



**POLITECNICO**  
MILANO 1863

# Yield Management part 2 - solutions

Prof. Alberto Portioli Staudacher  
Bassel Kassem

Lean excellence Centre [www.lean.polimi.it](http://www.lean.polimi.it)

Politecnico di Milano

Dep. Management, Economics and Industrial Engineering

[bassel.kassem@polimi.it](mailto:bassel.kassem@polimi.it)

- ✦ Exploiting the historical demand data, it is possible to avoid idle capacity by introducing overbooking policies
- ✦ It is essential to have a comprehensive database to study customers' behaviour

- ✦ Understand how to exploit no-show phenomena through overbooking
- ✦ Not all customers are behaving as a Normal distribution

Marcus, a famous British automaker, is going to propose to the market a new super-car model, limited edition. The company decided to produce only 250 cars of this model.

Marcus, to give exclusivity to its new model, made a contract with the well-known English circuit of "Brands Hatch" flat-rate fee of 700 € / service package, to allow the prospective customer to access, on certain dates, the circuit. This means that, when Marcus requires access to the circuit, he will pay the discounted rate of 700 €. Marcus established that the margin associated with this service is 200 € / package (customers will pay Marcus € 900 to access the circuit).

Moreover, because Marcus participates in the FIA "Le Mans Series" championship, the organizer reserved to Marcus the right to buy the tickets for the opening race in Monza at a privileged price of € 60 each. Marcus intends to take the opportunity because he is sure to sell all the tickets at € 100 each.

Finally, the company set to give the car keys in a precious box of brier-root, with the symbol of Marcus, prepared in advance for all 250 cars.

The sales function decided to sell the car and all the services (access to "Brands Hatch" circuit and ticket for the "Le Mans Series" opening) at the discounted price of 150,000 € to customers that book the vehicle ahead of its official launch at the London Motor Show. The production cost of the vehicle, regardless if it is sold at discounted or at full price, is 70,000 €.

The price of bookings that will be made after this event will be € 170,000, services not included.

Customers will have the opportunity to separately buy the services. It is estimated that the services will be purchased by 70% of this customer base.

It is also estimated that the demand for cars at full price, following the presentation at the show, can be described by a normal distribution with a mean of 150 and a variance of 15. After 3 months, the effect of the London Motor Show is finished and unsold cars have to be sold through another channel, which requires an additional cost of 30,000 € / car. The selling price and the opportunity to get additional services to remain unaffected.

The Marcus costs structure is specified in the table below.

London Motor Show participation	10.000 €
Stand preparation	50.000 €
Staff (hostess, steward)	15.000 €
precious box of brier-root	100€/box
Opening "Le mans series" ticket	60 €/ticket
Access to "Brands Hatch" circuit	700 €/access
Advertising banner	200 €/banner
registration "Le Mans Series" championship	20.000 €
gadget e merchandising for events	5.000 €

- Is it correct to set a protection level for after the London motor show booking? How many cars?
- How much does the protection level change if the distribution of after London motor show demand is described as a normal distribution with a mean of 150 and probability of sell between 110 and 190 cars equals 76%?



Question 1: compute full price car protection level.

Question 2: How much does the protection level change, if the distribution of after london motor show demand is described as a normal distribution with mean 150 and probability of sell between 110 and 190 cars equals to 76%?

Marginal analysis

$$P(X_1 \geq S_1) * C_u \geq P(X_1 < S_1) * C_o$$

$$[1 - P(X_1 < S_1)] * C_u \geq P(X_1 < S_1) * C_o$$

$$P(X_1 < S_1) \leq \frac{C_u}{C_u + C_o}$$

$C_o$  = cost related to have dedicated too many cars to after London exhibition sales, company wasn't able to sell all cars and has to pay additional costs to sell them through other channels

$C_u$  = cost related to have allocated too cars to discounted rate, so company loses the opportunity to sell cars at an higher price after the London exhibition.

$$\begin{aligned} \bullet C_o &= 0,7 * [(100-60)+(900-700)] + (150.000 - 60 - \\ &700) + (30.000 - 170.000) = \\ &\quad \underline{9.072 \text{ €}} \end{aligned}$$

$$\begin{aligned} \bullet C_u &= 170.000 + 0,7 * [(100-60)+(900-700)] \\ &- (150.000-60-700) = \\ &\quad \underline{20.928 \text{ €}} \end{aligned}$$



$$C_u = 20.928 \text{ €}$$

$$C_o = 9.072 \text{ €}$$

$$P(X_1 \geq S_1) * C_u \geq P(X_1 < S_1) * C_o$$

$$[1 - P(X_1 < S_1)] * C_u \geq P(X_1 < S_1) * C_o$$

$$P(X_1 < S_1) \leq \frac{C_u}{C_u + C_o}$$

$$P(X_1 < S_1) \leq \frac{20928 \text{ €}}{20928 \text{ €} + 9072 \text{ €}} = 0,6976$$

$$F(Z\alpha) = 0,6976$$

$$Z\alpha = 0,52$$

Full rate cars protection level :

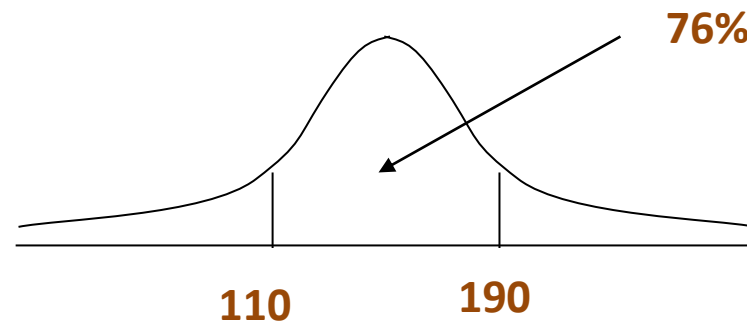
$$S_1 = 150 + 0,52 * \sqrt{15} = 152$$

152 cars allocated to after London motor show sales.

**98 cars to sell before London motor show  
at discounted rate**

## Marcus Spa → question 2

After-exhibition demand is described by a normal distribution with mean 150 and probability of sell a number of cars between 110 and 190 equals to 76%.

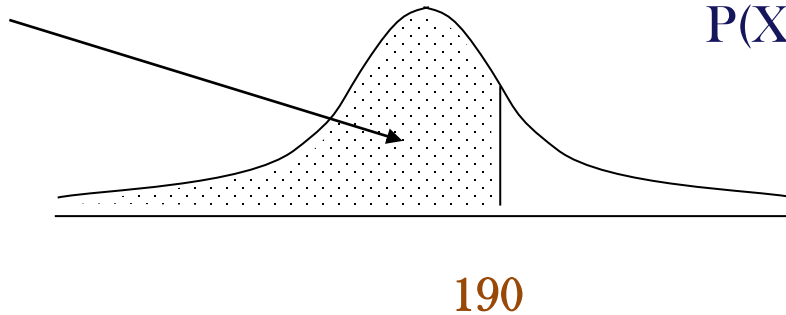


How much is the standard deviation?

## Marcus Spa → question 2

- The probability that the demand of cars after london exhibition is lower than 190 is equals to:

88%



$$P(X_1 \leq 190) = (1-0,76)/2 + 0,76 = 0,88$$

$$P\left(\frac{X_1 - 150}{\sigma} \leq \frac{190 - 150}{\sigma}\right) = 0,88$$

$$\begin{aligned} F(Z\alpha) &= 0,88 \\ Z\alpha &= 1,175 \end{aligned}$$

$$\frac{190 - 150}{\sigma} = 1,175 \quad \rightarrow \quad \sigma = \frac{40}{1,175} = 34,04$$

New protection level:

$$S_1 = 150 + 0,52 * 34,04 = 168$$

168 cars to sell at full price after London exhibition.  
Only 82 cars to sell before the event at discounted rate.

- ✦ Understand how to exploit no-show phenomena through overbooking
- ✦ Not all customers are behaving as a Normal distribution

## How many reservations

The Astoria-Hotels is a chain of luxury hotels.

Engineer Bianchi, Head of Operations at the Villa Reale Hotel in Milan, has noted that in some days many rooms of the hotel remained empty. It is strange because the hotel receives so many reservation requests that it partially refuses. That's why he asked the engineer Cantu to reconsider the optimal level of overbooking.

Customers of the Villa Reale Hotel can book by phone or internet and at the time of booking, they have to pay the entire booking price. This amount is not refundable under any circumstances. There is a single rate of 145 € per room per night, the total number of rooms of the Villa Reale Hotel is 120 and Engineer Cantu estimates that the No-show rate is about 10%.

The cost structure of the Villa Reale Hotel is shown in Figure 1

staff	1.500€/day
breakfast**	15€/meal
newspapers***	3€/room
Bedsheets and towel washing****	5€/set

\*\*breakfast is included in the booking rate (145€). There is not the possibility to have lunch or dinner.

\*\*\*It is included in the booking rate.

\*\*\*\*The set is cleaned only if the room is used. Washing is given in out-sourcing.

FIGURE 1

When a customer that bought a reservation arrives at the Villa Reale Hotel and she finds it full the operators at the reception try to find an alternative solution.

In 60% of cases, it is possible to find a free room in another hotel, Futura Hotel, belonging to the Astoria-Hotels chain, the remaining 40% of cases receptionist should contact other hotels.

For customers who go to Futura Hotel, it is paid the taxi to the hotel (average travel cost of 15 €) and a dinner at the Golf Club Milano restaurant that costs 70 €. In addition, the Villa Reale Hotel has to pay 25 €/day to Futura Hotel.

For customers who go to a hotel not belonging to the Astoria-Hotels chain, Villa Reale Hotel pays the taxi to the hotel (but in this case the average travel rate is 20 €) and offers the same dinner whose cost is 70 €. Usually, these hotels belong to the same class of Astoria-Hotels. The price of accommodation in these hotels is directly paid by Villa Reale Hotel and it is on average 120 €. In any case, the sum of 145 € for the reservation is not refunded to the customer.

- Compute the optimal overbooking level (number of reservations that should be accepted by the Villa Reale Hotel).
- How does the overbooking level change considering the cost of loss of image because the hotel doesn't guarantee the room that the customer has booked? (estimated loss of image cost 250 €)

### Attached 1

Binomial distribution parameters  $n$  and  $p$ ,  $n$  = hotel capacity (120 rooms) e  $p$  = probability of success (in this case 0,9 because of 10% of no-show).

$Q=120$  rooms  $p=0,9$

#acc. book	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145
$P(X \leq Q)$	1	1	1	1	1	1	0,99	0,98	0,95	0,91	0,85	0,77	0,68	0,58	0,47	0,37	0,28	0,21	0,15	0,1	0,07	0,04	0,03	0,02	0,01	0

The number of reservations to accept is related to the probability resulted from marginal analysis.



How many reservations should Villa Reale hotel accept?

## Marginal analysis adapted to overbooking

$C_u$  = cost of underestimating No-show customers

$C_o$  = cost of overestimating No-show customers

$$C_u P(Ovb \leq NS) \geq C_o * P(Ovb > NS)$$

$\Rightarrow$

$$C_u P(Ovb \leq NS) \geq C_o [1 - P(Ovb \leq NS)]$$

$\Rightarrow$

$$P(Ovb \leq NS) \geq \frac{C_o}{C_u + C_o} \quad \Leftrightarrow \quad P(X \leq C) \geq \frac{C_o}{C_u + C_o}$$

$C_u \rightarrow$  a room remains empty

$$C_u = 145 \text{ €} - 15\text{€} - 3\text{€} - 5\text{€} = 122\text{€}$$

in 2nd scenario  $\Rightarrow + 250$

$C_o$  = company has to assure a room to the customer that booked a room

$$C_o = 0,6 * (15\text{€} + 70\text{€} + 25\text{€}) + 0,4 * (20\text{€} + 70\text{€} + 120\text{€}) - 145\text{€} = 5 \text{ €}$$

# ASTORIA HOTELS → question 1

$$P(X \leq C) \geq \frac{C_o}{C_u + C_o} = 5 / (122+5) = 0,04$$

#	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145
P(X≤Q)	1	1	1	1	1	1	0,99	0,98	0,95	0,91	0,85	0,77	0,68	0,58	0,47	0,37	0,28	0,21	0,15	0,1	0,07	0,04	0,03	0,02	0,01	0

Villa Reale Hotel will accept 141 reservation requests

## ASTORIA HOTELS → question 2

$$P(X \leq C) \geq \frac{C_o}{C_u + C_o} = 255 / (122 + 255) = 0,68$$

#	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145
P(X≤Q)	1	1	1	1	1	1	0,99	0,98	0,95	0,91	0,85	0,77	0,68	0,55	0,47	0,37	0,28	0,21	0,15	0,1	0,07	0,04	0,03	0,02	0,01	0

**Villa Reale Hotel will accept 132 reservation requests**



**POLITECNICO**  
MILANO 1863