Ajani Stewart

Problem 1 $I = I(x)$ is a 1-D signal

$$F(u) = \int_{-\infty}^{\infty} I(x) e^{-i2\pi ux} dx$$

$I'(x)$ is the derivative of the signal

$$G(u) = \int_{-\infty}^{\infty} I'(x) e^{-i2\pi ux} dx$$
$$= I(x) e^{-i2\pi ux} \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} I(x) e^{-i2\pi ux} \cdot -i2\pi u \, dx$$
$$= i2\pi u \int_{-\infty}^{\infty} I(x) e^{-i2\pi ux} dx$$
$$= i2\pi u F(u)$$
$$|G(u)| = 2\pi |u F(u)|$$

For large values of u, the magnitude of the signal will be amplified, corresponding to amplifying the larger components of the signal

b) $n = n(x) = \varepsilon \sin(\omega x)$ for some small ε and large ω

$$\frac{d(I+n)(x)}{dx} = I'(x) + \omega \varepsilon \cos(\omega x)$$

$$(I+n)'(x) - I'(x) = I'(x) + \omega\varepsilon\cos(\omega x) - I'(x)$$
$$= \omega\varepsilon\cos(\omega x)$$

~~For very large values of ω~~, A large value of ω can lead to a large change in derivative, even if ε is small

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Problem 2

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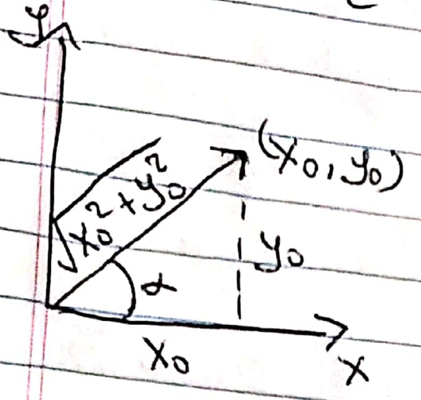

$\cancel{x\cos\theta}$

$$x\cos\theta + y\sin\theta = \rho$$

$$x_0\cos\theta + y_0\sin\theta = \rho$$

$$\cos\alpha = \frac{x_0}{\sqrt{x_0^2+y_0^2}}$$

$$\sin\alpha = \frac{y_0}{\sqrt{x_0^2+y_0^2}}$$

$$\rho = x_0\cos\theta + y_0\sin\theta$$

$$= \sqrt{x_0^2+y_0^2}\left(\frac{x_0}{\sqrt{x_0^2+y_0^2}}\cos\theta + \frac{y_0\sin\theta}{\sqrt{x_0^2+y_0^2}}\right)$$

$$= \sqrt{x_0^2+y_0^2}\,(\cos\alpha\cos\theta + \sin\alpha\sin\theta)$$

By sum-difference formula

$$\rho = \sqrt{x_0^2+y_0^2}\cos(\theta-\alpha)$$

The amplitude of the resultant sinusoid is equal to the magnitude/length of the vector $\langle x_0, y_0 \rangle$. The phase is shifted by the direction of the vector which is equal to $\tan^{-1}(y_0/x_0)$. The period and freqrency remains the same.