

Chapter Two: Mathematical fundamental for studying ANN

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Vector and Matrix fundamentals

- Matrices, vectors and vector function provide framework for visualization and computation
- Many concepts involve visualization of points in d -dimensional space
- Its easy if d is less than or equal to 3 but for $d > 3$ its hard

Elementary matrices

- A $n \times m$ dimensional matrix arranges mn entities in an array of n rows each with m elements

$$A = [a_{ij}] \quad i=1,2,\dots,n \quad j=1,2,\dots,m$$

Properties

- If $m=n$ —square matrix
- If $m=1$ ---column vector
- If $n=1$ ---- row vector
- Vectors are represented by underline e.g x , length of vector is also called dimension in vector space within which the vector resides
- If $m=n=1$ the matrix is scalar
- Transpose of A denoted by A^T is $[a_{ji}]$ $j=1,2,\dots,m$ and $i=1,2,\dots,n$
- If $A = A^T$ matrix is called symmetric and A must be square for that

- Matrix partition

$$\begin{bmatrix} \begin{matrix} p \times q \\ A_1 \end{matrix} & \begin{matrix} p \times (m-q) \\ A_2 \end{matrix} \\ \begin{matrix} (n-p) \\ A_3 \end{matrix} & \begin{matrix} (n-p)(m-q) \\ A_4 \end{matrix} \end{bmatrix}$$

p and q are real and positive integer

Matrix A of order $m \times n$ is partitioned as above. Such partition is useful for visualization and computation

- A column vector most important partition of a matrix is into an array of column vectors as

$$A = [\underline{a}_1, \underline{a}_2, \dots, \underline{a}_m]$$

here \underline{a}_i is a $n \times 1$ dimensional column vector

Elementary matrix operation

- Addition (subtraction)
- Multiplication
- Scaling

$C = A \times B$ A is $m \times n$ and B is $n \times p$ order matrix

For this $c_{ij} = \sum a_{ik} b_{kj}$ k runs from 1 to n

Vectors

- For any positive integer 'd' let R^d represents set of all ordered n tuples of the form

$$\{x_1, x_2, \dots, x_d\}$$

- They can be viewed as the co-ordinates of a point x in d – dimensional space- and it is necessary that the co-ordinate system is specified
- A x_i co-ordinate can be represented using vector as

$$x = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{Bmatrix}$$

Linearity

- If a mapping function holds superposition then it is said to be linear

$$x = ax_1 + bx_2 \text{ implies that } f(x) = f(x_1) + f(x_2)$$

- Vector Matrix equation

$$\begin{matrix} n \times d & d \times 1 \\ A & x \end{matrix} = \begin{matrix} n \times 1 \\ y \end{matrix}$$

- Multiple interpretation of this equations

