1493. Longest Subarray of 1's After Deleting One Element

Problem Description

The problem presents a binary array `nums`, comprised of `0`s and `1`s. The task is to find the size of the largest contiguous subarray of `1`s that could be obtained by removing exactly one element from the array. You are also informed that if there is no such subarray made up entirely of `1`s after removing an element, the function should return `0`.

Intuition

The intuition for solving this problem involves looking at each segment of `1`s as a potential candidate and checking the length we can obtain after removing a single element. We want to maximize this length by strategically picking an element for removal. The key insight here is that if there are consecutive `1`s, removing one `1` does not impact the length of the subarray of `1`s on either side of it.

We can approach the solution by precomputing and storing the lengths of continuous `1`s at any index from the left and right directions separately. This is done in two linear scans using the `left` and `right` auxiliary arrays:

1. **From Left to Right**: Starting at the index `1`, if the previous element is `1`, increment the value in the `left` array at the current index by `1` plus the value at the previous index. This will give us the number of continuous `1`s to the left of each index in the array `nums`.

2. **From Right to Left**: Starting from the second to last index and going backwards, if the next element is `1`, increment the value in the `right` array at the current index by `1` plus the value at the next index. This gives the number of continuous `1`s to the right of each index.

The final result, after having populated both `left` and `right` arrays, is the maximum sum of corresponding elements from `left` and `right` arrays across all indices. This effectively simulates the removal of one element (either a `1` or `0`) and joining the contiguous `1`s from both left and right sides. However, the result should be the length after the removal of one element, so another -1 is implicitly considered in the max operation [since we don't actually remove the element but calculate what the length would be as if we did].

By applying this method, we can find the longest subarray containing only `1`s after deleting exactly one element.

Solution Approach

The solution to this problem relies primarily on dynamic programming to keep track of the length of continuous `1`s on both sides of each index in the array `nums`.

Here is a breakdown of the implementation details:

1. **Initialization**: Two lists, `left` and `right`, each of the same length as `nums`, are created and initialized with zeroes. These lists will store the length of continuous `1`s to the left and right of each index, respectively.

2. **Populating the `left` list**: We start iterating over the array `nums` from the second element onwards (index `1` because we consider `0` as the base case which is already initialized to zero). For each index `i`, if the element at the previous index (`i - 1`) is `1`, we set `left[i]` to `left[i - 1] + 1`. Effectively, this records the length of a stretch of `1`s ending at the current index.

3. **Populating the `right` list**: We do a similar iteration, but this time from right to left, starting from the second-to-last element (index `n - 2` where `n` is the length of `nums`). For each index `i`, if the element at the next index (`i + 1`) is `1`, we set `right[i]` to `right[i + 1] + 1`. This captures the length of a stretch of `1`s starting right after the current index.

4. **Finding the longest subarray**: Once we have both `left` and `right` lists populated, we iterate over all possible remove positions (these positions do not have to be `1`; they can be `0` as well, as we will merge stretches of `1`s around them). For each index, we calculate the sum of `left[i] + right[i]`, which approximates the length of the subarray of `1`s if we removed the element at the current index. However, since we are supposed to actually remove an element to create the longest subsequence, the sum implicitly considers this by not adding an additional `1` even though we are summing the lengths from both sides.

5. **Returning the result**: The maximum value from these sums corresponds to the size of the longest subarray containing only `1`s after one deletion. This is found using the `max()` function applied to the sums of `left[i] + right[i]` for each index `i`.

The algorithm completes this in linear time with two passes through `nums`, and the space complexity is governed by the two additional arrays `left` and `right`, which are linear in the size of the input array `nums`.

Example Walkthrough

Let's use an example to illustrate the solution approach.

Suppose the input binary array `nums` is `[1, 1, 0, 1, 1, 1, 0, 1, 1]`.

Step by Step Solution

1. **Initialization**:
   We initialize two lists, `left` and `right`, with the same length as `nums`.
   ```
   left: [0, 0, 0, 0, 0, 0, 0, 0, 0]
   right: [0, 0, 0, 0, 0, 0, 0, 0, 0]
   ```

2. **Populating the `left` list**:
   We iterate from left to right over the array `nums`, starting from index 1.

   - `nums[1]` is `1`, and `nums[0]` is `1`, so `left[1] = left[0] + 1 -> 0 + 1 = 1`.
   - As we continue, `left` becomes `[0, 1, 0, 1, 2, 3, 0, 1, 2]`.

3. **Populating the `right` list**:
   Iterating from right to left, starting from index 7.

   - `nums[7]` is `1`, and `nums[8]` is `1`, so `right[7] = right[8] + 1 -> 0 + 1 = 1`.
   - Continuing this process, the `right` list fills up as `[2, 1, 0, 4, 3, 2, 0, 1, 0]`.

4. **Finding the longest subarray**:
   We calculate the potential longest subarrays after a deletion for each index, by summing the `left` and `right` values at each index.

   - At index `2` (the first `0`), the sum is `left[2] + right[2] = 0 + 0 = 0`.
   - At index `6` (the second `0`), the sum is `left[6] + right[6] = 3 + 0 = 3`.
   - The longest subarray occurs at either index `2` or `6`, where we would be removing the `0` to join the `1`s.
   - Considering every index, we would have sums as `[2, 1, 4, 5, 3, 2, 4, 1, 2]`.

5. **Returning the result**:
   The maximum value from the sum of `left` and `right` for all indexes gives us the result.
   In this case, the maximum is 5, corresponding to index `1`. Thus, the size of the longest subarray containing only `1`s after one deletion is `5`.

This approach allows us to solve the problem with a single pass for `left` and another for `right`, totaling linear time complexity, O(n), with `n` being the number of elements in `nums`. The use of the two lists `left` and `right` gives us a space complexity of O(n) as well.

Python Solution

```python
from typing import List

class Solution:
    def longestSubarray(self, nums: List[int]) -> int:
        # Determine the length of the nums list
        n = len(nums)

        # Initialize two lists to keep track of consecutive ones to the left and right of each index
        left_ones_count = [0] * n
        right_ones_count = [0] * n

        # Calculate the consecutive ones to the left of each index
        for i in range(1, n):
            if nums[i - 1] == 1:
                left_ones_count[i] = left_ones_count[i - 1] + 1

        # Calculate the consecutive ones to the right of each index
        for i in range(n - 2, -1, -1):
            if nums[i + 1] == 1:
                right_ones_count[i] = right_ones_count[i + 1] + 1

        # Find the maximum length subarray formed by summing up counts of left and right ones.
        # Note that the question assumes we can remove one zero to maximize the length.
        # So, connecting two streaks of ones effectively means removing one zero between them.
        max_length = max(a + b for a, b in zip(left_ones_count, right_ones_count))

        return max_length
```

Java Solution

```java
class Solution {

    // Function to find the length of the longest subarray consisting of 1s after deleting exactly one element.
    public int longestSubarray(int[] nums) {
        int length = nums.length;

        // Arrays to store the count of consecutive 1s to the left and right of each index in nums
        int[] leftOnesCount = new int[length];
        int[] rightOnesCount = new int[length];

        // Count consecutive 1s from left to right, starting from the second element
        for (int i = 1; i < length; ++i) {
            if (nums[i - 1] == 1) {
                // If the previous element is 1, increment the count
                leftOnesCount[i] = leftOnesCount[i - 1] + 1;
            }
        }

        // Count consecutive 1s from right to left, starting from the second-to-last element
        for (int i = length - 2; i >= 0; --i) {
            if (nums[i + 1] == 1) {
                // If the next element is 1, increment the count
                rightOnesCount[i] = rightOnesCount[i + 1] + 1;
            }
        }

        // Variable to store the answer, the maximum length of a subarray
        int maxSubarrayLength = 0;

        // Loop to find the maximum length by combining the left and right counts of 1s
        for (int i = 0; i < length; ++i) {
            // Compute the length of subarray by removing the current element, hence adding left and right counts of 1s.
            // Since one element is always removed, the combined length of consecutive 1s from left and right
            // should not be equal to the total length of the array (which implies no 0 was in the array to begin with).
            maxSubarrayLength = Math.max(maxSubarrayLength, leftOnesCount[i] + rightOnesCount[i]);
        }

        // Reduce the length by 1 if the length of consecutive 1s equals the array length, since we need to remove one element.
        if (maxSubarrayLength == length) {
            maxSubarrayLength--;
        }

        // Return the maximum length of subarray after deletion
        return maxSubarrayLength;
    }
}
```

C++ Solution

```cpp
class Solution {
public:
    int longestSubarray(vector<int>& nums) {
        // Get the size of the input array
        int size = nums.size();

        // Create two vectors to keep track of consecutive 1's on the left and right
        vector<int> left(size, 0);
        vector<int> right(size, 0);

        // Fill the left array with the counts of consecutive 1's from the left
        for (int i = 1; i < size; ++i) {
            if (nums[i - 1] == 1) {
                left[i] = left[i - 1] + 1;
            }
        }

        // Fill the right array with the counts of consecutive 1's from the right
        for (int i = size - 2; i >= 0; --i) {
            if (nums[i + 1] == 1) {
                right[i] = right[i + 1] + 1;
            }
        }

        // Initialize the variable to store the maximum length of the subarray
        int max_length = 0;

        // Iterate over the input array to compute the maximum subarray length
        for (int i = 0; i < size; ++i) {
            // The longest subarray is the sum of consecutive 1's to the left and right of the current element
            max_length = max(max_length, left[i] + right[i]);
        }

        // Return the computed maximum subarray length
        return max_length;
    }
};
```

Typescript Solution

```typescript
function longestSubarray(nums: number[]): number {
    // Get the size of the input array
    const size: number = nums.length;

    // Create two arrays to keep track of consecutive 1's to the left and right
    let left: number[] = new Array(size).fill(0);
    let right: number[] = new Array(size).fill(0);

    // Fill the left array with the counts of consecutive 1's from the left
    for (let i = 1; i < size; i++) {
        if (nums[i - 1] === 1) {
            left[i] = left[i - 1] + 1;
        }
    }

    // Fill the right array with the counts of consecutive 1's from the right
    for (let i = size - 2; i >= 0; i--) {
        if (nums[i + 1] === 1) {
            right[i] = right[i + 1] + 1;
        }
    }

    // Initialize the variable to store the maximum length of the subarray
    let maxLength: number = 0;

    // Iterate over the input array to compute the maximum subarray length
    for (let i = 0; i < size; i++) {
        // The longest subarray is the sum of consecutive 1's to the left and right of the current element
        maxLength = Math.max(maxLength, left[i] + right[i]);
    }

    // Return the computed maximum subarray length
    return maxLength;
}
```

Time and Space Complexity

The given code aims to find the length of the longest subarray of 1s after deleting one element from the array. Here's the analysis of its computational complexity:

Time Complexity:

- The time complexity of this algorithm is `O(n)`, where `n` is the length of the input array `nums`. The reasoning behind this is that there are two separate for loops that each iterate over the array exactly once. The first loop starts from index 1 and goes up to `n-1`, incrementing by 1 on each iteration. The second loop starts from `n-2` and goes down to 0, decrementing by 1 on each iteration. In both of these loops, only a constant amount of work is done (simple arithmetic and array accesses), so the time complexity for each loop is linear with respect to the size of the array.

- Furthermore, there's a final step that combines the results of the left and right arrays using `max` and `zip`, which is also a linear operation since it involves a single pass over the combined arrays, making it `O(n)`.

Adding up these linear time operations (`3 * O(n)`) still results in an overall time complexity of `O(n)`.

Space Complexity:

- The space complexity of this algorithm is `O(n)` as well. This is because two additional arrays, `left` and `right`, are created to store the lengths of continuous ones to the left and right of each index, respectively. Each of these arrays is the same length as the input array `nums`.

- Besides the two arrays `left` and `right`, there are only a constant number of variables used (i.e., `n`, `i`, `a`, `b`), so they do not add more than `O(1)` to the space complexity.

Thus, the total space complexity is determined by the size of the two additional arrays, which gives us `O(n)`.