Contents

Preface **vii**

Chapter 1. Probability Theory **1**

1.1 Probability Spaces and Random Elements 1

1.1.1 σ-fields and measures 1

1.1.2 Measurable functions and distributions 6

1.2 Integration and Differentiation 9

1.2.1 Integration 9

1.2.2 Radon-Nikodym derivative 14

1.3 Distributions and Their Characteristics 17

1.3.1 Useful probability densities 17

1.3.2 Moments and generating functions 25

1.4 Conditional Expectations 30

1.4.1 Conditional expectations 30

1.4.2 Independence 34

1.4.3 Conditional distributions 36

1.5 Asymptotic Theorems 38

1.5.1 Convergence modes and stochastic orders 38

1.5.2 Convergence of transformations 42

1.5.3 The law of large numbers 45

1.5.4 The central limit theorem 47

1.6 Exercises 49

Chapter 2. Fundamentals of Statistics **61**

2.1 Populations, Samples, and Models 61

2.1.1 Populations and samples 61

2.1.2 Parametric and nonparametric models 64
2.1.3 Exponential and location-scale families 66
2.2 Statistics and Sufficiency 70
2.2.1 Statistics and their distributions 70
2.2.2 Sufficiency and minimal sufficiency 73
2.2.3 Complete statistics 79
2.3 Statistical Decision Theory 83
2.3.1 Decision rules, loss functions, and risks 83
2.3.2 Admissibility and optimality 86
2.4 Statistical Inference 92
2.4.1 Point estimators 92
2.4.2 Hypothesis tests 95
2.4.3 Confidence sets 99
2.5 Asymptotic Criteria and Inference 101
2.5.1 Consistency 102
2.5.2 Asymptotic bias, variance, and mse 105
2.5.3 Asymptotic inference 109
2.6 Exercises 112

Chapter 3. Unbiased Estimation **127**
3.1 The UMVUE 127
3.1.1 Sufficient and complete statistics 128
3.1.2 A necessary and sufficient condition 132
3.1.3 Information inequality 135
3.1.4 Asymptotic properties of UMVUE's 138
3.2 U-Statistics 140
3.2.1 Some examples 140
3.2.2 Variances of U-statistics 142
3.2.3 The projection method 144
3.3 The LSE in Linear Models 148
3.3.1 The LSE and estimability 148
3.3.2 The UMVUE and BLUE 152
3.3.3 Robustness of LSE's 155
3.3.4 Asymptotic properties of LSE's 159
3.4 Unbiased Estimators in Survey Problems 161

3.4.1 UMVUE's of population totals 161
3.4.2 Horvitz-Thompson estimators 165
3.5 Asymptotically Unbiased Estimators 170
3.5.1 Functions of unbiased estimators 170
3.5.2 The method of moments 173
3.5.3 V-statistics 176
3.5.4 The weighted LSE 179
3.6 Exercises 182

Chapter 4. Estimation in Parametric Models **193**
4.1 Bayes Decisions and Estimators 193
4.1.1 Bayes actions 193
4.1.2 Empirical and hierarchical Bayes methods 198
4.1.3 Bayes rules and estimators 201
4.1.4 Markov chain Monte Carlo 207
4.2 Invariance 213
4.2.1 One-parameter location families 213
4.2.2 One-parameter scale families 217
4.2.3 General location-scale families 219
4.3 Minimaxity and Admissibility 223
4.3.1 Estimators with constant risks 223
4.3.2 Results in one-parameter exponential families 227
4.3.3 Simultaneous estimation and shrinkage estimators . . 229
4.4 The Method of Maximum Likelihood 235
4.4.1 The likelihood function and MLE's 235
4.4.2 MLE's in generalized linear models 241
4.4.3 Quasi-likelihoods and conditional likelihoods 245
4.5 Asymptotically Efficient Estimation 248
4.5.1 Asymptotic optimality 248
4.5.2 Asymptotic efficiency of MLE's and RLE's 252
4.5.3 Other asymptotically efficient estimators 257
4.6 Exercises 261

Chapter 5. Estimation in Nonparametric Models **277**
5.1 Distribution Estimators 277
5.1.1 Empirical c.d.f.'s in i.i.d. cases 278

5.1.2 Empirical likelihoods 281
5.1.3 Density estimation 288
5.2 Statistical Functionals 291
5.2.1 Differentiability and asymptotic normality 291
5.2.2 L-, M-, R-estimators and rank statistics 296
5.3 Linear Functions of Order Statistics 304
5.3.1 Sample quantiles 304
5.3.2 Robustness and efficiency 308
5.3.3 L-estimators in linear models 311
5.4 Generalized Estimating Equations 312
5.4.1 The GEE method and its relationship with others . . 313
5.4.2 Consistency of GEE estimators 317
5.4.3 Asymptotic normality of GEE estimators 321
5.5 Variance Estimation 325
5.5.1 The substitution method 326
5.5.2 The jackknife 329
5.5.3 The bootstrap 334
5.6 Exercises 337

Chapter 6. Hypothesis Tests **345**
6.1 UMP Tests 345
6.1.1 The Neyman-Pearson lemma 346
6.1.2 Monotone likelihood ratio 349
6.1.3 UMP tests for two-sided hypotheses 353
6.2 UMP Unbiased Tests 356
6.2.1 Unbiasedness and similarity 356
6.2.2 UMPU tests in exponential families 358
6.2.3 UMPU tests in normal families 362
6.3 UMP Invariant Tests 369
6.3.1 Invariance and UMPI tests 369
6.3.2 UMPI tests in normal linear models 374
6.4 Tests in Parametric Models 380
6.4.1 Likelihood ratio tests 380
6.4.2 Asymptotic tests based on likelihoods 383
6.4.3 χ^2-tests 387

6.4.4 Bayes tests 392
6.5 Tests in Nonparametric Models 394
6.5.1 Sign, permutation, and rank tests 394
6.5.2 Kolmogorov-Smirnov and Cramér-von Mises tests . . 398
6.5.3 Empirical likelihood ratio tests 401
6.5.4 Asymptotic tests 404
6.6 Exercises 406

Chapter 7. Confidence Sets **421**
7.1 Construction of Confidence Sets 421
7.1.1 Pivotal quantities 421
7.1.2 Inverting acceptance regions of tests 427
7.1.3 The Bayesian approach 430
7.1.4 Prediction sets 432
7.2 Properties of Confidence Sets 434
7.2.1 Lengths of confidence intervals 434
7.2.2 UMA and UMAU confidence sets 438
7.2.3 Randomized confidence sets 441
7.2.4 Invariant confidence sets 443
7.3 Asymptotic Confidence Sets 445
7.3.1 Asymptotically pivotal quantities 445
7.3.2 Confidence sets based on likelihoods 447
7.3.3 Results for quantiles 451
7.4 Bootstrap Confidence Sets 453
7.4.1 Construction of bootstrap confidence intervals 453
7.4.2 Asymptotic correctness and accuracy 457
7.4.3 High-order accurate bootstrap confidence sets 463
7.5 Simultaneous Confidence Intervals 467
7.5.1 Bonferroni's method 468
7.5.2 Scheffé's method in linear models 469
7.5.3 Tukey's method in one-way ANOVA models 471
7.5.4 Confidence bands for c.d.f.'s 473
7.6 Exercises 475

Appendix A. Abbreviations **489**

Appendix B. Notation **491**

References **493**

Author Index **505**

Subject Index **509**