Symbols

n=number of customers in the whole system; **λ**=arrival rate; **μ**=service rate; **N**=maximum number of customers in the system; **c**=number of servers; $\mathbf{P_n}$=probability there are n customers in the system; $\mathbf{L_s}$=average number of customers in the system; $\mathbf{L_q}$=average number of customers waiting in the queue; $\mathbf{L_b}$=average number of customers in a queue in a busy system; $\mathbf{W_s}$=average time spent by a customer in the system; $\mathbf{W_q}$=average time spent by a customer in the queue; $\mathbf{W_b}$=average time spent in a queue by a customer in a busy system

Standard Model M/M/1 (0< ρ<1)

$P_0 = 1 - \rho$

$P(n \geq k) = \rho^k$

$P_n = P_0 \rho^n$

$L_s = \frac{\lambda}{\mu - \lambda}$

$L_q = \frac{\rho\lambda}{\mu - \lambda}$

$L_b = \frac{\lambda}{\mu - \lambda}$

$W_s = \frac{1}{\mu - \lambda}$

$W_q = \frac{\rho}{\mu - \lambda}$

$W_b = \frac{1}{\mu - \lambda}$

Standard Model M/M/c (0< ρ<c)

$$P_o = \frac{1}{\left(\sum_{i=0}^{c-1} \frac{\rho^i}{i!}\right) + \frac{\rho^c}{c!(1-\rho/c)}}$$

$$P_n = \begin{cases} \frac{\rho^n}{n!} P_o & \text{per} \quad 0 \leq n \leq c \\ \frac{\rho^n}{c!c^{n-c}} P_0 & \text{per} \quad n \geq c \end{cases}$$

$P(n \geq c) = \frac{\rho^c \mu c}{c!(\mu c - \lambda)} P_o$

$L_s = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2} P_0 + \rho$

$L_q = L_s - \rho$

$L_b = \frac{L_q}{P(n \geq c)}$

$W_s = \frac{L_q}{\lambda} + \frac{1}{\mu}$

$W_q = \frac{L_q}{\lambda}$

$W_b = \frac{W_q}{P(n \geq c)}$

Standard M/G/1
(V(t) = service time variance)

$L_s = L_q + \rho$

$L_q = \frac{\rho^2 + \lambda^2 V(t)}{2(1-\rho)}$

$W_s = \frac{L_s}{\lambda}$

$W_b = \frac{L_q}{\lambda}$

Ws with Prehemptive priority

$$E(S_1) = \frac{1/\mu}{1-\rho_1},$$

$$E(S_2) = \frac{E(L_2)}{\lambda_2} = \frac{1/\mu}{(1-\rho_1)(1-\rho_1-\rho_2)}.$$

Ws with NON Prehemptive priority

$$E(S_1) = \frac{(1+\rho_2)/\mu}{1-\rho_1},$$

$$E(S_2) = \frac{(1-\rho_1(1-\rho_1-\rho_2))/\mu}{(1-\rho_1)(1-\rho_1-\rho_2)}.$$

Model M/M/1 limited queue

$$P_0 = \begin{cases} \dfrac{1-\rho}{1-\rho^{N+1}} & \text{per } \lambda \neq \mu \\[2ex] \dfrac{1}{N+1} & \text{per } \lambda = \mu \end{cases}$$

$$P(n>0) = 1 - P_0$$

$P_n = P_0\rho^n$ per n≤N

$$L_s = \begin{cases} \dfrac{\rho}{1-\rho} - \dfrac{(N+1)\rho^{N+1}}{1-\rho^{N+1}} & \text{per } \lambda \neq \mu \\[2ex] \dfrac{N}{2} & \text{per } \lambda = \mu \end{cases}$$

$$L_q = L_s - (1 - P_0)$$

$$L_b = \frac{L_q}{1-P_0}$$

$$W_s = \frac{L_q}{\lambda(1-P_N)} + \frac{1}{\mu}$$

$$W_q = W_s - \frac{1}{\mu}$$

$$W_b = \frac{W_q}{1-P_0}$$

Model M/M/c limited queue (0< ρ<c)

$$P_o = \frac{1}{\left(\sum_{i=0}^{c-1}\frac{\rho^i}{i!}\right) + \left(\frac{1}{c!}\right)\left(\sum_{i=c+1}^{N}\frac{\rho^i}{c^{i-c}}\right)}$$

$$P_n = \begin{cases} \dfrac{\rho^n}{n!}P_o & \text{per} \quad 0 \le n \le c \\[2ex] \dfrac{\rho^n}{c!\,c^{n-c}}P_0 & \text{per} \quad c \le n \le N \end{cases}$$

$$P(n \ge c) = 1 - P_0\sum_{i=0}^{c-1}\frac{\rho^i}{i!}$$

$$L_s = \frac{P_0\rho^{c+1}}{(c-1)!(c-\rho)^2}\left[1 - \left(\frac{\rho}{c}\right)^{N-c} - (N-c)\left(\frac{\rho}{c}\right)^{N-c}\left(1-\frac{\rho}{c}\right)\right] + \rho(1-P_N)$$

$$L_q = L_s - \rho(1-P_N)$$

$$L_b = \frac{L_q}{P(n \ge c)}$$

$$W_s = \frac{L_q}{\lambda(1-P_N)} + \frac{1}{\mu}$$

$$W_q = W_s - \frac{1}{\mu}$$

$$W_b = \frac{W_q}{P(n \ge c)}$$

Self Service Model M/G/∞ (e=2.718)

$P_n = \dfrac{e^{-\rho}}{n!}\rho^n$ per n≥0

$$L_s = \rho$$

$$W_s = \frac{1}{\mu}$$

Lq results of model M/M/c

λ/μ	c=1	c=2	c=3	c=4	c=5	c=6	c=7	c=8
0,15	0,026	0,001						
0,20	0,050	0,002						
0,25	0,083	0,004						
0,30	0,129	0,007						
0,35	0,188	0,011						
0,40	0,267	0,017						
0,45	0,368	0,024	0,002					
0,50	0,500	0,033	0,003					
0,55	0,672	0,045	0,004					
0,60	0,900	0,059	0,006					
0,65	1,207	0,077	0,008					
0,70	1,633	0,098	0,011					
0,75	2,250	0,123	0,015					
0,80	3,200	0,152	0,019					
0,85	4,817	0,187	0,024	0,003				
0,90	8,100	0,229	0,030	0,004				
0,95	18,050	0,277	0,037	0,005				
1,0		0,333	0,045	0,007				
1,1		0,477	0,066	0,011				
1,2		0,675	0,094	0,016	0,003			
1,3		0,951	0,130	0,023	0,004			
1,4		1,345	0,177	0,032	0,006			
1,5		1,929	0,237	0,045	0,009			
1,6		2,844	0,313	0,060	0,012			
1,7		4,426	0,409	0,080	0,017			
1,8		7,674	0,532	0,105	0,023			
1,9		17,587	0,688	0,136	0,030	0,007		
2,0			0,889	0,174	0,040	0,009		
2,1			1,149	0,220	0,052	0,012		
2,2			1,491	0,277	0,066	0,016		
2,3			1,951	0,346	0,084	0,021		
2,4			2,589	0,431	0,105	0,027	0,007	
2,5			3,511	0,533	0,130	0,034	0,009	
2,6			4,933	0,658	0,161	0,043	0,011	
2,7			7,354	0,811	0,198	0,053	0,014	
2,8			12,273	1,000	0,241	0,066	0,018	
2,9			27,193	1,234	0,293	0,081	0,023	
3,0				1,528	0,354	0,099	0,028	0,008
3,1				1,902	0,427	0,120	0,035	0,010
3,2				2,386	0,513	0,145	0,043	0,012
3,3				3,027	0,615	0,174	0,052	0,015
3,4				3,906	0,737	0,209	0,063	0,019
3,5				5,165	0,882	0,248	0,076	0,023
3,6				7,090	1,055	0,295	0,091	0,028
3,7				10,347	1,265	0,349	0,109	0,034
3,8				16,937	1,519	0,412	0,129	0,041
3,9				36,859	1,830	0,485	0,153	0,050
4,0					2,216	0,570	0,180	0,059
4,1					2,703	0,668	0,212	0,070
4,2					3,327	0,784	0,248	0,083
4,3					4,149	0,919	0,289	0,097
4,4					5,268	1,078	0,337	0,114
4,5					6,862	1,265	0,391	0,133
4,6					9,289	1,487	0,453	0,156
4,7					13,382	1,752	0,525	0,181
4,8					21,641	2,071	0,607	0,209
4,9					46,566	2,459	0,702	0,242