Probability Assignment

EE22BTECH11028-Katherapaka Nikhil*

Question:If X follows a binomial distribution with parameters n = 5, p and $p_X(2) = 9p_X(3)$ then p is?

Solution:

$$\mu = np \tag{1}$$
$$= 5p \tag{2}$$

$$\sigma^2 = np(1-p) \tag{3}$$
$$= 5p(1-p) \tag{4}$$

$$Y \sim N(\mu, \sigma) \tag{5}$$

Using the condition $p_Y(2) = 9p_Y(3)$, we get:

$$e^{-\frac{1}{2}(\frac{2-\mu}{\sigma})^2} = 9e^{-\frac{1}{2}\left(\frac{3-\mu}{\sigma}\right)^2} \tag{6}$$

$$\implies e^{-\frac{1}{2}\left(\frac{2-5p}{\sqrt{5p(1-p)}}\right)^2} = 9e^{-\frac{1}{2}\left(\frac{3-5p}{\sqrt{5p(1-p)}}\right)^2} \tag{7}$$

$$\implies e^{-\frac{1}{2}\left(\frac{2-5p}{\sqrt{5p(1-p)}}\right)^2} = 9e^{-\frac{1}{2}\left(\frac{3-5p}{\sqrt{5p(1-p)}}\right)^2} \tag{8}$$

$$\implies e^{-\frac{1}{2}\left(\frac{(2-5p)^2-(3-5p)^2}{(\sqrt{5p(1-p)})^2}\right)} = 9 \tag{9}$$

Taking the natural logarithm of both sides, we have:

$$\implies -\frac{1}{2}\left(\frac{(2-5p)^2-(3-5p)^2}{5p(1-p)}\right) = \ln(9) \tag{10}$$

$$\implies 4 + 25p^2 - 20p - 9 - 25p^2 + 30p = -10p(1-p)\ln(9) \tag{11}$$

$$\implies 10p - 5 = -10p(1-p)\ln(9) \tag{12}$$

$$\implies 1 - 2p = (2p - 2p^2)\ln(9) \tag{13}$$

$$\implies 2p^2\ln(9) - 2p\ln(9) - 2p + 1 = 0 \tag{14}$$

$$p = \frac{2\ln(9) + 2 \pm \sqrt{(-2\ln(9)-2)^2 - 4(2\ln(9))(1)}}{2(2\ln(9))} \tag{15}$$

$$= \frac{2\ln(9) + 2 \pm \sqrt{4(\ln(9))^2 + 4}}{4\ln(9)} \tag{16}$$

$$= 0.178211588 \tag{17}$$