1589. Maximum Sum Obtained of Any Permutation

Medium Topics Array Prefix Sum Sorting

Problem Description

In this problem, we have an array `nums` of integers and an array of `requests`. Each request is represented by a pair `[start, end]`, which corresponds to a sum operation of the elements from `nums[start]` up to `nums[end]`, inclusive. The goal is to calculate the maximum possible sum of all these individual request sums when we are allowed to permute `nums`. Since the result could be very large, it is asked to return the sum modulo `10^9 + 7`, which is a common way to handle large numbers in programming problems to avoid integer overflow.

Intuition

To find the maximum total sum of all requests, we need to understand that the frequency of each element being requested affects the total sum. If an element is included in many requests, we want to assign a larger value from `nums` to that position to maximize the sum. Conversely, if an element is rarely requested or not at all, it should be assigned a smaller value.

The crux of the solution lies in the following insights:

1. Count the frequency of each index being requested: We can achieve this by using a "difference array" technique. In this approach, for each request `[start, end]`, we increment `f[start]` and decrement `f[end + 1]`. After that, a prefix sum pass over the `f` array gives us the number of times each index is included in the range of the requests.
2. Sort both the `nums` and the frequency array `f`: By sorting the frequency array, we have a non-decreasing order of frequencies. Sorting `nums` also arranges the values in non-decreasing order. The intuition here is that we want to assign the highest values to the most frequently requested indices.
3. Calculate the maximum sum: We pair each number from `nums` with the corresponding frequency from `f` (after sorting both arrays), multiply them together, and accumulate the result to get the total sum. This works because both arrays are sorted, so the largest numbers are paired with the highest frequencies, ensuring the maximum total sum.

Finally, we take the calculated sum modulo `10^9 + 7` to prevent overflow and return this value as the result.

Solution Approach

Here is the step-by-step breakdown of implementing the solution:

1. **Initialization of the difference array**: We initialize an array `d` of zeros with the same length as `nums`. This array will help us keep track of how many times each index in `nums` is requested.
2. **Populate the difference array**: For each request `[l, r]` in `requests`, increment `d[l]` by `1` and decrement `d[r + 1]` by `1` (being careful not to go out of bounds). This is the difference array technique where `d[i]` represents the change at index `i`, and `d[r + 1]` represents the change just after index `r`.
3. **Calculate frequencies through prefix sums**: We convert the difference array into a frequency array by calculating the prefix sum. In essence, `for i in range(1, n): d[i] += d[i - 1]` converts the difference array into a frequency array, which tells us how many times each index is involved in a request after summing up the contributions from `d[0]` up to `d[i]`.
4. **Sort the arrays**: We sort both `nums` and the frequency array `d` in non-decreasing order using `nums.sort()` and `d.sort()`. By sorting these arrays, we ensure that the largest numbers in `nums` are lined up with the indices that occur most frequently in the requests.
5. **Calculate the total sum**: We calculate the total sum using `sum(a * b for a, b in zip(nums, d))`. Here, we multiply each number in `nums` with its corresponding frequency in `d` and accumulate the sum. Since both arrays are sorted, the highest frequency gets paired with the largest number, which is critical for maximizing the total sum.
6. **Return the result modulo `10^9 + 7`**: To avoid overflow issues and comply with the problem constraints, we take the sum modulo `10**9 + 7`, which we defined earlier as `mod`, and return this result.

The space complexity of the solution is O(n) due to the additional array `d`, and the time complexity is O(n log n) because we sort the arrays, which is the most time-consuming part of the algorithm.

Example Walkthrough

Let's walk through the solution approach with a small example.

Suppose our `nums` array is `[3, 5, 7, 9]` and our `requests` array is `[[0, 1], [1, 3], [0, 2]]`.

Step 1: Initialization of the difference array

We start with an array `d` of the same length as `nums`, plus one for the technique to work (length of `nums` plus one). Here `d = [0, 0, 0, 0, 0]`.

Step 2: Populate the difference array

We go through each request and update our difference array `d` accordingly:

- For the request `[0, 1]`, we increment `d[0]` and decrement `d[2]`: `d = [1, 0, -1, 0, 0]`.
- For the request `[1, 3]`, we increment `d[1]` and decrement `d[4]`: `d = [1, 1, -1, 0, -1]`.
- For the request `[0, 2]`, we increment `d[0]` and decrement `d[3]`: `d = [2, 1, -1, -1, -1]`.

Step 3: Calculate frequencies through prefix sums

Now we convert the difference array to a frequency array:

- `d = [2, 1, -1, -1, -1]` becomes `d = [2, 3, 2, 1, 0]` after calculating prefix sums.

Step 4: Sort the arrays

We sort `nums` (it is already sorted) and we sort `d`: `nums = [3, 5, 7, 9]` and `d = [0, 1, 2, 2, 3]`.

Step 5: Calculate the total sum

We calculate the sum by multiplying each element in `nums` by its corresponding frequency in `d` in their respective sorted orders:

- sum = `3*0 + 5*1 + 7*2 + 9*2 = 0 + 5 + 14 + 18 = 37`

Step 6: Return the result modulo `10^9 + 7`

Finally, we take our sum `37` modulo `10^9 + 7`. Since `37` is much less than `10^9 + 7`, our final result is `37`.

Therefore, the maximum possible sum of all request sums given the permutation of `nums` is `37`.

Solution Implementation

Python

```python
from typing import List

class Solution:
    def maxSumRangeQuery(self, nums: List[int], requests: List[List[int]]) -> int:
        # Get the length of the nums array.
        length_of_nums = len(nums)

        # Initialize a difference array with the same length as nums.
        difference_array = [0] * length_of_nums

        # Iterate over each request to populate the difference array.
        for start, end in requests:
            # Increment the start index to indicate a new range starts at this index.
            difference_array[start] += 1

            # Decrement the element after the end index to indicate the range ends at this index.
            if end + 1 < length_of_nums:
                difference_array[end + 1] -= 1

        # Calculate the prefix sum of the difference array to get the actual frequency array.
        for i in range(1, length_of_nums):
            difference_array[i] += difference_array[i - 1]

        # Sort both the nums array and the frequency array in non-decreasing order.
        nums.sort()
        difference_array.sort()

        # Initialize the modulus to avoid overflow issues with very large integers.
        modulus = 10**9 + 7

        # Calculate the total sum by summing the product of corresponding elements
        # from the sorted nums array and the sorted frequency array.
        total_sum = sum(num * frequency for num, frequency in zip(nums, difference_array))

        # Return the total sum modulo to ensure the result fits within the required range.
        return total_sum % modulus
```

Java

```java
class Solution {
    public int maxSumRangeQuery(int[] nums, int[][] requests) {
        // Define the length of the nums array.
        int length = nums.length;

        // Create an array to keep track of the number of times each index is included in the ranges.
        int[] frequency = new int[length];

        // Iterate over all the requests to calculate the frequency of each index.
        for (int[] request : requests) {
            int start = request[0], end = request[1];
            frequency[start]++;
            // Decrease the count for the index right after the end of this range
            if (end + 1 < length) {
                frequency[end + 1]--;
            }
        }

        // Convert the frequency array to a prefix sum array to get how many times each index is requested.
        for (int i = 1; i < length; ++i) {
            frequency[i] += frequency[i - 1];
        }

        // Sort both the nums array and the frequency array.
        Arrays.sort(nums);
        Arrays.sort(frequency);

        // Modulo value to be used for not to exceed integer limits during the calculation.
        long int mod = (int) 1e9 + 7;

        // Variable to store the result of the maximum sum.
        long maxSum = 0;

        // Compute the maximum sum by pairing the largest numbers with the highest frequencies.
        for (int i = 0; i < length; ++i) {
            maxSum = (maxSum + (long) nums[i] * frequency[i]) % mod;
        }

        // Return the maximum sum as an integer.
        return (int) maxSum;
    }
}
```

C++

```cpp
#include <vector>  // Required for using the std::vector
#include <algorithm>  // Required for using the std::sort algorithm
#include <cstring>  // Required for using the memset function

class Solution {
public:
    int maxSumRangeQuery(std::vector<int>& nums, std::vector<std::vector<int>>& requests) {
        int n = nums.size();

        // Array to keep track of frequency of indices being requested
        std::vector<int> frequency(n, 0);

        // Increment start index and decrement just past the end index for each request
        for(const auto& req : requests) {
            int start = req[0], end = req[1];
            frequency[start]++;
            if (end + 1 < n) {
                frequency[end + 1]--;
            }
        }

        // Convert frequency values into prefix sum to get the total count for each index
        for (int i = 1; i < n; ++i) {
            frequency[i] += frequency[i - 1];
        }

        // Sort the input array and frequency array
        std::sort(nums.begin(), nums.end());
        std::sort(frequency.begin(), frequency.end());

        long long totalSum = 0;
        const int MOD = 1e9 + 7;  // Modulo value for result

        // Compute the maximum sum of all ranges
        for (int i = 0; i < n; ++i) {
            totalSum = (totalSum + 1LL * nums[i] * frequency[i]) % MOD;
        }

        // Casting to int as the Problem statement expects an int return type
        return static_cast<int>(totalSum);
    }
};
```

TypeScript

```typescript
function maxSumRangeQuery(nums: number[], requests: number[][]): number {
    const length = nums.length; // Length of the array of elements in nums
    const frequency = new Array(length).fill(0); // Array to keep track of frequency of each index

    // Loop through each request to build the frequency array
    for (const [start, end] of requests) {
        frequency[start]++;
        if (end + 1 < length) {
            frequency[end + 1]--;
        }
    }

    // Convert frequency array to prefix sum array
    for (let i = 1; i < length; i++) {
        frequency[i] += frequency[i - 1];
    }

    // Sort the arrays to maximize the sum
    nums.sort((a, b) => a - b);
    frequency.sort((a, b) => a - b);

    let answer = 0; // Variable to store the final answer
    const modulo = 10 ** 9 + 7; // Use modulo to avoid overflow

    // Calculate the maximum sum of all range queries
    for (let i = 0; i < nums.length; ++i) {
        answer = (answer + nums[i] * frequency[i]) % modulo;
    }

    return answer; // return the final answer
}
```

Python

```python
from typing import List

class Solution:
    def maxSumRangeQuery(self, nums: List[int], requests: List[List[int]]) -> int:
        # Get the length of the nums array.
        length_of_nums = len(nums)

        # Initialize a difference array with the same length as nums.
        difference_array = [0] * length_of_nums

        # Iterate over each request to populate the difference array.
        for start, end in requests:
            # Increment the start index to indicate a new range starts at this index.
            difference_array[start] += 1

            # Decrement the element after the end index to indicate the range ends at this index.
            if end + 1 < length_of_nums:
                difference_array[end + 1] -= 1

        # Calculate the prefix sum of the difference array to get the actual frequency array.
        for i in range(1, length_of_nums):
            difference_array[i] += difference_array[i - 1]

        # Sort both the nums array and the frequency array in non-decreasing order.
        nums.sort()
        difference_array.sort()

        # Initialize the modulus to avoid overflow issues with very large integers.
        modulus = 10**9 + 7

        # Calculate the total sum by summing the product of corresponding elements
        # from the sorted nums array and the sorted frequency array.
        total_sum = sum(num * frequency for num, frequency in zip(nums, difference_array))

        # Return the total sum modulo to ensure the result fits within the required range.
        return total_sum % modulus
```

Time and Space Complexity

Time Complexity

The time complexity of the code can be broken down as follows:

1. Initializing the array `d`: It is created with `n` zeros, where `n` is the length of `nums`. This operation takes `O(n)` time.
2. Populating the `d` array with the frequency of each index being requested: The for loop runs for each request, and each request updates two elements in `d`. The number of requests is the length of `requests`, let's say `m`. Hence, this step would take `O(m)` time.
3. Prefix sum of `d` array: This for loop iterates `n-1` times, updating the `d` array with the cumulative frequency. This step takes `O(n)` time.
4. Sorting `nums` and `d`: Sorting an array of `n` elements takes `O(n log n)` time. Since both `nums` and `d` are sorted, this step takes `2 * O(n log n)` time, which simplifies to `O(n log n)`.
5. Calculating the sum product of `nums` and `d`: The zip operation iterates through both arrays once, so this takes `O(n)` time.

The most time-consuming operation here is the sorting step. Therefore, the overall time complexity of the algorithm is `O(n log n)` due to the sorting of `nums` and `d`.

Space Complexity

The space complexity of the code can be analyzed as follows:

1. Additional array `d`: This array is of size `n`, taking up `O(n)` space.
2. Sorting `nums` and `d`: In Python, the `sort` method sorts the array in place, so no additional space is needed for this step beyond the input arrays.
3. Temporary variables used for calculations and the sum operation (`mod`, loop variables, etc.) take `O(1)` space.

Therefore, the additional space used by the algorithm is for the `d` array, which gives us a space complexity of `O(n)`.