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## NUMBERING SYSTEM

The numbering system is designed to enable quick location of theorems, examples, and so on, which are referred to in the text. Theorems, Lemmas, Definitions, and so on, are numbered sequentially along with equations and expressions within each Section (e.g.: Definition 1.2.1 is in Chapter 1, Section 2; it is followed by expression (1.2.2); and so on). [This is simpler to use than the common system wherein Theorems, Lemmas, Definitions, and so on are numbered sequentially in a *separate* system from the equations and expressions; such a system can have a Definition 1.2.1 as well as an expression (1.2.1), often separated by several pages.]

Figures in the text have a separate numbering system (necessitated by the fact that their placement in the text must often differ from their placement in the manuscript that preceded it); for example Figure 1.2-2 is the second figure in Section 2 of Chapter 1. Tables, except for those appearing at the back of the book, are numbered similarly in a separate system. Footnotes are numbered sequentially (1, 2, . . .) within each chapter.

MATHEMATICAL NOTATIONS\*

$\{x: \mathcal{A}(x)\}$	the set of all $x$ such that $\mathcal{A}(x)$ holds; for example, $\{x: x \text{ is an integer}\} = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$
$\in$	is a member of (belongs to)
$\notin$	is not a member of (does not belong to)
$\Rightarrow$	implies
$\nRightarrow$	does not imply
$\ni$	such that
iff	if and only if
$\Leftrightarrow$	if and only if
$\geq$	greater than or equal to
$>$	greater than
$\leq$	less than or equal to
$<$	less than
$\gg$	much larger than
$\equiv$	identically equal to
$\emptyset$	the empty set

\*Common notations (such as  $\neq$  for not equal,  $\infty$  for infinity,  $\prod$  for a product, and  $\Sigma$  for a summation) are not listed. Specialized notations (such as  $!$  for a factorial) are given in the Glossary of Symbols.

GREEK ALPHABET

A	$\alpha$	alpha	N	$\nu$	nu
B	$\beta$	beta	$\Xi$	$\xi$	xi
$\Gamma$	$\gamma$	gamma	O	$o$	omicron
$\Delta$	$\delta$	delta	$\Pi$	$\pi$	pi
E	$\epsilon$	epsilon	P	$\rho$	rho
Z	$\zeta$	zeta	$\Sigma$	$\sigma$	sigma
H	$\eta$	eta	T	$\tau$	tau
$\Theta, \theta$	$\theta$	theta	$\Upsilon$	$\upsilon$	upsilon
I	$\iota$	iota	$\Phi$	$\phi, \varphi$	phi
K	$\kappa$	kappa	X	$\chi$	chi
$\Lambda$	$\lambda$	lambda	$\Psi$	$\psi$	psi
M	$\mu$	mu	$\Omega$	$\omega$	omega