

Contents

Preface *xiii*

I Foundations

Introduction 3

1 The Role of Algorithms in Computing 5

- 1.1 Algorithms 5
- 1.2 Algorithms as a technology 11

2 Getting Started 16

- 2.1 Insertion sort 16
- 2.2 Analyzing algorithms 23
- 2.3 Designing algorithms 29

3 Growth of Functions 43

- 3.1 Asymptotic notation 43
- 3.2 Standard notations and common functions 53

4 Divide-and-Conquer 65

- 4.1 The maximum-subarray problem 68
- 4.2 Strassen's algorithm for matrix multiplication 75
- 4.3 The substitution method for solving recurrences 83
- 4.4 The recursion-tree method for solving recurrences 88
- 4.5 The master method for solving recurrences 93

★ 4.6 Proof of the master theorem 97

5 Probabilistic Analysis and Randomized Algorithms 114

- 5.1 The hiring problem 114
- 5.2 Indicator random variables 118
- 5.3 Randomized algorithms 122

★ 5.4 Probabilistic analysis and further uses of indicator random variables 130

II *Sorting and Order Statistics*

	Introduction	147
6	Heapsort	151
6.1	Heaps	151
6.2	Maintaining the heap property	154
6.3	Building a heap	156
6.4	The heapsort algorithm	159
6.5	Priority queues	162
7	Quicksort	170
7.1	Description of quicksort	170
7.2	Performance of quicksort	174
7.3	A randomized version of quicksort	179
7.4	Analysis of quicksort	180
8	Sorting in Linear Time	191
8.1	Lower bounds for sorting	191
8.2	Counting sort	194
8.3	Radix sort	197
8.4	Bucket sort	200
9	Medians and Order Statistics	213
9.1	Minimum and maximum	214
9.2	Selection in expected linear time	215
9.3	Selection in worst-case linear time	220

III *Data Structures*

	Introduction	229
10	Elementary Data Structures	232
10.1	Stacks and queues	232
10.2	Linked lists	236
10.3	Implementing pointers and objects	241
10.4	Representing rooted trees	246
11	Hash Tables	253
11.1	Direct-address tables	254
11.2	Hash tables	256
11.3	Hash functions	262
11.4	Open addressing	269
★ 11.5	Perfect hashing	277

12	Binary Search Trees	286
12.1	What is a binary search tree?	286
12.2	Querying a binary search tree	289
12.3	Insertion and deletion	294
★ 12.4	Randomly built binary search trees	299
13	Red-Black Trees	308
13.1	Properties of red-black trees	308
13.2	Rotations	312
13.3	Insertion	315
13.4	Deletion	323
14	Augmenting Data Structures	339
14.1	Dynamic order statistics	339
14.2	How to augment a data structure	345
14.3	Interval trees	348

IV Advanced Design and Analysis Techniques

	Introduction	357
15	Dynamic Programming	359
15.1	Rod cutting	360
15.2	Matrix-chain multiplication	370
15.3	Elements of dynamic programming	378
15.4	Longest common subsequence	390
15.5	Optimal binary search trees	397
16	Greedy Algorithms	414
16.1	An activity-selection problem	415
16.2	Elements of the greedy strategy	423
16.3	Huffman codes	428
★ 16.4	Matroids and greedy methods	437
★ 16.5	A task-scheduling problem as a matroid	443
17	Amortized Analysis	451
17.1	Aggregate analysis	452
17.2	The accounting method	456
17.3	The potential method	459
17.4	Dynamic tables	463

V *Advanced Data Structures*

	Introduction	481
18	B-Trees	484
18.1	Definition of B-trees	488
18.2	Basic operations on B-trees	491
18.3	Deleting a key from a B-tree	499
19	Fibonacci Heaps	505
19.1	Structure of Fibonacci heaps	507
19.2	Mergeable-heap operations	510
19.3	Decreasing a key and deleting a node	518
19.4	Bounding the maximum degree	523
20	van Emde Boas Trees	531
20.1	Preliminary approaches	532
20.2	A recursive structure	536
20.3	The van Emde Boas tree	545
21	Data Structures for Disjoint Sets	561
21.1	Disjoint-set operations	561
21.2	Linked-list representation of disjoint sets	564
21.3	Disjoint-set forests	568
★ 21.4	Analysis of union by rank with path compression	573

VI *Graph Algorithms*

	Introduction	587
22	Elementary Graph Algorithms	589
22.1	Representations of graphs	589
22.2	Breadth-first search	594
22.3	Depth-first search	603
22.4	Topological sort	612
22.5	Strongly connected components	615
23	Minimum Spanning Trees	624
23.1	Growing a minimum spanning tree	625
23.2	The algorithms of Kruskal and Prim	631

24	Single-Source Shortest Paths	643
24.1	The Bellman-Ford algorithm	651
24.2	Single-source shortest paths in directed acyclic graphs	655
24.3	Dijkstra's algorithm	658
24.4	Difference constraints and shortest paths	664
24.5	Proofs of shortest-paths properties	671
25	All-Pairs Shortest Paths	684
25.1	Shortest paths and matrix multiplication	686
25.2	The Floyd-Warshall algorithm	693
25.3	Johnson's algorithm for sparse graphs	700
26	Maximum Flow	708
26.1	Flow networks	709
26.2	The Ford-Fulkerson method	714
26.3	Maximum bipartite matching	732
★	26.4 Push-relabel algorithms	736
★	26.5 The relabel-to-front algorithm	748

VII Selected Topics

	Introduction	769
27	Multithreaded Algorithms	772
27.1	The basics of dynamic multithreading	774
27.2	Multithreaded matrix multiplication	792
27.3	Multithreaded merge sort	797
28	Matrix Operations	813
28.1	Solving systems of linear equations	813
28.2	Inverting matrices	827
28.3	Symmetric positive-definite matrices and least-squares approximation	832
29	Linear Programming	843
29.1	Standard and slack forms	850
29.2	Formulating problems as linear programs	859
29.3	The simplex algorithm	864
29.4	Duality	879
29.5	The initial basic feasible solution	886

30	Polynomials and the FFT	898
30.1	Representing polynomials	900
30.2	The DFT and FFT	906
30.3	Efficient FFT implementations	915
31	Number-Theoretic Algorithms	926
31.1	Elementary number-theoretic notions	927
31.2	Greatest common divisor	933
31.3	Modular arithmetic	939
31.4	Solving modular linear equations	946
31.5	The Chinese remainder theorem	950
31.6	Powers of an element	954
31.7	The RSA public-key cryptosystem	958
★ 31.8	Primality testing	965
★ 31.9	Integer factorization	975
32	String Matching	985
32.1	The naive string-matching algorithm	988
32.2	The Rabin-Karp algorithm	990
32.3	String matching with finite automata	995
★ 32.4	The Knuth-Morris-Pratt algorithm	1002
33	Computational Geometry	1014
33.1	Line-segment properties	1015
33.2	Determining whether any pair of segments intersects	1021
33.3	Finding the convex hull	1029
33.4	Finding the closest pair of points	1039
34	NP-Completeness	1048
34.1	Polynomial time	1053
34.2	Polynomial-time verification	1061
34.3	NP-completeness and reducibility	1067
34.4	NP-completeness proofs	1078
34.5	NP-complete problems	1086
35	Approximation Algorithms	1106
35.1	The vertex-cover problem	1108
35.2	The traveling-salesman problem	1111
35.3	The set-covering problem	1117
35.4	Randomization and linear programming	1123
35.5	The subset-sum problem	1128

VIII Appendix: Mathematical Background

	Introduction	1143
A	Summations	1145
	A.1 Summation formulas and properties	1145
	A.2 Bounding summations	1149
B	Sets, Etc.	1158
	B.1 Sets	1158
	B.2 Relations	1163
	B.3 Functions	1166
	B.4 Graphs	1168
	B.5 Trees	1173
C	Counting and Probability	1183
	C.1 Counting	1183
	C.2 Probability	1189
	C.3 Discrete random variables	1196
	C.4 The geometric and binomial distributions	1201
★	C.5 The tails of the binomial distribution	1208
D	Matrices	1217
	D.1 Matrices and matrix operations	1217
	D.2 Basic matrix properties	1222
	Bibliography	1231
	Index	1251
