
# QUEUE MANAGEMENT

# EX SET #4

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# Theory link



- The queuing theory is necessary for companies in order take better business decisions in terms of resources
- Companies can better manage their resources and understanding which is the need of farther resources in order to provide a service.
- It studies the waiting times of customers/products

# Key messages from the lesson

- Focus on a network of systems
- Understanding in practical terms how to apply the queuing theory tanking into account all the possible criticalities
  - How to map a complex system
  - How to manage different types of customers/products
  - How to map/understand/evaluate the customers/products paths
  - How to calculate occurrences
  - How to manage paths in case of scraps
  - How to manage products/customers with different priorities

MASTER Spa has to deliver 500 good components per hour to the customer, respectively 300 type A pieces and 200 type B pieces. The production system of MASTER Spa is basically a job shop and production flow for the two types of components is described below.
Both types of product are processed in the first stage of rough-cut. It consists of two technologically identical machines. These resources don't need set-up to change product type, they can indifferently work both product families. The next item waiting to be processed in the queue will be worked by the first free machine. After this stage the production flows of the two types are separated, they meet on the last stage of testing.
Type A products first pass the surface finishing stage and further the drilling stage. Downstream of the drilling stage pieces join the testing stage queue.
Type B products, after initial roughing stage in common with type A product, are processed in a work center. After being processed by the work center, they are placed in the testing stage queue.
The product B has a particular shape, which easily creates problems during the process. Because of this, frequently type B products have to be reworked or discarded. The quality control is made by the same work center. In particular, 5% of pieces worked are no longer workable and therefore they are discarded, while 15% of the pieces that this stage works have to be reworked to become good products.
These B-products have to be reworked on traditional machines, so they are sent to the queue upstream of the surface finishing machine that works products of type A. Once the product is reprocessed by surface finishing and drilling machines, it is considered good. In surface finishing and drilling stages products of type B move forward with a lower priority than type A, according to a not preemptive logic.
Last stage is formed by the testing machine. In the testing stage there aren't any types of priority rule. The testing machine is old and sometimes it identifies as defective a good product. For this reason, a product been identified as defective first time has to be tested again. On Average 15% of products tested by the machine (both types) has to be tested again. 50% of the pieces tested twice are discarded (both types). The other products are good and they can be delivered to the customer.

The table shows the service rates of the resources of MASTER Spa. The service rates and the arrival rates are distributed according to a negative exponential.

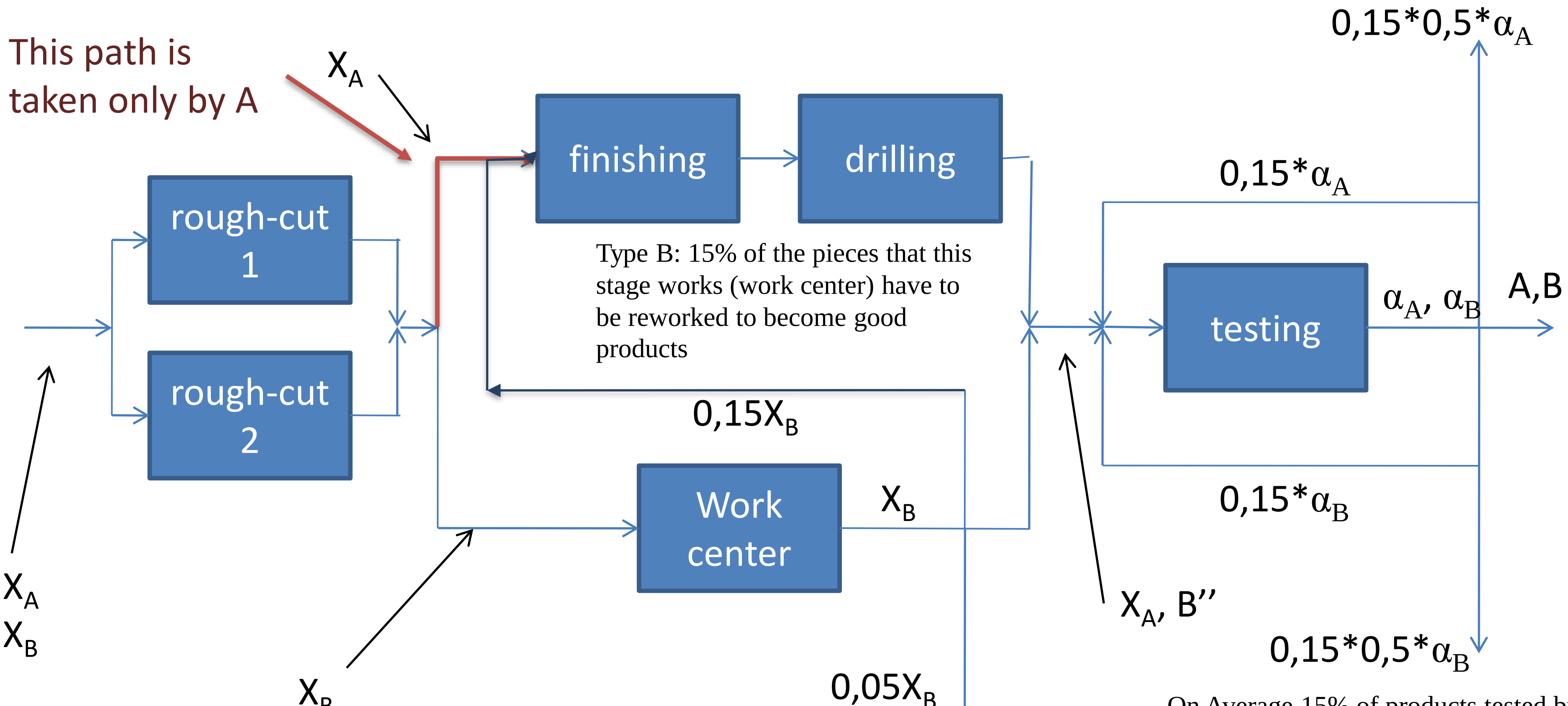
| Service rate | Pieces/hour |
|---|---|
| Rough-cut stage (1 machine) | 300 |
| Work center stage | 250 |
| Surface finishing stage | 380 |
| Drilling stage | 380 |
| testing | 660 |

Company's requests are:
1) Model the production system of the Master Spa, calculate all relevant parameters of the system, with particular attention to the calculation of the necessary input for each component in order to meet the production target.
2) Calculate the expected throughput time of the production system (not considering the products that need to be eliminated because defective)
3) Calculate the raw material input reduction if the defective products percentage at testing machine drops to 0 (in the second test all the products result conformed). What are the effects on the expected throughput time?

Input a = Xa?
Input b =Xb?

Output a = A = 300 p/h
Output b = B = 200 p/h

# Master S.p.A.



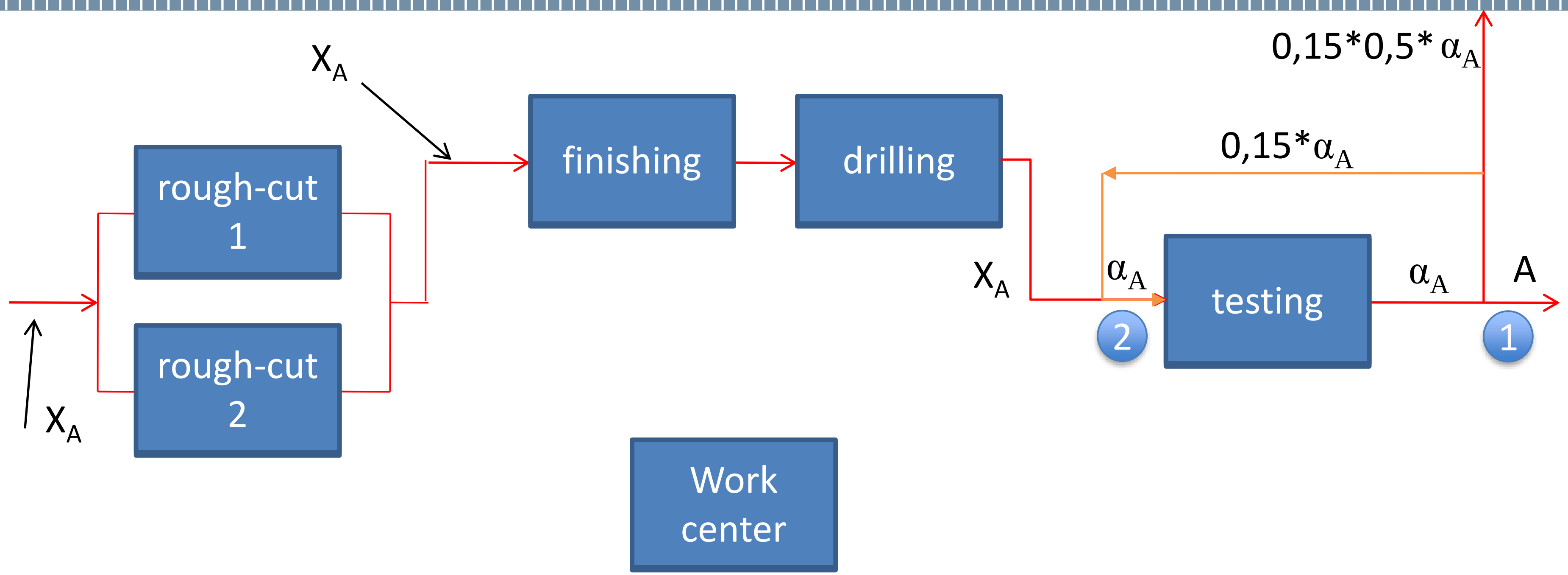

Type B: 15% of the pieces that this stage works (work center) have to be reworked to become good products

Type B: 5% of pieces worked (in the work center) are no longer workable and therefore they are discarded

On Average 15% of products tested by the machine (both types) has to be tested again. 50% of the pieces tested twice are discarded (both types)

Goal:
- 300 type A pieces/hour
- 200 type B pieces/hour

# Input A

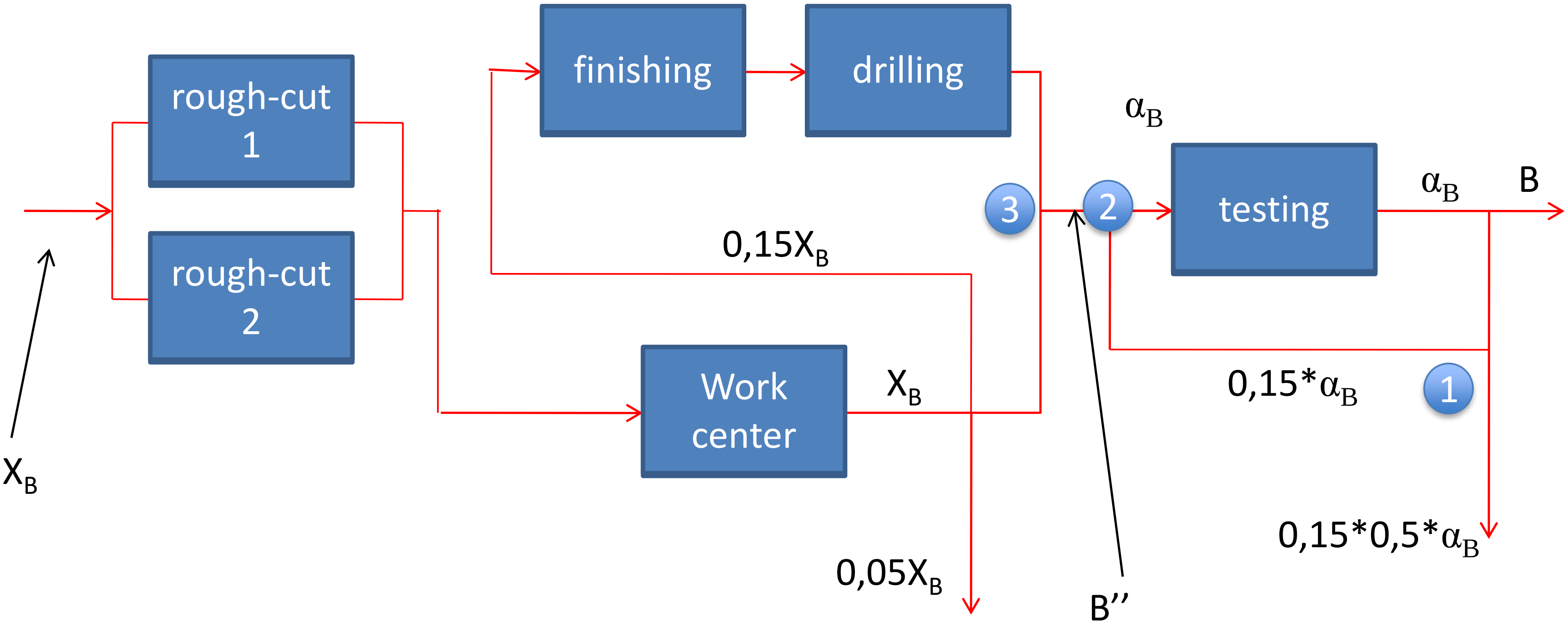



A=300

1 $\left[\begin{array}{l} A+0{,}15*0{,}5*\alpha_A+0{,}15*\alpha_A=\alpha_A \rightarrow 0{,}775*\alpha_A=300 \\ \alpha_A=387{,}097 \text{ pieces/hour} \end{array}\right.$

2 $\left[\begin{array}{l} X_A+0{,}15*\ \alpha_A=\alpha_A \rightarrow X_A=0{,}85*\ \alpha_A\ =329{,}032 \text{ pieces/hour} \\ \mathbf{X_A \text{ INPUT «A» : } 329{,}032 \text{ pieces / hour}} \end{array}\right.$

# Products B

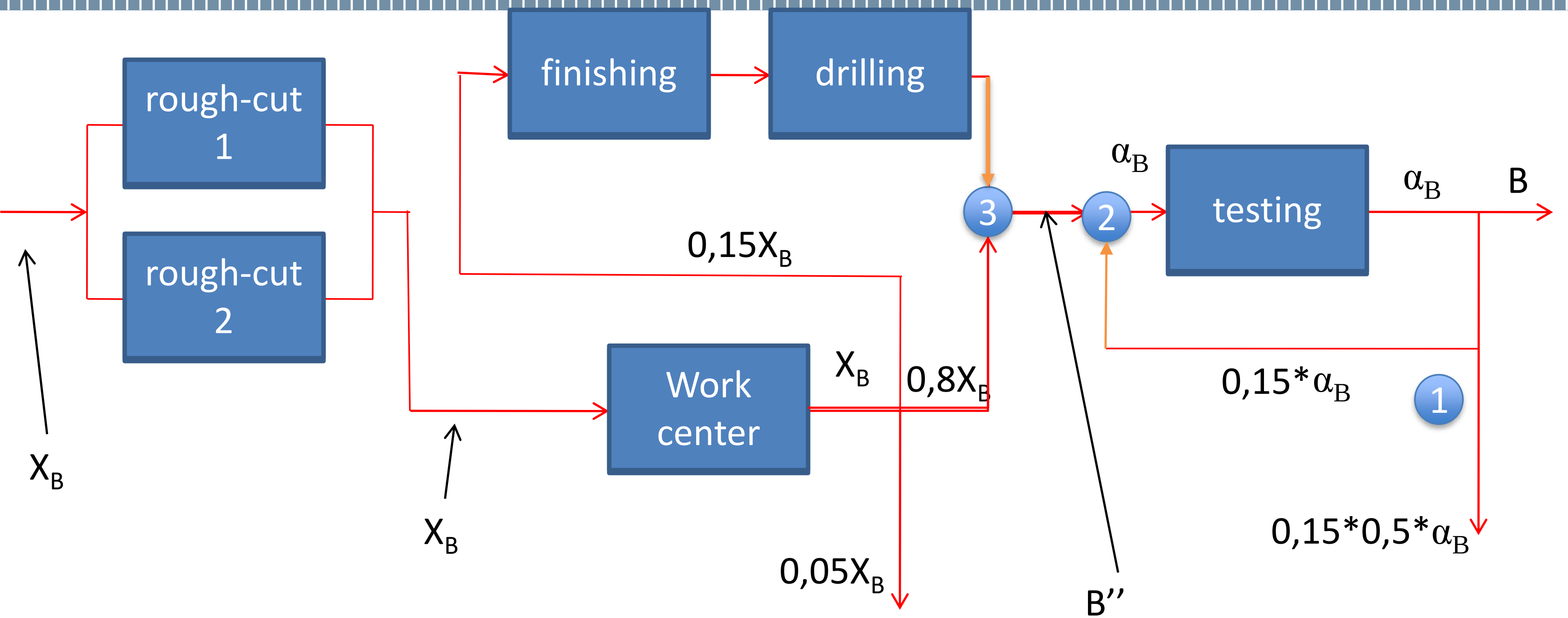



B=200

1 $B+0,15*0,5*\alpha_B+0,15*\alpha_B=\alpha_B \rightarrow 0,775*\alpha_B=200$

$\alpha_B$=258,064 pieces/hour

# Input B

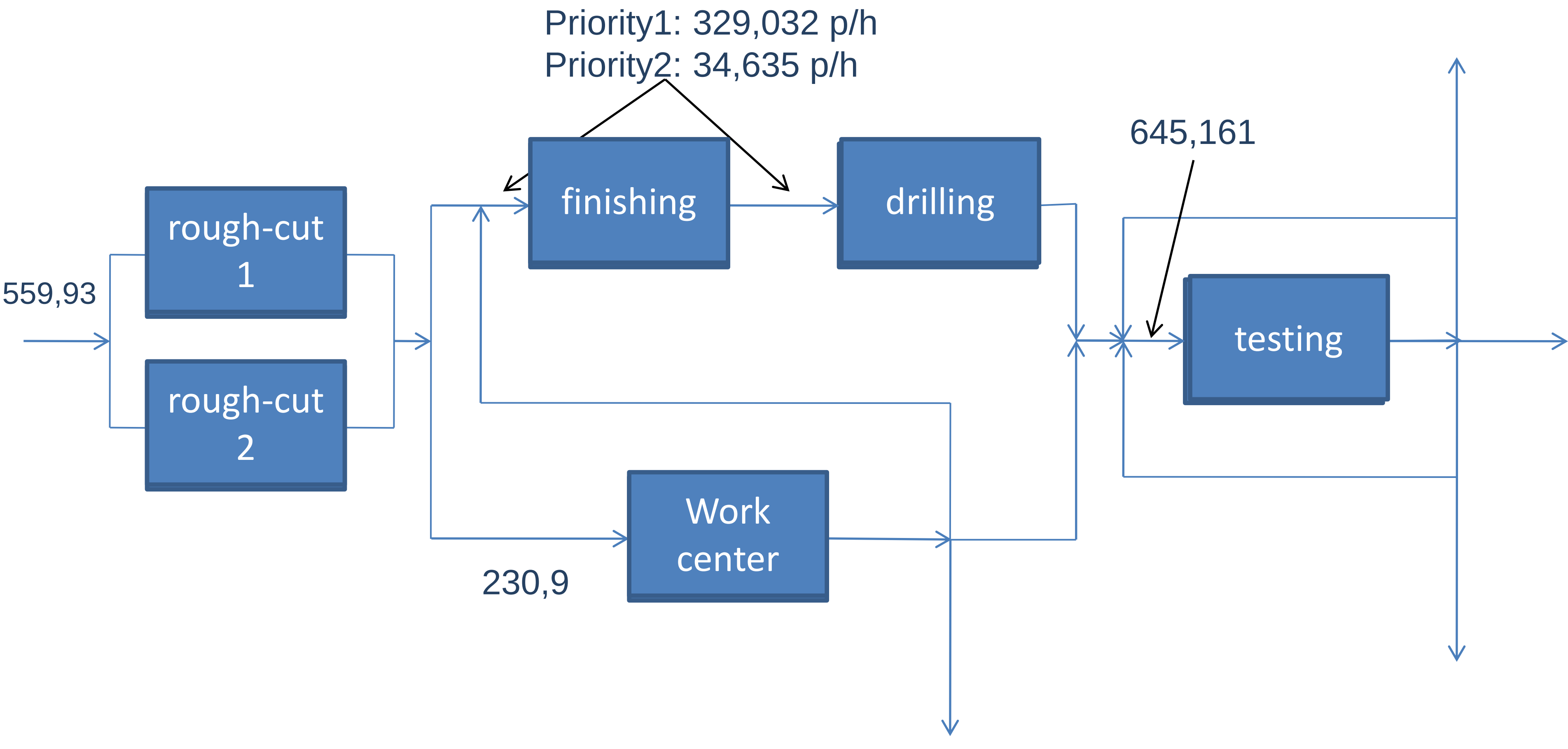



2 B'' $+0,15\ \alpha_B = \alpha_B$ → B''$= 0,85\ \alpha_B = 219,4$ pieces/hour

3 B'' $= 0,15X_B + 0,8\ X_B$

**$X_B$ INPUT «B» = 230,9 pieces/hour**

# Master S.p.A. – arrival rates



# Master S.p.A. – parameters

| | rough-cut | finishing | drilling | Work center | testing |
|---|---|---|---|---|---|
| λ (pz/hour) | 559,93 | Priority1: 329,032 Priority2: 34,635 | Priority1: 329,032 Priority2: 34,635 | 230,9 | 645,161 |
| μ (pz/hour) | 300 (1 machine) | 380 | 380 | 250 | 660 |
| Priority rule | - | NON PREHEMPTIVE | NON PREHEMPTIVE | - | - |
| Queue type | M/M/2 | M/M/1 | M/M/1 | M/M/1 | M/M/1 |

The next item waiting to be processed in the queue will be worked by the first free machine

# Throughput time

| | rough-cut | finishing | drilling | Work center | testing |
|---|---|---|---|---|---|
| λ (pz/hour) | 559,93 | A:Priority1: 329,032<br>B:Priority2: 34,635 | A:Priority1: 329,032<br>B:Priority2: 34,635 | 230,9 | 645,161 |
| μ (pz/hour) | 300<br>(1 machine) | 380 | 380 | 250 | 660 |
| Priority rule | - | NON PREHEMPTIVE | NON PREHEMPTIVE | - | - |
| queue | M/M/2 | M/M/1 | M/M/1 | M/M/1 | M/M/1 |
| Ws (min) | 2,08 | Priority1: 1,284<br>Priority2: 26,369 | Priority1: 1,284<br>Priority2: 26,369 | 3,141 | 4,043 |

# Ws of each stage

## Rough cut M/M/2

$Ws = Lq / \lambda + 1 / \mu$

$\lambda = 559{,}93 \qquad \mu = 300 \rightarrow \lambda/\mu = 1{,}866$

$C=2 \rightarrow Lq = 17{,}587$

$Ws = (17{,}587/559{,}93 + 1/300) \; {*}60 = 2{,}08 \text{ min}$

Lq results of model M

| λ/μ | c=1 | c=2 |
|---|---|---|
| 0,15 | 0,026 | 0,001 |
| 0,20 | 0,050 | 0,002 |
| 0,25 | 0,083 | 0,004 |
| 0,30 | 0,129 | 0,007 |
| 0,35 | 0,188 | 0,011 |
| 0,40 | 0,267 | 0,017 |
| 0,45 | 0,368 | 0,024 |
| 0,50 | 0,500 | 0,033 |
| 0,55 | 0,672 | 0,045 |
| 0,60 | 0,900 | 0,059 |
| 0,65 | 1,207 | 0,077 |
| 0,70 | 1,633 | 0,098 |
| 0,75 | 2,250 | 0,123 |
| 0,80 | 3,200 | 0,152 |
| 0,85 | 4,817 | 0,187 |
| 0,90 | 8,100 | 0,229 |
| 0,95 | 18,050 | 0,277 |
| 1,0 | | 0,333 |
| 1,1 | | 0,477 |
| 1,2 | | 0,675 |
| 1,3 | | 0,951 |
| 1,4 | | 1,345 |
| 1,5 | | 1,929 |
| 1,6 | | 2,844 |
| 1,7 | | 4,426 |
| 1,8 | | 7,674 |
| 1,9 | | 17,587 |

# Ws of each stage

**Finishing** M/M/1 Non preemptive priority

λ1 =329,032

λ2 = 34,635

μ = 380

$$E(S_1) = \frac{(1+\rho_2)/\mu}{1-\rho_1},$$

$$E(S_2) = \frac{(1-\rho_1(1-\rho_1-\rho_2))/\mu}{(1-\rho_1)(1-\rho_1-\rho_2)}.$$

E(S1) = 60*(( 1+ 34,635/380)/380) / (1- 329,032/380)= 1,284 min

E(S2) = 60*(1-329,032/380(1-329,032/380-34,635/380))/380 / ((1-329,032/380)*(1-329,032/380-34,635/380))= 26,369 min

# Ws of each stage

**Drilling** M/M/1 Non prehentive priority

λ1 =329,032

λ2 = 34,635

μ = 380

$$E(S_1) = \frac{(1+\rho_2)/\mu}{1-\rho_1},$$

$$E(S_2) = \frac{(1-\rho_1(1-\rho_1-\rho_2))/\mu}{(1-\rho_1)(1-\rho_1-\rho_2)}.$$

E(S1) = 60*(( 1+ 34,635/380)/380) / (1- 329,032/380)= 1,284 min

E(S2) = 60*(1-329,032/380(1-329,032/380-34,635/380))/380 / ((1-329,032/380)*(1-329,032/380-34,635/380))= 26,369 min

# Ws of each stage

**Work center** M/M/1

λ = 230,9

μ = 250

Ws= 1/(μ - λ) = 1/ (250-230,9)*60 = 3,141

**Testing** M/M/1

λ = 645,161

μ = 660

Ws= 1/(μ - λ) = 1/ (660-645,161) * 60 = 4,043

# Throughput time

## Which are the possible paths?

| | rough-cut | finishing | drilling | Work center | scrap | Testing 1 | Testing 2 | scrap | Final customer |
|---|---|---|---|---|---|---|---|---|---|
| A1 | X | X | X | | | X | | | X |
| A2 | X | X | X | | | X | X | | X |
| A3 | X | X | X | | | X | X | X | |
| B1 | X | | | X | | X | | | X |
| B2 | X | | | X | X | | | | |
| B3 | X | | | X | | X | X | | X |
| B4 | X | | | X | | X | X | X | |
| B5 | X | X | X | X | | X | | | X |
| B6 | X | X | X | X | | X | X | | X |
| B7 | X | X | X | X | | X | X | X | |

# Occurrences of type A

| | rough-cut | finishing | drilling | Work center | scrap | Testing 1 | Testing 2 | scrap | Final customer |
|---|---|---|---|---|---|---|---|---|---|
| A1 | × | × | × | | | × | | | × |
| A2 | × | × | × | | | × | × | | × |
| A3 | × | × | × | | | × | × | × | |

A1: $(X_A - 0{,}15\,\alpha_A)/X_A = 0{,}82$

A2: $((1-0{,}5)*0{,}15\,\alpha_A)/X_A = 0{,}09$

A3: $(0{,}5*0{,}15\,\alpha_A)/X_A = 0{,}09$

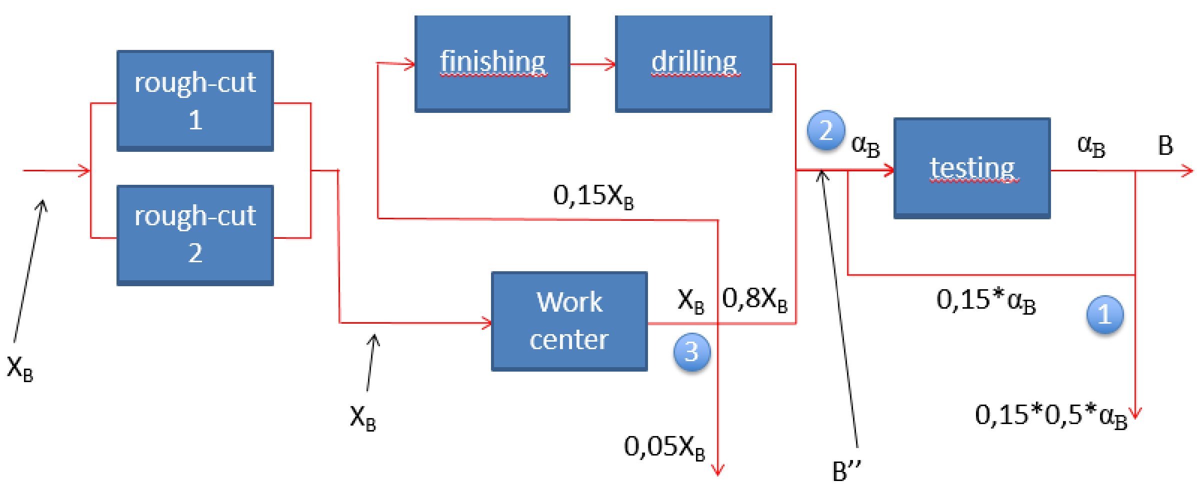
# Occurrences of type B (1/7)

| | rough-cut | finishing | drilling | Work center | scrap | Testing 1 | Testing 2 | scrap | Final customer |
|---|---|---|---|---|---|---|---|---|---|
| B1 | X | | | X | | X | | | X |
| B2 | X | | | X | X | | | | |
| B3 | X | | | X | | X | X | | X |
| B4 | X | | | X | | X | X | X | |
| B5 | X | X | X | X | | X | | | X |
| B6 | X | X | X | X | | X | X | | X |
| B7 | X | X | X | X | | X | X | X | |

# Occurrences of type B (2/7)

| | rough-cut | finishing | drilling | Work center | scrap | Testing 1 | Testing 2 | scrap | Final customer |
|---|---|---|---|---|---|---|---|---|---|
| B2 | X | | | X | X | | | | |

$$B2 = 0{,}05 * X_B / X_B = 0{,}05$$



# Occurrences of type B (3/7)

| | rough-cut | finishing | drilling | Work center | scrap | Testing 1 | Testing 2 | scrap | Final customer |
|---|---|---|---|---|---|---|---|---|---|
| B1 | × | | | × | | × | | | × |
| B3 | × | | | × | | × | × | | × |
| B4 | × | | | × | | × | × | × | |

In B1, B3 and B4 paths the pieces don't have to be reworked by finishing and drilling machines.

The total amount of these pieces is $0,8*X_{B}$.

$0,8*X_B$ is distributed among good pieces at first test, good pieces at 2° test and scraps at second test.

# Occurrences of type B (4/7)

$$\alpha_B = \boxed{0{,}8/0{,}95}\ \alpha_B + \boxed{0{,}15/0{,}95}\ \alpha_B$$

Part of pieces reworked by finishing and drilling machines

Part of pieces directly arrived to testing stage from work center

The quantities of pieces that enter in the testing phase are $0{,}8 * X_B$ that arrives directly from work center and $0{,}15 * X_B$ is reworked by traditional machines

This proportion is kept also for the pieces processed by the testing machine and the pieces that have to be tested again.

$0{,}8/(0{,}8+0{,}15) = 0{,}8/0{,}95$ not-reworked

$0{,}15/(0{,}8+0{,}15) = 0{,}15/0{,}95$ reworked

Reworked= «reworked by finishing and drilling machines», Not «pieces tested twice»!!)

# Occurrences of type B (5/7)

%Reworked in the testing

$$B1 = ( 0{,}8\ X_B - 0{,}8/0{,}95 * 0{,}15\ \alpha_B ) / X_B \quad = 152{,}12/230{,}9 = 0{,}659$$

%Reworked in the testing - %scraps

$$B3 = [\ 0{,}8/0{,}95 * (1 - 0{,}5) * 0{,}15\ \alpha_B\ ] / X_B \quad = \quad 16{,}3/230{,}9 = 0{,}071$$

%Scraps

$$B4 = [\ 0{,}8/0{,}95 * 0{,}5 * 0{,}15\ \alpha_B\ ] / X_B \quad = 16{,}3/230{,}9 = 0{,}071$$

# Occurrences of type B (6/7)

| | rough-cut | finishing | drilling | Work center | scrap | Testing 1 | Testing 2 | scrap | Final customer |
|---|---|---|---|---|---|---|---|---|---|
| B5 | × | × | × | × | | × | | | × |
| B6 | × | × | × | × | | × | × | | × |
| B7 | × | × | × | × | | × | × | × | |

In B5, B6 and B7 paths the pieces have to be reworked by finishing and drilling machines.
The total amount of these pieces is 0,15*XB.

0,15*XB is distributed among good pieces at first test, good pieces at 2°test and scraps at second test.

# Occurrences of type B (7/7)

B5 = ( 0,15 $X_B$ – 0,15/0,95* 0,15 $\alpha_B$ ) / $X_B$ = 28,52/230,9= 0,124

B6 = [ 0,15/0,95 *(1 – 0,5)*0,15 $\alpha_B$ ] / $X_B$ = 3,056/230,9= 0,013

B7 = [ 0,15/0,95 * 0,5*0,15 $\alpha_B$ ] / $X_B$ = 3,056/230,9= 0,013

# Paths

| | rough-cut | finishing | drilling | Work center | scrap | Testing 1 | Testing 2 | scrap | Final cust. | Pieces/ hour |
|---|---|---|---|---|---|---|---|---|---|---|
| A1 | × | × | × | | | × | | | × | 270.975 |
| A2 | × | × | × | | | × | × | | × | 29.025 |
| A3 | × | × | × | | | × | × | × | | 29.025 |
| B1 | × | | | × | | × | | | × | 152.12 |
| B2 | × | | | × | × | | | | | 11.545 |
| B3 | × | | | × | | × | × | | × | 16.30 |
| B4 | × | | | × | | × | × | × | | 16.30 |
| B5 | × | × | × | × | | × | | | × | 28.523 |
| B6 | × | × | × | × | | × | × | | × | 3.056 |
| B7 | × | × | × | × | | × | × | × | | 3.056 |
| | | | | | | | | | | **Total A 329.025**<br>**Total B 230.9** |

# Adjusted Occurrences

| | Pieces/hour | Occurrence | Adjusted occurrece |
|---|---|---|---|
| A1 | 270.975 | 0.8236 | 0.8236/0,9118= 0.9033 |
| A2 | 29.025 | 0.0882 | 0.0882/0,9118= 0.0967 |
| B1 | 152.12 | 0.6588 | 0.6588/0.8662= 0.7606 |
| B3 | 16.30 | 0.0706 | 0.0706/0.8662= 0.0815 |
| B5 | 28.523 | 0.1235 | 0.1235/0.8662= 0.1426 |
| B6 | 3.056 | 0.0132 | 0.0132/0.8662= 0.0153 |

0.9118=1-Occ (A3)

0.8662=1-[Occ (B2)+Occ(B4)+Occ(B7)]

# Throughput time

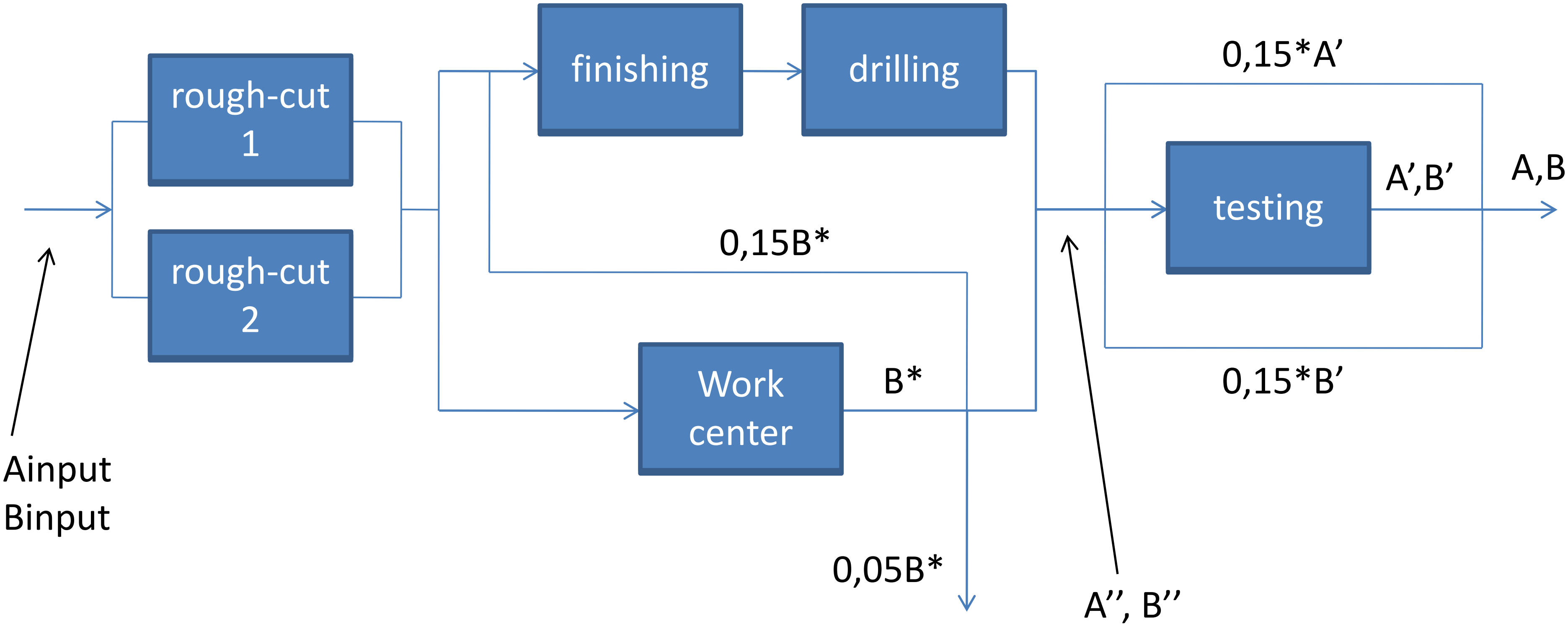
| | rough-cut | finishing | drilling | Work center | Testing 1 | Testing 2 | Throu. Time | Adj. Occ. | Weig. time |
|---|---|---|---|---|---|---|---|---|---|
| A 1 | 2.08 | 1,284 | 1,284 | | 4,043 | | 8.691 | 0.9033 | 7.85 |
| A 2 | 2,08 | 1,284 | 1,284 | | 4,043 | 4,043 | 12.734 | 0.0967 | 1.23 |
| B1 | 2,08 | | | 3,141 | 4,043 | | 9.264 | 0.7606 | 7.05 |
| B3 | 2,08 | | | 3,141 | 4,043 | 4,043 | 13.307 | 0.0815 | 1.08 |
| B5 | 2,08 | 26,369 | 26,369 | 3,141 | 4,043 | | 62.002 | 0.1426 | 8.84 |
| B6 | 2,08 | 26,369 | 26,369 | 3,141 | 4,043 | 4,043 | 66.045 | 0.0153 | 1.01 |

Type A occurrence: 300 / 500 = 0.6
Type B occurrence : 200 / 500 = 0.4

**Average throughput time:**

**0,6*(7.85 + 1.23)+0,4*(7.05+1.08+8.84+1.01) = 12.64 minutes**

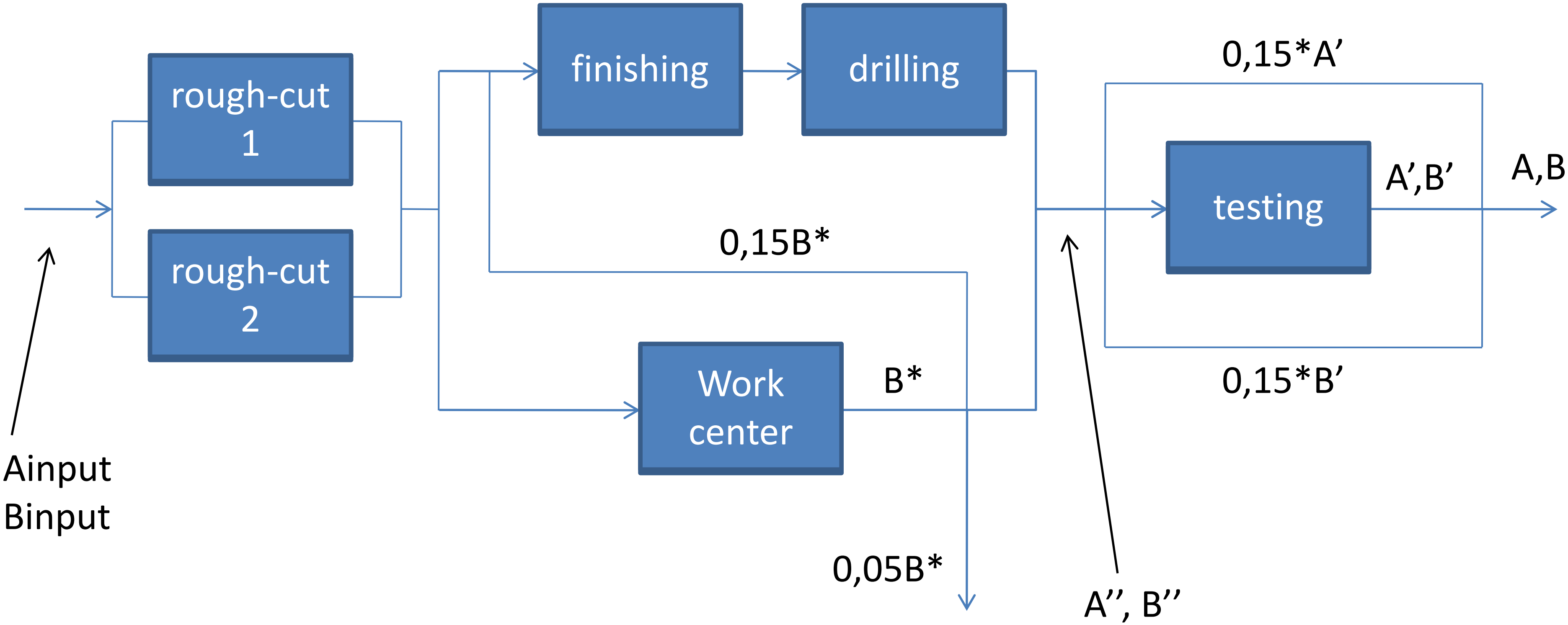
# No scraps from testing phase




Input A=300

How calculate it? → there aren't scraps

# No scraps from testing phase




Input B: B''=B=200

B''=0,15B* + 0.8B*

**B* = 211,54 pieces/hour = INPUT B**

# Some examples – How to shape the following systems?

1. In FIAT, each component of the various cars are processed through specific processes, the majority of them are automatic processes. In same case some components are discarded, others are reworked and others having not the right shape, after being reworked, are sold on a second market. The last processes are done by humans in order to ensure high quality of the product. Map the possible network of systems.

2. The Lamborghini plant has different lines where single components are processed. There are some customized products that have the priority over the other products. During the production process some problems can happen and some scraps is discarded. Try to map the possible network of systems

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