QUEUEING THEORY

EXERCISES 3

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Theory link

- The queuing theory is necessary for companies in order take better business decisions in terms of resources
- Companies can better manage their resources and understanding which is the need of farther resources in order to provide a service.
- It studies the waiting times of customers/products

Key messages from the lesson

- Focus on a network of systems
- Understanding in practical terms how to apply the queuing theory
  - How to map a complex system
  - How to manage different types of customers/products
  - How to map/understand/evaluate the customers/products paths
  - How to calculate occurrences and adjusted occurrences
  - How to manage paths in case of scraps

Verdi Spa

Verdi SPA has a final demand of 500 pieces/ hour.

The process is composed by an assembly line and a final packaging stage.

The assembly line is composed by 3 stages in series, decoupled by stock. There is not set-up in the assembly line. The product, at the beginning of the process, is checked by an automatic quality control stage. 15 % of the products provided by the supplier are identified as scrap. The 85% of the products provided by the supplier are good, so they can be processed by the assembly line After the stage 1 there is another quality control, in particular 20% of the product worked in stage 1 are categorized as nonstandard, so they have to be reworked by stage 1. Once the stage 1 reworks this 20%, it will be considers ok, so it can go to stage 2. After stage 2 there are 3 different cases :

1. The product is damaged so it becomes scrap (no possible to recover it) (5% of the products worked in stage 2).
2. The product is slightly damaged, so it can continue the process but it will be sold in a second market at lower price, and because of this it will continue with a lower priority just on stage 3. (15% of the products worked in stage 2). In the packaging stage, products have not different priorities. In stage 3, good products have non-preemptive priority on slightly damaged products.
3. The product is good quality.

Stage 3 is quite simple process, so it doesn't give any quality problem.

Packaging stage is composed by two different machines that work on parallel. Each one can work 300 pieces/hour. It is important that the input of packaging stage is 500 pieces/hour. The products are processed by FIFO and the configuration is a single queue with two servers (the two machines).

The three stages of the assembly line are manual and composed by an operator each one.

The table below is related to the service rates.

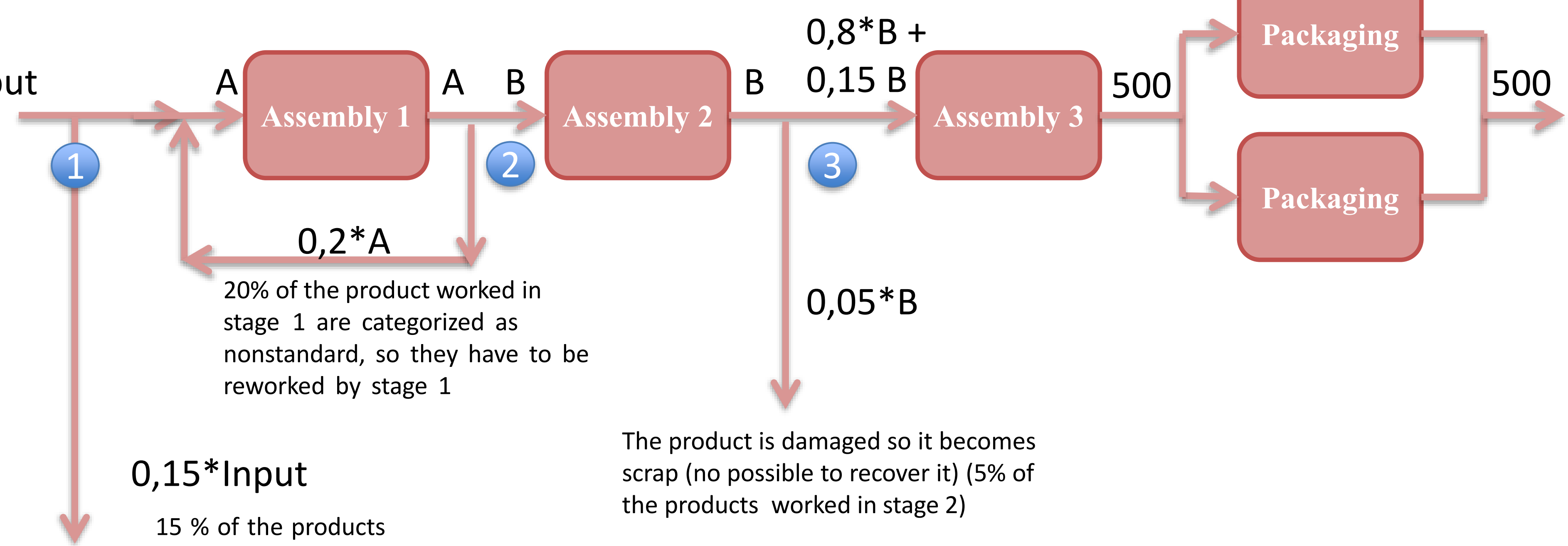
Service rates	Pcs/hour
Stage 1	700 p/h
Stage 2	600 p/h
Stage 3	600 p/h
Packaging (each machine)	300 p/h

1. You are required to model the system, highlighting all the important parameters.
2. Calculated Ws of the system (not considering scraps paths).
3. Comment the impact of using different priorities in stage 3.
4. Calculate average WIP before each stage of the assembly line.
5. How much is saturation in stage 2?

Verdi S.p.a. - MAP

The product is good quality

The product is slightly damaged, so it can continue the process but it will be sold in a second market at lower price, and because of this it will continue with a lower priority just on stage 3. (15% of the products worked in stage 2)

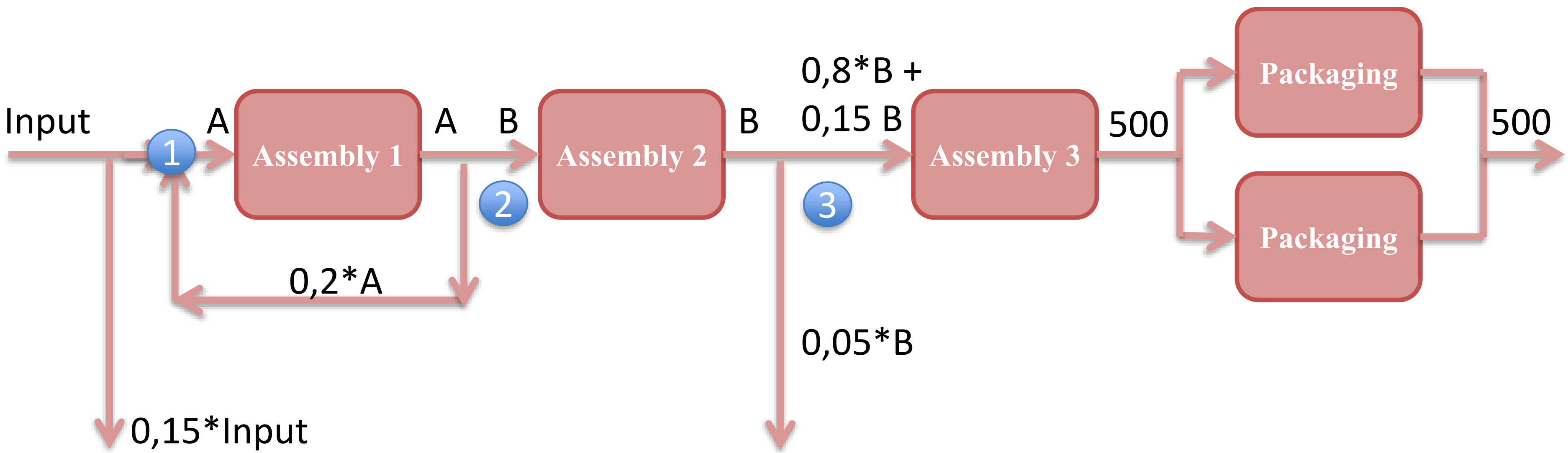


20% of the product worked in stage 1 are categorized as nonstandard, so they have to be reworked by stage 1

The product is damaged so it becomes scrap (no possible to recover it) (5% of the products worked in stage 2)

15 % of the products provided by the supplier are identified as scrap

Verdi S.p.a. - MAP



3. $500 = 0{,}95 * B \rightarrow B = 526{,}316$ pieces/hour
2. $B + 0{,}2 * A = A \rightarrow A = 657{,}895$ pieces/hour
1. $0{,}85 * Input + 0{,}2 * A = A \rightarrow Input = 619{,}19$ pieces/hour

Verdi S.p.a. - Relevant parameters

	Assembly 1	Assembly 2	Assembly 3	Packaging
λ (p/h)	657,895	526,316	Priority 1: 421,05 Priority 2: 78,95	500
μ (p/h)	700	600	600	300 (1 resource)
Priority	-	-	NON PREEMPTIVE	-
Queue system	M/M/1	M/M/1	M/M/1	M/M/2

Verdi S.p.a. - Throughput time

	Assembly 1	Assembly 2	Assembly 3	Packaging
λ (p/h)	657,895	526,316	Priority 1: 421,05 Priority 2: 78,95	500
μ (p/h)	700	600	600	300 (1 resource)
Priority	-	-	NON PREHEMPTIVE	-
Queue system	M/M/1	M/M/1	M/M/1	M/M/2
LT, Ws (min)	1/(μ - λ) = 1,425	1/(μ - λ) = 0,814	Priority 1: 0,379 Priority 2: 1,776 Priority Formulas	Lq / λ + 1 / μ = 0.7308

$$E(S_1) = \frac{(1+\rho_2)/\mu}{1-\rho_1},$$

$$E(S_2) = \frac{(1-\rho_1(1-\rho_1-\rho_2))/\mu}{(1-\rho_1)(1-\rho_1-\rho_2)}.$$

See next slide

Lq results of model M/M/c

λ/μ	c=1	c=2	c=3	c=4	c=5	c=6	c=7	c=8
0,15	0,026	0,001						
0,20	0,050	0,002						
0,25	0,083	0,004						
0,30	0,129	0,007						
0,35	0,188	0,011						
0,40	0,267	0,017						
0,45	0,368	0,024	0,002					
0,50	0,500	0,033	0,003					
0,55	0,672	0,045	0,004					
0,60	0,900	0,059	0,006					
0,65	1,207	0,077	0,008					
0,70	1,633	0,098	0,011					
0,75	2,250	0,123	0,015					
0,80	3,200	0,152	0,019					
0,85	4,817	0,187	0,024	0,003				
0,90	8,100	0,229	0,030	0,004				
0,95	18,050	0,277	0,037	0,005				
1,0		0,333	0,045	0,007				
1,1		0,477	0,066	0,011				
1,2		0,675	0,094	0,016	0,003			
1,3		0,951	0,130	0,023	0,004			
1,4		1,345	0,177	0,032	0,006			
1,5		1,929	0,237	0,045	0,009			
1,6		2,844	0,313	0,060	0,012			
1,7		4,426	0,409	0,080	0,017			
1,8		7,674	0,532	0,105	0,023			
1,9		17,587	0,688	0,136	0,030	0,007		
2,0			0,889	0,174	0,040	0,009		
2,1			1,149	0,220	0,052	0,012		
2,2			1,491	0,277	0,066	0,016		
2,3			1,951	0,346	0,084	0,021		
2,4			2,589	0,431	0,105	0,027	0,007	
2,5			3,511	0,533	0,130	0,034	0,009	
2,6			4,933	0,658	0,161	0,043	0,011	
2,7			7,354	0,811	0,198	0,053	0,014	
2,8			12,273	1,000	0,241	0,066	0,018	
2,9			27,193	1,234	0,293	0,081	0,023	
3,0				1,528	0,354	0,099	0,028	0,008
3,1				1,902	0,427	0,120	0,035	0,010
3,2				2,386	0,513	0,145	0,043	0,012
3,3				3,027	0,615	0,174	0,052	0,015
3,4				3,906	0,737	0,209	0,063	0,019
3,5				5,165	0,882	0,248	0,076	0,023
3,6				7,090	1,055	0,295	0,091	0,028

Packaging

M/M/2

ρ = λ/μ = 500/300=1,667
(Arrival rate/Service rate)

c=2
(Number of servers)

Lq=4,426

Verdi S.p.a. - Paths

Path	Input scrap	Assem 1	Assem 1 (2° time)	Assem 2	Scrap	Assem 3	packaging
A1	X						
A2		X		X		X -mkt1	X
A3		X		X		X - mkt2	X
A4		X	X	X		X -mkt1	X
A5		X	X	X		X - mkt2	X
A6		X		X	X		
A7		X	X	X	X		

Verdi S.p.a. - Paths & occurrences

Path	Input scrap	Assem 1	Assem 1 (2° time)	Assem 2	Scrap	Assem 3	packaging
A1	X						

A1 occurrence is:

*A1: 0,15*input/input =0,15*

Verdi S.p.a. - Paths & occurrences

Path	Input scrap	Assem 1	Assem 1 (2° time)	Assem 2	Scrap	Assem 3	packaging
A2		X		X		X -mkt1	X
A3		X		X		X - mkt2	X
A6		X		X	X		

Scraping 15% of pieces, it remains the 85% of the Input….

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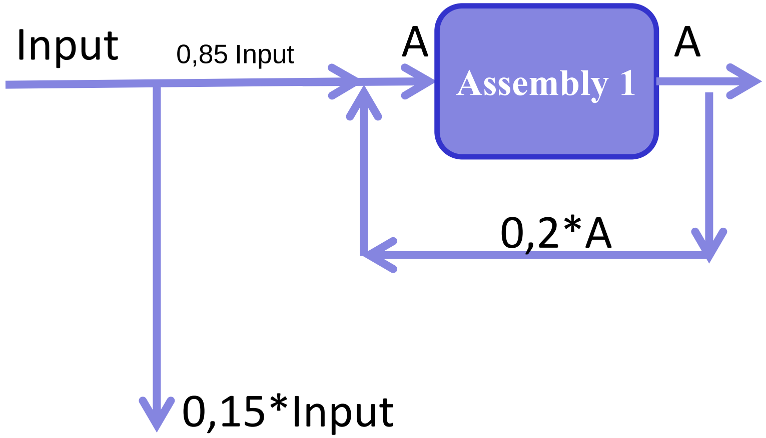


Verdi S.p.a. - Paths & occurrences

Path	Input scrap	Assem 1	Assem 1 (2° time)	Assem 2	Scrap	Assem 3	packaging
A2		X		X		X -mkt1	X
A3		X		X		X - mkt2	X
A6		X		X	X		

The quantity of pieces that moves directly from Assembly-1 to Assembly-2, avoiding the loop, is 85% of *input* decreased by 20% of A.
The population of pieces that follows the loop is different, isolated from the population which does not enter in the loop.
(0,85**input.- 0,2*A) items follow* A2, A3 and A6 paths

Verdi S.p.a. – Paths & occurrences

Path	Input scrap	Assem 1	Assem 1 (2° time)	Assem 2	Scrap	Assem 3	packaging
A2		X		X		X -mkt1	X
A3		X		X		X - mkt2	X
A6		X		X	X		

*A2: [(0,85*Input- 0,2*A)* 0,8]/Input = 0,51*

*A3: [(0,85*Input- 0,2*A)* 0,15]/Input = 0,096*

*A6: [(0,85*Input- 0,2*A)* 0,05]/Input = 0,032*

Verdi S.p.a. - Paths & occurrences

Path	Input scrap	Assem 1	Assem 1 (2° time)	Assem 2	Scrap	Assem 3	packaging
A4		X	X	X		X -mkt1	X
A5		X	X	X		X - mkt2	X
A7		X	X	X	X		

To find the occurrences of A4, A5 and A7 paths it is necessary to consider only the items that entered in the loop of assembly 1 stage.
The quantity of these items is 0,2*A.

*A4: 0,2*A*0,8/input = 0,17*
*A5: 0,2*A*0,15/input = 0,032*
*A7: 0,2*A*0,05/input = 0,010*

Verdi S.p.a. - Paths & occurrences

Path	Input scrap	Assem 1	Assem 1 (2° time)	Assem 2	Scrap	Assem 3	packaging	Pieces/ hour
A1	X							92.878
A2		X		X		X -mkt1	X	315.7896
A3		X		X		X - mkt2	X	59.2095
A4		X	X	X		X -mkt1	X	105.264
A5		X	X	X		X - mkt2	X	19.737
A6		X		X	X			19.737
A7		X	X	X	X			6.579

Verdi S.p.a. - Paths & occurrences

	Pieces	occurrence	Adjusted occurrence*
A2	315.784	0.51	0.63
A3	59.2095	0.096	0.12
A4	105.264	0.17	0.21
A5	19.737	0.032	0.04

*Adjusted occurrence i= Occurrence i/ Tot Occurrence without scraps
e.g. 0,63=0,51/(0,51+0,096+0,17+0,032)

Verdi S.p.a. - Throughput time

	Assem 1	Assem 1 (2° time)	Assem 2	Assem 3	packaging	Adjusted occurrence	Weighted time
A1	1,425		0,814	0,379	0.7308	0.63	2.11
A2	1,425	1,425	0,814	0,379	0.7308	0.12	0.57
A3	1,425		0,814	1,776	0.7308	0.21	0.99
A4	1,425	1,425	0,814	1,776	0.7308	0.04	0.25

Average throughput time: 3,92 minutes

WIP in the system and inactivity time for stage 2

Lq= $\dfrac{\rho\lambda}{\mu-\lambda}$

I stage → (657,895/700* 657,895)/(700- 657,895)=14,68 pieces

II stage → (526,316/600* 526,316)/(600- 526,316)=6,266 pieces

III stage → (421,05+78,95)/600* (421,05+78,95)/(600- (421,05+78,95))=4,167 pieces

Inactivity time stage 2:

1 – (526.316 / 600) = 0.1228

0.1228*60 = 7,3684 minutes/hour= 7.4 minutes / hour

Meccanica Spa

MECCANICA Spa is a little manufacturing company. The customers ask the company a rate of 100 pieces per hour.

The selection is composed by 3 types of products: type A (50% of the demand), type B (30% of the demand) and type C (20% of the demand).

The production system is simple, it is formed by:

1) Milling (2 milling machines. Service rate: 90 pieces per hour).
2) Drilling (3 drilling machines. Service rate: 75 pieces per hour).
3) Testing (1 testing machine). The service rate of the operator dedicated to the testing machine is 160 pieces per hour.
4) Packaging (1 packaging line with one operator able to package 130 pieces per hour).

The assumption is that all the service times are described by negative exponential distribution.

The flow is the same for the each type of products:

MILLING → DRILLING → TESTING → PACKAGING

In particular, at the milling stage, both the machines are able to work all types of products and the company decided to dedicate one machine to type A products and the other machine to products of type B and type C.

In the drilling stage there are three identical machines, each one dedicated to work on one type of product.

The testing machine reveals that sometimes type B and type C products have to be worked again because of an error during the drilling process. These products have to be processed again by the drilling machine and the re-working time is the same of the first time.

On average, 20% of type B and 10% of type C products that are introduced in the system have to be worked twice.

About the twice worked pieces, 85% are good and 15% are rejected and trashed.

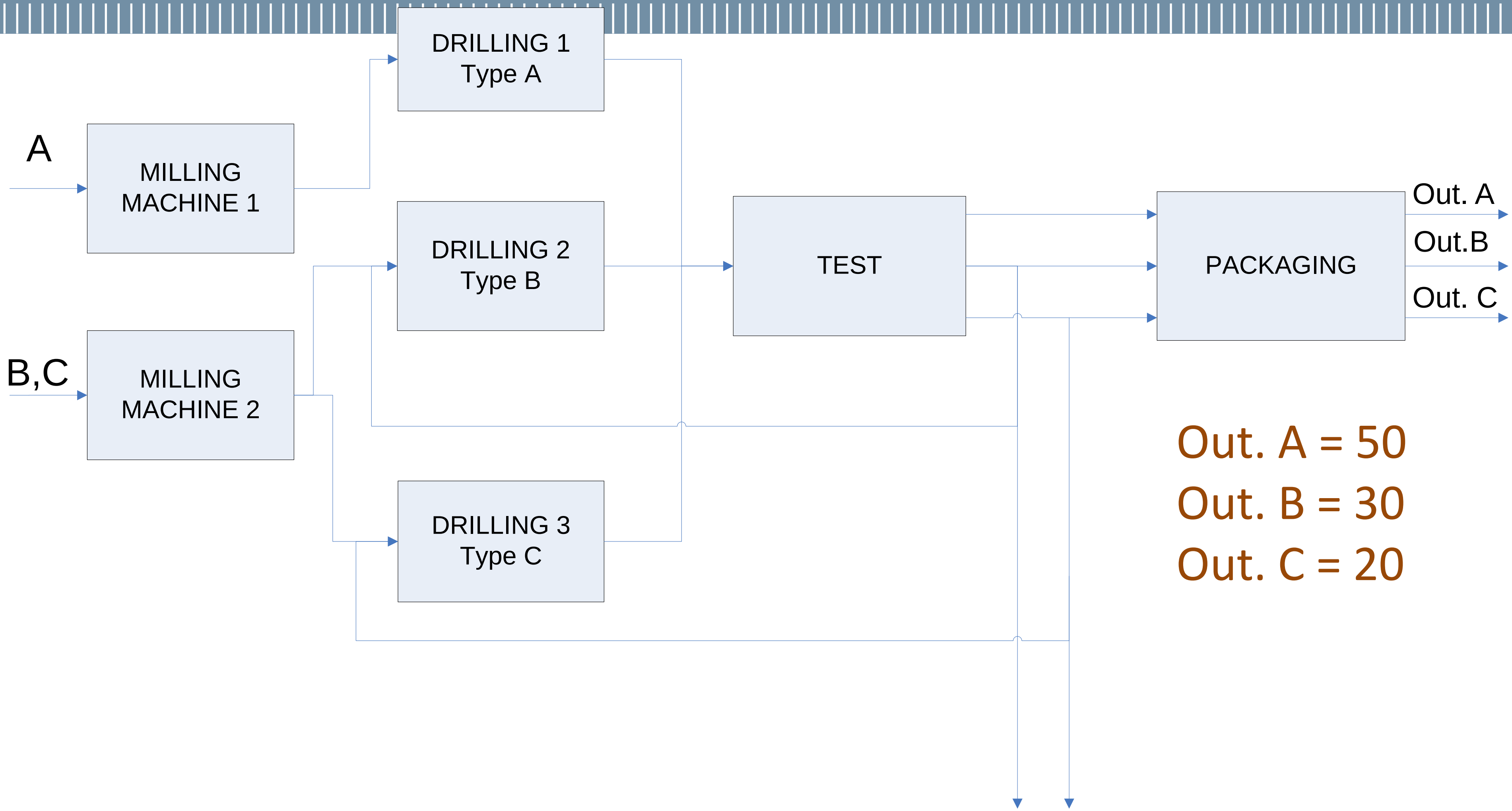
At the packaging stage a product of type C has a priority: when a server is free type C product is next to be processed.

Shape Meccanica S.p.a. System, define relevant parameters (arrival rates, service rates, queue type, inactivity time of resources) and map all the flows and paths of products.

MECCANICA S.p.a.

- We have to deliver 100 good pieces each hour, divided as:
  - 50 type A
  - 30 type B
  - 20 type C

System Modelling - Map

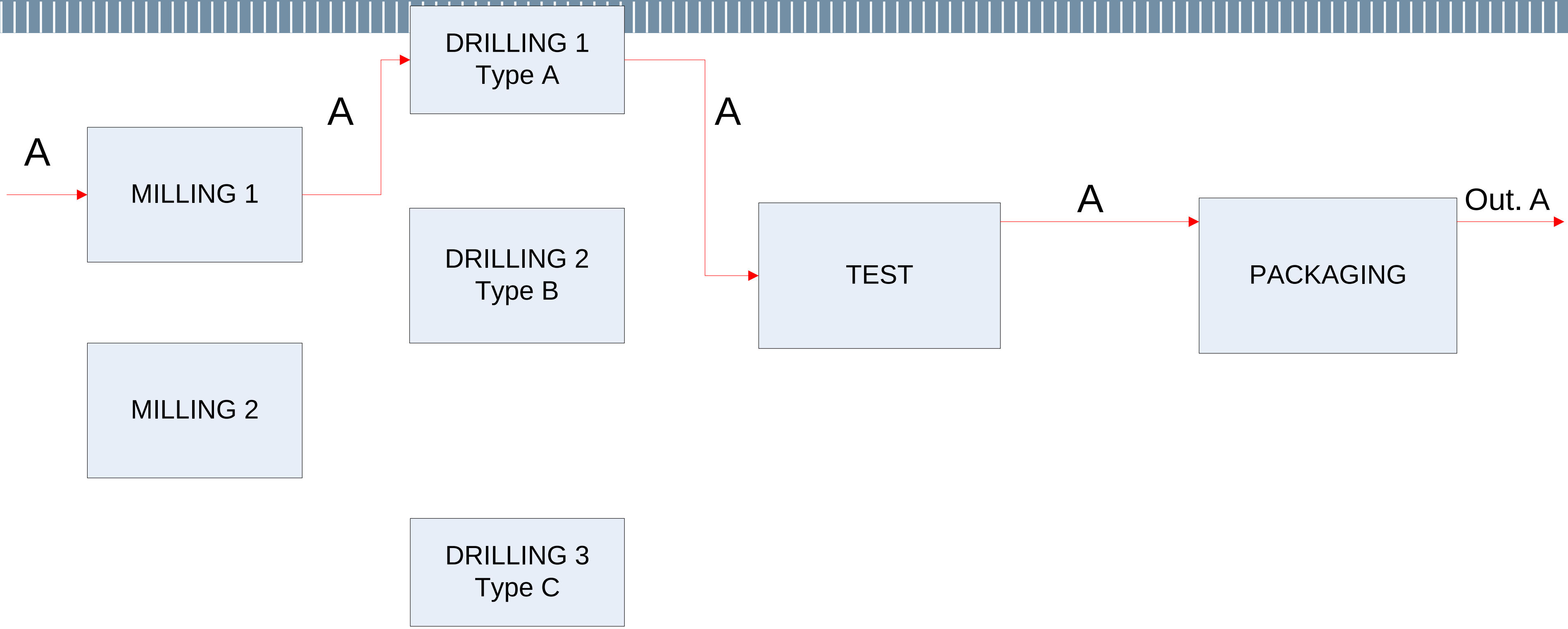


Out. A = 50

Out. B = 30

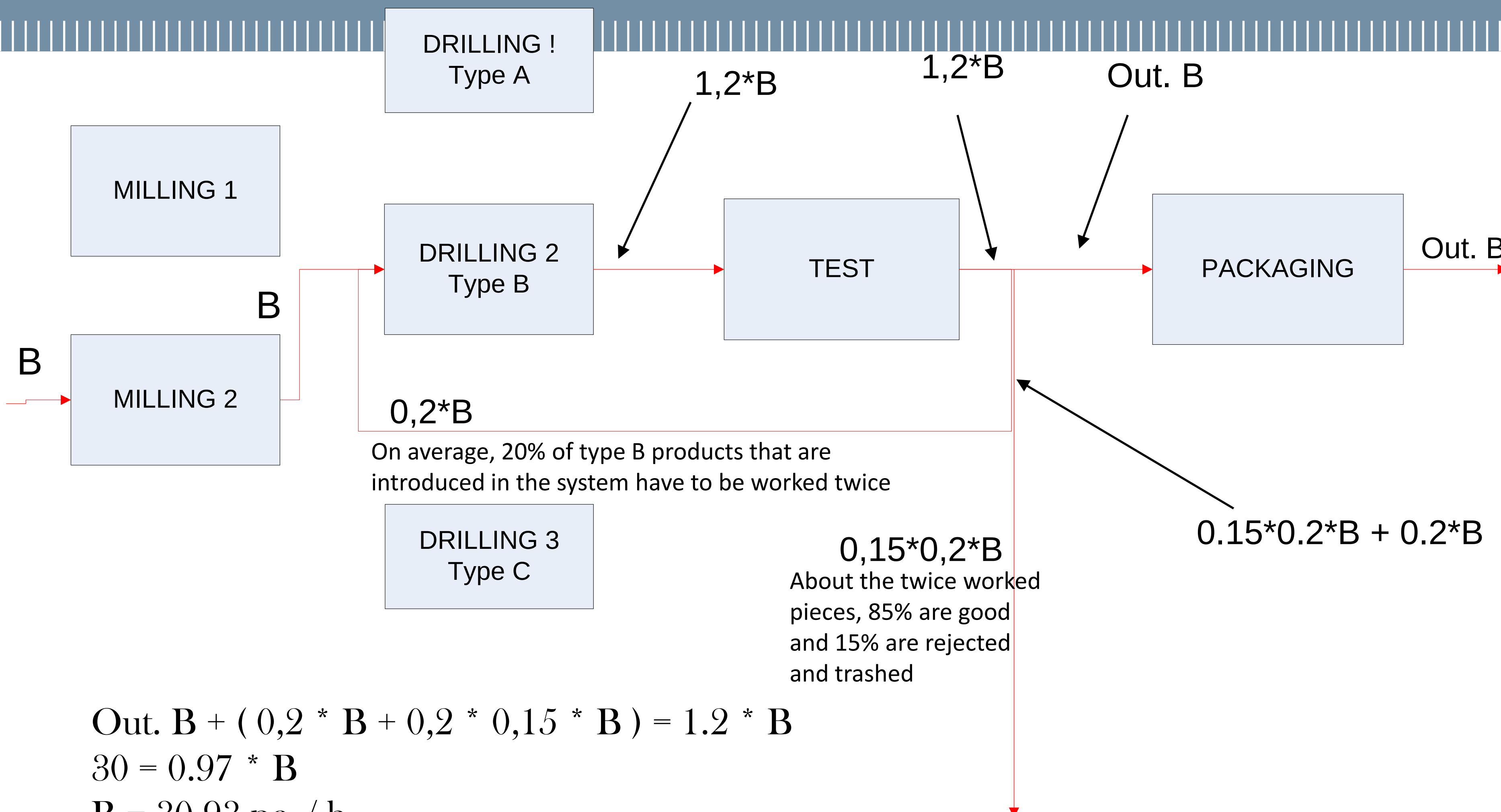
Out. C = 20

System Modelling - Paths



A = Out. A = 50 pc. / h

System Modelling - Paths

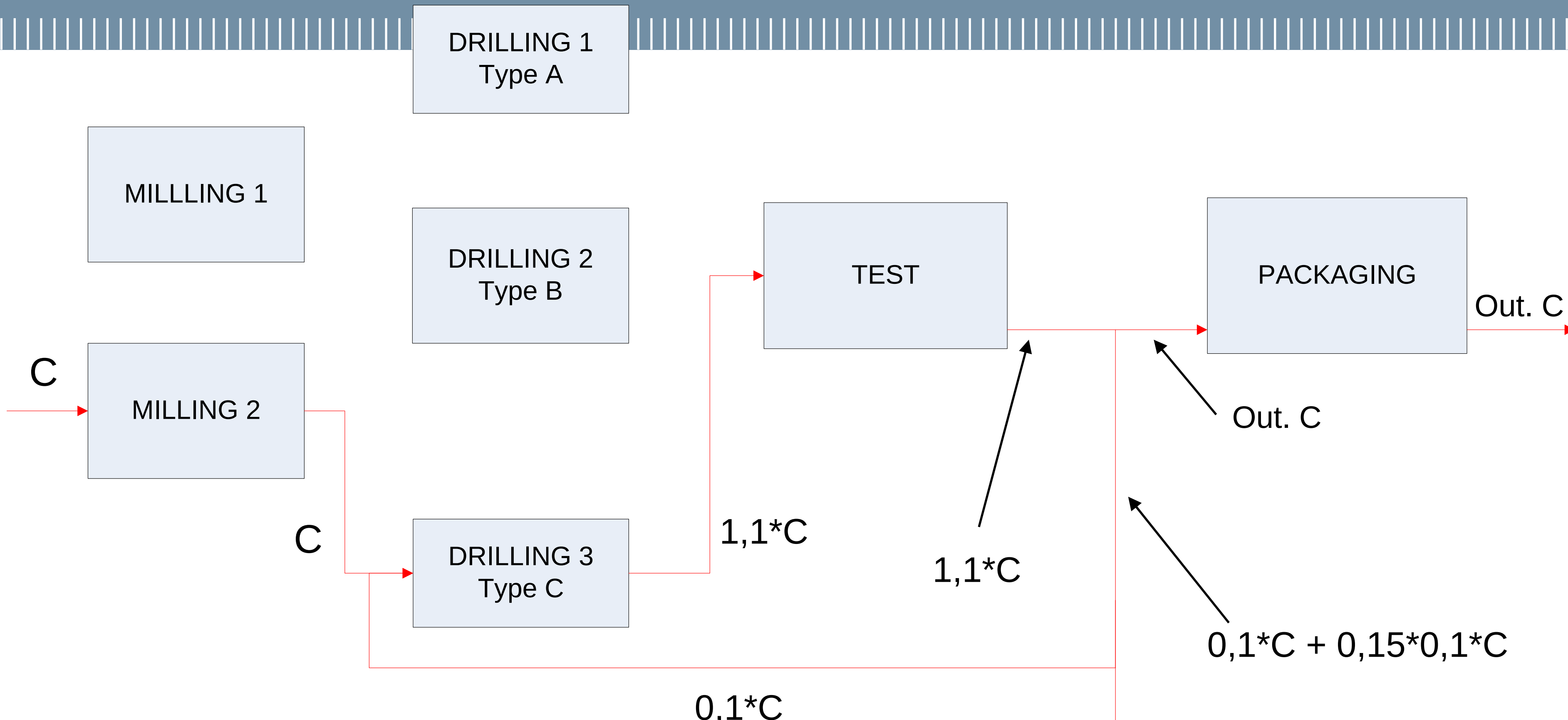


$$Out.\ B + (0{,}2 * B + 0{,}2 * 0{,}15 * B) = 1.2 * B$$

$$30 = 0.97 * B$$

$$B = 30.93\ pc. / h$$

System Modelling - Paths



On average, 10% of type C products that are introduced in the system have to be worked twice

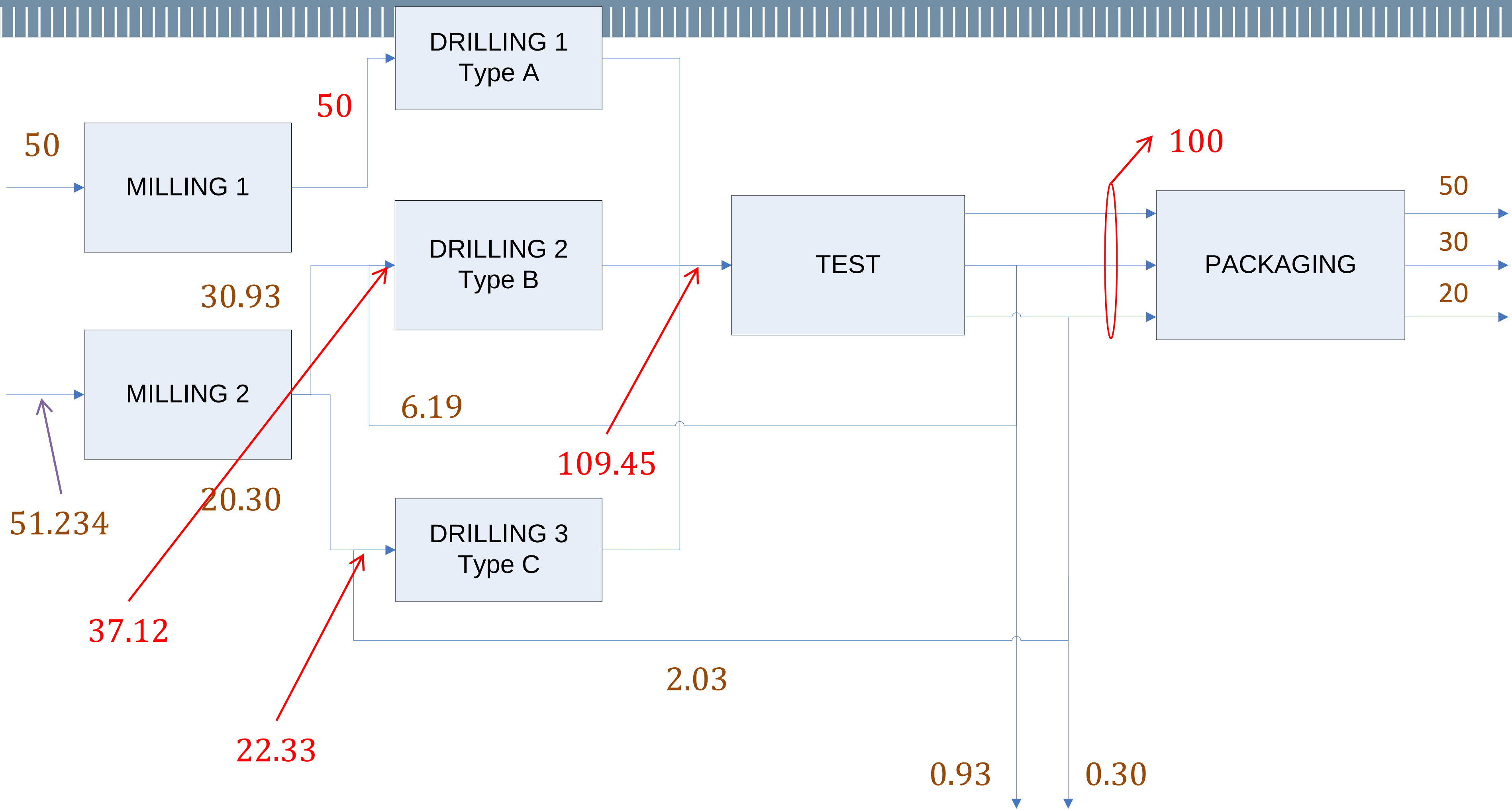
About the twice worked pieces, 85% are good and 15% are rejected and trashed

Out. C + 0,1 * C + 0,15 * 0,1 * C = 1,1 * C

20 = C – 0.015 * C = 0.985 * C

C = 20.304 pc. / h

System Modelling - Paths



System Modelling – Relevant parameters

	Milling1 (pdt. A)	Milling 2 (pdt B,C)	Drilling pdt. A	Drilling pdt. B	Drilling pdt. C	Testing	Packaging
λ (pc./h)	50	51,23	50	37,12	22,33	109,45	100
μ (pc./h)	90	90	75	75	75	160	130
Queue	M/M/1	M/M/1	M/M/1	M/M/1	M/M/1	M/M/1	M/M/1
Priority	-	-	-	-	-	-	Non Pre
Idle time (min/h)	26,67	25,85	20	30,30	42,136	18,96	13,85

Idle time = 1- λ /μ

Few messages for system modelling

1. Pay attention how to choose variables
2. Pay attention to text point of vue! In this case the focus of percentages is related to the input (it could be related to other points, as example the number of pieces processed by a stage).
3. Pay attention to the relation between paths and class of products.
4. Node balancing as a control method: evrything enters must exits in someway.

MAPPING PHASE IS FOUNDAMENTAL TO UNDERSTAND THE SYSTEM AND ITS INEFFICIENCES

Some examples – How to shape the following systems?

1. In a supermarket, each day, some batch of fruits arrive in the magazine, then there is a quality check and thus parts of the fruits are discarded. Those fruits that are good are put in the store where customers can buy them. During the evening, the staff check the fruits and discard what is no more good for the purchase. Map the system, considering the paths of the various fruits.

2. In the Kinder Factory, each product has its specific line. Some products share some machine but then they continue their path on their specific line. At the end, after the packaging machine, there is a quality check. In case a packaging does not properly close the snack, there is a human control. According to severity of the problem, there are three possibilities: the product is discarded, the product is sold in a second market after having fixed the problem, the product is re-packed and sold on the main market.

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