Lab 2.1 - ARMD Trial

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$$\text{VISUAL}_{it} = \beta_{0t} + \beta_1 \cdot \text{VISUAL0}_i + \beta_{2t} \cdot \text{TREAT}_i + \epsilon_{it}\ , \quad \epsilon_{it} \sim \mathcal{N}(0, \sigma_{it}^2)$$

for patient i $(i = 1, ..., 234)$
at time t with $t = 1\,(4 \text{ weeks}),\ 2\,(12 \text{ weeks}),\ 3\,(24 \text{ weeks}),\ 4\,(52 \text{ weeks})$

```
library(nlme)
lm1.form <- visual ~ -1 + visual0 + time.f + treat.f:time.f
```

Model 6.1

$\epsilon_{it} \sim \mathcal{N}(0, \sigma_t^2)$, such that

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4$$

which means $\epsilon_{it} \sim \mathcal{N}(0, \sigma^2)$

```
fm6.1 <- gls(lm1.form, data = armd)
```

Model 9.0 - λ_is known

$\epsilon_{it} \sim \mathcal{N}(0, \sigma_t^2)$ where $\sigma_t = \sigma \cdot \sqrt{\text{time}}$, i.e.,

$$\sigma_t = \begin{cases} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{cases} = \begin{cases} \sigma \cdot \sqrt{4} & \text{if } t = 1 \text{ (4 weeks);} \\ \sigma \cdot \sqrt{12} & \text{if } t = 2 \text{ (12 weeks);} \\ \sigma \cdot \sqrt{24} & \text{if } t = 3 \text{ (24 weeks);} \\ \sigma \cdot \sqrt{52} & \text{if } t = 4 \text{ (52 weeks).} \end{cases}$$

```
weights = varFixed(value = ~time)
# the variance covariate needs to be continuous: if we put time.f, it doesn't work!
fm9.0 <- gls(lm1.form, weights = weights, data = armd)
```

Model 9.1 - $< \delta >$-group

Time-specific variance: $\epsilon_{it} \sim \mathcal{N}(0, \sigma_t^2)$

$$\sigma_t = \begin{cases} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{cases} = \begin{cases} \sigma \cdot 1 & \text{if } t = 1 \text{ (4 weeks);} \\ \sigma \cdot \delta_2 & \text{if } t = 2 \text{ (12 weeks);} \\ \sigma \cdot \delta_3 & \text{if } t = 3 \text{ (24 weeks);} \\ \sigma \cdot \delta_4 & \text{if } t = 4 \text{ (52 weeks).} \end{cases}$$

we get: $\delta_2 = \frac{\sigma_2}{\sigma_1}$; $\delta_3 = \frac{\sigma_3}{\sigma_1}$; $\delta_4 = \frac{\sigma_4}{\sigma_1}$

```
weights = varIdent(form = ~1|time.f)
fm9.1 <- gls(lm1.form, weights = weights, data = armd)
```

Model 9.2 - $< \delta >$-group

$\epsilon_{it} \sim \mathcal{N}(0, \sigma_{it}^2)$
varPower($\cdot$) time
$\underline{\delta} = \delta$ (scalar) since we do not include any stratification in the model

$$\begin{aligned} \sigma_{it} &= \sigma \cdot \lambda_{it} \\ &= \sigma \cdot \lambda(\ \delta,\ \text{TIME}_{it}\ ) \\ &= \sigma \cdot |\text{TIME}_{it}|^{\delta} \quad \text{since } \lambda \text{ is varPower}(\cdot) \end{aligned}$$

```
weights = varPower(form = ~time)
fm9.2 <- gls(lm1.form, weights = weights, data = armd)
```

Model 9.3 - $< \delta >$-group

$\epsilon_{it} \sim \mathcal{N}(0, \sigma_{it}^2)$
varPower($\cdot$) time by treat.f
$\underline{\delta} = [\delta_1, \delta_2]'$ since we include stratification by treatment group

$$\begin{aligned} \sigma_{it} &= \sigma \cdot \lambda_{it} \\ &= \sigma \cdot \lambda(\ \underline{\delta},\ \text{TIME}_{it}\ ) \\ &= \sigma \cdot \lambda(\ [\delta_1, \delta_2]',\ \text{TIME}_{it}\ ) \\ &= \begin{cases} \sigma \cdot |\text{TIME}_{it}|^{\delta_1} & \text{if active} \\ \sigma \cdot |\text{TIME}_{it}|^{\delta_2} & \text{if placebo} \end{cases} \end{aligned}$$

```
weights = varPower(form = ~time|treat.f)
fm9.3 <- gls(lm1.form, weights = weights, data = armd)
```

Model 9.4 - $< \delta, \mu >$-group

$\epsilon_{it} \sim \mathcal{N}(0, \sigma_{it}^2)$
varPower($\cdot$) μ
$\underline{\delta} = \delta$ (scalar) since we do not include any stratification in the model

$$\begin{aligned} \sigma_{it} &= \sigma \cdot \lambda_{it} \\ &= \sigma \cdot \lambda(\ \mu_{it}, \delta\ ) \\ &= \sigma \cdot |\mu_{it}|^{\delta} \end{aligned}$$

where μ_{it} is the predicted (mean) value for VISUAL_{it}.

```
weights = varPower()
fm9.4 <- gls(lm1.form, weights = weights, data = armd)
```

Model 9.5 - $< \mu >$-group

$\epsilon_{it} \sim \mathcal{N}(0, \sigma_{it}^2)$
varPower($\cdot$) μ
$\underline{\delta} = \delta = 1$ (scalar) since we do not include any stratification in the model and we set it to 1

$$\begin{aligned} \sigma_{it} &= \sigma \cdot \lambda_{it} \\ &= \sigma \cdot \lambda(\ \mu_{it}\ ) \\ &= \sigma \cdot |\mu_{it}|^{1} \end{aligned}$$

which means that $\sigma = \frac{\sigma_{it}}{\mu_{it}}$, where μ_{it} is the predicted (mean) value for VISUAL_{it}.

```
weights = varPower(fixed=1)
fm9.5 <- gls(lm1.form, weights = weights, data = armd)
```