2096. Step-By-Step Directions From a Binary Tree Node to Another

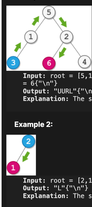
You are given the `root` of a **binary tree** with `n` nodes. Each node is uniquely assigned a value from `1` to `n`. You are also given an integer `startValue` representing the value of the start node `s`, and a different integer `destValue` representing the value of the destination node `t`.

Find the **shortest path** starting from node `s` and ending at node `t`. Generate step-by-step directions of such path as a string consisting of only the **uppercase** letters `'L'`, `'R'`, and `'U'`. Each letter indicates a specific direction:

- `'L'` means to go from a node to its **left child** node.
- `'R'` means to go from a node to its **right child** node.
- `'U'` means to go from a node to its **parent** node.

Return *the step-by-step directions of the **shortest path** from node `s` to node `t`.*

Example 1:



```
Input: root = [5,1,2,3,null,6,4], startValue = 3, destValue = 6
Output: "UURL"
Explanation: The shortest path is: 3 → 1 → 5 → 2 → 6.
```

Example 2:


```
Input: root = [2,1], startValue = 2, destValue = 1
Output: "L"
Explanation: The shortest path is: 2 → 1.
```

Constraints:

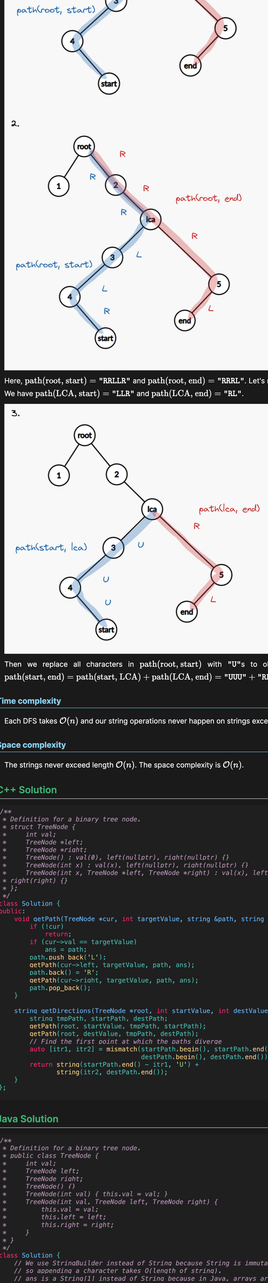
- The number of nodes in the tree is `n`.
- `2 <= n <= 10^5`
- `1 <= Node.val <= n`
- All the values in the tree are **unique**.
- `1 <= startValue, destValue <= n`
- `startValue != destValue`

Solution

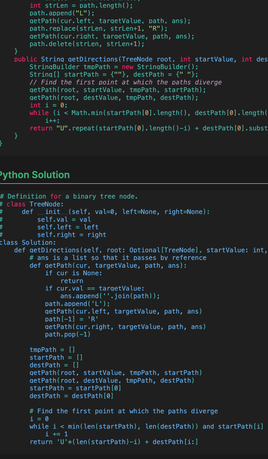
Let path(node1, node2) denote the path from node1 to node2.

First consider the case where path(start, end) goes through the root. Let's split this into path(start, root) + path(root, end). We can perform a DFS (depth first search) to get path(root, end). This path consists of 'L's and 'R's. We can do another DFS to get path(root, start). Replacing the 'L's and 'R's of path(root, start) with 'U's gives us path(start, root). Now we can concatenate path(start, root) and path(root, end) to get the answer.

In the general case, path(start, end) may not go through the root. Notice that this path goes up a non-negative number of times ("U"s) before going down a non-negative number of times ("L"s or "R"s). The highest node in this path is known as the LCA (lowest common ancestor) of start and end.



Here, path(root, start) = "RRLLR" and path(root, end) = "RRRL". Let's remove their longest common prefix, which is "RR". We have path(LCA, start) = "LLR" and path(LCA, end) = "RL".



Then we replace all characters in path(root, start) with "U"s to obtain path(start, lca) = "UUU". Finally, we get path(start, end) = path(start, LCA) + path(LCA, end) = "UUU" + "RL" = "UUURL".

Time complexity

Each DFS takes $O(n)$ and our string operations never happen on strings exceeding length $O(n)$. The time complexity is $O(n)$.

Space complexity

The strings never exceed length $O(n)$. The space complexity is $O(n)$.

C++ Solution

```
/**
 * Definition for a binary tree node.
 * struct TreeNode {
 *     int val;
 *     TreeNode *left;
 *     TreeNode *right;
 *     TreeNode() : val(0), left(nullptr), right(nullptr) {}
 *     TreeNode(int x) : val(x), left(nullptr), right(nullptr) {}
 *     TreeNode(int x, TreeNode *left, TreeNode *right) : val(x), left(left),
 * right(right) {}
 * };
 */
class Solution {
public:
    void getPath(TreeNode *cur, int targetValue, string &path, string &ans) {
        if (!cur)
            return;
        if (cur->val == targetValue)
            ans = path;
        path.push_back('L');
        getPath(cur->left, targetValue, path, ans);
        path.back() = 'R';
        getPath(cur->right, targetValue, path, ans);
        path.pop_back();
    }

    string getDirections(TreeNode *root, int startValue, int destValue) {
        string tmpPath, startPath, destPath;
        getPath(root, startValue, tmpPath, startPath);
        getPath(root, destValue, tmpPath, destPath);
        // Find the first point at which the paths diverge
        auto [it1, it2] = mismatch(startPath.begin(), startPath.end(),
                                   destPath.begin(), destPath.end());
        return string(startPath.end() - it1, 'U') +
               string(it2, destPath.end());
    }
};
```

Java Solution

```
/**
 * Definition for a binary tree node.
 * public class TreeNode {
 *     int val;
 *     TreeNode left;
 *     TreeNode right;
 *     TreeNode() {}
 *     TreeNode(int val) { this.val = val; }
 *     TreeNode(int val, TreeNode left, TreeNode right) {
 *         this.val = val;
 *         this.left = left;
 *         this.right = right;
 *     }
 * }
 */
class Solution {
    // We use StrinaBuilder instead of String because String is immutable,
    // so appending a character takes O(length of string).
    // ans is a String[1] instead of String because in Java, arrays are passed by reference.
    void getPath(TreeNode cur, int targetValue, StringBuilder path, String[] ans) {
        if (cur == null)
            return;
        if (cur.val == targetValue)
            ans[0] = path.toString();
        int strLen = path.length();
        path.append("L");
        getPath(cur.left, targetValue, path, ans);
        path.replace(strLen, strLen+1, "R");
        getPath(cur.right, targetValue, path, ans);
        path.delete(strLen, strLen+1);
    }
    public String getDirections(TreeNode root, int startValue, int destValue) {
        StringBuilder tmpPath = new StringBuilder();
        String[] startPath = {""}, destPath = {""};
        // Find the first point at which the paths diverge
        getPath(root, startValue, tmpPath, startPath);
        getPath(root, destValue, tmpPath, destPath);
        int i = 0;
        while (i < Math.min(startPath[0].length(), destPath[0].length()) && startPath[0].charAt(i) == destPath[0].ch
            i++;
        return "U".repeat(startPath[0].length()-i) + destPath[0].substring(i);
    }
}
```

Python Solution

```
# Definition for a binary tree node.
# class TreeNode:
#     def __init__(self, val=0, left=None, right=None):
#         self.val = val
#         self.left = left
#         self.right = right
class Solution:
    def getDirections(self, root: Optional[TreeNode], startValue: int, destValue: int) -> str:
        # ans is a list so that it passes by reference
        def getPath(cur, targetValue, path, ans):
            if cur is None:
                return
            if cur.val == targetValue:
                ans.append(''.join(path));
            path.append('L')
            getPath(cur.left, targetValue, path, ans)
            path[-1] = 'R'
            getPath(cur.right, targetValue, path, ans)
            path.pop(-1)

        tmpPath = []
        startPath = []
        destPath = []
        getPath(root, startValue, tmpPath, startPath)
        getPath(root, destValue, tmpPath, destPath)
        startPath = startPath[0]
        destPath = destPath[0]

        # Find the first point at which the paths diverge
        i = 0
        while i < min(len(startPath), len(destPath)) and startPath[i] == destPath[i]:
            i += 1
        return 'U'*(len(startPath)-i) + destPath[i:]
```