Politecnico di Milano
Scuola di Ingegneria Industriale e dell'Informazione
Applied Statistics
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Problem n.2

The telecommunications company *ConnectCom* offers cellular services with a 5G network to its subscribers. However, the strength and reliability of the **5G** signal are not consistently assured and the network may intermittently switch to a **4G** signal. Furthermore, contingent on the current hidden congestion level of the network, the quality of network coverage provided can be classified into two primary states: **Standard (S)** and **Enhanced (E)**. *ConnectCom* seeks to construct a Hidden Markov Model to depict this situation and supplies the following information:

- The presence of a reliable 5G signal is assessed as having a probability of 0.75 in the S state, while the probability rises to 0.85 in the E state;
- When in S state, the probability of remaining in S state is assessed as 0.6. When in E state, the probability of remaining in E state is assessed 0.7.

Answer the following questions:

a) Identify the hidden states of the model and explain your reasoning. Provide the Transition and Emission matrices based on the provided information.

b) On the 15th of April 2023, ConnectCom collected network quality data per minute from a subscriber over a 30-minute period. Given the recorded sequence of network: X = (5, 5, 5, 4, 4, 4, 5, 4, 4, 5, 5, 5, 5, 4, 4, 4, 5, 5, 5, 5, 4, 4, 5, 5, 5, 4, 4, 5, 4, 5), provide the most probable path of hidden states (use defaults parameters and assume equal start probabilities). How did you estimate it?

Hint: Use the following shortcut for copy-pasting data in R and be careful to conflicts between packages!

```
observation = paste( c(5, 5, 5, 4, 4, 4, 5, 4, 4, 5, 5, 5, 5, 4, 4, 4, 5, 5, 5, 5, 4, 4, 5, 5,
5, 4, 4, 5, 4, 5), "G", sep = "")
```

c) Based on the data provided by the sequence X of observed signals, derive the optimal estimates of the Transition and Emission matrices for the Hidden Markov Model.

d) Given the Transition Matrix computed at point b), define a Markov Chain (`markovchain` object) naming the states as done in point a) and provide a graphical representation of the `markovchain`.

Hint: Recall that rows of a transition matrix should sum to 1. If this doesn't happen from the transition matrix `trans_matr` obtained in b), do scale your probabilities through the command `trans_matr / rowSums(trans_matr)`.

e) Identify:

1. The long-term probability that the system will be in each state (i.e., the steady states),
2. The recurrent states and
3. The transient states.
4. Are there any absorbing states?
5. Is the chain irreducible? Report the periodicity.

Upload your solution here