

EE22BTECH11029 - Komakula Sreeja

Question 9.3.16

Suppose that 90% of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.

Solution: Given that 90% of the people are right-handed.

TABLE 1: Description of random variables

Parameters	Values	Description
n	10	Sample space
p	0.9	Probability that the person is right-handed
Y	$0 \leq Y \leq 10$	Number of people thatare right-handed
$\mu = np$	9	Mean
$\sigma = \sqrt{np(1-p)}$	0.9	Standard deviation

Gaussian Distribution

Central limit theorm:

$$Y \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \quad (1)$$

(2)

CDF of Y is

$$F_Y(y) = \Pr(Y \leq y) \quad (3)$$

We know that

$$Q(x) = \Pr(X > x), x > 0, X \sim N(0, 1) \quad (4)$$

$$Q(-x) = \Pr(X > -x), x < 0, X \sim N(0, 1) \quad (5)$$

$$= 1 - Q(x) \quad (6)$$

Hence,

CDF :

$$F_Y(y) = \begin{cases} 1 - Q\left(\frac{y-\mu}{\sigma}\right), & \text{if } y > \mu \\ 1 - Q\left(\frac{y-\mu}{\sigma}\right) = Q\left(\frac{\mu-y}{\sigma}\right), & \text{if } y < \mu \end{cases} \quad (7)$$

With a 0.9 correction:

$$F_Y(6) = \Pr(Y < 6.9) \quad (8)$$

$$= 1 - Q\left(\frac{6.9 - 9}{\sqrt{0.9}}\right) \quad (9)$$

$$= Q\left(\frac{2.1}{0.9487}\right) \quad (10)$$

$$= Q(2.21) \quad (11)$$

$$= 0.013553 \quad (12)$$

Without correction:

$$F_Y(6) = \Pr(Y \leq 6) \quad (13)$$

$$= 1 - Q\left(\frac{6-9}{\sqrt{0.9}}\right) \quad (14)$$

$$= Q\left(\frac{3}{0.9487}\right) \quad (15)$$

$$= Q(3.1622) \quad (16)$$

$$= 0.000783 \quad (17)$$

TABLE 2: Comparision

Number of people(RH)	Binomial	Gaussian	Gaussian(C)	Error(%)	Error(C)(%)
Atmost 6	0.012795	0.000783	0.013553	-93.88	55.92

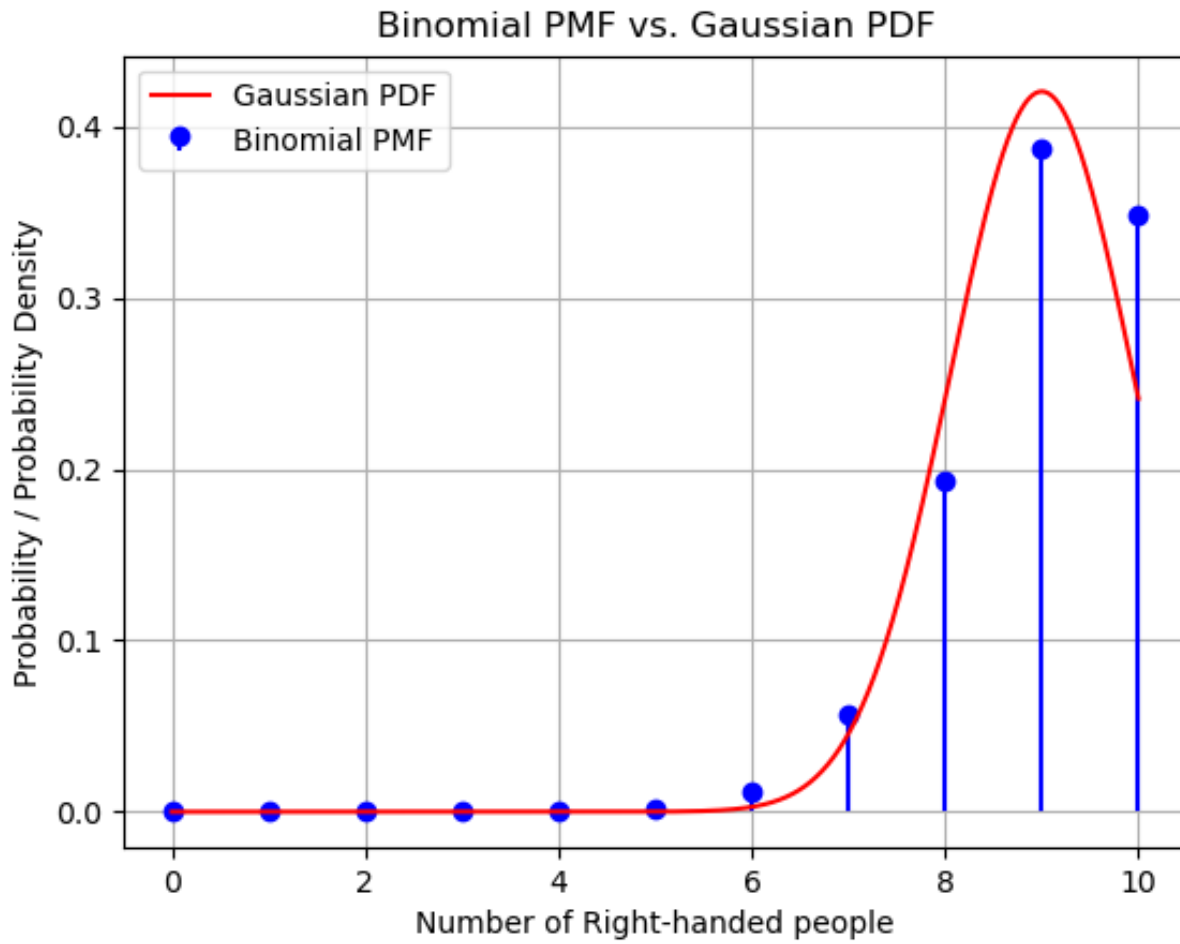


Fig. 1: Binomial vs Gaussian