method to find the missing rolls given the mean of all the rolls
    public int[] missingRolls(int[] rolls, int mean, int n) {
        // m represents the number of given rolls
        int m = rolls.length;

        // The total sum S that all rolls (both missing and given) should sum up to
        int totalSumRequired = (n + m) * mean;

        // Subtract the sum of given rolls from the total sum required
        // to find the sum needed from the missing rolls
        for (int rollValue : rolls) {
            totalSumRequired -= rollValue;
        }

        // If the total sum needed is impossible (either too low or too high given the constraints),
        // return an empty array
        if (totalSumRequired < n * 6 || totalSumRequired < n) {
            return new int[0];
        }

        // Initialize the array to hold the missing rolls
        int[] missingRolls = new int[n];

        // Fill the missing rolls array with the quotient of the sum needed and n
        // which ensures the mean is maintained
        Arrays.fill(missingRolls, totalSumRequired / n);

        // Distribute the remainder of the sum needed evenly across the first
        // `totalSumRequired % n` elements by adding one to each
        for (int i = 0; i < totalSumRequired % n; ++i) {
            ++missingRolls[i];
        }

        // Return the missing rolls array
        return missingRolls;
    }
}
```

C++ Solution

```cpp
#include <vector>
using namespace std;

class Solution {
public:
    vector<int> missingRolls(vector<int>& rolls, int mean, int n) {
        int numberOfExistingRolls = rolls.size(); // The number of existing rolls in the sequence

        // The total sum required to reach the desired mean for all rolls (both existing and missing)
        int totalSumNeeded = (n + numberOfExistingRolls) * mean;

        // Subtract the sum of existing rolls from the total sum needed to find the sum of the missing rolls
        for (int rollValue : rolls) {
            totalSumNeeded -= rollValue;
        }

        // Return an empty vector if the total sum of missing rolls exceeds the max or min possible sum
        if (totalSumNeeded > n * 6 || totalSumNeeded < n) {
            return {};
        }

        // Create the result vector with default values (totalSumNeeded / n)
        vector<int> missingRolls(n, totalSumNeeded / n);

        // Distribute the remaining values (totalSumNeeded % n) evenly across the first few elements
        for (int i = 0; i < totalSumNeeded % n; ++i) {
            ++missingRolls[i];
        }

        // Return the completed vector of missing rolls
        return missingRolls;
    }
};
```

Typescript Solution

```typescript
function missingRolls(rolls: number[], targetMean: number, n: number): number[] {
    // Calculate the desired total sum based on the target mean and total number of rolls
    const expectedTotalSum = (rolls.length + n) * targetMean;

    // Calculate the current sum of given rolls
    const currentSumOfRolls = rolls.reduce((previousValue, currentValue) => previousValue + currentValue, 0);

    // Calculate the sum needed from the missing rolls
    const requiredSumForMissing = expectedTotalSum - currentSumOfRolls;

    // Establish the range for the possible sum of the missing rolls
    const minPossibleSum = n;
    const maxPossibleSum = n * 6;

    // If the required sum is outside the range of possible sums, return an empty array
    if (requiredSumForMissing < minPossibleSum || requiredSumForMissing > maxPossibleSum) {
        return [];
    }

    // Initialize an array to hold the missing rolls
    const missingRollsArray = new Array(n).fill(0);

    // Calculate the average roll needed, rounded down
    const averageMissingRoll = Math.floor(requiredSumForMissing / n);

    // Fill the array with the average roll value
    missingRollsArray.fill(averageMissingRoll);

    // Distribute the remainder of the required sum across the missing rolls
    let remainder = requiredSumForMissing - averageMissingRoll * n;
    for (let i = 0; i < n && remainder > 0; i++) {
        if (missingRollsArray[i] < 6) {
            missingRollsArray[i]++;
            remainder--;
        }
    }

    // Return the array representing the missing rolls
    return missingRollsArray;
}
```

Time and Space Complexity

The time complexity of the provided code is $O(m)$, where m is the number of elements in the `rolls` list. This time complexity comes from the need to calculate the sum of the `rolls` list with the `sum(rolls)` function. The rest of the operations, including the division and modulus operations, as well as assigning values to the `ans` list, have a constant time complexity or a time complexity of $O(n)$ which is dominated by the $O(m)$ complexity due to the summing operation, assuming n is much smaller than or at most equal to m.

The space complexity is $O(n)$, because we are creating a list `ans` which contains n integers. The space taken up by n is negligible compared to m because we are given that we're generating n new roll results, and no other data structures are utilized that are dependent on the size of them.