# 43. Multiply Strings

Medium Math String Simulation

## Problem Description

In this problem, we are given two non-negative integer numbers represented as strings, `num1` and `num2`, and our task is to calculate the product of these two numbers and return the result as a string. The constraint is that we are not allowed to use any built-in library that can handle big integers, nor can we simply convert the strings to integers and multiply them in the standard way. This challenges us to think about how we can perform multiplication manually, mimicking the way one might do it on paper.

## Intuition

The intuition behind the solution is based on how we perform multiplication by hand between two numbers. More precisely, when we multiply, say, a three-digit number by a two-digit number, we do it digit by digit and keep track of the carries. This process results in a series of partial products, which are then added together to form the final product.

To implement this in code, we need a data structure to store intermediate results. The approach is to use an array `arr` to store the digits of the partial products. The length of this array is the sum of the lengths of `num1` and `num2` because that's the maximum possible length of the result (e.g., 99 * 99 is 9801, four digits long, which is the sum of the lengths of the numbers being multiplied).

Next, we iterate over each digit of `num1` and `num2` in nested loops, and for each pair of digits, we multiply them and add the result to the corresponding position in `arr`. The key formula for the index in `arr` where we should accumulate the product of digits at positions `i` and `j` is `arr[i+j+1]`.

After we have all the partial products, we then iterate through the `arr` array to handle carries at each position, adjusting each digit so that it represents a proper digit in a number (less than 10), and propagating the carry to the next position.

Finally, we need to return the string representation of the number stored in the array, but we skip any leading zeroes, as they don't contribute to the magnitude of the number. We join the remaining digits together to form the resulting product string to be returned.

## Solution Approach

The solution follows these steps:

1. **Check for Zero**: If either `num1` or `num2` is "0", the product is "0". We catch this case early to simplify further logic.
2. **Initialization**: Determine the lengths `m` and `n` of `num1` and `num2`, respectively. Initialize an array `arr` of length `m + n` to hold the digits of the product.
3. **Digit-by-Digit Multiplication**:
   - Iterate through the digits of `num1` and `num2` in descending order using two nested loops, with indices `i` and `j`.
   - Convert current digits to integers and multiply them: `a * b`.
   - Add the multiplication result to an appropriate position in the array `arr`: `arr[i + j + 1] += a * b`.
   - The position `i + j + 1` comes from the fact that when you multiply a digit at position `i` of `num1` with a digit at position `j` of `num2`, the result will contribute to the digits at positions `i + j` and `i + j + 1` of the product.
4. **Handling Carries**:
   - Iterate backwards through `arr`, starting from the end, to process carries.
   - For each position, if the value is greater than 9, divide by 10 to find the carry and keep the remainder:
     - `arr[i - 1] += arr[i] // 10`: This propagates the carry to the next higher position.
     - `arr[i] %= 10`: This ensures that the current position has only a single digit.
5. **Converting Array to String**:
   - If the digit at the highest position (`arr[0]`) is zero, it's a leading zero and should be omitted. Determine the starting index for the conversion (`i`), which is `0` if there's no leading zero, and `1` otherwise.
   - Join the digits from the `arr` starting at the right index `i` to form a string without leading zeros: `"".join(str(x) for x in arr[i:])`.

This implementation doesn't use any special algorithms or data structures—it uses simple arrays and elementary math operations to simulate digit-by-digit multiplication, carefully considering the placement of each partial product and the handling of carries, just like manual multiplication on paper.

### Example Walkthrough

To illustrate the solution approach, let's walk through a small example where `num1 = "13"` and `num2 = "24"`.

1. **Check for Zero**: Neither `num1` nor `num2` are "0", so we proceed.
2. **Initialization**: The length `m` of `num1` is 2, and the length `n` of `num2` is also 2. We initialize an array `arr` of length `m + n = 4` to `[0, 0, 0, 0]`.
3. **Digit-by-Digit Multiplication**:
   - We iterate over `num1` and `num2` in reverse order so we will have the indices `i=1,0` for `num1` and `j=1,0` for `num2`.
   - For `i=1` (`num1[1] = "3"`) and `j=1` (`num2[1] = "4"`), we multiply the digits and add the result to `arr[i + j + 1]`, which is `arr[3] = 0 + (3 * 4) = 12`.
   - We store `2` in `arr[3]` and carry over `1` to `arr[2]`, so `arr` becomes `[0, 0, 1, 2]`.
   - We repeat this for all pairs of digits:
     - `i=1, j=0`: `arr[2] += 3` (`num1[1] * 2` (`num2[0]`); `arr` becomes `[0, 0, 7, 2]`.
     - `i=0, j=1`: `arr[1] += 1` (`num1[0]) * 4` (`num2[1]`); `arr` becomes `[0, 4, 7, 2]`.
     - `i=0, j=0`: `arr[1] += 1` (`num1[0]) * 2` (`num2[0]`; `arr` becomes `[0, 6, 7, 2]`.
4. **Handling Carries**:
   - Starting from the end of `arr`, we process the carries:
     - At `arr[2] (7)`: We add `7 // 10` to `arr[1]` and set `arr[2]` to `7 % 10`. `arr` stays `[0, 6, 7, 2]` as `7` is less than `10`.
     - At `arr[1] (6)`: We do the same. There's no carry since `6` is also less than `10`.
5. **Converting Array to String**:
   - `arr` is `[0, 6, 7, 2]`, so we omit the leading zero.
   - The final product string is `"672"` as we join the digits from `arr` starting from index `1`.

The final result of multiplying "13" by "24" using our manual algorithm results in "672", which matches what we expect from standard multiplication.

## Python Solution

```python
class Solution:
    def multiply(self, num1: str, num2: str) -> str:
        # If either number is "0", return "0" because the product will also be "0"
        if num1 == "0" or num2 == "0":
            return "0"

        # Determine the lengths of the input strings
        length_num1, length_num2 = len(num1), len(num2)

        # Create a result list to store the product digits
        result = [0] * (length_num1 + length_num2)

        # Reverse process of multiplication, processing digits from the end
        for i in range(length_num1 - 1, -1, -1):
            digit_num1 = int(num1[i])
            for j in range(length_num2 - 1, -1, -1):
                digit_num2 = int(num2[j])
                # Add product of current digits to the previously stored value in result list
                result[i + j + 1] += digit_num1 * digit_num2

        # Handle carrying over digits > 9 to the next place
        for i in range(length_num1 + length_num2 - 1, 0, -1):
            result[i - 1] += result[i] // 10  # Carry over
            result[i] %= 10                   # Keep only the last digit

        # Skip leading zeros in the result list
        start_index = 0 if result[0] != 0 else 1

        # Convert the result list to string
        return "".join(str(digit) for digit in result[start_index:])

```

## Java Solution

```java
class Solution {
    public String multiply(String num1, String num2) {
        // If either number is 0, the product will be 0.
        if ("0".equals(num1) || "0".equals(num2)) {
            return "0";
        }

        // Get lengths of both numbers.
        int length1 = num1.length(), length2 = num2.length();

        // Initialize an array to store the product of each digit multiplication.
        int[] productArray = new int[length1 + length2];

        // Loop over each digit in num1 and num2 and multiply them.
        for (int i = length1 - 1; i >= 0; --i) {
            int digit1 = num1.charAt(i) - '0';
            for (int j = length2 - 1; j >= 0; --j) {
                int digit2 = num2.charAt(j) - '0';

                // Add the product of the two digits to the corresponding position.
                productArray[i + j + 1] += digit1 * digit2;
            }
        }

        // Normalize the productArray so that each position is a single digit.
        for (int i = productArray.length - 1; i > 0; --i) {
            productArray[i - 1] += productArray[i] / 10; // Carry over the tens to the next left cell.
            productArray[i] %= 10; // Keep the units in the current cell.
        }

        // Skip the leading 0 in the product array if it exists.
        int startIndex = productArray[0] == 0 ? 1 : 0;

        // Convert the product array into a string.
        StringBuilder product = new StringBuilder();
        for (int i = startIndex; i < productArray.length; ++i) {
            product.append(productArray[i]);
        }
        return product.toString();
    }
}
```

## C++ Solution

```cpp
class Solution {
public:
    string multiply(string num1, string num2) {
        // Check if either input is "0", if yes, then result is "0"
        if (num1 == "0" || num2 == "0") {
            return "0";
        }

        // Initialize the sizes of the input numbers
        int length1 = num1.size(), length2 = num2.size();

        // Create a vector to store the multiplication result
        vector<int> result(length1 + length2, 0);

        // Multiply each digit of num1 with each digit of num2
        for (int i = length1 - 1; i >= 0; --i) {
            int digit1 = num1[i] - '0'; // Convert char to integer
            for (int j = length2 - 1; j >= 0; --j) {
                int digit2 = num2[j] - '0'; // Convert char to integer
                // Add to the corresponding position in the result vector
                result[i + j + 1] += digit1 * digit2;
            }
        }

        // Handle carrying over the value for digits greater than 9
        for (int i = result.size() - 1; i > 0; --i) {
            result[i - 1] += result[i] / 10; // Carry over
            result[i] %= 10; // Remainder stays at current position
        }

        // Skip any leading zeros in the result vector
        int startIndex = result[0] == 0 ? 1 : 0;

        // Convert the result vector to a string
        string resultStr;
        for (int i = startIndex; i < result.size(); ++i) {
            resultStr += '0' + result[i]; // Convert integer to char and append to resultStr
        }

        // Return the final product as a string
        return resultStr;
    }
};

```

## Typescript Solution

```typescript
// Multiplies two non-negative integers represented as number strings and returns the product as a string.
function multiply(num1: string, num2: string): string {
    // If either number is '0', the product will be '0'.
    if (!num1 || !num2 || num1 === '0' || num2 === '0') return '0';

    // Get the lengths of the two number strings.
    const length1 = num1.length;
    const length2 = num2.length;
    let answer = '';

    // Iterate over each digit in the first number.
    for (let i = 0; i < length1; i++) {
        let currentDigit1 = parseInt(num1.charAt(length1 - i - 1), 10);
        let partialSum = '';

        // Iterate over each digit in the second number.
        for (let j = 0; j < length2; j++) {
            let currentDigit2 = parseInt(num2.charAt(length2 - j - 1), 10);

            // Multiply current digits and add them to the partial sum with proper offset.
            partialSum = addString(partialSum, (currentDigit1 * currentDigit2) + '0'.repeat(j));
        }

        // Construct the answer with the accumulated partial sum with proper offset.
        answer = addString(answer, partialSum + '0'.repeat(i));
    }

    // Return the final product as a string.
    return answer;
}

// Adds two number strings and returns the sum as a string.
function addString(num1String: string, num2String: string): string {
    const length1 = num1String.length;
    const length2 = num2String.length;
    let answerArray = [];
    let carry = 0;

    // Add the two strings together, digit by digit, from the end to the beginning.
    for (let i = 0; i < Math.max(length1, length2) || carry > 0; i++) {
        let digit1 = i < length1 ? parseInt(num1String.charAt(length1 - i - 1), 10) : 0;
        let digit2 = i < length2 ? parseInt(num2String.charAt(length2 - i - 1), 10) : 0;

        // Calculate the sum for current place and maintain the carry for the next iteration.
        let sum = digit1 + digit2 + carry;
        answerArray.unshift(sum % 10);
        carry = Math.floor(sum / 10);
    }

    // Join the computed digits to form the final sum string.
    return answerArray.join('');
}

```

## Time and Space Complexity

**Time Complexity**: The time complexity of the given code is $O(m * n)$, where $m$ and $n$ are the lengths of the input strings `num1` and `num2` respectively. The code involves a double loop where the outer loop runs $m$ times and the inner loop runs $n$ times, leading to $m * n$ multiplication operations. Additionally, there is a loop for carrying over the values, which runs in $O(m + n)$ time. However, since $O(m * n)$ dominates $O(m + n)$, the overall time complexity stays $O(m * n)$.

**Space Complexity**: The space complexity is $O(m + n)$, as an additional array `arr` of size $m + n$ is used to store the intermediate results of the multiplication before they are converted to the final string result.