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Assessing lifetime profitability of customers with purchasing cycles

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Abstract

Purpose – The purpose of this paper is to propose a method to help firms assess lifetime profitability of customers whose buying behaviors are characterized by purchasing cycles, which are determined by both intrinsic purchasing cycles and cumulative effects of firms' marketing solicitations.

Design/methodology/approach – This paper first proposes a probability model to predict customers' responses to firms' marketing solicitations in which a customer's inter-purchase times are assumed to follow a Poisson distribution, whose parameters vary across customers and follow a gamma distribution. The paper then proposes a customer profitability scoring model that uses customers' responses as an input to assess their lifetime profitability at a given point of time.

Findings – The paper illustrates the proposed method using individual-level purchasing data of 529 customers from a catalog firm. The paper shows that the proposed model outperforms the benchmark model in terms of both explaining and predicting customers' purchases. The paper also demonstrates significant profit consequences to the firm if incorrect methods are used instead of the proposed method.

Practical implications – The proposed method can help firms select or eliminate customers based on their lifetime profitability so that firms can focus their marketing efforts in a more targeted manner to increase total profits.

Originality/value – The proposed Gamma-Poisson probability model and the profitability scoring method are easy to implement due to the attractive conjugacy property. It is valuable for firms' customer relationship management applications from the standpoint of making customer selection and inventory management decisions.

Keywords CRM, Profitability, Database marketing, Customer selection, Gamma-Poisson, Purchasing cycles

Paper type Research paper

1. Introduction

Assessing customer profitability is integral to the functioning of many firms, especially from the standpoint of customer relationship management. Understanding the lifetime values of individual customers enables firms to focus their marketing efforts in a more targeted manner, and hence boosts the company's bottom-line (Rust *et al.*, 2004). For direct marketers such as the Wisconsin-based American clothing retailer, Lands' End, which sells clothes directly to end customers through catalogs, internet and its own retail stores, quantifying the expected lifetime profitability of their customers can help understand whether it is worthwhile to incur or continue the costs of periodically contacting these customers with direct marketing solicitations. Suppose it costs Lands' End \$2 to send a catalog mailing to John Doe each month, plus another \$1 for related costs such as support-staff time etc. Unless John Doe has a reasonable chance to respond to the Lands' End catalogs in a short to medium term, this effort will not be worthwhile. Therefore, the challenge for any marketer is to identify and separate "gold", "silver" customers from "lead" customers, so that they can cut down wasteful marketing expenditures incurred by contacting "lead" customers, and instead channel these expenditures into better serving their "gold" and "silver" customers over time. This principle of "weeding out the losers and hanging on to the winners" is the cornerstone of database marketing (Reichheld, 1996).

For the purpose of assessing the profitability of a customer, customer-scoring models are used. These models give each customer a score in terms of his or her expected profitability

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of responding to marketing solicitations, and only if the customer's score is high enough to yield expected revenues that are sufficient to offset the firm's costs of contacting the customer, does the firm continue its dialogue with the customer. For a prospect, i.e., a customer with whom the firm has never interacted in the past, the score is typically a function of the prospect's demographic profile and is constructed based on what the firm has understood from demographically similar customers in the past. For example, past data may reveal that high-income prospects are more likely to respond to the marketing offer than low-income prospects. For a customer, i.e., a prospect who has "converted" to buying from the firm, the score depends on not only the customer's demographic profile but also his or her history of responses to previous solicitations from the firm. For example, a customer who buys frequently would have a higher score than the other customer who buys infrequently, even if both have identical demographic profiles.

This study is intended to help direct marketing firms estimate the expected lifetime profitability of customers in any given time, whose buying behaviors are characterized by purchasing cycles. The purchasing cycles are determined by both customers' intrinsic purchasing cycles, which are often dictated by inventory pressure arising from the depletion of the stock of the products, and customers' responsiveness to periodic marketing solicitations. For example, because of inventory effects, a customer is not likely to purchase from Land's End again immediately after an apparel purchase from the catalog, but he or she will start desiring for apparel and return to Lands' End to make purchases again after a while. Alternatively, the customer's interest in buying from Lands' End may be gradually built by being exposed to multiple catalogs, which can eventually "trigger" his or her next purchase. Not recognizing such intrinsic purchasing cycles and cumulative effects of marketing solicitations in customers' buying behaviors may lead firms to falsely conclude that a customer's non-response to marketing solicitations within a short period is indicative of a low lifetime profitability thus prematurely delete the customer, who may indeed make purchases in the future and thus be profitable in a long run.

To assess the expected lifetime profitability of its customers, a firm first needs to estimate the probability that a customer responds to its marketing solicitations with purchases in a period. In this study, we first build a probability model to predict the responses to firms' regular marketing solicitations by customers whose buying behaviors are characterized by purchasing cycles. We assume that a customer's inter-purchase times follow a Poisson distribution which has an "interior" mode (i.e. the parameter of the Poisson distribution) that represents the customer's average inter-purchase time. Since the conditional probabilities derived from the Poisson distribution increase over time, the model nicely captures the increasing inventory pressure, which a customer may feel as time elapses since last purchase, and the cumulative effects of marketing solicitations. We use a gamma distribution to capture the variations in purchasing cycles across customers. This assumption also results in simplicity in computation, making the model easy to implement by marketing practitioners. We name the proposed model as the Gamma-Poisson model.

We next show how to use the proposed Gamma-Poisson model to score a firm's customers in terms of their lifetime profitability based on their expected purchase probabilities over time. The proposed procedure enables the firm to track profitability scores of its customers that vary over time. We discuss possible ways of dealing with optimal customer prospecting and elimination of unprofitable customers using these profitability scores.

We illustrate the proposed method using individual-level purchasing data from 529 customers of a catalog firm. First, we estimate the parameters of the Gamma-Poisson model. We then demonstrate the substantive benefits of employing the proposed method in terms of how one could effectively segment customers based on their expected profitability scores. We believe that the method will greatly assist marketing efforts of firms that possess databases of customers whose purchases are characterized by purchasing cycles.

The rest of the paper is organized as follows. In Section 2, we review the relevant literature and explain the novelty of the research. In Section 3, we describe the proposed Gamma-Poisson model and explain how to use the model to generate profitability scores for prospects and customers. We also discuss how to estimate the parameters of the model. In Section 4, we use a real-world customer database to demonstrate how to use the proposed method and compare its performance with the benchmark model and demonstrate its value in practice. Section 5 summarize the research and discuss the situations when the proposed model is applicable. Finally, we discuss limitations and future research in Section 6.

2. The research background

In our proposed Gamma-Poisson model of customers' buying behaviors, customers' inter-purchase times are assumed to be distributed Poisson and the parameter of this Poisson distribution is assumed to be heterogeneous across customers according to a gamma distribution. Compared to previous studies in the marketing literature, our proposed model has three appealing features. We explain this next.

The first attractive feature of the proposed model lies in the ability of the Poisson distribution to handle customers' purchasing cycles. This is made possible by the fact that the Poisson distribution has an "interior" mode, which can parsimoniously represent customers' average inter-purchase time, and an initially increasing hazard function, which nicely captures increasing inventory pressure that customers may feel as time elapses since last purchase and the cumulative effects of multiple marketing solicitations sent by the firm. Neither of these attractive properties is found in commonly used distributions of inter-purchase times in existing probability models.

One of two commonly used distributions for inter-purchase times in the literature is the exponential distribution in the widely used negative binomial distribution (NBD) model (Schmittlein *et al.*, 1987, Schmittlein and Peterson, 1994; Fader *et al.*, 2005a; Reinartz and Kumar, 2000, 2003; Ho *et al.*, 2006; Abe, 2009; Lam and Mizerski, 2009; Jerath *et al.*, 2011). In the NBD model, a customer's total number of purchases within a given time interval is assumed to be distributed Poisson, implying that the customer's inter-purchase times are distributed exponential. The second commonly used distribution for inter-purchase times is the geometric distribution that underlies customers' inter-purchase times in the beta-geometric model (Rao and Steckel, 1995; Fader *et al.*, 2005b, 2010). Both exponential and geometric distributions have a "left-corner" mode, which implies that a customer's average inter-purchase time is 0, with the likelihood of larger values of inter-purchase times getting progressively smaller, and a flat hazard function, which implies that a customer's conditional probability of buying the product is the same regardless of the time elapsed since last purchase. These features make neither NBD nor beta-geometric model appropriate to be applied to situations when customers' buying behaviors are characterized by purchasing cycles. Ignoring such purchasing cycles when they in fact exist will lead to spurious estimates of a customer's purchase probabilities over time. For example, if the true distribution of inter-purchase times is more regular (as in the presence of purchasing cycles), forcing it to be exponential or geometric (as in the NBD or beta-geometric model, respectively) will lead to over-estimates of purchase probabilities immediately following a purchase, and underestimates at later times since last purchase.

The second attractive feature of the proposed Gamma-Poisson model lies in the flexibility that the gamma distribution can accommodate heterogeneity across customers. It has been used for this purpose in a large number of empirical applications of stochastic models of buyer behavior (e.g. Fader *et al.*, 2005a, b; Batislam *et al.*, 2007; Abe, 2009; Fader and Hardie, 2009; Jerath *et al.*, 2011; Van Oest and Knox, 2011 etc.).

The third attractive feature of the proposed Gamma-Poisson model is that the gamma distribution is conjugate to the Poisson distribution. This aspect of conjugacy in the Gamma-Poisson model renders the posterior estimates of customer-specific purchasing cycles (as will be shown in Section 3.3) to have an analytical closed form, substantially aiding practical computation. Such conjugacy does not characterize many probability models in the marketing literature that also allow for a monotonic increasing hazard, for example, the Weibull Proportional Hazard model (Bhattacharjee *et al.*, 2007; Jung *et al.*, 2012) or the Gompertz Proportional Hazard model (Bemmaor and Glady, 2012). For these reasons, the proposed Gamma-Poisson model is very easy for marketing practitioners to implement, which is a notable practical strength.

To summarize, while the widely used NBD or beta-geometric model is appropriate when customers' conditional probabilities of buying the product are independent of time elapsed since previous purchases, our proposed Gamma-Poisson model is an appropriate alternative when customers' buying behaviors are characterized by purchasing cycles, where customers' conditional probabilities of buying the product increase with time elapsed since previous purchase. In the Gamma-Poisson model, the Poisson distribution is able to handle customers' purchasing cycles on account of the unimodality of its density function, and the gamma distribution is very flexible in terms of accommodating heterogeneity across customers. Just like the NBD or the beta-geometric model, our proposed Gamma-Poisson model also has an analytical closed form, which renders it easy to implement for marketing practitioners.

3. Methodology

We consider a firm that sends marketing solicitations to customers in its mailing list at regular intervals (say, every month). The question of interest is: how does the firm estimate the lifetime profitability of each customer at a given point of time?

We answer this question in five sub-sections: in Section 3.1, we propose a probability model to predict customers' responses to a firm's regular marketing solicitations; in Section 3.2, we present a model of customer profitability and plug in the expected responses of customers; in Section 3.3, we discuss how to estimate customers' expected purchasing cycles according to the Bayesian updating rule; in Section 3.4, we discuss the estimation of model parameters; in Section 3.5, we discuss how a firm can implement the proposed approach to score customers according to their expected lifetime profitability.

3.1 Customer probability model

Our proposed Gamma-Poisson model is built on the premise that different customers require a different number of cumulative marketing solicitations from a firm before their purchasing interests in the product category are "triggered." Additionally, different customers have different intrinsic purchasing cycles for the product category.

Suppose X_n stands for the number of solicitations (since a purchase from the firm) that it will take to induce a purchase from customer n ($n = 1, \ldots, N$). This is a random variable from the standpoint of the firm. We assume that X_n follows a shifted Poisson distribution as shown below:

$$\Pr(X_n = x | \lambda_n) = \frac{e^{-\lambda_n} \lambda_n^{x-1}}{(x-1)!}, \quad (x = 1, 2, \ldots) \tag{1}$$

where $\lambda_n > 0$ is the parameter of the Poisson process governing customer n's purchasing behavior. It represents customer n's purchasing cycle that is determined by both the intrinsic purchasing cycle of the customer and responses to the firm's regular marketing solicitations.

Further, we assume that the Poisson parameter λ_n is different among customers and follows a gamma distribution across customers, as shown below:

$$g_G(\lambda|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda}, \tag{2}$$

where $\alpha,\ \beta > 0$ are parameters of the gamma distribution representing the extent of heterogeneity in λ_n across customers in the market.

3.2 Customer profitability model

Suppose that customer n has already received j marketing solicitations from the firm since the customer's previous purchase from the firm. The expected lifetime profitability of the customer at this moment can be expressed as the expected profitability of the customer's next purchase from the firm as follows[1]:

$$\pi_{nj} = \sum_{k_{nj}=1}^{\infty}\left[P_n - k_{nj}c_n\right]\Pr{}_{n,j+k_{nj}} = P_n - c_n\sum_{k_{nj}=1}^{\infty} k_{nj}\Pr{}_{n,j+k_{nj}}, \tag{3}$$

where P_n is the expected gross profit associated with the product (assumed to be constant across time) from customer n, c_n is the cost of the customer to the firm per contact (also assumed to be constant across time), k_{nj} stands for the number of additional solicitations required to elicit the next purchase from customer n after j solicitations, which follows a shifted Poisson distribution specified in Equation (1), and $\Pr_{n,\,j+k_{nj}}$ stands for the probability that customer n's next purchase will occur following k_{nj} additional solicitations, conditional on whether customer n makes a purchase in response to the jth solicitation.

Depending on whether customer n responds to the jth solicitation with a purchase, we obtain the following expressions for customer profitability under the two cases:

(1) If the customer makes a purchase in response to the jth solicitation, the customer's profitability is:

$$\begin{aligned}\pi_{nj} &= P_n - c_n\sum_{k_{nj}=1}^{\infty} k_{nj}\Pr{}_{n,j+k_{nj}} \\ &= P_n - c_n\sum_{k_{nj}=1}^{\infty}\left(k_{nj}\cdot\frac{e^{-E(\lambda_n)}E(\lambda_n)^{k_n-1}}{(k_n-1)!}\right) \\ &= P_n - c_n(E(\lambda_n)+1).\end{aligned} \tag{4}$$

(2) If the customer does not respond to the jth solicitation, the customer's profitability is:

$$\begin{aligned}\pi_{nj} &= P_n - c_n\sum_{k_{nj}=1}^{\infty} k_{nj}\Pr{}_{n,j+k_{nj}} \\ &= P_n - \frac{c_n(E(\lambda_n)-y_n)}{1-y_n} + c_{n(j-j_0-1)},\end{aligned} \tag{5}$$

where[2]:

$$y_n = \sum_{u=1}^{j-j_0}\frac{e^{-E(\lambda_n)}E(\lambda_n)^{(u-1)}}{(u-1)!}.$$

3.3 Customer-specific expected purchasing cycles

In this section, we derive the customer-specific expected purchasing cycle, $E(\lambda_n)$, in Equations (4) and (5). We consider four levels of information that the firm may have on a customer: no information, demographic information only, purchasing behavior only, and full information (i.e. demographics and purchasing behavior). For each level, we show below how the firm can learn about the expected purchasing cycle of customer n, $E(\lambda_n)$.

Case 1: no information on a customer. If the firm has neither demographic information nor historical purchasing behavior of customer n, $E(\lambda_n)$ is taken by the firm to be equal to the mean of the gamma distribution in Equation (2). In other words:

$$E(\lambda_n) = \frac{\alpha}{\beta}. \tag{6}$$

Case 2: demographic information only. If the firm knows the demographic information but not the historical purchasing behavior of customer n, we can allow the parameters of the gamma distribution to be functions of customer demographics, as shown below (see Gupta and Chintagunta, 1994):

$$\alpha_n = \alpha + Z_n\phi_1,$$
$$\beta_n = \beta + Z_n\varphi_1, \tag{7}$$

where Z_n is a vector of demographic variables characterizing customer n, while ϕ_1 and φ_1 are corresponding vectors of parameters. This, in turn, yields the following:

$$E(\lambda_n) = \frac{\alpha + Z_n\phi_1}{\beta + Z_n\varphi_1}. \tag{8}$$

Case 3: purchasing behavior only. If the firm knows the historical purchasing behavior but not the demographic profile, of a particular customer n, it can employ a Bayesian updating procedure that exploits the attractive property that a gamma distribution is a conjugate prior to a Poisson distribution. Suppose T_n stands for the total number of historical purchases made by the customer in the past, and M_{Tn} stands for the total number of solicitations required by the firm to induce these T_n purchases. The expected customer purchasing cycle from the "prior" mean of gamma distribution given in Equation (2), is Bayesian updated to obtain the following "posterior" mean of gamma distribution:

$$E(\lambda_n) = \frac{\alpha + M_{T_n}}{\beta + T_n}. \tag{9}$$

Case 4: full information. If the firm knows both the demographic profile, as well as the historical purchasing behavior, of customer n, it can use the information to Bayesian update the expected purchasing cycle of the customer as below:

$$E(\lambda_n) = \frac{\alpha + Z_n\phi_1 + M_{T_n}}{\beta + Z_n\varphi_1 + T_n}. \tag{10}$$

3.4 Estimation

We next discuss how to estimate model parameters, i.e., α, β, ϕ_1, φ_1 using the training sample. Suppose the customer database spans the calendar period (t_1, t_2). Let us also suppose that new customer acquisition activity has ceased so that no new customers are acquired by the firm during this period. In this case, the customer database is left-truncated in that the first purchase made by a customer during the study period, in general, is at some point following calendar time t_1. The customer database is also right-truncated in that the most recent purchase of a customer, in general, is at some point prior to calendar time t_2. Within the study period, we observe each purchase made by each customer in the training sample. Recognizing these properties of the training sample, we set up the likelihood function for customer n as shown below:

$$L_n = \int_0^\infty \Pr(X_{n1} > LEFT_n|\lambda_n)\left[\prod_{t=2}^{T_n}\Pr(X_{nt} = x_t|\lambda_n)\right]$$
$$\Pr\left(X_{n,T_{n+1}} > RIGHT_n|\lambda_n\right)g_G\left(\lambda_n|\alpha, \beta\right)d\lambda_n, \quad (11)$$

where X_{nt} stands for the number of solicitations that it takes customer n to make the tth purchase. T_n stands for the total number of purchases made by customer n during the study period. $LEFT_n$ is the number of solicitations received by the customer from t_1 until the time of customer n's first purchase during the study period, $RIGHT_n$ stands for the number of solicitations received by the customer between the last purchase during the study period and t_2, and $g_G(.)$ is the density of the gamma distribution given in Equations (2)[3].

After pooling data across all customers in the training sample, the following sample likelihood function can be maximized to obtain estimates of model parameters, i.e., α, β, ϕ_1, φ_1.

3.5 Implementation algorithm for the firm

A firm can score its existing customers on expected lifetime profitability. The profitability scores are customer-specific and generated by the updated average inter-purchase time of a customer using his or her observed purchases, and the time elapsed since the customer's last purchase with the firm. From the practical standpoint, the firm can proceed as follows:

- Step 1: estimate the parameters of the gamma distribution given in Equations (2) and (11) using historical purchasing data from a "training sample" of customers in the firm's customer database.
- Step 2: depending on the level of information that the firm has on a particular "holdout" customer n, whose profitability the firm wants to assess, use one among Equations (6)-(10) to estimate the expected purchasing cycle of customer n, $E(\lambda_n)$.
- Step 3: depending on whether or not customer n, whose profitability the firm wants to assess, made a purchase in response to the most recent solicitation (jth) from the firm, use Equation (4) or (5) to assess the expected lifetime profitability of the customer, i.e., π_{nj}.

4. Data and empirical application

4.1 Data

In order to demonstrate a real-world application of the proposed approach, we use a customer database of a company (whose identity cannot be revealed for the purpose of confidentiality) that publishes safety-, wellness-, and health-related promotional education materials for both the healthcare industry and the workplace. The company provides these materials – in the form of brochures, booklets, videos, workbooks, newsletters and calendars and electronic products – to hospitals, healthcare providers, care management organizations and government agencies. The major sales channel is through catalogs. Each catalog includes a key code that a customer could use when placing the order. The key code helps the company track the performance of the catalog sent to the customer.

The data contain information on the ordering of brochures by 529 healthcare professionals who made two or more orders from July 2002 to August 2003. The data set also contains information on catalogs sent to those customers during this period, including both the number and types of catalogs. Even though the catalogs are mailed to all existing customers at the same frequency, different customers may have received different number of catalogs because different customers "enter" the customer database at different times during the study period.

The average number of catalogs that a customer receives over the period is 7.8, and the average number of catalogs to which each customer responds is 2.2. We observe only whether or not a customer made a purchase in response to a catalog mailing but not the exact date of the order.

This renders inter-purchase times in our data set to be discrete (rather than continuous). Each customer is observed to make either no purchase or one purchase in response to a catalog mailing, therefore, multiple orders from a customer between successive catalog mailings does not occur. The average quantity ordered per response is 33 brochures and the average gross profit when a customer responds to the catalog is $58.60, where the gross profit is defined as the revenues that are generated from the transaction minus the costs of producing the products sold, which are about $0.10 per product. The average mailing cost per catalog is about $1.80. We randomly select 424 customers as the calibration sample and the remaining 105 customers as the holdout sample. The data set also includes two demographic characteristics of customers:

(1) Practice type: private practice customers with fewer than six doctors, vs group practice customers with six or more doctors. We code practice type using an indicator variable that takes the value 1 for a private practice customer and the value 0 otherwise.

(2) Practice Area: primary doctors vs specialists vs others. We code practice area using two indicator variables. One takes the value 1 for a primary doctor and the value 0 otherwise while the other takes the value 1 for a specialist and the value 0 otherwise.

4.2 Benchmark model

We use the beta-geometric model (Rao and Steckel, 1995; Fader *et al.*, 2005b, 2010) as the benchmark model for three reasons: the beta-geometric model is a widely used model in the literature (e.g. Colombo, 2000; Batislam *et al.*, 2007; Van Oest and Knox, 2011; Gopalakrishnan *et al.*, 2016, to name a few); inter-purchase times in our data are discrete, which necessitates the use of discrete-time distributions; just like the Gamma-Poisson model, the beta-geometric model also has the attractive conjugacy property, i.e., beta is conjugate to geometric distribution.

4.3 Empirical results

First, we estimate the proposed Gamma-Poisson model using the calibration sample of 424 customers and applying the maximum likelihood technique. We also estimate the benchmark beta-geometric model. We then compute a validation log-likelihood measure for the observed purchases in the holdout sample, based on the estimates from the calibration sample. The fit results are reported in Table I. Based on fit criteria of log-likelihood values, we conclude that the proposed Gamma-Poisson model is superior to the benchmark model in terms of both explaining the observed purchases in the calibration sample (−1490.82 vs −1967.63) and predicting observed purchases in the holdout sample (−369.81 vs −489.02). The results lend strong empirical support to the Gamma-Poisson model over the benchmark model, indicating that the effects of purchasing cycles are important in our empirical context.

Next, we report the estimates yielded by the Gamma-Poisson model in Table II[4]. Our estimated value of α is found to be 3.81. This implies that the estimated baseline (i.e. ignoring customer demographics) average purchasing cycle across the 424 customers in the calibration sample is equal to $(E(\lambda_n)+1) = (\alpha/\beta+1) = (3.81+1) = 4.81$ mailings. In contrast, the benchmark model implies a baseline purchasing probability of 0.27 for the customer, implying a baseline average purchasing cycle of 2.66 mailings. In other words, the benchmark model is over-optimistic about customer inter-purchasing cycles than the Gamma-Poisson model. Further, we find in Table II that none of the parameters in the Gamma-Poisson model that are associated with customer demographics (i.e. ϕ_1) is statistically significant. This implies that there is no observed heterogeneity across customers though there is significant unobserved heterogeneity.

Next, we score our 105 holdout customers based on their expected lifetime profitability using the proposed algorithm detailed in Section 3. In our empirical application, since we observe both demographic profiles and historical purchasing behavior of all 105 customers in the holdout sample, we first compute each customer's average inter-purchasing cycle. We then score each holdout customer at the time of the last mailing from the firm to the customer. We generate the cumulative response curve for the firm by rank-ordering the 105 holdout customers in decreasing order of their expected profitability, and then plot the expected cumulative profits as a function of the number of customers contacted (where we assume that customers are contacted in decreasing order of expected profitability). We generate these plots for rank-ordering implied by three models: the proposed model, the benchmark model, and a naïve model that ranks customers at random[5]. In Figure 1, we plot the difference in expected cumulative profits that will accrue to the firm from using the Gamma-Poisson model, rather than using the benchmark model or the naïve model, to rank its customers. Figure 1(a) reports the plot for the calibration sample, while Figure 1(b) reports it for the holdout sample.

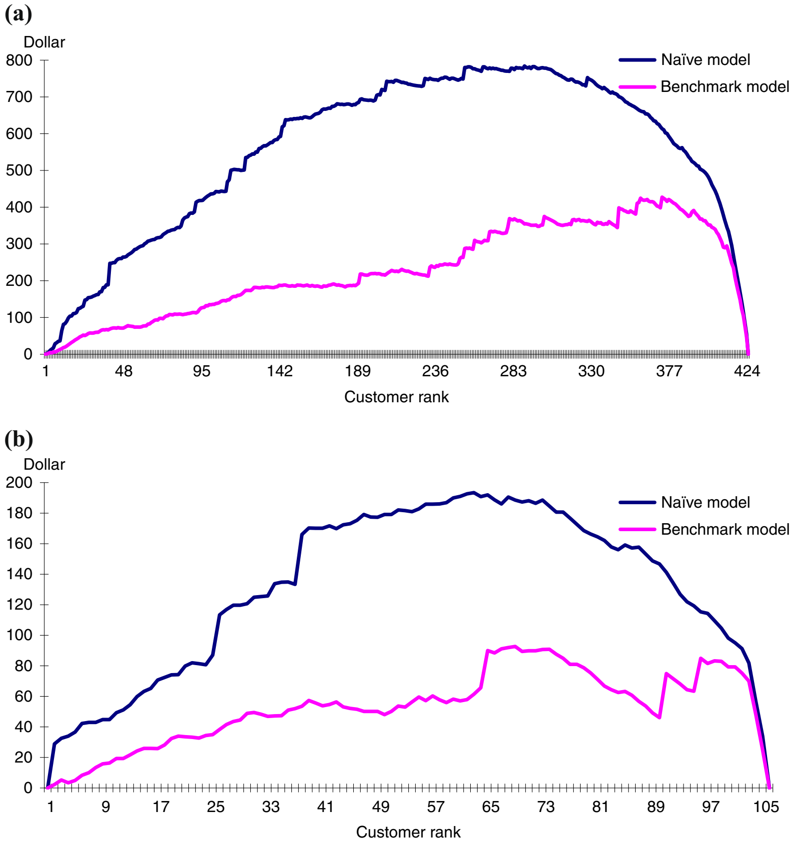
First, it is useful to note that as long as the firm either does not contact any customer at all, or decides to contact all customers in the sample, the Gamma-Poisson model offers no benefit over either the beta-geometric model or the naïve model. What is important about the plots is that the difference in cumulative profits, between not only the Gamma-Poisson model and naive model but also the Gamma-Poisson model and beta-geometric model, is always positive as long as one is not at the end-points of the curve. In other words, as long as the firm wants to exploit the rank-ordering of customers and contact only a subset of its customer database, the Gamma-Poisson model yields higher profits than both the beta-geometric model and the naïve model. The difference between the Gamma-Poisson

Table I.
Fit results

Fit criterion	Fit value
In-sample log-likelihood for Gamma-Poisson model	−1490.82
In-sample log-likelihood for beta-binomial model	−1967.63
Holdout validation log-likelihood for Gamma-Poisson model	−369.81
Holdout validation log-likelihood for beta-binomial model	−489.02

Table II.
Estimation results for gamma-poisson model (Standard Errors within Parentheses)

Parameter	Estimate
Constant (α)	3.81 (0.35)
$\phi_{1,\ \text{Private Practice}}$	0.09 (0.36)
$\phi_{1,\ \text{Primary Doctors}}$	0.17 (0.97)
$\phi_{1,\ \text{Specialists}}$	−0.11 (0.25)



Notes: (a) when ranking customers in calibration sample between proposed and comparison models; (b) when ranking customers in holdout sample between proposed and comparison models

Figure 1. Difference in expected cumulative profits

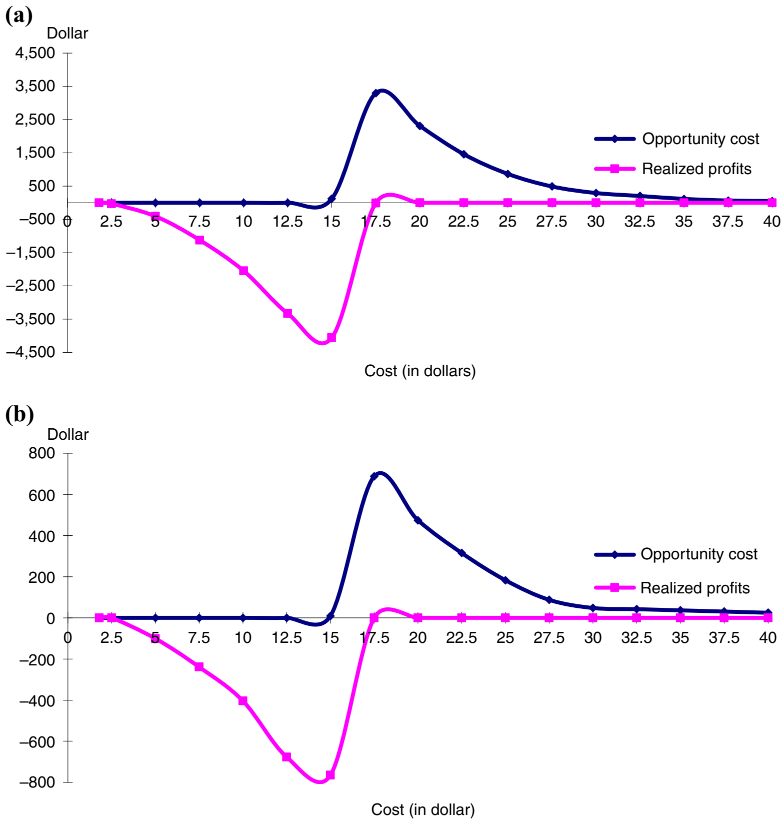
model and beta-geometric model in absolute profit terms is most pronounced if the firm decides to contact the top 380 customers in the calibration sample. The resulting difference in profits is about $425. The percentage differences between the Gamma-Poisson model and beta-geometric model are found to vary from 2-5 percent depending on what fraction of the customer database is chosen for mailing.

To the extent that the beta-geometric model itself is clearly a modeling improvement over the naïve scoring models typically used in practice, an incremental profit lift of 2-5 percent speaks well of the practical efficacy of the Gamma-Poisson model. In relation to the naïve model, the Gamma-Poisson model wins even more convincingly, as is made clear by Figure 1. In percentage terms, the difference between the profits implied by the Gamma-Poisson model and the profits implied by the naïve model is found to be as high as 51 percent.

In order to understand the substantive consequences to the firm of using the wrong distribution of inter-purchase times when calculating customer profitability metrics across customers, we compute the realized loss as the sum of the expected losses across all customers who will be wrongly included by the beta-geometric model in the mailing list when the Gamma-Poisson model shows that they are unprofitable. We also compute the opportunity cost as the sum of the expected lost profits across all customers who will be wrongly excluded by the beta-geometric model from the mailing list when the Gamma-Poisson model shows that they are profitable. We compute both measures under various assumptions of a customer's mailing cost, *c,* and report them in Figure 2. Interestingly, we see that both the realized loss and the opportunity cost are the highest for an intermediate value of *c*. This can be understood as follows: for very small values of *c*, practically every customer will be profitable; similarly, for very large values of *c*, practically every customer will be unprofitable; therefore, the value of the Gamma-Poisson model in terms of correctly segmenting customers in both scenarios is likely to be small. For intermediate values of *c,* the corresponding value of the Gamma-Poisson model to correctly include good customers and exclude bad customers is significant.

5. Summary and conclusions

In this study, we propose a customer profitability scoring method that can help firms, whose customers' buying behaviors are characterized by purchasing cycles, select or eliminate



Notes: (a) when using benchmark model to determine the inclusion of customers in calibration sample; (b) when using benchmark model to determine the inclusion of customers in holdout sample

Figure 2.
The opportunity cost and realized profits

customers based on their expected lifetime profitability at a given time point to implement more targeted marketing effort to increase profits. The proposed method is built on a Gamma-Poisson model to predict customers' responses to the firms' regular marketing solicitations. In the model, customers' inter-purchase times are assumed to follow a Poisson distribution, whose parameters vary across customers and follow a gamma distribution. The attractive property of a gamma distribution with a conjugate prior to a Poisson distribution results in simplicity in computation, which is valuable in practice.

We illustrate the proposed method using individual-level purchasing data from a catalog firm. We show that the proposed Gamma-Poisson model outperforms the benchmark beta-geometric model in both explaining and predicting customers' purchases. We also demonstrate significant profit consequences to the firm if incorrect methods of customer selection are used.

Our proposed method is appropriate for direct marketing applications when a firm sends regular marketing solicitations to its customers or prospects whose purchase responses are affected by their own intrinsic purchasing cycles and these solicitations. The marketing solicitations can be in various forms, such as face-to-face selling, catalogs, direct mails, print or e-mail coupons, and online or mobile product recommendations, etc. The model should be used by firms whose objective is to decide which customers to select for marketing solicitations. To implement our model, the firm should possess a customer database, which contains customer purchase information such as when an individual customer responds to the firm's regular marketing solicitations with purchases. The firm also needs to know the gross profit from a customer's purchase and cost of its marketing solicitations. It is also helpful to have descriptive information about individual customers such as demographics.

6. Limitations and future research

There are some limitations of this study and a few potential areas for future research. In the study, we assume that the gross profit from a customer to a firm is constant over time, but it can also be a random variable from the firm's perspective. It may be useful to add an expenditure model to our proposed inter-purchase timing model and, correspondingly, develop a more comprehensive customer profitability model. Extending this point further, customers' purchasing decisions may be multi-dimensional, for example, manifesting themselves as whether or not to buy from the firm, which product to choose (Li *et al.*, 2005) and how much to buy etc. Incorporating such multi-dimensional decision-making within the proposed framework will be useful. Finally, it will be of interest to develop a model that can be used to optimize customer-specific marketing solicitation frequencies for the firm (Gonul *et al.*, 2000; Gonul and Hofstede, 2006; Simester *et al.*, 2006).

We do not address the above issues in this study for three main reasons: our interest lies in demonstrating the importance of modeling customers' purchasing cycles, and this can be done using an inter-purchase timing model such as our proposed Gamma-Poisson model; our empirical data does not show significant variations in customers' purchase quantities, expenditures or gross profits over time; and we want to exploit the conjugacy of the gamma and Poisson distributions to propose an analytically appealing and easy-to-implement modeling approach for firms. We believe that our proposed customer profitability framework will greatly assist direct marketing applications of firms that possess customer databases and would like to implement marketing strategies based on data. We also hope that our study spurs future research in this area.

Notes

1. The proof is available from authors upon request.
2. The derivation of this expression is available from authors upon request.

3. The detailed expression of the likelihood function is available from authors upon request.
4. We estimate the standard gamma distribution, which restricts the scale parameter of the Gamma distribution, β, to be equal to 1 and only estimates the shape parameter, α in the empirical application.
5. Under the benchmark model, although we rank-order customers based on its implied customer profitability metrics, we must still calculate cumulative profits for the y-axis based on what is implied by the proposed Gamma-Poisson model. Under the naïve model, there are no implied profits, so it is easier to understand that the cumulative profits are calculated based on the Gamma-Poisson model.

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Further reading

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