Simple Derivation rules

1. **Power Derivative**

$$\frac{dx^n}{dx} = n.x^{n-1}$$

Similarly

$$\frac{dx^{-n}}{dx} = -n.x^{-(n+1)}$$

2. **Log Derivative**

$$\frac{dlog(x)}{dx} = \frac{1}{x}$$

3. **Exponential Derivative**

$$\frac{de^{ax}}{dx} = a.e^{ax}$$

Similarly,

$$\frac{de^{-ax}}{dx} = -a.e^{-ax}$$

Lets come back the derivative for sigmoid

$$\sigma = \frac{1}{1+e^{-z}}$$

Lets write the same as a power

$$\sigma = (1+e^{-z})^{-1}$$

Lets substitute $(1+e^{-z})$ this with u, thus

$$\sigma = (u)^{-1}$$

And lets use power rule here and take the derivative of sigmoid equation in terms u w.r.t to z

$$\frac{du^{-1}}{dz} = -u^{-2}.\frac{du}{dz}$$

Lets work seperately on calulcating $\frac{du}{dx}$

$$\frac{du}{dz} = \frac{d(1+e^{-z})}{dz}$$

Lets do the same thing, lets take the derivative of above equation in terms z w.r.t to x first, and then multiply $\frac{dz}{dx}$

$$\frac{du}{dz} = \frac{d(1)}{dz} + \frac{d(e^{-z})}{dz}$$

First term, becomes zero.

We already know that derivative of negative exponent $\frac{de^{-ax}}{dx} = -a.e^{-ax}$

$$\frac{du}{dz} = -e^{-z}$$

Now subsituting the value of $\frac{du}{dz}$ to equation $\sigma'(z) = \frac{du^{-1}}{dz} = -u^{-2}.\frac{du}{dz}$

$$\sigma'(z) = -u^{-2}.-e^{-z}$$

Replacing u with $1+e^{-z}$

$$\sigma'(z) = \frac{1.e^{-z}}{(1+e^{-z})^2}$$

Now rewriting the equation a bit

$$\sigma'(z) = \frac{1}{1+e^{-z}}\frac{e^{-z}}{1+e^{-z}}$$

And since 1-1 = 0 , we can do a little tweek in the equation

$$\sigma'(z) = \frac{1}{1+e^{-z}}\frac{e^{-z}+1-1}{1+e^{-z}}$$

Rewriting equation a bit

$$\sigma'(z) = \frac{1}{1+e^{-z}}[\frac{e^{-z}+1}{1+e^{-z}} - \frac{1}{1+e^{-z}}]$$

Equation becomes:

$$\sigma'(z) = \frac{1}{1+e^{-z}}[1 - \frac{1}{1+e^{-z}}]$$

As we know $\sigma(z) = \frac{1}{1+e^{-z}}$, Equation becomes:

$$\sigma'(z) = \sigma(z)[1-\sigma(z)]$$