Fuzzy logic

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Fuzzy logic is a form of many-valued logic that deals with approximate, rather than fixed and exact reasoning. Compared to traditional binary logic (where variables may take on true or false values), fuzzy logic variables may have a truth value that ranges in degree between 0 and 1. Fuzzy logic has been extended to handle the concept of partial truth, where the truth value may range between completely true and completely false.[1] Furthermore, when linguistic variables are used, these degrees may be managed by specific functions.[2]

The term "fuzzy logic" was introduced with the 1965 proposal of fuzzy set theory by Lotfi A. Zadeh.[3][4] Fuzzy logic has been applied to many fields, from control theory to artificial intelligence. Fuzzy logic had, however, been studied since the 1920s, as infinite-valued logic—notably by Łukasiewicz and Tarski.[5]

Contents

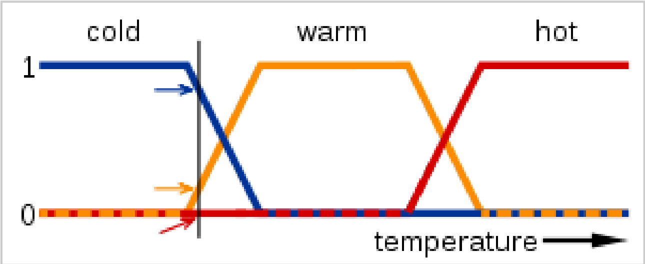
Overview

Classical logic only permits propositions having a value of truth or falsity. The notion of whether 1+1=2 is an absolute, immutable and mathematical truth. However, there exist certain propositions with variable answers, such as asking various people to identify a colour. The notion of truth doesn't fall by the wayside, but rather on a means of representing and reasoning over partial knowledge when afforded, by aggregating all possible outcomes into a dimensional spectrum.

Both degrees of truth and probabilities range between 0 and 1 and hence may seem similar at first. For example, let a 100 ml glass contain 30 ml of water. Then we may consider two concepts: empty and full. The meaning of each of them can be represented by a certain fuzzy set. Then one might define the glass as being 0.7 empty and 0.3 full. Note that the concept of emptiness would be subjective and thus would depend on the observer or designer. Another designer might, equally well, design a set membership function where the glass would be considered full for all values down to 50 ml. It is essential to realize that fuzzy logic uses truth degrees as a mathematical model of the vagueness phenomenon while probability is a mathematical model of ignorance.

Applying truth values

A basic application might characterize various sub-ranges of a continuous variable. For instance, a temperature measurement for anti-lock brakes might have several separate membership functions defining particular temperature ranges needed to control the brakes properly. Each function maps the same temperature value to a truth value in the 0 to 1 range. These truth values can then be used to determine how the brakes should be controlled.



Fuzzy logic temperature

In this image, the meanings of the expressions *cold*, *warm*, and *hot* are represented by functions mapping a temperature scale. A point on that scale has three "truth values" — one for each of the three functions. The vertical line in the image represents a particular temperature that the three arrows (truth values) gauge. Since the red arrow points to zero, this temperature may be interpreted as "not hot". The orange arrow (pointing at 0.2) may describe it as "slightly warm" and the blue arrow (pointing at 0.8) "fairly cold".

Linguistic variables

While variables in mathematics usually take numerical values, in fuzzy logic applications, the non-numeric are often used to facilitate the expression of rules and facts.[6]

A linguistic variable such as *age* may have a value such as *young* or its antonym *old*. However, the great utility of linguistic variables is that they can be modified via linguistic hedges applied to primary terms. These linguistic hedges can be associated with certain functions.

Early applications

The Japanese were the first to utilize fuzzy logic for practical applications. The first notable application was on the high-speed train in Sendai, in which fuzzy logic was able to improve the economy, comfort, and precision of the ride.[7] It has also been used in recognition of hand written symbols in Sony pocket computers; flight aid for helicopters; controlling of subway systems in order to improve driving comfort, precision of halting, and power economy; improved fuel consumption for auto mobiles; single-button control for washing machines, automatic motor control for vacuum cleaners with recognition of surface condition and degree of soiling; and prediction systems for early recognition of earthquakes through the Institute of Seismology Bureau of Metrology, Japan.[8]

Example

Hard science with IF-THEN rules

Fuzzy set theory defines fuzzy operators on fuzzy sets. The problem in applying this is that the appropriate fuzzy operator may not be known. For example, a simple temperature regulator that uses a fan might look like this:

```
IF temperature IS very cold THEN stop fan
IF temperature IS cold THEN turn down fan
IF temperature IS normal THEN maintain fan
IF temperature IS hot THEN speed up fan
```

There is no "ELSE" – all of the rules are evaluated, because the temperature might be "cold" and "normal" at the same time to different degrees.

The AND, OR, and NOT operators of boolean logic exist in fuzzy logic, usually defined as the minimum, maximum, and complement; when they are defined this way, they are called the *Zadeh operators*. So for the fuzzy variables x and y:

```
NOT x = (1 - truth(x))
x AND y = minimum(truth(x), truth(y))
x OR y = maximum(truth(x), truth(y))
```

There are also other operators, more linguistic in nature, called *hedges* that can be applied. These are generally adverbs such as "very", or "somewhat", which modify the meaning of a set using a mathematical formula.

Define with multiply

```
x AND y = x*y
x OR y = 1-(1-x)*(1-y)
```

1-(1-x)*(1-y) comes from this:

```
x OR y = NOT( AND( NOT(x), NOT(y) ) )
x OR y = NOT( AND(1-x, 1-y) )
x OR y = NOT( (1-x)*(1-y) )
x OR y = 1-(1-x)*(1-y)
```

Define with sigmoid

```
sigmoid(x)=1/(1+e^-x)
sigmoid(x)+sigmoid(-x) = 1
(sigmoid(x)+sigmoid(-x))*(sigmoid(y)+sigmoid(-y))*(sigmoid(z)+sigmoid(-z)) = 1
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Logical analysis

In mathematical logic, there are several formal systems of "fuzzy logic"; most of them belong among so-called t-norm fuzzy logic.

Propositional fuzzy logics

The most important propositional fuzzy logics are:-

- Monoidal t-norm-based propositional fuzzy logic MTL is an axiomatization of logic where conjunction is defined by a left continuous t-norm and implication is defined as the residuum of the t-norm. Its models correspond to MTL-algebras that are pre-linear commutative bounded integral residuated lattices.
- Basic propositional fuzzy logic BL is an extension of MTL logic where conjunction is defined by a continuous t-norm, and implication is also defined as the residuum of the t-norm. Its models correspond to BL-algebras.
- Łukasiewicz fuzzy logic is the extension of basic fuzzy logic BL where standard conjunction is the Łukasiewicz t-norm. It has the axioms of basic fuzzy logic plus an axiom of double negation, and its models correspond to MV-algebras.
- Gödel fuzzy logic is the extension of basic fuzzy logic BL where conjunction is Gödel t-norm. It has the axioms of BL plus an axiom of idempotence of conjunction, and its models are called G-algebras.
- Product fuzzy logic is the extension of basic fuzzy logic BL where conjunction is product t-norm. It has the axioms of BL plus another axiom for cancellativity of conjunction, and its models are called product algebras.
- Fuzzy logic with evaluated syntax (sometimes also called Pavelka's logic), denoted by EVŁ, is a further generalization of mathematical fuzzy logic. While the above kinds of fuzzy logic have traditional syntax and many-valued semantics, in EVŁ is evaluated also syntax. This means that each formula has an evaluation. Axiomatization of EVŁ stems from Łukasziewicz fuzzy logic. A generalization of classical Gödel completeness theorem is provable in EVŁ.

Predicate fuzzy logics

These extend the above-mentioned fuzzy logics by adding universal and existential quantifiers in a manner similar to the way that predicate logic is created from propositional logic. The semantics of the universal (resp. existential) quantifier in t-norm fuzzy logics is the infimum (resp. supremum) of the truth degrees of the instances of the quantified subformula.

Decidability issues for fuzzy logic

The notions of a "decidable subset" and "recursively enumerable subset" are basic ones for classical mathematics and classical logic. Thus the question of a suitable extension of these concepts to fuzzy set theory arises. A first proposal in such a direction was made by E.S. Santos by the notions of *fuzzy Turing machine*, *Markov normal fuzzy algorithm* and *fuzzy program* (see Santos 1970). Successively, L. Biacino and G. Gerla argued that the proposed definitions are rather questionable and therefore they proposed the following ones. Denote by $Ü$ the set of rational numbers in [0,1]. Then a fuzzy subset $s : S \longrightarrow [0,1]$ of a set S is *recursively enumerable* if a recursive map $h : S \times N \longrightarrow Ü$ exists such that, for every x in S, the function $h(x,n)$ is increasing with respect to n and $s(x) = \lim h(x,n)$. We say that s is *decidable* if both s and its complement $-s$ are recursively enumerable. An extension of such a theory to the general case of the L-subsets is possible (see Gerla 2006). The proposed definitions are well related with fuzzy logic. Indeed, the following theorem holds true (provided that the deduction apparatus of the considered fuzzy logic satisfies some obvious effectiveness property).

Theorem. Any axiomatizable fuzzy theory is recursively enumerable. In particular, the fuzzy set of logically true formulas is recursively enumerable in spite of the fact that the crisp set of valid formulas is not recursively enumerable, in general. Moreover, any axiomatizable and complete theory is decidable.

It is an open question to give supports for a *Church thesis* for fuzzy mathematics the proposed notion of recursive enumerability for fuzzy subsets is the adequate one. To this aim, an extension of the notions of fuzzy grammar and fuzzy Turing machine should be necessary (see for example Wiedermann's paper). Another open question is to start from this notion to find an extension of Gödel's theorems to fuzzy logic.

It is known that any boolean logic function could be represented using a truth table mapping each set of variable values into set of values $\{0, 1\}$. The task of synthesis of boolean logic function given in tabular form is one of basic tasks in traditional logic that is solved via disjunctive (conjunctive) perfect normal form.

Each fuzzy (continuous) logic function could be represented by a choice table containing all possible variants of comparing arguments and their negations. A choice table maps each variant into value of an argument or a negation of an argument. For instance, for two arguments a row of choice table contains a variant of comparing values x_1, $\neg x_1$, x_2, $\neg x_2$ and the corresponding function value $f(x_2 \leq \neg x_1 \leq x_1 \leq \neg x_2) = \neg x_1$.

The task of synthesis of fuzzy logic function given in tabular form was solved in.[9] New concepts of constituents of minimum and maximum were introduced. The sufficient and necessary conditions that a choice table defines a fuzzy logic function were derived.

Criticisms

Fuzzy databases

Once fuzzy relations are defined, it is possible to develop fuzzy relational databases. The first fuzzy relational database, FRDB, appeared in Maria Zemankova's dissertation. Later, some other models arose like the Buckles-Petry model, the Prade-Testemale Model, the Umano-Fukami model or the GEFRED model by J.M. Medina, M.A. Vila et al. In the context of fuzzy databases, some fuzzy querying languages have been defined, highlighting the SQLf by P. Bosc et al. and the FSQL by J. Galindo et al. These languages define some structures in order to include fuzzy aspects in the SQL statements, like fuzzy conditions, fuzzy comparators, fuzzy constants, fuzzy constraints, fuzzy thresholds, linguistic labels and so on.

Much progress has been made to take fuzzy logic database applications to the web and let the world easily use them, for example: http://sullivansoftwaresystems.com/cgi-bin/fuzzy-logic-match-algorithm.cgi?SearchString=garia This enables fuzzy logic matching to be incorporated into a database system or application.

Comparison to probability

Fuzzy logic and probability address different forms of uncertainty. While both fuzzy logic and probability theory can represent degrees of certain kinds of subjective belief, fuzzy set theory uses the concept of fuzzy set membership, i.e., *how much* a variable is in a set (there is not necessarily any uncertainty about this degree), and probability theory uses the concept of subjective probability, i.e., *how probable* is it that a variable is in a set (it either entirely is or entirely is not in the set in reality, but there is uncertainty around whether it is or is not). The technical consequence of this distinction is that fuzzy set theory relaxes the axioms of classical probability, which are themselves derived from adding uncertainty, but not degree, to the crisp true/false distinctions of classical Aristotelian logic.

Bruno de Finetti argues that only one kind of mathematical uncertainty, probability, is needed, and thus fuzzy logic is unnecessary. However, Bart Kosko shows in Fuzziness vs. Probability (http://sipi.usc.edu/~kosko/Fuzziness_Vs_Probability.pdf) that probability theory is a subtheory of fuzzy logic, as questions of degrees of belief in mutually-exclusive set membership in probability theory can be represented as certain cases of non-mutually-exclusive graded membership in fuzzy theory. In that context, he also derives Bayes' theorem from the concept of fuzzy subsethood. Lotfi A. Zadeh argues that fuzzy logic is different in character from probability, and is not a replacement for it. He fuzzified probability to fuzzy probability and also generalized it to possibility theory. (cf.[10])

More generally, fuzzy logic is one of many different extensions to classical logic intended to deal with issues of uncertainty outside of the scope of classical logic, the inapplicability of probability theory in many domains, and the paradoxes of Dempster-Shafer theory. See also probabilistic logics.

Relation to ecorithms

Leslie Valiant, a winner of the Turing Award, uses the term "ecorithms" to describe how many less exact systems and techniques like fuzzy logic (and "less robust" logic) can be applied to learning algorithms. Valiant essentially redefines machine learning as evolutionary. Ecorithms and fuzzy logic also have the common property of dealing with possibilities more than probabilities, although feedback and feed forward, basically stochastic "weights," are a feature of both when dealing with, for example, dynamical systems.

In general use, ecorithms are algorithms that learn from their more complex environments (Hence Eco) to generalize, approximate and simplify solution logic. Like fuzzy logic, they are methods used to overcome continuous variables or systems too complex to completely enumerate or understand discretely or exactly. See in particular p. 58 of the reference comparing induction/invariance, robust, mathematical and other logical limits in computing, where techniques including fuzzy logic and natural data selection (à la "computational Darwinism") can be used to short-cut computational complexity and limits in a "practical" way (such as the brake temperature example in this article).[11]

Compensatory Fuzzy Logic

The CFL (Compensatory Fuzzy Logic) is a branch of Fuzzy Logic. This is a new multivalent system that breaks with traditional axioms of such systems to achieve better semantic behaviour to classical systems.

In processes involving decision making, trade with the experts leads to obtaining complex and subtle formulations and requires compound predicates. The truth values obtained on these compound predicates must possess sensitivity to changes in the truth values of basic predicates.

This need is met by the use of the CFL, waiving compliance of the classical properties of conjunction and disjunction and rather opposing to them the idea that the increase or decrease of the truth value of the conjunction or disjunction caused by change the truth value of one of its components can be compensated with a corresponding decrease or increase in the other. This increase or decrease in truth may be offset by the increase or decrease in another component. This notion makes the CFL logical and useful. There are cases in which compensation is not possible. This occurs when certain thresholds are violated and there is a veto preventing compensation.

Compensatory Fuzzy Logic consists of four continuous operators: conjunction (c); disjunction (d); fuzzy strict order (or); and negation (n). The conjunction is the geometric mean and its dual as conjunctive and disjunctive operators.[12]

See also

- Adaptive neuro fuzzy inference system (ANFIS)
- Artificial neural network
- Defuzzification
- Expert system
- False dilemma
- Fuzzy architectural spatial analysis
- Fuzzy classification
- Fuzzy concept
- Fuzzy Control Language
- Fuzzy control system
- Fuzzy electronics
- Fuzzy subalgebra
- FuzzyCLIPS
- High Performance Fuzzy Computing
- IEEE Transactions on Fuzzy Systems
- Interval finite element
- Machine learning
- Neuro-fuzzy
- Noise-based logic
- Rough set
- Sorites paradox
- Type-2 fuzzy sets and systems
- Vector logic

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External links

- Formal fuzzy logic (http://en.citizendium.org/wiki/Formal_fuzzy_logic) - article at Citizendium
- Fuzzy Logic (http://www.scholarpedia.org/article/Fuzzy_Logic) - article at Scholarpedia
- Modeling With Words (http://www.scholarpedia.org/article/Modeling_with_words) - article at Scholarpedia
- Fuzzy logic (http://plato.stanford.edu/entries/logic-fuzzy/) - article at Stanford Encyclopedia of Philosophy
- Fuzzy Math (http://blog.peltarion.com/2006/10/25/fuzzy-math-part-1-the-theory) - Beginner level introduction to Fuzzy Logic
- Fuzzylite (http://www.fuzzylite.com/) - A cross-platform, free open-source Fuzzy Logic Control Library written in C++. Also has a very useful graphic user interface in QT4.
- Online Calculator based upon Fuzzy logic (http://www.cirvirlab.com/simulation/fuzzy_logic_calculator.php) – Gives online calculation in educational example of fuzzy logic model.

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