Fuzzy Logic

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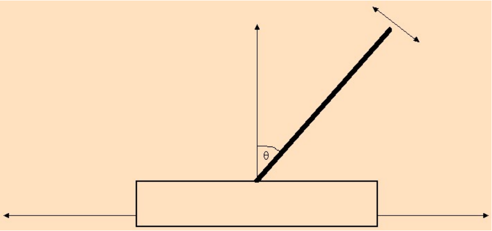
1. Introduction

1.1. Motivation: The classical engineering problem of balancing an inverted pendulum



Inverted Pendulum

- A solid pendulum is hinged at its base to a platform which can move in opposite directions.
- Pendulum can move in the same plane as the platform.
- The objective is to keep the pendulum upright by compensating for the tilt of the pendulum by corresponding movements of the platform.

1.2. Practical Application

1.3. "Classical" Controller

- Input Measurements:
  - the angle the pendulum makes with the upright,
  - the angular velocity of the pendulum must be measured,
  - the rate of change of this velocity.
- Output:
  - the direction, velocity, and
  - change in velocity of the platform.
- Find a suitable relation between these variables. Can become very complicated (Newtonian mechanics)
- Demand a lot of computing power.

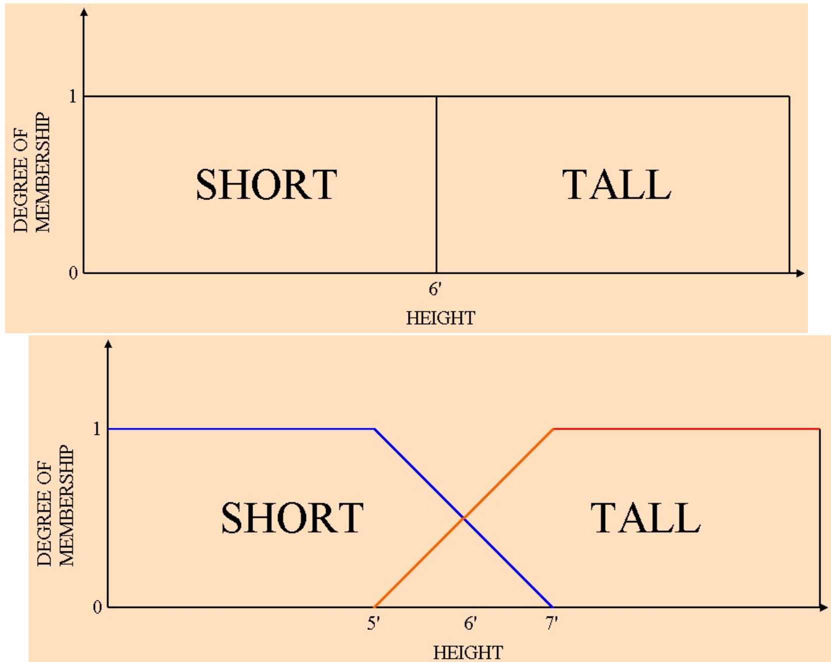
1.4. "Human" Controller

- When the pendulum tilts we measure the nature of this movement.
- How much the broomstick has moved, what direction it is moving in and how quickly it is moving?
- We automatically make the corresponding movement to compensate. We don't actually quantify these factors, rather we make quick, instinctive estimations.
- If we were to examine the thought pattern in such a situation it might read like this:
  - The pendulum is tilting, to the right, a little , so I must move my hand, to the right, a little.
  - The pendulum is tilting, quickly, forward, a lot, so I must move my hand, quickly, forwards, a lot.

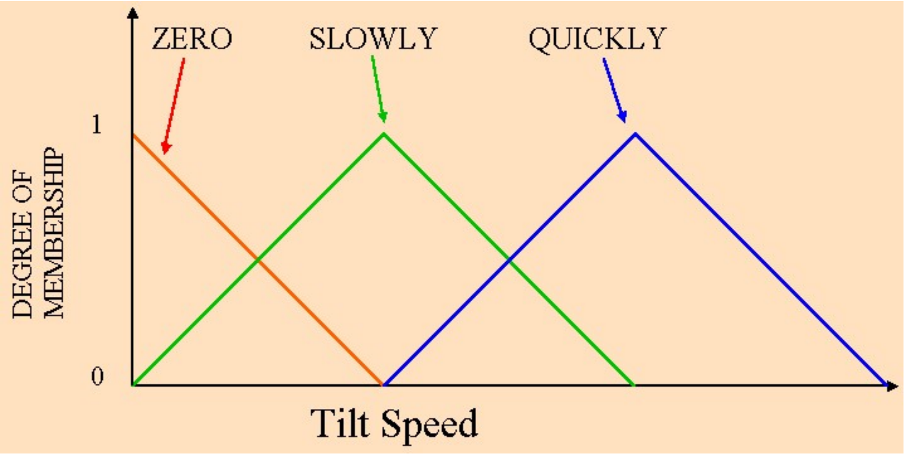
1.5. Sufficient Accurate Estimates

- And so on. The key thing here is our ability to make sufficiently accurate estimations about the nature of the situation, build a number of rules with them and act on these rules accordingly.
- We find it easy to do this. We make use of abstract concepts such as a little, a lot, quickly, slowly and apply them to the variables of pendulum tilt and hand movement.
- Fuzzy logic provides a means by which computers can imitate the way we humans make these kinds of estimations
  - rather than A or NOT A,
  - we can say MOSTLY A or SLIGHTLY A.

2. Fuzzy Sets



- Fuzzy Sets - a matter of degree

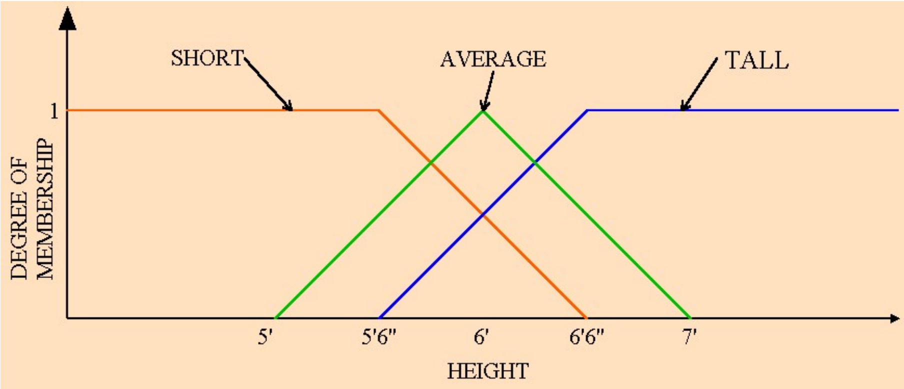


- Fuzzy truth, T, likely hood of a predicate to be true, given a crisp input value.
- Degree of membership, $\mu(x)$, of a given crisp input value to a fuzzy set.

2.1. Representation of Knowledge: Sets

- Fuzzy Sets: define attributes
  - Height: TALL, AVERAGE HIGHT, SHORT
  - Build: FAT, MEDIUM, SLIM
  - Weight, HEAVY, MEDIUM, LIGHT

Membership

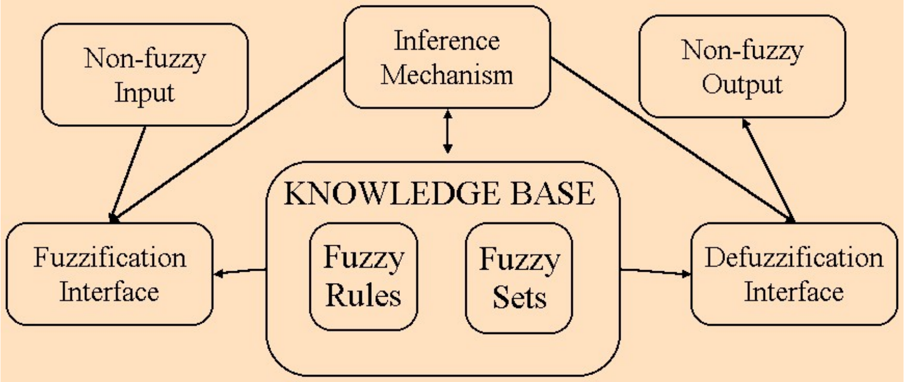


2.2. Representation of Knowledge: Rules

- Linguistic rules:
  - if a man is tall and fat, then he will be heavy in weight.
  - if a man is tall and slim, then he will be average in weight.
  - if a man is tall and of medium build, then he will be heavy in weight.
  - if a man is short and fat, then he will be average in weight. if a man is short and slim, then he will be light in weight.
  - if a man is short and of medium build, then he will be light in weight.
  - if a man is of average height and fat, then he will be heavy in weight.
  - if a man is of average height and slim, then he will be light in weight.
  - if a man is of average height and of medium build, then he will be medium in weight.

3. Logic

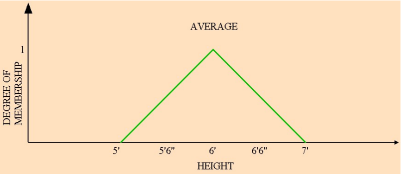
3.1. Fuzzy Rule Based Systems

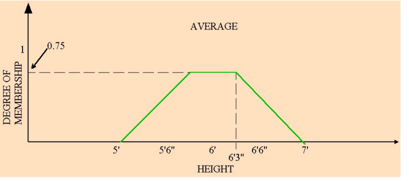


3.2. Fuzzification

- Process by which crisp, non-fuzzy, input values are converted into their fuzzy representations.

- Example: a crisp input value of 6'3" for height, fuzzification of this input entails applying this value to the fuzzy set average.

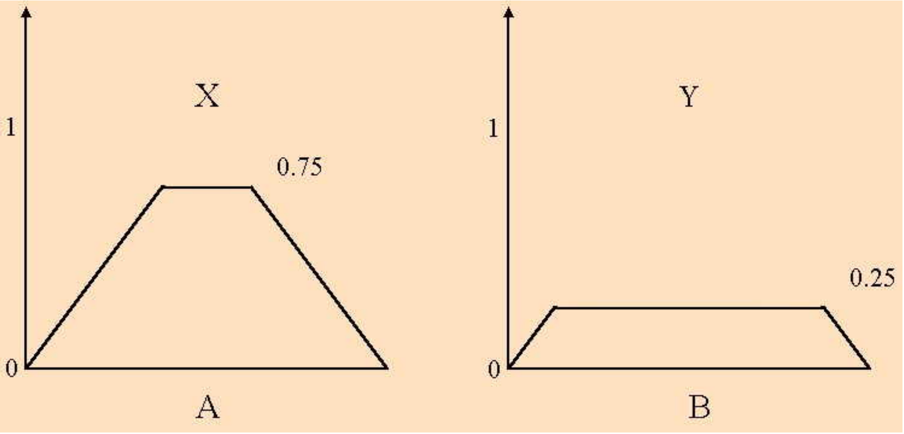




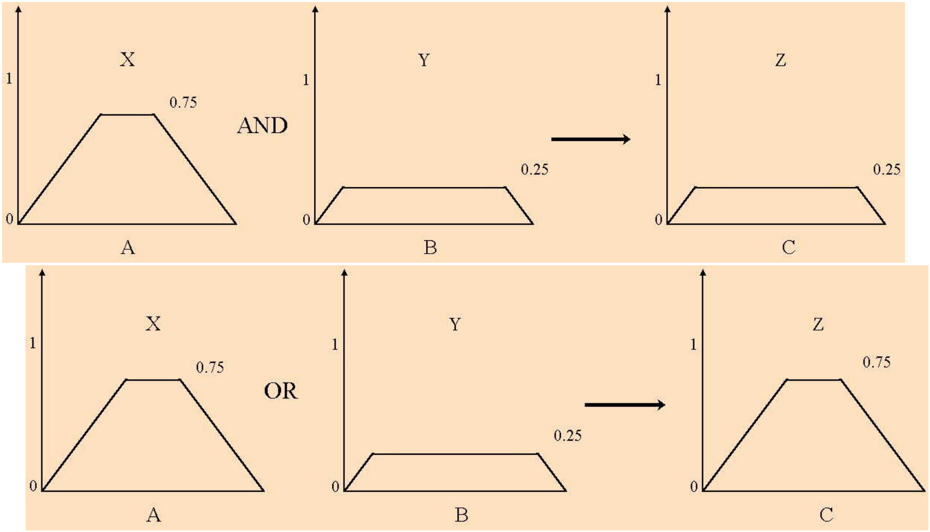
- The fuzzification process will take place for all inputs in all corresponding fuzzy sets, yielding fuzzified membership functions for use in each rule.

3.3. Inference in Fuzzy Logic

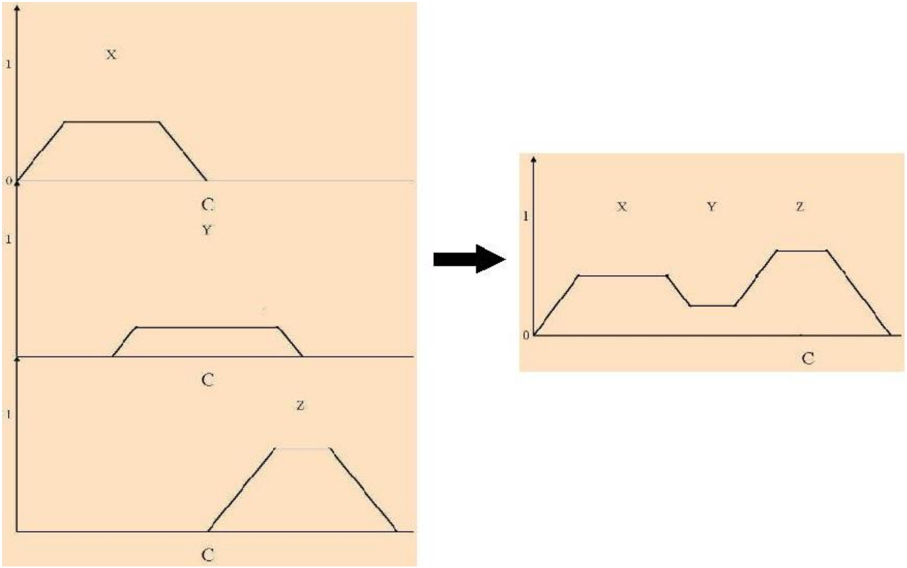
- Min-Max inference (Lotfi Zadeh)
  - if A is X and B is Y, then C is min(X,Y)
  - if A is X or B is Y, then C is max(X,Y)
- Example X=0.75, Y=0.25



- Logic Operators

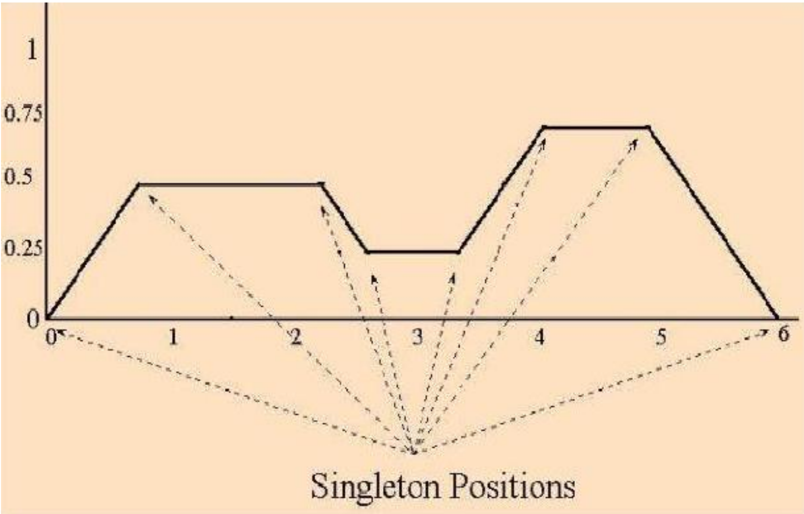


* Compute all rules that apply, i.e. for any set that any input value is a member of.
* Combine results: unify sets.
* Defuzzify result: calculating a crisp output value.



3.4. Defuzzification

- The opposite of fuzzification; entails the rationalization of a fuzzified output to obtain a crisp value for the output.
- Several methods can be used, one of them: the Center of Gravity method.
- Finding the center of gravity of the fuzzified output membership function and returning the crisp value that corresponds to this point.



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4. Example

Inverted Pendulum

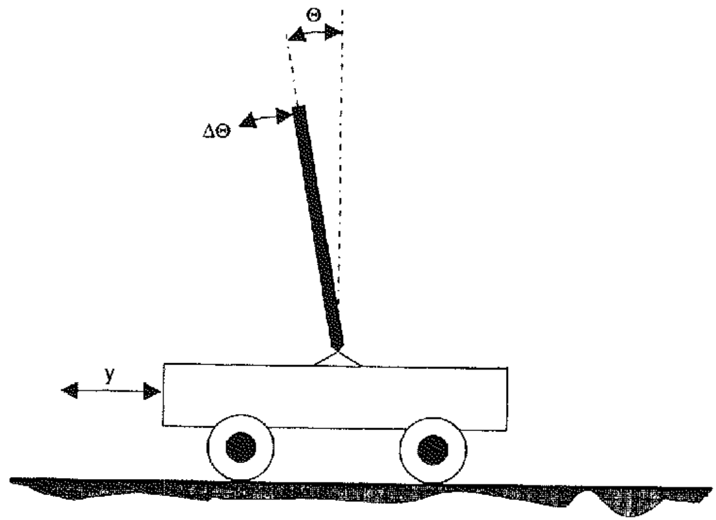

Figure 1.32
The Inverted Pendulum case example—a physical presentation of an experimental system.

IF Θ is PM AND $\Delta\Theta$ is ZR, THEN y is PM,

IF Θ is PS AND $\Delta\Theta$ is PS, THEN y is PS,

IF Θ is PS AND $\Delta\Theta$ is NS, THEN y is ZR,

IF Θ is NM AND $\Delta\Theta$ is ZR, THEN y is NM,

IF Θ is NS AND $\Delta\Theta$ is NS, THEN y is NS,

IF Θ is NS AND $\Delta\Theta$ is PS, THEN y is ZR,

IF Θ is ZR AND $\Delta\Theta$ is ZR, THEN y is ZR.

Figure 1.33
Seven heuristic rules for balancing the inverted pendulum. (Adapted with permission from Yamakawa 1989.)

ΔΘ \ Θ	NL	NM	NS	ZR	PS	PM	PL
PL							
PM							
PS			ZR		PS		
ZR		NM		ZR		PM	
NS			NS		ZR		
NM							
NL							

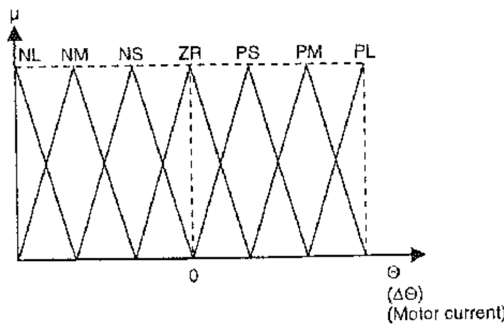

Figure 3.42
A set of fuzzy rules and membership functions for the Inverted Pendulum case problem. (Adapted with permission from Yamakawa 1989.)

5. References and Further Reading

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- This presentation is intended solely for students at Clark Atlanta University.
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