What "practical" examples of using linear algebra and basic multivariable calculus can I use to motivate my students?

Principal Component Analysis of some cool dataset can look like real magic in many fields.

Running a pca on political opinions or votes and showing how different political parties can be visualized as clusterings on a field thanks to a projection that maximizes variance should impress political science majors. Biology majors should love how that same thing can be done to flower measurements (the standard "Iris dataset" you'll find online).

The pagerank algorithm draws heavily from linear algebra. It is also quite easy to explain if the students have learned about eigenvectors.

Another good example is to talk about transformation matrices that are used in computer graphics (computer games in particular) to rotate/scale or position a 3d object.

Are there any real life application of the rank of a matrix?

The rank of a matrix is only meaningful if you have a matrix of exact numbers. If you have noisy data, then there is no meaningful concept of rank, because non-full-rank matrices have measure zero. In other words, if you have a matrix with less than the maximum possible rank for its size, then you can make an arbitrarily small tweak to the entries and get a full-rank matrix.

Needless to say, this rules out applying the notion of rank directly to anything consisting of measured data.

However, numerically unstable concepts like rank can be vitally important in understanding the mathematics used to build a model. Singularities, for instance, are important throughout mathematics and science, and they can be precisely characterized as points where the derivative does not have full rank.

Speaking mathematically rank represents the number of independent rows or columns of a matrix.
If a matrix have low rank then it means that it has less number of independent rows or columns. Which is other way of saying that the data represented in the matrix has low degree of freedom.
There are very practical applications of above concept. If there is some unknown data to be recovered in matrix form and it is know that it has low rank then it can be recovered very efficiently.
Read about low rank matrix minimization and application if u are interested

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So far as I know, there are many, especially in the computer world. Matrix rotating for your BattleField3 game; Social networking clustering of Facebook; Trading optimization of Wall Street, etc. They are all in real life.

Is there any use of matrices in real life?

At very simple level, Digital Images are nothing but the spaced collection of pixels or image elements arranged in 2d space and in computer we can represent them as "Matrices", those matrices contains different values of integer which represents the level of brightness or intensity or similar properties.
Similarly various other fields also requires the same.

What use is the inverse matrix?

Inverse matrices are really useful for a variety of things, but they really come into their own for 3D transformations.

Concatenating a series of matrices together appropriately, you can represent in a single matrix the translation, rotation, skewing and scaling of a single point in space with respect to the origin.

This is great for 3d games! Define your object as a mesh of triangles with each point being a coordinate in 3d space. Then apply that entire matrix to each of the points in the object in turn, and you can tumble, roll it, fly it around, make it grow and shrink, and any number of things that you've seen a billion times in video games.

The inverse matrix? Also incredibly useful! With the inverse matrix, you canundo one of these operations. This is very useful in animation, where you might have a base point (usually called the root), and then each bone in the animation's skeleton applies another forward transform that is applied to the mesh of the object, until you get the new position of the point in 3d world space.

But you need to figure out where something is with respect to the local space of (say) Master Chief's hand? Well, you can either do that calculation in world space, or in local space. Either way, being able to walk up and down the stack of matrix transforms in the animation hierarchy is very useful - and this is where the inverse comes in. It performs the translation/scale/rotation/skew in the opposite direction, undoing it.

Or another example: Say you want to find out if a point's in a box. Sure, you could test against every plane that the box is made of... or, if you've got hardware that does fast matrix operations and bounding comparisons, you can do the following:

Figure out the matrix (M) that would take a canonical 1-unit width/height/depth cube and move it/rotate it/scale it so that it matched the box you want to test against.

Now invert this matrix, and store it. You can now take any point in world space, and multiply it by this inverse matrix. This will move the point into the space of the canonical box - so if it would be inside the box in world space, it will be inside the box in canonical box space. That is, you can simply test if each of its transformed coordinates are between -0.5 and +0.5 - if they are, it's inside the box.

A lot of matrix/linear algebra operations can be considered as compositions of transformations of a point in space. Transforming by the matrix takes you in one direction, transforming by the inverse of the matrix takes you into the opposite direction. If you can get your head around this idea, it's probably one of the most useful mental pictures you can have of matrices in general.