Why study linear algebra?

Simply as the title says. I've done some research, but still haven't arrived at an answer I am satisfied with. I know the answer varies in different fields, but in general, why would someone study linear algebra?

(linear-algebra) (soft-question) (motivation)

edited Dec 12 '12 at 0:39

asked Dec 12 '12 at 0:04
AaronAAA
376 ● 1 ■ 4 ▲ 6

32 1. Because they need it: it's an indispensable tool in many areas both within and without mathematics, as much so as calculus. 2. Because they like it. – Brian M. Scott Dec 12 '12 at 0:07

20 Because it's a hero science and engineering needs but not the one it deserves..... – Inquest Dec 12 '12 at 0:41

1 My answer details some of its uses in Engineering. Learning how to use linear algebra is to Engineering what ditching your stone-age spear and buying an AK-47 is to hunting! – user1158559 Dec 12 '12 at 2:00

2 I don't remember life without linear algebra, and I don't want to live without it ^^. It is so incredibly useful to a mathematician, and to many applied sciences, there is just no way around it. The answers below are more on topic though. – Olivier Bégassat Dec 12 '12 at 2:23

7 Possible duplicate of Importance of Linear Algebra, Purpose of Linear Algebra, What is the usefulness of matrices?, and probably several others. – BlueRaja - Danny Pflughoeft Dec 12 '12 at 9:09

12 Answers

Linear algebra is vital in multiple areas of **science** in general. Because linear equations are so easy to solve, practically every area of modern science contains models where equations are approximated by linear equations (using Taylor expansion arguments) and solving for the system helps the theory develop. Beginning to make a list wouldn't even be relevant ; you and I have no idea how people abuse of the power of linear algebra to approximate solutions to equations. Since in most cases, solving equations is a synonym of solving a practical problem, this can be VERY useful. Just for this reason, linear algebra has a reason to exist, and it is enough reason for any scientific to know linear algebra.

More specifically, in mathematics, linear algebra has, of course, its use in abstract algebra ; vector spaces arise in many different areas of algebra such as group theory, ring theory, module theory, representation theory, Galois theory, and much more. Understanding the tools of linear algebra gives one the ability to understand those theories better, and some theorems of linear algebra require also an understanding of those theories ; they are linked in many different intrinsic ways.

Outside of algebra, a big part of analysis, called *functional analysis*, is actually the infinite-dimensional version of linear algebra. In infinite dimension, most of the finite-dimension theorems break down in a very interesting way ; some of our intuition is preserved, but most of it breaks down. Of course, none of the algebraic intuition goes away, but most of the analytic part does ; closed balls are never compact, norms are not always equivalent, and the structure of the space changes a lot depending on the norm you use. Hence even for someone studying analysis, understanding linear algebra is vital.

In other words, if you wanna start thinking, learn how to think straight (linear) first. =)

Hope that helps,

answered Dec 12 '12 at 0:18
Patrick Da Silva
22.1k ● 1 ■ 32 ▲ 65

2 This, totally. I was trying to figure out how to correct colors from a camera. Turns out that color spaces can be mapped to vectorial space, then solved using linear algebra. Talk about artsy stuff totally unrelated to math. – Eric Dec 12 '12 at 5:35

2 @Eric : Numerical processing of images is one of the infinite number of applications of linear algebra. Not just what you described, but for instance when you take a very high-quality file and you want to compress it (say in a .jpeg file, which is a compressed format), the algorithm that selects which information to keep usually uses discrete Fourier transforms. When you hear "Fourier transform", think "linear algebra in Hilbert spaces". It's closely related. Signal processing too uses Fourier transforms, and there are a whole load of things in real life that require signal processing. – Patrick Da Silva Dec 12 '12 at 5:38

It is very useful in linear control theory. Systems are described by so called state which is vector and change of state is described by matrices. Than questions about stability becomes questions about eigenvalues. etc. – boucekv Dec 12 '12 at 11:33

Because its interesting! – Belgi Dec 12 '12 at 12:18

Computer graphics and Machine learning (Artificial Intelligence) are totally dependent on Linear Algebra. – SunnyShah Sep 17 '14 at 8:00

Having studied Engineering, I can tell you that Linear Algebra is fundamental and an extremely powerful tool in **every single** discipline of Engineering.

If you are reading this and considering learning linear algebra then I will first issue you with a warning: Linear algebra is mighty stuff. You should be both manically excited and scared by the awesome power it will give you!!!!!!

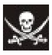
In the abstract, it allows you to manipulate and understand whole systems of equations with huge numbers of dimensions/variables on paper without any fuss, and solve them computationally. Here are some of the real-world relationships that are governed by linear equations and some of its applications:

- Load and displacements in structures
- Compatability in structures
- Finite element analysis (has Mechanical, Electrical, and Thermodynamic applications)
- Stress and strain in more than 1-D
- Mechanical vibrations
- Current and voltage in LCR circuits
- Small signals in nonlinear circuits = amplifiers
- Flow in a network of pipes
- Control theory (governs how state space systems evolve over time, discrete and continuous)
- Control theory (Optimal controller can be found using simple linear algebra)
- Control theory (Model Predictive control is heavily reliant on linear algebra)
- Computer vision (Used to calibrate camera, stitch together stereo images)
- Machine learning (Support Vector Machine)
- Machine learning (Principal component analysis)
- Lots of optimization techniques rely on linear algebra as soon as the dimensionality starts to increase.
- Fit an arbitrary polynomial to some data.

Arbitrarily large problems of the types listed above can be converted into simple matrix equations, and most of those equations are of the form `A x = b`. Nearly all other problems are of the form `A x = λ x`. Yep, you read that right! Nearly all engineering problems, no matter how huge, can be reduced to one of two equations!

Linear algebra is so powerful that it also deals with small deviations in lots of non-linear systems! A typical engineering way to deal with a non-linear system might be to linearize it, then use Linear Algebra to understand it!

answered Dec 12 '12 at 1:55



user1158559
441 3 3

2 For a mathematician, pretty much all that you just listed goes into the "applications" section. I am not even surprised to see so many applications. If you are impressed with the list you have, just know that for mathematicians, your list is a sub-point in one of my lists. =P But I agree with you that your answer shows that linear algebra is mighty stuff. =) – Patrick Da Silva Dec 12 '12 at 3:10

1 Then you might ask, "What's the point in maths?" .... – user1158559 Dec 18 '12 at 18:19

I am going for a Ph.D. in math because I'm passioned about it and I like to make my brain work. I am not interested at all in applications. Mathematics have the power to make me think of very abstract things which allows me to solve so many problems. At some point, you lose interest in explicitly solving problems (i.e. applications) ; all I want to do now is just gain more tools and develop them. – Patrick Da Silva Dec 18 '12 at 21:51

2 Of course you needn't view applied math as inferior to pure math or subordinate to profit. Many great mathematicians have used instances from physical sciences as motivation for math, for the sake of understanding. Philosophy concerns the real world, even when it is abstract. Certainly, such a list belongs in the answer of "why study ---?" – Andrew Marshall Dec 30 '13 at 19:58

TL;DR version

Linear algebra is your ticket to multidimensional space. Study it if you are into economics, computer graphics, physics, chemistry, statistics or anything quantitative (in today's world, that's everything).

Meaning of "Linear" and why it is "Easy"

Since you are asking the question, perhaps you would benefit from a discussion of what "linear" means, and why it is "easy", as mentioned in some answers above.

The word "linear" is to be understood as in a linear function (a line) in calculus, such as

```
g(x) = m*x + n,
```

where `m`, `n`, and `x` are numbers.

You will know that in calculus, the Taylor expansion of a function `f` is

```
f(x) = f(x₀) + f'(x₀)*(x - x₀) + ½ f"(x₀) (x - x₀)^2 + ...
```

If you use only the first two terms of the Taylor expansion of `f`, you have a linear approximation to your function `f`:

```
f(x) ≈ f(x₀) + f'(x₀)*(x - x₀)
     = f'(x₀)*x + (f(x₀) - f'(x₀)*x₀)
```

If you set

```
m = f'(x₀), n = f(x₀) - f'(x₀)*x₀,
```

you can see that this is a linear function just like `g(x)`. This approximation works reasonably well near `x₀` (think of the sine function, for example). It is the simplest approximation to `f` beyond saying "near `x₀`, it's probably something like `f(x₀)`". That's why the linear approximation is called "easy".

If you want to do slightly better, use the first two terms of the Taylor expansion of `f`. You get a quadratic approximation to `f`.

How Linear Algebra Comes In

Now what do we do if we have a function that depends on several variables (or, which is the same, a variable x that is a vector containing these variables as components)? This is called multivariate calculus (because, multiple variables).

In that case, `f'(x₀)` becomes a vector that has to be multiplied (how?) with the vector `(x - x₀)`. `m` and `n` become vectors, too.

`f"(x₀)` becomes a matrix (actually, a bilinear tensor) that has to be multiplied twice with the vector `(x - x₀)`. (How?)

As you have seen, the linear and quadratic approximations are the easiest ways to think about complicated functions `f`.

Linear algebra deals with vectors, matrices, and tensors, and how to operate them with each other so that everything makes sense and is useful.

In other words, linear algebra is *your ticket to multidimensional space*, such as the three-dimensional space we inhabit (e.g. computer graphics, physics) or the space of many variables a function depends on (economics, more physics).

Examples

Economics example: Price (our function `f`) depends on supply and demand (the two components of the two-dimensional vector `x`). If you do economics, you do linear algebra.
Politics example: If more roads are built, capacity goes up (good for business) but quality of life goes down. People move away, which results in more traffic, which results in less excess capacity, which is bad for business. Many variables and complex connections lead to multivariate calculus. Sociologists and politics majors ought to know linear algebra to think about problems like these.
Physics example: If you don't watch a moving car from the outside but sit in the car, you are changing your frame of reference. This corresponds to a change of basis in linear algebra, which naturally pops out the centrifugal force (and the Coriolis force) from your force expression.

For the mathematician

Linear algebra is of course a rich field in its own right but I wanted to write a motivating explanation to aspiring students who do not yet know what it is and how it is relevant to their lives.

edited Dec 16 '12 at 19:30

answered Dec 12 '12 at 2:01
Andreas
271 1 5

Patrick Da Silva gives a good answer, however I will expand. Well worth looking at his post when (or if) you do get into Linear Algebra and come back to this post a year later or so with some knowledge under your belt, as you can easily extend your studies for a couple of years by what he stated.

Your beginning motivation to study linear algebra is to put together what you initially learn in (mathematically) geometry and calculus and in (education) high school, college, or even first year or second year of university.

Imagine there's some "room" where everything you played around with in A-Levels or high-school was contained; $\sin(x)$, $\cos(x)$, $ln(x)$ were there, a definition of a *field* and some set theory.

Now suppose we want to understand this room. Suppose instead of looking at each little "person" in the room, we know that if they are in that room, they behave under a specific way.

Linear Algebra (and Algebra in general) **allows us to classify and understand many objects, situations, spaces, to some basic context**. At the a high undergraduate level, a remarkable theorem in *Hilbert Spaces* allows you to identify any separable Hilbert space to some specific, well-known Hilbert Space. This is just the work of the last paragraph in application.

Therefore, if you want structure, you need algebra. Linear algebra specifically so you can understand the basic structure.

One application of Linear Algebra is in the use of *eigenvalues*.

We can understand some specific systems; weather predictions, behaviour of people, game situations, etc, and the long run behaviour (as it approaches ∞ so as to speak) by instead of looking at some function $f(x)$, we look at some real value λ such that $f(x) = \lambda x$ at some meaningful λ.

Linear Algebra allows us to start understanding basic linear systems with use of matrices and vectors.

Lastly, for a purely computational reason, Linear Algebra gives you many tools for proving many key theorems. The infinite dimensional Pythagorean theorem, Cauchy-Schwarz inequality, Bessel Inequality, etc, are frequently used in many areas of mathematics to prove key theorems. This is good motivation for any subject.

answered Dec 12 '12 at 0:32
Arbias Hashani
465 2 6

I am pretty sure a solution to a system of linear equations and their representations as matrices is part of linear algebra. These kind of systems are used in just about every discipline. Also, things like vectors are...just too ubiquitous for me to articulate. The field of linear algebra is just the ground work of almost everything in science and higher math.

answered Dec 12 '12 at 0:21
Valtteri
1,215 4 21

Personally I found that Linear Algebra is very sound - in particular compared to Calculus - and fun to study. Its soundness gives it a certain beauty :-)

answered Dec 12 '12 at 11:45
sorencito
121 2

In addition to the good points already made, one function of the linear course at many schools is that this is often the first place where serious attention is paid to reading and constructing rigorous proofs at a level beyond the all-too-common "handwaving" pseudo-proofs that appear in some calculus courses. As a result, at such places a linear algebra course can also provide a way for students to assess whether or not mathematics is going to be a good fit for them. Whether it is or isn't, most students come out of the course with a solid start towards mathematical maturity.

answered Dec 12 '12 at 0:40
Rick Decker
6,178 ● 3 ■ 15 ▲ 35

Linear Programming is also a term that i want to throw in. Many problems (no list here) can be expressed as a linear optimization problem. Being able to solve those efficiently means being able to solve an incredible amount of real-world problems . Although in some cases, there exist more efficient ways to solve a specific problem, the linear program and the theory behind it often gives rise to new ways to approach a problem.

Does linear optimizaion still count as *Linear Algebra*?

answered Dec 12 '12 at 8:42
kutschkem
205 ■ 1 ▲ 9

Because it's fun!

Linear algebra is stuff you see around every day: electricity, economics, they way your car handles.

Since most people never get to see this, it also gives you a view of the world that most people don't see. That adds an interesting human dimension to the knowledge.

Taking my examples, you understand why the un-sprung weight of your car is important? What would Marx think about adding friction to high-frequency trading? Why is your electricity supply AC instead of DC?

answered Dec 12 '12 at 11:22
david
119 ▲ 2

Right now, I am sitting at my desk at a aerospace company and due to the fact that no one solve the Navier-Stokes equations so far, our cluster is doing its best to solve all the liner systems which are results from our computation. Linear algebra is in my opinion the most important part of math in applied industry. Everything comes down to $Ax = b$, so you better pay attention when you're taught how to solve them ;)

answered Dec 12 '12 at 12:29
sonystarmap
2,956 ■ 8 ▲ 28

When you learn ODE, solve some surface integral,do the circuit analysis or numerical analysis,systems control or so you will find it is useful. You will only find the true value of it when it's the time you relly need it.If you do not tend to a field links to engineering or math,it may not have vaule.It may like the ancient style prose, when you come to a situation like the ancient says you will find what they have said is ture and moral-philosophical.

answered Dec 12 '12 at 16:16
Jebei
308 ■ 1 ▲ 11

Good question. I didn't understand Linear Algebra for a while.

In other areas of math - Calculus, Trigonometry, etc. - you learn complicated functions of one variable. $f(x) = x^4 + 5x^2 + 15$ Yes, you occasionally get equations with multiple variables, but that is never the goal. In Linear Algebra, you get simple functions with many variables. That's the key: simple functions of many variables.

Now, simple functions are boring. A huge piece of the study of Linear Algebra is figuring out how to use the complex functions with many variables. (I wish there was a good engineering textbook that presented this clearly!) One way is to apply the complex functions to the data before putting it into linear algebra. (This is done when curve-fitting data.) Another is to use partial derivatives to do a linear approximation of a complex function near a particular point in space. (Useful in physics.) Lastly, if you understand properties the matrices and the complex function, you can sometimes combine them directly. (E.g., applying the Taylor series expansion to the eigendecomposition.)

So, Linear Algebra is useful any time you have multiple variables. It's useful when you have many moving parts in an engine, concentrations of multiple chemicals in a test tube, many regions of atmosphere in a climate simulation, prices of stocks in a market, etc.. Even though some of those are non-linear systems, there are techniques that may make Linear Algebra useful.

POST SCRIPT: Linear Algebra textbooks are, in general, awful. They apply transpose to vectors. They focus on calculation rather than use. (E.g., how to compute a determinant.) They focus on irrelevant topics. (E.g., determinants.) Some adequate ones are by Axler and by Hoffman & Kunze, but both of those are for mathematician, not engineers.

edited Dec 30 '13 at 19:37
user93957

answered Dec 30 '13 at 18:53
user118316
1

Halmos' *Finite-Dimensional Vector Spaces* is also a fine little book on linear algebra, also for the mathematician and not the engineer. – Phoenix Stormcrow Feb 7 '14 at 1:24