2670. Find the Distinct Difference Array

Easy Array Hash Table

Problem Description

The problem presents us with an integer array called `nums`, indexed from 0, which means the first element of the array is at index 0. Our task is to calculate a new array, `diff`, also of the same length as `nums`, based on a specific rule. For each position `i` in the `nums` array, we have to find the count of distinct elements in two different parts of `nums`:

1. The "prefix" which includes all elements from the start of `nums` up to and including the element at index `i`.
2. The "suffix" which includes all elements after index `i` up to the end of the `nums`.

`diff[i]` is then defined as the number of distinct elements in the prefix minus the number of distinct elements in the suffix.

Note that when working with subarrays or slices of an array, `nums[i, ..., j]` means we are considering the elements of `nums` from index `i` through to index `j`, inclusive. If `i` is greater than `j`, it means the subarray is empty.

Intuition

To construct the `diff` array as described, we need to keep track of the distinct elements as we move through the `nums` array. Using two sets, we can capture the unique elements in the prefixes and suffixes of `nums` at each index.

The algorithm includes the following steps:

1. Initialize an extra array `suf` with the same length as `nums` plus one, to record the number of distinct elements in the suffix part starting from each index. We add an extra space since we are including the empty subarray when `i > j`.
2. Traverse the `nums` array in reverse (right to left), starting from the last element, and with each element, we encounter, add it to a set to ensure only distinct elements are counted. Update the corresponding `suf` entry with the length of this set to reflect the current number of distinct elements in the suffix up to the current index.
3. Clear the set (for counting distinct elements in the prefixes) and then traverse the `nums` array once more, from left to right. With each element, we add it to the set to keep track of distinct elements and calculate `diff[i]` by subtracting the number of distinct elements in the suffix (obtained from `suf[i + 1]`) from the number of distinct elements in the prefix (length of the set at this point in the traversal).

Solution Approach

The solution uses a couple of Python data structures - lists and sets - and leverages their properties to solve the problem efficiently:

1. **Lists**: They are used to store the input (`nums`) and output (`ans` for the distinct difference array, and `suf` for tracking the number of distinct elements in suffixes).
2. **Sets**: A set is used to keep track of distinct elements because sets naturally eliminate duplicates.

The implementation follows these steps:

- Initialize the `suf` list to have `n + 1` zeros, where `n` is the length of `nums`. This is done to handle the suffix count of distinct elements as well as accounting for the empty subarray at the end.
- Start by populating the `suf` array from the end of `nums` moving towards the beginning. For each index `i`, add `nums[i]` to a set (this ensures only distinct elements are counted). Then, set `suf[i]` to the current size of the set, which represents the count of distinct elements from `nums[i]` to the end of the array.
- Once the `suf` array is complete, reset (clear) the set to use it for counting distinct elements in prefixes.
- Create an answer list `ans` initialized with zeros of length `n` to hold the final result.
- Iterate through `nums` from start to end. For each index `i`, add `nums[i]` to the set (again, ensuring uniqueness) and subtract the number of distinct elements in the suffix (obtained from `suf[i + 1]`) from the number of distinct elements in the prefix (the current size of the set). This gives the `diff[i]` value.
- Assign this result to `ans[i]`, building the answer as the loop progresses.
- Finally, return the `ans` list once the loop is complete.

The key algorithmic insight here is to use two passes, one for suffixes and one for prefixes, with the help of sets to dynamically maintain the count of distinct elements efficiently. This negates the need to re-calculate the number of distinct elements for each index from scratch, thus improving the time complexity significantly.

Here's the main loop in the code explained:

```
for i in range(n - 1, -1, -1):
    s.add(nums[i])
    suf[i] = len(s)
s.clear()
for i, x in enumerate(nums):
    s.add(x)
    ans[i] = len(s) - suf[i + 1]
```

In the first loop, we fill the `suf` array with the count of distinct elements for the suffix starting from each index. In the second loop, we use the `suf` array and the prefix count of distinct elements to calculate the `diff` for each index, which is stored in `ans`.

Example Walkthrough

Let's illustrate the solution approach using a small example. Suppose we have the following integer array `nums`:

```
nums = [4, 6, 4, 3]
```

We need to calculate the `diff` array where `diff[i]` is the count of distinct elements from the start of `nums` up to `i`, minus the count of distinct elements from `i+1` to the end of `nums`. Here's how we would apply the solution step by step:

- Initialize the `suf` list to length `n+1` with zero values. `n` is the length of `nums`, so `suf` initially will be `[0, 0, 0, 0, 0]`.
- Initialize an empty set `s` to record distinct elements as we go.

We first populate the `suf` array in backward fashion:

- For `i = 3`, `nums[3] = 3`. We add this to the set `s`, which becomes `{3}`, and `suf[3] = 1`.
- For `i = 2`, `nums[2] = 4`. We add this to the set `s`, which becomes `{3, 4}`, and `suf[2] = 2`.
- For `i = 1`, `nums[1] = 6`. We add this to the set `s`, which becomes `{3, 4, 6}`, and `suf[1] = 3`.
- For `i = 0`, `nums[0] = 4`. The set `s` is already `{3, 4, 6}`, so adding `4` doesn't change it, and `suf[0]` remains at `3`.

Now our `suf` array looks like this: `[3, 3, 2, 1, 0]`.

Next, we'll go over `nums` from left to right to build our `diff` array:

- Clear the set `s`.
- For `i = 0`, we add `nums[0] = 4` to `s`, which becomes `{4}`. We calculate `diff[0] = len(s) - suf[1] = 1 - 3 = -2`.
- For `i = 1`, we add `nums[1] = 6` to `s`, which becomes `{4, 6}`. We calculate `diff[1] = len(s) - suf[2] = 2 - 2 = 0`.
- For `i = 2`, we add `nums[2] = 4` to `s`, which doesn't change it as `4` is already in the set. Thus, we calculate `diff[2] = len(s) - suf[3] = 2 - 1 = 1`.
- For `i = 3`, we add `nums[3] = 3` to `s`, which becomes `{3, 4, 6}`. We calculate `diff[3] = len(s) - suf[4] = 3 - 0 = 3`.

Our final `diff` array is `[-2, 0, 1, 3]` which is the result of our algorithm.

Solution Implementation

Python

```
from typing import List

class Solution:
    def distinctDifferenceArray(self, nums: List[int]) -> List[int]:
        # Get the length of the input list
        length = len(nums)
        # Initialize a suffix array of length 'length + 1' with zeros
        suffix_count = [0] * (length + 1)
        # Create an empty set to store unique elements
        unique_elements = set()

        # Populate the suffix_count array with count of unique elements from the end
        for i in range(length - 1, -1, -1):
            unique_elements.add(nums[i])
            suffix_count[i] = len(unique_elements)

        # Clear the set for re-use
        unique_elements.clear()
        # Initialize the answer array with zeros
        answer = [0] * length

        # Calculate the distinct count difference for each position
        for i, number in enumerate(nums):
            unique_elements.add(number)
            # Calculate the difference between the number of unique elements seen
            # so far and the number of unique elements that will be seen in the suffix
            answer[i] = len(unique_elements) - suffix_count[i + 1]

        # Return the answer array
        return answer
```

Java

```
class Solution {
    public int[] distinctDifferenceArray(int[] nums) {
        // The length of the input array
        int length = nums.length;

        // Suffix array to store the number of distinct elements from index 'i' to the end
        int[] suffixDistinctCount = new int[length + 1];

        // A set to keep track of distinct numbers as we traverse the array from the end
        Set<Integer> distinctNumbers = new HashSet<>();

        // Populate the suffixDistinctCount array with the count of distinct numbers
        // starting from the end of nums array
        for (int i = length - 1; i >= 0; --i) {
            distinctNumbers.add(nums[i]);
            suffixDistinctCount[i] = distinctNumbers.size();
        }

        // Clear the set to reuse it for the next loop
        distinctNumbers.clear();

        // Resultant array to store the desired distinct differences
        int[] result = new int[length];

        // Traverse the nums array and calculate the distinct difference
        for (int i = 0; i < length; ++i) {
            distinctNumbers.add(nums[i]);
            // The current distinct difference is the size of the set
            // minus the size of the suffixDistinctCount at the next index
            result[i] = distinctNumbers.size() - suffixDistinctCount[i + 1];
        }

        // Return the computed distinct difference array
        return result;
    }
}
```

C++

```
#include <vector>
#include <unordered_set>
using namespace std;

class Solution {
public:
    // Function to return a vector that contains the number of distinct integers
    // in the array minus after removing the elements to the right of the current element
    vector<int> distinctDifferenceArray(vector<int>& nums) {
        int n = nums.size();

        // Create a suffix array to store the count of distinct numbers from the current index to the end
        vector<int> distinctCountSuffix(n + 1);

        // Use a set to store distinct numbers to facilitate counting
        unordered_set<int> distinctNumbers;

        // Fill the suffix array with distinct number count, starting from the end of the vector
        for (int i = n - 1; i >= 0; --i) {
            distinctNumbers.insert(nums[i]);
            distinctCountSuffix[i] = distinctNumbers.size();
        }

        // Clear the set for reuse from the start of the vector
        distinctNumbers.clear();

        // Initialize the answer vector to store the result
        vector<int> ans(n);

        // Calculate the distinct difference for each position
        for (int i = 0; i < n; ++i) {
            // Insert number into the set
            distinctNumbers.insert(nums[i]);
            // The distinct difference is the count of unique numbers we've seen so far
            // (in the prefix) minus the count of unique numbers from the next index onwards
            ans[i] = distinctNumbers.size() - distinctCountSuffix[i + 1];
        }

        // Return the final answer vector
        return ans;
    }
};
```

TypeScript

```
function distinctDifferenceArray(nums: number[]): number[] {
    // The length of the input array
    const length = nums.length;

    // This array will hold the count of distinct numbers from the current index to the end.
    const suffixDistinctCount = new Array(length + 1).fill(0);

    // A set to keep track of distinct numbers seen so far.
    const seenNumbers = new Set<number>();

    // Populate the suffixDistinctCount array with distinct number counts by iterating backwards.
    for (let i = length - 1; i >= 0; --i) {
        seenNumbers.add(nums[i]);
        suffixDistinctCount[i] = seenNumbers.size;
    }

    // Clear the set to reuse it for the prefix pass.
    seenNumbers.clear();

    // The resulting array that will hold the final difference counts.
    const result = new Array(length);

    // Iterate through the input array to calculate the difference in the distinct count
    // before the current index and after the current index.
    for (let i = 0; i < length; ++i) {
        seenNumbers.add(nums[i]);
        // The result is the difference at the current index is the difference between the count of
        // distinct numbers seen so far and the distinct numbers from the next index to the end.
        result[i] = seenNumbers.size - suffixDistinctCount[i + 1];
    }

    return result;
}
```

```
from typing import List

class Solution:
    def distinctDifferenceArray(self, nums: List[int]) -> List[int]:
        # Calculate the length of the input list
        length = len(nums)
        # Initialize a suffix array of length 'length + 1' with zeros
        suffix_count = [0] * (length + 1)
        # Create an empty set to store unique elements
        unique_elements = set()

        # Populate the suffix_count array with count of unique elements from the end
        for i in range(length - 1, -1, -1):
            unique_elements.add(nums[i])
            suffix_count[i] = len(unique_elements)

        # Clear the set for re-use
        unique_elements.clear()
        # Initialize the answer array with zeros
        answer = [0] * length

        # Calculate the distinct count difference for each position
        for i, number in enumerate(nums):
            unique_elements.add(number)
            # Get the difference between the number of unique elements seen
            # so far and the number of unique elements that will be seen in the suffix
            answer[i] = len(unique_elements) - suffix_count[i + 1]

        # Return the answer array
        return answer
```

Time and Space Complexity

The `distinctDifferenceArray` function aims to calculate a list where each element at index `i` reflects the count of unique numbers from index `i` to the end of the list `nums`, minus the count of unique numbers from index `i+1` to the end.

Time Complexity

The function comprises two main loops which both iterate over the array `nums`:

1. The first loop runs backward from `n-1` to `0`. In each iteration, it adds the current element into the set `s`. The length of the set is then recorded in the `suf` array. Since adding an element to the set and checking its length are O(1) operations, this loop has a complexity of O(n), where `n` is the number of elements in `nums`.
2. The second loop runs forward from `0` to `n`. Similar to the first loop, elements are added to a cleared set `s`, and the length difference between the sets is stored in the `ans` array. The complexity of this loop is also O(n).

There are no nested loops, so the overall time complexity is the sum of the two loops, which is O(n) + O(n) = O(2n). Simplified, the time complexity is O(n).

Space Complexity

The space complexity includes the memory used by the data structures that scale with the input size:

1. The two sets, `s` and `suf`: `s` will contain at most `n` unique elements, thus O(n) space complexity. `suf` is an array of size `n+1`, which also gives O(n).
2. The `ans` array: Created to store the resulting differences and is of size `n`, contributing another O(n) space complexity.
3. Auxiliary variables like `i`, `x`, and `n`: These use constant space, O(1).

Summing these up, the total space complexity is O(n) + O(n) + O(n) which simplifies to O(3n). However, in Big O notation, constant factors are discarded, so the space complexity simplifies to O(n).