

ASSIGNMENT

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Question 12.13.6.11 In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.

Solution: Random variables defined as

Random Variable	Values	Description
X	$\{0, 1, 2, 3\}$	6 appeared roll

k be roll on which 6 appeared. Let m_k be money recieved till 6 appeared.

$$p_X(k) = \begin{cases} \left(\frac{5}{6}\right)^{k-1} \frac{1}{6} & \text{if } k \in \{1, 2, 3\} \\ \left(\frac{5}{6}\right)^3 & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$m_k = \begin{cases} (-1)(k-1) + 1 & \text{if } k \in \{1, 2, 3\} \\ -3 & \text{if } k = 0 \end{cases} \quad (2)$$

Calculating the expected value

$$\text{Expected value} = \sum_{k=0}^3 m_k p_X(k) \quad (3)$$

$$= \left(1 \times \frac{1}{6}\right) + \left(0 \times \frac{5}{36}\right) + \left(-1 \times \frac{25}{216}\right) + \left(-3 \times \frac{125}{216}\right) \quad (4)$$

$$= \frac{1}{6} - 0 + \left(-\frac{25}{216}\right) - \frac{375}{216} \quad (5)$$

$$= \frac{36}{216} - 0 + \left(-\frac{25}{216}\right) - \frac{375}{216} \quad (6)$$

$$= \frac{36 - 0 - 25 - 375}{216} \quad (7)$$

$$= -\frac{364}{216} \quad (8)$$

$$\approx -1.685 \quad (9)$$