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POLITECNICO DI MILANO

BUSINESS & INDUSTRIAL ECONOMICS: Wrap-up - Fundamentals of Economics

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Demand and price elasticity

Demand curve

The demand curve tells the **quantity** buyers wish to purchase at **various prices**

Law of Demand: the demand curve has a **negative slop** (when price falls, the quantity demanded increases)

Direct demand curve: given the price, how many units are demanded at that price?

- **Q*i*(p*i*,z)** quantity demanded of good *i*, is a function of price and other possible variables affecting the demand of good *i* (e.g., price of all other goods, income)

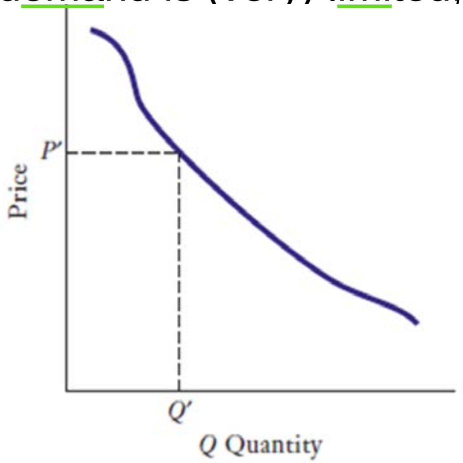
Inverse demand curve: given the quantity, what is the price at which that quantity is sold?

- ***Pi*(q*i*,z)** the price of good *i*, is a function of quantity and other possible variables

Implications: which is the % **decrease** in demand when a % **increase** in **price** occurs?

If the decrease in demand is (very) limited, an increase in price leads to an increase in firm's revenues

Direct: we have P
we need Q
Inverse: we have Q
we ned P



Wrapping up
- Law of demand
- Direct vs. inverse demand
- Implications for firms
- Exceptions
- Movement vs. shift in the demand

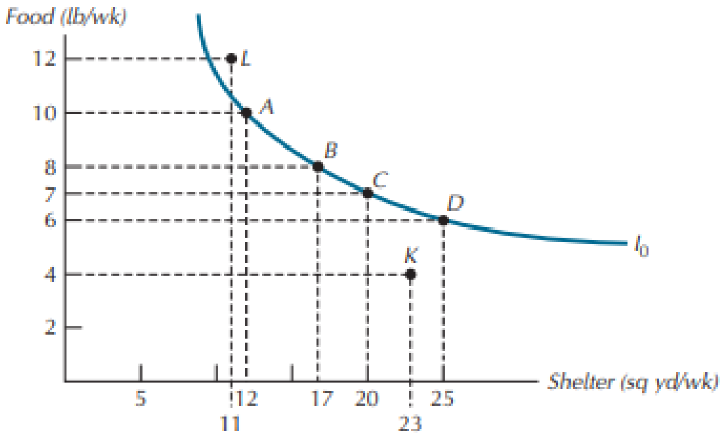
What is the inverse demand function of Q(P) = 240 – 2P?

IMPORTANT: in Module E you will need to master switching from direct to inverse demand and viceversa

Indifference curve map

Indifference curve show all combinations of two goods that give the consumer equal utility

- **Diminishing marginal utility:** indifference curves are convex as the utility decreases with extra units of the same good
- **Marginal rate of substitution (MRS):** slope of the indifference curve expressing the amount of one good the consumer is willing to trade for another



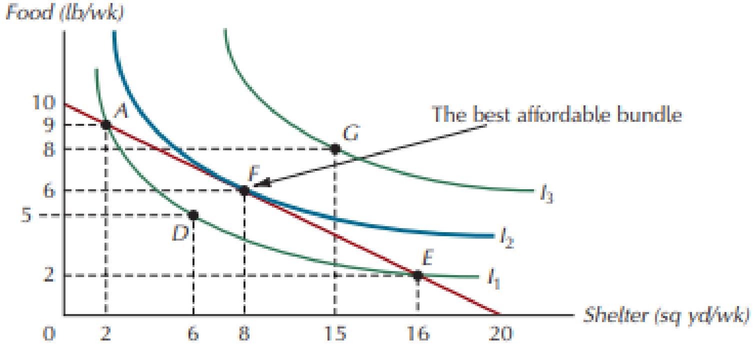
$$MRS = -\frac{\Delta F}{\Delta S} = -\frac{\frac{\partial U}{\partial q1}}{\frac{\partial U}{\partial q2}}$$

Budget line

$$p1q1 + p2q2 \leq Y$$

Combination of goods that can be afforded with the current income

- **Optimal bundle**: the consumer maximize utility where the highest indifference curve is tangential to the budget line
- **Marginal rate of transformation (MRT):** slope of the budget line, how many units of one good the consumer must give up to purchase more of the other given the current prices



$$MRS = -\frac{MU1}{MU2} = -\frac{p1}{p2} = MRT$$

Price elasticity: percentage change in the quantity of a good demanded that results from a 1 percent change in its price

$$\varepsilon = \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} \Rightarrow \frac{\Delta q}{\Delta p} \cdot \frac{p}{q} \Rightarrow \frac{\partial q}{\partial p} \cdot \frac{p}{q}$$

1. Elastic demand: |ε| > 1

- **High demand elasticity**: an **increase** in **price** causes a **sizable reduction** of **demand**

a lot change in quantity demand with little change in price like luxury goods

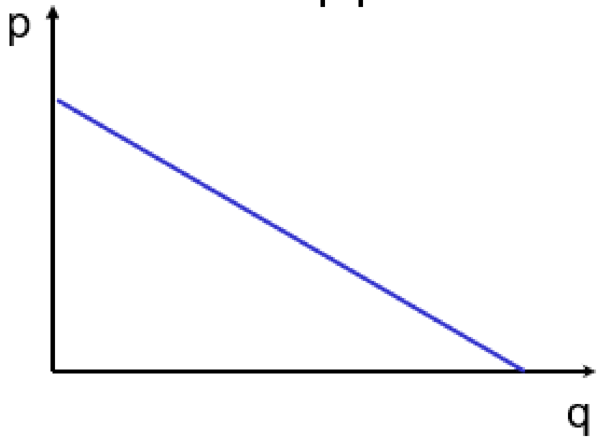
- **Flat** demand curve

2. Inelastic demand: |ε| < 1

- **Low demand elasticity**: an **increase** in **price** causes a **limited reduction** of **demand**
- **Steep** demand curve

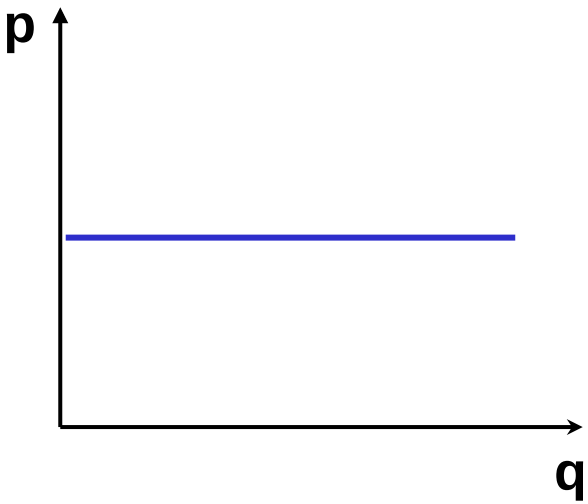
Price elasticity of demand (2/2)

Elastic $|\varepsilon| > 1$

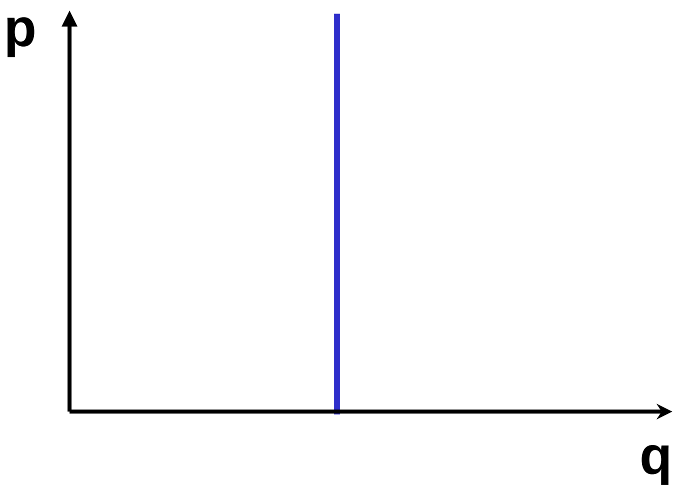


Inelastic $|\varepsilon| < 1$

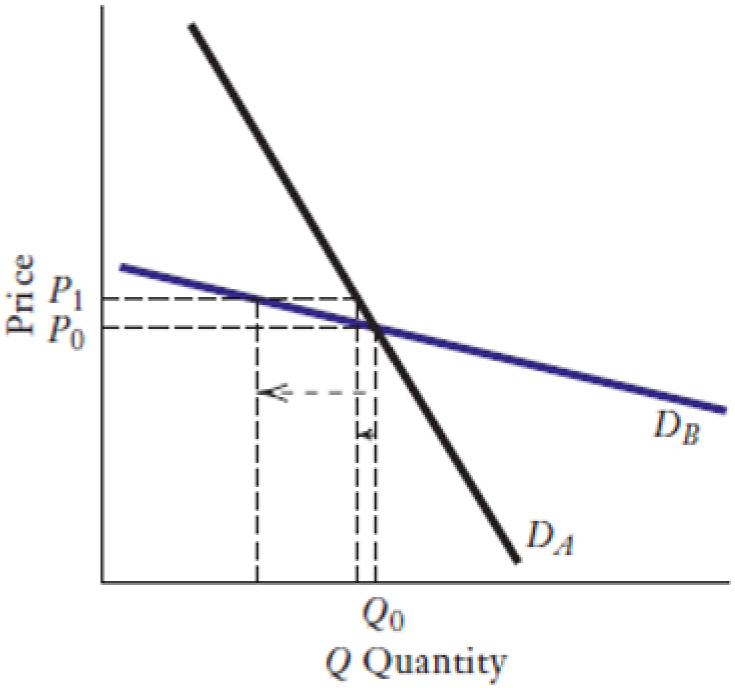
Perfectly elastic : $|\varepsilon| \rightarrow \infty$



Perfectly inelastic : $|\varepsilon| = 0$



Implications of the demand elasticity (1/3)



D

DA: inelastic demand curve

DB: elastic demand curve

The demand curve and its elasticity may influence the success of a firm's pricing decisions

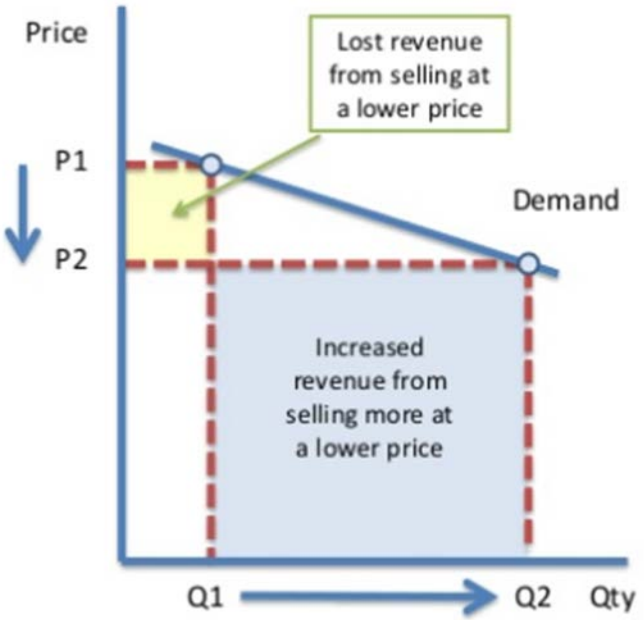
- Pricing strategies
- Sales forecasting

Take away message:

Firms aim to reduce the demand elasticity of their products because, in case of an inelastic demand, increases in prices lead to increases in revenues

Implications of the demand elasticity (2/3)

Elastic demand curve

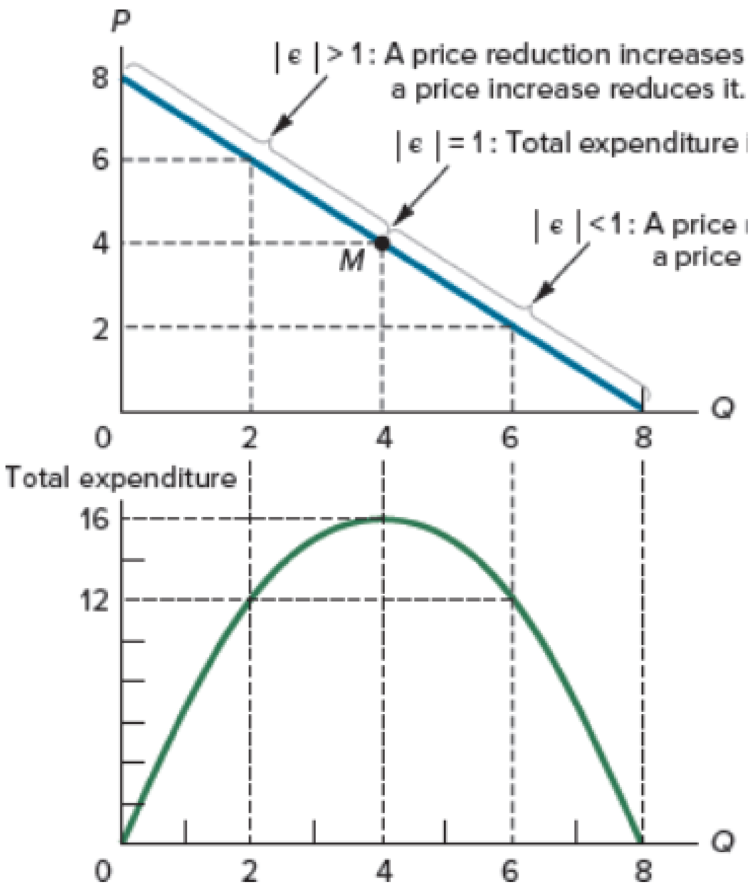


A decrease in the price will lead to an increase in the revenues generated from selling the product

Inelastic demand curve

A rise in the price will lead to an increase in the revenues generated from selling the product

Implications of the demand elasticity (3/3)
Relationship between price elasticity and total expenditures



Factors influencing price elasticity of demand

- Characteristics of the product
- Existence of close substitutes
- Proportion of the expenditure for the good
- Customers' loyalty to the product
- Length of time period considered
- Peak and off-peak demand

Interpretation of the graph:
Total expenditure starts at zero when Q is zero and increases to its maxmum value at the quantity corresponding to the midpoint of the demand curve (M)

How firms reduce the demand elasticity?

It is possible to **reduce** the **demand elasticity** of a product by **differentiating** it from **other products** (of competitors and/or of the firm)

Product differentiation is

- Conducive to **market power**: it allows setting high prices without losing customers
- A widespread **competitive strategy**
  - **Real differentiation:** by changing the real characteristics of the product, for instance through innovation
  - **Perceived differentiation:** by changing consumers' perception of the characteristics of the product, for instance through advertising

Cross price elasticity of demand

Cross price elasticity: percentage change in the quantity demanded of one good that results from a 1 percent change in the price of the other

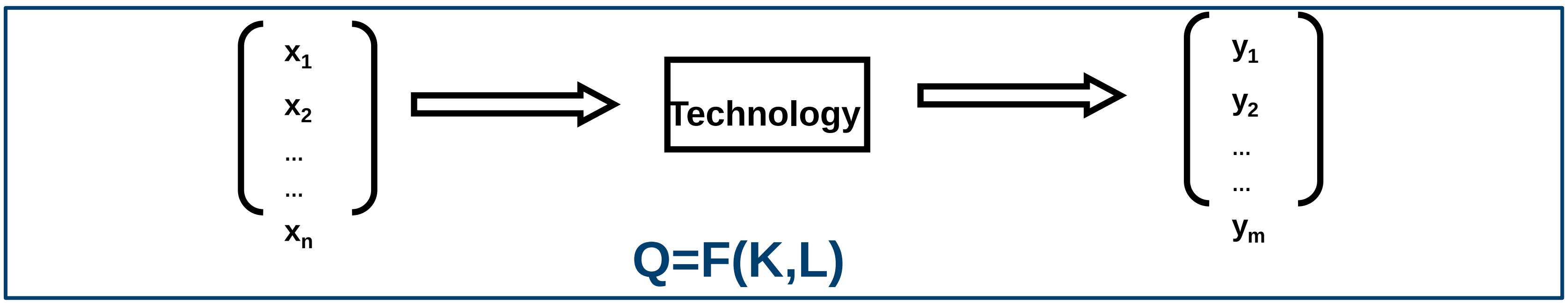
$$\epsilon_{xz} = \frac{\Delta Q_x / Q_x}{\Delta P_z / P_z}$$

- The cross price elasticity can be either positive or negative
- Good x and z are complements if $\in_{xz} < 0$
- Good x and z are substitute if $\in_{xz} > 0$

Technology and production

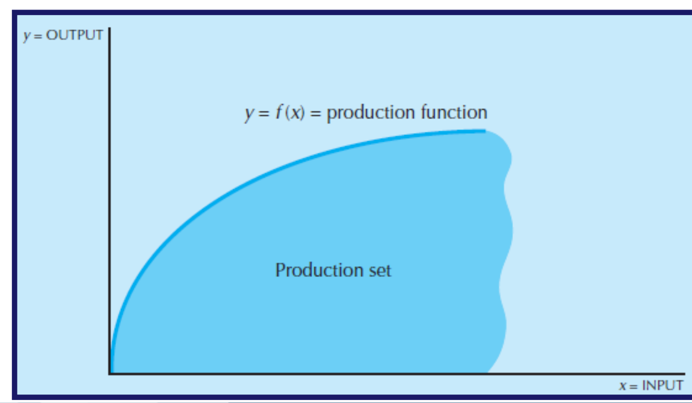
When choosing quantities to produce and prices to charge, firms face **constraints**

- Customers (**demand**), competitors (**market structure**), nature of the production process (**technology**)
  - **Technology**: the set of processes that a firm can use to turn a vector X of **N inputs** into a vector Y of **M outputs**



Production set: the set of all combinations of inputs and outputs that are technologically feasible

Production function: the maximum possible outputs from a given vector of inputs. It is the boundary of the production set



Marginal Product (MP)
Production in the short run

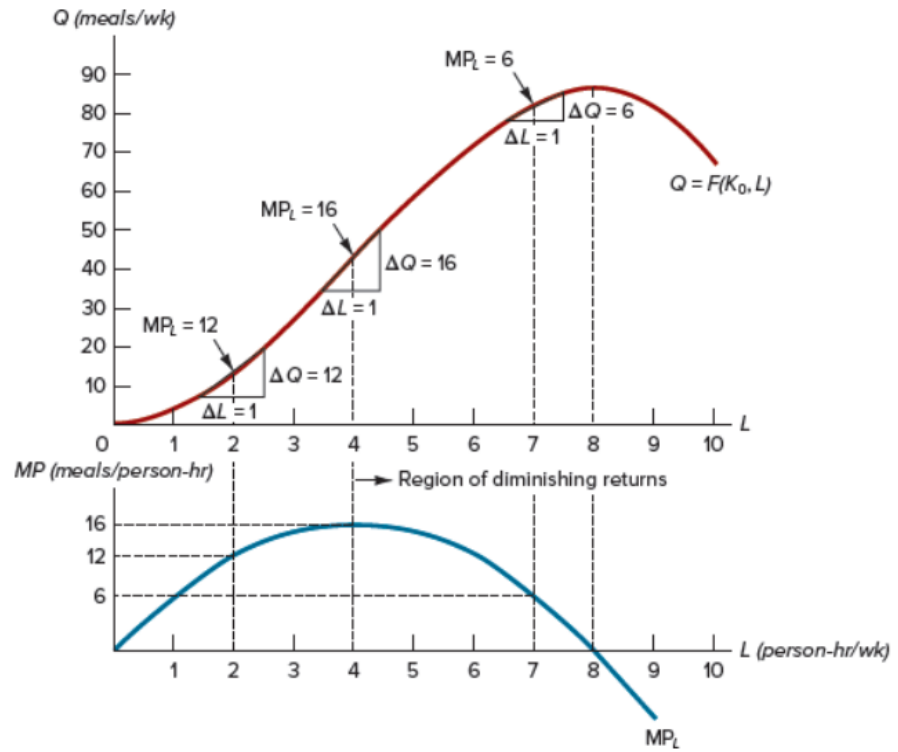
The **marginal product (MP)** of an input is the **output variation** due to the variation of **1 unit of this input**, holding all other inputs constant:

$$MP_i = \frac{\partial y}{\partial x_i}$$

Rate of change of output as the level of input *i* changes

Law of diminishing marginal product (common feature of many production processes): the marginal product of an input decreases as the level of that input increases, holding other inputs constant. In general

$$\frac{\partial MP_i}{\partial x_i} = \frac{\partial^2 f(x_i, x_j)}{\partial x_i^{\,2}} < 0$$

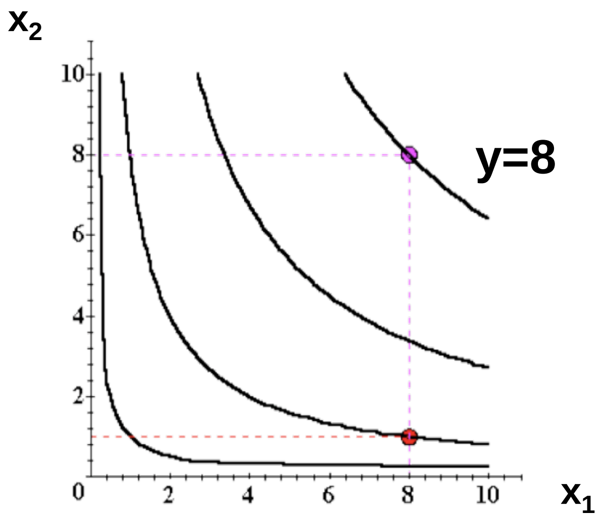


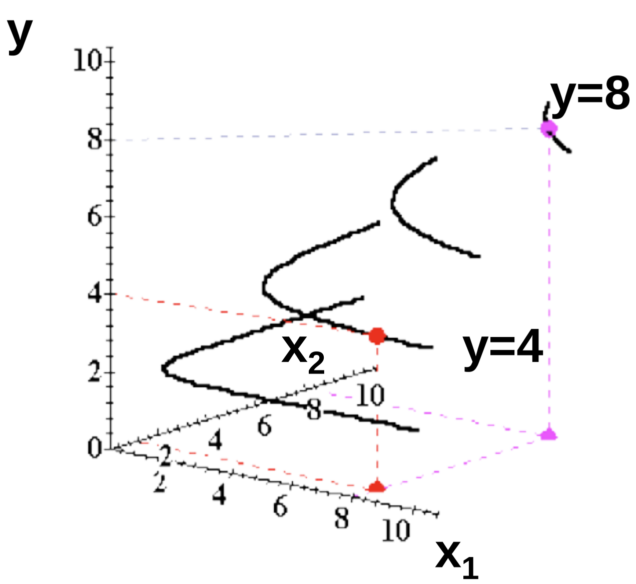
1. If there is 1 input x and 1 output y, the production function is **y=f(x)**

2. With 2 inputs, x_1 and x_2, and 1 output y, the production function is **$y=f(x_1, x_2)$**

Isoquant: set of all input bundles that produce the same output level *y*

- Isoquants can be graphed both in a 2D plan or in 3D plan

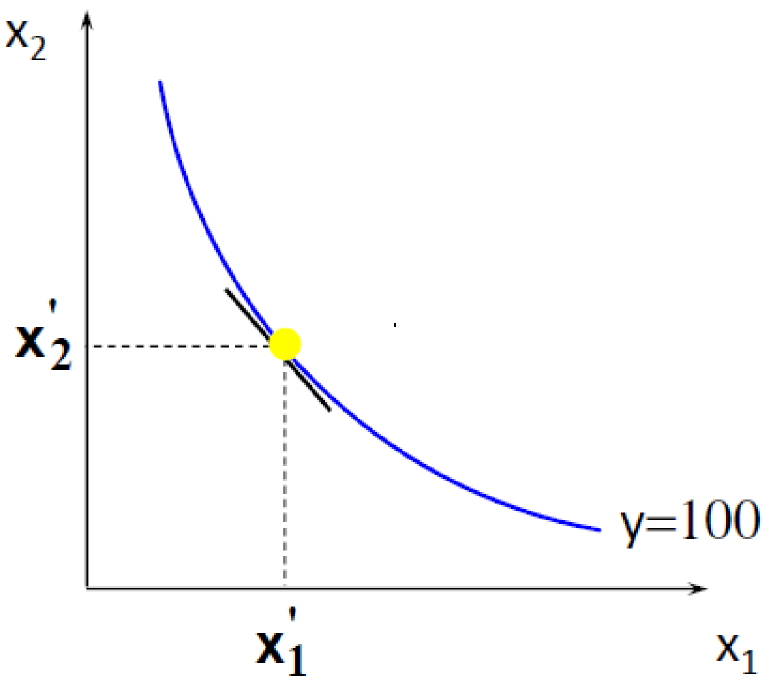




Technical Rate of Substitution (TRS)

TRS measures the **trade-off between the two inputs**

- It is the rate at which input 2 can be substituted with input 1 to keep the **output level constant**
  - Graphically, it is the **slope** of the isoquant
- TRS: the amount by which the quantity of one input has to be reduced ($-\Delta x_2$) when one extra unit
- of another input is used ($\Delta x_1=1$), s
- that output remains constant



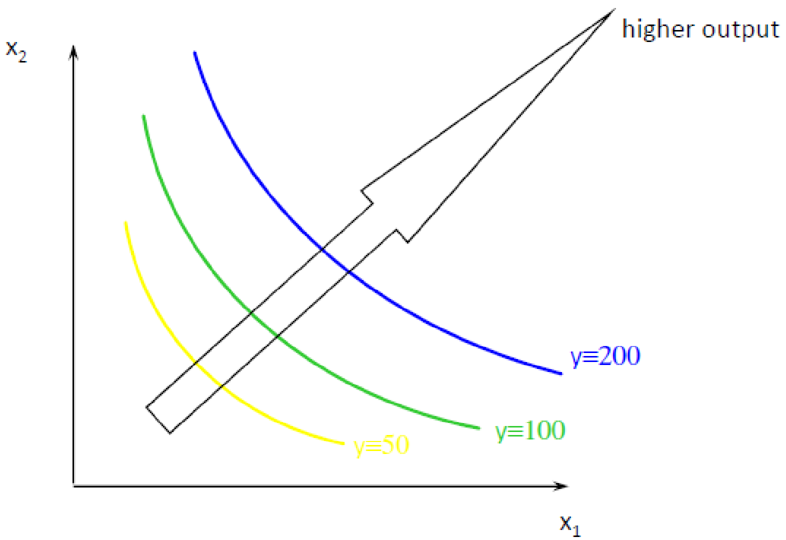
$$\begin{aligned} dy &= \frac{\partial y}{\partial x_1}dx_1 + \frac{\partial y}{\partial x_2}dx_2 \\ &= MP_1 dx_1 + MP_2 dx_2 = 0 \end{aligned}$$

$$\Rightarrow = \frac{dx_2}{dx_1} = \left|TRS(x_1, x_2)\right| = \left|\frac{MP_1}{MP_2}\right|$$

Assumption of **diminishing TRS**: the slope of the isoquant decreases (in absolute value) when moving to the right along the isoquant

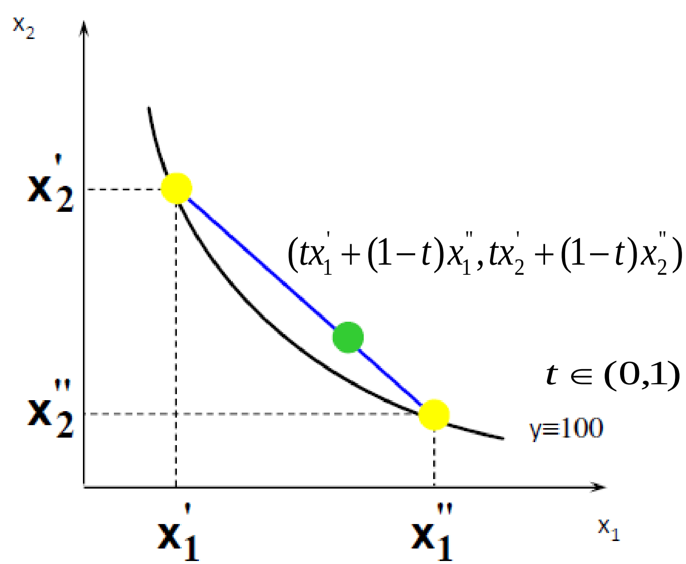
Monotonicity

Production increases if one input increases, while the other input stays constant



Convexity

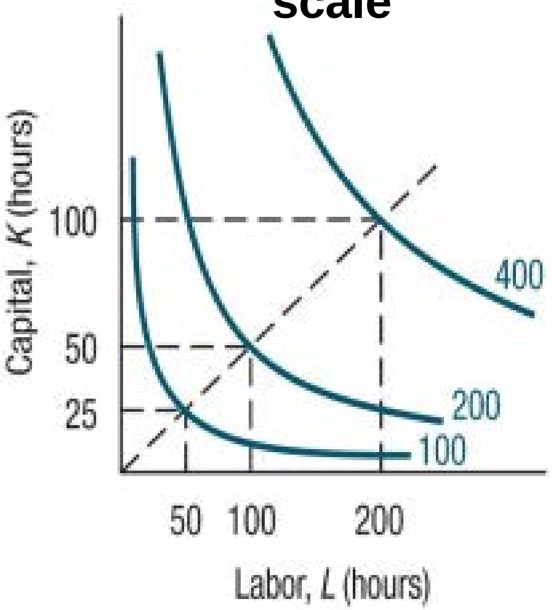
If two bundles of inputs, (x1', x2') and (x1'', x2'') both produce y units of output, then their weighted average produces at least y units of output



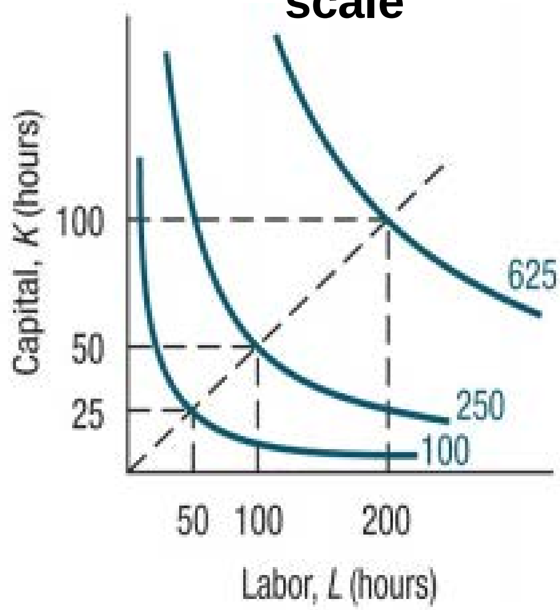
Returns to Scale (1/2)

- **Marginal products** describe the change in output as **one input** changes
- **Returns to scale** describe how output changes as **all inputs change** in the **same proportion** (*e.g.*, all inputs double, or halve)
  - → Returns to scale may vary along the production function
  - → It is a long run concept

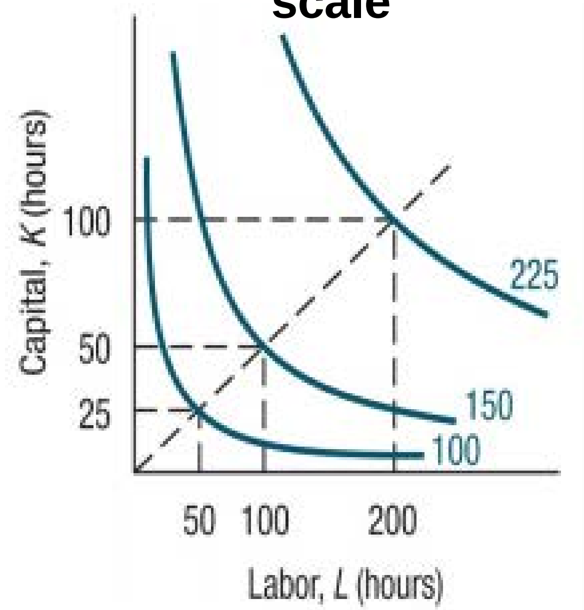
Constant returns to scale



Increasing returns to scale

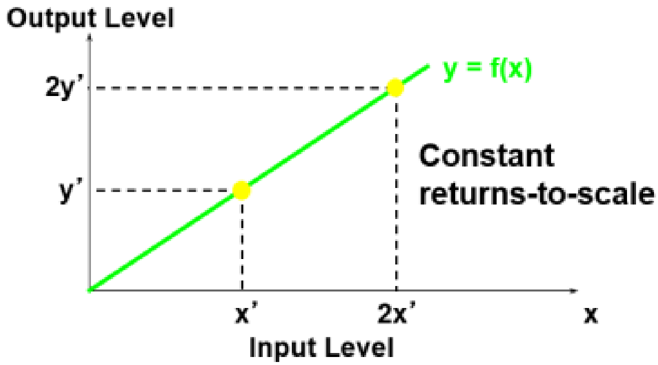


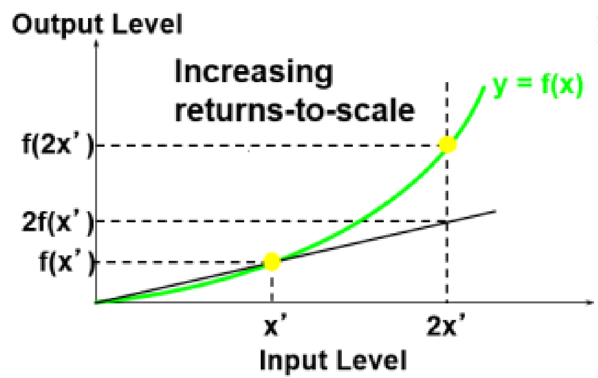
Decreasing returns to scale

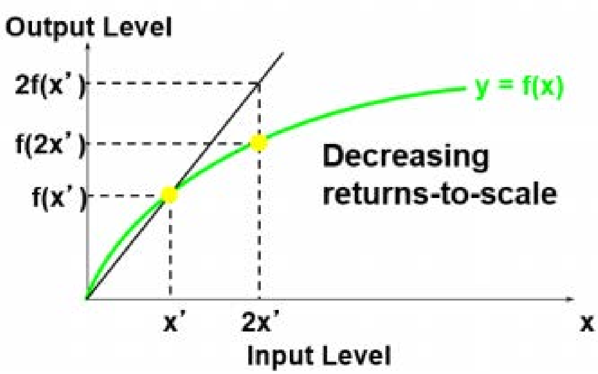


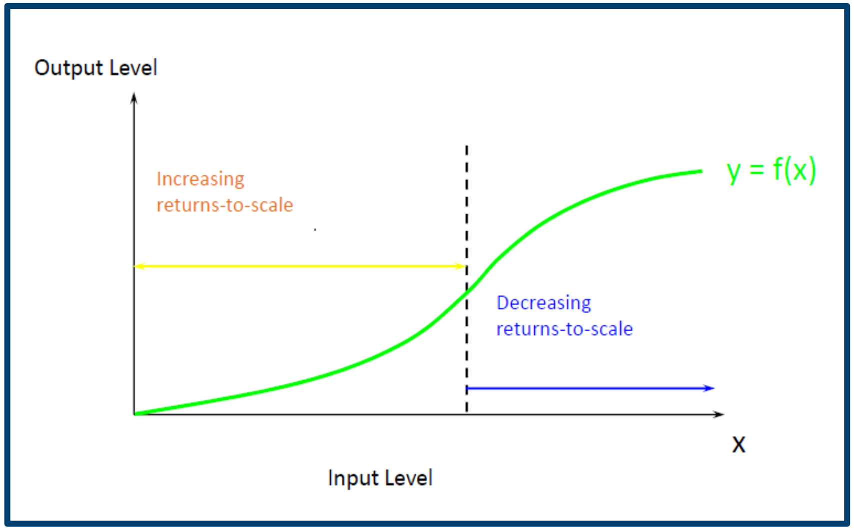
Returns to scale (2/2)

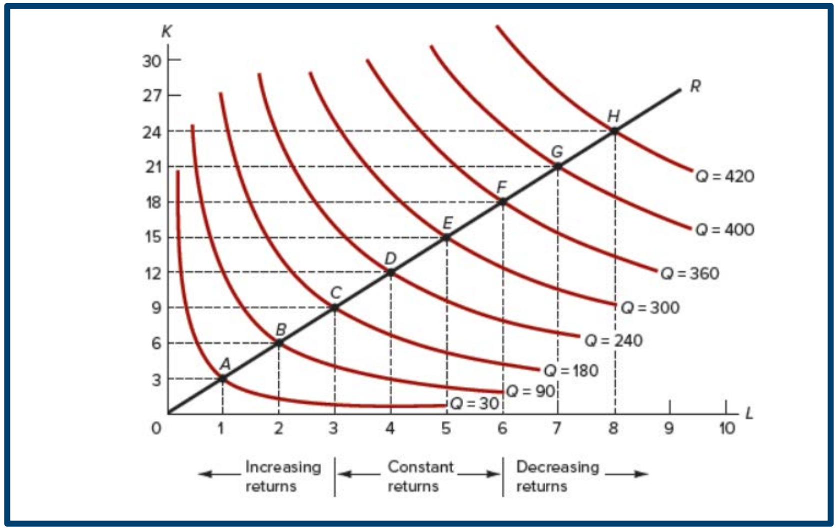
A single technology can locally exhibit different returns-to-scale











Cost functions

Cost Functions: Taxonomy

In the **short run**, we have

- Total cost function: C(y)
- Fixed cost function: FC
- Variable cost function: VC(y)
- Average total cost function: ATC(y)
- Average variable cost function: AVC(y)
- Average fixed cost function: AFC(y)
- Marginal cost function: MC(y)

In the **long run**, we have

- Average cost function: AC(y)
- Long-run marginal cost function: LMC(y)

How do these cost functions relate to each other?

- Specifically, how do long-run and short-run cost functions relate?

Note: y is quantity produced also called level of output, thus you will also see costs function expressed as C(q)

IMPORTANT: refreshing the basic of the cost functions will help you understanding the concepts of Module E

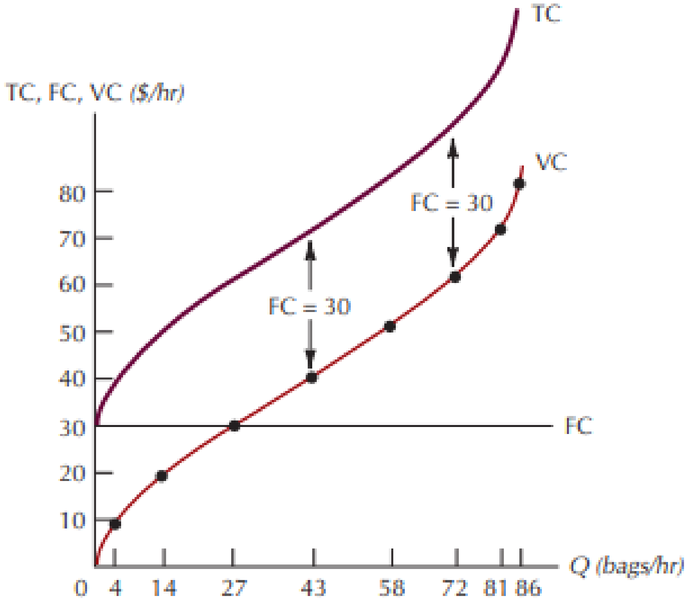
Short run: Total, variable, and fixed costs

- **Total cost function, C(y)**: is the minimum cost of all inputs (fixed and variable) when producing y units of output

- **Fixed cost function, FC**: is the cost of inputs, which are fixed in the short run. FC does not vary with the firm's output

- **Variable cost function, VC(y):** is the cost of variable inputs when producing y units of output. It varies with the firm's output and depends on the level of the fixed input

$$\boxed{C(y) = F + VC(y)}$$



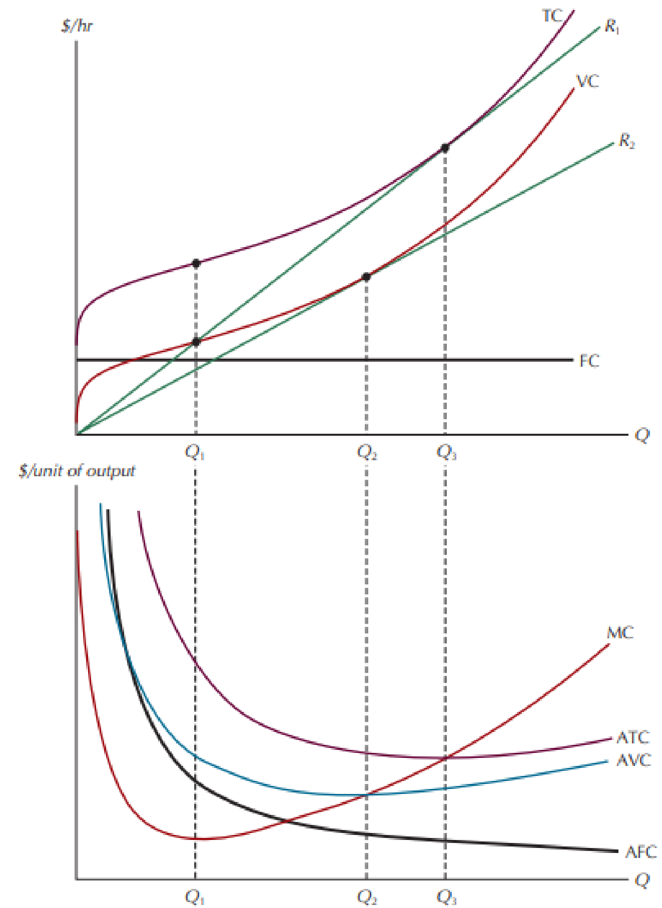
Can you provide some examples of fixed costs? And of variable costs?

Short run: Average costs

- **Average total cost function, AC(y)**: is the **total cost of each unit** of output

$$\boxed{AC(y) = \frac{C(y)}{y} = \frac{F}{y} + \frac{VC(y)}{y} = AFC(y) + AVC(y)}$$

- **Average variable cost function, AVC(y):** is the variable cost of each unit of output
- AVC(y) increases with output. If the MP is
  - Increasing, VC(y) increases at a decreasing rate as y increases: AVC(y) is decreasing
  - Decreasing, VC(y) increases at an increasing rate as y increases: AVC(y) is increasing
- Usually, along the production function, the MP is first increasing and then decreasing → Thus, AVC(y) is U-shaped, AC(y) likewise
- **Average fixed cost function, AFC(y):** is the fixed cost of each unit of output. It decreases as output increases

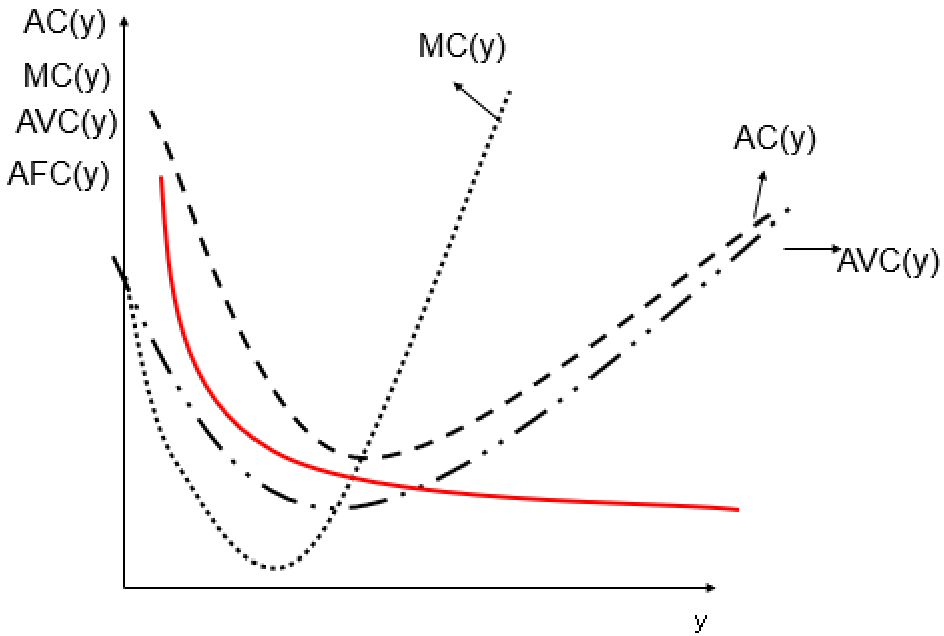


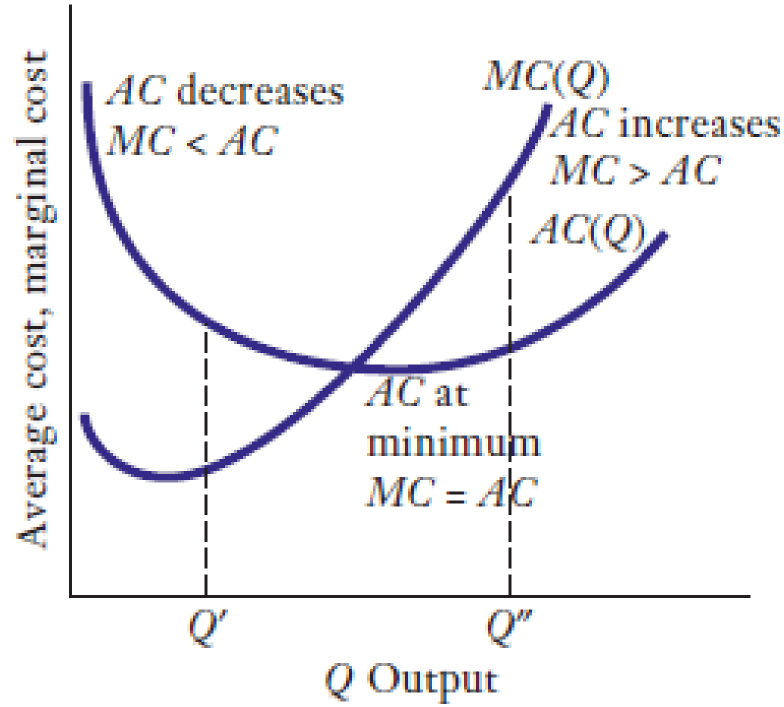
- **Marginal cost function, MC(y)**: is the **change in the** (total/variable) **cost** associated to a **unitary change in output**
- Considering **infinitesimal changes,** and noting that fixed cost *FC* does not vary with the output level *y*, we have

$$\boxed{MC(y) = \frac{\partial C(y)}{\partial y} = \frac{\partial VC(y)}{\partial y}}$$

- In other words, **MC(y)** is the **derivative of total cost with respect to quantity**, which is equivalent to the derivative of the variable cost
- It gives the **slope** of the C(y) and VC(y) curves as output changes

Average and Marginal Costs Functions: Graphical representation





- AC decreases when MC<AC
- AC increases when MC>AC
- AC is the same when MC=AC

Long-Run: Total cost curves

Suppose that there are 3 possible levels of input *2*

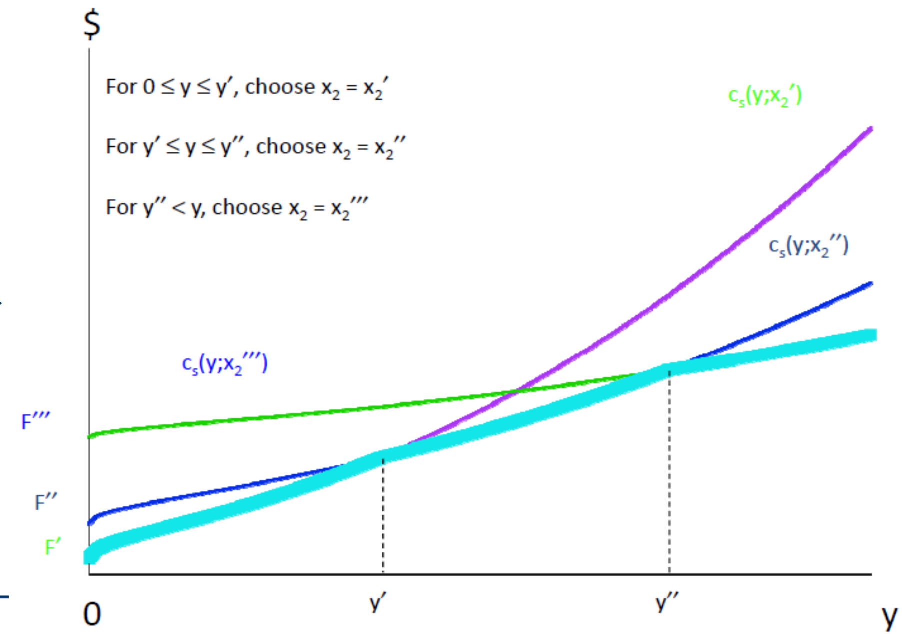
$$x_2 = x_2', \; x_2 = x_2'', \; x_2 = x_2''' \text{ with } x_2' < x_2'' < x_2'''$$

In the short run, the firm can choose **just one of them** and has a **different short-run total cost curve** for each possible level of input 2 (and thus of fixed costs)

A larger amount of the fixed input increases the fixed cost (F'<F''<F''')

In the long-run, the firm can choose among the 3 levels of input 2, depending on the quantity *y* to be produced. The result is the long-run total cost curve (light blue line)

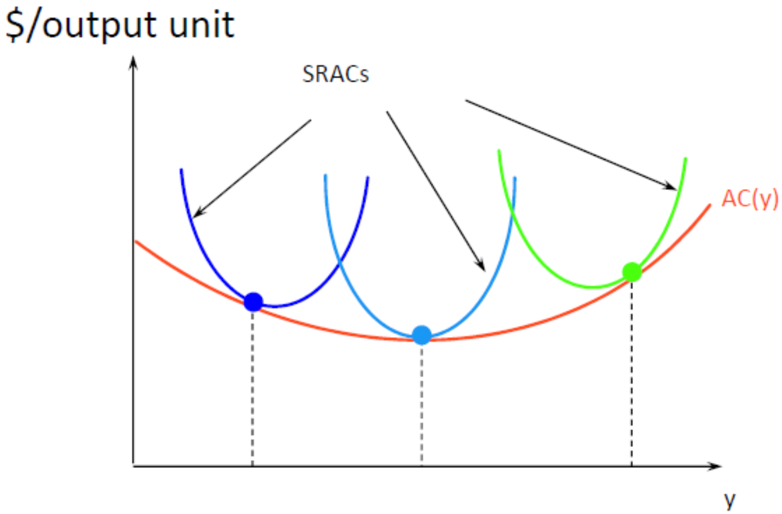
The firm's **long-run total cost curve** is the **lower envelope** of the short-run total cost curves

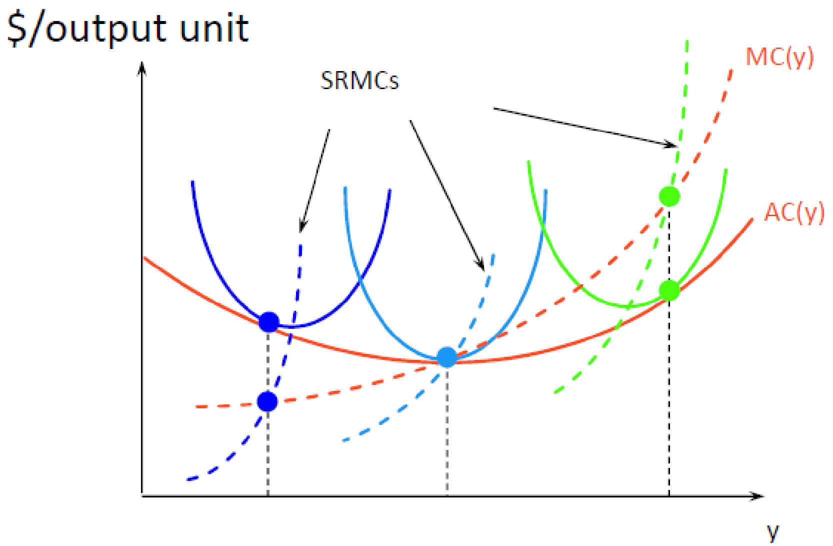


Long-Run: Average and marginal cost curves

For **any output level *y***, the long-run total cost curve always gives the **lowest possible total production cost**

- Similarly, the **long-run average total cost curve** gives the **lowest possible average total cost** → The **long-run average total cost curve** is the **lower envelope** of all the short-run average total cost curves
- Similarly, the **long-run marginal cost curve** gives the **lowest possible marginal cost** → The **long-run marginal cost curve** is the **higher envelope** of all of the short-run marginal cost curves





Profit maximization and cost minimization

IMPORTANT: see Cabral (2017) Chapter 3 explaining the calculus behind profit maximization function (Chapter available on WeBeep, folder “additional readings”)

Profit Maximization

- A firm profits (π) are given by the difference of revenues minus costs of production

$$\pi=\sum_{i=1}^{n} p_i\, y_i \;-\sum_{i=1}^{m} \omega_i\, x_i$$

- In order to maximize profits, the firm needs to choose the amount of output to produce and the production plan to employ
  - A firm uses inputs $x = 1, 2, \cdots, m$ to make products $i = 1, 2, \cdots, n$
  - Output levels are $y_1, \cdots y_n$
  - Input levels are $x_1, \cdots x_m$
  - Product prices are $p_1, \cdots p_n$
  - Input prices are $\varpi_1, \cdots \varpi_m$
- We will study the profit-maximization problem of a firm that faces competitive markets for both factors of production and output
- The competitive firm takes all output prices $p_1, \cdots p_n$ and all input prices $\varpi_1, \cdots \varpi_m$ as given constants

IMPORTANT: you need to refresh the basics of profit maximization and cost minimization to properly understand the concept of Module E

Profit Maximization in the Short Run

- Profit function in the short run is $\boxed{\pi = py - w_1 x_1 - w_2 \bar{x}_2}$
- In the short run, input 2 is **fixed**
- Consider a firm which produces output *y* using the production function **y=f(x$_1$, x$_2$)**, where input 2 is fixed at a certain level $\overline{x_2}$

With *p* the product price and ϖ_1 and ϖ_2 the input prices, the profit maximization problem of the firm is

$$\boxed{\max_{x_1} \; p\underbrace{f(x_1, \bar{x}_2)}_{y=f(x_1,\, x_2)} - \omega_1 x_1 - \omega_2 \bar{x}_2}$$

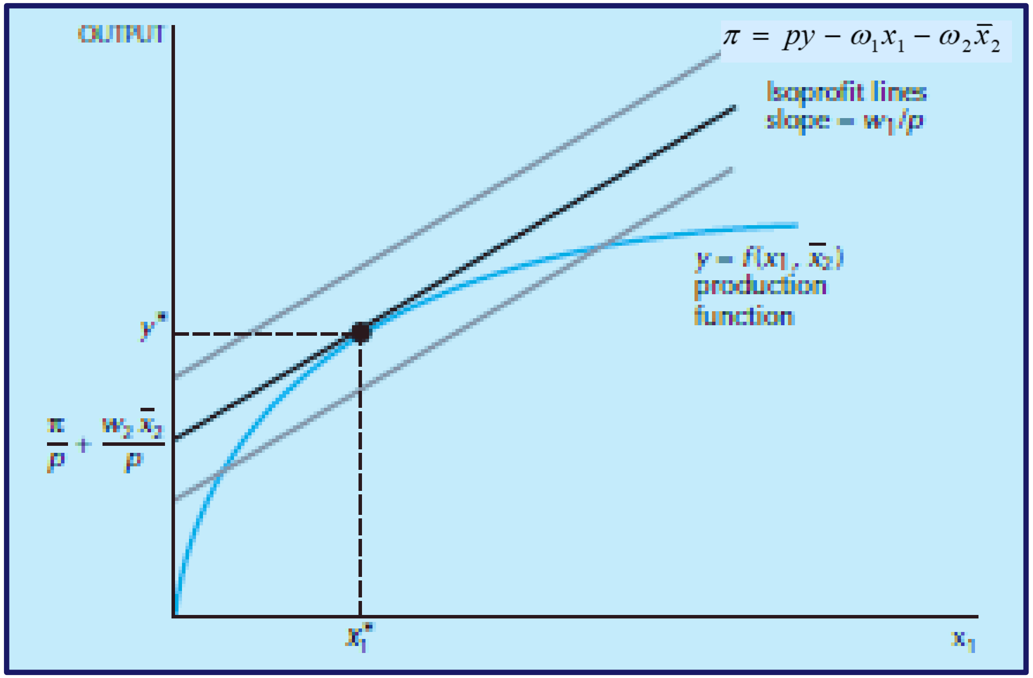
Applying the first-order condition we have:

$$\boxed{p\frac{\partial f(x_1, \bar{x}_2)}{\partial x_1} - w_1 = 0}$$

The firm maximizes profit by choosing the level of input 1 (and producing the level of output) at which the marginal revenue product of input 1 equals its price (e.g., wage of input 1)

$$\boxed{pMP_1(x_1^*, \bar{x}_2) = \omega_1}$$

- We can solve the problem **graphically**
- Solving for y from the (short-run) profit function, we obtain the equation describing the **isoprofit lines** representing all the combinations of output (*y*) and input (x_1) that give a constant level of profit



$$\pi = py - \omega_1 x_1 - \omega_2 \bar{x}_2$$

$$y = \frac{\pi}{p} + \frac{\omega_2}{p}\bar{x}_2 + \frac{\omega_1}{p}x_1$$

$$MP_1 = \frac{\omega_1}{p}$$

The slope of the isoprofit line equals the slope of the production function

Profit Maximization in the Long Run

- In the long run, the firm can choose the level of all inputs, thus the **profit maximization problem** is

$$\boxed{\max_{x_1, x_2} pf(x_1, x_2) - \omega_1 x_1 - \omega_2 x_2}$$

- Analogously to the short run case, the firm maximizes profits by choosing the level of inputs (and producing the level of output) at which

$$\boxed{\begin{aligned} pMP_1(x_1^*, x_2^*) &= \omega_1 \\ pMP_2(x_1^*, x_2^*) &= \omega_2 \end{aligned}}$$

In the long-run the value of the marginal product of each factors should equal its price

Cost Minimization in the long run

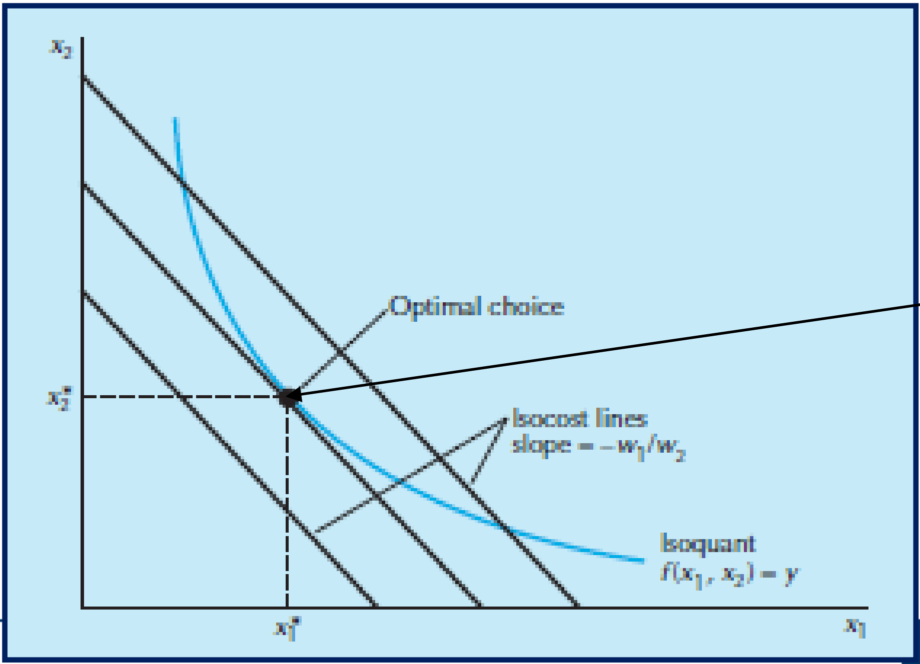
Key question: how to minimize the costs of producing a certain output *y*?

The **cost minimization problem** can be written as

$$\boxed{\begin{aligned}&\min_{x_1, x_2} \omega_1 x_1 + \omega_2 x_2 \\ &\textit{such that } f(x_1, x_2) = y\end{aligned}}$$

- The solution gives the **cost function**: **minimum cost** of producing *y*, **given input prices** $\boxed{c(\omega_1, \omega_2, y)}$

We obtain the solution **graphically**, by considering **isoquants** and **isocosts**, *i.e.*, set of all the input bundles having the same cost *C*



$$\boxed{\omega_1 x_1 + \omega_2 x_2 = C \Rightarrow x_2 = \frac{C}{\omega_2} - \frac{\omega_1}{\omega_2} x_1}$$

$$\boxed{\left| TRS(x_1^*, x_2^*) \right| = \frac{\omega_1}{\omega_2}}$$

Basics of market structures

Introduction

It is possible to define different **market structures** depending on

- The **number of firms** operating on a market
- The **ways in which these firms interact** when they make their pricing and output decisions
- Q produced depends on how firms behave in these different market structures

- **Perfect competition** → **benchmark**
- Monopoly
- Monopolistic competition
- Oligopoly

Imperfect competition

Perfect competition

In **perfect competition**

1. There are **many (infinite) firms on the market**, N→∞, where N is the number of firms → they are small relative to the market and unable to affect the market price

2. Firms produce **a homogeneous good**, having access to the same technology and thus having the same cost curves (i.e., firms sell a standardize product or substitute products) → thus they compete on price, the only relevant variable

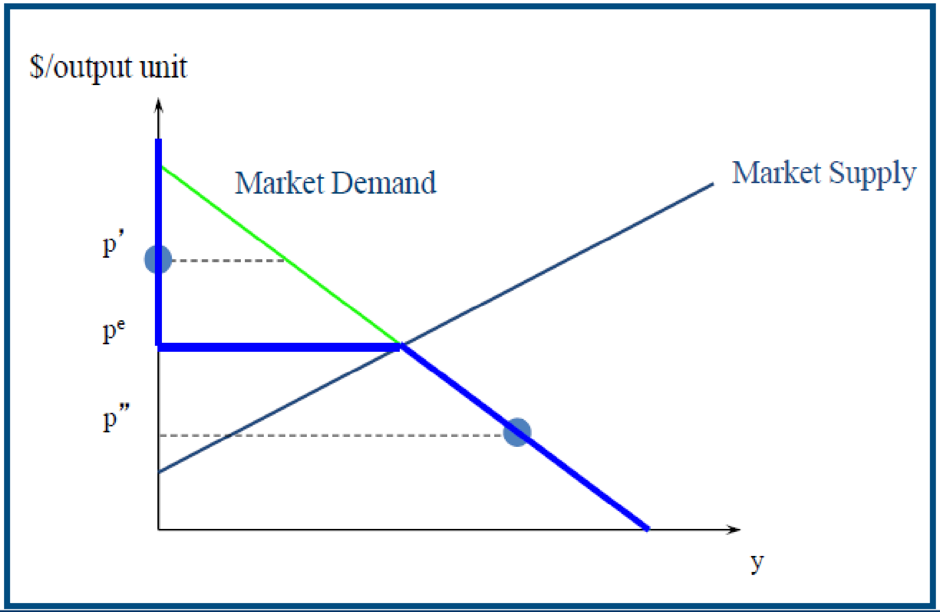
3. In consequence of **1** and **2**, firms have no influence over the market price → They are **price-takers**

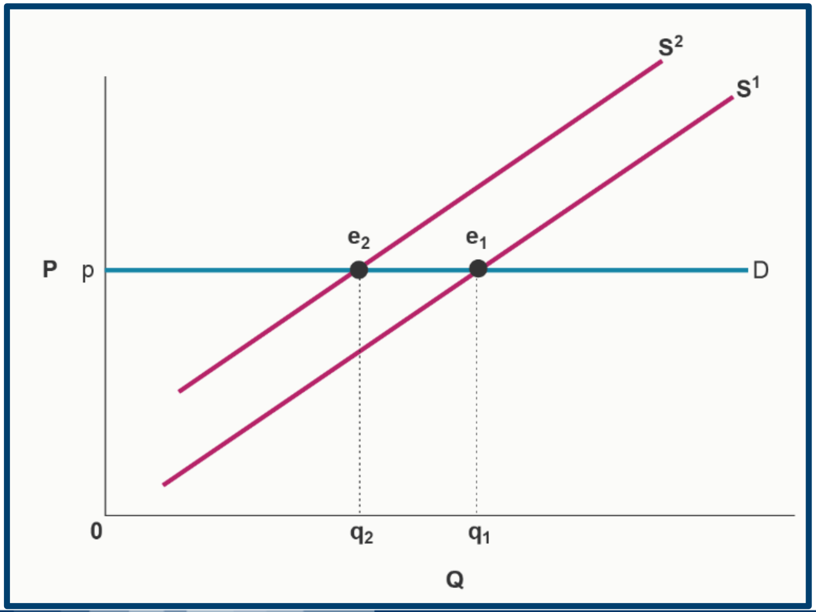
4. Firms can **entry and exit** the market at no cost, there are no barriers to the establishment of a new firm. If new firm enter, they incur in the same costs as the incumbent

5. **Perfect information** → all firms knows about prices and product characteristics

Perfect Competition
Demand Curve

- If ***p' > p^e***, the firm has no demand
- At the equilibrium price ***p^e***, market demand is equal to market supply
- If *p" < p^e*, the firm faces the entire market demand
- But it cannot serve it due to its very limited productive capacity
- **Atomistic hypothesis**: the firm's technology allows it to supply just a small part of the market demand
- If the price of one of the input goes up they still cannot charge a higher price for its product





Short Run: Profit Maximization Problem

- Given *p* (price-taking assumption), each firm offers the **quantity** that **maximizes** its **profit**

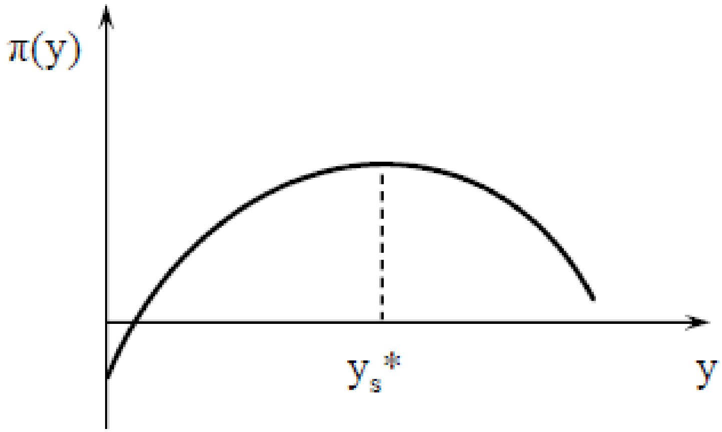
$$\max_{y\geq 0} \Pi(y) = R(y) - C(y) = py - C(y)$$

- **First** and **second order conditions** for profit maximisation

$$(i) \quad \frac{\partial \Pi(y)}{\partial y} = p - MC(y) = 0 \quad \Rightarrow \quad p = MC(y^*)$$

$$(ii) \quad \frac{\partial^2 \Pi(y)}{\partial y^2} = \frac{d}{dy}\left(p - MC(y)\right) = -\frac{dMC(y)}{dy} < 0 \quad at\ y = y_s^* \quad \Rightarrow \quad \frac{dMC(y^*)}{dy} > 0$$

The second order condition states that MC is increasing



The firm offers the quantity at which the MR of producing that quantity equals the MC

In perfect competition, marginal revenue is equal to price, thus p=MC

Short Run: the shut down

We have to **compare** the *y*>0* **solution with the no production case** (*y=0*)

- The firm's profit function is $\boxed{\Pi_s(y) = py - C(y) = py - F - VC(y)}$
- If the firm chooses *y = 0* then its profit is $\boxed{\Pi(y) = 0 - F - VC(0) = -F}$

- The firm chooses an output *y>0* only if

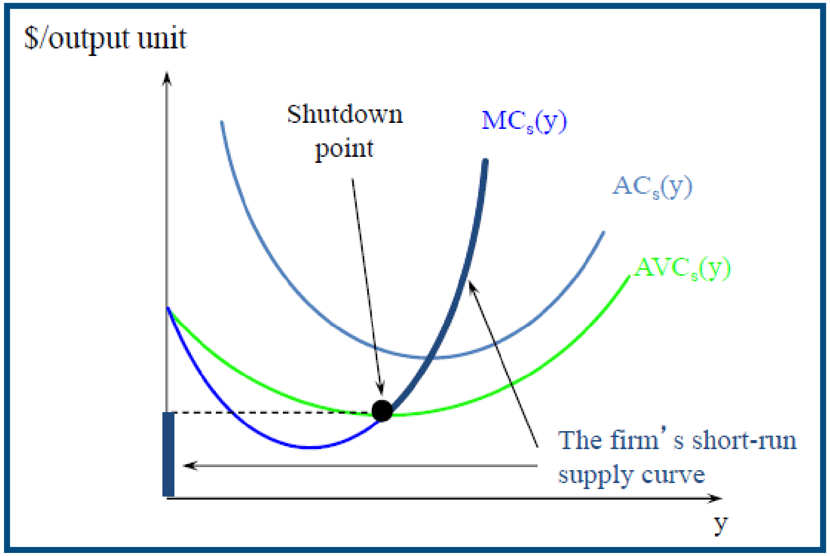
$$\boxed{\Pi(y) = py - F - VC(y) \geq -F} \Rightarrow \boxed{py - VC(y) \geq 0} \Rightarrow \boxed{p \geq \frac{VC(y)}{y} = AVC(y)}$$

- If $\boxed{p > AVC(y) \rightarrow y^* > 0}$

the firm has a positive **production level**

- If $\boxed{p < AVC(y) \rightarrow y^* = 0}$

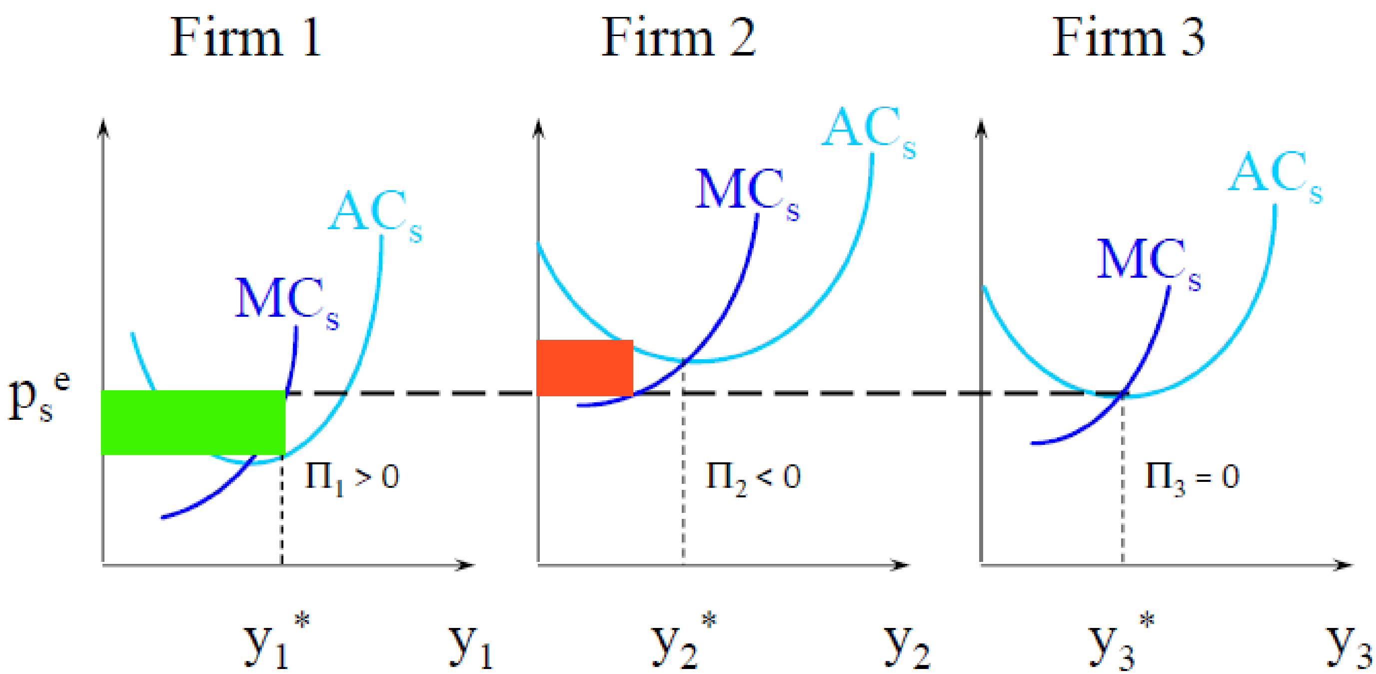
the firm **shut-down**, producing no output



In the short run firms maximize profit by setting p=MC provided that p is higher than the minimum value of AV

Short Run: Economic profits and losses

In the short run, firms in perfect competition can make economic profits or report losses



Firm 1 wishes to remain in the industry

Firm 2 wishes to exit from the industry

Firm 3 is indifferent

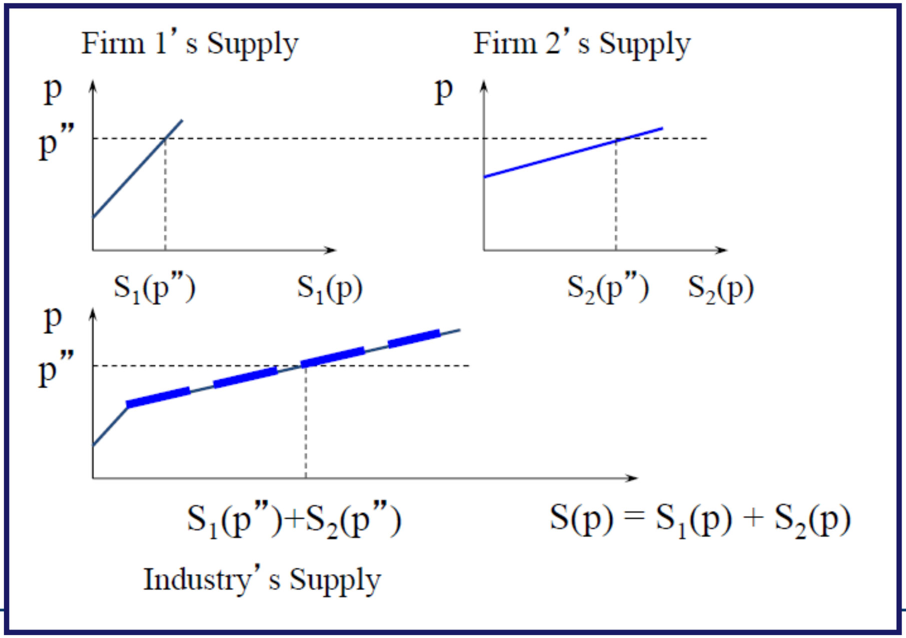
Short Run: Industry Supply

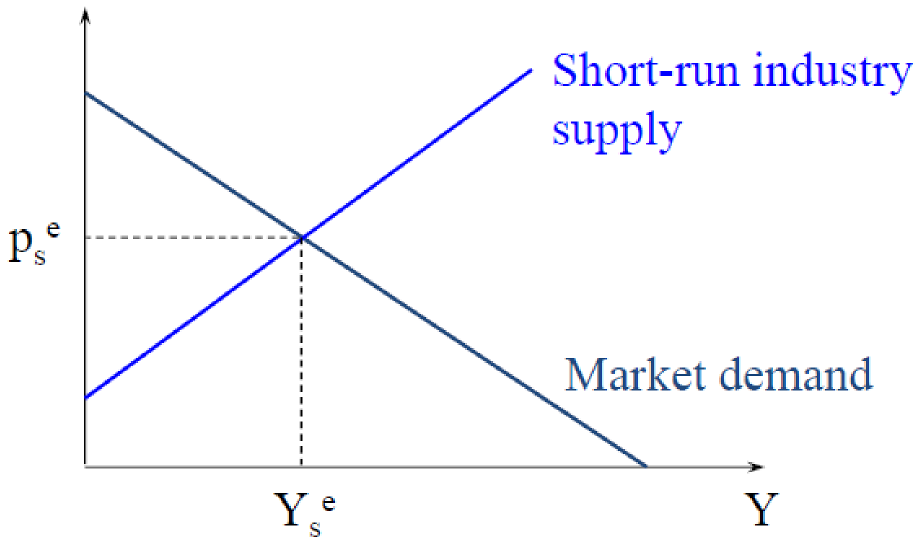
The **total quantity supplied** in the industry at the market price is the **sum of quantities supplied** at that price by each firm

- The **short-run industry supply** is

$$S(p) = \sum_{i=1}^{N} S_i(p)$$

Where N is the number of firms, $i = 1, \dots, N$, which is temporarily **fixed** in the short-run, and $S_i(p)$ is the firm i' s supply function





Long Run: Profit Maximization Problem

- In the long-run, all inputs are variable, thus, the cost *C(y)* of producing y units of output consists only of **variable costs**. The firm's **long-run profit function** is

$$\boxed{\Pi(y) = py - C(y)}$$

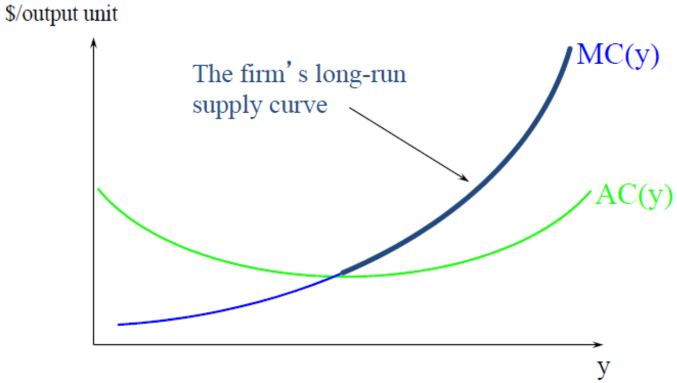
- The **profit maximization** problem is $\boxed{\max_{y \geq 0} \Pi(y) = py - C(y)}$

- The **first and second order conditions** are

$$\boxed{\begin{array}{l} p = MC(y) \quad and \\ \dfrac{dMC(y)}{dy} > 0 \end{array}}$$

- Additionally, the firm must not report losses otherwise it would **exit** the industry. So

$$\boxed{\begin{array}{l} \Pi(y) = py - C(y) \geq 0 \\ \Rightarrow p \geq \dfrac{C(y)}{y} = AC(y) \end{array}}$$

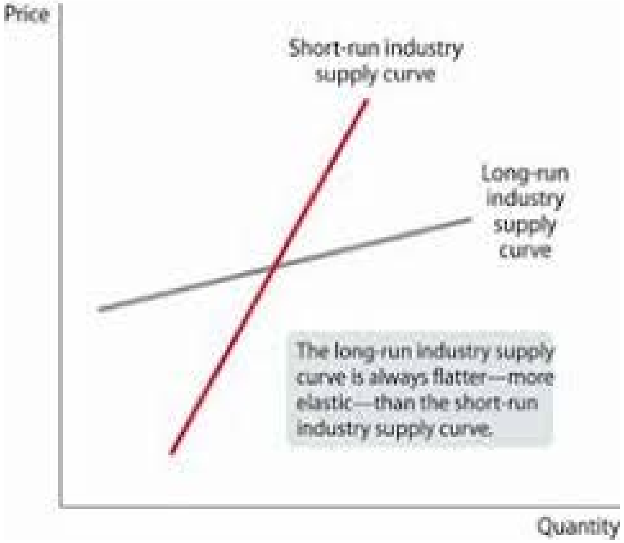


Long Run: Industry Supply

The **industry long run supply function** is the **sum** of **firms' supply function.** In the long run, **firms in the industry** are **free to exit** and **firms outside the industry** are **free to enter**, this dynamic causes N to vary. Specifically:

- Firms **enter** when incumbents gain economic profits, and this happens when ***p> min AC(y)*** → **N increases**
- Entry increases industry supply, *p* falls causing some firms to make losses and exit the industry → **N decreases**

The process ends when the long-run market equilibrium price emerges $p^e = min\ AC(y)$, thus firms make zero profit, this defines the number of firms in the industry in the long run



Monopoly

Monopoly

- There is **one firm** in the industry, which faces the market demand as its unique constraint
- **Monopolies unfold through**
  - A legal permission *(e.g.,* the salt monopoly)
  - A patent (*e.g.*, on a new drug)
  - Sole ownership of a resource (*e.g.*, a toll highway)
  - The formation of a cartel (*e.g.*, OPEC)
  - Control over key inputs

The monopolist wants to maximize its economic profit

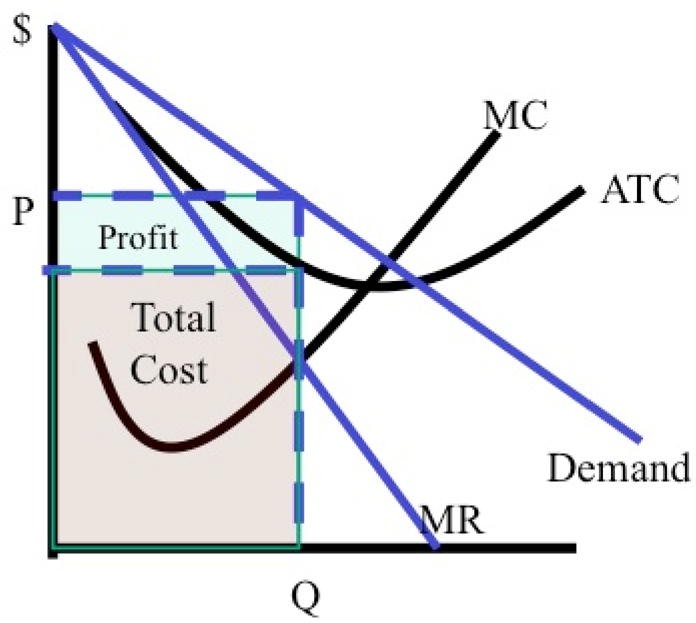
$$\boxed{\max_y \Pi(y) = r(y) - C(y) = p(y)y - C(y)}$$

- It produces the output *y**, at which marginal revenue equals marginal cost

$$\boxed{MR\left(y^*\right) = MC\left(y^*\right)}$$

Monopoly
Graphical overview

- Marginal revenue *MR(y)* is the **change in revenues for one unitary change in output**
- At the profit-maximizing output *y**, *MR(y*) = MC(y*)*



Profit max: MR=MC

Total revenue: P*Q

Total costs: ATC*Q

Profit: TR-TC

IMPORTANT: knowing the basics of Monopoly will help you understand collusion in Module E

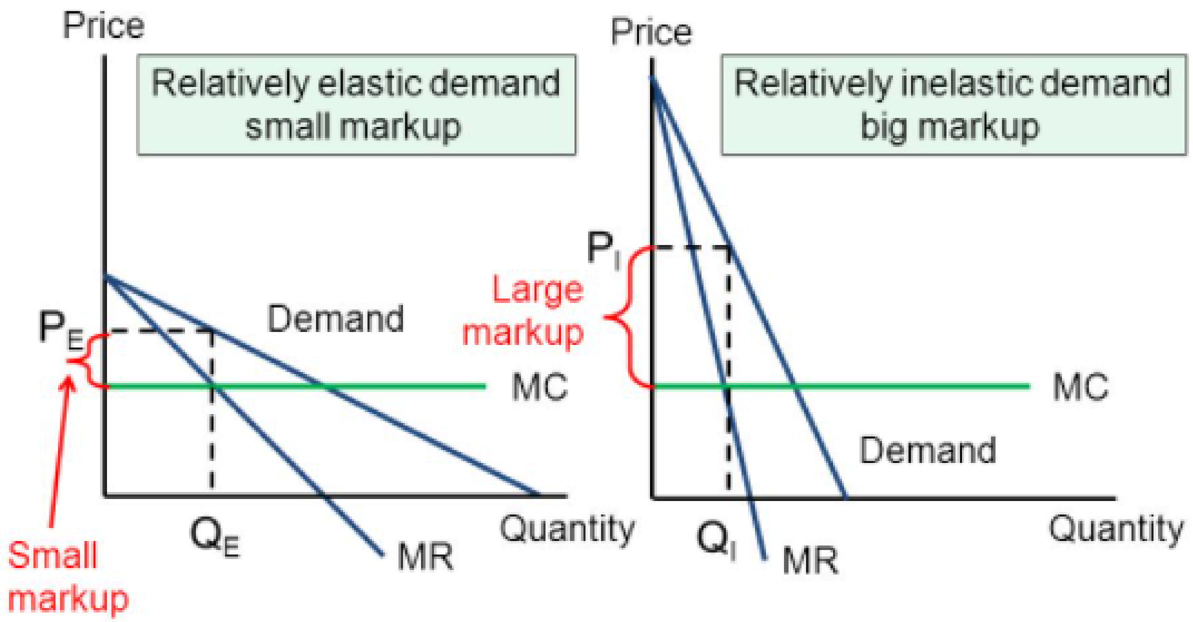
Monopoly Price and demand Elasticity

It is possible to express the **profit maximizing condition** in **monopoly** in **terms of demand elasticity**

$$\boxed{\frac{p - MC}{p} = -\frac{1}{\varepsilon}}$$

The monopolist

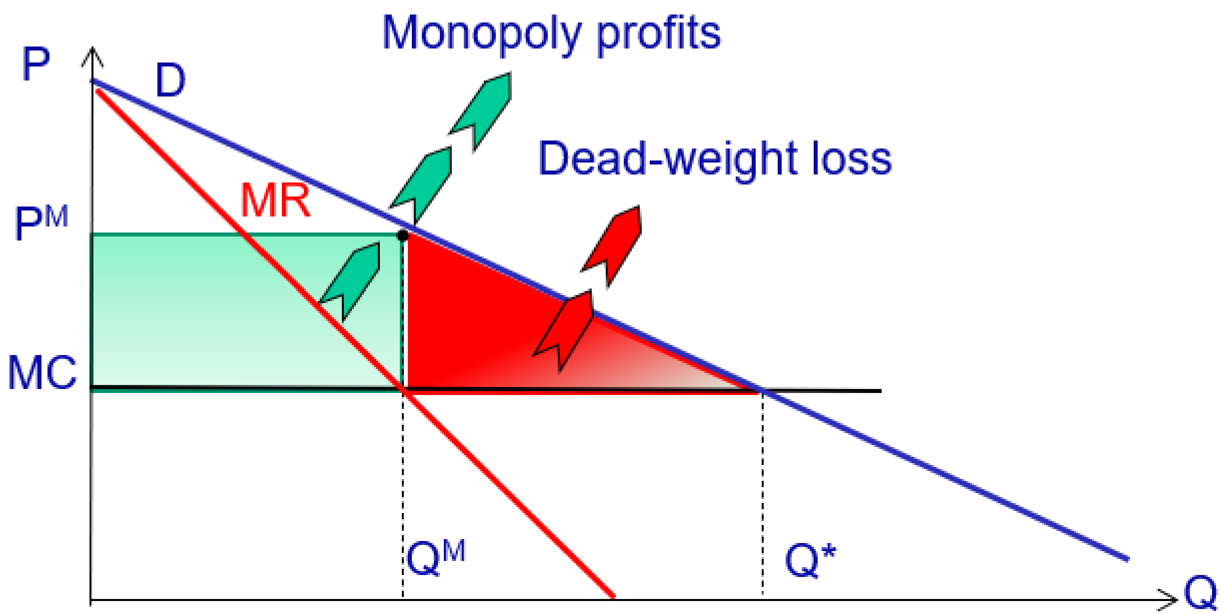
- Can charge a **higher price** the **lower** the **demand elasticity**
- Has **market power** (as p>MC) and this **market power** is **higher the lower** the **demand elasticity**



Monopoly vs. Perfect competition

Compared to **perfect competition** (where the price is equal to marginal cost), the **monopoly** offers a **lower quantity** at a **higher price (p>MC)**

- This leads to the **MONOPOLY DEAD-WEIGHT LOSS**



Readings and useful material

- Stengel, D. N. (2011). Managerial economics: Concepts and principles (chapter 3, 4, 6,7) (included in the WeBeep folder additional readings)
- Cabral (2017) Chapter 3 explaining the calculus behind profit maximization function. (included in the WeBeep folder additional readings)

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