1122. Relative Sort Array

Problem Description

In this problem, we are provided with two integer arrays, `arr1` and `arr2`. The elements within `arr2` are distinct, which means no element repeats itself, and all these elements are present within `arr1`. Our goal is to sort `arr1` so that the order of elements matches how they appear in `arr2`, while any elements that are not found in `arr2` should be appended to the end of `arr1` in ascending order.

For example, if we have `arr1 = [2,3,1,3,2,4,6,7,9,2,19]` and `arr2 = [2,1,4,3,9,6]`, the first step is to arrange the elements of `arr1` that are also in `arr2` in the order they appear in `arr2`: `[2,2,2,1,4,3,3,9,6]`. After that, we take the remaining elements that are not in `arr2` (which are 7, 19, and 7 in this case) and place them at the end in sorted order: `[7,19,7]`. The final output array would be the concatenation of these two processes, giving us `[2,2,2,1,4,3,3,9,6,7,7,19]`.

The challenge lies in figuring out a strategy that allows us to sort `arr1` with these constraints in mind.

Intuition

The key to solving this problem efficiently is to recognize that we can use a custom sorting strategy. The default sort mechanisms most programming languages provide can often be customized with a user-defined key function that determines the sort order.

Given that the elements in `arr2` are the first to be sorted and they are in a specific order, we need to have this order play a significant role in our custom sorting strategy. For elements that are in `arr2`, we want them to appear first and in the order they are found in `arr2`.

To achieve this, we can map each element of `arr2` to its position (or index) within `arr2`. This can be done using a dictionary or a hash map that takes the element as a key and the position of that element in `arr2` as the value.

In Python, this mapping can be created as follows: `pos = {x: i for i, x in enumerate(arr2)}`

For the elements of `arr1` that are not found in `arr2`, we need to ensure they come after any element that is in `arr2`. One way to do this is to assign them an index that is larger than any index that would be assigned to the elements that are in `arr2`. Since indices in Python are zero-based and `arr2` can have, at most, the same number of elements as `arr1`, all the elements not in `arr2` can be given an index based on a large number (for example, 1000) plus their value, which ensures they will be sorted in ascending order after the elements of `arr2`.

Now, using Python's `sorted` function, we can sort `arr1` with a custom key. This key is defined by a lambda function that takes an element `x` and returns its corresponding index from the dictionary `pos` if it's in `arr2`, or `1000 + x` otherwise.

Here's the implementation in Python, which reflects our approach:

```
sorted(arr1, key=lambda x: pos.get(x, 1000 + x))
```

By using this custom sorting key, we align `arr1` elements per `arr2`'s order and then append the remaining elements in ascending order right after. The overall runtime complexity of the solution is O(NlogN), where N is the length of `arr1`.

Solution Approach

The solution is implemented using a dictionary for direct mapping and the sorting algorithm that Python provides. Here's a step-by-step explanation, referencing the provided code:

1. **Create a Mapping of `arr2`:** The solution starts by creating a dictionary called `pos`. This dictionary maps each element `x` of `arr2` to its index `i` in the array. This is done using a dictionary comprehension:

   ```
   pos = {x: i for i, x in enumerate(arr2)}
   ```

   By doing this, we are setting up a quick reference so that we can look up the index of any element from `arr2` instantly. This index acts as a custom priority value for the sorting algorithm.

2. **Custom Sort with a Lambda Function:** We need to sort `arr1`, but not by its elements' natural order. We will use Python's `sorted` function, which allows us to specify a custom key function:

   ```
   sorted(arr1, key=lambda x: pos.get(x, 1000 + x))
   ```

   The `key` parameter is crucial here. It takes a lambda function which determines how the elements of `arr1` should be compared during sorting:

   - `lambda x:` is an anonymous function that takes `x`, an element from `arr1`.
   - `pos.get(x, 1000 + x)` is the function's body that returns a value for each `x` that will be used to compare during sorting.
   - If `x` is found in `arr2` (and, as a result, the `pos` dictionary), `pos.get(x)` returns the index of `x` from `arr2`.
   - If `x` is not found in `arr2`, the method `pos.get(x, 1000 + x)` provides a default value which is the sum of 1000 and the element `x` itself. This ensures that:
     - The elements not in `arr2` are ordered at the end because their sorting key is greater than any index assigned from `arr2`.
     - They are sorted in ascending order amongst themselves because if two elements are not found in `arr2`, their sorting key is simply their value (`1000 + their own value` respects the natural ascending order).

By using this sorting strategy, we can maintain the relative ordering based on another array `arr2` while also logically appending and sorting elements not found in `arr2`.

Thus, using a combination of dictionary for direct access and sorting algorithm with a custom key function, this solution efficiently achieves the required array manipulation adhering to the conditions of the problem statement.

Example Walkthrough

Let's illustrate the solution approach using a smaller example. Suppose `arr1` contains `[8, 4, 5, 4, 6]` and `arr2` contains `[4, 6]`. We want to sort `arr1` such that the order of `arr2` is preserved for common elements, and the rest are sorted in ascending order at the end.

Step 1: Create a Mapping for `arr2` Elements:

First, we create a dictionary that maps each element of `arr2` to its index:

```
pos = {x: i for i, x in enumerate(arr2)}
# pos will be {4: 0, 6: 1}
```

Here, `4` is at index `0` in `arr2` and `6` is at index `1`.

Step 2: Sort `arr1` with a Custom Sort Key:

We now sort `arr1` using Python's `sorted` function with the custom key:

```
sorted(arr1, key=lambda x: pos.get(x, 1000 + x))
```

Breaking down how the key function works for each element in `arr1`:

- `8` is not found in `arr2`, so the key would be `1000 + 8 = 1008`.
- The first `4` is found in `arr2` and its index is `0`; hence, the key would be `0`.
- `5` is not found in `arr2`, so its key is `1000 + 5 = 1005`.
- The second `4` is found and its key is again `0`.
- `6` is found and its index is `1`, so the key is `1`.

The sorted order using these keys would be `[4, 4, 6, 5, 8]`.

Step 3: Review the Final Sorted Array:

So, the elements from `arr1` that were also in `arr2` are now sorted in `arr2` order: `4` comes first, followed by `6`. The remaining elements, not in `arr2`, follow at the end in ascending order: `5` and then `8`.

In conclusion, after applying the custom key sorting strategy to `arr1`, we get the final sorted array: `[4, 4, 6, 5, 8]`, successfully matching the required conditions.

Python Solution

```
class Solution:
    def relative_sort_array(self, arr1: List[int], arr2: List[int]) -> List[int]:
        # Create a dictionary to map elements of arr2 to their indices.
        position_map = {value: index for index, value in enumerate(arr2)}

        # Sort arr1 based on the condition that elements of arr2 should come first
        # in the order they appear in arr2, followed by the remaining elements in
        # ascending order.
        sorted_arr1 = sorted(arr1, key=lambda x: position_map.get(x, 1000 + x))

        # The lambda function inside sorted uses the 'get' method to find the position
        # of x from position_map if x exists in arr2. Otherwise, it assigns a number
        # greater than 1000 to ensure that x is positioned after all elements of arr2.
        # This is based on the assumption that the elements of arr1 do not exceed 1000.

        return sorted_arr1

```

Java Solution

```
class Solution {
    public int[] relativeSortArray(int[] arr1, int[] arr2) {
        // Create a hashmap to store the positions of each element in arr2
        Map<Integer, Integer> elementToIndexMap = new HashMap<>(arr2.length);
        for (int i = 0; i < arr2.length; ++i) {
            // Fill the hashmap with the position of each element
            elementToIndexMap.put(arr2[i], i);
        }

        // Create a new 2D array to hold elements and their corresponding positions
        int[][] elementPositionPairs = new int[arr1.length][0];
        for (int index = 0; index < elementPositionPairs.length; ++index) {
            // Use the position from arr2 if present, or else use the value from arr1 plus the length of arr2
            elementPositionPairs[index] = new int[]{arr1[index], elementToIndexMap.getOrDefault(arr1[index], arr2.length + arr1[index]}
        }

        // Sort the 2D array based on the positions
        Arrays.sort(elementPositionPairs, (pair1, pair2) -> pair1[1] - pair2[1]);

        // Place the sorted elements back into arr1
        for (int index = 0; index < elementPositionPairs.length; ++index) {
            arr1[index] = elementPositionPairs[index][0];
        }

        // Return the sorted arr1
        return arr1;
    }
}
```

C++ Solution

```
#include <vector>
#include <unordered_map>
#include <algorithm>

class Solution {
public:
    // Function to sort arr1 with respect to the order in arr2
    std::vector<int> relativeSortArray(std::vector<int>& arr1, std::vector<int>& arr2) {
        // We will use an unordered_map to store the position (index) of
        // each element in arr2; this position is used when sorting arr1
        std::unordered_map<int, int> elementToIndex;
        for (int i = 0; i < arr2.size(); ++i) {
            elementToIndex[arr2[i]] = i; // Map the element to its index in arr2
        }

        // Vector to temporarily store pairs of position and value
        // from original array arr1. If the element does not exist in arr2,
        // we assign the index as the size of arr2 which is effectively
        // putting it at the end of the sorted array.
        std::vector<std::pair<int, int>> tempArray;
        for (int value : arr1) {
            // Find the index if value exists in arr2; for elements not in arr2, set index to arr2's size
            int index = elementToIndex.count(value) ? elementToIndex[value] : arr2.size();
            // Create a pair of (index, value) for each element in arr1
            tempArray.emplace_back(index, value);
        }

        // Sort the tempArray based on the previously stored positions (indices)
        // If two elements have the same position, they're compared by their values
        std::sort(tempArray.begin(), tempArray.end());

        // Reassign sorted values back to arr1
        for (int i = 0; i < arr1.size(); ++i) {
            arr1[i] = tempArray[i].second; // Set arr1[i] with the value from the sorted pair
        }

        // Return the sorted arr1
        return arr1;
    }
};
```

Typescript Solution

```
// Given two arrays arr1 and arr2, the function sorts elements of arr1
// such that the relative ordering of items in arr1 are the same as in arr2.
// Elements not present in arr2 will be placed at the end of arr1 in ascending order.
function relativeSortArray(arr1: number[], arr2: number[]): number[] {
    // Create a mapping of element to its position for quick access
    const elementToIndex: Map<number, number> = new Map();
    // Fill the map with elements of arr2 and their corresponding indices
    for (let index = 0; index < arr2.length; ++index) {
        elementToIndex.set(arr2[index], index);
    }

    // Initialize an array to hold pairs of position in arr2 and the value
    const sortedPairs: [number, number][] = [];

    // Map each element of arr1 to its corresponding index in arr2, with a fallback index
    for (const element of arr1) {
        // Get the index position of the element from the map or use default if not present
        const indexInArr2 = elementToIndex.get(element) ?? arr2.length;
        sortedPairs.push([indexInArr2, element]);
    }

    // Sort the array: first by the index position and then by the element values
    sortedPairs.sort((a, b) => a[0] - b[0] || a[1] - b[1]);

    // Map the sorted array back to a single array of values
    return sortedPairs.map(pair => pair[1]);
}

```

Time and Space Complexity

Time Complexity

The time complexity of the given code primarily depends on the sorting algorithm used by the `sorted` function in Python.

- Creating the `pos` dictionary has a time complexity of `O(M)` where `M` is the length of `arr2`, since we iterate over each element of `arr2` once.
- The `sorted` function has a time complexity of `O(N log N)` where `N` is the length of `arr1`. This is because Python uses the TimSort algorithm for sorting, which is a hybrid sorting algorithm derived from merge sort and insertion sort.

However, since the key function in the sorted algorithm uses a lookup in the `pos` dictionary, which is `O(1)` time complexity for each element, together with the condition to handle elements not in `pos`, the overall time complexity of the sorting with these operations is still `O(N log N)`.

Therefore, the total time complexity of the algorithm is `O(M + N log N)`.

Space Complexity

The space complexity of the algorithm consists of the space needed for the `pos` dictionary and any additional space used by the sorting algorithm:

- The `pos` dictionary has a space complexity of `O(M)` because it contains as many entries as there are elements in `arr2`.
- The `sorted` function returns a new list and internally uses additional space which depends on the specifics of the implementation. In the worst case, this could be `O(N)` space.

The total space complexity is therefore `O(M + N)`.