## 1

## EE22BTECH11029 - Komakula Sreeja

## **Ouestion 9.3.16**

Suppose that 90% of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.

**Solution:** Given that 90% of the people are right-handed.

TABLE 1: Description of random variables

Parameters	Values	Description	
n	10	Sample space	
p	0.9	Probability that the person is right-handed	
Y	$0 \le Y \le 10$	Number of people thatare right-handed	
$\mu = np$	9	Mean	
$\sigma = \sqrt{np(1-p)}$	0.9	Standard deviation	

## **Gaussian Distribution**

Central limit theorm:

$$Y \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$
 (1)

(2)

CDF of Y is

$$F_Y(y) = \Pr(Y \le y) \tag{3}$$

We know that

$$Q(x) = \Pr(X > x), x > 0, X \sim N(0, 1)$$
(4)

$$Q(-x) = \Pr(X > -x), x < 0, X \sim N(0, 1)$$
(5)

$$=1-Q(x) \tag{6}$$

Hence,

CDF:

$$F_{Y}(y) = \begin{cases} 1 - Q\left(\frac{y - \mu}{\sigma}\right), & \text{if } y > \mu \\ 1 - Q\left(\frac{y - \mu}{\sigma}\right) = Q\left(\frac{\mu - y}{\sigma}\right), & \text{if } y < \mu \end{cases}$$
 (7)

With a 0.9 correction:

$$F_Y(6) = \Pr(Y < 6.9)$$
 (8)

$$=1-Q\left(\frac{6.9-9}{\sqrt{0.9}}\right) \tag{9}$$

$$=Q\left(\frac{2.1}{0.9487}\right) \tag{10}$$

$$= Q(2.21)$$
 (11)

$$=0.013553$$
 (12)

Without correction:

$$F_Y(6) = \Pr(Y \le 6) \tag{13}$$

$$=1-Q\left(\frac{6-9}{\sqrt{0.9}}\right)\tag{14}$$

$$= 1 - Q\left(\frac{6-9}{\sqrt{0.9}}\right)$$
 (14)  
=  $Q\left(\frac{3}{0.9487}\right)$  (15)

$$= Q(3.1622) \tag{16}$$

$$= 0.000783$$
 (17)

TABLE 2: Comparision

Number of people(RH)	Binomial	Gaussian	Gaussian(C)	Error(%)	Error(C)(%)
Atmost 6	0.012795	0.000783	0.013553	-93.88	55.92

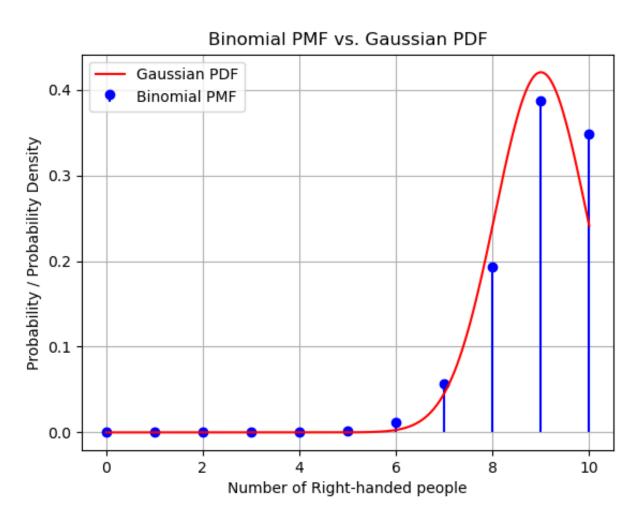


Fig. 1: Binomial vs Gaussian