

Exercise 2

The deviation from the nominal center to center distance of a piston rod is known to be characterized by:

- $\mu = 0.4417 \mu\text{m}$
- $\sigma = 3.4914 \mu\text{m}$

A sample of size $n = 5$ is acquired on a daily basis. The measurements of 25 consecutive days are reported in the file `ESE09_ex2.csv`.

1. Design a Xbar-S control chart for the process.
2. Design a CUSUM control chart ($h = 4, k = 0.5$).
3. Design an EWMA control chart ($\lambda = 0.2$).

```
In [ ]: # Import the necessary libraries
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from scipy import stats
import qda

# Import the dataset
data = pd.read_csv('ESE09_ex2.csv')

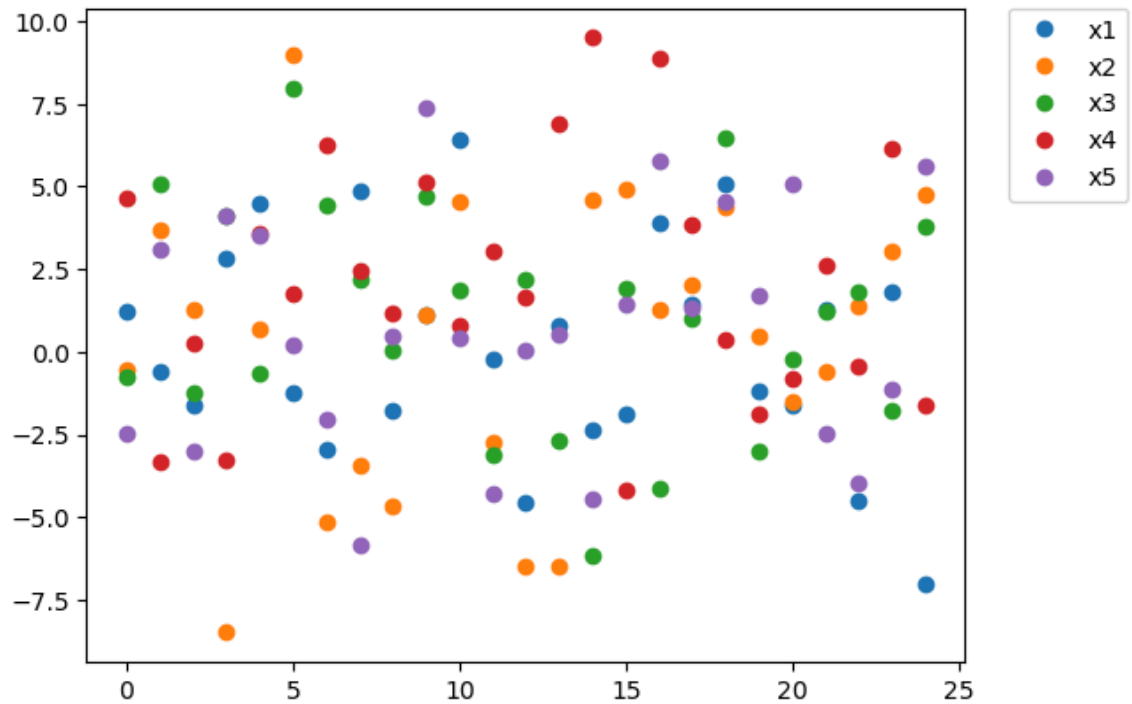
# Inspect the dataset
data.head()
```

```
Out[ ]:
```

	X1	X2	X3	X4	X5
0	1.2102	-0.5621	-0.7336	4.6353	-2.4700
1	-0.5686	3.6728	5.1017	-3.3084	3.0759
2	-1.6336	1.3034	-1.2234	0.2847	-3.0211
3	2.8496	-8.4910	4.1368	-3.2575	4.1078
4	4.4853	0.6767	-0.6653	3.6053	3.5055

Visualize the dataset.

```
In [ ]: # Make a scatter plot of all the columns against the index
plt.plot(data['X1'], linestyle='none', marker='o', label = 'x1')
plt.plot(data['X2'], linestyle='none', marker='o', label = 'x2')
plt.plot(data['X3'], linestyle='none', marker='o', label = 'x3')
plt.plot(data['X4'], linestyle='none', marker='o', label = 'x4')
plt.plot(data['X5'], linestyle='none', marker='o', label = 'x5')
# place the legend outside the plot
plt.legend(bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
plt.show()
```



Point 1

Design a Xbar-S control chart for the process.

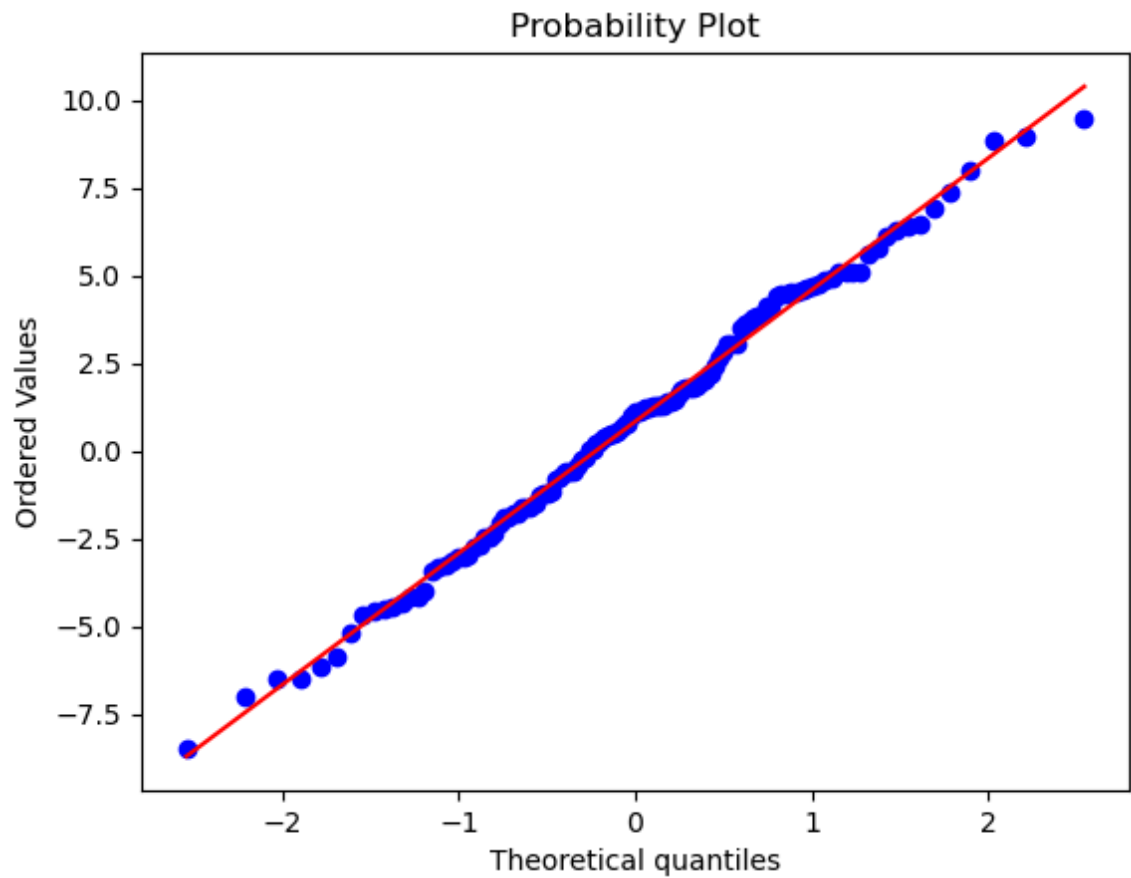
Check if the data is normally distributed.

```
In [ ]: # Stack the data into a single column
data_stack = data.stack()

# Check the normality assumption
# We can use the Shapiro-Wilk test
_, p_value_SW = stats.shapiro(data_stack)
print('p-value of the Shapiro-Wilk test: %.3f' % p_value_SW)

# QQ-plot
stats.probplot(data_stack, dist="norm", plot=plt)
plt.show()
```

p-value of the Shapiro-Wilk test: 0.841

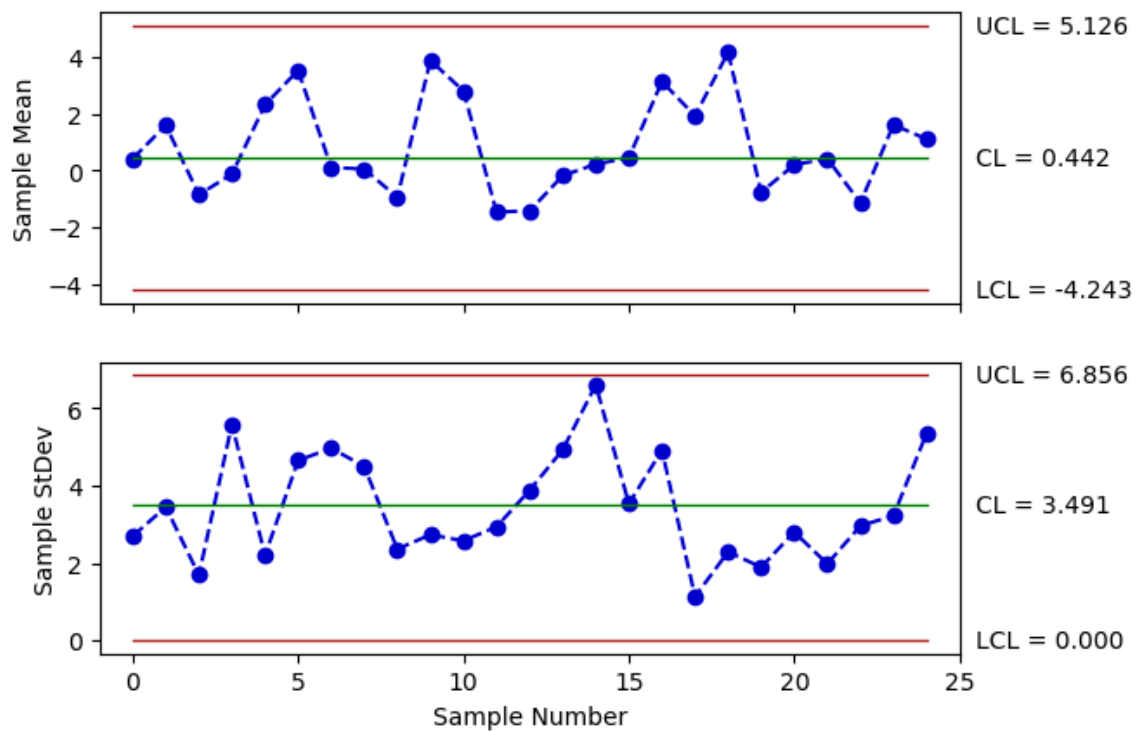


No information is given about the acquisition order of the data. Randomness is only qualitatively assessed from the scatter plot. Let's design an Xbar-S control chart for the process.

```
In [ ]: # Input the known mean and standard deviation
mean = 0.4417
stdev = 3.4914
n = 5

data_XS = qda.ControlCharts.XbarS(data, mean = mean, sigma = stdev)
```

Xbar-S charts



The process is in control.

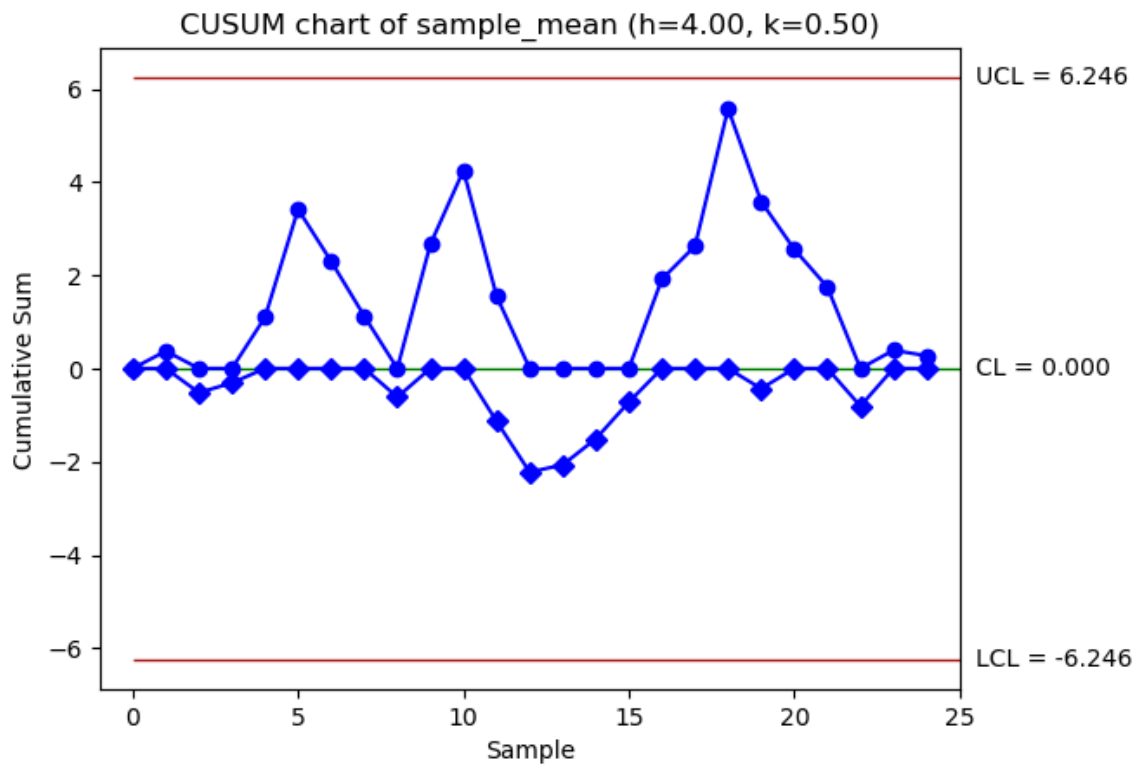
Point 2

Design a CUSUM control chart ($h = 4, k = 0.5$) and a CUSUM FIR chart.

```
In [ ]: # input the parameters of the CUSUM control chart
h = 4
k = 0.5

# extract the sample_mean column from data_XS and put it in a new dataframe
data_mean = pd.DataFrame(data_XS['sample_mean'])

# Design the CUSUM control chart
data_CUSUM = qda.ControlCharts.CUSUM(data_mean, 'sample_mean', params=(h,k), mea
```

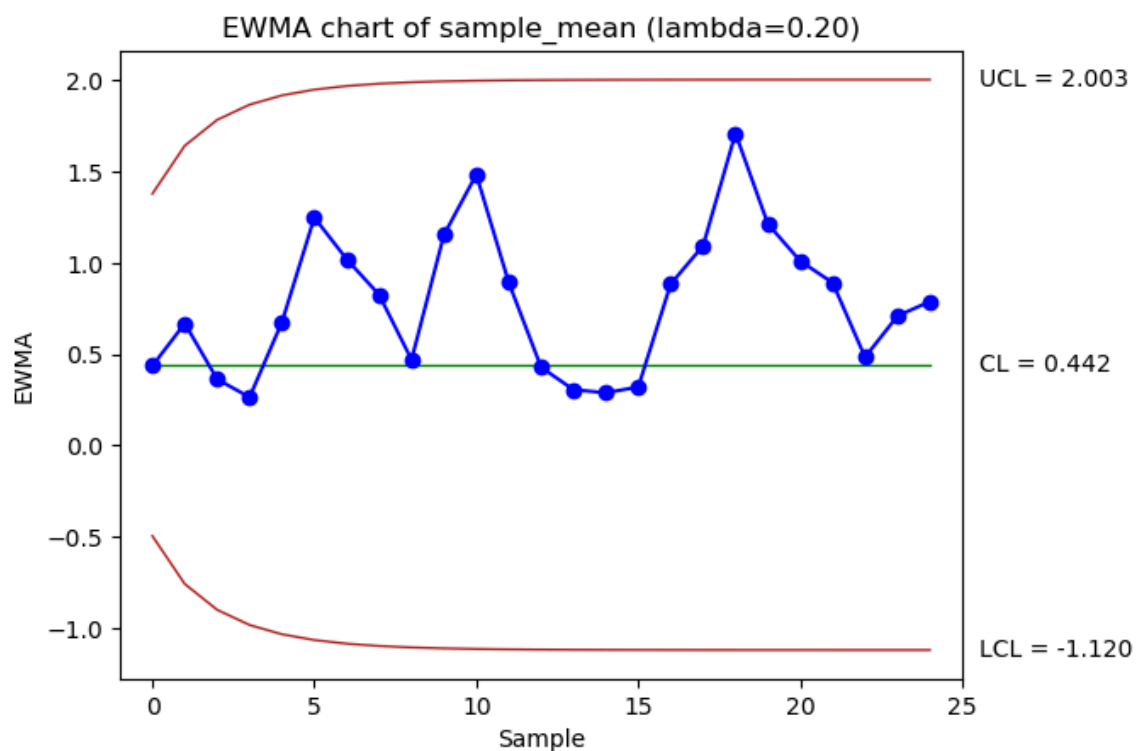


Point 3

Design an EWMA control chart ($\lambda = 0.2$).

```
In [ ]: # Input the parameters of the EWMA control chart
lambda_ = 0.2

# Design the EWMA control chart
data_EWMA = qda.ControlCharts.EWMA(data_mean, 'sample_mean', params=(lambda_), m
```



Exercise 2.1

Import the 5 additional samples that were collected in phase 2. Determine if the process is still in control using the three control charts.

```
In [ ]: # Import the dataset
data_p2 = pd.read_csv('ESE09_ex2_phase2.csv')
data_p2.head()
```

```
Out[ ]:
```

	X1	X2	X3	X4	X5
0	3.8871	-0.8182	3.6395	4.5988	8.86949
1	3.9634	-5.1245	1.6469	5.7935	2.23434
2	3.2074	0.2740	4.4500	-4.2358	5.71082
3	6.7545	9.4610	6.6967	-0.3139	0.92709
4	-0.2426	3.7593	7.0754	7.5853	-2.80609

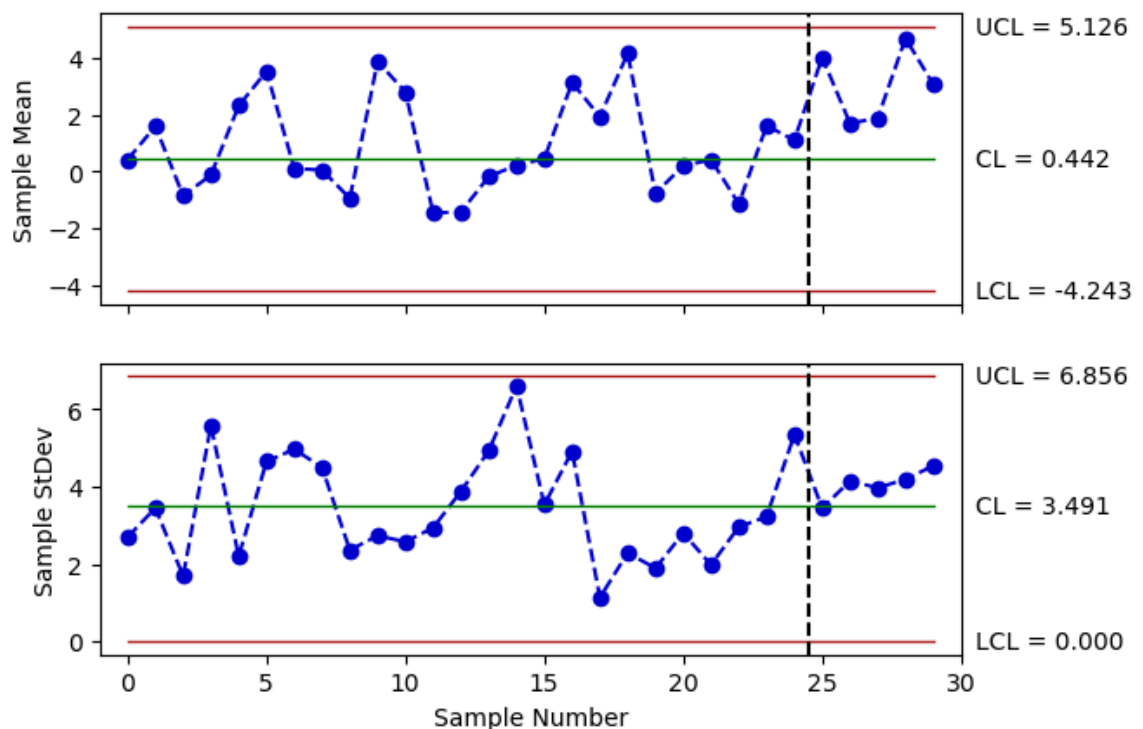
```
In [ ]: phase1_size = len(data)
print(phase1_size)

25
```

```
In [ ]: data_all = pd.concat([data, data_p2], ignore_index=True)
```

```
In [ ]: data_XS = qda.ControlCharts.XbarS(data_all, mean = mean, sigma = stdev, subset_s
```

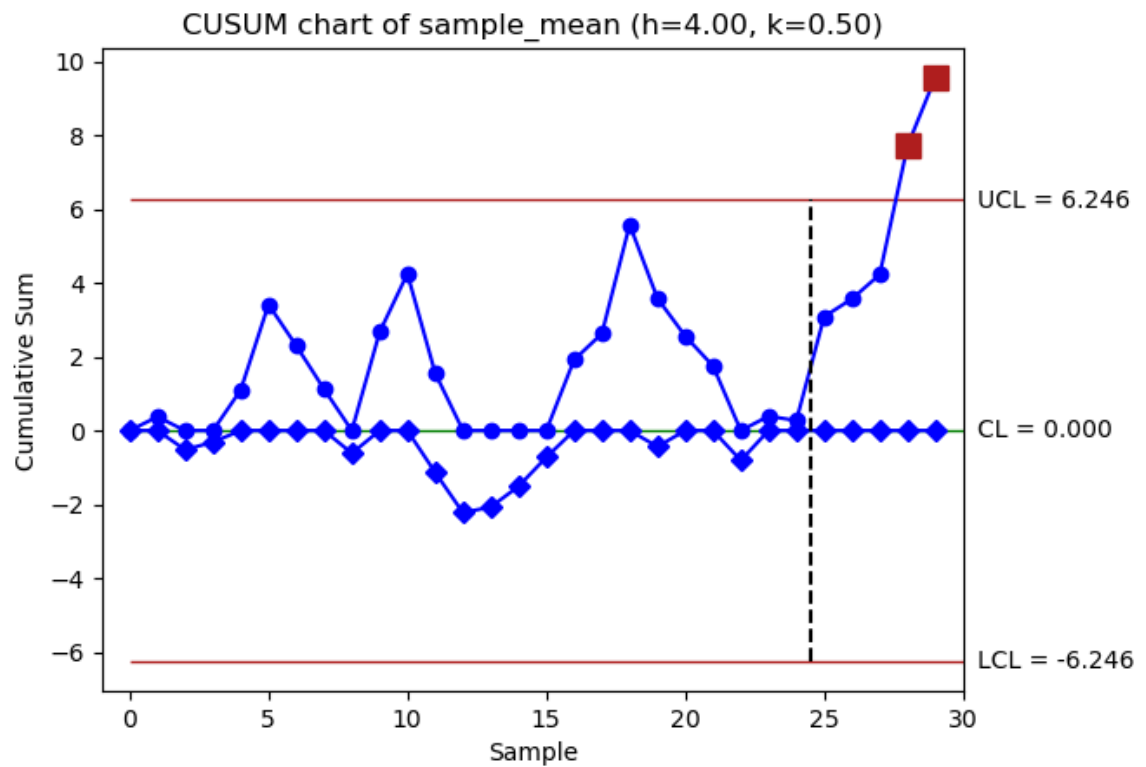
Xbar-S charts



```
In [ ]: # extract the sample_mean column from data_XS and put it in a new dataframe
data_mean = pd.DataFrame(data_XS['sample_mean'])
```

```
# Design the CUSUM control chart
```

```
data_CUSUM = qda.ControlCharts.CUSUM(data_mean, 'sample_mean', params=(h,k), mea
```



```
In [ ]: data_EWMA = qda.ControlCharts.EWMA(data_mean, 'sample_mean', params=(lambda_), m
```

