1331. Rank Transform of an Array

Easy Array Hash Table Sorting

Problem Description

The problem states that we have an array of integers called `arr`. We need to transform this array so that instead of the original numbers, each element is replaced by its "rank". The rank of an element in the array is determined by the following rules:

- Rank is assigned starting from 1.
- The bigger the number, the higher the rank it should have.
- If two numbers are the same, they should have the same rank.
- The rank should be as small as possible, meaning that the smallest number gets rank 1, the second smallest gets rank 2, and so on, without any gaps in the ranks.

Therefore, the task is to rewrite the array in such a way that the numbers are replaced with their corresponding ranks.

Intuition

To come up with a solution to this problem, we can use a step-by-step approach. The first part of the problem suggests a sorting operation since we need to determine the order of the elements (to assign ranks properly). However, we must also handle duplicates (elements of equal value should have the same rank). To do this effectively, we can utilize the `set` data structure in Python, which stores only unique elements. Here is the intuition broken down:

- **First**, sort the unique elements of the array. This can be done by converting the array to a set to remove duplicates, and then converting it back to a list to sort it. This gives us a sorted list of the unique elements in ascending order.
- **Second**, to find the rank of the original elements, we need to determine their position in this sorted list of unique elements.
- **Third**, we use the `bisect_right` function from the `bisect` module in Python, which is designed to find the position in a sorted list where an element should be inserted to maintain sorted order. However, in our case, it effectively gives us the number of elements that are less than or equal to our target number. Since our list is already sorted, this is equivalent to finding the rank.
- **Finally**, apply the `bisect_right` operation to each element in the original array to replace it with its rank.

The elegance of this solution lies in its simplicity and efficiency. By using sorting and binary search (which `bisect_right` employs), we arrive at a solution that is both clear and optimized for performance.

Solution Approach

The implementation of the solution can be understood by breaking down the steps and the algorithms or data structures used in each.

The code begins by defining a method `arrayRankTransform` within the `Solution` class which takes one argument, the array `arr`.

The first line within the method:

```
t = sorted(set(arr))
```

involves two operations:

- `set(arr)`: Converts the list `arr` into a set, removing any duplicate elements.
- `sorted(...)`: Takes the set of unique elements and returns a sorted list of these elements, `t`.

Now that `t` holds a sorted list of unique elements from the original array, the next step is to find out the rank of each element in the original array `arr` according to its position in `t`.

The following line:

```
return [bisect_right(t, x) for x in arr]
```

uses a list comprehension to create a new list by going through each element `x` in the original array `arr`.

- For each element `x`, `bisect_right(t, x)` is called, which uses a binary search to find the insertion point for `x` in `t` to the right of any existing entries. Essentially, it finds the index of the first element in the sorted list `t` that is greater than `x`.

Since `t` is sorted and contains every unique value from `arr`, the index returned by `bisect_right` corresponds to the number of elements less than or equal to `x`. This is exactly the rank of `x` because:

- The smallest element in `arr` will have a rank of 1 (because there will be 0 elements smaller than it in `t`).
- If elements in `arr` are equal, they will have the same rank since `bisect_right` will return the same index for them.
- Every unique element will have a unique rank in increasing order.

By combining `set`, `sorted`, and `bisect_right`, the solution effectively creates a ranking system for the elements in the array `arr`. It does so without the need for cumbersome loops or manually implementing a binary search, instead relying on Python's built-in high-performance algorithms.

Finally, the method returns the array which consists of ranks corresponding to each element in `arr`. This list represents the transformed version of the input where each value is replaced by its rank, effectively solving the problem.

Example Walkthrough

Let's illustrate the solution approach with a small example. Consider the following array:

```
arr = [40, 20, 10, 10]
```

Following the solution approach step by step, we first convert the `arr` into a set to remove duplicates, and then sort it:

1. Convert to set:

   ```
   {10, 20, 40}
   ```

   This set does not include duplicates and now contains only the unique values `10`, `20`, and `40`.

2. Sort the unique elements:

   ```
   t = [10, 20, 40]
   ```

   Now we have a sorted list `t` with the unique elements from the array in ascending order.

The next step is to find the rank for each element in `arr` by determining its position in the sorted list `t` using the binary search provided by the `bisect_right` function:

- For the first element `40` in `arr`, `bisect_right(t, 40)` returns `3` because there are two elements in `t` that are less than or equal to `40`. Hence, its rank is `3`.
- For the element `20` in `arr`, `bisect_right(t, 20)` returns `2`, as there is only one element less than or equal to `20`. Hence, its rank is `2`.
- For the first `10` in `arr`, `bisect_right(t, 10)` returns `1`, since there are no elements in `t` that are less than `10`. Hence, its rank is `1`.
- The second `10` in `arr` will have the same rank as the first `10` since they are equal. Therefore, `bisect_right(t, 10)` again returns `1`.

When we replace each element in `arr` with its calculated rank, we get the final transformed array:

```
arr = [3, 2, 1, 1]
```

Each number in the original array has now been replaced with its respective rank, completing the process. The code executing the described steps for the provided example would look like this:

```
from bisect import bisect_right

def arrayRankTransform(arr):
    t = sorted(set(arr))
    return [bisect_right(t, x) for x in arr]

# Example array
arr = [40, 20, 10, 10]

# Transforming the array
ranked_arr = arrayRankTransform(arr)
# ranked_arr is now [3, 2, 1, 1]
```

This demonstrates how the algorithm effectively translates the initial array into one representing the ranks of each element.

Solution Implementation

Python

```
from typing import List
from bisect import bisect_right

class Solution:
    def arrayRankTransform(self, arr: List[int]) -> List[int]:
        # Create a sorted list of the unique elements in `arr`
        unique_sorted_arr = sorted(set(arr))

        # For every element `x` in `arr`, find its rank. The rank is determined by the
        # position of `x` in the `unique sorted arr` also one because bisect_right will
        # give us the insertion point which is the number of elements that are less than or equal to `x`.
        # Since ranks are 1-indexed, the position itself is the rank.
        ranks = [bisect_right(unique_sorted_arr, x) for x in arr]

        # Return the list of ranks corresponding to each element in `arr`
        return ranks
```

Java

```
class Solution {

    public int[] arrayRankTransform(int[] arr) {
        // Get the size of the input array
        int arrayLength = arr.length;

        // Create a copy of the array to sort and find unique elements
        int[] sortedArray = arr.clone();
        Arrays.sort(sortedArray);

        // This will be the actual size of the array of unique elements
        int uniqueSize = 0;

        // Loop through the sorted array to filter out the duplicates
        for (int i = 0; i < arrayLength; ++i) {
            // If it's the first element or it's different from the previous, keep it
            if (i == 0 || sortedArray[i] != sortedArray[i - 1]) {
                sortedArray[uniqueSize++] = sortedArray[i];
            }
        }

        // Create an array to hold the answers
        int[] ranks = new int[arrayLength];

        // Assign the rank to each element in the original array
        for (int i = 0; i < arrayLength; ++i) {
            // Binary search finds the index of the current element in the unique array
            // Since array indexing starts at 0, add 1 to get the correct rank
            ranks[i] = Arrays.binarySearch(sortedArray, 0, uniqueSize, arr[i]) + 1;
        }

        // Return the ranks array containing the ranks of each element in arr
        return ranks;
    }
}
```

C++

```
#include <vector>
#include <algorithm>

class Solution {
public:
    // Function to transform an array into ranks of elements
    vector<int> arrayRankTransform(vector<int>& arr) {
        // Create a copy of the original array to sort and find unique elements
        vector<int> sortedArr = arr;
        sort(sortedArr.begin(), sortedArr.end());
        // Remove duplicates from sortedArr to get only unique elements
        sortedArr.erase(unique(sortedArr.begin(), sortedArr.end()), sortedArr.end());

        // Prepare a vector to store the answer
        int arrSize = arr.size();
        // For each element in the original array, find its rank
        for (int element : arr) {
            // Find the position (rank) of the element in the sorted unique array
            // The rank is the index + 1 because ranks are 1-based
            int rank = upper_bound(sortedArr.begin(), sortedArr.end(), element) - sortedArr.begin();
            ranks.push_back(rank);
        }
        // Return the vector of ranks corresponding to the original array's elements
        return ranks;
    }
};
```

TypeScript

```
function arrayRankTransform(arr: number[]): number[] {
    // Sort the array while preserving the original via spreading
    const sortedUniqueElements = [...arr].sort((a, b) => a - b);
    let uniqueCounter = 0;

    // Count unique elements in the sorted array
    for (let i = 0; i < sortedUniqueElements.length; ++i) {
        if (i === 0 || sortedUniqueElements[i] !== sortedUniqueElements[i - 1]) {
            sortedUniqueElements[uniqueCounter++] = sortedUniqueElements[i];
        }
    }

    // Binary search function to find the rank of an element
    const binarySearch = (sortedArray: number[], arrayLength: number, target: number) => {
        let leftIndex = 0;
        let rightIndex = arrayLength;
        while (leftIndex < rightIndex) {
            const midIndex = (leftIndex + rightIndex) >> 1;
            if (sortedArray[midIndex] >= target) {
                rightIndex = midIndex;
            } else {
                leftIndex = midIndex + 1;
            }
        }
        return leftIndex + 1; // The rank is index + 1
    };

    // Create an array for the answer
    const ranks: number[] = [];

    // Compute the rank for each element in the original array
    for (const element of arr) {
        ranks.push(binarySearch(sortedUniqueElements, uniqueCounter, element));
    }

    // Return the array of ranks
    return ranks;
}
```

```
from typing import List
from bisect import bisect_right

class Solution:
    def arrayRankTransform(self, arr: List[int]) -> List[int]:
        # Create a sorted list of the unique elements in `arr`
        unique_sorted_arr = sorted(set(arr))

        # For every element `x` in `arr`, find its rank. The rank is determined by the
        # position of `x` in the `unique sorted arr` also one because bisect_right will
        # give us the insertion point which is the number of elements that are less than or equal to `x`.
        # Since ranks are 1-indexed, the position itself is the rank.
        ranks = [bisect_right(unique_sorted_arr, x) for x in arr]

        # Return the list of ranks corresponding to each element in `arr`
        return ranks
```

Time and Space Complexity

Time Complexity

The time complexity of the given code can be broken down into the following parts:

1. `sorted(set(arr))`: Creating a set from the array removes duplicates and takes $O(n)$ time, where n is the number of elements in `arr`. Sorting the set will take $O(k \times \log(k))$ time, where k is the number of unique elements in `arr` which is less than or equal to n. Therefore, this step takes $O(n + k \times \log(k))$.
2. `[bisect_right(t, x) for x in arr]`: For each element `x` in the original `arr`, we perform a binary search using `bisect_right`. Binary search takes $O(\log(k))$ time per search, and since we are performing it for each element n times, this step takes $O(n \times \log(k))$ time.

Thus, the overall time complexity is $O(n + k \times \log(k) + n \times \log(k))$, which simplifies to $O(n + \log(k))$ because n will typically be larger than k and hence $n \times \log(k)$ would be the dominant term.

Space Complexity

The space complexity can be considered as follows:

1. `t = sorted(set(arr))`: Creating a set and then a sorted list from the array takes $O(k)$ space where k is the number of unique elements.
2. The list comprehension does not use additional space that depends on the size of the input (other than space for the output list, which is always required for the problem). The output list itself takes $O(n)$ space.

In conclusion, the space complexity is $O(n + k)$, which simplifies to $O(n)$ if we do not count the output space as extra space (as is common in complexity analysis), or if we assume that k is at most n.