# EX4 – QUEUEING THEORY

## Exercise 1

MASTER Spa has to deliver 500 good components per hour to the customer, respectively 300 type A pieces and 200 type B pieces. The production system of MASTER Spa is basically a job shop and production flow for the two types of components is described below.

Both types of product are processed in the first stage of rough-cut. It consists of two technologically identical machines. These resources don't need set-up to change product type, they can indifferently work both product families. The next item waiting to be processed in the queue will be worked by the first free machine. After this stage the production flows of the two types are separated, they meet on the last stage of testing.

Type A products first pass the surface finishing stage and further the drilling stage. Downstream of the drilling stage pieces join the testing stage queue.

Type B products, after initial roughing stage in common with type A product, are processed in a work center. After being processed by the work center, they are placed in the testing stage queue.

The product B has a particular shape, which easily creates problems during the process. Because of this, frequently type B products have to be reworked or discarded. The quality control is made by the same work center. In particular, 5% of pieces worked are no longer workable and therefore they are discarded, while 15% of the pieces that this stage works have to be reworked to become good products.

These B-products have to be reworked on traditional machines, so they are sent to the queue upstream of the surface finishing machine that works products of type A. Once the product is reprocessed by surface finishing and drilling machines, it is considered good. In surface finishing and drilling stages products of type B move forward with a lower priority than type A, according to a not preemptive logic.

Last stage is formed by the testing machine. In the testing stage there aren't any types of priority rule. The testing machine is old and sometimes it identifies as defective a good product. For this reason, a product been identified as defective first time has to be tested again. On Average 15% of products tested by the machine (both types) has to be tested again. 50% of the pieces tested twice are discarded (both types). The other products are good and they can be delivered to the customer.

The table shows the service rates of the resources of MASTER Spa. The service rates and the arrival rates are distributed according to a negative exponential.

| Service rate | Pieces/hour |
|---|---|
| Rough-cut stage (1 machine) | 300 |
| Work center stage | 250 |
| Surface finishing stage | 380 |
| Drilling stage | 380 |
| testing | 660 |

Company's requests are:
1) Model the production system of the Master Spa, calculate all relevant parameters of the system, with particular attention to the calculation of the necessary input for each component in order to meet the production target.
2) Calculate the expected throughput time of the production system (not considering the products that need to be eliminated because defective)
3) Calculate the raw material input reduction if the defective products percentage at testing machine drops to 0 (in the second test all the products result conformed). What are the effects on the expected throughput time?

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## Exercise 2

**DESPACITO Srl**

A company has a central document-copying service where employees of this company go occasionally to make copies. Arrivals are assumed to follow the Poisson probability distribution, with a mean rate of 15 per hour. Also, service times are assumed to follow the Poisson distribution. With the present copying equipment, the average service time is 3 minutes. A new machine is available that will have a mean service time of 2 minutes. The average wage of the employees who bring the documents to be copied is €8 an hour. Employees need to wait at the machine while it's copying, and all employees are fully saturated during their work.

**1**: If the machine can be rented for €2 per hour more than the old machine, should the company rent the new machine? Consider lost productive time of employees as the time they spent waiting in the system.

**2**: The area around the copy machine can accommodate only three employees. What percentage of time will this area be inadequate for waiting employees when using just the old copy machine?

**3**: Due to the very low prices, the company thinks about purchasing many small-scale copy machines. Assume the small-scale copy machines are set up with a first come first serve queue policy in a single queue system, and that each small-scale copy machine has an average service rate of 5 employees per hour. Determine the smallest number of copy machines the company needs to provide to keep the average waiting time in the system for the employees lower than 15 minutes

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## Exercise 3

The production system of SPEED Spa is very simple. It is shaped by three different stations. Also the production flow is extremely simple because the company works only one type of product
The production flow is composed of one stage of mechanical process and a following stage of finishing. Both machines in the mechanical process have an extremely high availability that is equal to 100%.

The stage of finishing has an automatic control system which is able to separate good pieces from defective pieces. The good items are moved to the special treatment section, in another area of the plant. This area is not the subject of the study and therefore it can be ignored. 600 good pieces per hour are to be supplied to special treatment section.

The pieces considered defective by finishing stage are carried to a quality control stage that has to identify the type of defect of the pieces resulted non-compliant after finishing job. Three different options:

a) The piece erroneously considered defective. It is good and it is delivered to special treatment section.
b) The piece has small damage and it must be reworked by both machines. It is reinserted in the production flow from the first station.
c) The piece is very damaged, so it is rejected and thrown away

The automatic control system of the finishing machine is not infallible. In fact, it generally considers 10% of the pieces good as non-compliant. It never fails to identify damaged pieces, which are on average 15% of the finishing stage output.

The machine of quality control stage has 100% correct identification of pieces quality. Usually, the 20% of the pieces coming from the finishing stage to quality control stage result to be discarded.

All arrival and service rates are described by a negative exponential distribution. The times given below refer to a single machine or operator.

| | |
|---|---|
| Mechanical machine | 413 pieces/hour |
| Finishing machine | 800 pieces/hour |
| Quality control machine | 188 pieces/hour |

The stage of mechanical working is composed of two identical machines that work in parallel. The product that is waiting in the queue will take the first free machine. Finishing and quality control stages are composed of a single machine each.
With the data available:

1) Calculate the minimum allowable arrival rate of raw material input to the system in order to ensure to special treatment section 600 pieces / hour.
2) Calculate the average WIP and waiting time in line in the queue upstream of quality control stage.
3) Calculate the probability that a product which just joined the queue of the finishing stage exits from same stage in less than 45 seconds. (The pieces are processed in FIFO logic).
4) Calculate the average throughput time of the system under the following assumptions:
    1- The automatic control machine of the finishing stage can separate 100% good parts from the damaged pieces.
    2- Once the piece has been reprocessed it is conformed.