QUEUEING THEORY

EXERCISES 4-c

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Theory link

- The queuing theory is necessary for companies in order take better business decisions in terms of resources
- Companies can better manage their resources and understanding which is the need of farther resources in order to provide a service.
- It studies the waiting times of customers/products

Key messages from the lesson

- Focus on a network of systems
- Understanding in practical terms how to apply the queuing theory tanking into account all the possible criticalities
  - How to map a complex system
  - How to manage different types of customers/products
  - How to map/understand/evaluate the customers/products paths
  - How to calculate occurrences
  - How to manage paths in case of scraps
  - How to manage products/customers with different priorities

The production system of SPEED Spa is very simple. It is shaped by three different stations. Also the production flow is extremely simple because the company works only one type of product

The production flow is composed of one stage of mechanical process and a following stage of finishing. Both machines have an extremely high availability that is equal to 100%.

The stage of finishing has an automatic control system which is able to separate good pieces from defective pieces. The good items are moved to the special treatment section, in another area of the plant. This area is not the subject of the study and therefore it can be ignored. Must be supply 600 good pieces per hour to special treatment section.

The pieces considered defective by finishing stage are carried to a quality control stage that has to identify the type of defect of the pieces resulted non-compliant after finishing job. Three different options:

a) The piece erroneously considered defective. It is good and it is delivered to special treatment section.
b) The piece has small damage and it has to be reworked by both machines. It is reinserted in the production flow from the first station.
c) The piece is very damaged so it is rejected and thrown away

The automatic control system of the finishing machine is not infallible. In particular, it generally considers 10% of the pieces really good as non-compliant. It never fails to identify damaged pieces, which are on average 15% of the finishing stage output.

The machine of quality control stage has 100% correct identification of pieces quality. Usually, the 20% of the pieces coming from the finishing stage to quality control stage results to be discarded.

All arrival and service rates are described by a negative exponential distribution. The times given below refer to a single machine or operator.

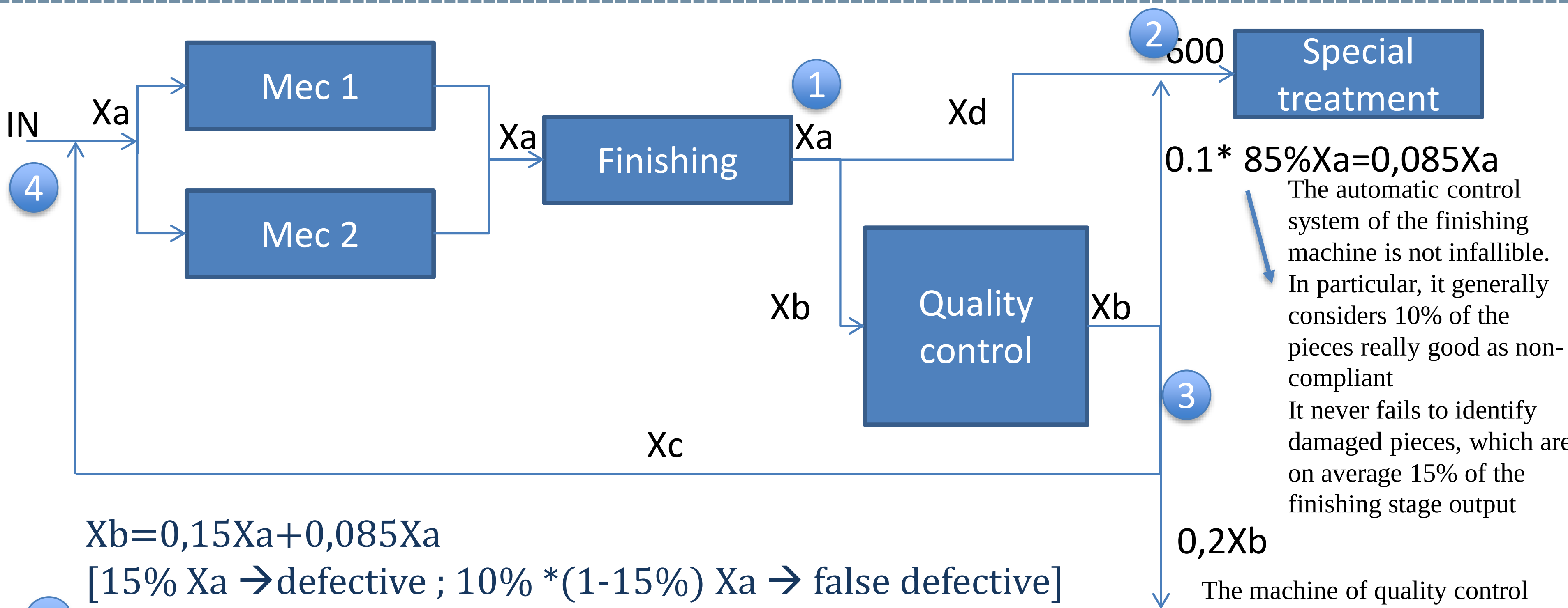
Mechanical machine	413 pieces/hour
Finishing machine	800 pieces/hour
Quality control machine	188 pieces/hour

The stage of mechanical working is composed of two identical machines that work in parallel. The product that is waiting in the queue will take the first free machine. Finishing and quality control stages are composed of a single machine each.

With the data available:

1) Calculate the minimum allowable arrival rate of raw material input to the system in order to ensure to special treatment section 600 pieces / hour.
2) Calculate the average WIP and waiting time in line in the queue upstream of quality control stage.
3) Calculate the probability that a product which just joined the queue of the finishing stage exits from same stage in less than 45 seconds. (The pieces are processed in FIFO logic).
4) Calculate the average throughput time of the system under the following assumptions:
    1- The automatic control machine of the finishing stage can separate 100% good parts from the damaged pieces.
    2- Once the piece has been reprocessed it is conformed.

Speed S.p.A. – Q1



Xb=0,15Xa+0,085Xa

[15% Xa →defective ; 10% *(1-15%) Xa → false defective]

1 Xd=Xa-Xb=0.85Xa-0.1*.85Xa

2 600=0,085Xa+ Xa(1-0,085-0,15) → Xa=705,88

Xb=165,88 ; Xd=540

3 Xb=0,085Xa+0,2Xb+Xc → Xc=72,7

4 IN+Xc=Xa → **IN=633,18** (Minumum arrival rate or raw material)

The machine of quality control stage has 100% correct identification of pieces quality. Usually, the 20% of the pieces coming from the finishing stage to quality control stage results to be discarded

Q2 - WIP and Wq at quality control

Quality Control Stage

M/M/1

λ=Xb= 165, 88p/h

μ=188p/h

Wq

- Wq = (λ / μ) / (μ – λ) =

  60*(165,88/188) / (188-165,88) = 2,39 min

Wip (only items in the queue)

- Lq = λ*(λ / μ) / (μ – λ) =

  165,88 *(165,88/188) / (188-165,88) = 6,617 pieces

Q3 - Probability of Ws(finishing) lower than 45 seconds

$P(Ws_{finishing} < 45 \text{ sec}) = ?$

$Ws_{finishing} = WIP_{finishing} * 1/\lambda$ (Little law)

$Ws_{finishing} = n * 1/\lambda,$

n = number of products in the finishing subsystem = $Ws_{finishing} * \lambda$

$P(Ws * \lambda < 45 * \lambda) =$

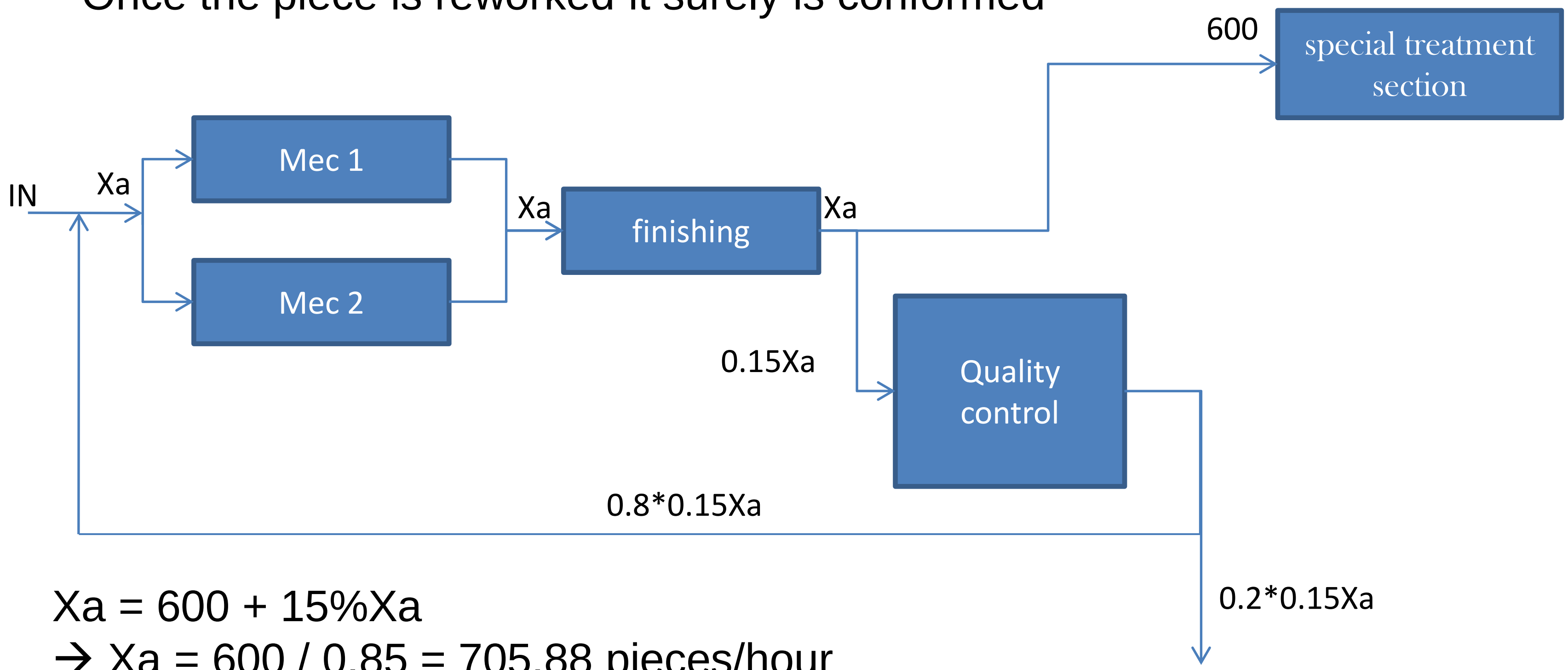
$P(n < 45 \text{ sec} * 705{,}88 / 3600 \text{ p / sec}) =$

$P(n < 8{,}824) = 1 - p\ (n \geq 8{,}824) = 1 - (705{,}88/800)^{\wedge}8{,}824 = 0{,}669$

Q4 – average throughput time

Assumptions:

- Automatic control after finishing correctly define damaged and not-damaged pieces
- Once the piece is reworked it surely is conformed



Xa = 600 + 15%Xa

→ Xa = 600 / 0.85 = 705.88 pieces/hour

Q4 – average throughput time

Assumptions:

- Automatic control after finishing correctly define damaged and not-damaged pieces
- Once the piece is reworked it surely is conformed

	Mechanical Process	Finishing	Quality control
λ	705.88	705.88	105.882
μ	413	800	188
Queue system	M/M/2	M/M/1	M/M/1
Ws	c=2; λ/μ=1,709 Lq= 4,426 Ws= 4,426/705,88+ 1/413= 31,3 sec	=1 / ($\mu - \lambda$) = 60*60*1/(800-705,88) = 38,25 sec	=1 / ($\mu - \lambda$)= 60*60*1/(188-105,882)= 43,84 sec

Q4 – average throughput time

	Mec. process	Finishing	Quality control	Mec. process	Finishing	Scrap	Number of products	Occurr.
Path 1	x	x					515.29	0.8295
Path 2	x	x	x	x	x		84.71	0.1364
Path 3	x	x	x			x	21.176	0.0341
								1

Path1: (Input-0,15Xa)/Input = 515,29 /621,176
Path2: 0,8*0,15Xa/Input = 84,71/621,176=0,1364
Path3: 0,2*0,15Xa/Input= 21,176/621,176=0,0341

Input= 600+0,2*0,15*705,88=621,176

Q4 – average throughput time

	Mec. process	Finishing	Quality control	Mec. process	Finishing	Occorr.	Adjusted Occorr.	Ws*Occ.
Path 1	31,3	38,25				0.8295	0.8588	59.73
Path 2	31,3	38,25	43,84	31,3	38,25	0.1364	0.1412	25.83
TOTAL (sec)								85.56

Some examples – How to shape the following systems?

1. In FIAT, each component of the various cars are processed through specific processes, the majority of them are automatic processes. In same case some components are discarded, others are reworked and others having not the right shape, after being reworked, are sold on a second market. The last processes are done by humans in order to ensure high quality of the product. Map the possible network of systems.

2. The Lamborghini plant has different lines where single components are processed. There are some customized products that have the priority over the other products. During the production process some problems can happen and some scraps is discarded. Try to map the possible network of systems

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