Machine Learning
Associative rules

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Association rules

Find interesting **REGULAR PATTERNS** and **RECURRENCES** within a large set of transactions

Some application domains:

- **market basket analysis**
  find recurrent rules relating the purchase of a group of products to the purchase of other products
- **credit card transactions analysis**
  analyze the purchase via credit cards to direct future promotions (product and services are virtually infinite)
- **web mining**
  understand the patterns of navigation paths
- **fraud detection**
  identify potential fraudulent behavior from a set of transactions consisting of incident reports and applications for compensation

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Association rules: some definitions

Consider a set of n objects (items):

$$\mathcal{O} = \{o_1, o_2, \ldots, o_n\}$$

A **K-ITEMSET** is a generic subset $L \subseteq \mathcal{O}$ containing k objects

A **TRANSACTION** is a generic itemset recorded in a database in a single activity

The data set is composed by a list of m transactions T_i, each associated to a unique identifier t_i

Transaction T contains itemset L if $L \subseteq T$

Data set of transactions

List of transactions

identifier t_i	transaction T_i
001	$\{a, c\}$
002	$\{a, b, d\}$
003	$\{b, d\}$
004	$\{b, d\}$
005	$\{a, b, c\}$
006	$\{b, c\}$
007	$\{a, c\}$
008	$\{a, b, e\}$
009	$\{a, b, c, e\}$
010	$\{a, e\}$

$$\mathcal{O} = \{a, b, c, d, e\} = \{\text{bread, milk, cereals, coffee, tea}\}$$

Data set of transactions

The list of m transactions may be expressed by means of a bi-dimensional matrix:

- columns correspond to objects in set $\mathcal{O}$
- rows correspond to transactions
- the generic elements of the matrix is given by $x_{ij} = \begin{cases} 1 & \text{if } o_j \text{ belongs t} T_i \\ 0 & \text{otherwise} \end{cases}$

Matrix of transactions

identifier t_i	a	b	c	d	e
001	1	0	1	0	0
002	1	1	0	1	0
003	0	1	0	1	0
004	0	1	0	1	0
005	1	1	1	0	0
006	0	1	1	0	0
007	1	0	1	0	0
008	1	1	0	0	1
009	1	1	1	0	1
010	1	0	0	0	1

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Association rules

The EMPIRICAL FREQUENCY of itemset L is defined as the number of transactions in the data set containing L

When dealing with a large sample, the ratio $f(L)/m$ approximates the probability of occurrence of the itemset L

$\mathcal{O} = \{a, b, c, d, e\} = \{\text{bread, milk, cereals, coffee, tea}\}$

identifier t_i	a	b	c	d	e
001	1	0	1	0	0
002	1	1	0	1	0
003	0	1	0	1	0
004	0	1	0	1	0
005	1	1	1	0	0
006	0	1	1	0	0
007	1	0	1	0	0
008	1	1	0	0	1
009	1	1	1	0	1
010	1	0	0	0	1

$L = \{a, c\} \quad \Rightarrow \quad f(L) = 4 \quad \Rightarrow \quad \Pr(L) \approx 4/10 = 0.4$

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What is a rule?

Let Y and Z be two propositions which may be true or false

A **RULE** is an implication in the form Y $\Rightarrow$ Z : “ if Y is true, then Z is also true ”

A rule is called **PROBABILISTIC** if the validity of Z is associated with a certain probability p : “ if Y is true, then Z is also true with probability p ”

Consider two disjoint itemsets, $L \subset \mathcal{O}$ and $H \subset \mathcal{O}$, and the transaction T

An **ASSOCIATIVE RULE** is a probabilistic implication denoted as L $\Rightarrow$ H with the following meaning:

“ if L is contained in T , then H is also contained in T with probability p ”

$\downarrow$ body $\qquad$ $\downarrow$ head

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Confidence and support

CONFIDENCE $\quad p = \mathrm{conf}\{L \Rightarrow H\}$

- proportion of transactions containing H among those including L (reliability of the rule)
- it approximates the conditional probability that H belongs to T given that L belongs to T

SUPPORT $\quad s = \mathrm{supp}\{L \Rightarrow H\}$

- proportion of transactions containing both H and L (frequency of the pair L-H)
- it approximates the probability that L and H are both contained in a future transaction

$\mathcal{O} = \{a, b, c, d, e\} = \{$bread, milk, cereals, coffee, tea$\}$

$L = \{a, c\}$

$H = \{b\}$

identifier t_i	a	b	c	d	e
001	1	0	1	0	0
002	1	1	0	1	0
003	0	1	0	1	0
004	0	1	0	1	0
005	1	1	1	0	0
006	0	1	1	0	0
007	1	0	1	0	0
008	1	1	0	0	1
009	1	1	1	0	1
010	1	0	0	0	1

$p = \mathrm{conf}\{L \Rightarrow H\} =$ $\qquad$ $s = \mathrm{supp}\{L \Rightarrow H\} =$

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Strong association rules

Objective $\Rightarrow$ find the **STRONG** association rules:

$$s \geq s_{min} \qquad p \geq p_{min}$$

what about extracting all the rules?...

The number of possible rules increases exponentially as the number of objects increases …

$\Downarrow$

… a two-phase procedure is applied:

- generation of **FREQUENT ITEMSETS**
- generation of strong rules

From a set of 20 objects it is possible to generate about 3.5 billions of association rules!

Strong association rules

Strong rules are always meaningful?

A retailer whishes to analyze a set of transactions to identify associations between sales of color printers and sales of digital cameras:

- 1000 transactions
- 600 include digital cameras
- 750 include color printers
- 400 include both products

probability of purchasing a printer conditioned to the purchase of a camera

$\uparrow$

The rule { digital camera } $\Rightarrow$ { color printer } has support 0.4 and confidence 0.66

What is the probability of purchasing a printer?...0.75, which is greater than 0.66!

$\downarrow$

printers and cameras show a negative correlation…
…the purchase of a camera reduces the probability of buying a printer!

Lift index

A third measure of significance is the LIFT :

$$l = \text{lift}\{L \Rightarrow H\} = \frac{\text{conf}\{L \Rightarrow H\}}{\Pr(H)} = \frac{f(L \cup H)}{f(L)f(H)/m}.$$

- the lift is greater than 1: body and head are ***positively associated***
  the rule is effective in predicting the presence of the head in a given transaction
- the lift is lower than 1: body and head are ***negatively associated***
  the rule is less effective than the estimate obtained by the frequency of the head

For the former example:

$f(L \cup H)$ = 400
$f(L)$ = 600
$f(H)$ = 750

$\text{lift}\{L \Rightarrow H\}$ = 400/(600*750/1000) ≈ 0.88

Association rules

The support of the rule L ⇒ H only depends on the set $L \cup H$ given by the union of the itemsets L and H

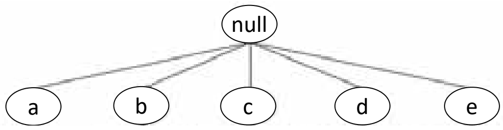
⇓

If the itemset $L \cup H$ is not frequent, we can exclude from the analysis all the rules obtained by using all the proper subsets $L \cup H$ ...

$$\begin{array}{ll} \{a,b\} \Rightarrow \{c\} & \{a,c\} \Rightarrow \{b\} \\ \{b,c\} \Rightarrow \{a\} & \{a\} \Rightarrow \{b,c\} \\ \{b\} \Rightarrow \{a,c\} & \{c\} \Rightarrow \{a,b\} \end{array}$$

... the real problem, therefore, is how to determine the frequent itemsets !

Itemset grid



Association rules

A data set of *m* transactions defined over a set of *n* objects may contain up to 2^n-1 frequent itemsets (excluding the empty set) ...exhaustive enumeration is impracticable...

The APRIORI ALGORITHM is an effective method for extracting strong rules.
It relies on the following property (APRIORI PRINCIPLE):

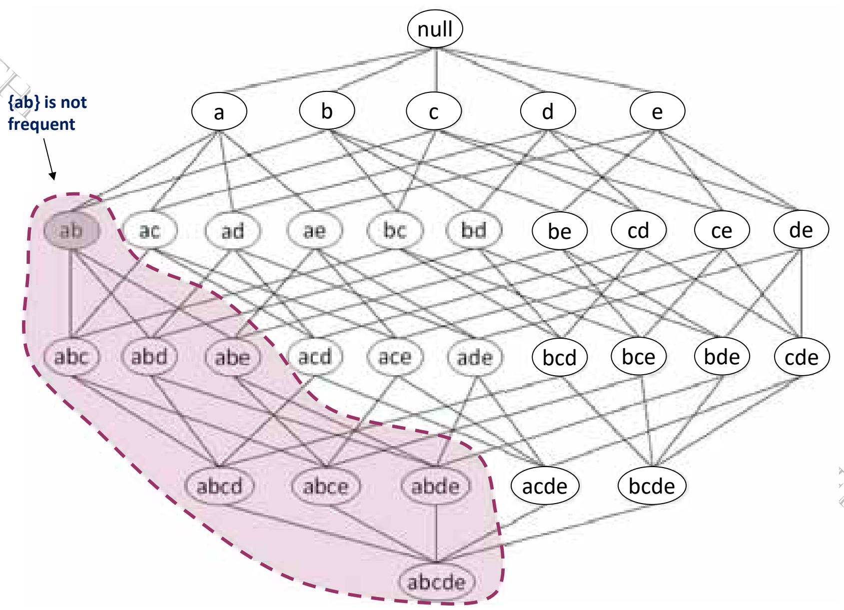
"if an itemset is frequent, then all its subsets are also frequent"

$\Downarrow$

if an itemset is not frequent, then each of the itemset containing it must turn out to be not frequent!

Once a non-frequent itemset is identified in the course of the algorithm, all the other itemsets (with greater cardinality) containing it are implicitly eliminated and excluded from the analysis

Itemset grid



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Apriori algorithm: phase 1

The Apriori algorithm generates the frequent itemsets iteratively, starting from the frequent 1 – itemset to the frequent k – itemsets

- **Phase 1 - initialization**
  Compute the relative frequency of each object.
  Discard the objects with a frequency smaller than the given minimum threshold.

$\mathcal{O} = \{a, b, c, d, e\}$ = {bread, milk, cereals, coffee, tea}

itemset	relative frequency	status
$\{a\}$	$7/10 = 0.7$	frequent
$\{b\}$	$7/10 = 0.7$	frequent
$\{c\}$	$5/10 = 0.5$	frequent
$\{d\}$	$3/10 = 0.3$	frequent
$\{e\}$	$3/10 = 0.3$	frequent

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Apriori algorithm: phase 1

- **Phase 1 - iteration 1**
  The candidate 2 – itemsets are obtained from the 1 – itemset found during the former iteration.
  Non-frequent itemsets are discarded.

$\mathcal{O} = \{a, b, c, d, e\}$ = {bread, milk, cereals, coffee, tea}

itemset	relative frequency	status
$\{a, b\}$	$4/10 = 0.4$	frequent
$\{a, c\}$	$4/10 = 0.4$	frequent
$\{a, d\}$	$1/10 = 0.1$	not frequent
$\{a, e\}$	$3/10 = 0.3$	frequent
$\{b, c\}$	$3/10 = 0.3$	frequent
$\{b, d\}$	$3/10 = 0.3$	frequent
$\{b, e\}$	$2/10 = 0.2$	frequent
$\{c, d\}$	$0/10 = 0.0$	not frequent
$\{c, e\}$	$1/10 = 0.1$	not frequent
$\{d, e\}$	$0/10 = 0.0$	not frequent

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Apriori algorithm: phase 1

- **Phase 1 - iteration 2**
  The candidate 3 – itemsets are obtained from the 2 – itemsets found during the former iteration.
  Non-frequent itemsets are discarded.

$\mathcal{O} = \{a, b, c, d, e\}$ = {bread, milk, cereals, coffee, tea}

itemset	relative frequency	status
$\{a, b, c\}$	$2/10 = 0.2$	frequent
$\{a, b, e\}$	$2/10 = 0.2$	frequent

The first phase of the algorithm terminates since no candidate 4 – itemsets can be generated!

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Apriori algorithm: phase 2

- **Phase 2 - step 1**
  Split each frequent itemset B into two disjoint non-empty subsets L and H = B – L , according to each possible combination
  Generate all the corresponding candidate rules ⟶ **given a frequent k-itemesets it is possible to generate 2^k-2 candidate rules which may by strong rules (excluding those with empty body or head)**

- **Phase 2 - step 2**
  For each candidate rule L ⇒ H compute the confidence
  ↓
  very fast step – the frequencies of (L∪H) and L are known at the end of the first phase

- **Phase 2 - step 3**
  Include into the list of strong rules all the rules provided with a confidence greater than the prefixed minimum threshold

Apriori algorithm: phase 2

Strong rules generation

itemset	rule	confidence	status
$\{a, b\}$	$\{a \Rightarrow b\}$	$p = 4/7 = 0.57$	strong
$\{a, b\}$	$\{b \Rightarrow a\}$	$p = 4/7 = 0.57$	strong
$\{a, c\}$	$\{a \Rightarrow c\}$	$p = 4/7 = 0.57$	strong
$\{a, c\}$	$\{c \Rightarrow a\}$	$p = 4/5 = 0.80$	strong
$\{a, e\}$	$\{a \Rightarrow e\}$	$p = 3/7 = 0.43$	not strong
$\{a, e\}$	$\{e \Rightarrow a\}$	$p = 3/3 = 1.00$	strong
$\{b, c\}$	$\{b \Rightarrow c\}$	$p = 3/7 = 0.43$	not strong
$\{b, c\}$	$\{c \Rightarrow b\}$	$p = 3/5 = 0.60$	strong
$\{b, d\}$	$\{b \Rightarrow d\}$	$p = 3/7 = 0.43$	not strong
$\{b, d\}$	$\{d \Rightarrow b\}$	$p = 3/3 = 1.00$	strong
$\{b, e\}$	$\{b \Rightarrow e\}$	$p = 2/7 = 0.29$	not strong
$\{b, e\}$	$\{e \Rightarrow b\}$	$p = 2/3 = 0.67$	strong
$\{a, b, c\}$	$\{a, b \Rightarrow c\}$	$p = 2/4 = 0.50$	not strong
$\{a, b, c\}$	$\{c \Rightarrow a, b\}$	$p = 2/5 = 0.40$	not strong
$\{a, b, c\}$	$\{a, c \Rightarrow b\}$	$p = 2/4 = 0.50$	not strong
$\{a, b, c\}$	$\{b \Rightarrow a, c\}$	$p = 2/7 = 0.29$	not strong
$\{a, b, c\}$	$\{b, c \Rightarrow a\}$	$p = 2/3 = 0.67$	strong
$\{a, b, c\}$	$\{a \Rightarrow b, c\}$	$p = 2/7 = 0.29$	not strong
$\{a, b, e\}$	$\{a, b \Rightarrow e\}$	$p = 2/4 = 0.50$	not strong
$\{a, b, e\}$	$\{e \Rightarrow a, b\}$	$p = 2/3 = 0.67$	strong
$\{a, b, e\}$	$\{a, e \Rightarrow b\}$	$p = 2/3 = 0.67$	strong
$\{a, b, e\}$	$\{b \Rightarrow a, e\}$	$p = 2/7 = 0.29$	not strong
$\{a, b, e\}$	$\{b, e \Rightarrow a\}$	$p = 2/2 = 1.00$	strong
$\{a, b, e\}$	$\{a \Rightarrow b, e\}$	$p = 2/7 = 0.29$	not strong

Association rules

The computational effort required grows exponentially as the number *n* of objects increases

To improve the efficiency:

- resort to advanced data structures (dictionaries, binary trees)
- split the data set into disjoint subsets of transactions and apply the algorithm to each subset (local frequent itemsets)
- use the algorithm on a given sample of transactions
- resort to hierarchies of objects
- discard less interesting objects in the preprocessing phase

Association rules

The aforementioned rules are *binary*, *asymmetric* and *one-dimensional* rules

binary ↓ presence vs. absence; *asymmetric* ↓ presence is more relevant

More general association rules:

- rules for binary symmetric variables ({male,female}, {yes,no}, …)
  → two asymmetric binary variables are introduced for each symmetric variable
- rules for categorical variables (education, profession,…)
  → a set of asymmetric binary variables is introduced for each categorical variable
- rules for continuous variables (age, income,…)
  → discretization is first applied to obtain a categorical variable
- multi-dimensional rules
- sequential rules (when transactions are recorded according to a specific temporal sequence)