1570. Dot Product of Two Sparse Vectors

Medium **Design** Array **Hash Table** Two Pointers Leetcode Link

Problem Description

The task is to create a class, `SparseVector`, which encapsulates the concept of a sparse vector and implements a method to compute the dot product of two sparse vectors. A sparse vector is defined as a vector that contains mostly zeros and therefore should not be stored in a typical dense format as that would waste space. The goal is to store such vectors efficiently and perform operations on them.

To efficiently store the sparse vector, we can use a dictionary to hold only the non-zero elements, where the keys are the indices of these elements, and the values are the elements themselves. This way we aren't storing all the zero values, which can dramatically reduce memory usage for very sparse vectors.

The `SparseVector` class has two methods:

1. `SparseVector(nums)`: The constructor takes a list of integers `nums` and initializes the sparse vector using a dictionary comprehension that filters out the zero values.
2. `dotProduct(vec)`: This method computes the dot product between the vector represented by the current instance of `SparseVector` and another sparse vector, `vec`.

The dot product of two vectors is computed by multiplying corresponding entries and summing those products. In the case of sparse vectors, most of these products will be zero, because they involve multiplications by zero, so we only need to consider the non-zero entries.

The follow-up question asks about efficiently computing the dot product if only one of the two vectors is sparse.

Intuition

For the solution, the idea is to leverage the sparsity of the vectors to optimize the dot product computation. Given that most of the elements in the vectors are zeros, we want to perform multiplications only for the non-zero elements. By converting the vectors into a dictionary structure with non-zero elements, we can quickly identify which elements actually need to be multiplied.

In the `dotProduct` method, we iterate over the items in the smaller dictionary (to optimize the number of iterations) and multiply values by the corresponding values in the other dictionary, if they exist. If an index does not exist in the other dictionary, it means that the value for that index in the other vector is zero and thus does not contribute to the dot product. Therefore, we use the `.get` method to handle such cases, which allows us to specify a default value of `0` when an index is not found.

When dealing with one sparse and one non-sparse vector, the current approach still works efficiently because the dot product will focus on iterating over the non-zero elements of the sparse vector and lookup the corresponding values in the non-sparse vector.

Solution Approach

The implementation of the `SparseVector` solution makes use of two main aspects: Python dictionaries and the concept of dictionary comprehension for building a structure to represent the sparse vectors.

Algorithm and Data Structure

- **Dictionary for Sparse Representation**: A Python dictionary is an ideal data structure for representing a sparse vector. It allows storing key-value pairs where the key is the index of a non-zero element in the original vector, and the value is the non-zero element itself. This structure is memory efficient since we only store entries for non-zero elements.
- **Constructor `__init__`**:
  - We use a dictionary comprehension in the constructor to iterate over `nums` using `enumerate` to get both the index `i` and the value `v` together.
  - The condition `if v` ensures that we're only storing the non-zero values (as zero is considered `False` in Python).
  - The resulting dictionary `self.d` holds only the elements of the input list that are non-zero, along with their indices.
- **`dotProduct` Method**:
  - We receive another `SparseVector` object, `vec`, and we want to compute the dot product with the current sparse vector instance.
  - We access the internal dictionaries of the current instance (`a`) and the `vec` (`b`) — these dictionaries store the indices and values of the non-zero entries.
  - By comparison of the lengths of these two dictionaries, we choose to iterate over the smaller dictionary (here represented as `a`). This is an optimization step; since the dot product will be zero for all indices not present in both vectors, we can skip the zero values of the larger vector and only iterate over the potential non-zero counterparts.
  - We then use a generator expression to iterate over the items of `a`: `for i, v in a.items()`.
  - For each element, we calculate the product `v * b.get(i, 0)`. The `get` method of the dictionary is very handy in this case, as it will return `0` if the index `i` doesn't exist in `b`—a common occurrence with sparse vectors, and also safe considering the default value for any index not in the dictionary would be zero.
  - Finally, the `sum` function accumulates all the products to give us the result of the dot product.

Pattern Used

- **Efficient Computation with Sparse Representation**: By representing the vectors in a sparse form, computations are made more efficient since we ignore all zero-product cases which would contribute nothing to the final sum.
- **Iterating Over a Smaller Set**: Choosing to iterate over the smaller set of elements to reduce the number of operations is a common optimization strategy in algorithms involving collection processing.

Combining these algorithms and patterns, the `SparseVector` class efficiently implements the computation of a dot product between two sparse vectors, considering only the meaningful, non-zero elements, and thus avoiding unnecessary computations.

By using a compressed representation for the vectors with dictionaries and strategically leveraging the sparsity, the implementation maximizes efficiency in terms of both time and space complexity.

Example Walkthrough

Let's walk through a small example to illustrate the solution approach.

Suppose we have two sparse vectors represented as follows:

- Vector A: [1, 0, 0, 2, 0]
- Vector B: [0, 3, 0, 4, 0]

Using the given solution approach, we first convert these lists into `SparseVector` objects. Let's call these objects `sparseA` and `sparseB`. When initializing these objects, our dictionary comprehension will filter out the zeroes and store only the non-zero elements and their indices:

- sparseA's internal dictionary will have the elements {0: 1, 3: 2}, corresponding to indices and values (index 0 has value 1, and index 3 has value 2).
- sparseB's internal dictionary will have the elements {1: 3, 3: 4}.

To find the dot product of `sparseA` and `sparseB`, we invoke the dotProduct method on one of them, let's say `sparseA.dotProduct(sparseB)`.

Here's a step-by-step explanation:

1. Inside the `dotProduct` method, we compare the sizes of the internal dictionaries of `sparseA` and `sparseB`. Since both have two elements, we can choose either one to iterate over, but for the sake of this example, we will iterate over `sparseA` because we called the method on it.
2. We now loop over the items in `sparseA`'s dictionary. For each item, we look up whether the corresponding index is in `sparseB`'s dictionary. We have two iterations in our loop:
   - First iteration: For index 0 in `sparseA`, there is no corresponding index in `sparseB`. When we attempt to multiply `1 (from sparseA)` with `sparseB.get(0, 0)` (using `.get` to specify a default value of `0`), the result is `0` since index 0 is not present in `sparseB`.
   - Second iteration: For index 3 in `sparseA`, there is a matching index in `sparseB` which has the value 4. We perform the multiplication `2 (from sparseA) * 4 (from sparseB)` which equals `8`.
3. We sum the results of the multiplications. In this example, the only non-zero result came from the second iteration (`8`), so the sum and thus the dot product of A and B is `8`.

In conclusion, the `SparseVector` class successfully computes the dot product of `sparseA` and `sparseB` as `8` using an efficient approach, taking advantage of the sparse representation by only considering non-zero elements and their indices in the computations.

Python Solution

```python
from typing import List

class SparseVector:
    def __init__(self, nums: List[int]):
        # Store the non-zero elements with their indices as keys
        self.non_zero_elements = {i: v for i, v in enumerate(nums) if v != 0}

    # Return the dot product of two sparse vectors
    def dotProduct(self, vec: "SparseVector") -> int:
        # For efficiency, iterate through the smaller vector
        smaller_vector, larger_vector = (self.non_zero_elements, vec.non_zero_elements) \
            if len(self.non_zero_elements) < len(vec.non_zero_elements) \
            else (vec.non_zero_elements, self.non_zero_elements)

        # Calculate the dot product by summing the product of the
        # overlapping elements of the two sparse vectors
        return sum(value * larger_vector.get(index, 0) for index, value in smaller_vector.items())

# Example usage:
# v1 = SparseVector(nums1)
# v2 = SparseVector(nums2)
# ans = v1.dotProduct(v2)

```

Java Solution

```java
import java.util.HashMap;
import java.util.Map;

class SparseVector {
    // Using a HashMap to efficiently store non-zero elements and their positions
    private Map<Integer, Integer> nonZeroElements = new HashMap<>();

    // Constructor to populate the map with non-zero elements from the input array
    SparseVector(int[] nums) {
        for (int i = 0; i < nums.length; ++i) {
            if (nums[i] != 0) {
                nonZeroElements.put(i, nums[i]);
            }
        }
    }

    // Return the dotProduct of two sparse vectors
    public int dotProduct(SparseVector vec) {
        // Reference to the smaller of the two maps to iterate over for efficiency
        Map<Integer, Integer> smallerMap = nonZeroElements;
        Map<Integer, Integer> largerMap = vec.nonZeroElements;

        // Swap if 'vec's map has fewer elements to iterate over the smaller map
        if (largerMap.size() < smallerMap.size()) {
            Map<Integer, Integer> temp = smallerMap;
            smallerMap = largerMap;
            largerMap = temp;
        }

        int productSum = 0; // The result of the dot product operation
        // Iterating through the smaller map and multiplying the matching values
        for (var entry : smallerMap.entrySet()) {
            int index = entry.getKey();
            int value = entry.getValue();
            productSum += value * largerMap.getOrDefault(index, 0);
        }
        return productSum; // Return the computed dot product
    }
}

// Example of usage:
// SparseVector v1 = new SparseVector(nums1);
// SparseVector v2 = new SparseVector(nums2);
// int ans = v1.dotProduct(v2);

```

C++ Solution

```cpp
#include <vector>
#include <unordered_map>
using namespace std;

// Class to represent a Sparse Vector
class SparseVector {
public:
    // This map will store the non-zero values of the sparse vector associated with their indices
    unordered_map<int, int> indexToValueMap;

    // Constructor which takes a vector of integers and populates the indexToValueMap
    SparseVector(vector<int>& nums) {
        for (int i = 0; i < nums.size(); ++i) {
            if (nums[i] != 0) {
                indexToValueMap[i] = nums[i];
            }
        }
    }

    // Return the dot product of this sparse vector with another sparse vector vec
    int dotProduct(SparseVector& vec) {
        // Using references to the internal maps for easier access
        auto& thisVectorMap = indexToValueMap;
        auto& otherVectorMap = vec.indexToValueMap;

        // Optimize by iterating over the smaller map
        if (thisVectorMap.size() > otherVectorMap.size()) {
            swap(thisVectorMap, otherVectorMap);
        }

        int total = 0;

        // Compute dot product by only considering non-zero elements
        for (auto& [index, value] : thisVectorMap) {
            if (otherVectorMap.count(index)) {
                total += value * otherVectorMap[index];
            }
        }

        return total;
    }
};

// Usage example:
// vector<int> nums1 = { ... };
// vector<int> nums2 = { ... };
// SparseVector v1(nums1);
// SparseVector v2(nums2);
// int product = v1.dotProduct(v2);

```

Typescript Solution

```typescript
// Object to encapsulate sparse vector data and operations
const sparseVectorOperations = {
    sparseVectorData: new Map<number, Map<number, number>>(),

    // Function to initialize a sparse vector from a given array of numbers
    createSparseVector: function(nums: number[], vectorId: number): void {
        const sparseData = new Map<number, number>();
        for (let index = 0; index < nums.length; ++index) {
            if (nums[index] !== 0) {
                sparseData.set(index, nums[index]);
            }
        }
        this.sparseVectorData.set(vectorId, sparseData);
    },

    // Function to calculate the dot product of two sparse vectors identified by their IDs
    calculateDotProduct: function(vectorId1: number, vectorId2: number): number {
        const vec1Data = this.sparseVectorData.get(vectorId1) || new Map();
        const vec2Data = this.sparseVectorData.get(vectorId2) || new Map();

        // Swap if vec1Data has more elements than vec2Data to minimize iterations
        let smallerMap = vec1Data;
        let largerMap = vec2Data;
        if (vec1Data.size > vec2Data.size) {
            smallerMap = vec2Data;
            largerMap = vec1Data;
        }

        let result = 0;
        // Calculate dot product by iterating through the map with fewer elements
        for (const [index, value] of smallerMap) {
            if (largerMap.has(index)) {
                result += value * (largerMap.get(index) || 0);
            }
        }

        return result;
    }
};

// Example usage:
sparseVectorOperations.createSparseVector([1, 0, 0, 2, 3], 1);
sparseVectorOperations.createSparseVector([0, 3, 0, 4, 0], 2);
const dotProductResult = sparseVectorOperations.calculateDotProduct(1, 2); // Should calculate the dot product
console.log(dotProductResult); // Outputs the result of the dot product calculation

```

Time and Space Complexity

Time Complexity:

The constructor `__init__` has a time complexity of `O(n)` where `n` is the number of elements in `nums`, as it needs to iterate through all elements to create the dictionary with non-zero values.

The `dotProduct` function has a time complexity of `O(min(k, l))` where `k` and `l` are the number of non-zero elements in the two SparseVectors, respectively. This is because the function iterates over the smaller of the two dictionaries (after ensuring `a` has the smaller length, swapping if necessary) and attempts to find matching elements in the larger one. The `get` operation on a dictionary has an average case time complexity of `O(1)`.

Space Complexity:

The space complexity of the `__init__` function is `O(k)`, where `k` is the number of non-zero elements in `nums`, since the space required depends on the stored non-zero elements.

The `dotProduct` function operates in `O(1)` space complexity because it calculates the sum on the fly and does not store intermediate results or allocate additional space based on input size, other than a few variables for iteration and summing.