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List of Symbols

A_{ij}	the ij -th entry of the matrix A	page 9
A^{-1}	the inverse of the matrix A	page 100
A^\dagger	the pseudoinverse of the matrix A	page 414
A^*	the adjoint of the matrix A	page 331
\tilde{A}_{ij}	the matrix A with row i and column j deleted	page 210
A^t	the transpose of the matrix A	page 17
$(A B)$	the matrix A augmented by the matrix B	page 161
$B_1 \oplus \cdots \oplus B_k$	the direct sum of matrices B_1 through B_k	page 320
$\mathcal{B}(\mathbf{V})$	the set of bilinear forms on \mathbf{V}	page 422
β^*	the dual basis of β	page 120
β_x	the \mathbf{T} -cyclic basis generated by x	page 526
\mathbb{C}	the field of complex numbers	page 7
C_i	the i th Gerschgorin disk	page 296
$\text{cond}(A)$	the condition number of the matrix A	page 469
$C^n(R)$	set of functions f on R with $f^{(n)}$ continuous	page 21
C^∞	set of functions with derivatives of every order	page 130
$C(R)$	the vector space of continuous functions on R	page 18
$C([0, 1])$	the vector space of continuous functions on $[0, 1]$	page 331
C_x	the \mathbf{T} -cyclic subspace generated by x	page 525
D	the derivative operator on C^∞	page 131
$\det(A)$	the determinant of the matrix A	page 232
δ_{ij}	the Kronecker delta	page 89
$\dim(\mathbf{V})$	the dimension of \mathbf{V}	page 47
e^A	$\lim_{m \rightarrow \infty} \left(I + A + \frac{A^2}{2!} + \cdots + \frac{A^m}{m!} \right)$	page 312
e_i	the i th standard vector of F^n	page 43
E_λ	the eigenspace of \mathbf{T} corresponding to λ	page 264
F	a field	page 6
$f(A)$	the polynomial $f(x)$ evaluated at the matrix A	page 565
F^n	the set of n -tuples with entries in a field F	page 8

$f(T)$	the polynomial $f(x)$ evaluated at the operator T	page 565
$\mathcal{F}(S, F)$	the set of functions from S to a field F	page 9
H	space of continuous complex functions on $[0, 2\pi]$	page 332
I_n or I	the $n \times n$ identity matrix	page 89
I_V or I	the identity operator on V	page 67
K_λ	generalized eigenspace of T corresponding to λ	page 485
K_ϕ	$\{x: (\phi(T))^p(x) = 0 \text{ for some positive integer } p\}$	page 525
L_A	left-multiplication transformation by matrix A	page 92
$\lim_{m \rightarrow \infty} A_m$	the limit of a sequence of matrices	page 284
$\mathcal{L}(V)$	the space of linear transformations from V to V	page 82
$\mathcal{L}(V, W)$	the space of linear transformations from V to W	page 82
$M_{m \times n}(F)$	the set of $m \times n$ matrices with entries in F	page 9
$\nu(A)$	the column sum of the matrix A	page 295
$\nu_j(A)$	the j th column sum of the matrix A	page 295
$N(T)$	the null space of T	page 67
$\text{nullity}(T)$	the dimension of the null space of T	page 69
O	the zero matrix	page 8
$\text{per}(M)$	the permanent of the 2×2 matrix M	page 448
$P(F)$	the space of polynomials with coefficients in F	page 10
$P_n(F)$	the polynomials in $P(F)$ of degree at most n	page 18
ϕ_β	standard representation with respect to basis β	page 104
R	the field of real numbers	page 7
$\text{rank}(A)$	the rank of the matrix A	page 152
$\text{rank}(T)$	the rank of the linear transformation T	page 69
$\rho(A)$	the row sum of the matrix A	page 295
$\rho_i(A)$	the i th row sum of the matrix A	page 295
$R(T)$	the range of the linear transformation T	page 67

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