Chapter-01
Introduction to Control sys.

* Consider liquid level control system whose control objective is to keep the water level into tank at a height 'h'.

* Controller is a automatic device with error signal E(s) as i/p & controller o/p P(s) affecting the dynamics of plant to acheive the control objective.

  Therefore, controller o/p $p = f(e)$

  where e = steady state error.

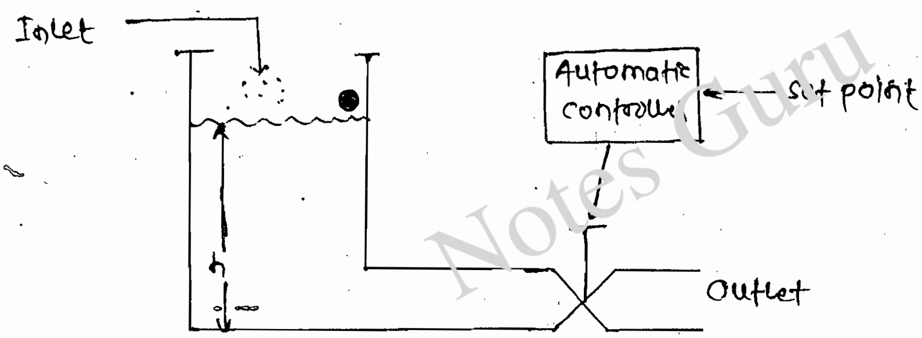
* The different modes of controller operation are proportional, proportional + integral & proportional + integral + derivative.

* There are 2 basic control loop configurations:-

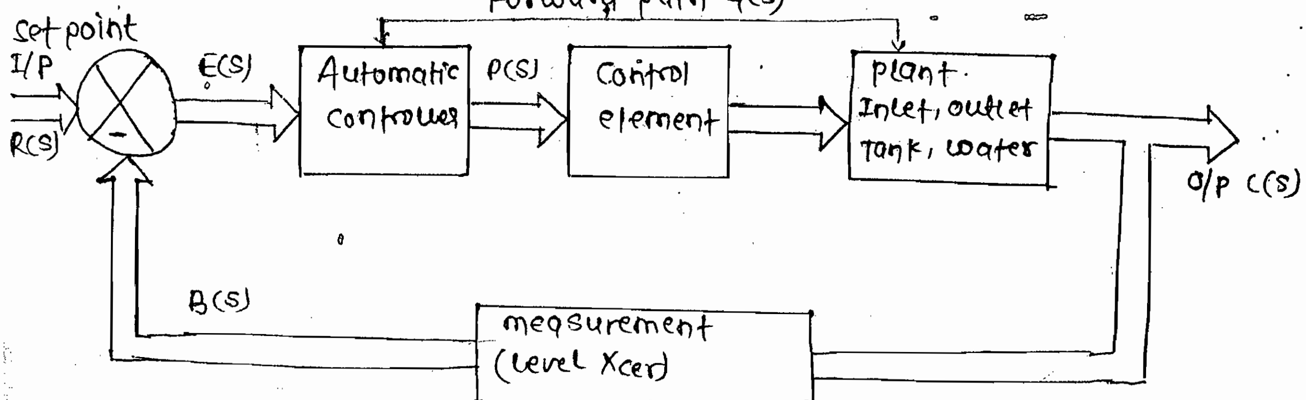
  (1.) Closed loop (or) Feedback control system

  ⇒ In this configuration the changes in the o/p are measured through F/B & compared with the i/p (or) set point to acheive control objective.

  ⇒ Feeback employs measurement (sensor or Xcer)



(control element)



Backward path H(s)

$$E(s) = R(s) - B(s)$$

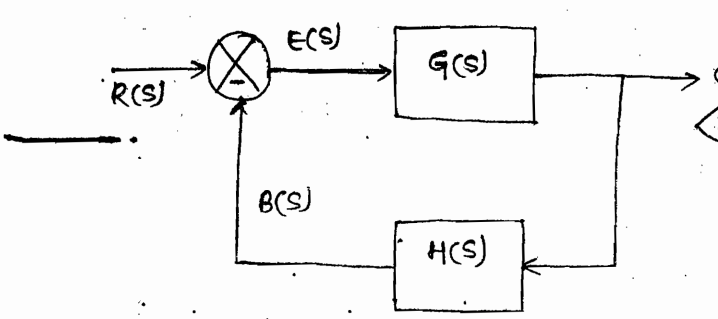
$$\frac{C(s)}{G(s)} = R(s) - C(s) \cdot H(s)$$

$$C(s) = G(s)R(s) - G(s) \cdot H(s)\, C(s)$$

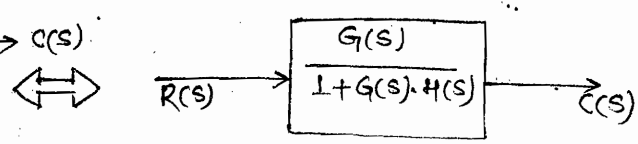
$$C(s)\left[1 + G(s) \cdot H(s)\right] = G(s)\, R(s)$$

$$\boxed{C(s) = \left[\frac{G(s)}{1 + G(s) \cdot H(s)}\right] R(s)}$$

Control canonical form →



Eq. mathematical model →



* Sensitivity Analysis →

Let α = A variable that changes its value

β = A parameter that changes the value of 'α'

$$S_{\beta}^{\alpha} = \frac{\%\text{ change in } \alpha}{\%\text{ change in } \beta}$$

$$\boxed{S_{\beta}^{\alpha} = \frac{\frac{\partial \alpha}{\alpha}}{\frac{\partial \beta}{\beta}} = \frac{\beta}{\alpha} \cdot \frac{\partial \alpha}{\partial \beta}}$$

Sensitivity of a closed loop control system (CLCS) $\rightarrow$

Let $\alpha = m(s) =$ CLCS

$\beta =$ disturbances in forward path elements

$$S^{m(s)}_{G(s)} = \frac{G(s)}{m(s)} \cdot \frac{\partial m(s)}{\partial G(s)}$$

Since $m(s) = \dfrac{G(s)}{1+G(s)\cdot H(s)}$

$$\frac{G(s)}{m(s)} = 1+G(s)\cdot H(s)$$

$$\frac{\partial m(s)}{\partial G(s)} = \frac{\partial}{\partial G(s)}\left[\frac{G(s)}{1+G(s)\cdot H(s)}\right]$$

$$= \frac{1+G(H)\cdot H(s) - G(s)\cdot H(s)}{[1+G(s)\cdot H(s)]^2}$$

$$= \frac{1}{[1+G(s)\cdot H(s)]^2}$$

$$S^{m(s)}_{G(s)} = \cancel{[1+G(s)\cdot H(s)]} \times \frac{1}{[1+G(s)\cdot H(s)]^{\cancel{2}}}$$

$$\boxed{S^{m(s)}_{G(s)} = \frac{1}{1+G(s)\cdot H(s)}}$$

$1+G(s)\cdot H(s) =$ Noise reduction factor (OR) Return diff.

* Sensitivity of ~~clcs~~ CLCS with respect to disturbances in H(s) i.e. F/b elements $\rightarrow$

$\alpha = m(s) =$ CLCS

$\beta =$ disturbances in F/b elements i.e. $H(s)$

$$S^{m(s)}_{H(s)} = \frac{H(s)}{m(s)} \cdot \frac{\partial m(s)}{\partial H(s)}$$

Since $m(s) = \dfrac{G(s)}{1+G(s)\cdot H(s)}$

$$\frac{M(s)}{H(s)} = \frac{G(s)}{H(s)[1+G(s)\cdot H(s)]}$$

$$\frac{H(s)}{M(s)} = \frac{H(s)[1+G(s)\cdot H(s)]}{G(s)}$$

$$\frac{\partial M(s)}{\partial H(s)} = \frac{\partial}{\partial H(s)}\cdot\left[\frac{G(s)}{1+G(s)\cdot H(s)}\right]$$

$$= \frac{[1+G(s)\cdot H(s)]\times 0 - G(s)\cdot G(s)}{[1+G(s)\cdot H(s)]^2}$$

$$= \frac{-[G(s)]^2}{[1+G(s)\cdot H(s)]^2}$$

$$S^{M(s)}_{H(s)} = \frac{H(s)\cdot[1+\cancel{G(s)\cdot H(s)}]}{G(s)} \times \frac{-[G(s)]^2}{\cancel{[1+G(s)\cdot H(s)]^2}}$$

$$\boxed{S^{M(s)}_{H(s)} = \frac{-G(s)\cdot H(s)}{1+G(s)\cdot H(s)}}$$

$$\boxed{\left|S^{M(s)}_{H(s)}\right| = \frac{G(s)\cdot H(s)}{1+G(s)\cdot H(s)}}$$

Note → The CLCS is more sensitive to disturbances in F/b elements i.e. $H(s)$ then forward path element i.e. $G(s)$.

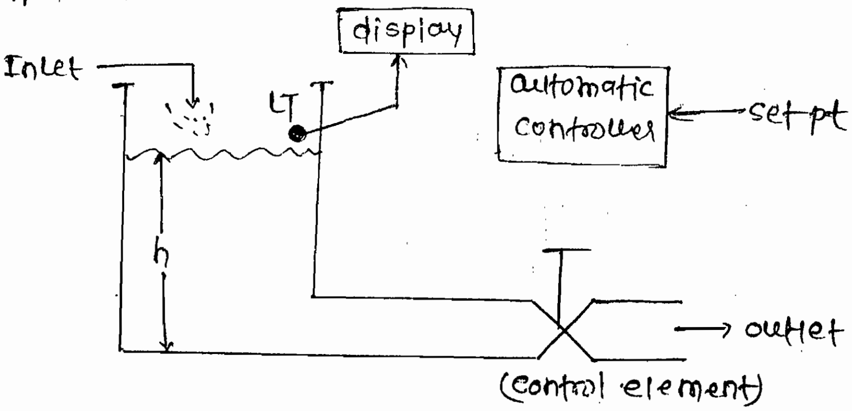
Open loop Control system (OLCS) →

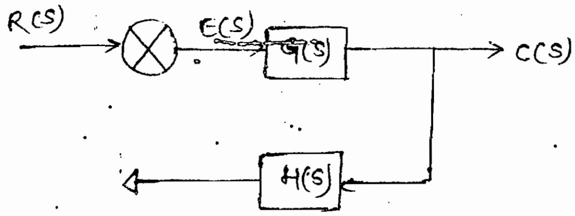
(1.) They are conditional control sys. formulated under the basic condn that the sys. is not subjected to any type of disturbances.

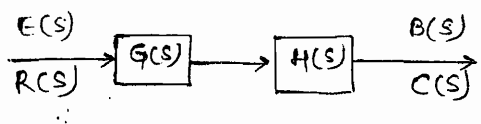
(2.) In this configuration the F/b (or) measurement is not connected to forward path (or) controller (open loop).

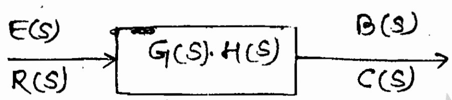
(3.) F/b in open loop sys. except for displaying the information about o/p do not have any major significance. This insignificantness of F/b is known as elimination of F/b

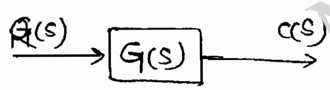
4.) Performance analysis is not applicable to these systems because they are not subjected to any type of disturbances. & give out a desired o/p for the desired i/p.











Sensitivity of OLCS wrt disturbances in G(s)

Let $\alpha = m(s) = \text{OLCS}$

$\beta = G(s)$.

$$S^{m(s)}_{G(s)} = \frac{G(s)}{m(s)} \cdot \frac{\partial m(s)}{\partial G(s)}$$

$$m(s) = G(s) \cdot H(s)$$

$$\frac{G(s)}{H(s)} = \frac{1}{G(s)}$$

$$\frac{\partial M(s)}{\partial G(s)} = \frac{\partial}{\partial G(s)}\left[G(s)\cdot H(s)\right] = H(s)$$

$$S^{M(s)}_{G(s)} = \frac{1}{\cancel{H(s)}} \times \cancel{H(s)}$$

$$\boxed{S^{M(s)}_{G(s)} = 1}$$

Que.→ The OL & CL sys. are shown in Figs
For 10% change in the forward path, the responces of open loop sys. & CL sys. will be affected by:-

(a.) 10%, 1%
(b) 10%, 0.1%
(c) 1%, 10%
(d) 1%, 0.1%

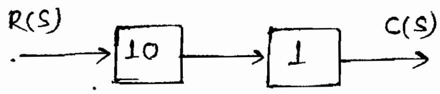


Fig. 01

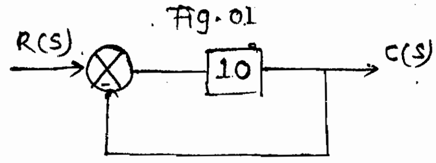


Fig:-02

Sol^n→

OPCS → 10%

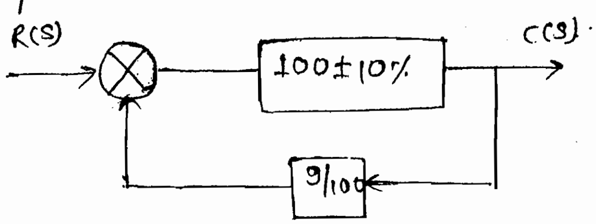
CLCS → $\frac{1}{1+G(s)\cdot H(s)} \times 10\%$

$= \frac{1}{1+10\times1} \times 10\%$

$= 0.1\%$

Ans. (b)

Que.(2.) The closed loop gain when f/b of gain 9/100 is connected in the sys. as shown below will be:-



(a.) 10±10% (b.) 10±1% (c) 100±10% (d) 100+1%

Solⁿ →

$$\text{C.L. gain} = \frac{G}{1+GH}$$

$$= \frac{100}{1+100\times\frac{9}{100}}$$

$$\text{CL gain} = 10$$

$$= \frac{1}{1+GH}\times 10\%$$

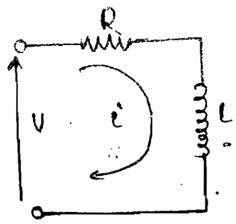
$$= \frac{1}{1+100\times\frac{9}{100}}\times 10\%$$

$$= \frac{1}{10}\times 10\% = 1\%$$

* Concept of TF →

A TF is a mathematical model representing the CS relating i/p & o/p in the form of a ratio i.e. o/p devided by i/p.

Ex:-



$$V = iR + L\frac{di}{dt}$$

Applying L.T

$$V(S) = I(S)\cdot R + LS\,I(S)$$

$$V(S) = I(S)\,[R+LS]$$

$$\frac{I(S)}{V(S)} = \frac{1}{R+LS}$$

$$\frac{I(S)}{V(S)} = \frac{1/L}{S+R/L}$$

$$TF = F(S) = \frac{C(S)}{R(S)} = \frac{N(S)}{D(S)} = \frac{K(S+Z_1)(S+Z_2)\ldots}{(S+P_1)(S+P_2)\ldots} \quad \text{——— (i)}$$

where K = sys. gain

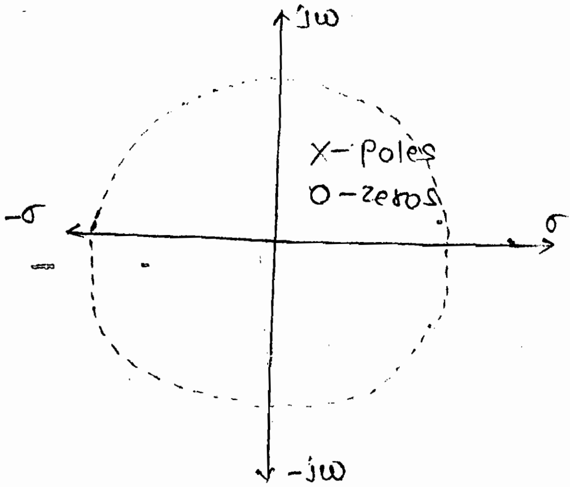
Time constant form $(1+TS)$

$$TF = F(S) = \frac{C(S)}{R(S)} = \frac{N(S)}{D(S)} = \frac{K'(1+T_aS)(1+T_bS)}{(1+T_1S)(1+T_2S)} \quad \text{——— (ii)}$$

where; $K' = \frac{K\cdot Z_1 Z_2}{P_1\cdot P_2}$ = sys. dc gain

$$T_a = \frac{1}{Z_1},\ T_b = \frac{1}{Z_2}$$

$$T_1 = \frac{1}{P_1},\ T_2 = \frac{1}{P_2}$$



s-plane
space plane

$S = \lim_{R \to \infty} Re^{j\theta}$

$(\theta = 0 - 360^\circ)$

From eqn (i) the TF of LTI sys. may be defined as the ratio of Laplace Xform of o/p to Laplace Xform of i/p under the assumption that the sys. initials condn are zero.

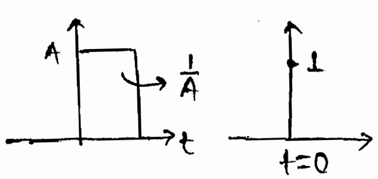
Poles & zeros are those critical freq. which make the TF ∞ (OR) zero.

* Impulse Responce & TF →

*** Impulse signal**

$r(t) = 1 \quad , t = 0$

$\quad\;\; = 0 \quad , t \neq 0$



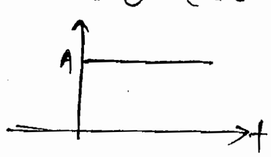
$L.T. \rightarrow R(S) = \frac{1}{S}$

*** Step signal**

$r(t) = A\,u(t)$

$u(t) = 1 \quad t > 0$

$\quad\;\; = 0 \quad t < 0$

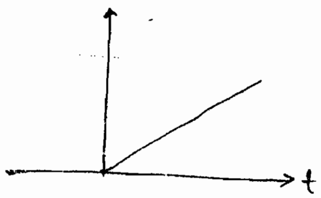


$L.T. \rightarrow R(S) = \frac{A}{S}$

* **Ramp signal**

$r(t) = At \; , \; t>0$

$\quad = 0 \; , \; t<0$

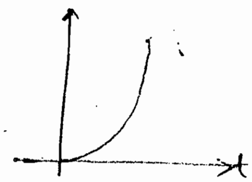


$L.T. \rightarrow R(s) = \frac{A}{s^2}$

* **Parabolic signal**

$r(t) = A \cdot \frac{t^2}{2} , \; t>0$

$\quad = 0 \quad t<0$



$L.T. \rightarrow R(s) = \frac{A}{s^3}$

$$T.F. = F(s) = \frac{C(s)}{R(s)}$$

$$C(s) = F(s).R(s)$$

let $R(s)$ = Impulse Responce = 1

$C(s)$ = Impulse Responce = $F(s) \times 1$

$= T.F. \quad \text{——— (3.)}$

$L\begin{pmatrix}\text{Impulse}\\\text{Responce}\end{pmatrix} = T.F.$ "weighting function"

$\frac{d}{dt}$(Parabolic Responce) = Ramp Responce

$\frac{d}{dt}$(Ramp Responce) = Step Responce

$\frac{d}{dt}$(Step Responce) = Impulse Responce

$$\boxed{L\begin{pmatrix}\text{Impulse}\\\text{Responce}\end{pmatrix} = T.F.}$$

Chap.(1.)

Que-1.) The IR of a system is

$$c(t) = -te^{-t} + 2e^{-t} \quad (t>0)$$

Its open loop TF will be :-

(a.) $\frac{2s+1}{(s+1)^2}$ (b.) $\frac{2s+1}{s^2}$ (c.) $\frac{2s+1}{s}$ (d.) $\frac{2s+1}{(s+1)}$

Soln → L(IR) = T.F.

$$TF = \frac{-1}{(s+1)^2} + \frac{2}{(s+1)}$$

$$TF = \frac{-1+2(s+1)}{(s+1)^2}$$

$$\frac{G(s)}{1+G(s).H(s)} = \frac{2s+1}{(s+1)^2}$$

$$G(s).H(s) = 1$$

put $H(s) = 1$

$$\frac{G(s)}{1+G(s)} = \frac{2s+1}{(s+1)^2}$$

$$G(s)\left[(s+1)^2-(2s+1)\right] = 2s+1$$

$$\boxed{G(s) = \frac{2s+1}{s^2}}$$

Shortcut method

(for unity f/b sys.)

$$\text{open loop TF} = \frac{Num.}{Den - Num}$$

$$G(s) \quad TF = \frac{2s+1}{(s+1)^2-(2s+1)}$$

$$\boxed{G(s) = \frac{2s+1}{s^2}}$$

Chap.(2.)

Que(5.) What is the open loop dc gain of unity -ve f/b sys. having closed loop TF. $\frac{s+4}{s^2+7s+13}$

(a.) $\frac{4}{13}$ (b.) $\frac{4}{9}$ (c.) 4 (d.) 13

Soln →

$$\text{Open loop TF. } G(s) = \frac{s+4}{s^2+7s+13-s-4}$$

$$= \frac{s+4}{s^2+6s+9}$$

$$= \frac{4(1+\frac{s}{4})}{9(1+\frac{6s}{9}+\frac{s^2}{9})}$$

$$\boxed{K=\frac{4}{9}}$$

Que.→ Find the dc gain of TF.

$$\frac{C(s)}{R(s)} = \frac{10(s+5)}{s(s+1)(s+2)}$$

Soln→

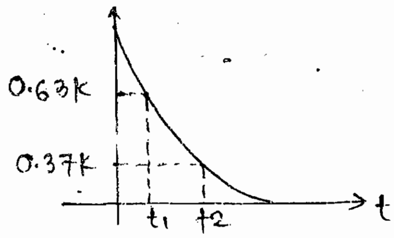
$$\frac{C(s)}{R(s)} = \frac{10\times5(1+0.2s)}{(s)\,1(1+s)\,2(1+0.5s)}$$

$$= \frac{25(1+0.2s)}{s(1+s)(1+0.5s)}$$

$$\boxed{K=25}$$

Que.→ The IR of the sys. is

$$\frac{C(s)}{R(s)} = \frac{K}{s+\alpha}$$ is shown in fig.



The value of α will be

(a.) t_1 (b.) $1/t_1$ (c) t_2 (d) $1/t_2$

Soln→ Impulse Responce $= \mathcal{L}^{-1}(T.F.)$

$$= \mathcal{L}^{-1}\left(\frac{K}{s+\alpha}\right)$$

$$= Ke^{-\alpha t}$$

At $t = t_2$

$Ke^{-\alpha t_2} = K(0.37)$

Since $e^{-1} = 0.37$

$Ke^{-\alpha t_2} = Ke^{-1}$

$\alpha t_2 = 1$

$$\boxed{\alpha = \frac{1}{t_2}}$$

Chap(01)
Que.(2.)

$$R(s) \rightarrow \boxed{\frac{1}{s+1}} \rightarrow \boxed{\frac{1}{s}} \rightarrow C(s)$$

Impulse Responce = ?

Ans.→ Impulse Responce $= \mathcal{L}^{-1}(T.F.)$

$$= \mathcal{L}^{-1}\frac{1}{s(s+1)} = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{1}{s+1}\right]$$

$$= e^{t} - e^{-t}$$

Q.→ A certain cs has i/p $r(t)$ & o/p $c(t)$. If the i/p is 1st passed through a block whose TF is e^{-s} & then applied to the sys. the modefied o/p will be.

(a) $c(t)\,u(t-1)$ (c.) $c(t-1)\,u(t-1)$

(b) $c(t-1)\,u(t)$ (d.) None.

Soln→

$$R(s) \rightarrow \boxed{F(s)} \rightarrow C(s)$$

$$C(s) = R(s)\,F(s)$$

$$R(s) \rightarrow \boxed{e^{-s}} \rightarrow \boxed{F(s)} \rightarrow C_m(s)$$

$$C_m(s) = R(s)\,e^{-s}F(s)$$

$$C_m(s) = C(s)\,e^{-s}$$

$$\boxed{\mathcal{L}^{-1}F(s)\,e^{-as} = f(t-a)\,u(t-a)}$$

$$\boxed{C_m(t) = c(t-1)\,u(t-1)}$$

Cha(2)(1)
(59) Ans.(b)

(2.) $CL\ gain = \frac{G}{1+GH}$

$$100 = \frac{10^3}{1+10^3 \times B}$$

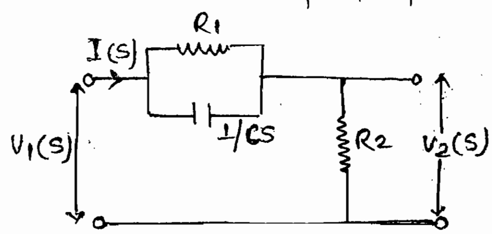
$$\boxed{B = 9\times10^{-3}}$$

* TF for compensators →

* Compensators in CS are used for improving the performings specifications i.e. transient & steady state responce c/s.

* There are 3 type:-

(i) **Lead compensator** → It is used for improving the transient state or speed of responce of sys.



$$V_i(s) = I(s)\left[\frac{R_1}{R_1CS+1}\right] + I(s)\cdot R_2$$

$$= I(s)\left[\frac{R_1+R_2+R_1R_2CS}{R_1CS+1}\right]$$

$$V_0(s) = I(s)R_2$$

$$\boxed{\frac{V_0(s)}{V_i(s)} = \frac{R_2(R_1CS+1)}{R_1+R_2+R_1R_2CS}}$$

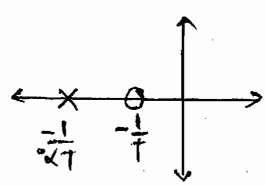
$T = R_1C$; $\alpha = \frac{R_2}{R_1+R_2}$ $(\alpha < 1)$

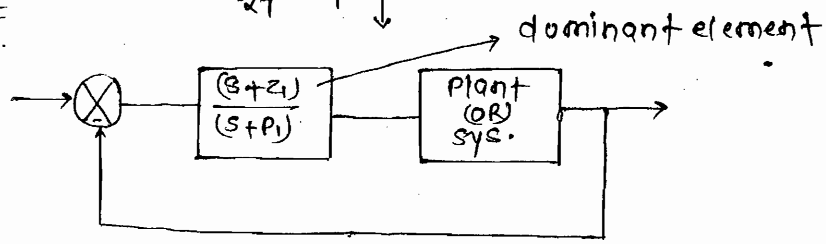
$$\frac{R_2(R_1CS+1)}{R_1+R_2\left[1+\frac{R_1R_2CS}{R_1+R_2}\right]}$$

$$\boxed{\frac{V_0(s)}{V_i(s)} = \frac{\alpha(1+Ts)}{(1+\alpha Ts)}}$$

Zero at $s = \frac{-1}{T}$.

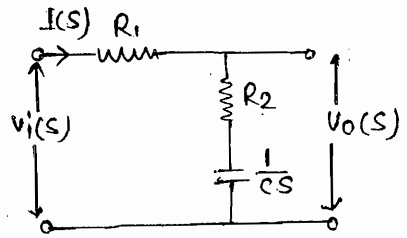
Poles at $s = \frac{-1}{\alpha T}$





Note → Adding a zero to a sys. TF in terms of compensators represents a lead compensator.

(2.) Lag compensator → It is used for improving steady state responce c/s of the sys. i.e. elemination of steady state error b/w i/p & o/p.



$$V_i(s) = I(s)\left[R_1 + R_2 + \frac{1}{Cs}\right]$$

$$= I(s)\left[\frac{R_1Cs + R_2Cs + 1}{Cs}\right]$$

$$V_o(s) = I(s)\left[R_2 + \frac{1}{Cs}\right]$$

$$V_o(s) = I(s)\left[\frac{R_2Cs + 1}{Cs}\right]$$

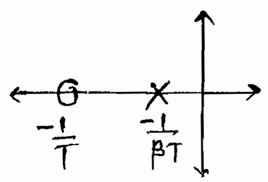
$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{R_2Cs + 1}{R_1Cs + R_2Cs + 1}}$$

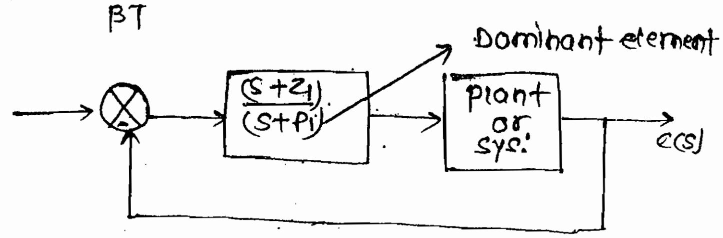
$$T = R_2C \; ; \quad \beta = \frac{R_1 + R_2}{R_2} = \frac{1}{\alpha} \quad (\beta > 1)$$

$$\frac{R_2Cs + 1}{R_2Cs\left(\frac{R_1 + R_2}{R_2}\right) + 1}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1 + Ts}{1 + \beta Ts}}$$

zero at $s = \frac{-1}{T}$; pole at $s = \frac{-1}{\beta T}$

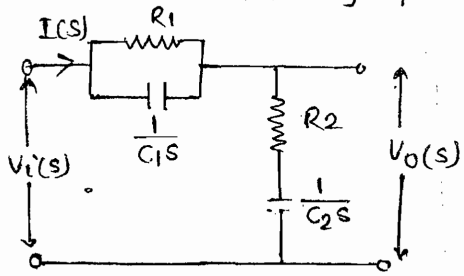




Note → Adding the pole to a sys. TF in terms of compensators represent lag compensator.

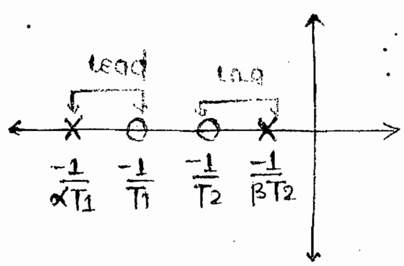
(3.) LAG-LEAD compensator → It improves both transient & steady state responce c/s.

* It exibits both lead & lag c/s in its freq. responce.



	Lead	Lag
	$\alpha = \dfrac{R_2}{R_1+R_2}$	$\beta = \dfrac{R_1+R_2}{R_2}$
	$T_1 = R_1C_1$	$T_2 = R_2C_2$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+T_1s)(1+T_2s)}{(1+\alpha T_1s)(1+\beta T_2s)}}$$



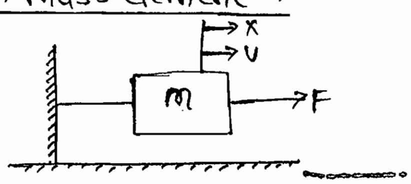
* mechanical systems → All mech. sys. are classified into 2 types :-

(1.) mechanical translational sys. →

I/P = Force (F) ; O/P = Linear disp (x) (or) Linear velocity (v)

* The 3 ideal elements are :-

(1.) mass element →



(a) $F = m\dfrac{dv}{dt}$

(b.) $F = m\dfrac{d^2x}{dt^2}$

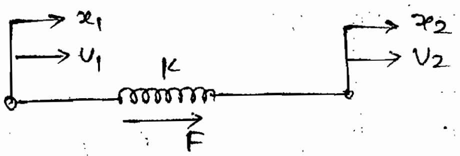
(2.) Damper element →

f (OR) B

(a) $F = f(v_1 - v_2) = fv \quad [\because v = v_1 - v_2]$

(b) $F = f\dfrac{d}{dt}(x_1 - x_2) = f\dfrac{dx}{dt} \quad (x = x_1 - x_2)$

(3.) Spring element →



(a) $F = K\int(v_1-v_2)\,dt = K\int v\,dt \quad (v=v_1-v_2)$

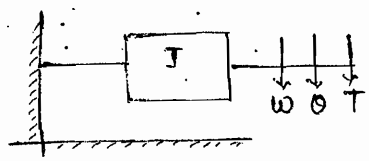
(b) $F = K(x_1-x_2) = Kx \quad (x=x_1-x_2)$

(2.) Mech. Rotational sys. →

I/p = torque (T) ; o/p = Angular disp (θ) (or) Angular velocity (ω)

3 ideal elements are →

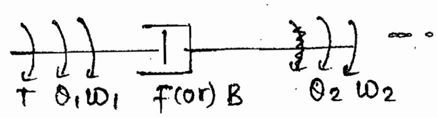
(1.) Inertia element



(a) $T = J\dfrac{d\omega}{dt}$

(b.) $T = J\dfrac{d^2\theta}{dt^2}$

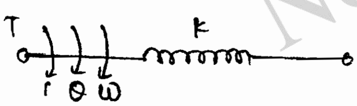
(2.) Torsional Damper element (Friction)



(a) $T = f(\omega_1-\omega_2) = f\omega \quad (\omega_1-\omega_2=\omega)$

(b.) $T = f\dfrac{d}{dt}(\theta_1-\theta_2) = f\dfrac{d\theta}{dt} \quad (\theta=\theta_1-\theta_2)$

(3.) Torsional Spring element (stiffness)



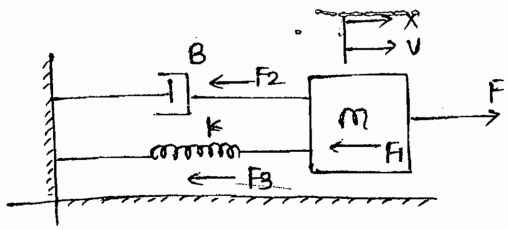
(a) $T = K\int\omega\,dt$

(b.) $T = K\theta$

* Analogeous system →

The electrical eq: of mech. elements are known as analogeous sys.

(1.) mech. translational sys →

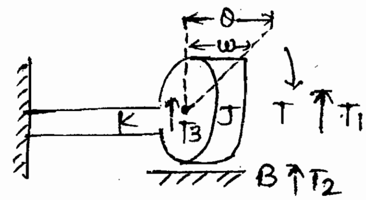


$$F = F_1 + F_2 + F_3$$

$$F = m\frac{dv}{dt} + BV + K\int v\,dt$$

$$F = m\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \text{ —}$$

(2.) mech. Rotational sys →

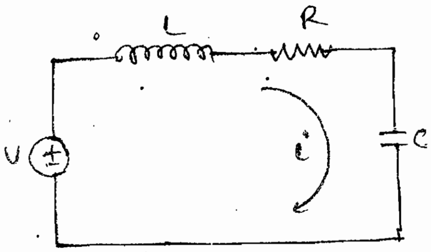


$$T = T_1 + T_2 + T_3$$

$$T = J\frac{d\omega}{dt} + B\omega + K\int \omega\,dt$$

$$T = J\frac{d^2\theta}{dt^2} + \frac{Bd\theta}{dt} + K\theta \quad \text{——— (ii)}$$

(3.) Electrical system →



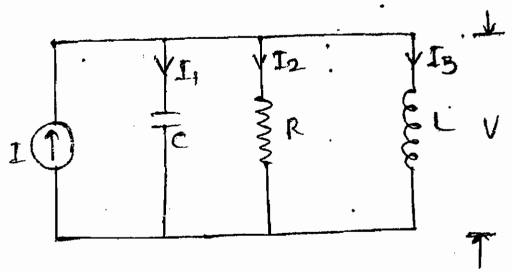
$$V = V_1 + V_2 + V_3$$

$$V = L\frac{di}{dt} + iR + \frac{1}{C}\int i\,dt$$

$$\left[i = \frac{dq}{dt},\ q = \text{charge}\right]$$

$$V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} \quad \text{——— (iii)}$$

(4.) Electrical system →



$$I = I_1 + I_2 + I_3$$

$$I = C\frac{dv}{dt} + \frac{V}{R} + \frac{1}{L}\int v\,dt$$

$$\left[V = \frac{d\phi}{dt};\ \phi = \text{Flux}\right]$$

$$I = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L} \quad \text{——— (iv)}$$

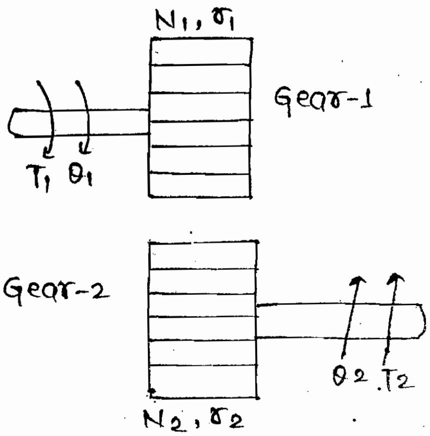
Comparing eqn (i) – (4.)

(1.) F - T - V Analogy (2.) F - T - I Analogy

F	T	V	I
m	J	L	C
B	B	R	$\frac{1}{R}$
k	k	$\frac{1}{C}$	$\frac{1}{L}$
V	ω	i	v
x	θ	q	ϕ

GEARS →

* These are mech. devices which are used as intermediate elements b/n electrical motor & load.
* They are used for stepping up (or) stepping down either torqe (or) speed.
* They are analogeous to electrical TF.



N = No. of teeths

r = Radius of gear wheel (m)

T = Torque on gear wheel (N-m)

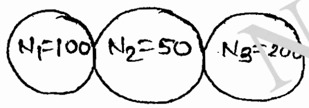
θ = Angular disp (radians)

Dynamics of gear train

$$\frac{N_1}{N_2} = \frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1}$$

$$\boxed{\frac{N_x}{N_y} = \frac{T_x}{T_y} = \frac{r_x}{r_y} = \frac{\theta_y}{\theta_x} = \frac{\omega_y}{\omega_x} = \frac{\alpha_y}{\alpha_x}}$$

Ex:—



(i) $T_1 = 10$ N-m, Find T_2 & T_3?

(ii) $\omega_1 = 20$ r/s (cw), Find ω_2 & ω_3?

(i) $\frac{N_1}{N_2} = \frac{T_1}{T_2} = \frac{100}{50} = \frac{10}{T_2} = T_2 = 5$ N-m

$\frac{N_1}{N_3} = \frac{T_1}{T_3} = \frac{100}{200} = \frac{10}{T_3} = T_3 = 20$ N-m

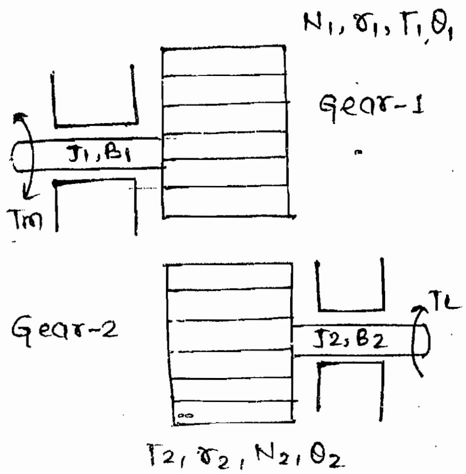
(ii) $\frac{N_1}{N_2} = \frac{\omega_2}{\omega_1} = \frac{100}{50} = \frac{\omega_2}{20} = \omega_2 = 40$ r/s (ccw)

$\frac{N_1}{N_3} = \frac{\omega_3}{\omega_1} = \frac{100}{200} = \frac{\omega_3}{20} = \omega_3 = 10$ r/s (cw)

Observations →

(1) $N_1 > N_2 \Rightarrow T\downarrow, \omega\uparrow$

(2) $N_1 = N_2 \Rightarrow$ There will be no change on T & ω (speed & torqe)



T_m = motor torque (N-m)

T_1 = Torque on gear-1 due to T_m (N-m)

T_2 = Torque on gear-2 due to T_1 (N-m)

T_L = Torque on load due to T_2 (N-m)

J, B = Inertia & Friction of gears.

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + T_1$$

$$T_2 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$\frac{N_1}{N_2} = \frac{T_1}{T_2} \; ; \quad T_1 = \left(\frac{N_1}{N_2}\right) T_2$$

$$T_1 = \left(\frac{N_1}{N_2}\right) J_2 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) \cdot T_L$$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) J_2 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

* (I) = eq. Inertia & Friction of motor side gear (gear-1)

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1} , \quad \dot{\theta}_2 = \left(\frac{N_1}{N_2}\right)\dot{\theta}_1 \; ; \quad \ddot{\theta}_2 = \left(\frac{N_1}{N_2}\right)\ddot{\theta}_1$$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right)^2 J_2 \frac{d^2\theta_1}{dt^2} + \left(\frac{N_1}{N_2}\right)^2 B_2 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$T_m = \left[J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2\right] \frac{d^2\theta_1}{dt^2} + \left[B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2\right] \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$\boxed{J_{eq} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2} \quad \boxed{B_{eq} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2}$$

* (ii) Eq. inertia & friction of Load side gear (gear-2) →

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1} \; ; \; \dot{\theta}_1 = \left(\frac{N_2}{N_1}\right)\dot{\theta}_2 \; , \; \ddot{\theta}_1 = \left(\frac{N_2}{N_1}\right)\ddot{\theta}_2$$

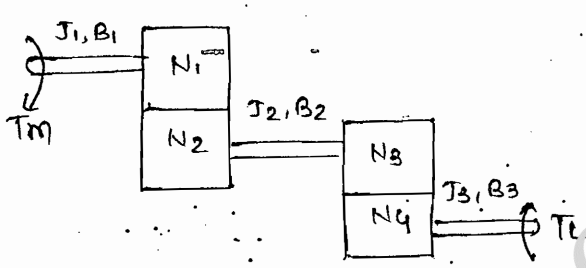
$$T_m = \left(\frac{N_2}{N_1}\right) J_1 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_2}{N_1}\right) B_1 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) \cdot \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$\left(\frac{N_2}{N_1}\right) T_m = \left(\frac{N_2}{N_1}\right)^2 J_1 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_2}{N_1}\right)^2 B_1 \frac{d\theta_2}{dt} + J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$\left(\frac{N_2}{N_1}\right) T_m = \left[\left(\frac{N_2}{N_1}\right)^2 J_1 + J_2\right] \frac{d^2\theta_2}{dt^2} + \left[\left(\frac{N_2}{N_1}\right)^2 B_1 + B_2\right] \frac{d\theta_2}{dt} + T_L$$

$$\boxed{J_{eq(2)} = \left(\frac{N_2}{N_1}\right)^2 J_1 + J_2} \qquad \boxed{B_{eq(2)} = \left(\frac{N_2}{N_1}\right)^2 B_1 + B_2}$$

Q. →



Soln → (i) motor side gear →

$$J_{eq} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2 + \left(\frac{N_1 N_3}{N_2 N_4}\right)^2 J_3$$

$$B_{eq} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2 + \left(\frac{N_1 N_3}{N_2 N_4}\right)^2 B_3$$

(ii) Load side gear →

$$J_{eq} = \left(\frac{N_2 N_4}{N_1 N_3}\right)^2 J_1 + \left(\frac{N_4}{N_3}\right)^2 J_2 + J_3$$

$$B_{eq} = \left(\frac{N_2 N_4}{N_1 N_3}\right)^2 B_1 + \left(\frac{N_4}{N_3}\right)^2 B_2 + B_3$$

* NODAL METHOD →

(i) No. of nodes = No. of displacements.

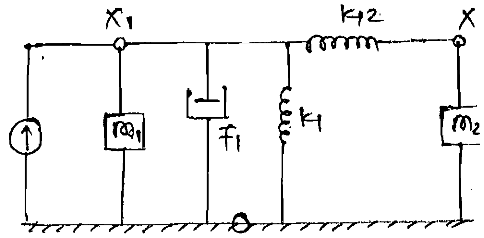
(ii) Take an additional Node which is a ref node.

(iii) Connect the mass or inertia elements b/n the principal node & ref node only.

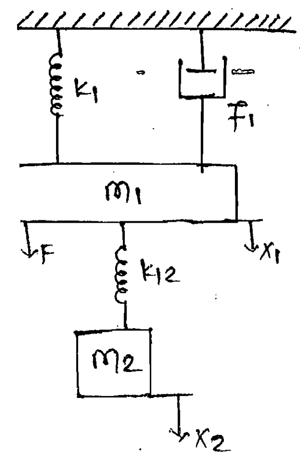
(iv) Connect an spring & damping elements either b/n the principal node (or) b/n the principal node & ref. depending on there position.

(v) Obtain the nodel dia. & write the describing diffn eqn at each node.

*** Nodal diagram →**
(mech. eq. diagram)



*** mech. system →**



At Node x_1

$$F = m_1\frac{d^2x_1}{dt^2} + f_1\frac{dx_1}{dt} + k_1x_1 + k_{12}(x_1 - x_2)$$

at node x_2

$$0 = m_2\frac{d^2x_2}{dt^2} + k_{12}(x_2 - x_1)$$

*** Transfer function →**

$$F(s) = [m_1s^2 + f_1s + k_1 + k_{12}]X_1(s) - k_{12}X_2(s)$$

$$0 = [m_2s^2 + k_{12}]X_2(s) - k_{12}X_1(s)$$

$$X_2(s) = \left[\frac{k_{12}}{m_2s^2 + k_{12}}\right]X_1(s)$$

$$\boxed{\frac{X_1(s)}{X_2(s)} = \frac{m_2s^2 + k_{12}}{(m_1s^2 + f_1s + k_1 + k_{12})(m_2s^2 + k_{12}) - k_{12}^2}}$$

*** Order →**

(1) mass element → Order (2)

(2) mass element → order - 4

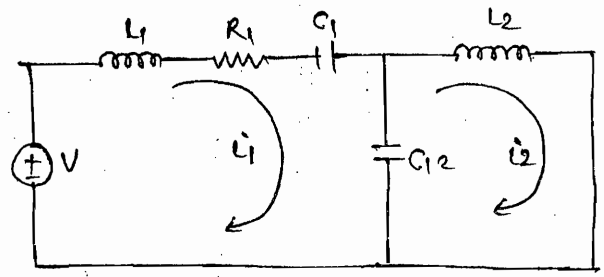
3 mass element → order - 6

n mass element → order - 2n

F-V Analogy →

$$V = L_1\frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1\,dt + \frac{1}{C_{12}}\int (i_1 - i_2)\,dt$$

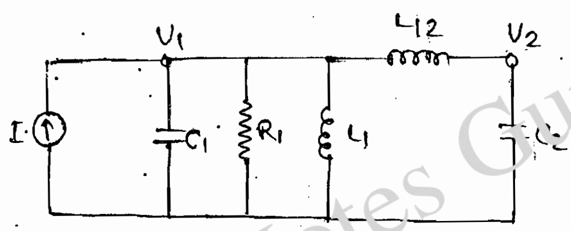
$$0 = L_2\frac{di_2}{dt} + \frac{1}{C_{12}}\int (i_2 - i_1)\,dt$$



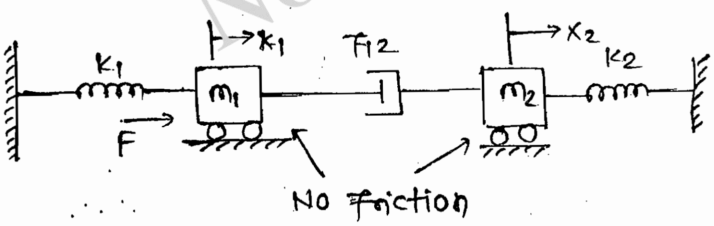
F-I Analogy →

$$I = C_1\frac{dv_1}{dt} + \frac{v_1}{R_1} + \frac{1}{L_1}\int v_1\,dt + \frac{1}{L_{12}}\int (v_1 - v_2)\,dt$$

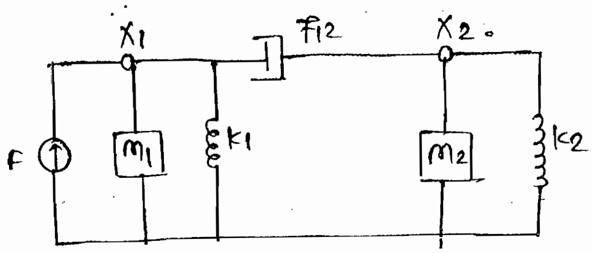
$$0 = C_2\frac{dv_2}{dt} + \frac{1}{L_{12}}\int (v_2 - v_1)\,dt$$



Conv.(2)
Q.(2)



Solⁿ → . Nodal diagram → (mech. eq. dia.)



At node $x_1 \rightarrow$

$$F = m_1 \frac{d^2x_1}{dt^2} + f_{12}\frac{d}{dt}(x_1 - x_2) + K_1 x_1$$

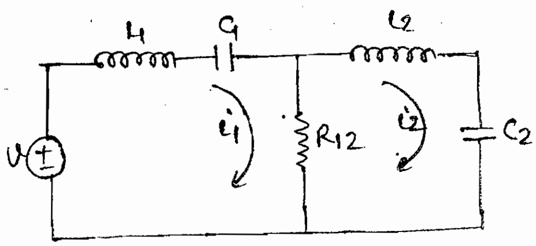
at node $x_2 \rightarrow$

$$0 = m_2 \frac{d^2x_2}{dt^2} + f_{12}\frac{d}{dt}(x_2 - x_1) + K_2 x_2$$

F-V analogy $\rightarrow$

$$V = L_1 \frac{di_1}{dt} + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int i_1 dt$$

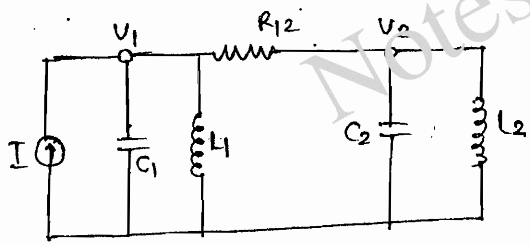
$$0 = L_2 \frac{di_2}{dt} + R_{12}(i_2 - i_1) + \frac{1}{C_2}\int i_2 dt$$



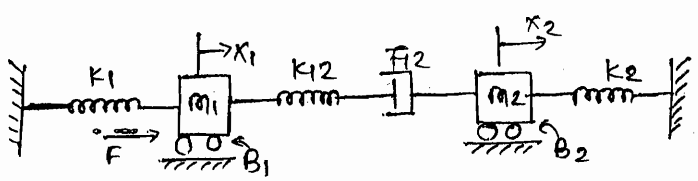
F-I analogy $\rightarrow$

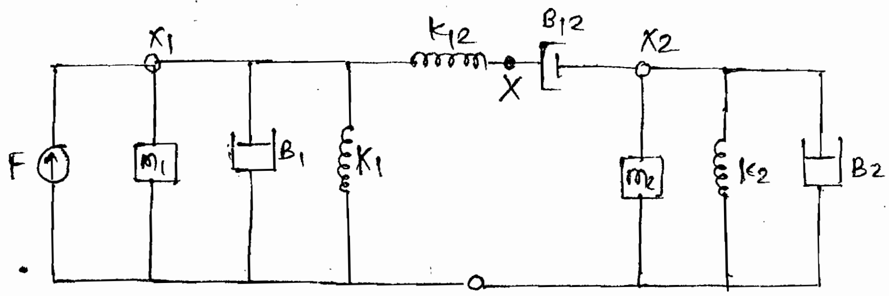
$$I = C_1 \frac{dv_1}{dt} + \frac{V_1 - V_2}{R_{12}} + \frac{1}{L_1}\int V_1 dt$$

$$0 = C_2 \frac{dv_2}{dt} + \frac{V_2 - V_1}{R_{12}} + \frac{1}{L_2}\int V_2 dt$$



Conv. Que.





at node $x_1 \rightarrow$

$$F = m_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + K_1x_1 + K_{12}(x_1 - x_2)$$

at dummy node $x \rightarrow$

$$0 = K_{12}(x - x_1) + B_{12}\frac{d}{dt}(x - x_2)$$

at Node $x_2 \rightarrow$

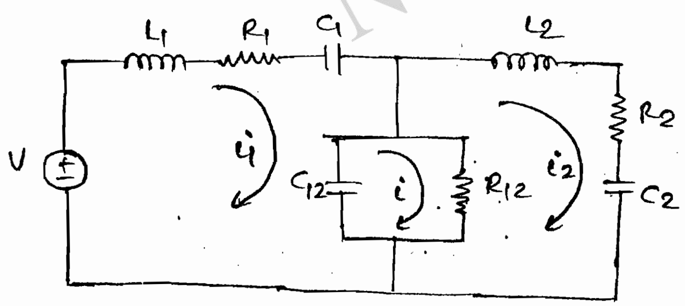
$$0 = m_2\frac{d^2x_2}{dt^2} + B_{12}\frac{dx_2}{dt} + K_2x_2 + B_{12}\frac{d}{dt}(x_2 - x)$$

F-V analogy $\rightarrow$

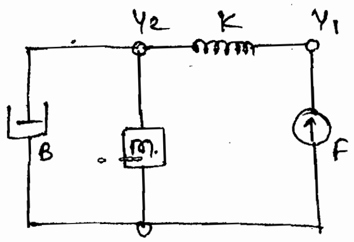
$$V = L_1\frac{di_1}{dt} + R_1i_1 + \frac{1}{C_1}\int i_1dt + \frac{1}{C_{12}}\int(i_1 - i)\,dt$$

$$0 = \frac{1}{C_{12}}\int(i - i_1)\,dt + R_{12}(i - i_2)$$

$$0 = L_2\frac{di_2}{dt} + R_2i_2 + \frac{1}{C_2}\int i_2dt + R_{12}(i_2 - i)$$



Q.5 / 55



at dummy node (y_1)

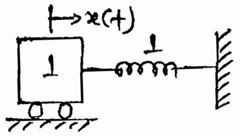
$$F = k(y_1 - y_2)$$

at node (y_2)

$$0 = m\frac{d^2y_2}{dt^2} + B\frac{dy_2}{dt} + k(y_2 - y_1)$$

$$\boxed{k(y_1 - y_2) = m\frac{d^2y_2}{dt^2} + B\frac{dy_2}{dt}}$$

4 / 55



$$F = m\frac{d^2x}{dt^2} + kx$$

Given $m = k = 1$

$$F = \frac{d^2x}{dt^2} + x$$

$$F(s) = (s^2+1)\,X(s)$$

$$X(s) = \frac{1}{s^2+1} \cdot F(s)$$

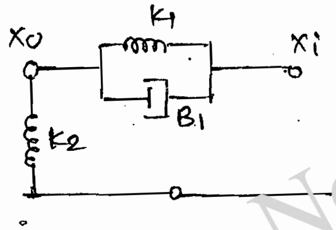
Given:

$F(s)$ = unit impulse force = 1

$$X(s) = \frac{1}{s^2+1} \times (1)$$

$$\boxed{x(t) = \sin t}$$

3 / 55



at node $x_i \rightarrow$

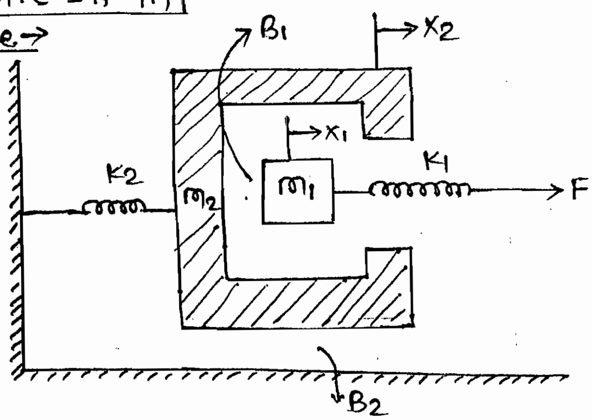
$$0 = k_2 x_0 + k_1(x_0 - x_i) + B_1\frac{d}{dt}(x_0 - x_i)$$

$$[k_2 + k_1 + B_1 s]\,X_0(s) = X_i(s)\,[B_1(s)] + k_1$$

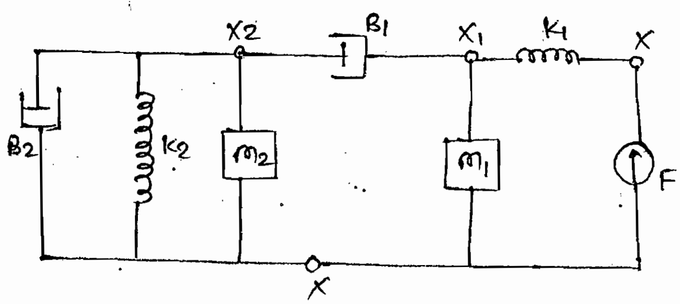
$$\frac{X_0(s)}{X_i(s)} = \frac{k_1 + B_1(s)}{(k_1 + k_2 + B_1 s)}$$

DATE-14/11/14

Que→



Solⁿ→ Nodal diagram→



At dummy node 'x'→

$$F = K_1(x - x_1) \quad \text{——(i)}$$

at node 'x₁'→

$$0 = m_1\frac{d^2x_1}{dt^2} + B_1\frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x)$$

$$K_1(x - x_1) = m_1\frac{d^2x_1}{dt^2} + B_1\frac{d}{dt}(x_1 - x_2)$$

$$F = m_1\frac{d^2x_1}{dt^2} + B_1\frac{d}{dt}(x_1 - x_2) \quad \text{——(2)}$$

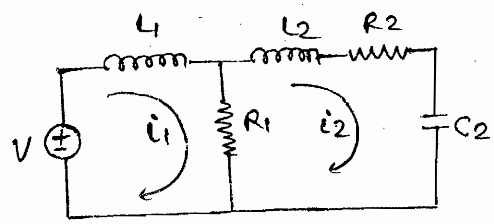
At node 'x₂'→

$$0 = m_2\frac{d^2x_2}{dt^2} + B_2\frac{dx_2}{dt} + K_2x_2 + B_1\frac{d}{dt}(x_2 - x_1) \quad \text{——(iii)}$$

F-V analogy→

$$V = L_1\frac{di_1}{dt} + R_1(i_1 - i_2)$$

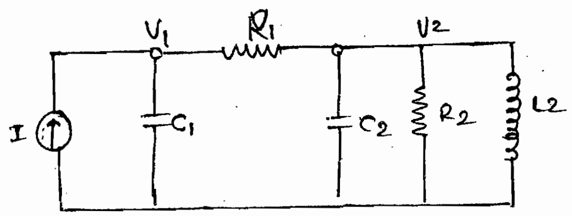
$$0 = L_2\frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + R_1(i_2 - i_1)$$



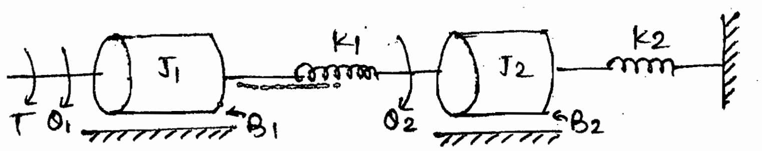
F-I Analogy →

$$I = C_1\frac{dv_1}{dt} + \frac{v_1 - v_2}{R_1}$$

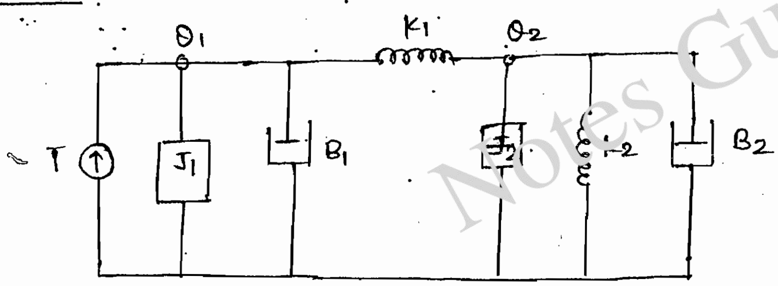
$$0 = C_2\frac{dv_2}{dt} + \frac{v_2}{R_2} + \frac{1}{L_2}\int v_2\,dt + \frac{v_2 - v_1}{R_1}$$



Que. →



Soln →



At node θ1 →

$$T = J_1\frac{d^2\theta_1}{dt^2} + B_1\frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2)$$

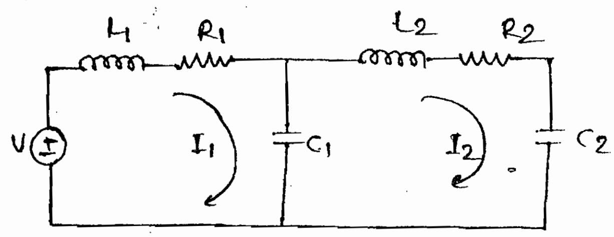
At node θ2 →

$$0 = J_2\frac{d^2\theta_2}{dt^2} + B_2\frac{d\theta_2}{dt} + K_2\theta_2 + K_1(\theta_2 - \theta_1)$$

T-V Analogy →

$$v = L_1\frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int (i_1 - i_2)\,dt$$

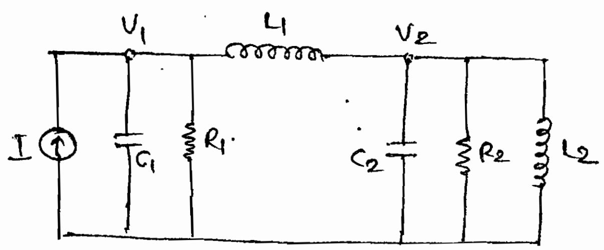
$$0 = L_2\frac{di_1}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + \frac{1}{C_1}\int (i_2 - i_1)\,dt$$



T-I analogy →

$$I = C_1\left(\frac{dv_1}{dt}\right) + \left(\frac{V_1}{R_1}\right) + \frac{1}{L_1}\int (V_1 - V_2)\,dt$$

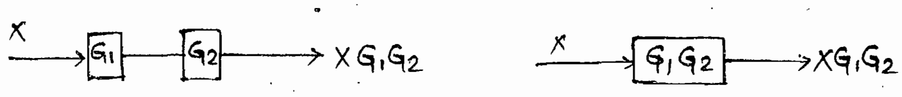
$$0 = C_2\frac{dV_2}{dt} + \left(\frac{V_2}{R_2}\right) + \frac{1}{L_2}\int (V_2 - V_1)\,dt$$



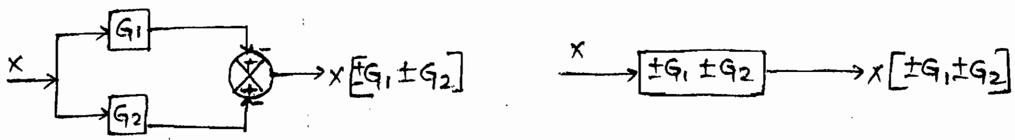
Block diagram →

Rules | **Original diagram** | **Eq. diagram**

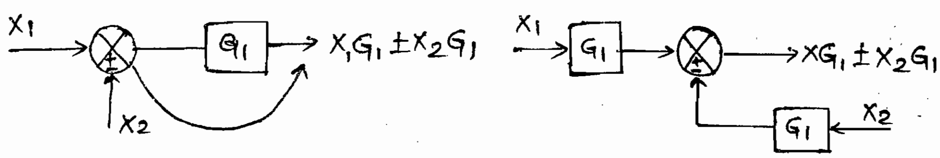
i) Combining bloks in series.



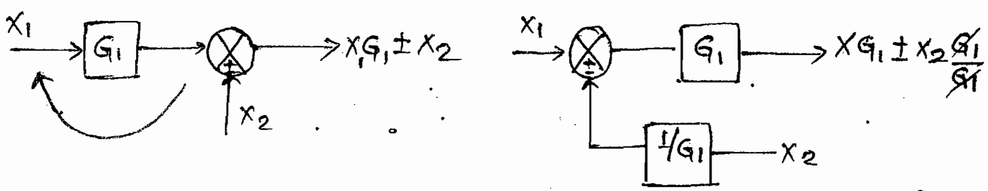
ii) Combining blocks in parallel.



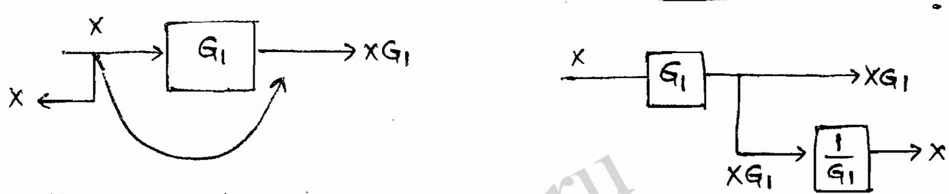
iii) Shifting the summing elements after the blocks.



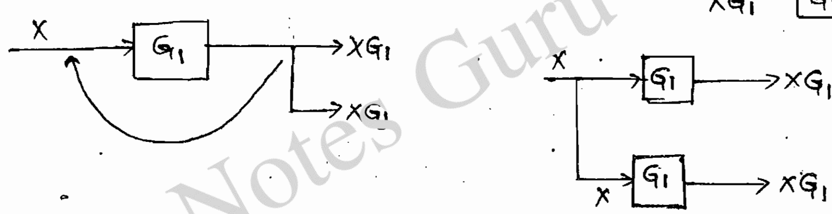
iv) Shifting the summing elements before blocks



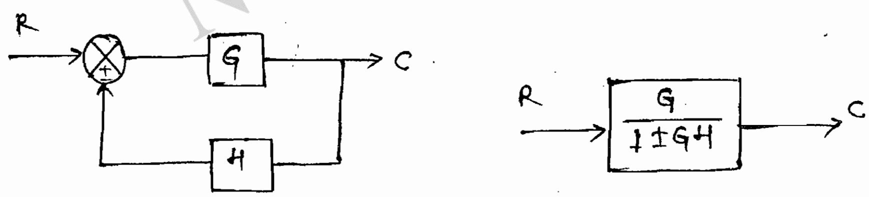
v) Shifting the take off point after the block.



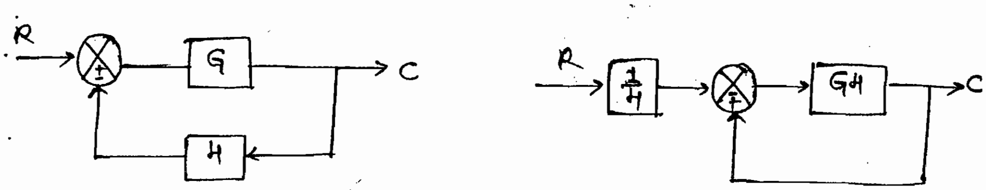
vi) —//— before the block.



vii). TF of CLCS.

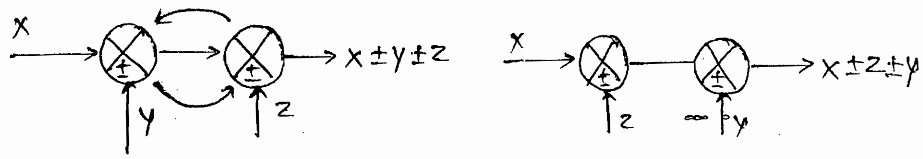


viii) Block dia. transformation



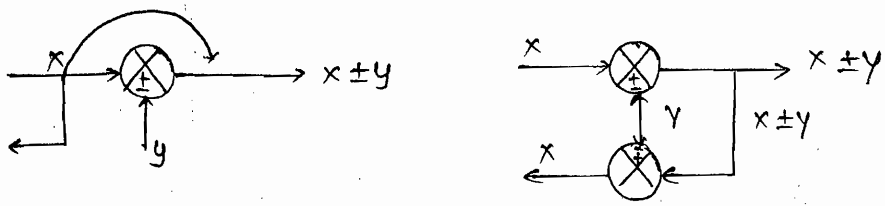
$$\boxed{\frac{C}{R} = \frac{1}{H} \times \frac{GH}{1 \pm GH}}$$

9.) Interchanging the summing elements.

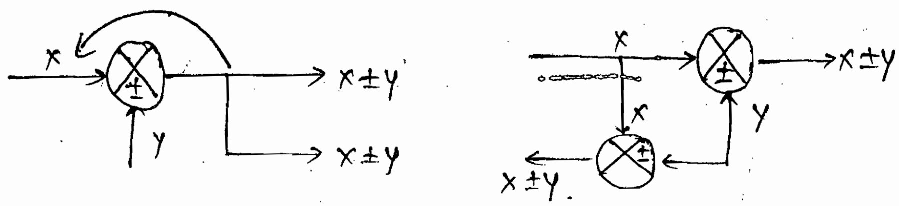


CRITICAL RULE →

10) shifting the take-off point after the summing element



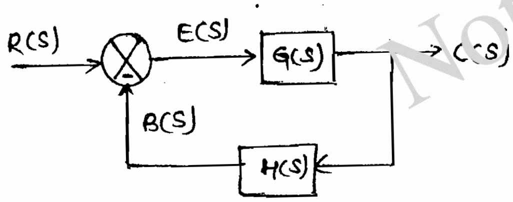
11) —11— Before

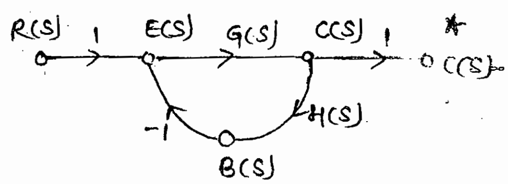


7/56 (d.) 8/56 (b.) 9/56 (c.)

* SIGNAL FLOW GRAPH → It is the graphical representation of cs in which nodes representing each of the system variable & are connected by direct branches.

SFG for CLCS →





Terminology of SFG's →

(1.) Node:- It represents sys. variable & is equal to the sum of all incoming signal at it.

(2.) I/p Node (OR) Source node → It is a node having only outgoing branches

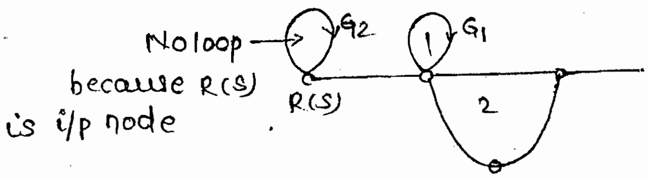
*(3) O/p Node (OR) Sink node → It is a node having only incoming branches.

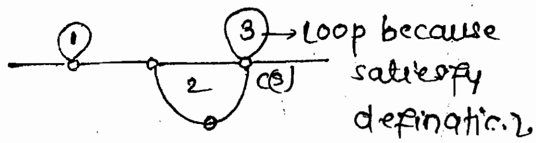
(4) mix (OR) chain node → It is a node having both incoming & outgoing branches.

(5.) Path → It is the traversal of the connected branches in the dirⁿ of branch arrow such that no node is traversed more than one.

(6.) Forward path → It is a path from i/p node to o/p node.

(7.) Loop → It is a path which starts & ends at the same node.





Note:- Self loops on the defined i/p nodes are not valid loops.

Loops (or) ~~self loops~~ on the defined o/p nodes are valid loops.

(8.) Non-touching loops → 2 (or) more loops are said to be non-touching if they do not have a common node.

* MASON'S GAIN Formula →

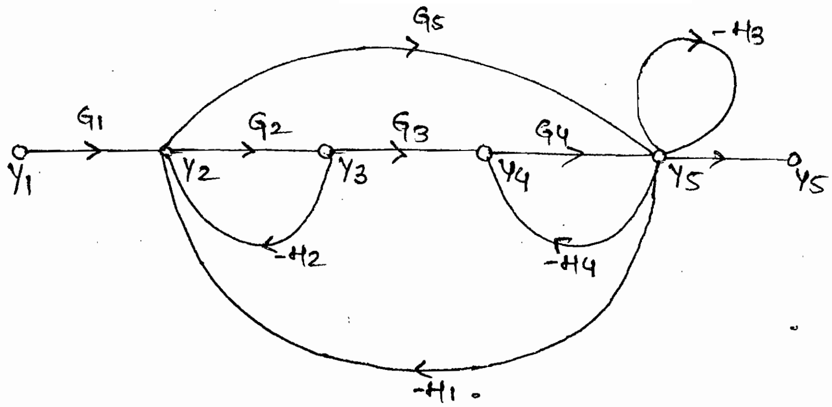
$$\text{The overall Gain (OR) transfer function} = \frac{\sum P_K \Delta_K}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2 + \cdots + P_R\Delta_R}{\Delta}$$

where; P_K = Path gain of k^{th} forward path.

$$\Delta = 1 - \begin{bmatrix} \text{sum of } \cancel{\text{all}} \text{ loop gains} \\ \text{of all individual} \\ \text{loops} \end{bmatrix} + \begin{bmatrix} \text{Sum of gain products} \\ \text{at 2 Non-touching} \\ \text{loops} \end{bmatrix}$$

$$- \begin{bmatrix} \text{Sum of gain products} \\ \text{of 3 non-touching loops} \end{bmatrix} + \cdots$$

Δ_K = It is that value of Δ obtained by removing all the loops touching k^{th} forward path.



Case-(i.) $\frac{Y_5}{Y_1}$

(i) Forward paths

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_1 G_5$

(ii) To Find $\Delta \rightarrow$

(a.) Individual loops

$I_1 = -G_2H_2$ $\quad I_3 = -H_3$ $\quad I_5 = -G_5H_1$

$I_2 = -G_4H_4$ $\quad I_4 = -G_2G_3G_4H_1$

(b.) Two NTL's $\rightarrow$

$L_1 = I_1 I_2 = G_2H_2 = G_2H_2H_4G_4$

$L_2 = I_1 I_3 = G_2H_2H_3$

(iii) To Find Δ_1 & Δ_2

$\Delta_1 = \Delta_2 = 1$

$\frac{Y_5}{Y_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$ $\qquad \frac{Y_5}{Y_1} = \frac{G_1G_2G_3G_4 + G_1G_5}{\Delta}$ ——(i)

$$\Delta = 1 - \begin{bmatrix} -G_2H_2 - G_4H_4 - H_3 \\ -G_2G_3G_4H_1 - G_5H_1 \end{bmatrix} + \begin{bmatrix} G_2H_2G_4H_4 \\ + G_2H_2H_3 \end{bmatrix}$$

Case(ii) $\rightarrow$ To Find $Y_5/Y_3 = \frac{\left(\frac{Y_5}{Y_1}\right)}{\left(\frac{Y_3}{Y_1}\right)}$

To Find $\frac{Y_3}{Y_1}$.

(i) Forward path

$P_1 = G_1 G_2$

(ii) To Find Δ

Δ is independant of Forward path

(iii) To Find Δ_1

$\Delta_1 = 1 - (-G_4H_4 - H_3)$

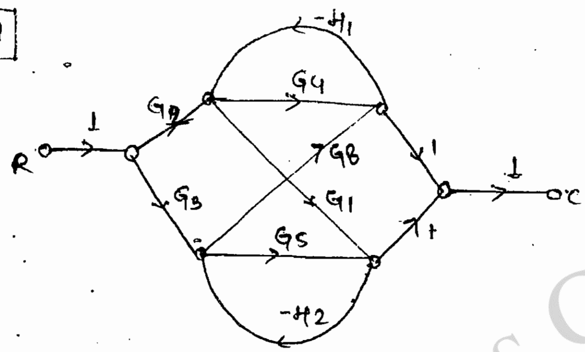
$$\frac{Y_3}{Y_1} = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1G_2(1+G_4H_4+H_3)}{\Delta} \quad \text{--- (2)}$$

$$\frac{Y_5}{Y_3} = \frac{G_1G_2G_3G_4 + G_1G_5}{G_1G_2(1+G_4H_4+H_3)} \quad \text{--- (3)}$$

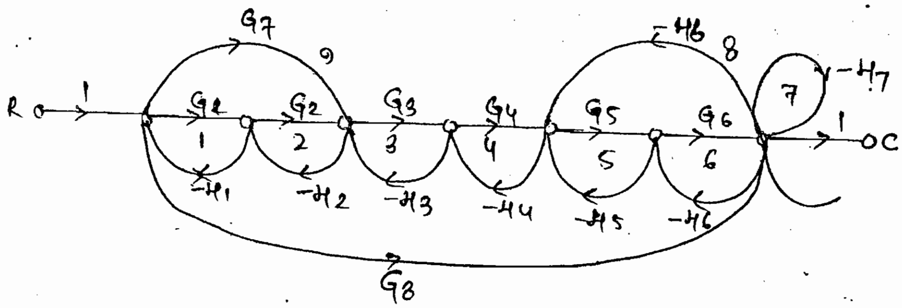
TE-15/01/14

4.1

8



$$\frac{C}{R} = \frac{G_2G_4(1+G_5H_2) + G_3G_5(1+G_4H_1) + G_2G_1(1) + G_3G_8(1) + G_2G_1(-H_2)G_8(1) + G_3G_8(-H_1)G_1(1)}{1 - [-G_4H_1 - G_5H_2 + G_1G_8H_1H_2] + [G_4H_1G_5H_2]}$$

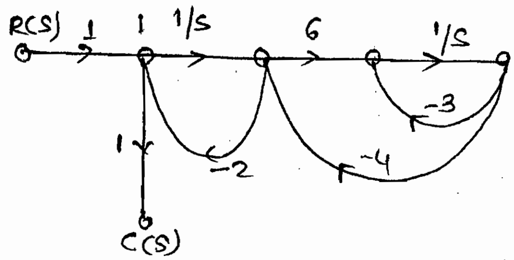


Sol^n →
$$L_{10} = G_8(-H_8)(-H_4)(-H_3)(-H_2)(-H_1)$$
$$L_{11} = G_8(-H_6)(-H_5)(-H_4)(-H_3)(-H_2)(-H_1)$$

No. of forward paths = 3

No. of loops = 11

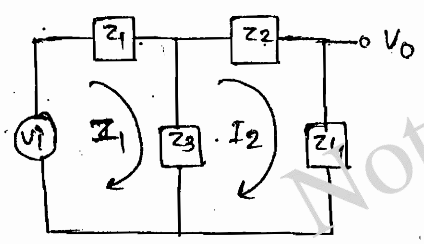
Q(11) / 56



calc. $\frac{C(S)}{R(S)} = ?$

Sol^n →
$$\frac{C(S)}{R(S)} = \frac{1\left\{1-\left(\frac{-3}{s}-\frac{24}{s}\right)\right\}}{1-\left[\frac{-2}{s}-\frac{3}{s}-\frac{24}{s}\right]+\frac{6}{s^2}}$$
$$= \frac{\frac{s+27}{s}}{\frac{s^2+29s+6}{s^2}} = \frac{s(s+27)}{s^2+29s+6}$$

12 / 56

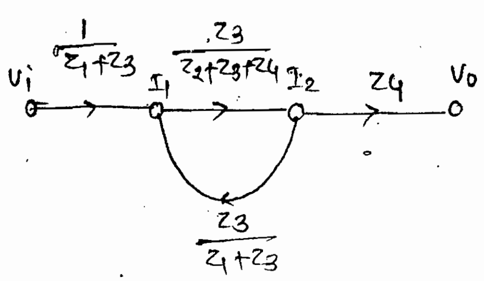


calc. G_2 & H = ?

Sol^n → Loop-(1) →
$$V_i = I_1 Z_1 + (I_1 - I_2) Z_3$$
$$V_i = I_1(Z_1+Z_3) - I_2 Z_3$$
$$I_1 = \frac{V_i}{Z_1+Z_3} + \frac{I_2 Z_3}{Z_1+Z_3}$$

loop(2) →
$$0 = I_2 Z_2 + I_2 Z_4 + (I_2 - I_1) Z_3$$
$$I_2(Z_2+Z_3+Z_4) = I_1 Z_3$$
$$I_2 = I_1 \cdot \frac{Z_3}{Z_2+Z_3+Z_4}$$

Consider the variables as node in the given que. eq,



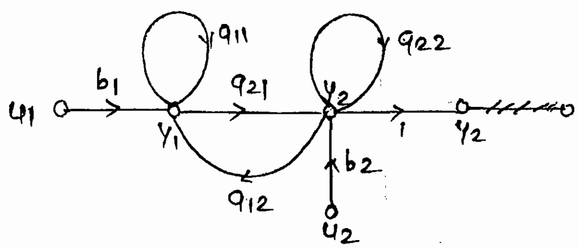
$$\boxed{G_2 = \frac{Z_3}{Z_2+Z_3+Z_4} \qquad H = \frac{Z_3}{Z_1+Z_3}}$$

Q.→ Find $\frac{Y_2}{U_1}$ & $\frac{Y_2}{U_2}$ using SFG

$$Y_1 = a_{11}Y_1 + a_{12}Y_2 + b_1U_1$$

$$Y_2 = a_{21}Y_1 + a_{22}Y_2 + b_2U_2$$

Solⁿ→



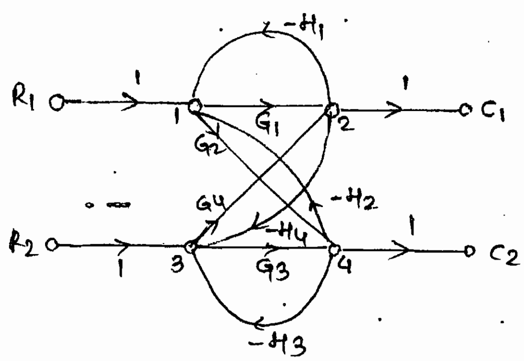
case(1)→ $\left.\frac{Y_2}{U_1}\right|_{U_2=0}$

$$\frac{Y_2}{U_1} = \frac{b_1 a_{21}(1)}{1-(a_{11}+a_{22}+a_{12}a_{21})+(a_{11}a_{22})}$$

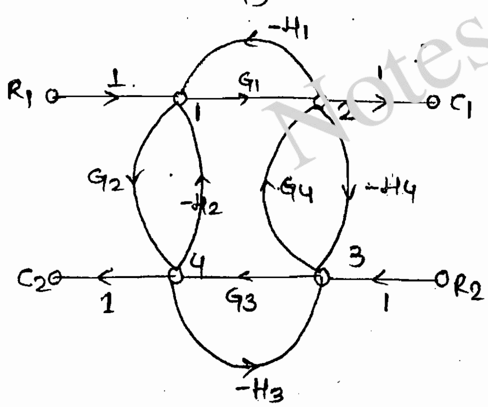
case(2) $\left.\frac{Y_2}{U_2}\right|_{U_1=0}$

$$\frac{Y_2}{U_2} = \frac{b_2(1-a_{11})}{1-(a_{11}+a_{22}+a_{12}a_{21})+(a_{11}a_{22})}$$

Q.→



Solⁿ→



case(1) $\left.\frac{C_1}{R_1}\right|_{R_2=C_2=0}$

$$\frac{C_1}{R_1} = \frac{G_1(1+G_3H_3)+G_2(-H_3)G_4(1)}{1-[-G_1H_1-G_4H_4-G_2H_2-G_3H_3+G_2H_3G_4H_1+G_1H_2G_3H_4] + (G_1H_1G_3H_3+G_2H_2G_4H_4)}$$

case(2) $\left.\frac{C_2}{R_2}\right|_{C_1=R_1=0}$; $\frac{C_2}{R_2} = \frac{G_3(1+G_1H_1) + G_4(-H_1)G_2(1)}{\Delta}$

case(3) $\left.\frac{C_2}{R_1}\right|_{C_1=R_2=0}$; $\frac{C_2}{R_1} = \frac{G_3(1+G_4H_4) + G_1(-H_4)G_3(1)}{\Delta}$

case(4) $\left.\frac{C_1}{R_2}\right|_{C_2=R_1=0}$; $\frac{C_1}{R_2} = \frac{G_4(1+G_2H_2) + G_3(-H_2)G_1(1)}{\Delta}$

conv(3.) 58

$$F = m\frac{d^2x}{dt^2} + f\frac{dx}{dt} + kx$$

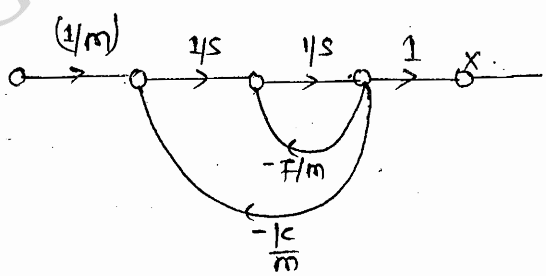
$$F(s) = (ms^2 + fs + k)\,X(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + fs + k}$$

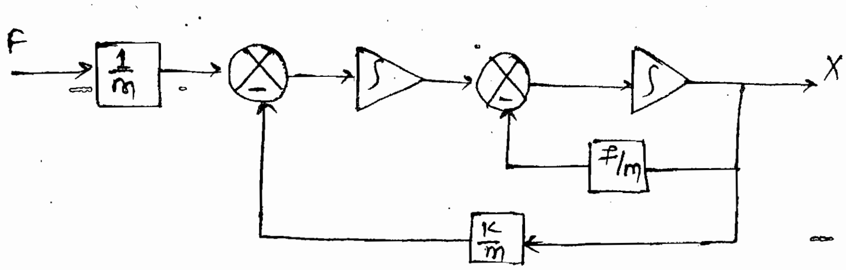
$$= \frac{(1/m)}{s^2 + \left(\frac{f}{m}\right)s + \left(\frac{k}{m}\right)}$$

State diagram →

$$\frac{\frac{(1/m)}{s^2}}{1 + \frac{(f/m)}{s} + \frac{(k/m)}{s^2}}$$



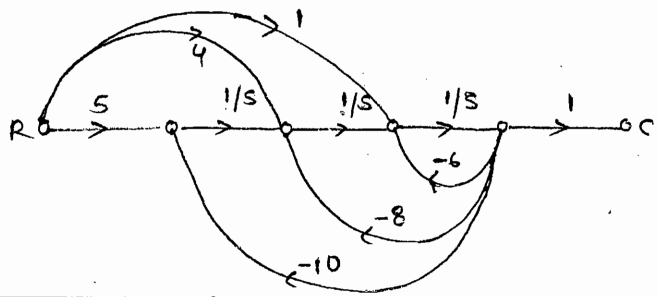
$$\frac{\frac{(1/m)}{s^2}}{1 - \left[\frac{-f/m}{s} - \frac{k/m}{s^2}\right]}$$

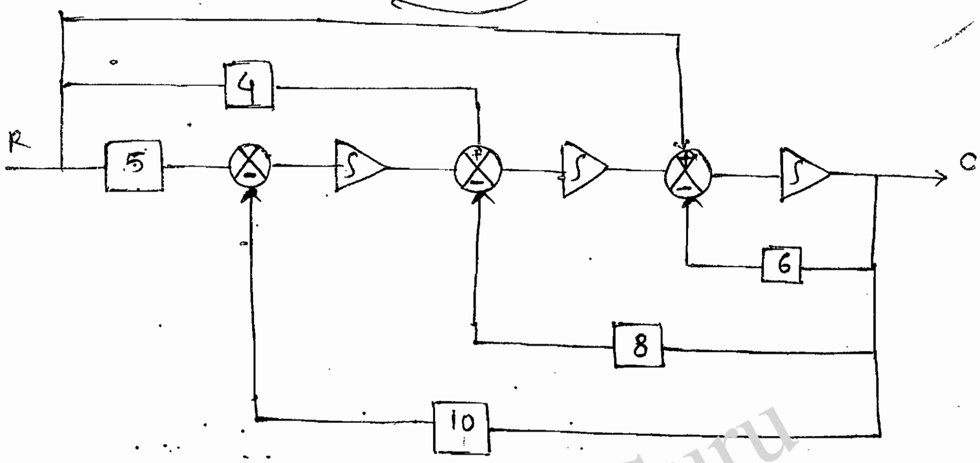


Q.→ $\frac{C(s)}{R(s)} = \frac{s^2+4s+5}{s^3+6s^2+8s+10}$

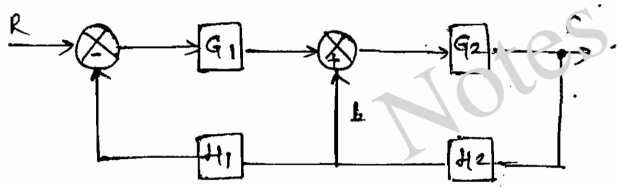
Sol^n→

$$\frac{C(s)}{R(s)} = \frac{\frac{s^2}{s^3}+\frac{4s}{s^3}+\frac{5}{s^3}}{1+\frac{6s^2}{s^3}+\frac{8s}{s^3}+\frac{10}{s^3}} = \frac{\frac{1}{s}+\frac{4}{s^2}+\frac{5}{s^3}}{1-\left(\frac{-6}{s}-\frac{8}{s^2}-\frac{10}{s^3}\right)}$$

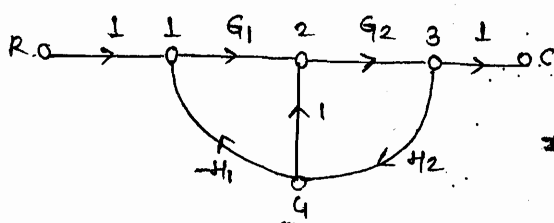


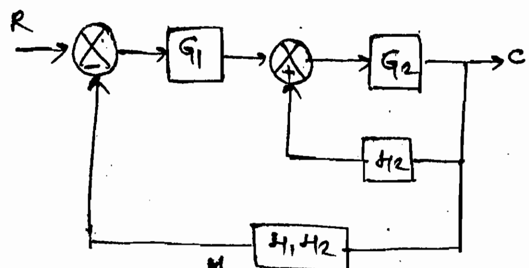


Q→

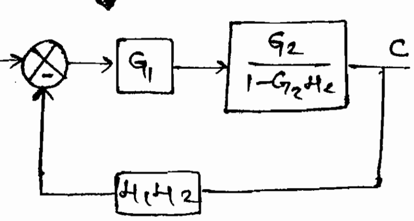


Sol^n→

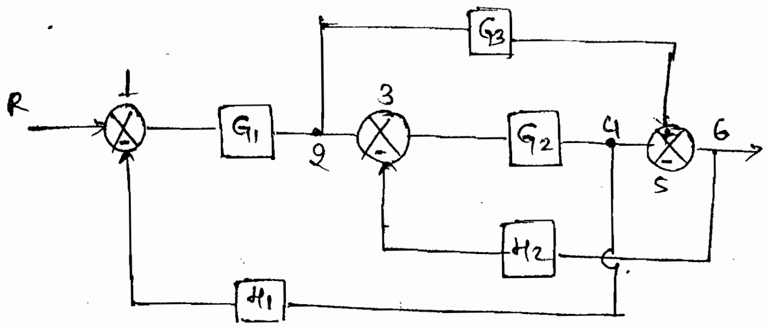




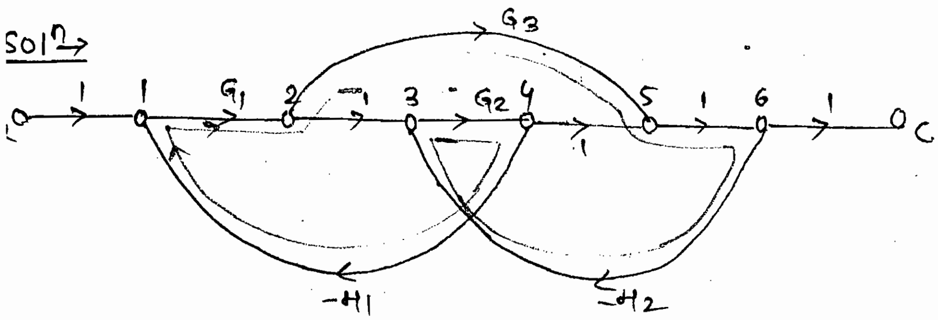
$$\frac{C}{R} = \frac{G_1G_2(1)}{1-(G_2H_2-G_1G_2H_1H_2)} = \frac{G_1G_2}{1-G_2H_2+G_1G_2H_1H_2}$$



1(Conv-)
56

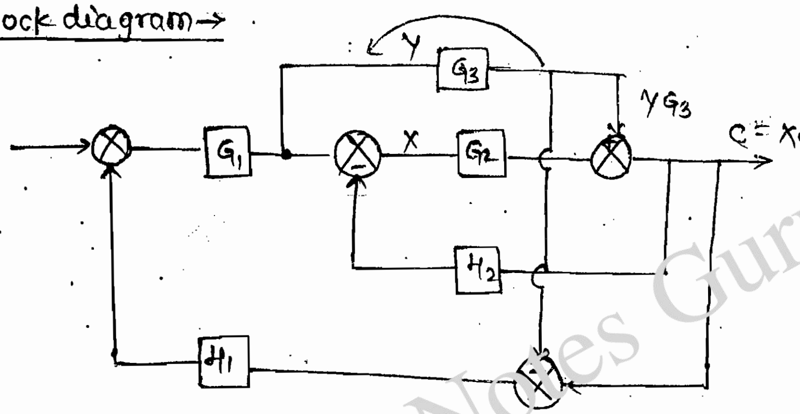


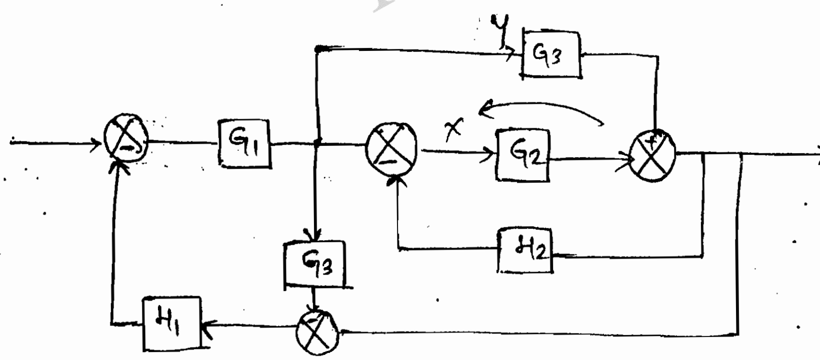
Sol^n →

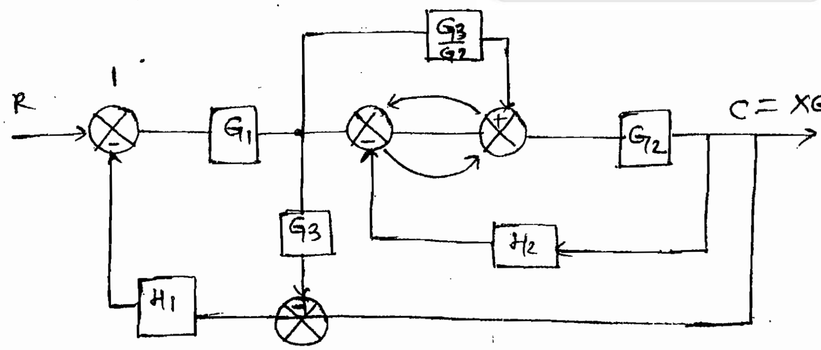


$$\frac{C}{R} = \frac{G_1G_2(1) + G_1G_3(1)}{1 - (-G_1G_2H_1 - G_2H_2 + G_1G_2G_3H_1H_2)}$$

Block diagram →

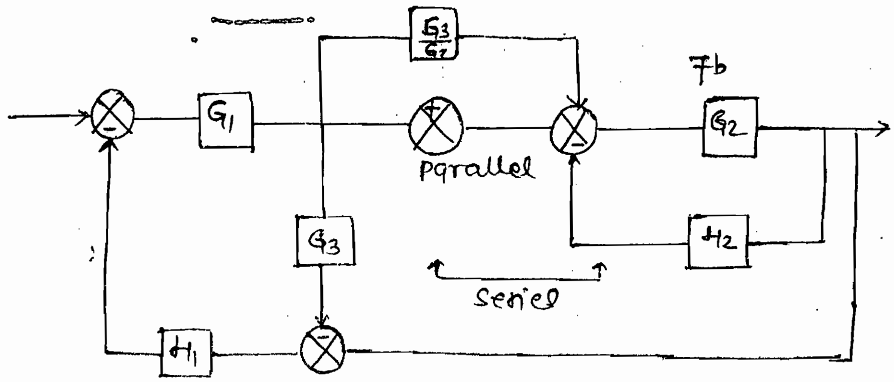






$C = XG_2 + YG_3$

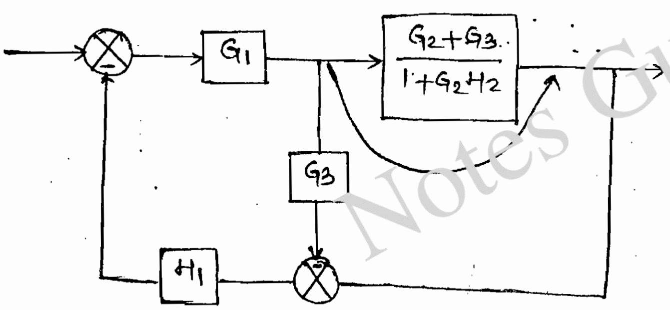
$\left[X + \frac{YG_3}{G_2}\right] G_2$

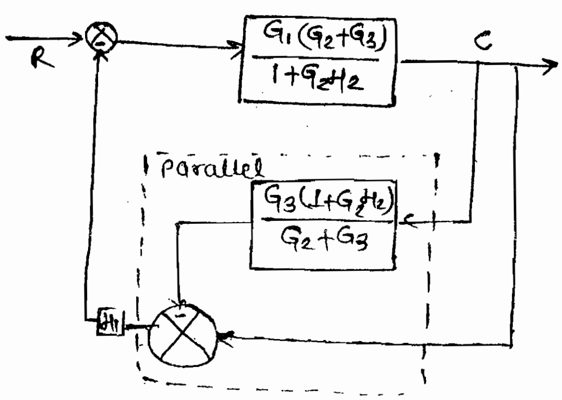


$$\text{Parallel} = 1 + \frac{G_3}{G_2} = \frac{G_2 + G_3}{G_2}$$

$$\text{Feedback} = \frac{G_2}{1 + G_2H_2}$$

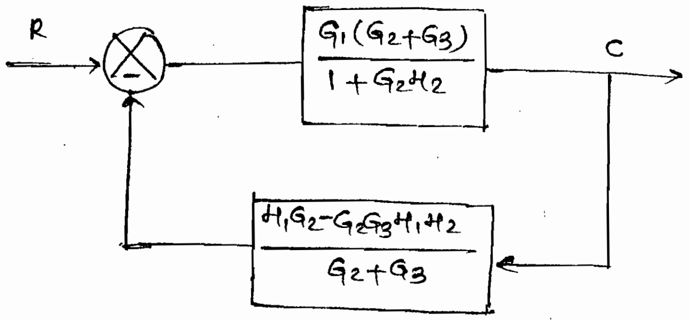
$$\text{series} = \frac{G_2 + G_3}{1 + G_2H_2}$$





Parallel

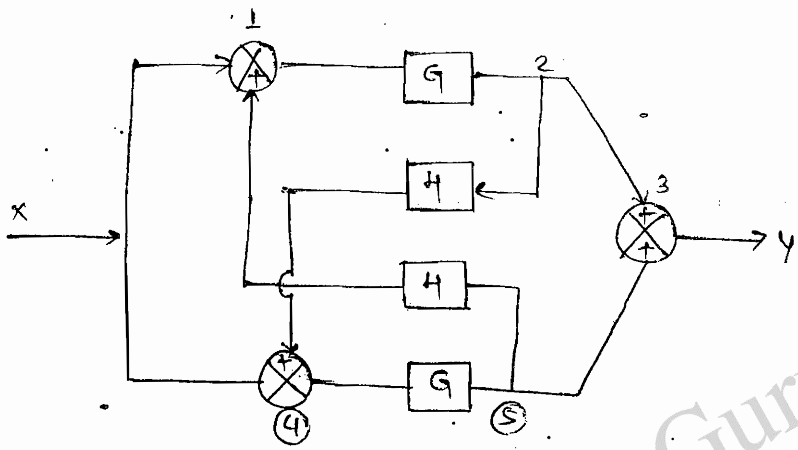
$$1 - \frac{G_3(1+G_2H_2)}{G_2+G_3} = \frac{G_2 + \cancel{G_3} - \cancel{G_3} - G_2G_3H_2}{G_2+G_3}$$



$$\frac{C}{R}=\frac{G_1(G_2+G_3)}{1+G_2H_2+G_1G_2H_1-G_1G_2G_3H_1H_2}$$

Q.→ $\frac{Y}{X}$ equals

(a.) $\frac{2G}{1-2GH}$ (b.) $\frac{2G}{1-GH}$ (c.) $\frac{G}{1-2GH}$ (d.) $\frac{G}{1-GH}$

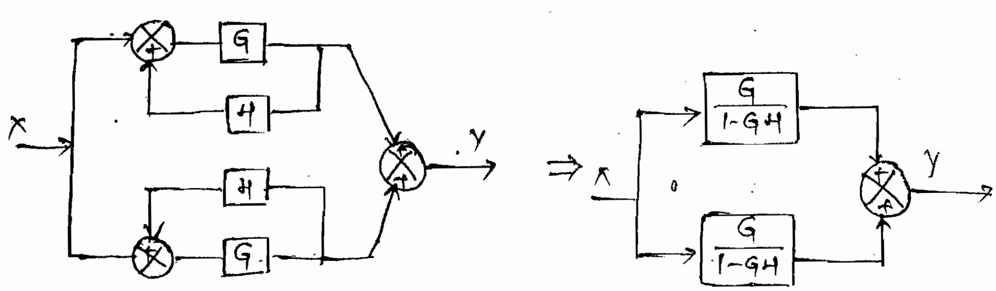


Sol^n→

$$\frac{Y}{X}=\frac{G(1)+G(1)+G^2H(1)+G^2H(1)}{1-G^2H^2}=\frac{2G+2G^2H}{1-G^2H^2}=\frac{2G\cancel{(1+GH)}}{\cancel{(1+GH)}(1-GH)}$$

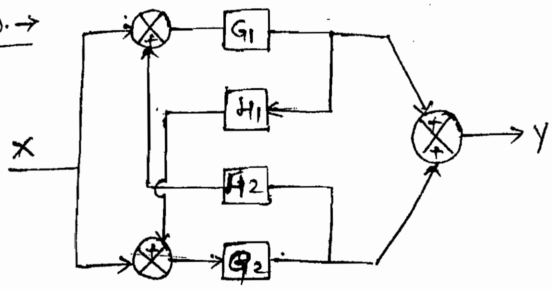
$$\boxed{\frac{Y}{X}=\frac{2G}{1-GH}}$$

By Block dia. reduction→



$$\boxed{\frac{Y}{X}=\frac{2G}{1-GH}}$$

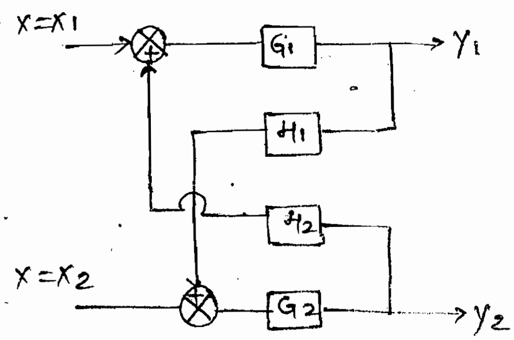
Q.→



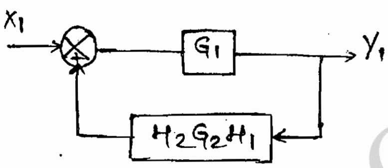
Sol^n→

$$\frac{Y}{X} = \frac{G_1 + G_2 + G_1H_1G_2 + G_2H_2G_1}{1 - G_1H_1G_2H_2}$$

By Block diagram →

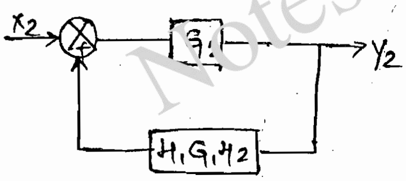


case(i) → $\left.\frac{Y_1}{X_1}\right|_{Y_2 = X_2 = 0}$



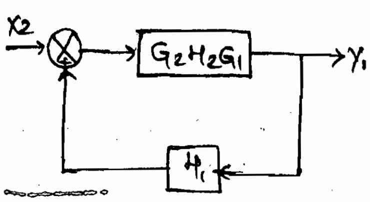
$$\frac{Y_1}{X_1} = \frac{G_1}{1 - G_1H_1G_2H_2}$$

Case(2) → $\left.\frac{Y_2}{X_2}\right|_{Y_1 = X_1 = 0}$



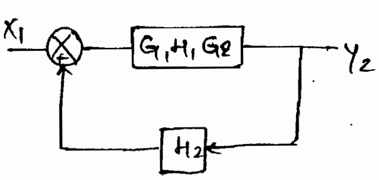
$$\frac{Y_2}{X_2} = \frac{G_2}{1 - G_1H_1G_2H_2}$$

Case(3) → $\left.\frac{Y_1}{X_2}\right|_{Y_2 = X_1 = 0}$



$$\frac{Y_1}{X_2} = \frac{G_1H_1G_2}{1 - G_1H_1G_2H_2}$$

Case(4) $\left.\frac{Y_2}{X_1}\right|_{X_2 = Y_1 = 0}$



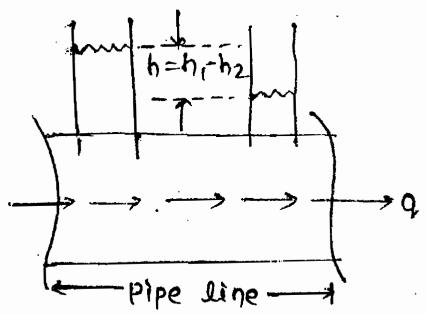
$$\frac{Y_2}{X_1} = \frac{G_2H_2G_1}{1 - G_1H_1G_2H_2}$$

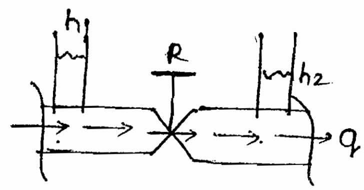
TF for physical systems →

* A general physical sys. is said to be constituted of 5 elements :-

1.) Resistance type element.
2.) Capacitance type elemet.
3.) Time-constant element.
4.) Oscillatory element.
5.) Dead time element.

1.) Resistance type element →

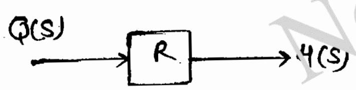




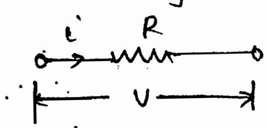
$$h \propto q$$

$$h = Rq \qquad R = \text{Hydraulic Resistance}$$

$$H(s) = RQ(s)$$

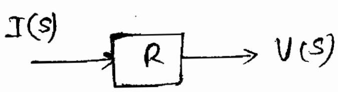


It is analouges to electrical resistance

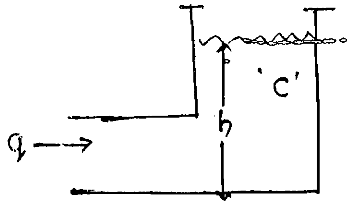


$$V = Ri$$

$$V(s) = RI(s)$$



(2.) Capacitance type element →



C = Hydraulic capacitance
= (Area/volume)

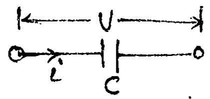
$$q \propto \frac{dh}{dt}$$

$$q = c\frac{dh}{dt}$$

$$Q(s) = cs\,H(s)$$

$$Q(s) \longrightarrow \boxed{1/cs} \longrightarrow H(s)$$

It is analouges to "electrical capacitance"

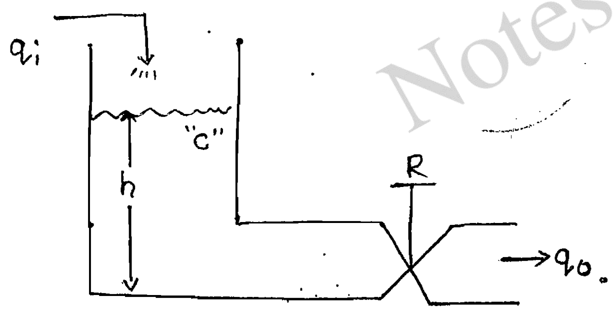


$$i = c\frac{dv}{dt}$$

$$I(s) = cs\,V(s)$$

$$I(s) \longrightarrow \boxed{\frac{1}{cs}} \longrightarrow V(s)$$

3.) Time-constant element →



$$q_i - q_o = \frac{c\,dh}{dt}$$

$$q_i = \frac{c\,dh}{dt} + q_o$$

since $h = Rq_o$

$$q_o = \frac{h}{R}$$

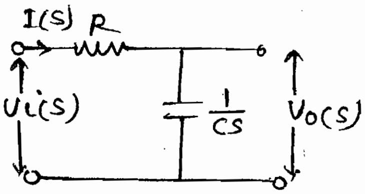
$$q_i = c\frac{dh}{dt} + \frac{h}{R}$$

$$Rq_i = Rc\frac{dh}{dt} + h$$

$$RQ_i(s) = (Rcs+1)\,H(s)$$

$$\boxed{\frac{H(s)}{Q_i(s)} = \frac{R}{Rcs+1}}$$

It is analogous to RC n/w



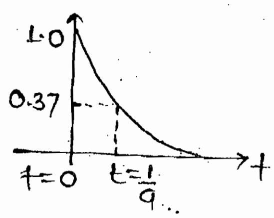
$$V_i(s) = I(s)\left[R + \frac{1}{CS}\right]$$

$$= I(s)\left[\frac{RCS+1}{CS}\right]$$

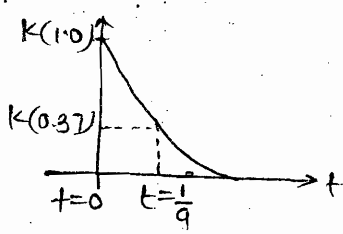
$$V_o(s) = I(s)\frac{1}{CS}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{RCS+1}}$$

Eg:- $\frac{1}{(S+a)} \Rightarrow e^{-at} \Rightarrow$

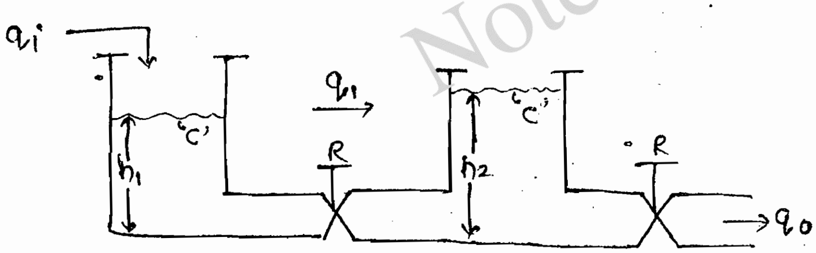


$\frac{k}{S+a} \Rightarrow ke^{-at} \Rightarrow$



~~(4) Oscillatory element→~~

* Interacting & non-interacting system→



$$q_i - q_1 = C\frac{dh_1}{dt}$$

$$Q_i(s) - Q_1(s) = CSH_1(s) \text{ ——— (i)}$$

$$h_1 - h_2 = Rq_1$$

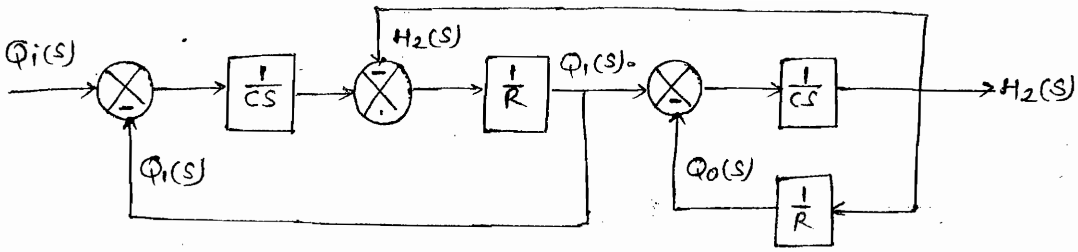
$$H_1(s) - H_2(s) = RQ_1(s) \text{ ——— (ii)}$$

$$q_1 - q_o = C\frac{dh_2}{dt}$$

$$Q_1(s) - Q_o(s) = CSH_2(s) \text{ ——— (iii)}$$

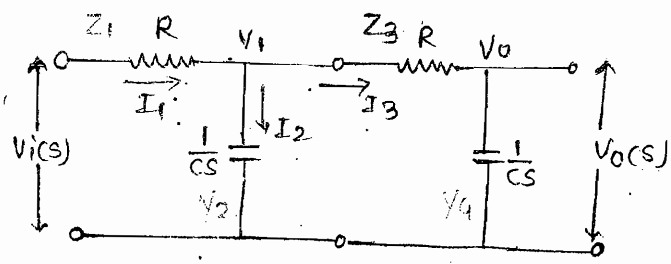
$$h_2 = Rq_o$$

$$H_2(s) = RQ_o(s) \text{ ——— (iv)}$$



$$\frac{H_2(s)}{Q_i(s)} = \frac{\frac{1}{R^2C^2s^2}(1)}{1-\left[\frac{-1}{RCs}-\frac{1}{RCs}-\frac{1}{RCs}\right]+\frac{1}{R^2C^2s^2}} = \frac{\frac{1}{R^2C^2s^2}}{\frac{R^2C^2s^2+3RCs+1}{R^2s^2C^2}}$$

$$\boxed{\frac{H_2(s)}{Q_i(s)} = \frac{R}{R^2C^2s^2+3RCs+1}}$$



(1.) Current through $Y_4\,(I_3) = V_0 \cdot Y_4 = V_0 Cs$.

(2.) To find V_1

$$I_3 = \frac{V_1 - V_0}{Z_3} \Rightarrow V_1 = I_3 Z_3 + V_0 = V_1 = V_0 Cs \cdot R + V_0$$

$$V_1 = V_0(1+RCs)$$

(3.) To find I_2

$$I_2 = V_1 \cdot Y_2 = V_0(1+RCs)\cdot Cs \Rightarrow V_0(Cs+RC^2s^2)$$

(4.) To find I_1

$$I_1 = I_2 + I_3 = V_0(Cs+RC^2s^2) + V_0 Cs = V_0(2Cs+RC^2s^2)$$

(5.) To find V_i

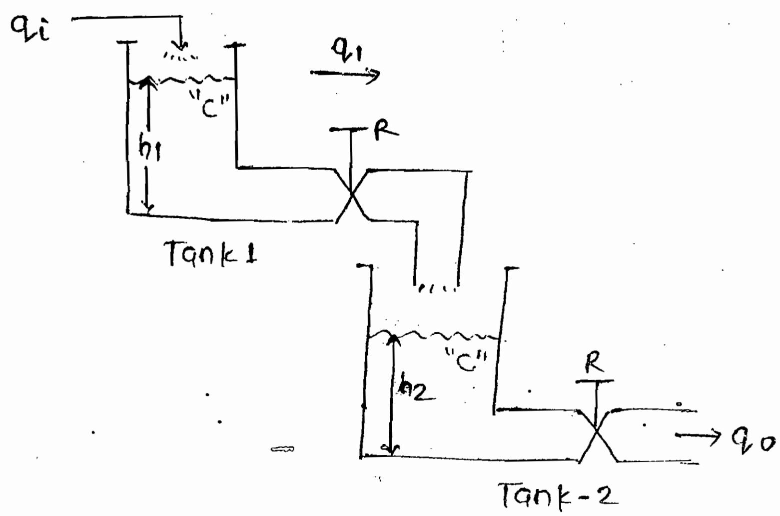
$$I_1 = \frac{V_i - V_1}{Z_1} \Rightarrow V_i = I_1 Z_1 + V_1 \Rightarrow V_0(2Cs+RC^2s^2)\,R + V_0(1+RCs)$$

$$V_i = V_0(2RCs + R^2C^2s^2 + 1 + RCs)$$

$$\boxed{\frac{V_0(s)}{V_i(s)} = \frac{1}{R^2C^2s^2+3RCs+1}}$$

Note→ When 2 time constant elements are cascaded interactively the overall TF of such an arrangement is not the product of 2 individual TF.

*** Non-interacting system →**



Tank-(01)

$$q_i - q_1 = c\frac{dh_1}{dt}$$

$$q_i = c\frac{dh_1}{dt} + q_1$$

Since $h_1 = Rq_1$

$$q_1 = h_1$$

$$q_i = c\frac{dh_1}{dt} + \frac{h_1}{R}$$

$$Rq_i = Rc\frac{dh_1}{dt} + h_1$$

$$RQ_i(s) = (Rcs+1)\,H_1(s) \quad \text{----- (i)}$$

Tank (02)

$$q_1 - q_0 = c\frac{dh_2}{dt}$$

$$q_1 = c\frac{dh_2}{dt} + q_0$$

$$\frac{h_1}{R} = c\frac{dh_2}{dt} + \frac{h_2}{R}$$

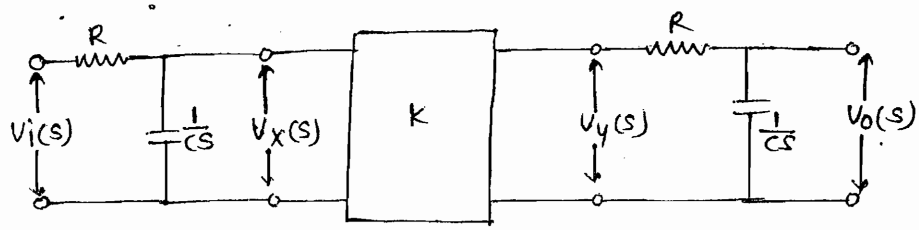
$(\because h_2 = Rq_0)$

$$h_1 = Rc\frac{dh_2}{dt} + h_2$$

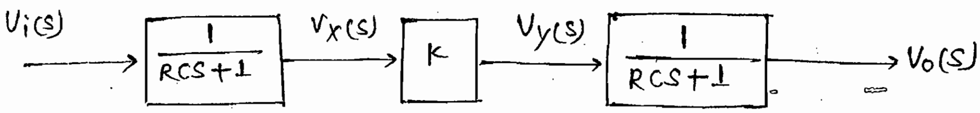
$$H_1(s) = (Rcs+1)\,H_2(s) \quad \text{----- (ii)}$$

$$Q_i(s) \longrightarrow \boxed{\frac{R}{Rcs+1}} \xrightarrow{H_1(s)} \boxed{\frac{1}{Rcs+1}} \xrightarrow{H_2(s)}$$

$$\boxed{\frac{H_2(s)}{H_1(s)} = \frac{R}{(Rcs+1)^2}}$$



$$\frac{V_x(s)}{V_i(s)} = \frac{1}{RCs+1} \qquad K = \frac{V_y(s)}{V_x(s)} \qquad \frac{V_o(s)}{V_y(s)} = \frac{1}{RCs+1}$$



$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{K}{(RCs+1)^2}}$$

Note→ when 2 time constant elements are cascaded non-interactively the overall TF of such an arrangement is the product of 2 individual TF.

* Control sys. componants→ CS, there are 2 types:-

(1) Tranducers → A Xcer is a device which when actuated with one form of energy is capable of converted it into any other related form; the conversion is usualy from non-electrical to electrical.

(2) Servomechanisms →* They are electromech. sys. whose i/p is ele. voltage & o/p is mech. position (OR) speed.

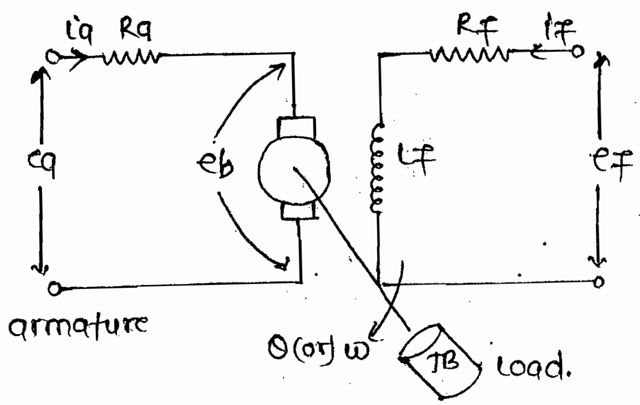
* They are also known as inverse Xcers.

servo - Low power low Freq. application

* The term servo indicates low power & low freq. appli^n. It also indicates that the i/p & o/p c/s must be approximately linear.

Servomotors
- Ac Servomotors
- dc servomotor
  - Armature Controlled
  - Field Controlled

* DC servomotor →



- armature Controlled dc servomotors →

Input = e_a ; output = θ (OR) ω

Principle of operation →

(i) Air gap flux (ϕ) $\propto$ Field current (i_f)

$$\phi = k_f \cdot i_f$$

(ii) $T_m \propto \phi \cdot i_a$

$T_m \propto k_f \cdot i_f \cdot i_a$

$T_m = \underbrace{k_1 k_f \cdot i_f} \cdot i_a$

k_T = motor torque const.

$T_m = k_T i_a$

$T_m(s) = k_T I_a(s)$ ——— (1)

(3.) Back emf (e_b) $\propto$ speed (ω)

$e_b = k_b \omega$

$E_b(s) = k_b \omega(s)$ ——— (2)

(4.) Analysis of arm. current

$e_a = i_a R_a + e_b$

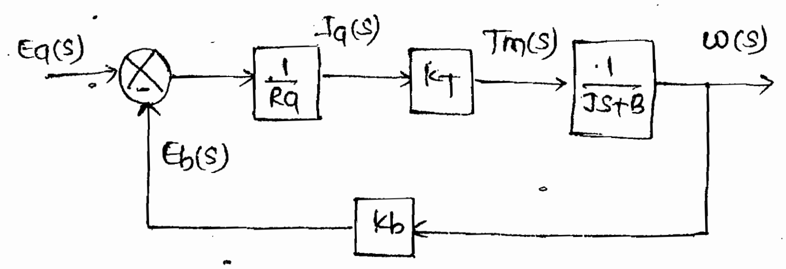
$e_a - e_b = i_a R_a$

$E_a(s) - E_b(s) = I_a(s) R_a$ ——— (3.)

(5.) At load

$T_m = J\dfrac{d\omega}{dt} + B\omega$

$T_m(s) = (Js+B)\,\omega(s)$ ——— (4.)



$$\frac{\omega(s)}{E_a(s)} = \frac{\frac{k_T}{R_a(JS+B)}}{1+\frac{k_T k_b}{R_a(JS+B)}} = \frac{k_T}{R_a(JS+B)+k_T k_b}$$

$$= \frac{k_T/R_a}{JS+\underbrace{B+\frac{k_T k_b}{R_a}}_{f}} = \frac{k_T/R_a}{JS+f}$$

$$\boxed{\frac{\omega(s)}{E_a(s)} = \frac{k_T/R_a}{JS+f}}$$

Note:- The TF of arm. control dc servomotor has a single time constant element & it represents the f/b control mechanism as seen from the block dia.

* Field control servomotor →

$I/p = e_f$ ~~, o/p~~ $= \theta$ (or) ω

Principle of operation →

(1.) Air gap flux (ϕ) $\propto$ field current (i_f)

$$\phi = k_f i_f$$

(2.) $T_m \propto \phi . i_a$

$$T_m \propto k_f i_f i_a$$

$$T_m = \underbrace{k_1 k_f i_a}_{k_T = \text{motor torque const.}} . i_f$$

$$T_m = k_T i_f$$

$$T_m(s) = k_T I_f(s) \quad \text{——— (1)}$$

(3.) Back emf (e_b) $\propto$ speed (ω)

$$e_b = k_b \omega$$

$$E_b(s) = k_b \omega(s) \quad \text{——— (2)}$$

(4) Analysis of field circuit

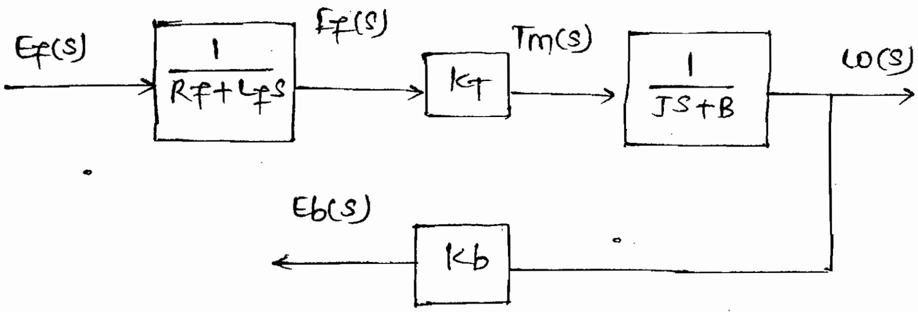
$$e_f = i_f R_f + L_f \frac{di_f}{dt}$$

$$E_f(s) = I_f(s)[R_f + L_f . s] \quad \text{——— (3.)}$$

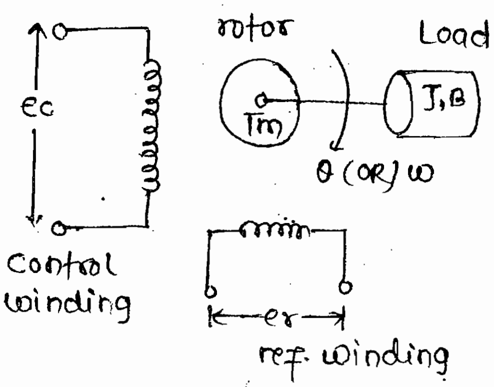
(5.) At load.

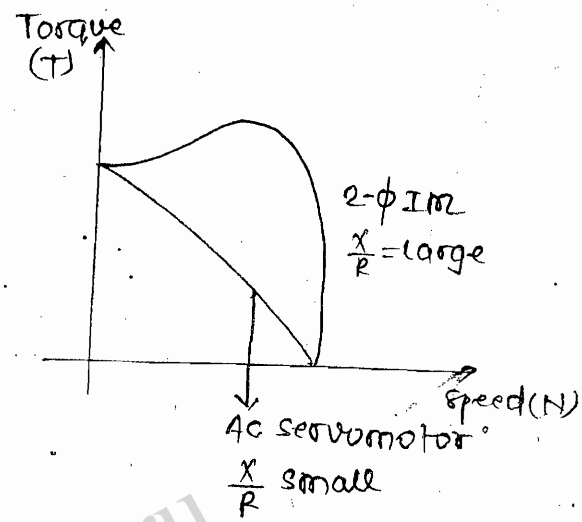
$$T_m = J\frac{d\omega}{dt} + B\omega$$

$$T_m(s) = (JS+B)\,\omega(s) \quad \text{——— (4)}$$



⇐ AC servomotor →





* It is constructionally similar to a 2-φ ind motor.

* Out of the 2 wdgs placed in quadrature. One of the wdg is excited by ~~for~~ a constant vol. & is known as ref wdg.

* The torque developed by the rotor is proportional to control winding vol.

* The rotor of AC servomotor is built with high resistance so that its $\frac{X}{R}$ ratio is small & torque-speed c/s are approximately linearised.

* In the TF modelling since the torque developed by the rotor is proportional to control winding vol.; the resistance & inductance of the control wdg is assumed to be negligible.

$T_m \propto e_c$

$T_m = k_m e_c$

$$\boxed{k_m = \frac{T_0}{e_c}}$$

where; T_0 = stall torque

$T_m(s) = k_m E_c(s)$ ------(i)

at load →

$$T_m = J\frac{d\omega}{dt} + B\omega$$

$T_m(s) = (Js+B)\cdot\omega(s)$ ------(2)

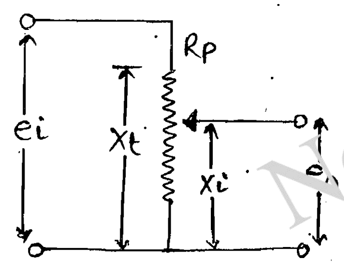
From eq^n (i) & (ii)

$k_m E_c(s) = (Js+B)\,\omega(s)$

$$\boxed{\frac{\omega(s)}{E_c(s)} = \frac{k_m}{Js+(B-m)}}$$

where; m = correction factor. slope of N-T charts.

* Potentiometer →



* It is a variable resistive displacement Xcer used as error detector in CS app^n

* A pair of potentiometer act as error detector.

Input = wiper disp. X_i , o/p = e_0

Total resistance of POT. = R_p

Resistance/~~for~~ unit length $= \frac{R_p}{X_t}$

Resistance for wiper disp of X_i units $= \frac{R_p}{X_t}\cdot X_i$

Applying voltage divider;

$$e_0 = \frac{\frac{R_P}{X_t} \cdot X_i}{R_P} \times e_i$$

$$e_0 = \frac{X_i}{X_t} \cdot e_i$$

Let K_p = POT gain $= \frac{e_i}{X_t} \left(\frac{V}{mm}\right)$

$$e_0 = K_p X_i$$

$$E_0(s) = K_p X_i(s)$$

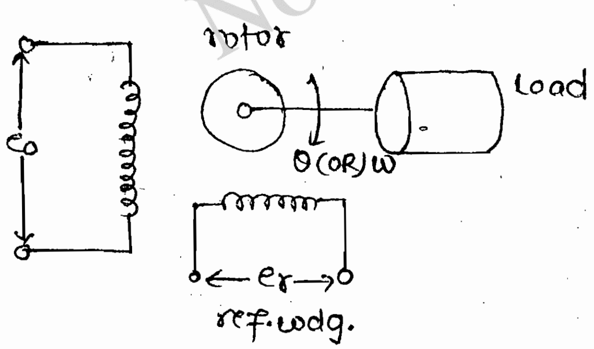
$$X_i(s) \longrightarrow \boxed{K_p} \longrightarrow E_0(s)$$

* Tachometer → * They are speed Xcer used as f/b elements in CS appn.

* There of 2 types :-

(i) DC tachometer → These is a small dc genr whose i/p is mech. speed & o/p is ele. voltage proportional to the speed.

(ii) AC tachometer →



* Out of the 2 wdgs placed in quadrature only one of the wdg is excited by a constant vol. & is known as ref. wdg.

* when the rotor is stationary the peripheral flux links the ref. wdg only. As the rotor rotates the rate of change of flux induces an emf

which is directly proportional to speed of rotor.
The ac tachometer is also known as drag cup genr because the rotor ~~Transfer fn~~ is of drag cup shape.

Transfer fn →

$e_0 \propto speed$

(a.) Input $= \theta$, O/p $= e_0$

$e_0 \propto \frac{d\theta}{dt}$

$e_0 = k_T \cdot \frac{d\theta}{dt}$

$E_0(s) = k_T . s \cdot \theta(s)$

$\theta(s) \longrightarrow \boxed{k_T s} \longrightarrow E_0(s)$

(b.) I/p $= \omega$

O/p $= e_0$

$e_0 \propto \omega$

$E_0(s) = k_T \, \omega(s)$

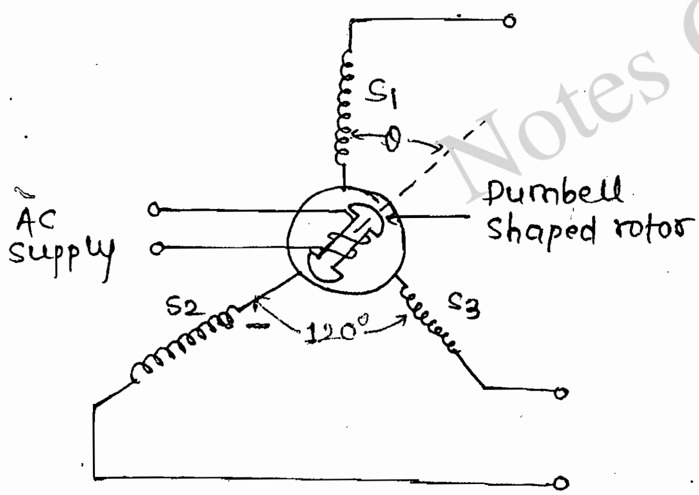
$\omega(s) \longrightarrow \boxed{k_T} \longrightarrow E_0(s)$

where; k_T = Tachometer gain

Ex. → The T.F. of a tachometer $\frac{E(s)}{\theta(s)}$ is

(a) ks (b) ks^2 (c) $\frac{k}{s}$ (d) k

* SYNCHRO →

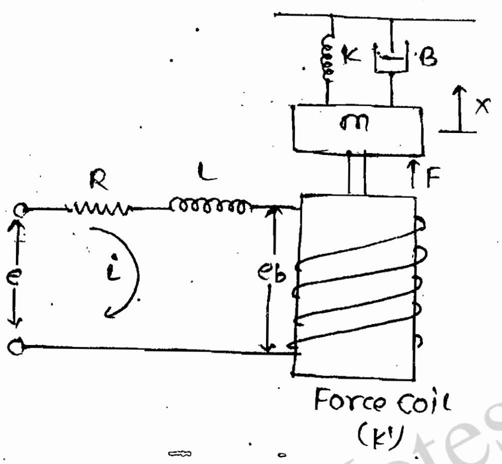


It is commercialy known as SELSYN (OR) autosyn.

It is an electromagnetic Xcer which converts angular portion of the rotor into proportional voltages.

* It is contructionaly similar to a 3-ϕ alternator but operationaly the principle is based on Xmer action.
* when the rotor is inclined along any one of the stator access completely then max^m vol. will be induced in that wdg. & negligible vol. is induced in the other 2 wdgs.
* Such a position of rotor is known as ele. zero position.
* A pair of synchro known as synchro Xmitter & synchro control Xmer (receiver) act as error detector.
* The Xmitter rotor is dumbell shape & th

Que.→



Soln→ I/p = e ; O/p = x

$$e = iR + L\frac{di}{dt} + e_b$$

$$e - e_b = iR + L\frac{di}{dt}$$

$$E(s) - E_b(s) = I(s)[R + Ls] \text{ ------ (i)}$$

e_b (tranduced vol.)

$$e_b \propto k_T \frac{dx}{dt}$$

$$E_b(s) = k_T s X(s) \text{ ------ (ii)}$$

where; k_T = Tachometer gain.

At Force coil

$$F \propto i$$

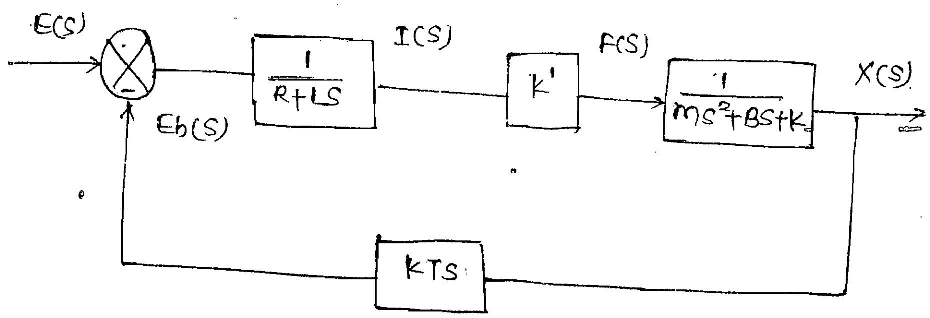
$$F = k'i$$

$$F(s) = k' I(s) \text{ ----- (iii)}$$

At mech. sys.

$$F = m\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

$$F(s) = (ms^2 + Bs + K) X(s) \text{ --- (iv)}$$

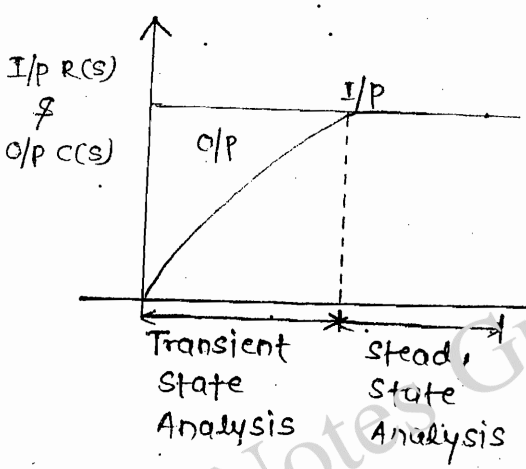

E(S)
I(S)
F(S)
X(S)
Eb(S)
1
R+LS
K'
1
ms²+BS+K
KTS

Chapter-02
Time domain analysis

* The time responce analysis is the analysis on time taken by the Responce of the sys. when subjected to an i/p.
* It is devided into 2 parts:-

(i) Transient state analysis → It deals with the nature of responce of sys. when subjected to an i/p.

(ii) Steady state responce analysis → It deals with the estimation of magnitude of steady state error b/n i/p & o/p.



* Standard test signals →

(1.) Sudden i/p → Step signal
(2.) Velocity type i/p → Ramp signal
(3) Acceleration type i/p → Parabolic signal
(4.) Sudden shocks → Impulse signal

Time domain analysis (1)–(3)
→ Stability (4)

Note:- The transient state analysis & the transient state specification are defined for step signal only because the magnitude of the i/p sig. should not change with time.

* Type & Order →

(1.) Every TF representing the cs has certain type & order.

(2.) Steady-state responce analysis depands on type of the cs.

(3.) The type of the sys. obtained from open loop TF $G(s)H(s)$, by observing the no. of open loop poles occuring at origin.

$$\text{Let } G(s)\cdot H(s) = \frac{K(1+T_a s)}{s^p(1+T_1 s)}$$

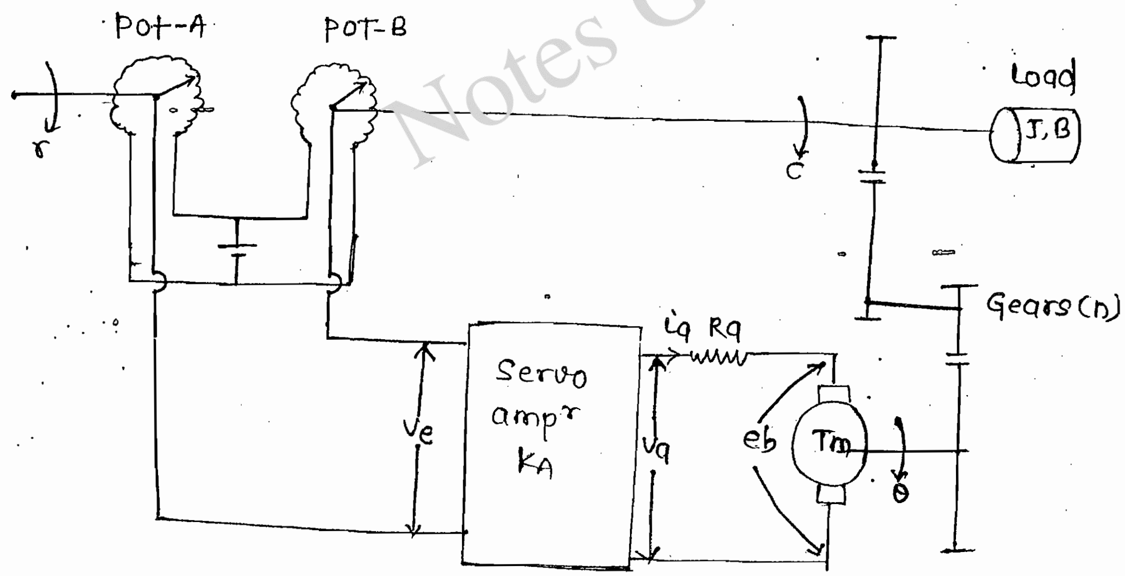
p=0, type-0 system
p=1, type-1 system
⋮ ⋮
p=n, type-n system

(4.) The transient state analysis depands upon the order of cs.

(5.) The order of sys is obtained from closed loop TF $\frac{G(s)}{1+G(s)H(s)}$ by observing the highest power of a c/s eqⁿ.

$$\boxed{1+G(s)\cdot H(s)=0}$$

Eg:- Position Control system



Arm. controlled
dc servomotor.

(1.) $I/p = r\,[R(s)]$, $o/p = c\,;\,[C(s)]$

(2.) Principle of operation

(i) At potentiometer

$V_e \propto (r-c)$

$V_e = k_p(r-c)$

$V_e(s) = k_p[R(s)-C(s)]$ ----(i)

(ii) At Ampr

$V_a \propto V_e$

$V_a = k_A V_e$

$V_a(s) = k_A V_e(s)$ ------(ii)

(iii) Analysis of arm. controlled dc servomotor

$T_m = k_T I_a(s)$ ------(iii)

$e_b = k_b \frac{d\theta}{dt}$

$E_b(s) = k_b(s)\,\theta(s)$ ------(iv)

$V_a = i_a R_a + e_b$

$V_a(s) - E_b(s) = I_a(s)\cdot R_a$ -----(v)

$T_m = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt}$

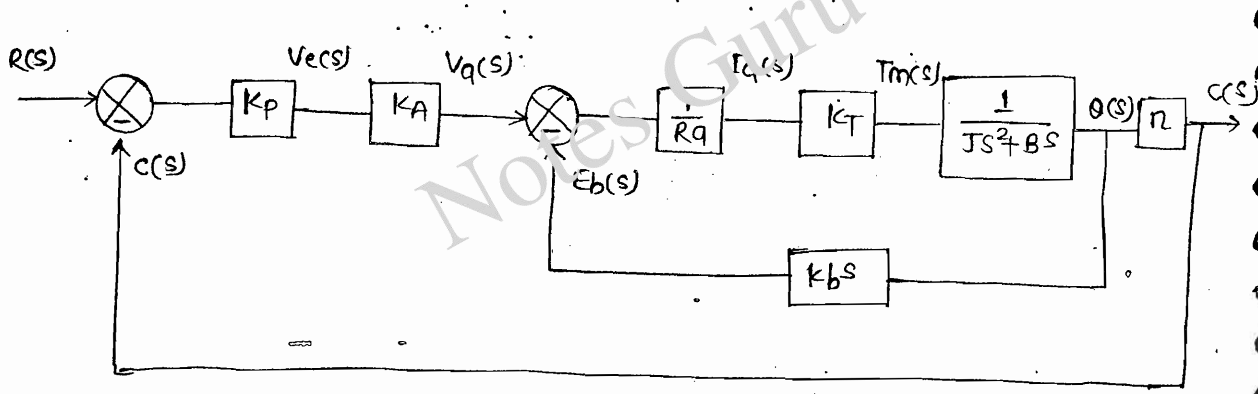
$T_m(s) = (Js^2+Bs)\,\theta(s)$ -----(vi)

(iv) At gears:-

$c \propto \theta$

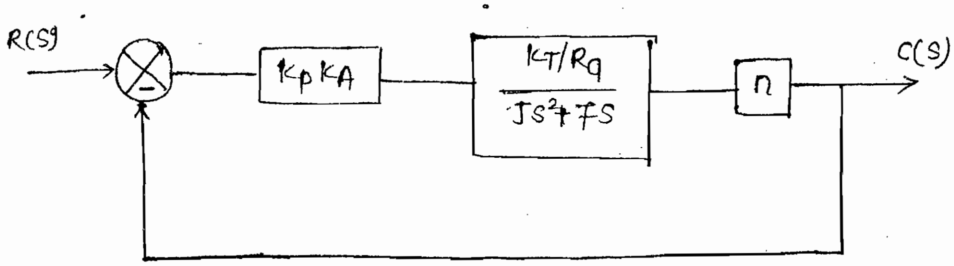
$c = n\theta$

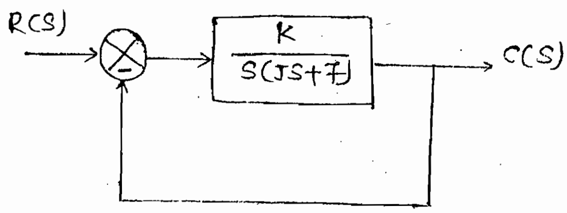
$C(s) = n\,\theta(s)$ -----(vii)



Inner f/b loop→

$$\frac{\frac{k_T}{R_a(Js^2+Bs)}}{1+\frac{k_T k_b\, s}{R_a(Js^2+Bs)}} = \frac{k_T}{R_a(Js^2+Bs)+k_T k_b s} = \frac{k_T/R_a}{Js^2+\underbrace{Bs+\frac{k_T k_b s}{R_a}}_{Fs}} = \frac{k_T/R_a}{Js^2+Fs}$$





where; $K = \dfrac{K_pK_AK_Tn}{R_a}$

For type of sys →

$$G(S) = \frac{K}{S(JS+f)} \quad ; H(S) = 1$$

$$G(S)\cdot H(S) = \frac{K}{S(JS+f)}$$

$\boxed{\text{Type} = 1}$

Order of the sys. →

$$1 + G(S)H(S) = 0$$

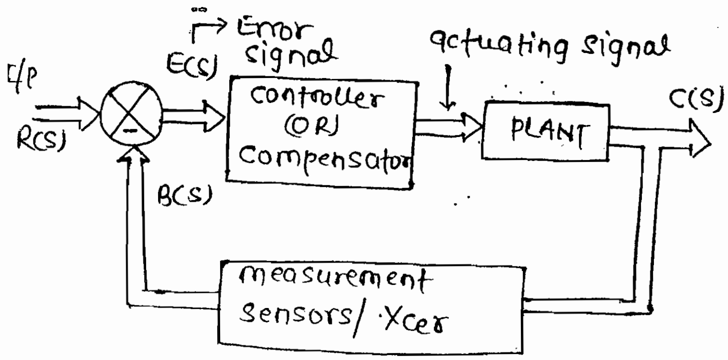
$$1 + \frac{K}{S(JS+f)} = 0$$

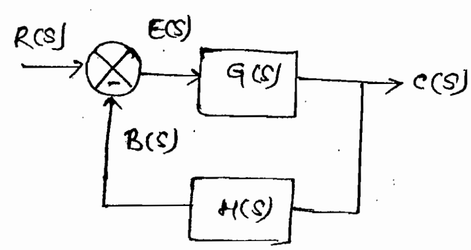
$$JS^2 + fS + K = 0$$

$\boxed{\text{Order} = 2}$

* Steady state Responce analysis → It deals with estimation of magnitude of steady state error b/n i/p & o/p & depends on type of CS.

Error Compensation →





To obtain an expression for error

$$E(s) = \underset{\downarrow}{R(s)} - \underset{\downarrow}{B(s)}$$
$$\quad \text{i/p} \qquad \text{o/p}$$

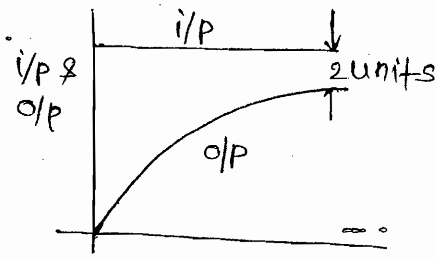
$$E(s) = R(s) - C(s).H(s)$$

$$E(s) = R(s) - E(s).G(s).H(s)$$

$$E(s)\left[1 + G(s).H(s)\right] = R(s)$$

$$\boxed{E(s) = \frac{R(s)}{1+G(s).H(s)}}$$

error Ratio



$$\lim_{t\to\infty} e(t) = 2\text{units} \quad (e_{ss})$$

$$\boxed{e_{ss} = \lim_{t\to\infty} e(t)}$$

By FVT

$$\boxed{e_{ss} = \lim_{s\to 0} s.E(s)}$$

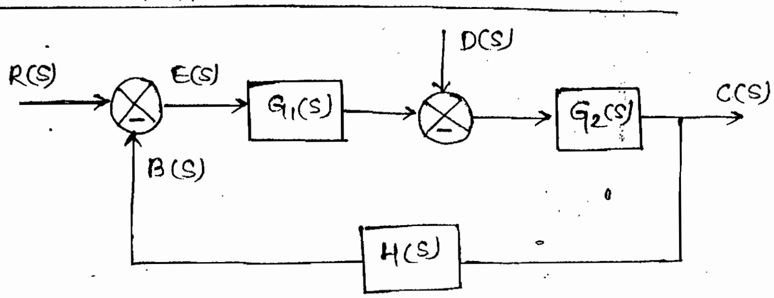
$$\therefore e_{ss} = \lim_{s\to 0} \frac{s.R(s)}{1+G(s).H(s)}$$

$$\boxed{e_{ss} = \frac{\lim\limits_{s\to 0} s.R(s)}{1+\lim\limits_{s\to 0} G(s).H(s)}}$$

Note:- * Since the steady state error is defined as the diff b/n i/p & o/p, to find the type of the sys. & hence steady state error the F/b gain should be unity [$H(s) = 1$]

* For non-unity F/b elements they should be specified as measuring element i.e sensors (OR) Xcer element.

* To obtain an exp^n for error with disturbance →



$$E(s) = R(s) - B(s)$$

$$E(s) = R(s) - C(s)\cdot H(s)$$

$$\because \; C(s) = \left[E(s)\,G_1(s) + D(s)\right] G_2(s)$$

$$C(s) = E(s)\cdot G_1(s)\,G_2(s) + D(s)\cdot G_2(s)$$

$$E(s) = R(s) - E(s)\cdot G_1(s)\cdot G_2(s)\cdot H(s) - D(s)\cdot G_2(s)\cdot H(s)$$

$$E(s)\left[1 + G_1(s)\cdot G_2(s)\cdot H(s)\right] = R(s) - D(s)\cdot G_2(s)\cdot H(s)$$

$$E(s) = \frac{R(s)}{1+G_1(s)\,G_2(s)\,H(s)} - \frac{D(s)\cdot G_2(s)\,H(s)}{1+G_1(s)\,G_2(s)\cdot H(s)}$$

$$e_{ss} = \lim_{s\to 0} s\cdot E(s)$$

$$\boxed{e_{ss} = \lim_{s\to 0} \frac{s\cdot R(s)}{1+G_1(s)\cdot G_2(s)\,H(s)} - \lim_{s\to 0} \frac{s\cdot D(s)\cdot G_2(s)\,H(s)}{1+G_1(s)\cdot G_2(s)\,H(s)}}$$

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$$e_{ss} = -\lim_{s\to 0} \frac{s\cdot D(s)\cdot G(s)}{1+G_1(s)\cdot G_2(s)}$$

$$= -\lim_{s\to 0} \frac{\cancel{s}\times \frac{1}{\cancel{s}}\cdot G_2(s)}{1+G_1(s)\cdot G_2(s)}$$

ans (c)

$$e_{ss} = \frac{-G_2}{G_1G_2+1} \Rightarrow |e_{ss}| = \frac{G_2}{1+G_1G_2}$$

$e_{ss}\downarrow$ by $G_1\uparrow$

* Steady state error for different types of i/p →

(1) Step i/p

$$R(s) = \frac{A}{s}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{A}{\cancel{s}}}{1+G(s)\cdot H(s)}$$

$$= \frac{A}{1+\lim\limits_{s\to 0} G(s)\cdot H(s)} = \frac{A}{1+K_p}$$

K_p = position error constant

$$= \lim_{s\to 0} G(s)\cdot H(s)$$

$$\boxed{e_{ss} = \frac{A}{1+K_p}}$$

(2.) Ramp signal

$$R(s) = \frac{A}{s^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{A}{s^{\cancel{2}}}}{1+G(s)\cdot H(s)}$$

$$= \lim_{s\to 0} \frac{A}{s + sG(s)\cdot H(s)}$$

$$= \frac{A}{\lim_{s\to 0} s + \lim_{s\to 0} s\cdot H(s)\cdot G(s)}$$

$$= \frac{A}{K_v}$$

K_v = Velocity error const.

$$= \lim_{s\to 0} s\cdot G(s)\cdot H(s)$$

(3.) Parabolic signal

$$R(s) = \frac{A}{s^3}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\cdot\frac{A}{s^3}}{1+G(s)\cdot H(s)}$$

$$\frac{A}{\lim_{s\to 0} s^2 + \lim_{s\to 0} s^2 G(s)\cdot H(s)} = \frac{A}{K_A}$$

K_A = acceleration error const.

$$= \lim_{s\to 0} s^2 G(s)\cdot H(s)$$

Steady state error for different types of system →

Type-0 →

$$G(s)\cdot H(s) = \frac{K(1+T_a s)}{(1+T_1 s)}$$

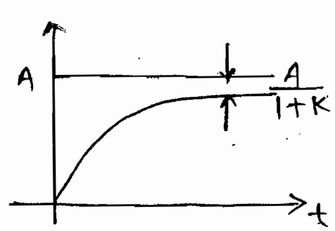
(1.) step i/p

$$R(s) = \frac{A}{s}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{A}{\cancel{s}}}{1+\frac{K(1+T_a s)}{(1+T_1 s)}}$$

$$= \frac{A}{1+\lim_{s\to 0}\frac{K(1+T_a s)}{(1+T_1 s)}}$$

$$= \frac{A}{1+K}$$



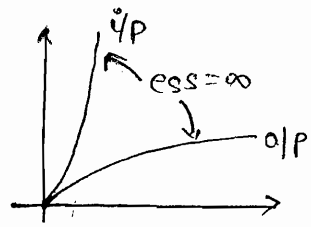
(2.) RAmp i/p

$$R(s) = \frac{A}{s^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{A}{s^{\cancel{2}}}}{1+\frac{K(1+T_a s)}{(1+T_1 s)}}$$

$$e_{ss} = \lim_{s\to 0} \frac{A}{s+\frac{Ks(1+T_a s)}{(1+T_1 s)}}$$

$$= \infty$$



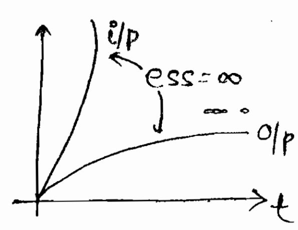
(3.) Parabolic i/p

$$R(s) = \frac{A}{s^3}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{A}{s^{\cancel{3}2}}}{1+K\left(\frac{1+T_a s}{1+T_1 s}\right)}$$

$$= \lim_{s\to 0} \frac{A}{s^2 + Ks^2\left(\frac{1+T_a s}{1+T_1 s}\right)}$$

$$= \infty$$



System	Step Input	Ramp input	Parabolic Input
TYPE-0	$\frac{A}{1+K}$ $K_p = K$	∞ $K_V = 0$	∞ $K_A = 0$
TYPE-1	0 $K_p = \infty$	$\frac{A}{K}$ $K_V = K$	∞ $K_A = 0$
TYPE-2	0 $K_p = \infty$	0 $K_V = \infty$	A/K $K_A = K$

1	∞	∞
0	1	∞
0	0	1

K_p	K_V	K_A
K	0	0
∞	K	0
∞	∞	K

Observations →

(1.) $e_{ss} \propto \frac{1}{K}$

As $K\uparrow$ $e_{ss}\downarrow$

(2.) The max^m type no. for a linear CS is 2. Beyond type-2 the sys. tends to become unstable & also exibits non-linear c/s more dominatly.

(3/61) $G(s) = \frac{10}{s^2(4+s)}$; $r(t) = 2+3t+4t^2$

$$R(s) = \frac{2}{s} + \frac{3}{s^2} + \frac{8}{s^3}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\left(\frac{2s^2+3s+8}{s^{\cancel{3}2}}\right)}{1+\frac{10}{s^2(4+s)}}$$

$$e_{ss} = \lim_{s\to 0} \frac{2s^2+3s+8}{s^2+s^2 \cdot \frac{10}{s^2(4+s)}}$$

$$= \frac{8\times 4}{10}$$

$$\boxed{e_{ss} = 3.2}$$

Shortcut method

type-2

$$R(s) = \frac{2}{s} + \frac{3}{s^2} + \frac{8}{s^3}$$

$$e_{ss} = 0 \quad 0 \quad \frac{A}{K}$$

$A = 8$

$$K = K_A = \lim_{s\to 0} \cancel{s^2} \cdot \frac{10}{\cancel{s^2}(4+s)} = \frac{10}{4}$$

$$e_{ss} = \frac{8}{\left(\frac{10}{4}\right)} = 3.2$$

(4/61) $r(t) = (1-t^2)\,3u(t)$

$R(s) = \frac{3}{s} - \frac{6}{s^2}$

$$e_{ss} = \lim_{s\to 0} \frac{s\left[\frac{3}{s} - \frac{6}{s^3}\right]}{1+G(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\left(\frac{3}{\cancel{s}}\right)}{1+G(s)} - \lim_{s\to 0} \frac{s\cdot\frac{6}{s^3}}{1+G(s)}$$

$$e_{ss} = \frac{3}{1+\lim\limits_{s\to 0} G(s)} - \frac{6}{\lim\limits_{s\to 0} s^2 G(s)}$$

$$\boxed{e_{ss} = \frac{3}{1+K_p} - \frac{6}{K_A}}$$

Shortcut method

$R(s) = \frac{3}{s} - \frac{6}{s^3}$

$= \frac{A}{1+K} - \frac{A}{K}$

$= \frac{3}{1+K_p} - \frac{6}{K_A}$

If I/p is not specified then finite error is taken.

(5/61) Type-0 $\xrightarrow{(1/s)}$ Type-1

$e_{ss} = \frac{1}{1+K}$

$0.2 = \frac{1}{1+K}$

$K = 4$

$e_{ss} = \frac{1}{K}$

$e_{ss} = \frac{1}{4} = 0.25$ units

$\boxed{e_{ss} = 0.25}$

(6/61) Type-1 system

$e_{ss} = \frac{1}{K}$

$e_{ss} = 5\% \Rightarrow \frac{5}{100} = \frac{1}{20}$

$\frac{1}{20} = \frac{1}{K}$

$\boxed{K = 20 \quad \text{Type-1}}$

(1/6) $G(s) = \frac{10}{s^2+14s+50}$

Type-0

$e_{ss} = \frac{1}{1+K}$

$K = K_p = \lim\limits_{s\to 0} \frac{10}{s^2+14s+50} = \frac{10}{50}$

$e_{ss} = \frac{1}{1+\frac{10}{50}} = \frac{50}{60} = 0.83$ units

$\boxed{e_{ss} = 0.83}$

3/59 $G(s) = \frac{K}{s(s+a)}$

Type-1 system

$e_{ss} = \frac{A}{K}$

$A=1,\ K=K_v = \lim_{s\to 0} s\cdot \frac{K}{s(s+a)}$

$= \frac{K}{a}$

$e_{ss} = \frac{a}{K}$

(i) $S_K^{e_{ss}} = \frac{K}{e_{ss}} \cdot \frac{\partial e_{ss}}{\partial K}$

$e_{ss} = \frac{a}{K}$

$\frac{e_{ss}}{K} = \frac{a}{K^2} \Rightarrow \frac{K}{e_{ss}} = \frac{K^2}{a}$

$\frac{\partial e_{ss}}{\partial K} = \frac{\partial}{\partial K}\left(\frac{a}{K}\right) = \frac{-a}{K^2}$

$S_K^{e_{ss}} = \frac{K^2}{a} \times \frac{-a}{K^2} = -1$

$\boxed{S_K^{e_{ss}} = -1}$

(ii) $S_a^{e_{ss}} = \frac{a}{e_{ss}} \cdot \frac{\partial e_{ss}}{\partial a}$

$e_{ss} = \frac{a}{K} \Rightarrow \frac{a}{e_{ss}} = K$

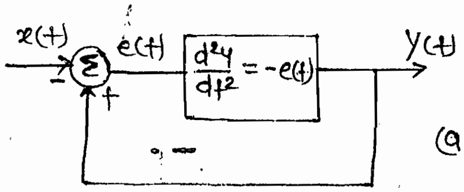
$\frac{\partial e_{ss}}{\partial a} = \frac{\partial}{\partial a}\left(\frac{a}{K}\right) = \frac{1}{K}$

$S_a^{e_{ss}} = K \times \frac{1}{K} = 1$

$\boxed{S_a^{e_{ss}} = 1}$

Note:- Sensitivity of ess wrt K & a is same for the above system.

Que.→



For $x(t) = t u(t)$ find $e(t) = ?$

(a) $\sin t$ (b) $\cos t$ (c) $-\sin t$ (d) $-\cos t$

Soln→

$e(t) = -x(t) + y(t)$

$E(s) = -X(s) + Y(s)$

$\frac{d^2y}{dt^2} = -e(t)$; $s^2 Y(s) = -E(s)$

$Y(s) = \frac{-E(s)}{s^2}$

$E(s) = -X(s) - \frac{E(s)}{s^2}$

$E(s) + \frac{E(s)}{s^2} = -X(s)$

$\left(\frac{s^2+1}{s^2}\right) E(s) = -X(s)$

$E(s) = \frac{-X(s) \cdot s^2}{s^2+1}$

Given $x(t) = t u(t)$

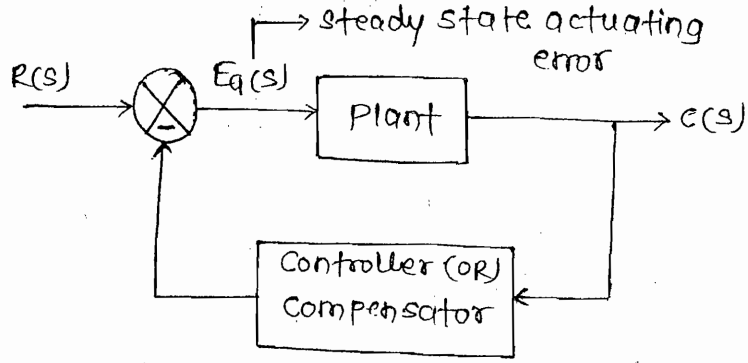
$X(s) = \frac{1}{s^2}$

$E(s) = \frac{-\frac{1}{s^2} \cdot s^2}{(s^2+1)}$

$= \frac{-1}{s^2+1}$

$\boxed{e(t) = -\sin t}$

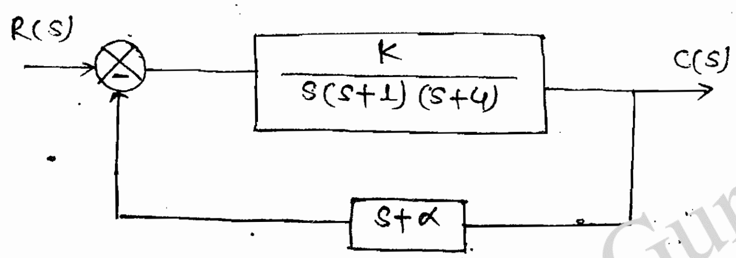
* Output Compensation →



* A controller/compensator placed in the f/b path compensates for changes in o/p & a steady state actuating error signal effects the dynamics of the plant to acheive the control objective.

∴ In such cases to find steady state error which is the diff. b/n i/p & o/p convert the CS into unity f/b system.

Que.→



(i) which i/p will yeild constant error?

(a.) Step i/p. (b) Ramp i/p (c) Parabolic i/p. (d) Impulse i/p

(ii) Find steady state error for the above i/p?

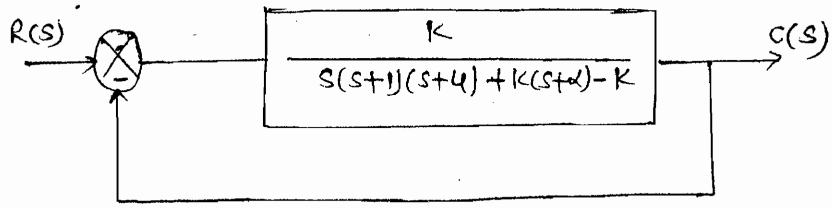
(a) $\frac{\alpha}{\alpha-1}$ (b) $\frac{\alpha-1}{\alpha}$ (c) α (d) $\alpha-1$

Soln→

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+1)(s+4)}}{1+\frac{K(s+\alpha)}{s(s+1)(s+4)}}$$

$$= \frac{K}{s(s+1)(s+4)+K(s+\alpha)}$$

$$G(s) = \frac{K}{s^0\{[(s+1)(s+4)s+K(s+\alpha)]-K\}}$$



$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{1}{\cancel{s}}}{1+\frac{K}{s(s+1)(s+4)+K(s+\alpha)-K}}$$

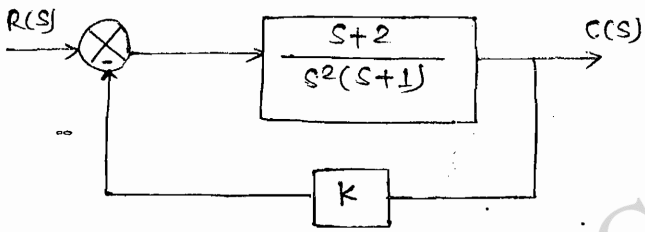
$$= \frac{1}{1+\frac{K}{K\alpha-K}} = \frac{K\alpha-K}{K\alpha-K+K}$$

$$e_{ss} = \frac{K(\alpha-1)}{K\alpha}$$

ans. (a) & (a)

$$\boxed{e_{ss} = \frac{\alpha-1}{\alpha}}$$

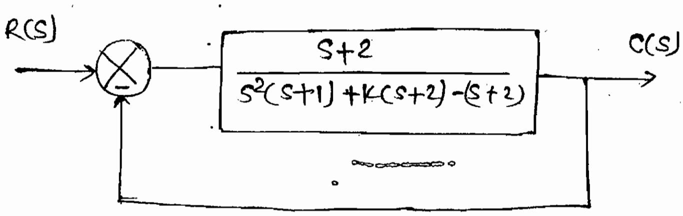
Que.→



Soln→

$$\frac{C(s)}{R(s)} = \frac{\frac{s+2}{s^2(s+1)}}{1+\frac{(s+2)K}{s^2(s+1)}} = \frac{s+2}{s^2(s+1)+(s+2)K}$$

$$G(s) = \frac{(s+2)}{s^0[s^2(s+2)+K(s+2)-(s+2)]}$$



$$e_{ss} = \lim_{s\to 0} \frac{s\cdot\frac{1}{s}}{1+\frac{(s+2)}{s^2(s+1)+K(s+2)-(s+2)}}$$

$$e_{ss} = \frac{1}{1+\frac{2}{2K-2}}$$

$$\boxed{e_{ss} = \frac{K-1}{K}}$$

* Error Series →

$$E(s) = \frac{R(s)}{1+G(s).H(s)}$$

Let; $F(s) = \frac{1}{1+G(s).H(s)}$

$$\mathcal{L}^{-1}E(s) = R(s).F(s)$$

$$\mathcal{L}^{-1}E(s) = \mathcal{L}^{-1}R(s).F(s)$$

$$e(t) = \int_0^\infty F(T).r(t-T)\,dT$$

Expanding $r(t-T)$ using Taylor series

$$r(t-T) = r(t) - T\dot{r}(t) + \frac{T^2}{2!}\ddot{r}(t) - \frac{T^3}{3!}\dddot{r}(t) + \ldots\ldots$$

$$e(t) = r(t)\int_0^\infty F(T)\,dT - \dot{r}(t)\int_0^\infty T.F(T).dT + \frac{\ddot{r}(t)}{2!}\int_0^\infty T^2 F(T)\,dT - \frac{\dddot{r}(t)}{3!}\int_0^\infty T^3 F(T)\,dT \ldots$$

Defining "Dynamic error const's"

$$K_0 = \int_0^\infty F(T).dT; \quad K_1 = -\int_0^\infty T F(T).dT, \quad K_2 = \int_0^\infty T^2 F(T).dT, \quad K_3 = -\int_0^\infty T^3 F(T)\,dT.$$

$$\boxed{e(t) = K_0 r(t) + K_1\dot{r}(t) + \frac{K_2}{2!}\ddot{r}(t) + \frac{K_3}{3!}\dddot{r}(t) + \ldots}$$

$$\boxed{e_{ss} = \lim_{t\to\infty} e(t).}$$

To find dynamic error constants →

$$\mathcal{L}\,F(T) = F(s) = \int_0^\infty F(T).e^{-sT}dT$$

$$\lim_{s\to 0} F(s) = \lim_{s\to 0}\int_0^\infty F(T).e^{-sT}dT = \int_0^\infty F(T).dT \Rightarrow K_0$$

$$\frac{d}{ds}F(s) = \frac{d}{ds}\int_0^\infty F(T).e^{-sT}dT = -\int_0^\infty T F(T)e^{-sT}dT$$

$$\lim_{s\to 0}\frac{d}{ds}F(s) = \lim_{s\to 0} -\int_0^\infty T F(T).e^{-sT}dT \Rightarrow -\int_0^\infty T.F(T).dT \Rightarrow K_1$$

$$K_0 = \lim_{s\to 0} F(s)$$

$$K_1 = \lim_{s\to 0} \frac{d}{ds} F(s)$$

$$K_2 = \lim_{s\to 0} \frac{d^2F(s)}{ds^2}$$

$$\vdots \qquad \vdots$$

where; $F(s) = \dfrac{1}{1+G(s)\cdot H(s)}$

* Relationship b/n static & dynamic error constants →

$$G(s)\cdot H(s) = \frac{100}{s(s+2)}$$

(I.) Static error constants :-

$$K_p = \lim_{s\to 0} \frac{100}{s(s+2)} = \infty$$

$$K_v = \lim_{s\to 0} \frac{\cancel{s}\cdot 100}{\cancel{s}(s+2)} = 50$$

$$K_A = \lim_{s\to 0} \frac{s^2\cdot 100}{s(s+2)} = 0$$

(II.) Dynamic error constants.

$$F(s) = \frac{1}{1+G(s)\cdot H(s)} = \frac{1}{1+\frac{100}{s(s+2)}}$$

$$K_0 = \lim_{s\to 0} F(s)$$

$$K_0 = \lim_{s\to 0} \frac{1}{1+\frac{100}{s(s+2)}}$$

$$K_0 = \frac{1}{1+\lim_{s\to 0} \frac{100}{s(s+2)}} = \frac{1}{1+\infty} = 0$$

$$\boxed{K_0 = \frac{1}{1+K_p}}$$

$$K_1 = \lim_{s\to 0} \frac{d}{ds} F(s)$$

$$\frac{d}{ds} F(s) = \frac{d}{ds} \frac{1}{1+\frac{100}{s(s+2)}} = \frac{d}{ds} \frac{s(s+2)}{s^2+2s+100}$$

$$= \frac{(s^2+2s+100)(2s+2) - s(s+2)(2s+2)}{(s^2+2s+100)^2}$$

$$K_1 = \lim_{s\to 0} \frac{d}{ds} F(s) = \frac{(0+0+100)(0+2)-0}{(0+0+100)^2} = \frac{100\times 2}{(100)^2} = \frac{1}{50}$$

$$\boxed{K_1 = \frac{1}{K_V}} \quad \boxed{K_2 = \frac{1}{K_A}}$$

Note:- Static & dynamic error constant are inversaly related to each other however they need not be direct reciprocal value because the dynamic error constant are defined for error series.

Que.→ $G(s).H(s) = \dfrac{100}{s(s+2)}$

Find ess for $r(t) = 5+2t$

Soln→ (i) Error ratio

$$R(s) = \frac{5}{s} + \frac{2}{s^2} = \frac{5s+2}{s^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\cdot\left(\frac{5s+2}{s^2}\right)}{1+\frac{100}{s(s+2)}}$$

$$e_{ss} = \frac{2\times 2}{100} = \frac{2}{50} \text{ units}$$

(ii) Error Series

$$e_{ss} = \lim_{t\to 0}\left[k_0 r(t) + k_1 \dot{r}(t) + \frac{k_2}{2!}\ddot{r}(t) + \cdots\right]$$

$$r(t) = 5+2t \Rightarrow k_0 = 0$$

$$\dot{r}(t) = 0+2 = 2 \Rightarrow k_1 = \frac{1}{50}$$

$$\ddot{r}(t) = 0$$

$$e_{ss} = \lim_{t\to\infty}\left[0\times(5+2t) + \frac{1}{50}\times 2\right] = \frac{2}{50}$$

(iii) Short cut methods

Type-(1)

$$R(s) = \frac{5}{s} + \frac{2}{s^2}$$

$$e_{ss} = 0 + \frac{A}{k}$$

$A = 2$, $k = k_V = 50$

$$e_{ss} = \frac{2}{50}$$

DATE-17/11/14.

*** Transient state analysis →** * It deals with the nature of responce of sys. when subjected to an i/p & depands on order of CS.

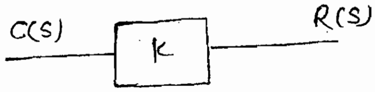
$$\frac{C(s)}{R(s)} = \frac{b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

(1.) Zero order system →

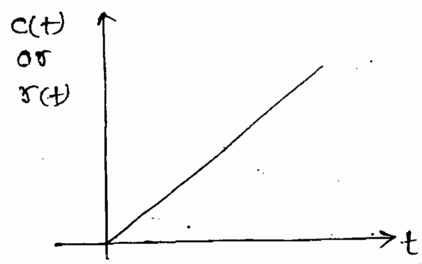
$$\frac{C(s)}{R(s)} = \frac{b_0}{a_0}$$

let $k = \text{gain} = \frac{b_0}{a_0}$

$$\frac{C(s)}{R(s)} = k$$



Ex:- Sensors / Xcers



(2.) 1st order sys →

$$\frac{C(s)}{R(s)} = \frac{b_0}{a_1 s + a_0}$$

$$= \frac{b_0/a_0}{\frac{a_1}{a_0} s + 1}$$

let $k = \cancel{\text{gain}} = \frac{b_0}{a_0}$

T = time constant element $= \frac{a_1}{a_0}$

$$\frac{C(s)}{R(s)} = \frac{k}{1+Ts}$$

Ex:- RC n/w

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{RCs+1} \qquad T = RC$$

Transient analysis →

let $R(s) = 1/s$

$$C(s) = \frac{K}{s(1+Ts)}$$

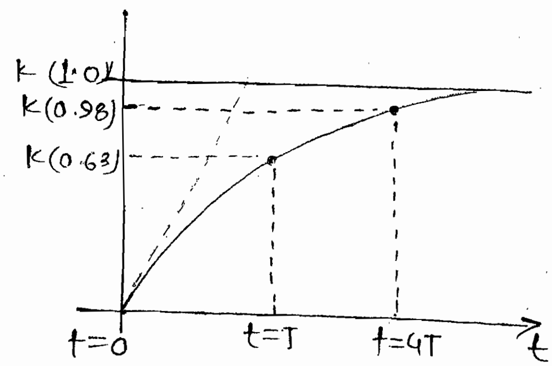
$$= \frac{K}{s(1+Ts)}$$

$$= K\left[\frac{1}{s} - \frac{T}{1+Ts}\right]$$

$$= K\left[\frac{1}{s} - \frac{1}{s+\frac{1}{T}}\right]$$

$$c(t) = K(1-e^{-t/T})$$

$$\lim_{t\to\infty} c(t) = K$$



at $t=0$, $c(t) = K(1-e^{0}) = 0$

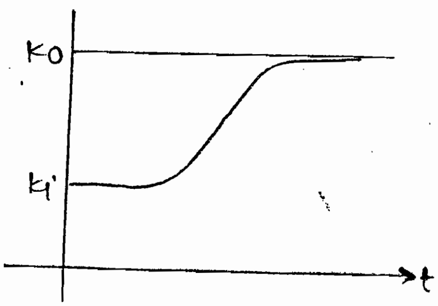
$t=T$, $c(t) = K(1-e^{-1}) = (0.63)K$

⋮

$t=4T$, $c(t) = K(1-e^{-4}) = (0.98)K$

The time const is defined as time taken by the responce of the sys. to reach 63% of the final value.

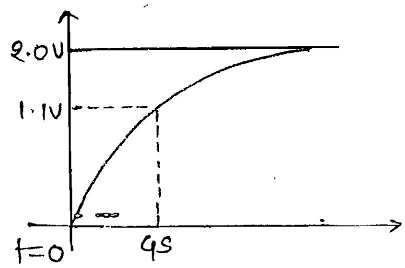
Liquid level sys., pneumatic sys., thermometers, RC (or) RL n/w are eg. of 1st order system.



$$c(t) = K_o + (K_i - K_o)e^{-t/T}$$

Que.→ Certain 1st order sys. is initialy at rest & subjected to sudden i/p at t=0, its responce reaches 1.1V is 4s & evantualy reaches a steady state of 2V. find the time const.

Soln→



$$c(t) = k(1 - e^{-t/T})$$

$$\lim_{t\to\infty} c(t) = k = 2$$

at $t = 4s$

$$1.1 = 2(1 - e^{-4/T})$$

$$2e^{-4/T} = 0.9$$

$$\boxed{T = 5s}$$

(17/63)

$$H(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$

$$r(t) = 10\,u(t)$$

$$R(s) = \frac{10}{s}$$

$$C(s) = \frac{10}{s(s+2)} = \frac{5}{s} - \frac{5}{s+2}$$

$$c(t) = 5(1 - e^{-2t})$$

$$\lim_{t\to\infty} c(t) = 5$$

$$\frac{99}{100} \times 5 = 4.95$$

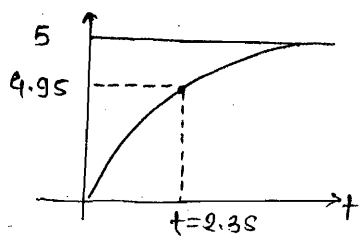
$$4.95 = 5(1 - e^{-2t})$$

$$5e^{-2t} = 0.05$$

$$e^{-2t} = 0.01$$

$$-2t = \ln(0.01)$$

$$t = 2.3s$$



Que.→ A thermometer having 1st order dynamics is subjected to sudden temp. change of 30°c – 150°c. If it has a time constant of 4s. what temp. it will indicate after 4s.

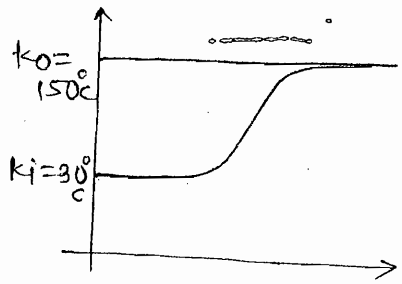
Soln →

$c(t) = k_0 + (k_i - k_0)e^{-t/T}$

at $t = 4s$.

$c(t) = 150 + (30 - 150)e^{-4/4}$

$c(t) = 105.6°c$



Que.→ The TF of the 1st order sys. is

$$\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$$

Its type & ess | unit ramp i/p are

(a) 0, ∞ (b) 0, T (c) 1, ∞ (d) 1, T

Soln →

$$G(s) = \frac{1}{1+Ts-1} = \frac{1}{Ts}$$

$\boxed{\text{Type-1 system}}$

$$ess = \frac{1}{K} \quad \text{where } K = K_v = \lim_{s\to0} \cancel{s} \cdot \frac{1}{T\cancel{s}} = \frac{1}{T}$$

$$\boxed{ess = T}$$

2nd order system →

The responce of 2nd order (or) higher order sys. exibits contineous (or) & sustained oscillation about the steady state value of i/p with a freq. known as undamped natural freq. ω_n.

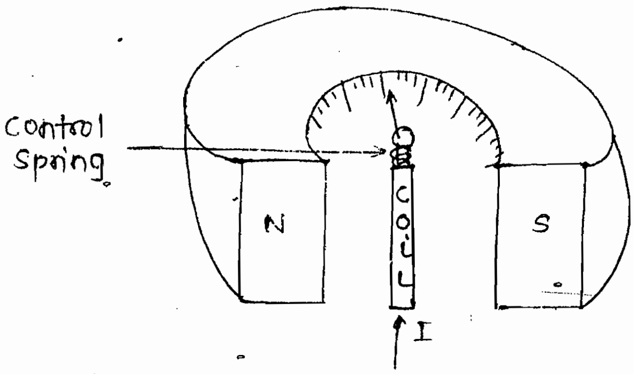
This oscillation in the responce are damped to the steady state value of i/p using appropriate damping methods, the damping is mathematically expressed as damping ratio ξ (ζ).

eg:- PMMC.

Undamped Natural ~~freq. ω_m~~ ω_n r/s

damping Ratio "ξ (zeta)"

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}}$$



I/p = deflecting torque (T_d)

o/p = angular deflection of pointer (θ)

J = Inertia of moving system , B = Inherent friction

k = spring constant.

$$T_d = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + k\theta \quad ; \quad T_d(s) = (Js^2 + Bs + k)\,\theta(s)$$

$$\frac{\theta(s)}{T_d(s)} = \frac{1}{Js^2 + Bs + k} = \frac{1/J}{s^2 + \frac{B}{J}s + \frac{k}{J}}$$

$$s^2 + \frac{B}{J}s + \frac{k}{J} = s^2 + 2\xi\omega_n s + \omega_n^2$$

$$\boxed{\omega_n = \sqrt{\frac{k}{J}}\ \text{r/s}} \quad ; \quad 2\xi\sqrt{\frac{k}{J}} = \frac{B}{J} \quad ; \quad \boxed{\xi = \frac{B}{2\sqrt{kJ}}} \quad \boxed{\xi \propto B}$$

k "TYPE" of std. 2nd order system →

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \cancel{\omega_n^2} - \cancel{\omega_n^2}}$$

$$G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)} \qquad \text{Type-1 sys}$$

Effect of damping on the nature of responce →

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$s^2 + 2\xi\omega_n s + 1 = 0$$

$$= \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

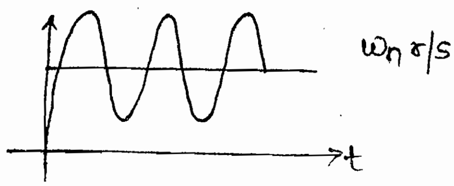
$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$D = \xi^2 - 1 = 0 \Rightarrow \xi = 1$

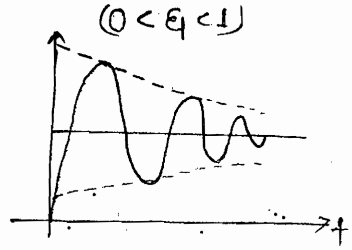
$D = \xi^2 - 1 < 0 \Rightarrow \xi < 1$

$D = \xi^2 - 1 > 0 \Rightarrow \xi > 1$

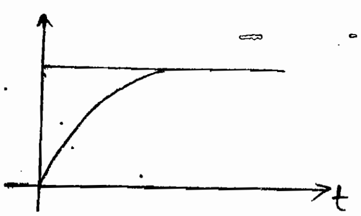
Case(1) → Un-damped case ($\xi = 0$)



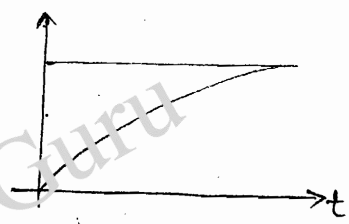
Case(2) → Under-damped case ($0 < \xi < 1$)



Case(3) → Critically damped case ($\xi = 1$)



Case(4) → Overdamped case ($\xi > 1$)



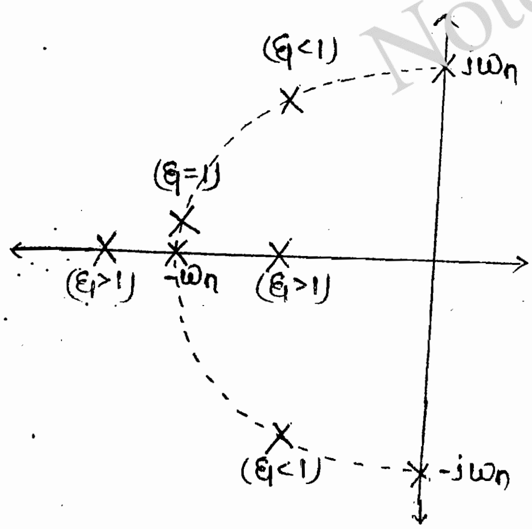


Fig :- Root locus.

* Most of the cs are designed $\xi<1$ because the responce can be analysised using more no. of performance specification (optimum values of ξ in cs design are b/n 0.3–0.8).

* The root-locus of 2nd order sys. obtained by varying the damping ratio ξ is a semicircular path with a radius of ω_n & breakaway point at $-\omega_n$ on the -ve real axis.

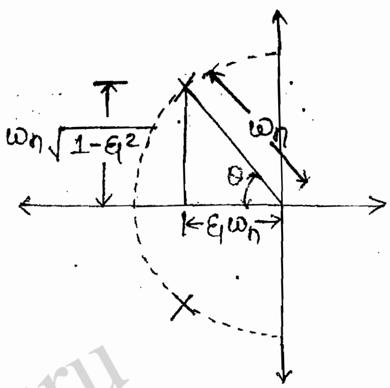
c/s of underdamped sys. →

$$s^2+2\xi\omega_n s+\omega_n^2=0$$

$$s=-\xi\omega_n \pm \sqrt{(\xi^2-1)}\ \omega_n$$

For $\xi<1$

1.) $\cos\theta = \frac{\xi\cancel{\omega_n}}{\cancel{\omega_n}} = \xi$



$$\boxed{\begin{aligned}\theta &= \cos^{-1}\xi\\ \sin\theta &= \sqrt{1-\xi^2}\\ \tan\theta &= \frac{\sqrt{1-\xi^2}}{\xi}\end{aligned}}$$

2.) Damping cofficient (or) actual damping (or) damping factor

$$\boxed{\alpha=\xi\omega_n}$$

3.) Timeconstant of underdamped responce

$$T=\frac{1}{\alpha}=\frac{1}{\xi\omega_n}$$

4.) Damped natural freq. $\omega_d=\omega_n\sqrt{1-\xi^2}$ r/s

) For $\xi<1$

$$s^2+2\xi\omega_n s+\omega_n^2=(s+\xi\omega_n)^2+\omega_d^2$$

Damping ratio $= \frac{\text{Actual damping}}{\text{critical damping}} = \frac{\xi\cancel{\omega_n}}{\cancel{\omega_n}} = \xi$

actual damping $=\xi\omega_n$, at ⇒ $\xi=1$

Actual damping becomes critical damping

critical damping $=1$

* Transient analysis → (Underdamped Responce)

let $R(s) = \frac{1}{s}$, $C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$

$$C(s) = \frac{1}{s} - \frac{(s+2\xi\omega_n)}{s^2+2\xi\omega_n s+\omega_n^2}$$

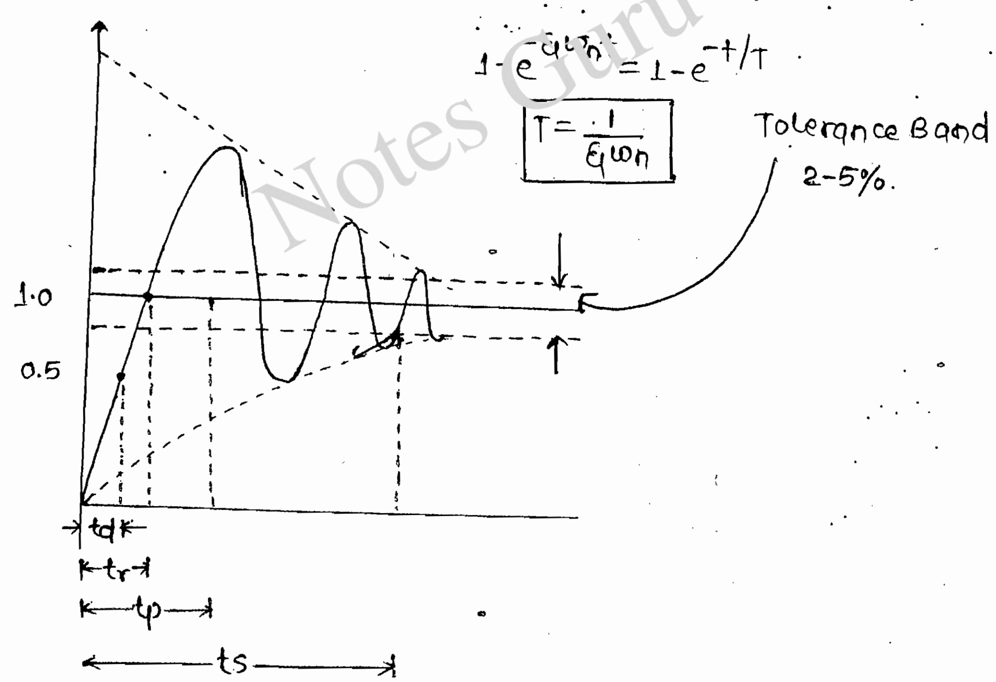
$$= \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2} \times \frac{\xi\cancel{\omega_n}}{\cancel{\omega_n}\sqrt{1-\xi^2}}$$

$$c(t) = 1 - e^{-\xi\omega_n t}\cdot\cos\omega_d t - e^{-\xi\omega_n t}\sin\omega_d t\cdot\frac{\xi}{\sqrt{1-\xi^2}}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sqrt{1-\xi^2}\cos\omega_d t + \xi\cdot\sin\omega_d t\right]$$

$\because A\sin\omega t + B\cos\omega t = \sqrt{A^2+B^2}\,\sin\left[\omega t + \tan^{-1}\left(\frac{B}{A}\right)\right]$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left[\omega_d t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]}$$



(1.) Delay time (t_d) →

$$t_d = \frac{1+0.7\xi}{\omega_n} \text{ secs.}$$

(2.) Rise time (t_r) →

At $t = t_r$; $c(t) = 1$

$$\therefore c(t)\Big|_{t=t_r} = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 1$$

$$\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0$$

since $\sin(\omega_d t_r + \theta) = 0$

$$\omega_d t_r + \theta = \pi$$

$$\boxed{t_r = \frac{\pi - \theta}{\omega_d}}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) \text{ rad.}$$

(3.) Settling time (t_s) →

For 2% of Tolerance band ; $t_s = 4T \rightarrow \frac{4}{\xi\omega_n}$ secs.

For 5% of tolerance band ; $t_s = 3T \rightarrow \frac{3}{\xi\omega_n}$ secs.

(4.) No. of cycles →

$$\omega_d = 2\pi f_d \;;\; f_d = \frac{\omega_d}{2\pi} \left(\frac{\text{cycles}}{\text{sec}}\right)$$

2% of (TB) → $t_s \times f_d \Rightarrow \frac{4 f_d}{\xi\omega_n}$ (cycles)

5% of TB. → $t_s \times f_d \Rightarrow \frac{3 f_d}{\xi\omega_n}$ (~~secs~~)(cycles)

(5.) Time period / Time interval of damped sinusoid →

$$\boxed{T = \frac{1}{f_d}} \text{ secs.}$$

(6.) Peak time (t_p) →

At $t = t_p$; $c(t) = \max^m$ value

$$\frac{d}{dt} c(t) = 0$$

$$\Rightarrow \frac{d}{dt}\left[1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\cdot \sin(\omega_d t+\theta)\right]=0$$

$$\Rightarrow 0-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\cdot\cos(\omega_d t+\theta)\omega_d+\sin(\omega_d t+\theta)\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}(-\xi\omega_n)=0$$

$$\Rightarrow \frac{\cancel{e^{-\xi\omega_n t}}}{\cancel{\sqrt{1-\xi^2}}}\cdot\cos(\omega_d t+\theta)\cdot\omega_d=\sin(\omega_d t+\theta)\cdot\frac{\cancel{e^{-\omega_n\xi t}}}{\cancel{\sqrt{1-\xi^2}}}\cdot\xi\omega_n$$

$$\Rightarrow \frac{\sin(\omega_d t+\theta)}{\cos(\omega_d t+\theta)}=\frac{\omega_d}{\xi\omega_n}=\frac{\cancel{\omega_n}\sqrt{1-\xi^2}}{\xi\cancel{\omega_n}}$$

$$\tan(\omega_d t+\theta)=\tan\theta$$

$$\tan(n\pi+\theta)=\tan\theta$$

$$\omega_d t=n\pi$$

$$\therefore\ t=t_p$$

$$\omega_d t_p=n\pi$$

$$\boxed{t_p=\frac{n\pi}{\omega_d}}\ \text{sec.}$$

<u>Maximum peak overshoot (m_p) →</u>

$$c(t)\Big|_{t=t_p=\frac{\pi}{\omega_d}}$$

$$c(t)=1-\frac{e^{-\xi\omega_n\cdot\frac{\pi}{\omega_d}}}{\sqrt{1-\xi^2}}\cdot\sin\left(\cancel{\omega_d}\cdot\frac{\pi}{\cancel{\omega_d}}+\theta\right)$$

$$c(t)=1+e^{-\xi\cancel{\omega_n}\cdot\frac{\pi}{\cancel{\omega_n}\sqrt{1-\xi^2}}}$$

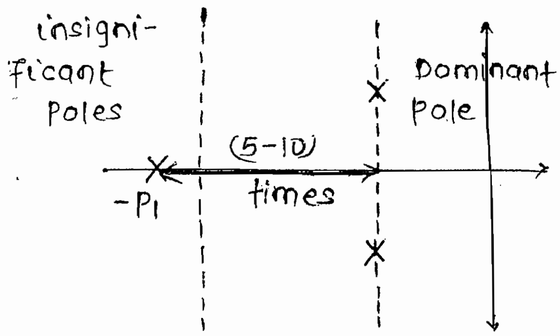
$$c(t)=1+e^{-\xi\pi/\sqrt{1-\xi^2}}$$

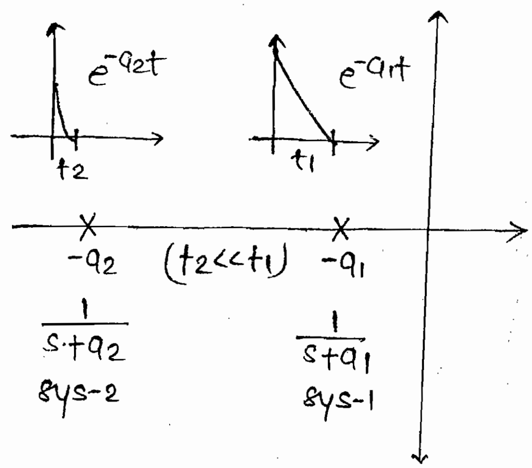
$$m_p=c(t)\Big|_{t=t_p}-1=e^{-\xi\pi/\sqrt{1-\xi^2}}\triangleq e^{-\xi n\pi/\sqrt{1-\xi^2}}$$

* Time Responce analysis for higher order sys. → Consider a 3^{rd} order c/s eqn.;

$$s^3+ps^2+qs+k=0$$

$$(s+p_1)(s^2+q_1s+k_1)=0$$





* The time responce analysis of higher order sys. is obtained by approximating to 2^{nd} order system. wrt dominant poles.

* The time domain specification obtained for approximated lower order sys. are valid for original higher order sys. also because poles lie in the insignificant region have insignificant affect on the time responce c/s.

Que → The 2^{nd} order approximation using dominant pole concept is

$$T(s) = \frac{10}{(s+5)(s^2+s+1)}$$

(a.) $\frac{10}{s^2+s+1}$ (b.) $\frac{2}{s^2+s+1}$ (c.) $\frac{10}{(s+5)(s+1)}$ (d.) $\frac{2}{(s+5)(s+1)}$

Soln → Note:- When approximating higher order TF to a lower order TF convert in time constant form before eliminating the insignificant pole.

$$T(s) = \frac{10}{(s+5)(s^2+s+1)}$$

$$= \frac{\cancel{10}\,2}{\cancel{5}(\frac{s}{5}+1)(s^2+s+1)}$$

$$T(s) = \frac{2}{s^2+s+1}$$

ATE-18/11/14

ue.→ The c/s eqⁿ of the sys. is

$$s(s^2+6s+13)+k=0$$

Find the value of k such that the c/s eqⁿ has a pair of complex roots with real part −1.

(a) 10 (b) 20 (c) 30 (d) 40

Solⁿ→

$$s^3+6s^2+13s+k=0 \quad ----(i)$$

$$(s+a)(s^2+bs+c)=0$$

Roots are $s=\frac{-b\pm\sqrt{b^2-4ac}}{2}$

Given $\frac{-b}{2}=-1$, $b=2$

$$(s+a)(s^2+2s+c)=0$$

$$s^3+as^2+2s^2+2as+sc+ac=0 \quad ---(ii)$$

From eqⁿ (i) & (ii)

$$s^3+s^2(2+a)+s(2a+c)+ac=0 \quad ---(iii)$$

$(2+a)=6$, $a=4$, $2a+c=13$, $c=5$ $\quad$ $ac=k$, $6\times5=k$, $k=30$

ue.→ The open loop TF of a unity f/b sys. is $G(s)=\frac{k(s+b)}{s^2(s+20)}$

For what value of b does all the 3 roots of the c/s eqⁿ converge at the same point on the real axis.

Solⁿ→

$$1+\frac{k(s+b)}{s^2(s+20)}=0$$

$$s^2(s+20)+k(s+b)=0$$

$$s^3+20s^2+ks+kb=0 \quad ----(i)$$

& $(s+a)(s+a)(s+a)=0$

$$s^3+s^2(3a)+s(3a^2)+a^3=0 \quad ----(ii)$$

$3a=20$ $\quad$ $a^3=3a^2b$

$3a^2=k$ $\quad$ $b=\frac{a}{3}$ $\quad$ $\because a=\frac{20}{3}$

$a^3=kb$

$$\boxed{b=\frac{20}{9}}$$

(5/64) $15\% < m_p < 30\%$, $t_s < 0.75s$.

* (a.) $\% m_p = 15\%$

$m_p = 0.15$

$e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.15$

$\xi = 0.55$

$\theta = \cos^{-1}\xi = 57^\circ$

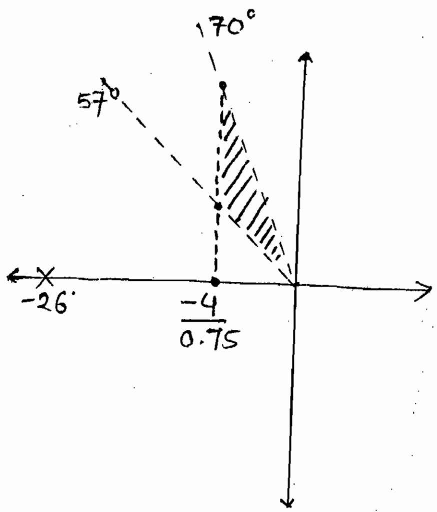
$\% m_p = 30\%$

$m_p = 0.3$

$e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.3$

$\xi = 0.35$

$\theta = \cos^{-1}(0.35) = 70^\circ$



$$t_s = \frac{4}{\xi\omega_n} = 0.75$$

$$\xi\omega_n = \frac{4}{0.75}$$

* (b.) 3rd roots ; $s = 5 \times \frac{-4}{0.75} = -26$

* (c.) $\% m_p = 30\%$; $\xi = 0.35$

$$t_s = 0.75s = \frac{4}{0.35 \times \omega_n} = 0.75$$

$$\omega_n = 15\ r/s.$$

$$(s+26)(s^2 + 2\times0.35\times15s + 225) = 0$$

$$s^3 + 36.5s^2 + 498s + 5850 = 0$$

$$1 + \frac{5850}{s^3 + 36.5s^2 + 498s} = 0$$

$$1 + G(s) = 0$$

$$\boxed{G(s) = \frac{5850}{s(s^2 + 36.5s + 498)}}$$

(3/63)

$$1 + \frac{K(s+2)}{s^3 + \alpha s^2 + 4s + 1} = 0$$

$$s^3 + \alpha s^2 + 4s + 1 + Ks + 2K = 0$$

$$s^3 + \alpha s^2 + s(4+K) + 2K + 1 = 0 \quad ----(i)$$

$\because (s+a)(s^2 + bs + c) = 0$

given $\xi = 0.2$, $\omega_n^2 = 9$

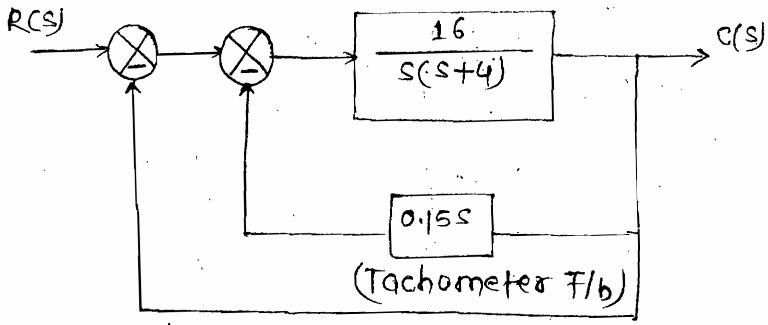
$c = \omega_n^2 = 9$, $b = 2\xi\omega_n = 2\times0.2\times3 = 1.2$

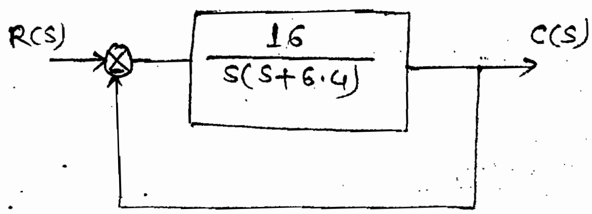
$$(s+a)(s^2 + 1.2s + 9) = 0$$

$$s^3 + 1.2s^2 + 9s + as^2 + 1.2as + 9a = 0$$

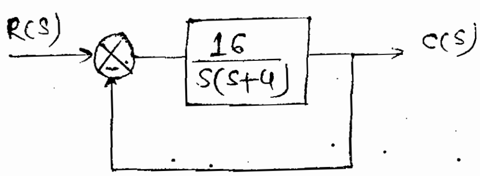
If the pre

* Effect of tachometer F/b on the performance c/s →





Case(1) → without tachometer F/b.



$$1+\frac{16}{S(S+4)}=0$$

$$S^2+4S+16=0$$

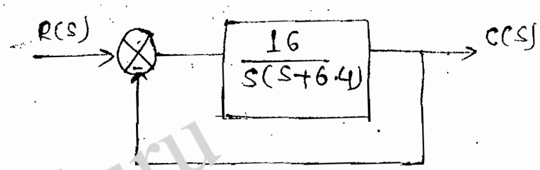
$$\omega_n=4,\quad 2\xi\times4=4$$

$$\omega_n=4;\quad \xi=0.5$$

$$T=\frac{1}{\xi\omega_n}=\frac{1}{0.5\times4}=0.5S$$

$$\boxed{\xi=0.5\ ;\ T=0.5S \quad \omega_n=4}$$

Case(2) → with tachometer F/b



$$1+\frac{16}{S(S+6.4)}=0$$

$$\omega_n=4,\quad \xi=0.8$$

$$T=\frac{1}{\xi\omega_n}=0.32S$$

$$\boxed{\xi=0.8,\ \omega_n=4 \quad T=0.32S}$$

ω_n = Remains fixed

$\xi = \xi\uparrow$

$T=0.32S;\ T\downarrow$

Responce is faster

19/63 (d)

13/62 $G(s) = \frac{10K}{s(1+0.1s)}$

$I/p = \frac{1}{2}$ rps

$e_{ss} < 0.2^\circ$

(a.) $I/p = \frac{1}{2}$ rps $\Rightarrow \pi$ r/s

$e_{ss} = \frac{0.2\pi}{180}$ radians

$$\frac{0.2\pi}{180} = \lim_{s\to 0} \frac{s \cdot \frac{\pi}{s^2}}{1+\frac{10K}{s(1+0.1s)}}$$

$$\frac{0.2\pi}{180} = \frac{\pi}{10K}$$

$\boxed{K=90}$

(b.) $1 + \frac{1000}{s(1+0.1s)} = 0$

$0.1s^2 + s + 900 = 0$

$s^2 + 10s + 9000 = 0$

$\omega_n = \sqrt{9000} = 95$ r/s.

$2\xi \times 95 = 10$

$\boxed{\xi = 0.05}$

12/62 $\frac{4d^2c(t)}{dt^2} + \frac{8dc(t)}{dt} + 16c(t) = 16u(t)$

$(4s^2 + 8s + 16)\,C(s) = 16\,U(s)$

$$\frac{C(s)}{U(s)} = \frac{16}{4s^2+8s+16}$$

$$= \frac{4}{s^2+2s+4}$$

$\omega_n = 2$ rad/s

$2\xi \times 2 = 2$

$\boxed{\xi = 0.5}$

11/62 $\frac{d^2y}{dt^2} + \frac{3dy}{dt} + 2y = x(t)$

at $t=0$

$y(t) = 0$

ans. (a)

10/62 $\omega_n = 4$ rad/s

$2\xi \times 4 = 4$

$\xi = 0.5$

$$t_p = \frac{3\pi}{4\sqrt{1-(0.5)^2}}$$

$\boxed{t_p = \frac{1.5\pi}{\sqrt{3}}}$

9/62 $\frac{d}{dt}(1 - e^{-5t} - 5te^{-5t})$

Imp. res. $= 25te^{-5t}$

$$TF = \frac{25}{(s+5)^2} = \frac{25}{s^2+10s+25}$$

$\boxed{\omega_n = 5\text{ r/s};\ \xi = 1}$ ans (d)

8/62 $\quad \dfrac{H(s+c)}{(s+a)(s+b)}$ $\quad$ (1) $u(t) = 2 + De^{-t} + Ee^{-3t}$

(2) $e^{-2t}u(t) = Fe^{-t} + Ge^{-3t}$

(i.) $$\frac{H(s+c)}{s(s+a)(s+b)} = \frac{k_1}{s} + \frac{k_2}{(s+a)} + \frac{k_3}{(s+b)}$$

$$= 2 + De^{-t} + Ee^{-3t}$$

$$a=1,\ b=3$$

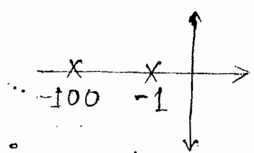
$$\lim_{s\to 0} \frac{s\cdot H(s+c)}{s(s+a)(s+b)} = \lim_{t\to\infty}\left[2+De^{-t}+Ee^{-3t}\right]$$

$$\frac{HC}{ab}=2\ ;\quad \frac{HC}{3}=2,\quad HC=6$$

(ii) $$\frac{H(s+c)}{(s+2)(s+a)(s+b)}$$

$$\boxed{\begin{matrix} C=2 \\ H=3 \end{matrix}}$$

7/61 $\quad TF = G(s) = \dfrac{100}{(s+1)(s+100)}$



Overdamped sys.

$$\frac{1}{(1+s)(1+\cancel{\frac{s}{100}})} = \frac{1}{(1+Ts)} \qquad T = 1\,\text{sec.}$$

$$2\%\ TB = 4T = 4\,\text{sec.}$$

2^{nd} pole is ~~domin~~ insignificant effect.

5) (A) $\quad$ 6/65 (C)

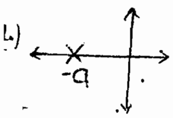
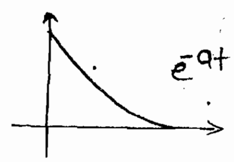
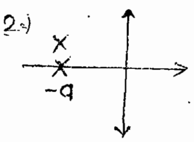
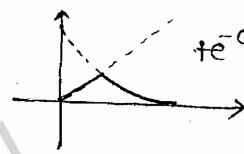
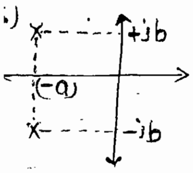
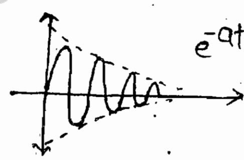
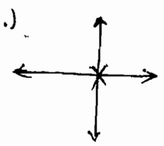
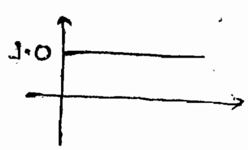
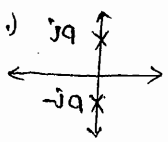
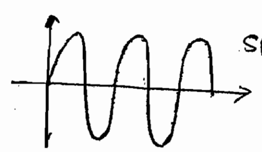
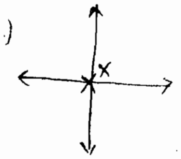
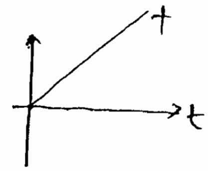
Chapter-03
Stability

* The stability of LTI system may be defined as when the sys. is subjected to bounded i/p the o/p should be bounded.
* BIBO implies the IR of the sys. should tend to zero as time t approaches ∞.
* The stability of a sys. depends on roots of the c/s eqn

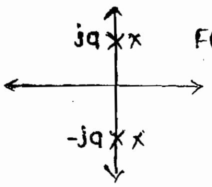
$$1+G(S)H(S)=0$$

i.e. closed loop poles.

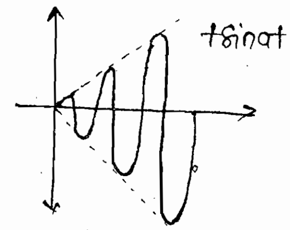
* IR & stability →

Closed loop pole locn		Stability criteria	Impulse Responce
1.) 	$F(S)=\frac{1}{S+a}$	absolutely stable	 e^{-at}
2.) 	$F(S)=\frac{1}{(S+a)^2}$	absolutety stable	 te^{-at}
3.) 	$F(S)=\frac{1}{(S+a)^2+b^2}$	absolutely stable	 $e^{-at}\sin bt$
4.) 	$F(S)=\frac{1}{S}$	marginaly stable/ criticaly stable	
5.) 	$F(S)=\frac{1}{S^2+a^2}$	marginaly/criticaly	 $\sin at$
6.) 	$F(S)=\frac{1}{S^2}$	Unstable	

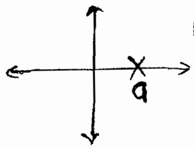
(7.)



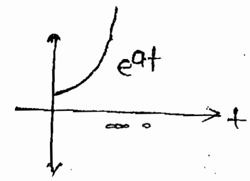
$F(s) = \dfrac{1}{(s^2+a^2)^2}$ Unstable



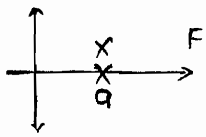
(8.)



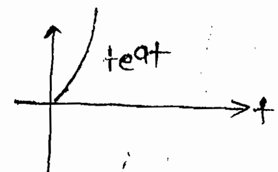
$F(s) = \dfrac{1}{(s-a)}$ Unstabe



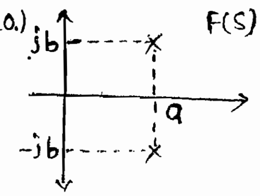
9.)



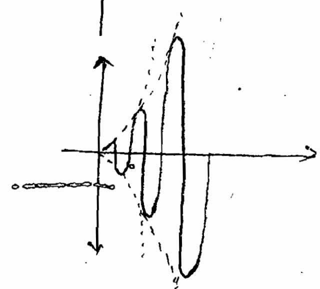
$F(s) = \dfrac{1}{(s-a)^2}$ Unstable



0.)



$F(s) = \dfrac{1}{(s-a)^2+b^2}$ Unstable



* Routh-Hurwitz Criteria →

2/5) $P(s) = s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$

ROUTH ARRAY =

s^4	1	18	5
s^3	8	16	0
s^2	$b_1 = 16$	$b_2 = 5$	0
s	$c_1 = 13.5$	0	0
s^0	$d_1 = 5$	0	0

$b_1 = \dfrac{8\times18 - 1\times16}{8} = 16$

$b_2 = \dfrac{8\times5 - 1\times0}{8} = 5$

$c_1 = \dfrac{16\times16 - 8\times5}{16} = 13.5$

$d_1 = \dfrac{13.5\times5 - 16\times0}{13.5} = 5$

3/5) $P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15$

s^5	1	2	3
s^4	1	2	15
s^3	$\cancel{0}\ \epsilon$	-12	0
s^2	$\frac{2\epsilon+12}{\epsilon}$	15ϵ	0
s^1	$\frac{-24\epsilon-144-15\epsilon^2}{2\epsilon+12}$		
s^0	15		

To check for sign changes

(1.) $\lim_{\epsilon\to 0} \frac{2\epsilon+12}{\epsilon} = \frac{2(0)+12}{0} = +\infty$

.) $\lim_{\epsilon\to 0} \frac{-24\epsilon-144-15\epsilon^2}{2\epsilon+12} = \frac{-144}{12} = -12$

sign changes: $+\infty \to -12$

$-12 \to +15$

2 poles are in RHS

Difficulty-1 → When the 1st element of any row is 0 while the rest of row has atleast one non-zero term then is such cases substitute $+\epsilon$ (small +ve no) in place of zero & evaluate the rest of RA (Routh array) in terms of ϵ.

: check for sign changes by taking $\lim_{\epsilon\to 0}$ for the 1st column elements to comment on stability.

$\frac{4}{65}$ $P(s) = s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$

s^6	1	8	20	16
s^5	2	12	16	20
s^4	2	12	16	0
s^3	$\cancel{0}$ 8	$\cancel{0}$ 24	0	0
s^2	6	16	0	0
s^1	2.6	0	0	0
s^0	16	0	0	0

(i) Construct an auxiliary eq^n $A(s)$

$A(s) = 2s^4+12s^2+16$

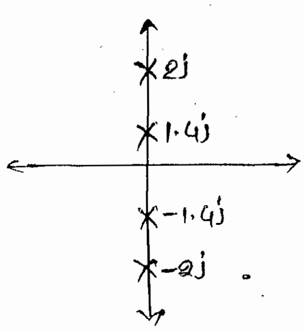
(ii) $\frac{d}{ds}A(s) = \cancel{8s^3+36s^2+16}\ 8s^3+24s$

$\boxed{\text{The roots of } A(s) = \text{poles symmetric about origin}}$

$\frac{-12\pm\sqrt{144-8\times 16}}{4} = -2, -4$

$(s^2+2)(s^2+4)=0$

$s = \pm j1.4, \pm j2$



5/65

s^5	2	4	2
s^4	1	2	1
s^3	$\cancel{0}$ 4	$\cancel{0}$ 4	0
s^2	1	1	0
s^1	$\cancel{0}$ 2	0	0
s^0	1	0	0

$A_1(s) = s^4 + 2s^2 + 1$

$\frac{d}{ds}A_1(s) = 4s^3 + 4s$

$A_2(s) = s^2 + 1$

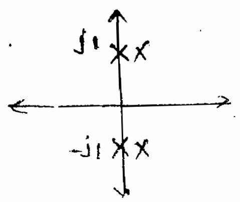
$\frac{d}{ds}A_2(s) = 2s$

The roots of $A_1(s)$

$$\frac{-2 \pm \sqrt{4-4}}{2} = -1, -1$$

$$(s^2+1)(s^2+1) = 0$$

$$s = \pm j, \pm j$$

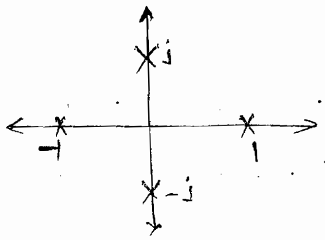


Unstable

Poles symmetric about origin

$$(s^2+1)(s^2-1) = 0$$

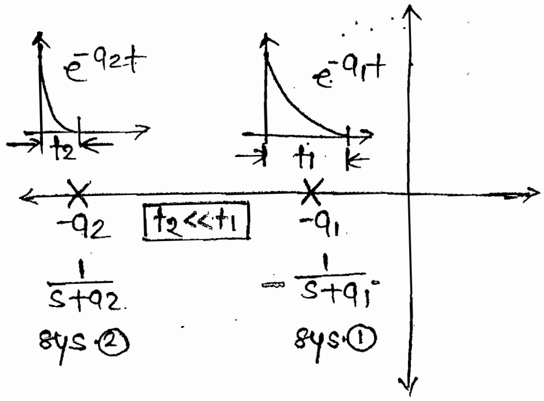
$$s = \pm j, \; s = \pm 1$$



Difficulty (02) → when one compete row of Routh array is 0, then in such cases construct an AE, A(s) differentiated to get new cofficient & evaluate rest of the RA.

* Check the roots of AE which are poles symmetric about origin to comment on stability.

Relative stability Analysis →

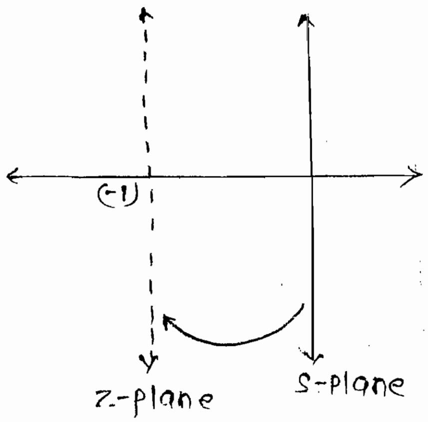


Both sys① & sys② are said to be absolutely stable

sys(2) is said to be relatively more stable than sys① bcoz $t_2 << t_1$

$P(s) = s^3 + 7s^2 + 25s + 39 = 0$

check weather the roots are lying more -vely wrt = 1?



$s+1 = z$

$s = z-1$

$P(z) = (z-1)^3 + 7(z-1)^2 + 25(z-1) + 39 = 0$

$P(z) = z^3 + 4z^2 + 14z + 20 = 0$

z^3	1	14
z^2	4	20
z	9	0
z^0	20	0

Shortcut → * Put $s=-1$ in given $P(s)$ & if +ve value is coming means all the roots are lying in LHS

If $P(s) = 0$; then only 4 values are lying on LHS.

Conditionaly stable → A sys. is said to be conditionaly stable if its stability depends on one (or) more parameters.

7/56) $P(s) = s^4 + 2s^3 + 3s^2 + 2s + k = 0$

s^4	1	3	k
s^3	2	2	0
s^2	2	k	0
s^1	$\frac{4-2k}{2}$	0	0
s^0	k	0	0

(i) $\frac{4-2k}{2} > 0$; $k < 2$

(ii) $k > 0$

$\boxed{0 < k < 2}$

At $k = k_{mar} = 2$

s^1 row $= 0$

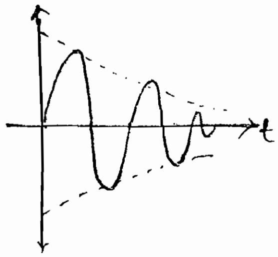
$A(s) = 2s^2 + k = 0$

$2s^2 + 2 = 0$

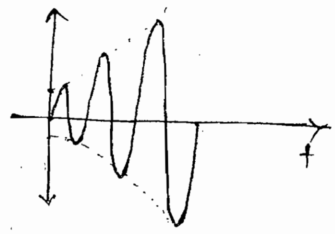
$s = \pm j \rightarrow (j\omega)$

$\boxed{\omega = \omega_{mar} = 1\ r/s}$

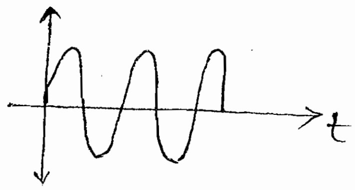
case(1) → $s = -\alpha \pm j\omega$



Case(2) → $s = \alpha \pm j\omega$



case(3) → $s = \pm j\omega$



9/06

$$1 + \frac{K(s-2)^2}{(s+2)^2} = 0$$

$$(s+2)^2 + K(s-2)^2 = 0$$

$$s^2(1+K) + s(4-4K) + (4+4K) = 0$$

s^2	$1+K$	$4+4K$
s^1	$4-4K$	0
s^0	$4+4K$	0

(i) $1+K>0$
$K>-1$

(2) $4-4K>0$
$K<1$

$\boxed{-1<K<1}$ $\boxed{0 \le K < 1}$

1/6

$$1 + \frac{10(K_p s + K_I)}{s(s^2+s+20)} = 0$$

$$s^3 + s^2 + s(20+10K_p) + 10K_I = 0$$

s^3	1	$20+10K_p$
s^2	1	$10K_I$
s	$20+10K_p-10K_I$	0
s^0	$10K_I$	0

(i) $10K_I>0$, $\boxed{K_I>0}$

(ii) $20+10K_p-10K_I>0$

$\boxed{K_p>K_I-2}$

2/66

$$1 + \frac{K(s+2)^2}{s(s^2+1)(s+4)} = 0$$

$$s^4 + 4s^3 + s^2(1+K) + s(4+4K) + 4K = 0$$

$$\begin{array}{c|ccc} s^4 & 1 & 1+K & 4K \\ s^3 & 4 & (4+4K) & 0 \\ s^2 & \cancel{\phi}\ \epsilon & 4K & 0 \\ s^1 & \frac{(4+4K)\epsilon-16K}{\epsilon} & 0 & 0 \\ s^0 & 4K & 0 & 0 \end{array}$$

$\boxed{\text{the sys. is unstable for all } K>0}$

DATE-19/11/14

8/66

$$1+\frac{K}{(s^2+2s+2)(s+2)}=0$$

<u>Soln</u> →

$$(s^2+2s+2)(s+2)+K=0$$

$$s^3+4s^2+6s+(4+K)=0$$

$$\begin{array}{c|ccc} s^3 & 1 & 6 & 0 \\ s^2 & 4 & (4+K) & 0 \\ s^1 & \frac{24-(4+K)}{4} & 0 & 0 \\ s^0 & 4+K & 0 & 0 \end{array}$$

(i) $\frac{24-(4+K)}{4}>0$

$K<20$

(ii) $4+K>0$

$K>-4$

$\boxed{-4<K<20}$

at $K=K_{mar}=20$

$$A(s)=4s^2+(4+K)=0$$

$$4s^2+(4+20)=0$$

$$s^2=-6$$

$$s=\pm j\sqrt{6}=j\omega$$

$$\boxed{\omega=\sqrt{6}\ r/s}$$

<u>Shortcut</u> → (3rd order system) (only when all coefficients of eqn are +ve)

$$s^3+\overbrace{4s^2+6s+(4+K)}^{A(s)}=0$$

(i) Product of external coefficients < product of internal = stable

(ii) Product of external coefficients > product of internal = unstable

(iii) " = " = marginaly stable

$$4+k=24$$

$$k_{max}=20$$

$$\cancel{4+20=24}$$

$$A(s) = 4s^2+(4+k)=0$$

$$4s^2+(4+20)=0$$

$$s^2=-6$$

$$s=\pm j\sqrt{6} \approx j\omega$$

$$\boxed{\omega=\sqrt{6}\ r/s}$$

10
66

$$1+\frac{K(s+1)}{s^3+a s^2+2s+1}=0$$

Sol^n →

$$s^3+as^2+s(2+k)+(k+1)=0$$

$$k+1=a(k+2) \text{ (Given)}$$

$$a=\frac{k+1}{k+2}$$

$$A(s)=as^2+(k+1)=0$$

$$s^2=\frac{-(k+1)}{a}$$

$$s^2=\frac{-(\cancel{k+1})\times(k+2)}{(\cancel{k+1})}$$

$$s=\pm j\sqrt{k+2} \approx j\omega$$

$$\omega=\sqrt{k+2}$$

$$2=\sqrt{k+2}$$

$$k=2$$

$$a=\frac{2+1}{2+2}=\frac{3}{4}$$

$$\boxed{k=2,\ a=\frac{3}{4}}$$

$\boxed{\text{conv(3)}\ 66}$ $\underline{\text{Soln} \rightarrow \text{(a.)}}$

$$1 + \frac{K(s+\alpha)}{s(s+2)(s+4)^2} = 0$$

$$s^4 + 10s^3 + 32s^2 + (K+32)s + K\alpha = 0$$

s^4	1	32	$K\alpha$
s^3	10	$32+K$	0
s^2	$\frac{288-K}{10}$	$K\alpha$	0
s^1	A	0	0
s^0	$K\alpha$	0	0

where $A = \dfrac{\dfrac{(288-K)(K+32)}{10} - 10K\alpha}{\left(\dfrac{288-K}{10}\right)}$

* (i) $\dfrac{288-K}{10} > 0$

$\boxed{K < 288}$

* (ii) $\dfrac{\dfrac{(288-K)(K+32)}{10} - 10K\alpha}{\left(\dfrac{288-K}{10}\right)} > 0$

$(288-K)(K+32) - 100K\alpha > 0$

$K\alpha < \dfrac{(288-K)(K+32)}{100}$

$$\boxed{0 < K\alpha < \frac{(288-K)(K+32)}{100}}$$

$\underline{\text{K (b.)}}$ $e_{ss} = \lim_{s \to 0} \dfrac{s \cdot \frac{1}{s^2}}{1 + \dfrac{K(s+\alpha)}{s(s+2)(s+4)^2}}$

$e_{ss} = \dfrac{32}{K\alpha}$ let $e_{ss} = 0.16$ (16%)

$0.16 = \dfrac{32}{200 \times \alpha}$ $\boxed{\because \alpha = 1}$ $\boxed{K = 200}$

$K\alpha = 200 \times 1 = 200$

$200 < \dfrac{(288-200)(200+32)}{100}$

$\boxed{200 < 204}$

THE ROOT LOCUS Technique

* The Root locus is defined as the Locus of closed loop poles obtained when sys. gain k is varied from 0 to ∞.
* The RL determines relative stability of the sys.

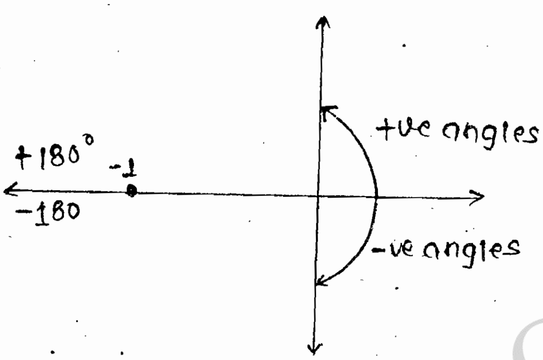
Angle & magnitude condⁿ →

- The angle condⁿ is used for checking weather certain points lie on RL (or) not & also the validity of RL for closed loop poles.

$$1 + G(S)\cdot H(S) = 0$$

$$G(S)\cdot H(S) = -1 + j0$$

$$\angle G(S)\cdot H(S) = +180^\circ - \tan^{-1}\left(\frac{0}{1}\right)\left(\frac{Img.}{Real}\right) = 180^\circ \text{ or } \pm 180^\circ \text{ or } \pm(2q+1)\,180^\circ$$



- The angle condⁿ may be stated as for a point to lie on RL the angle evaluated at that point must be odd multiple of ±180°.
- The magnitude condⁿ is used for finding the sys. gain k at any point on RL.

$$1 + G(S)\cdot H(S) = 0$$

$$G(S)\cdot H(S) = -1 + j0$$

$$|G(S)\cdot H(S)| = \sqrt{(-1)^2 + 0^2} = 1$$

$$\boxed{|G(S)\cdot H(S)| = 1}$$

(4/68) $s_1 = -3+4j$; $s_2 = -3-2j$

$$G(s).H(s) = \frac{k}{(s+1)^4}$$

$$G(s).H(s)\Big|_{(s=s_1=-3+4j)} = \frac{k+j0}{(-3+4j+1)^4} = \frac{0^\circ}{(-2+4j)^4} = \frac{0^\circ}{116.5\times 4} = -466^\circ$$

$$G(s).H(s)\Big|_{(s=s_2=-3-2j)} = \frac{k+j0}{(-3-2j+1)^4} = \frac{0^\circ}{-135\times 4} = +540^\circ$$

Because of the odd multiple is present in the s_2 then $180\times 3 = 540^\circ$ it is lying on RL.

(5/68)

$$G(s) = \frac{k}{s(s^2+7s+12)}$$

$s = -1+j1$

$$G(s)\Big|_{s=-1+j1} = \frac{k}{(-1+j)[(-1+j)^2+7(-1+j)+12]} = \frac{k+j0}{(-1+j)(5+5j)}$$

$$\angle G(s)\Big|_{s=-1+j} = \frac{0^\circ}{(135^\circ)(45^\circ)} = -180^\circ \text{ (lying on the R-L)}$$

$$|G(s)|_{s=-1+j} = 1 \Rightarrow \frac{\sqrt{k^2+0^2}}{\sqrt{(-1)^2+(1)^2}\sqrt{5^2+5^2}} \Rightarrow \frac{k}{\sqrt{2}\times\sqrt{50}} = 1 \quad \boxed{\therefore k = 10}$$

Que.→ The OLTF of U.FB sys. is $G(s) = \frac{k}{s(s+1)(s+3)}$

A zero is added to the sys. so that the locus passes through $-1+j$.

The locn of zero would be?

(a) -1.33 (b) -2.33 (c) -1.66 (d) -2.66.

Soln→ Locus passes through $-1+j$ means it is lying on R-L.

$$G(s) = \frac{k}{s(s+1)(s+3)}$$

$$G(s)\Big|_{s=-1+j} = \frac{k(-1+j+z)}{(-1+j)(-1+j+1)(-1+j+3)}$$

$$= \frac{(k+j0)(z-1+j)}{(-1+j)(0+j)(2+j)}$$

$$\angle G(j\omega) = \frac{(0^\circ)\tan^{-1}\left(\frac{1}{z-1}\right)}{(135^\circ)(90^\circ)(26.5^\circ)}$$

$$\tan^{-1}\left(\frac{1}{z-1}\right) - 251.5^\circ = 180^\circ$$

$$\tan^{-1}\left(\frac{1}{z-1}\right) = 180 + 251.5^\circ$$

$$\left(\frac{1}{z-1}\right) = \tan(431.5^\circ)$$

$$\left(\frac{1}{z-1}\right) = 3$$

$$z = 1.33$$

$$(s+z) = 0$$

$$(s+1.33) = 0$$

$$\boxed{s = -1.33}$$

* Construction Rules of R-L →

(1.) The R-L is symmetrical about real axis.

(2.) Let P = no. of open Loop poles ; &

z = no. of open Loop zeros

& $P > Z$; then the no. of branches of R-L = P

(3.) The no. of branches terminating at zeros = Z

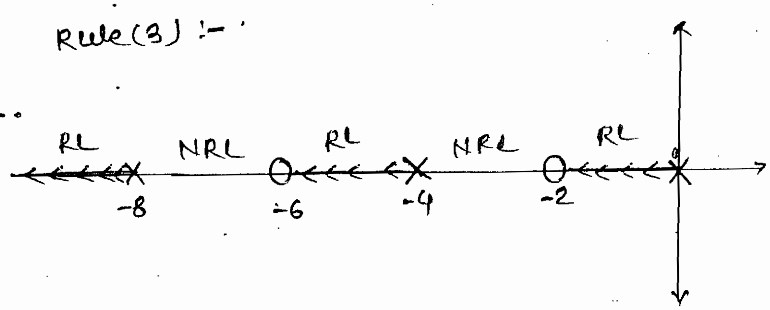
The no. of branches terminating at ∞ = P−Z

(3.) A point on Real axis is said to be on RL if to the right side of this point the sum of open loop poles & zeros is odd.

Eg:- $$G(s) = \frac{K(s+2)(s+6)}{s(s+4)(s+8)}$$

Rule (2) :- $P = 3, Z = 2, \quad P - Z = 1$

Rule (3) :-



$$\frac{G(s)}{1+G(s)} = \frac{\frac{K(s+2)(s+6)}{s(s+4)(s+8)}}{1+\frac{K(s+2)(s+6)}{s(s+4)(s+8)}}$$

$$= \frac{K(s+2)(s+6)}{s(s+4)(s+8)+K(s+2)(s+6)}$$

Closed loop poles $= s(s+4)(s+8)+K(s+2)(s+6)$

when $K=0$

closed loop poles $= 0, -4, -8$

(4.) Angle of Asymptodes → The p-z branches terminate at ∞ along certain straight line known as asymptodes of RL.

Therefore no. of asymptodes = p-z.

$$\theta = \frac{(2q+1)\,180^\circ}{(p-z)} \qquad q = 0, 1, 2, 3 \ldots\ldots$$

eg:- $p-z=2$

$$\theta_1 = \frac{[2(0)+1]\times 180^\circ}{2} = 90^\circ \quad ; \quad \theta_2 = \frac{[2(1)+1]\times 180^\circ}{2} = 270^\circ$$

6
68

$$s(s+4)(s^2+2s+5)+K(s+1)=0$$

$$1+\frac{K(s+1)}{s(s+4)(s^2+3s)}=0$$

$$1+G(s)\cdot H(s)=0$$

$$G(s)\cdot H(s) = \frac{K(s+1)}{s(s+4)(s^2+3s)} = \frac{K(s+1)}{s^2(s+4)(s+3)}$$

(2.) $p=4,\ z=1,\ p-z=3$

(3.) $\theta_1 = \frac{[2(0)+1]\times 180^\circ}{3} = 60^\circ \qquad \theta_2 = \frac{[2(2)+1]\times 180^\circ}{3} = 300^\circ$

$\theta_2 = \frac{[2(1)+1]\times 180^\circ}{3} = 180^\circ$

*** $\boxed{\text{Angle b/n asymptodes} = \frac{2\pi}{p-z}}$

5.) Centoriod → It is the intersection point of asymptodes on the real axis. It may (or) may not be a part of RL.

$$\text{Centoriod} = \frac{\sum \text{Real part of open Loop poles} - \sum \text{Real part of open Loop zeros}}{P-Z}$$

7/8

$$s^3 + 5s^2 + (K+6)s + K = 0$$

$$s^3 + 5s^2 + 6s + Ks + K = 0$$

$$s^3 + 5s^2 + 6s + K(s+1) = 0$$

$$1 + \frac{K(s+1)}{s^3+5s^2+6s} = 0$$

$$1 + G(s)H(s) = 0$$

$$G(s) \cdot H(s) = \frac{K(s+1)}{s(s^2+5s+6)} = \frac{K(s+1)}{s(s+3)(s+2)}$$

(2.) $P = 3$; $Z = 1$, $P - Z = 2$

(5.) Centoriod →

Zero at $s = -1+j0 = -1$

Poles at $s = 0+j0$, $-2+j0$, $-3+j0$

Sum of poles $= -5$

$$\text{Centoriod} = \frac{-5-(-1)}{2} = -2$$

Centoriod $= -2, 0$

K(6.) Break away points → They are those points where multiple roots of the c/s eqn occur.

Procedure → (1.) Construct $1 + G(s) \cdot H(s) = 0$

(2.) Write 'K' in terms of "s"

(3.) Find $\frac{dK}{ds} = 0$.

(4.) The roots of $\frac{dK}{ds} = 0$ will give break away points.

(5.) To test valid BA points substitute in step (2.)

If K is +ve ⇒ valid BA points.

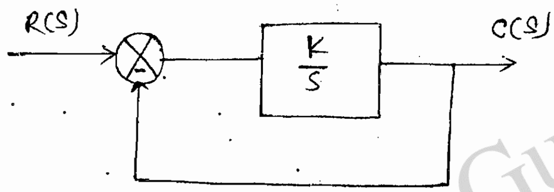
General predictions about BA points →

(1.) The branches of RL either approach (or) ~~live~~ leave the BA points at an angle of $\frac{\pm 180}{n}$, where n = no of branches approaching (or) leaving the BA point.

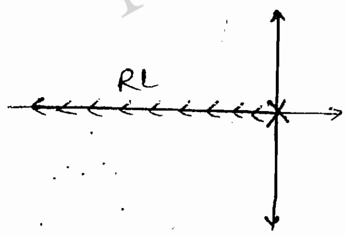
(2.) The complex congugate path for the branches of RL approaching (or) leaving the BA point is a circle.

(3.) whenever there are 2 adjecently placed poles on the real axis with the section of real axis b/n them as a part of RL then there exist some BA point b/n the adjecently placed poles.

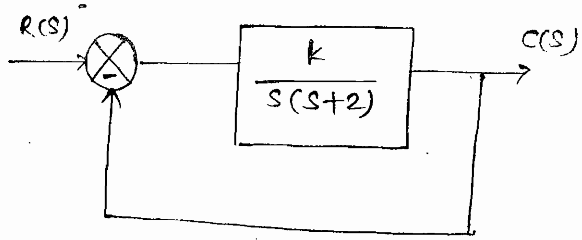
Con(4) 69 First order system →



$$\frac{C(S)}{R(S)} = \frac{K}{S+K} \qquad G(S) = \frac{K}{S}$$



* 2nd order system →

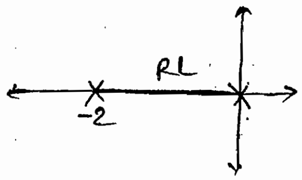


$$\frac{C(s)}{R(s)} = \frac{K}{s^2+2s+K}$$

$$G(s) = \frac{K}{s(s+2)}$$

(2.) $P = 2$; $Z = 0$; $P - Z = 2$

(3.)



(4.) $\theta_1 = 90^\circ$, $\theta_2 = 270^\circ$

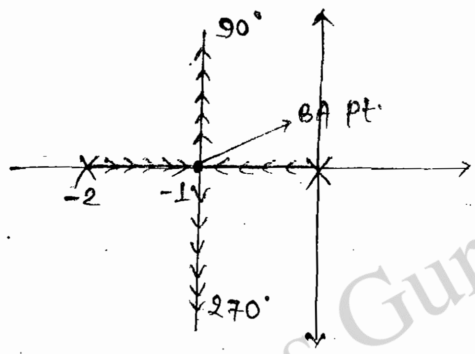
(5.) Centroid $= \frac{0+(-2)}{2} = -1$

(6.) BA point →

$s^2 + 2s + K = 0$

$K = -s^2 - 2s$

$\frac{dK}{ds} = 0$; $-2s - 2 = 0$; $\boxed{s = -1}$



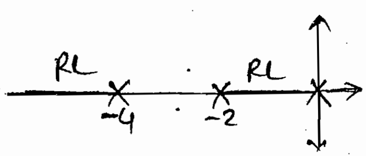
* 3rd order system →

$$G(s) = \frac{K}{s(s+2)(s+4)}$$

Effect of adding poles to a TF :-

(2) $P = 3$, $Z = 0$, $P - Z = 3$

(3)



(4.) $\theta_1 = 60^\circ$, $\theta_2 = 180^\circ$, $\theta_3 = 300^\circ$

(5.) $\frac{0+(-2)+(-4)-0}{3} = -2$

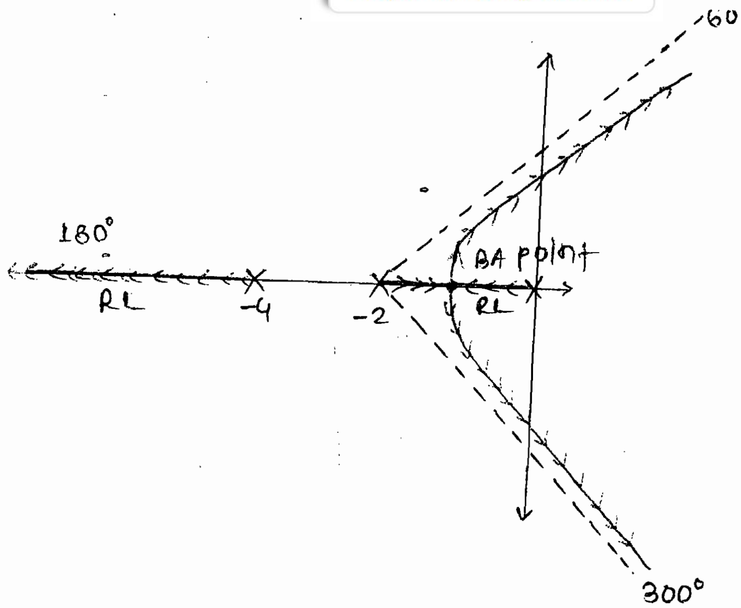
(6.) BA point

$s^3 + 6s^2 + 8s + K = 0$

$K = -s^3 - 6s^2 - 8s$

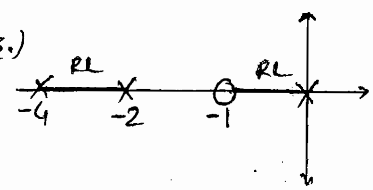
$\frac{dK}{ds} = 0$; $3s^2 + 12s + 8 = 0$

$s = -0.8, -3.15$ ✗



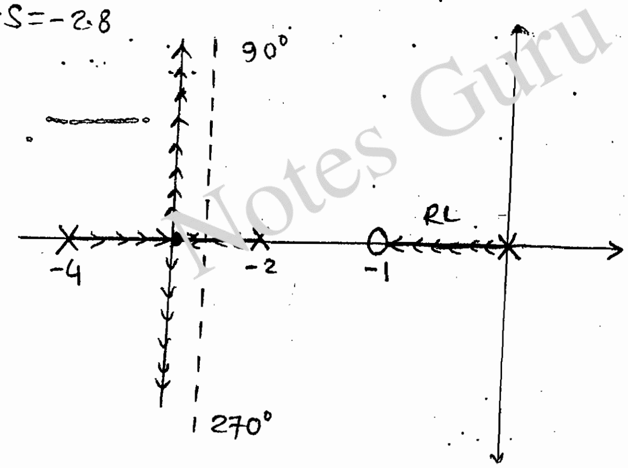
* $G(s) = \frac{k(s+1)}{s(s+2)(s+4)}$ Effect of adding zero to a TF.

(2.) P=3, Z=1, P-Z= 2 (3.)



(4.) $\theta_1 = 90°$, $\theta_2 = 270°$ (5.) $\frac{0+(-2)+(-4)-(-1)}{2} = -2.5$

(6.) BA point S=-2.8



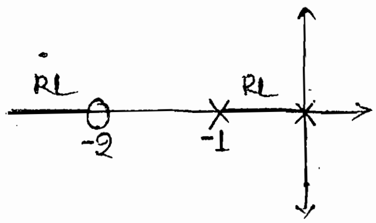
*** Stability ↑ RL shifts to LHS

Prediction (4.) Whenever there is a zero on real axis & to the left side of that zero there are no poles (or) zeros on the real axis with the entire section of real axis to the left side of zero as a part of RL then there exists a BA points to the left side of that zero.

Eg:- $G(s) = \frac{k(s+2)}{s(s+1)}$

(2.) $P = 2, \; Z = 1, \; P - Z = 1$

(3.)



(6.) BA points:-

$$s(s+1) + k(s+2) = 0$$

$$k = \frac{-s^2 - s}{s+2}$$

$$\frac{dk}{ds} = 0$$

$$\frac{(s+2)(-2s-1) - [(-s^2-s)\cdot 1]}{(s+2)^2} = 0$$

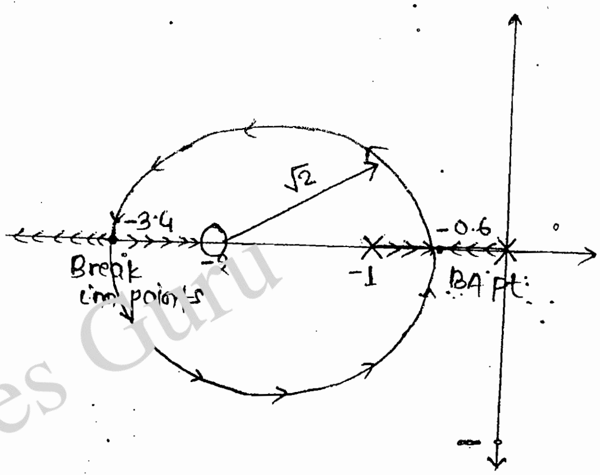
$$-2s^2 + s - 4s - 2 + s^2 + s = 0$$

$$-s^2 - 4s - 2 = 0$$

$$s^2 + 4s + 2 = 0$$

$$s = \frac{-4 \pm \sqrt{16-8}}{2}$$

$$s = -2 \pm \sqrt{2} = -0.6, -3.4$$



To evaluate the center & radius →

$$G(s) = \frac{k(s+b)}{s(s+a)}$$

let $s = x + jy$

$$G(s) = \frac{k[x+jy+b]}{(x+jy)(x+jy+a)}$$

$$G(s) = \frac{k[(x+b)+jy]}{x^2 + xjy + jyx - y^2 + ax + ajy}$$

$$G(s) = \frac{K[(x+b)+jy]}{x^2+ax-y^2+j(2xy+ay)}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x+b}\right) - \tan^{-1}\left(\frac{2xy+ay}{x^2+ax-y^2}\right) = 180^\circ$$

$$\Rightarrow \tan^{-1}\left(\frac{A-B}{1+AB}\right) = 180^\circ$$

$$\Rightarrow \frac{A-B}{1+AB} = \tan(180^\circ)$$

$$\Rightarrow \frac{A-B}{1+AB} = 0$$

$$\Rightarrow A-B = 0$$

$$\Rightarrow \left(\frac{\cancel{y}}{x+b}\right) - \left(\frac{2x\cancel{y}+a\cancel{y}}{x^2+ax-y^2}\right) = 0$$

$$\Rightarrow x^2+ax-y^2-(2x^2+2xb+ax+ab) = 0$$

$$\Rightarrow (x+b)^2+y^2 = b(b-a)$$

$$\Rightarrow \text{center} = -b, 0 \;, = -2, 0$$

$$\Rightarrow \text{Radius} = \sqrt{b(b-a)} = \sqrt{2(2-1)} = \sqrt{2}$$

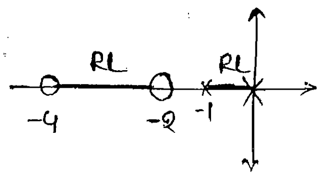
$\boxed{\text{BA points Shortcuts} = \text{Center} \pm \text{Radius}}$

Prediction (5.) whenever there are adjecently placed zeros on the real axis with the section of real axis b/n them as a part of RL then there exists BA point b/n the adjecently placed zeros.

eg:- $G(s) = \frac{K(s+2)(s+4)}{s(s+1)}$

(2.) $P=2$, $Z=2$; $P-Z=0$

(3.)



(6.) BA. points →

$$s(s+1) + K(s^2+6s+8) = 0$$

$$K = \frac{-s^2 - s}{s^2+6s+8}$$

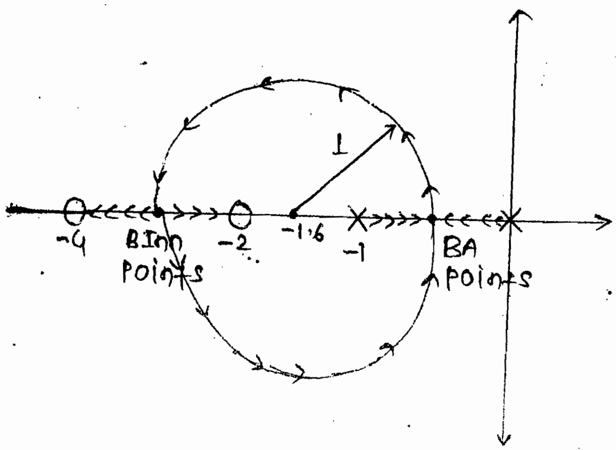
$$\frac{dK}{ds} = 0$$

$$\frac{(s^2+6s+8)(-2s-1) - [(-s^2-s)(2s+6)]}{(s^2+6s+8)^2} = 0$$

$$\Rightarrow 5s^2 + 16s + 8 = 0$$

$$s = \frac{-16 \pm \sqrt{256-160}}{10}$$

$s = -1.6 \pm 1 = -2.6, -0.6$
(Center Radius)



Rule(7) → Intersection of RL with imaginary axis :- The roots of the AE A(s) at $K = K_{max}$ from Routh Array gives the intersection of RL with imaginary axis.

eg:-

$$G(s) = \frac{K}{s(s+2)(s+4)}$$

$$s^3 + 6s^2 + 8s + K = 0$$

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	0
s^0	K	0

(i.) $\frac{48-K}{6} > 0$ (ii) $K > 0$

$K < 48$

$$\boxed{0 < K < 48}$$

$K = K_{max} = 48$

$A(s) = 6s^2 + K = 0$

$6s^2 + 48 = 0$

$s = \pm j\sqrt{8}$

$$\boxed{s = \pm j2.8}$$

Intersection of asymptodes with $j\omega$ axis

$$\tan\theta = \frac{y}{x}$$

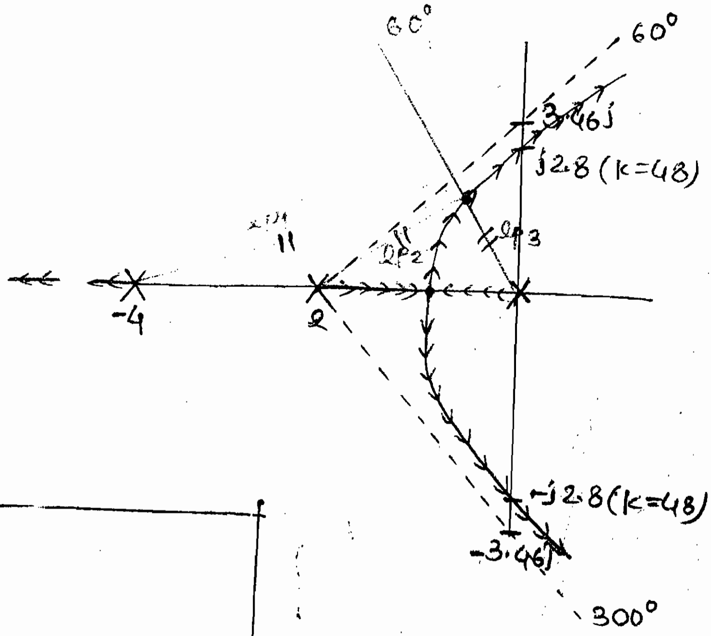
$$\tan 60^\circ = \frac{y}{2}$$

$$y = \tan 60^\circ \times 2$$

$$y = \sqrt{3} \times 2$$

$$= 3.46$$

$$= \pm 3.48j$$



Short cut method →

$$G(s) = \frac{k}{s(s+a)(s+b)}$$

Intersection of RL with $j\omega$ axis $= \pm j\sqrt{ab}$

Q.→ Find k when $\xi = 0.5$ from RL?

Soln →

$$\theta = \cos^{-1}(0.5)$$

$$\theta = 60^\circ$$

$$k = \frac{\text{Product of vector lengths of poles}}{\text{Product of vector lengths of zeros}}$$

$$= \frac{l_{p_1} \times l_{p_2} \times l_{p_3}}{1}$$

Linearity	$a_1x_1(t)+a_2x_2(t) \rightleftharpoons a_1c_1(n)+a_2c_2(n)$	$a_1x_1(t)+a_2x_2(t) \rightleftharpoons a_1X_1(\omega)+a_2X_2(\omega)$	$a_1f_1(t)+a_2f_2(t) \rightleftharpoons a_1F_1(s)+a_2F_2(s)$	$a_1x_1(n)+a_2x_2(n) \rightleftharpoons a_1X_1(z)+a_2X_2(z)$
Time-Reversal	$x(-t) \rightleftharpoons c_{(-n)}$	$x(-t) \rightleftharpoons X(-\omega)$	$f(-t) \rightleftharpoons F(-s)$	$x(-n) \rightleftharpoons X(z^{-1})$
Congugation	$x^*(t) \rightleftharpoons c^*_{-n}$	$x^*(t) \rightleftharpoons X^*(-\omega)$	$f^*(t) \rightleftharpoons F^*(s^*)$	$x^*(n) \rightleftharpoons X^*(z^*)$
Time-shifting	$x(t-t_0) \rightleftharpoons c_n e^{-jn\omega_0 t_0}$	$x(t-t_0) \rightleftharpoons X(\omega)e^{-j\omega t_0}$	$f(t-t_0) \rightleftharpoons F(s)e^{-st_0}$	$x(n-n_0) \rightleftharpoons X(z)z^{-n_0}$ $n_0=1 \Rightarrow x(n-1) \rightleftharpoons X(z)z^{-1}$
Freq. shifting	$x(t)e^{jm\omega_0 t} \rightleftharpoons c_{n-m}$ $x(t)e^{-jm\omega_0 t} \rightleftharpoons c_{n+m}$	$x(t)\cdot e^{j\omega_0 t} \rightleftharpoons X(\omega-\omega_0)$	$e^{-at}f(t) \rightleftharpoons F(s+a)$	
Convolution in time	$x_1(t) * x_2(t) \rightleftharpoons T_0(c_{1n}\cdot c_{2n})$ where $T_0 = \mathrm{LCM}(T_1, T_2)$	$x_1(t) * x_2(t) = X_1(\omega)\cdot X_2(\omega)$	$F_1(t) * F_2(t) \rightleftharpoons F_1(s)\cdot F_2(s)$	$x_1(n) * x_2(n) \rightleftharpoons X_1(z)X_2($
Multiplication in time	$x_1(t)\cdot x_2(t) \rightleftharpoons (c_{1n} * c_{2n})$	$x_1(t)\cdot x_2(t) = \frac{1}{2\pi}[X_1(\omega) \overset{*}{\ } X_2(\omega)]$	$f_1(t)\cdot f_2(t) \rightleftharpoons \frac{1}{2\pi j}[F_1 s * F_2(s)]$	$x_1(n)\cdot x_2(n) \rightleftharpoons \frac{1}{2\pi j}[X_1(z) * X_2(z$
Integration in time.	$\int_{-\infty}^{t} x(k)dk = \frac{c_n}{jn\omega_0}$	$\int_{-\infty}^{t} x(k)dk = \frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$ $X(0) = x(\omega)\vert_{\omega=0}$	$\int_{-\infty}^{t} x(t)\,dt \rightleftharpoons \begin{cases} \frac{F(s)}{s}; & \text{Bilateral LT.} \\ \frac{F(s)}{s} + \frac{\int_{-\infty}^{0} f(t)dt}{s} & \text{Unilateral LT.} \end{cases}$	$\sum_{k=-\infty}^{n} x(n) \rightleftharpoons \frac{X(z)}{1-z^{-1}}$
Diff. in time	$\frac{d^m x(t)}{dt^m} = (jn\omega_0)^m c_n$	$\frac{d^n x(t)}{dt^n} = (j\omega)^n X(\omega)$	$\frac{d^n f(t)}{dt^n} \rightleftharpoons \begin{cases} s^n F(s) & \text{Bilateral LT.} \\ s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) & \text{Uni LT} \end{cases}$	$\frac{dx(n)}{dn} = \frac{x(n)-x(n-1)}{n-(n-1)}$ $x(n)-x(n-1) \rightleftharpoons (1-z^{-1})X(z)$

Passeval's Power theorem	$P=\frac{1}{T_0}\int_{T_0}\lvert x(t)\rvert^2dt$ $P=\sum_{n=-\infty}^{\infty}\lvert C_n\rvert^2$	$E=\int_{-\infty}^{\infty}\lvert x(t)\rvert^2dt$ $=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2d\omega.$	Initial value theorem:- $f(0)=\lim_{s\to\infty}s\cdot F(s)$ applicable only causal sys. $f(t)=0,\ t<0$	Initial value theorem $x(0)=\lim_{z\to\infty}X(z)$ $x(0)=\lim_{s\to\infty}sX(s)$
Time scaling		$x(at)_{a\neq 0}=\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	$f(at)_{a\neq 0}\rightleftharpoons\frac{1}{\lvert a\rvert}F\left(\frac{s}{a}\right)$	Condn:- applicable only for causal type of sys. i.e. $x(n)=0,\ n<0$
modulation		$x(t)\cdot\cos\omega_0t=\frac{1}{2}\left[X(\omega+\omega_0)+X(\omega-\omega_0)\right]$ $x(t)\cdot\sin\omega_0t=\frac{j}{2}\left[X(\omega+\omega_0)-X(\omega-\omega_0)\right]$		Scaling of z:- $a^nx(n)\rightleftharpoons X(a^{-1}z)$
Differentiation in freq.		$t^nx(t)=j^n\frac{d^nX(\omega)}{d\omega^n}$	$t^nf(t)\rightleftharpoons(-1)^n\frac{d^nF(s)}{ds^n}$	$n\cdot x(n)\rightleftharpoons -z\frac{dX(z)}{dz}$
Area under time domain		$X(\omega)=\int_{-\infty}^{\infty}x(t)\cdot e^{-j\omega t}dt$ $\downarrow \omega=0$ $X(0)=\int_{-\infty}^{\infty}x(t)dt$ $x(t)=X(\omega)\vert_{\omega=0}$	Final value theorem $\rightarrow$ $f(\infty)=\lim_{s\to 0}s\cdot F(s)$ (i) causal type $f(t)=0,\ t<0$ (ii) $sF(s)$ should have only LHS poles in s-plane.	Final value theorem $\rightarrow$ $x(\infty)=\lim_{z\to 1}(1-z^{-1})X(z)$ $x(\infty)=\lim_{s\to 0}sX(s)$
Area under freq. domain		$X(\omega)=2\pi x(t)\vert_{t=0}$		Condn:- (i) applicable for causal signals i.e. $x(n)=0$, $n<0$
Integration in freq.			$\frac{f(t)}{t}\rightleftharpoons\int_{s}^{\infty}F(s)ds$	(ii) $(1-z^{-1})X(z)$ should have poles inside unit circle in z-plane.

DATE-20/11/14

* Rule no. (8.) → Angle of departure & arrival → The angle of departure is obtained when complex poles terminate at ∞

* The angle of arrival is obtained at complex zeros.

$$\phi_D = 180° + \phi \quad ; \phi_A = 180° - \phi$$

where; $\phi = \Sigma\phi_Z - \Sigma\phi_P$

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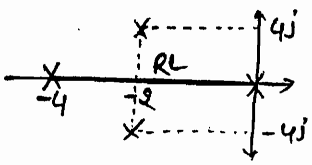
$$G(s) \cdot H(s) = \frac{K}{s(s+4)(s^2+4s+20)}$$

(2.) $P=4$; $Z=0$; $P-Z=4$

(3.)



(4.) $\theta_1 = 45°$, $\theta_2 = 135°$, $\theta_3 = 225°$, $\theta_4 = 315°$

(5.) $\dfrac{0+(-2)+(-2)+(-4)-0}{4} = -2$

(6) BA points →

(9.) Shortcut methods →

Avg. value of real poles $= \dfrac{0+(-4)}{2} = -2$

* If the avg. value of real poles = Real part of complex pole
There will be 3 B.A points.

* The avg. value of real poles ≠ Real part of complex poles.
There will be 1 BA. points

(b.) nature of BA points →

Absolute value of avg. value = 2
of real poles.

$2 \times x = 20$

$\boxed{x = 10}$

$x \geq 5 \Rightarrow$ There will be 1 real & 2 complex BA points

$x < 5 \Rightarrow$ There will be 3 real BA points.

* Original method →

$$s^4 + 8s^3 + 36s^2 + 80s + K = 0$$

$$K = -s^4 - 8s^3 - 36s^2 - 80s$$

$$\frac{dK}{ds} = -4s^3 - 24s^2 - 72s - 80 = 0$$

$$4s^3 + 24s^2 + 72s + 80 = 0$$

$$s = -2 ; -2 \pm 2.45j$$

Note:- To check the validity of complex BA points use angle condⁿ.

$$G(s)H(s)\Big|_{s=-2\pm 2.45j} = \frac{K}{(-2+2.45j)(2+j2.45)(0+6.45j)(0-1.55j)}$$

$$\angle G(s)\cdot H(s)\Big|_{s=-2+j2.45} = \frac{0^\circ}{(130^\circ)(50^\circ)(90^\circ)(-90^\circ)} = -180^\circ$$

(7.)

s^4	1	36	K
s^3	8	80	0
s^2	26	K	0
s^1	$\frac{2080-8K}{26}$	0	0
s^0	K	0	0

$$\frac{2080-8K}{26} > 0 \quad , \quad K > 0$$

$$K < 260$$

$$\boxed{0 < K < 260}$$

$$K_{mar} = 260$$

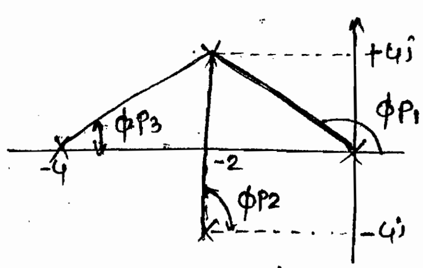
$$A(s) = 26s^2 + K = 0$$

$$26s^2 + 260 = 0$$

$$\boxed{s = \pm 3.16j}$$

$$\boxed{\begin{aligned} y &= \tan 45 \times 2 = 2 \\ &= \pm \hat{j}2 \end{aligned}}$$

(8.) Angle of departure →



$$\phi_{P_1} = 180^\circ - \tan^{-1}\left(\frac{4-0}{0-(-2)}\right)$$

$$= 116.6^\circ$$

$$\phi_{P_2} = 90^\circ$$

$$\phi_{P_3} = \tan^{-1}\left[\frac{4-0}{-2-(-4)}\right] = 63.4^\circ$$

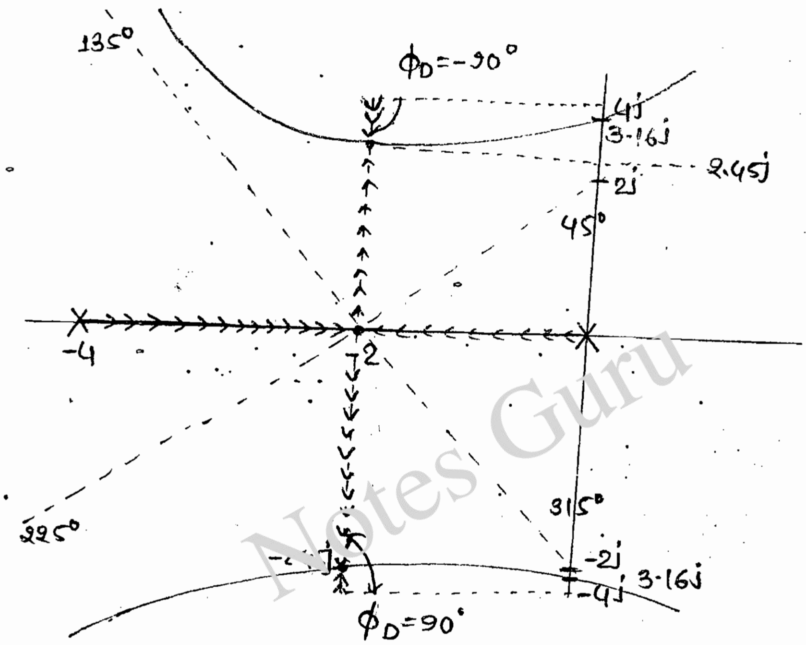
$$\phi = \Sigma\phi_z - \Sigma\phi_p$$

$$= 0 - [116.6 + 90 + 63.4]$$

$$\phi = -270^\circ$$

$$\phi_D = 180^\circ + \phi = 180^\circ - 270^\circ$$

$$\boxed{\phi_D = -90^\circ}$$

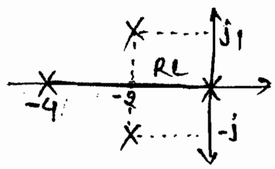


(9/68)

$$G(s)\cdot H(s) = \frac{K}{s(s+4)(s^2+4s+5)}$$

(1.) $P=4$; $Z=0$, $P-Z=4$

(3.)



(4.) $\theta_1 = 45^\circ$, $\theta_2 = 135^\circ$, $\theta_3 = 225^\circ$, $\theta_4 = 315^\circ$

(5.) $\dfrac{0+(-2)+(-2)+(-4)-0}{4} = -2$

(6.) BA points →

$$s^4 + 8s^3 + 21s^2 + 20s + K = 0$$

$$K = -s^4 - 8s^3 - 21s^2 - 20s$$

$$\frac{dK}{ds} = 0.$$

$$-4s^3 - 24s^2 - 42s - 20 = 0$$

$$4s^3 + 24s^2 + 42s + 20 = 0$$

$$\boxed{s = -0.78,\ -2,\ -3.22}$$

(7.)

s^4	1	21	K
s^3	8	20	0
s^2	18.5	K	0
s^1	$\frac{370-8K}{18.5}$	0	0
s^0	K	0	0

$\dfrac{370-8K}{18.5} > 0$, $K < 46.25$

$$\boxed{0 < K < 46.25}$$

$K = K_{max} = 46.25$

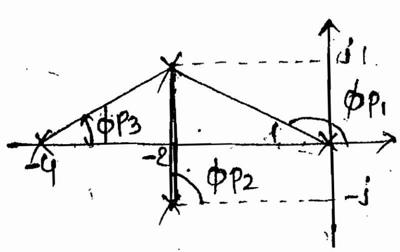
$$A(s) = 18.5s^2 + K = 0$$

$$18.5s^2 + 46.25 = 0$$

$$\boxed{s = \pm j1.58}$$

$$Y = \tan^{-1}45^\circ \times 2 = 2 = \pm 2j$$

(8.) Angle of departure →



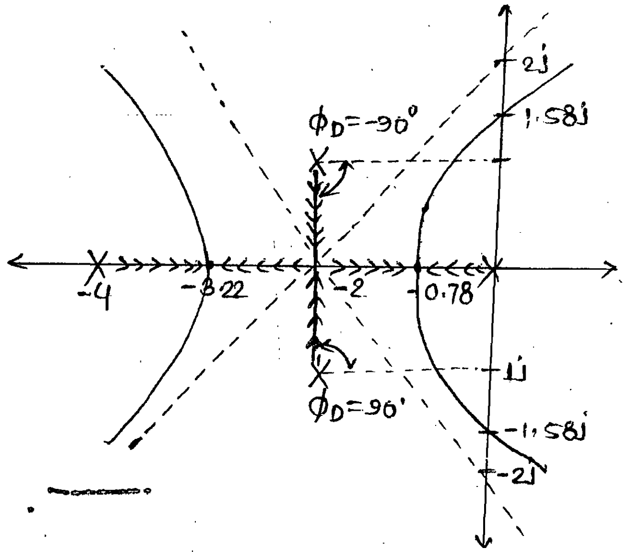
$$\phi P_1 = 180^\circ - \tan^{-1}\left(\frac{1-0}{0-(-2)}\right) = 153.5^\circ$$

$$\phi P_2 = 90^\circ ; \quad \phi P_3 = \tan^{-1}\left(\frac{1-0}{-2-(-4)}\right) = 26.5^\circ$$

$$\phi = 0^\circ - (153.5 + 90^\circ + 26.5) = -270^\circ$$

$$\phi_D = 180^\circ + \phi = 180 - 270^\circ = -90$$

$$\boxed{\phi_D = -90^\circ}$$

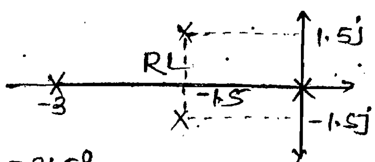


Conv (1.) 69

$$G(s) = \frac{K}{s(s+3)(s^2+3s+4.5)}$$

(2.) $P=4,\ Z=0,\ P-Z=4$

(3.)



(4.) $\theta_1 = 45^\circ,\ \theta_2 = 135^\circ,\ \theta_3 = 225^\circ,\ \theta_4 = 315^\circ$

(5.) $$\frac{0+(-1.5)+(-1.5)+(-3)-0}{4} = -1.5$$

(6.) BA points :-

$$s^4+6s^3+13.5s^2+13.5s+K=0$$

$$K = -s^4-6s^3-13.5s^2-13.5s$$

$$\frac{dK}{ds} = -4s^3-18s^2-27s-13.5=0$$

$$4s^3+18s^2+27s+13.5=0$$

$$\boxed{s = -1.5, -1.5, -1.5}$$

(7.)

s^4	1	13.5	K
s^3	6	13.5	0
s^2	11.25	K	0
s^1	$\frac{151.87-6K}{11.25}$	0	0
s^0	K	0	0

$$\frac{151.87-6K}{11.25} > 0$$

$$K < 25.3$$

$$\boxed{0 < K < 25.3}$$

at $K = K_{max} = 25.3$

$$A(s) = 11.25s^2 + K = 0$$

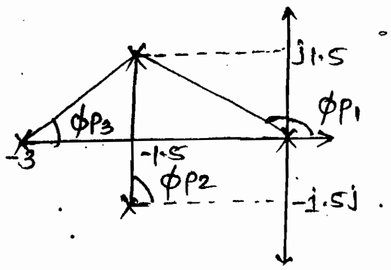
$$11.25s^2 + 25.3 = 0$$

$$\boxed{s = \pm j1.5}$$

$$Y = \tan 45^0 \times 1.5 = 1.5$$

$$= \pm 1.5j$$

(8.) Angle of departure →



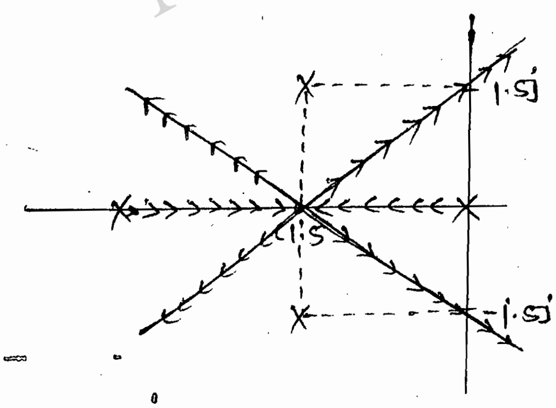
$$\phi P_1 = 180^0 - \tan^{-1}\left(\frac{1.50}{0-(-1.5)}\right) = 135^0$$

$$\phi P_2 = 90^0, \quad \phi P_3 = \tan^{-1}\left(\frac{1.5-0}{1.5-(-3)}\right) = 45^0$$

$$\phi = 0^0 - (135^0 + 90 + 45) = -270^0$$

$$\phi_D = 180^0 - 270^0 = -90^0$$

$$\boxed{\phi_D = -90^0}$$



conv.(2) 69

$$s(s+4)(s^2+2s+2)+K(s+1)=0$$

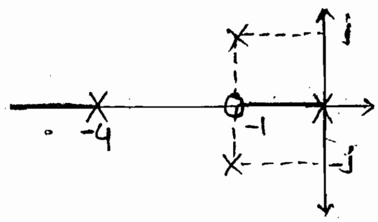
$$1+\frac{K(s+1)}{s(s+4)(s^2+2s+2)}=0$$

$$1+G(s)\cdot H(s)=0$$

$$G(s)\cdot H(s)=\frac{K(s+1)}{s(s+4)(s^2+2s+2)}$$

(2.) $P=4$; $Z=1$, $P-Z=3$

(3.)



(4.) $\theta_1=60^\circ$, $\theta_2=180^\circ$, $\theta_3=300^\circ$

(5.) $$\frac{0+(-1)+(-1)+(-4)-(-1)}{3}=-1.6$$

(6.) BA points:— Nill.

(7.) $s^4+6s^3+10s^2+(K+8)s+K=0$

s^4	1	10	K
s^3	6	$(K+8)$	0
s^2	$\frac{52-K}{6}$	K	0
s^1	$\frac{(52-K)(K+8)-6K}{6}$ / $\left(\frac{52-K}{6}\right)$	0	0
s^0	K	0	0

$$(52-K)(8+K)-36K=0$$

$$K^2-8K-416=0$$

$$K=24.78,\ -16.75$$

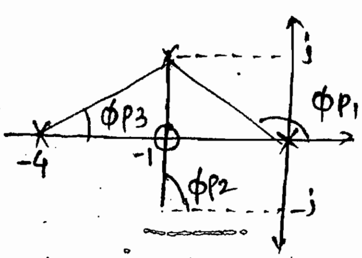
$$\boxed{K_{mar}=24.78}$$

$$A(s)=\left(\frac{52-24.78}{6}\right)s^2+24.78=0$$

$$\boxed{s=\pm j2.34}$$

$$y=\tan 60^\circ\times 1.6=\sqrt{3}\times 1.6=2.77$$
$$=\pm 2.77j$$

(8.)



$$\phi P_1=180^\circ-\tan^{-1}\left(\frac{1-0}{0-(-1)}\right)=135^\circ$$

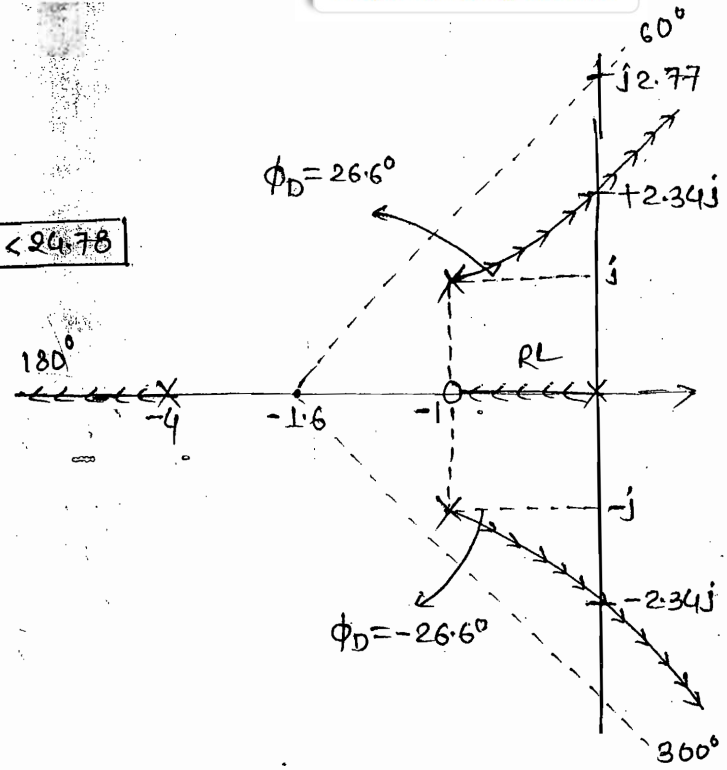
$$\phi P_2=90^\circ;\ \phi P_2=90^\circ$$

$$\phi P_3=\tan^{-1}\left(\frac{1-0}{-1-(-4)}\right)=18.4^\circ$$

$$\phi=90^\circ-(135^\circ+90+18.4)=-153.4^\circ$$

$$\phi_D=180-153.4^\circ=26.6^\circ$$

$\boxed{0<K<24.78}$

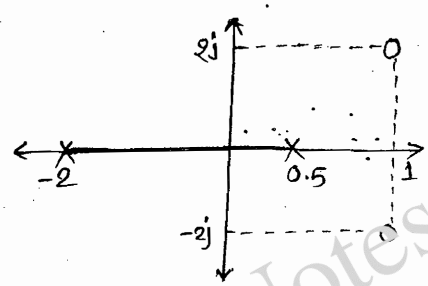


Q(3) 69

$$G(s) = \frac{K(s^2-2s+5)}{(s+2)(s-0.5)}$$

(2.) $P=2,\ Z=2,\ P-Z=0$

(3.)



(6.) BA. point →

$$(s+2)(s-0.5)+K(s^2-2s+5)=0$$

$$s^2+1.5s-1+K(s^2-2s+5)=0$$

$$K = \frac{-s^2-1.5s+1}{s^2-2s+5}$$

$$\frac{dK}{ds} = 0$$

$$\Rightarrow \quad 3.5s^2-12s-5.5=0$$

$$s = -0.4,\ 3.8$$

(7.) $s^2(1+K)+s(1.5-2K)+(5K-1)=0$

s^2	$(1+K)$	$5K-1$
s^1	$(1.5-2K)$	0
s^0	$5K-1$	0

$1.5-2K=0$

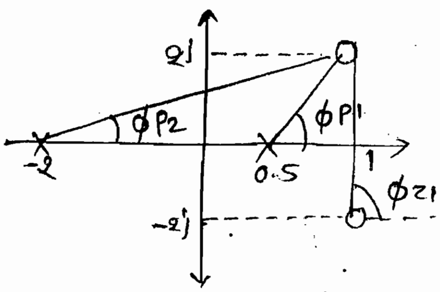
$K_{mar}=0.75$

$A(s)=(1+K)s^2+(5K-1)=0$

$(1+0.75)s^2+(5\times0.75-1)=0$

$\boxed{s=\pm j1.25}$

(8.) Angle of arrival→



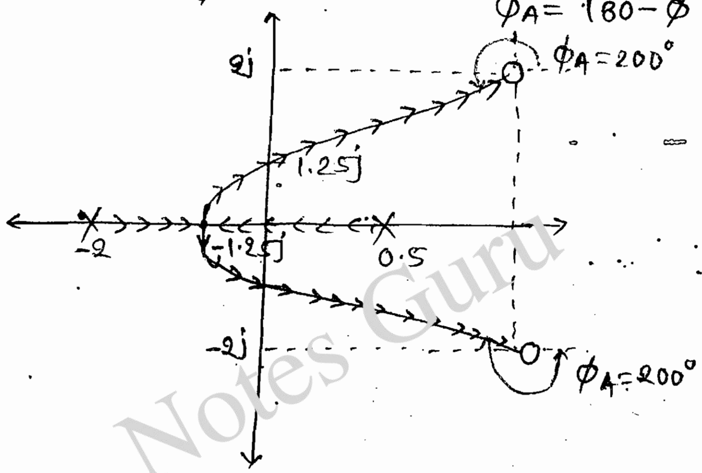
$\phi z_1 = 90^\circ$

$\phi P_1 = \tan^{-1}\left(\frac{2-0}{1-0.5}\right) = 76^\circ$

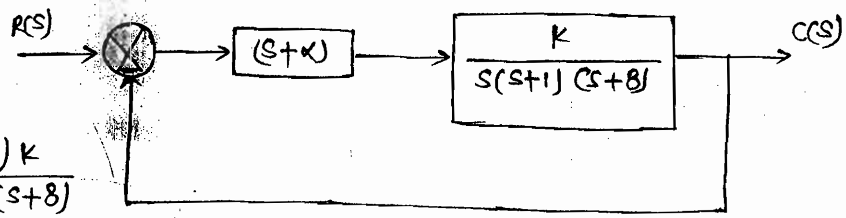
$\phi P_2 = \tan^{-1}\left(\frac{2-0}{1-(-2)}\right) = 34^\circ$

$\phi = 90^\circ-(76+34) = -20^\circ$

$\phi_A = 180^\circ-\phi = 180-(-20) = 200^\circ$



* Root contour →



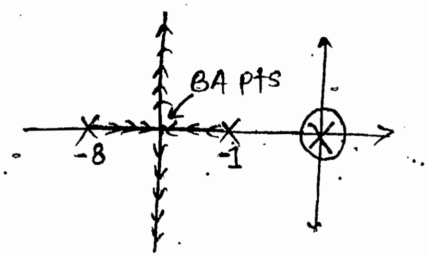
$$G(s) = \frac{K(s+\alpha)K}{s(s+1)(s+8)}$$

* These are multiple RL diagrams obtained by varying multiple parameter in a TF· drawn on same s-plane.

case(i) →

$\alpha = 0$

$$G(s) = \frac{K\cdot\cancel{s}}{\cancel{s}(s+1)(s+8)}$$



Case(2) →

$$1 + \frac{K(s+\alpha)}{s(s+1)(s+8)} = 0$$

$$s(s+1)(s+8) + K(s+\alpha) = 0$$

$$s(s+1)(s+8) + Ks + K\alpha = 0$$

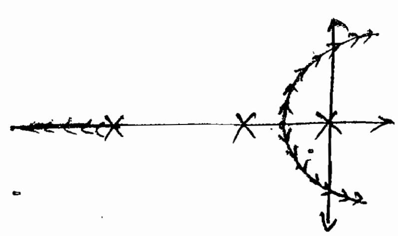
$$1 + \frac{K\alpha}{s(s+1)(s+8)+Ks} = 0$$

$$1 + G(s)\cdot H(s) = 0$$

$$G(s)\cdot H(s) = \frac{K\alpha}{s(s+1)(s+8)+Ks}$$

if value of K is not given then K=1

$$G(s)\cdot H(s) = \frac{\alpha}{s(s+1)(s+8)+s} = \frac{\alpha}{s(s^2+9s+9)}$$



Sagar sen
8871453536

Q.→ Find BA. points for $k=10$?

Soln→ $$G(s).H(s) = \frac{10\alpha}{s(s+1)(s+8)+10s}$$

let $10\alpha = k'$

$$= \frac{k'}{s(s+1)(s+8)+10s}$$

$$= \frac{k'}{s(s^2+9s+18)}$$

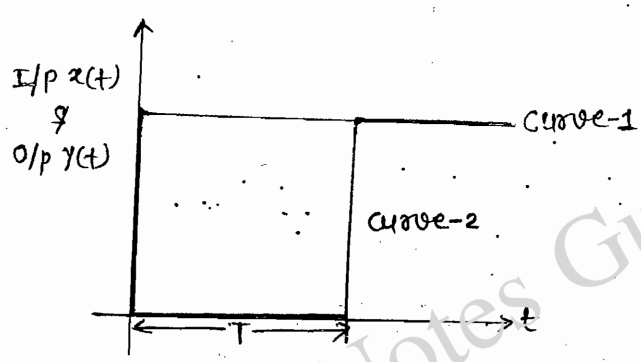
$$1 + \frac{k'}{s(s^2+9s+18)} = 0$$

$$s^3+9s^2+18s+k' = 0$$

$$k' = -s^3-9s^2-18s$$

$$\frac{dk'}{ds} = -3s^2-18s-18 = 0$$

* Analysis of sys. having DEAD TIME (or) TRANSPORTATION LAG →



For curve-(1)

O/p $y(t) =$ I/p $x(t)$

For curve-(2)

O/p $y(t) = x(t-T)$

$$Y(s) = e^{-Ts}X(s)$$

$$\boxed{\frac{Y(s)}{X(s)} = e^{-Ts}}$$

Time domain approximation → (Time domain analysis, RH, RL plots)

$$y(t) = x(t-T) = x(t) - T\dot{x}(t) + \frac{T^2}{2!}\ddot{x}(t) - \frac{T^3}{3!}\dddot{x}(t) + \cdots\cdots$$

$$y(t) = x(t) - T\dot{x}(t)$$

$$Y(s) = X(s)(1-Ts) \quad \& \quad Y(s) = X(s)\cdot e^{-Ts}$$

$$\boxed{e^{-Ts} \simeq (1-Ts)}$$

Ex:- $G(s) = \frac{ke^{-s}}{s(s+3)} = \frac{k(1-s)}{s(s+3)}$

* Dead time is one of the forms of non-linearity & is approximated as zero in RHS of s-plane.
* TF having poles (or) zeros in RHS of s-plane are known as non-min^m phase f^n.

Non min^m phase f^n →

$$\boxed{\angle F(s)\Big|_{\omega\to\infty} \neq -(P-Z).90^\circ}$$

Eg:- $G(s) = \frac{ke^{-s}}{s(s+3)} = \cancel{\frac{k(s-3)}{}}\ \frac{K(1-s)}{s(s+3)} = \frac{K(1-s)}{(s)\,3\left(1+\frac{s}{3}\right)}$

$$G(j\omega) = \frac{\left(\frac{k}{3}+j0\right)(1-j\omega)}{(0+j\omega)\left(1+\frac{j\omega}{3}\right)}$$

$$\angle G(j\omega) = \frac{(0^\circ)(-\tan^{-1}\omega)}{(90^\circ)\left(\tan^{-1}\frac{\omega}{3}\right)} = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{3}$$

$$\angle G(j\omega)\Big|_{\omega=\infty} = -90^\circ - 90 - 90 = -270^\circ \neq -(P-Z)90^\circ$$

* TF having poles & zeros in the LHS of s-plane are known as min^m phase f^n. LTI TF should be min^m phase f^n.

min^m phase f^n →

$$\boxed{\angle F(s)\Big|_{\omega\to\infty} = -(P-Z)90^\circ}$$

Eg:- $G(s) = \dfrac{K(1+s)}{s(s+3)} = \dfrac{(K/3)(1+s)}{s(1+\frac{s}{3})}$

$$G(j\omega) = \frac{\left(\frac{K}{3}+j0\right)(1+j\omega)}{(0+j\omega)\left(1+\frac{j\omega}{3}\right)}$$

$$\angle G(j\omega) = \frac{(0^\circ)\,[\tan^{-1}(\omega)]}{(90^\circ)\,\left(\tan^{-1}\frac{\omega}{3}\right)} = -90^\circ + \tan^{-1}\omega - \tan^{-1}\frac{\omega}{3}$$

$$\angle G(j\omega)\big|_{\omega=\infty} = -90 + 90^\circ - 90^\circ = -90$$

$$\boxed{-90^\circ = -(P-Z)90^\circ}$$

* Since s indicates time & can't be -ve (1-s) factor should be expressed as -(s-1) in time domain methods.

$$G(s) = \frac{Ke^{-s}}{s(s+3)} = \frac{K(1-s)}{s(s+3)} = \frac{-K(s-1)}{s(s+3)}$$

* Complemetry R-L (CRL) or Inverse RL (IRL) →

$$G(s)H(s) = 1+j0$$

(1.) Angle condn

$$\angle G(s)\cdot H(s) = 0^\circ = \pm(2q)\,180^\circ$$

(2.) Mag. condn

$$|G(s)\cdot H(s)| = 1$$

* Construction Rule of CRL →

Rule(1). The CRL is symmetrical about real axis.

Rule(2) Same as RL.

Rule(3) A point on real axis is said to be on CRL is to the right side of this point the sum of open loop poles & zeros is even.

Rule(4) Angle of asymptodes →

$$\boxed{\theta = \frac{(2q)\,180}{P-Z}}$$

where, $q = 0, 1, 2, \ldots$

Rule (5) Centroid → same as RL

Rule (6) BA points → same as RL

Rule (7) Intersection of CRL with $j\omega$ axis → same as RL

Rule (8) Angle of departure & arrival →

$$\phi_D = 0° + \phi$$

$$\phi_A = 0° - \phi$$

where;

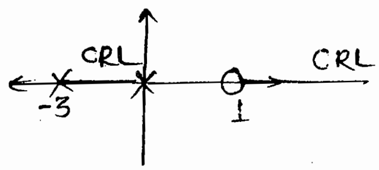
$$\phi = \Sigma\phi_z - \Sigma\phi_p$$

Que. →

$$G(s) = \frac{Ke^{-s}}{s(s+3)} = \frac{K(1-s)}{s(s+3)}$$

Soln → (2.) $P = 2$, $Z = 1$, $P - Z = 1$

(3.)



(6.) BA points:-

$$1 + \frac{K(1-s)}{s(s+3)} = 0$$

$$s(s+3) + K(1-s) = 0$$

$$K = \frac{-s^2 - 3s}{1-s}$$

$$\frac{dK}{ds} = 0$$

$$\frac{(1-s)(-2s-3) - [(-s^2-3s)(-1)]}{(1-s)^2} = 0$$

$$s^2 - 2s - 3 = 0$$

$$s = \frac{2 \pm \sqrt{4+12}}{2}$$

$$s = 1 \pm 2 = -1, 3$$

(7.) $s^2 + s(3-K) + K = 0$

s^2	1	K
s	$3-K$	0
s^0	K	0

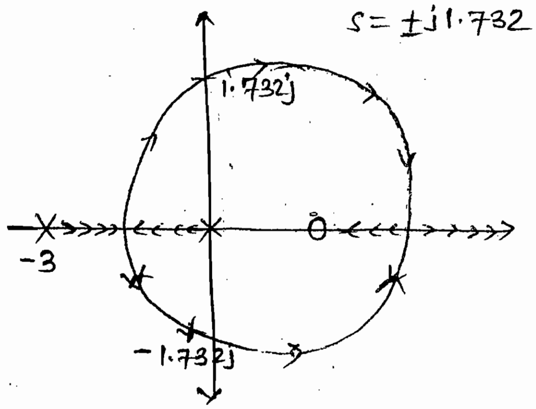
$3-K=0$

$K_{mar}=3$

$A(s)=s^2+K=0$

$s^2+3=0$

$s=\pm j1.732$



$\frac{1}{67}$

$$G(s)=\frac{K(s+a)}{s^2(s+b)}$$

ans (c)

$1+G(s)=0$

$$1+\frac{K(s+a)}{s^2(s+b)}=0$$

$s^3+bs^2+Ks+aK=0$

s^3	1	K
s^2	b	aK
s^1	$\frac{bK-aK}{b}$	0
s^0	aK	0

(i) $aK>0$

$K>0$

(ii) $\frac{bK-aK}{b}>0$

$K(b-a)>0$

$\boxed{K>0}$

$\frac{bK-aK}{b}=0$

$K(b-a)=0$

$K=K_{mar}=0$

$A(s)=bs^2+aK=0$

$bs^2+0=0$

$\boxed{s=0}$

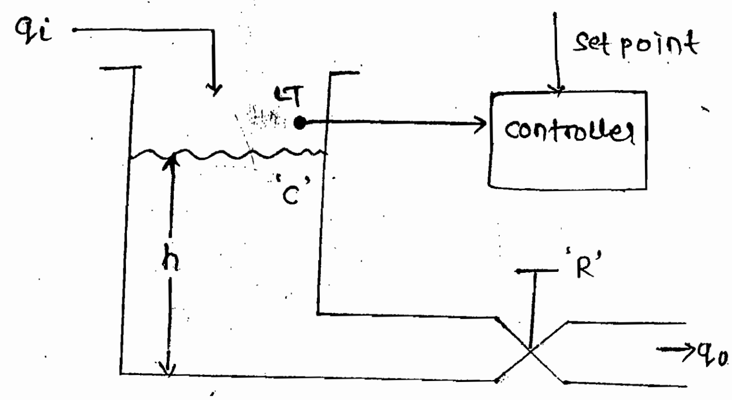
$\frac{10}{69}$

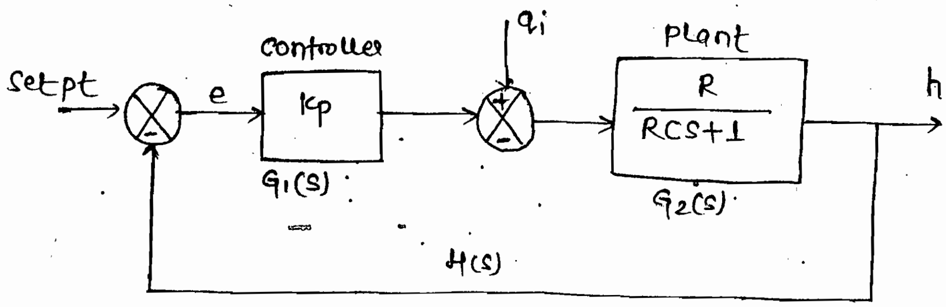
$0\le K<1 \rightarrow$ overdamped (1, 3) ans. (c)

$K>5$. overdamped

DATE-21/11/14

INDUSTRIAL CONTROLLER





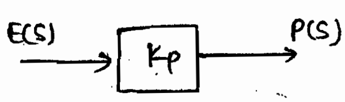
$$|e_{ss}| = \lim_{s\to 0} \frac{S \cdot Q_i(S) \cdot G_2(S)}{1+G_1(S)G_2(S)}$$

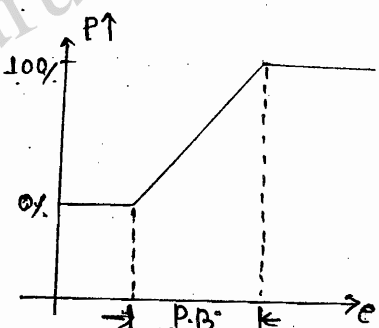
1) Proportional mode →

$p \propto e$

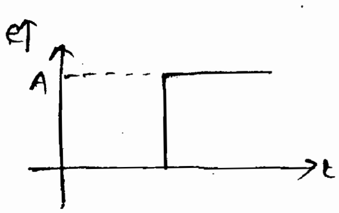
$p = K_p e$; K_p = proportional gain

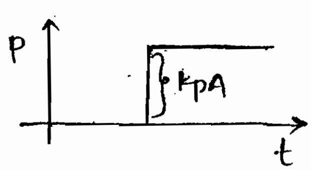
$P(S) = K_p E(S)$





$$\boxed{\text{Proportional Band (PB)} = \frac{100}{K_p}}$$





$p = K_p e \quad (e = A)$

$p = K_p A$

$$|e_{ss}| = \lim_{s\to 0} \frac{S \cdot \frac{A}{S} \cdot \frac{R}{RCS+1}}{1+\frac{RK_p}{RCS+1}}$$

$$\boxed{|e_{ss}| = \frac{AR}{1+RK_p}}$$

Offset

$$\boxed{\text{Offset} \propto \frac{1}{K_p}}$$

* It is a natural extension of ON/OFF controller.
* The band of error where every value of error has unique value of controller o/p is known as proportional band.
* The disadvantage of this controller is it exibits a permanant residual error known as 'offset'.

20/63

$$P = K_p e$$
$$100 = K_p \times 1$$
$$K_p = 100$$
$$PB = \frac{100}{K_p} = \frac{100}{100} = 1$$

$$\% PB = 1 \times 100\% = 100\%$$
$$100\%\ PB = 100V$$
$$20\%\ PB = ?$$
$$\frac{20}{100} \times 100 = 20V$$

$\boxed{1V, 20V}$

(2) Integral mode →

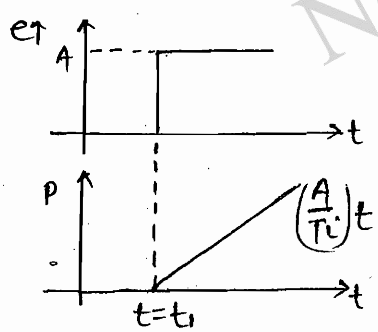
$$\frac{dP}{dt} \propto e$$
$$\frac{dP}{dt} = K_I e \quad (K_I = \text{Integral scaling})$$
$$P = K_I \int e\, dt$$
$$P(S) = \frac{K_I}{S} E(S)$$

$$E(S) \longrightarrow \boxed{\frac{K_I}{S}} \longrightarrow P(S)$$

Defining "RESET TIME!"

$$T_i = \frac{1}{K_I}$$
$$P = \frac{1}{T_i} \int e\, dt$$
$$P(S) = \frac{1}{T_i S} \cdot E(S)$$

$$E(S) \longrightarrow \boxed{\frac{1}{T_i S}} \longrightarrow P(S)$$



$$P = \frac{1}{T_i} \int A\, dt \quad (e = A)$$
$$P = \left(\frac{A}{T_i}\right) t$$

Let $e = \sin \omega t$

$$P = \frac{1}{T_i} \int \sin \omega t\, dt$$
$$P = \frac{1}{\omega T_i} (-\cos \omega t)$$
$$P = \frac{1}{\omega T_i} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$|e_{ss}| = \lim_{S \to 0} \frac{\cancel{S} \cdot \frac{A}{\cancel{S}} \times \frac{R}{RCS+1}}{1 + \frac{R}{T_i S (T+S)}}$$

$$\boxed{|e_{ss}| = \frac{AR}{1+\infty} = 0}$$

* The disadvantage of integral controller is its responce to errors is slow. However it is capable of eliminating the error completely in the sys.

3) Derivative (OR) Rate mode →

$$P \propto \frac{de}{dt}$$

$$P = K_D \frac{de}{dt}$$

K_D = Rate constant

$$P(s) = K_D s \cdot E(s)$$

$$E(s) \rightarrow \boxed{K_D s} \rightarrow P(s)$$

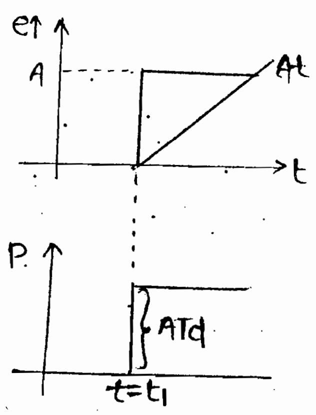
Defining "RATE TIME"

$$T_d = K_D$$

$$P = T_d \frac{de}{dt}$$

$$P(s) = T_d s \cdot E(s)$$

$$E(s) \rightarrow \boxed{T_d s} \rightarrow P(s)$$



$$P = T_d \frac{d}{dt}(A) \quad (e = A)$$

$$P = 0$$

$$P = T_d \frac{d}{dt}(At) \quad (e = At)$$

$$P = T_d A$$

$$|e_{ss}| = \lim_{s \to 0} \frac{s \cdot \frac{A}{s^2} \cdot \frac{R}{RCs+1}}{1 + \frac{T_d s \cdot R}{RCs+1}}$$

$$|e_{ss}| = \lim_{s \to 0} \frac{\frac{AR}{RCs+1}}{\frac{s + s^2 T_d R}{RCs+1}}$$

$$|e_{ss}| = \frac{AR}{0} = \infty$$

$$\boxed{|e_{ss}| = \infty}$$

* The disadvantage of this controller is it can't respond to sudden error.
* It is also called as anticipatary controller because it sends a control signal in anticipation of error.
* This anticipatry nature may result in large instablity in the system.

* Composite controller mode →

(1) P+I mode →

$$P = k_p e + \frac{k_p}{T_i}\int e\,dt$$

$$P(s) = \left[k_p\left(1 + \frac{1}{T_i s}\right)\right] E(s)$$

$$E(s) \longrightarrow \boxed{k_p\left(1+\frac{1}{T_i s}\right)} \longrightarrow P(s)$$

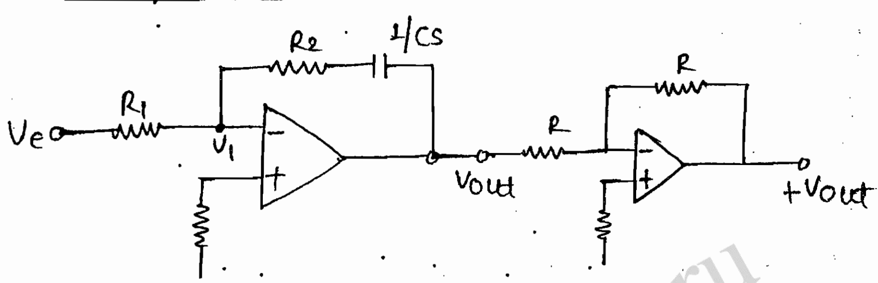
Effect on transient state

Let $e = \sin\omega t$

$$P = k_p \sin\omega t + \frac{k_p}{T_i}\int \sin\omega t\,dt$$

$$P = k_p \sin\omega t + \left(\frac{-k_p}{\omega T_i}\right)\cos\omega t$$

$$P = \sqrt{k_p^2 + \left(\frac{k_p}{\omega T_i}\right)^2}\,\sin\left(\omega t - \tan^{-1}\frac{1}{\omega T_i}\right)$$



$$\frac{V_e - V_1}{R_1} = \frac{V_1 - V_{out}}{\frac{R_2 CS + 1}{CS}}$$

$\because V_1 = 0$

$$\frac{V_e(R_2CS+1)}{R_1CS} = -V_{out}$$

$$-V_{out} = \frac{V_e R_2 \cancel{CS}}{R_1 \cancel{CS}} + \frac{V_e}{R_1 CS}$$

$$V_{out} = \frac{R_2}{R_1}V_e + \frac{R_2}{R_1}\frac{1}{R_2 C}\int V_e\,dt$$

$$\boxed{k_p = \frac{R_2}{R_1},\quad T_i = R_2 C}$$

* It is capable of improving steady state responce c/s of the sys. i.e. elimination of steady state error b/n i/p & o/p.
* The integral controller eliminates offset of proportional controller.
* It is also known as proportional + Reset controller because the rate of change of controller o/p can be reset by changing the value of reset time Ti.
* For sinusoidal i/p the phase of the controller o/p lags by $\tan^{-1}\left(\frac{1}{\omega T_i}\right)$. Hence it is similar to phase lag compensator.
* It terms of filtering property it acts as low pass filter.
* The P+I controller increases the type & order of system by one.

<u>effect on performance specification→</u>

1.) It increases rise time.

2.) It reduces BW.

3.) It reduces the stability of the system.

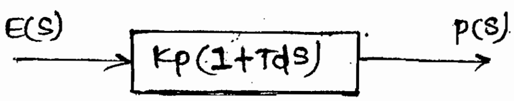
4.) It increase the damping ratio & hence reduces the peak overshoot.

5.) It eliminate steady state error.

<u>2.) P+D mode→</u>

$$P = K_p e + K_p T_d \frac{de}{dt}$$

$$P(s) = \left[K_p(1+T_d s)\right] E(s)$$



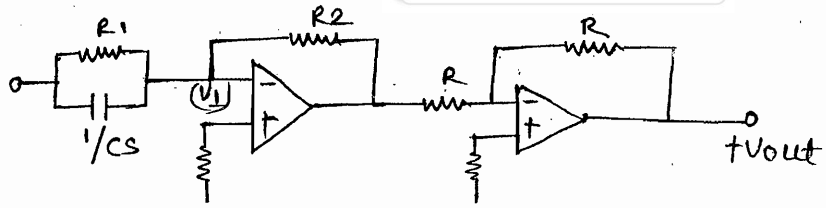
<u>effect on transient state</u>

Let $e = \sin\omega t$

$$P = K_p \sin\omega t + K_p T_d \frac{d}{dt} \sin\omega t$$

$$P = K_p \sin\omega t + \omega K_p T_d \cos\omega t$$

$$P = \sqrt{K_p^2 + (\omega K_p T_d)^2} \cdot \sin\left(\omega t + \tan^{-1}\omega T_d\right)$$



$$\frac{\dfrac{Ve - V_1}{\dfrac{R_1}{R_1CS+1}}} {} = \frac{V_1 - V_{out}}{R_2}$$

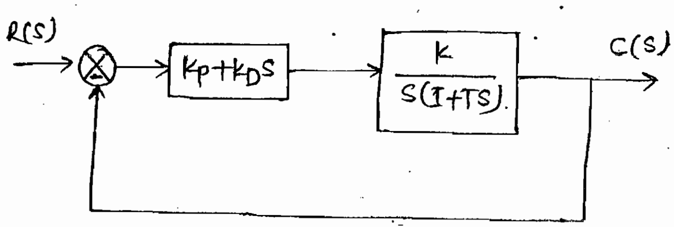
$$-V_{out} = \frac{Ve(R_1CS+1).R_2}{R_1}$$

$$\boxed{K_p = \frac{R_2}{R_1}\ ;\ T_d = R_1C}$$

$$-V_{out} = \frac{R_2}{R_1}Ve + \frac{R_2}{R_1}.R_1CSVe$$

$$+V_{out} = \frac{R_2}{V_1}Ve + \frac{R_2}{R_1}.R_1C.\frac{dVe}{dt}$$

Chapter(3.)
Conv.(4.)



(1.) Without P+D controller →

$$G(S) = \frac{K}{S(1+TS)}$$

Type-1 / order-2

with P+D controller

$$G(S) = \frac{K(K_p+K_DS)}{S(1+TS)}$$

Type-1 / order-2

(2.) with P-controller →

$$G(S) = \frac{KK_p}{S(1+TS)}$$

$$1 + \frac{KK_p}{S(1+TS)} = 0$$

$$S(1+TS) + KK_p = 0$$

$$TS^2 + S + KK_p = 0$$

$$S^2 + \frac{S}{T} + \frac{K.K_p}{T} = 0$$

$$\boxed{\omega_n = \sqrt{\frac{KK_p}{T}}\ r/s}$$

$$2\xi\sqrt{\frac{KK_p}{T}} = \frac{1}{T}$$

$$\boxed{\xi = \frac{1}{2\sqrt{KK_pT}}}$$

(III) With P+D controller →

$$G(s) = \frac{K(K_P + K_D s)}{s(1+Ts)}$$

$$1 + \frac{K(K_P + K_D s)}{s(1+Ts)} = 0$$

$$Ts^2 + s + K_D s + KK_P = 0$$

$$s^2 + \frac{s(1+K_D)}{T} + \frac{KK_P}{T} = 0$$

$$\boxed{\omega_n = \sqrt{\frac{KK_P}{T}}\ r/s}$$

~~$2\xi\omega_n$~~

$$2\xi\sqrt{\frac{KK_P}{T}} = \frac{1+KK_D}{T}$$

$$\boxed{\xi = \frac{1+KK_D}{2\sqrt{KK_P T}}}$$

(IV) ess| unit Ramp i/p →

$$e_{ss} = \lim_{s\to 0} \frac{s\cdot\frac{1}{s^2}}{1+\frac{K(K_P+K_D s)}{s(1+Ts)}}$$

$$\boxed{e_{ss} = \frac{1}{KK_P}}$$

* It is capable of improving the transient state c/s of the system only. i.e. it improves the speed of responce of sys.
* For sinusoidal i/p the phase of controller o/p leads by $\tan^{-1}\omega T_d$. Hence it is similar to phase lead compensator.
* In terms of filtering property it acts as HPF.
* The P+D controller does not affect the type & order of the system.

* effect on performance specification →

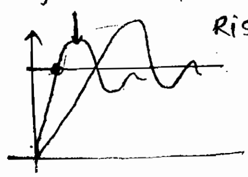
(1.) Reduces the Rise time.

(2.) Increases the BW.

(3.) It amplifies noise & hence reduces $\frac{S}{N}$ ratio.

(4.) It increases the stability of the sys.

(5.) It increases the damping ratio ξ & Hence reduces peak overshoot.



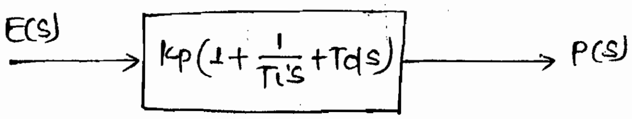
Rise time & peak overshoot ↓.

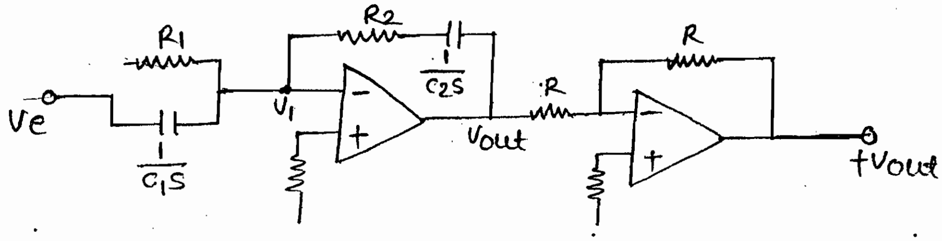
$\frac{14}{62}$ (d.) $\frac{15}{63}$ (c.) $\frac{16}{63}$ (c.) $\frac{18}{63}$ (c.) $\frac{8}{77}$ (b.) $\frac{9}{77}$ (a.)

(3.) PID mode →

$$p = k_p e + \frac{k_p}{T_i}\int e\,dt + k_p T_d \frac{de}{dt}$$

$$P(s) = \left[K_p\left(1 + \frac{1}{T_i s} + T_d s\right)\right] E(s)$$





$$\Rightarrow \frac{V_e - V_1}{\frac{R_1}{R_1C_1s+1}} = \frac{V_1 - V_{out}}{\frac{R_2C_2s+1}{C_2s}}$$

$$\Rightarrow -V_{out} = \frac{\cancel{V_e(R_1C_1}s+1)(R_2C_2s+1)}{R_1C_2s}$$

$$\Rightarrow -V_{out} = \frac{V_e(\cancel{R_1}C_1R_2\cancel{C_2}s^{\cancel{2}})}{\cancel{R_1}\cancel{C_2}\cancel{s}} + \frac{V_e\cancel{s}(R_1C_1+R_2C_2)}{R_1C_2\cancel{s}} + \frac{V_e}{R_1C_2s}$$

$$\Rightarrow -V_{out} = V_e\left[\frac{R_1C_1}{R_1C_2} + \frac{R_2C_2}{R_1C_2}\right] + \frac{V_e}{R_1C_2s} + R_2C_1sV_e$$

$$\Rightarrow +V_{out} = \frac{R_2}{R_1}V_e + \frac{R_2}{R_1}\cdot\frac{1}{R_2C_2}\int V_e\,dt + \frac{R_2}{R_1}\cdot R_1C_1\frac{dV_e}{dt}$$

$$\boxed{K_p = \frac{R_2}{R_1}, \quad T_i = R_2C_2, \quad T_d = R_1C_1}$$

* It improves both transient state & steady state responce c/s.
* It is similar to lag lead compensator.
* In terms of filtering property it acts as band reject filter.

*** Effect on performance specification →**

(1.) It reduces rise time.
2.) It increases BW.
3.) It amplifies noise & Hence reduces S/N ratio.
4.) It increases stability of the sys.
5.) It increases damping ratio ξ & Hence reduces peak overshoot.
6.) It eliminates steady state error b/n i/p & o/p.
7.) The PID controller increases type & order of sys. by 1.

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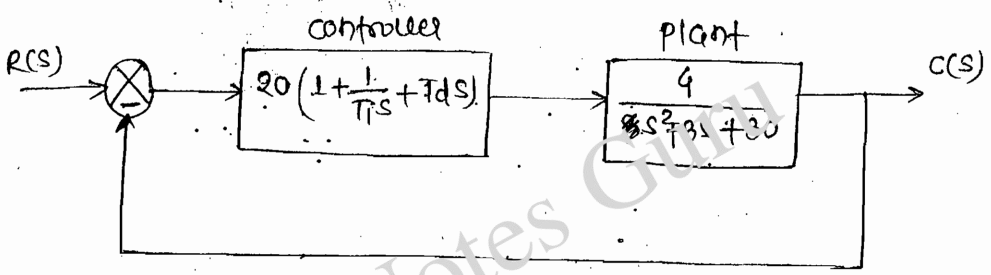
$$\theta_c(s) = \left\{20\left[1 + \frac{1}{T_i s} + T_d s\right]\right\} E(s)$$



(a.) $G(s) = \dfrac{4\times 20(1+T_d s)}{s^2+8s+80}$

($T_i s \to 0$ because given)

$1 + G(s) = 0$

$1 + \dfrac{80(1+T_d s)}{s^2+8s+80} = 0$

$s^2+8s+80+80(1+T_d s) = 0$

$s^2+s(8+80T_d)+160 = 0$

$\omega_n = \sqrt{160} = 12.64$ r/s

$2\xi \times 12.64 = 8+80T_d$

$\xi = 1$ (given)

$2\times 1\times 12.64 = 8+80T_d$

$\boxed{T_d = 0.2s}$

(b.) $G(s) = \dfrac{20(T_i s+20+4T_i s^2)\,4}{T_i s(s^2+8s+40)}$

$1+G(s) = 0$

$T_i s^3+8T_i s^2+80T_i s+80T_i s+80+16T_i s^2 = 0$

$T_i s^3+24T_i s^2+160T_i s+80 = 0$

$s^3+24s^2+160s+\dfrac{80}{T_i} = 0$

$\dfrac{80}{T_i} = 24\times 160$

$\boxed{T_i = 0.02s}$

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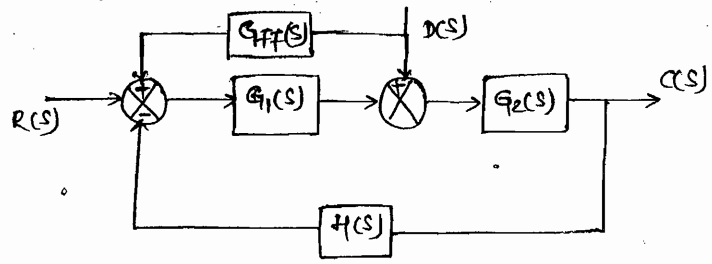
* Feed Forward Compensation →



$$\left.\frac{C(s)}{R(s)}\right|_{D(s)=0} = \frac{G_1(s)\cdot G_2(s)}{1+G_1(s)\cdot G_2(s)\cdot H(s)}$$

$$\left.\frac{C(s)}{D(s)}\right|_{R(s)=0} \quad \frac{G_2(s)+G_{ff}(s)\cdot G_1(s)\cdot G_2(s)}{1+G_1(s)\cdot G_2(s)\,H(s)}$$

$$C(s) = \frac{R(s)\cdot[G_1(s)\,G_2(s)] + D(s)\,[G_2(s)+G_{ff}(s)\,G_1(s)\,G_2(s)]}{1+G_1(s)\,G_2(s)\,H(s)}$$

To eliminate the effect of the disturbances in the sys., the condⁿ for feed forward controller is

$$\boxed{G_{ff}(s) = \frac{-1}{G_1(s)}}$$

Q (2) cha (2).

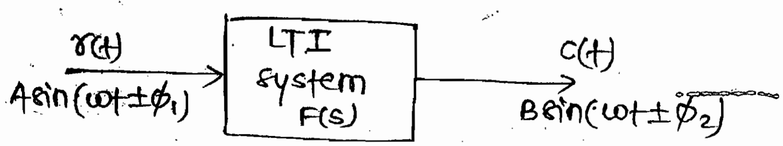
$$G_1(s) = \frac{k(s+a)(s+c)}{(s+b)(s+d)} \; ; \; G_2(s)=1$$

$$G_c(s) = \frac{1}{G_1(s)} = \frac{(s+b)(s+d)}{k(s+a)(s+c)}$$

$$\boxed{G_c(s) = \frac{(s+b)(s+d)}{k(s+a)(s+c)}}$$

Chapter-(04)
Freq. domain analysis

* When any sys. is subjected to sinusoidal i/p the o/p is also sinusoidal having diff magnitude & phase angle but same i/p freq. ω r/s.
* Freq. responce analysis emplies varying ω from $(0-\infty)$ & observing variation in magnitude & phase angle of the responce.



$$\boxed{\begin{aligned} B &= A\,|F(s)| \\ \phi_2 &= \pm\phi_1 + \angle F(s) \end{aligned}}$$

$$T.F. = F(s) = \frac{C(s)}{R(s)}$$

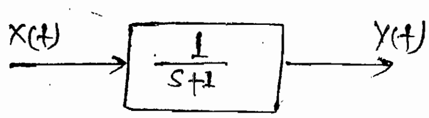
put $s = j\omega$

$$F(j\omega) = \text{sinusoidal TF}$$
$$\triangleq \text{sinusoidal Responce}$$

$$F(j\omega) = |F(j\omega)|\,\underline{\angle F(j\omega)} \longrightarrow \tan^{-1}\left(\frac{I.P.}{R.P.}\right)$$

$$|F(j\omega)| \to \sqrt{(R.P.)^2 + (I.P.)^2}$$

Ex:-
Chap.(06)
Q.(1)



$x(t) = \sin t$; find $y(t) = ?$

$$F(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}$$

$$F(j\omega) = \frac{1}{j\omega + 1} = \frac{1 + j0}{1 + j\omega}$$

$$|F(j\omega)| = \frac{\sqrt{1^2 + 0^2}}{\sqrt{1^2 + \omega^2}} = \frac{1}{\sqrt{1+\omega^2}}$$

$$\angle F(j\omega) = \frac{\tan^{-1}(0/1)}{\tan^{-1}(\frac{\omega}{1})} = -\tan^{-1}(\omega)$$

$$\therefore F(j\omega) = \frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1}(\omega)$$

Given; $\sin t$

$\Rightarrow \sin\omega t$; $\omega = 1$ r/s

$$F(j\omega) = \frac{1}{\sqrt{2}} \angle -45^\circ$$

$$\boxed{y(t) = \frac{1}{\sqrt{2}} \sin(t - 45^\circ)}$$

Ex:- Find $y(t)$ when $x(t) = 10\sin(t+30^\circ)$

$$\boxed{y(t) = \frac{10}{\sqrt{2}} \sin(t - 15^\circ)}$$

Q.(2.)

$$G(s) = \frac{(s^2+9)(s+2)}{(s+3)(s+4)(s+5)}$$

Soln →

$$G(j\omega) = \frac{(-\omega^2+9)(j\omega+2)}{(j\omega+3)(j\omega+4)(j\omega+5)} = 0$$

$\therefore \omega^2 = 9$, $\omega = 3$ r/s

* Freq. Responce plot →

(1.) Polar plots →

Absolute values of $|F(j\omega)|$ Vs ω

& $\angle F(j\omega)$ (degree)

(2.) Bode plot →

decibal values (db) of $|F(j\omega)|$ Vs $\log\omega$

$[20\log|F(j\omega)|]$

& $\angle F(j\omega)$ (degree)

Freq. responce analysis of 2nd order sys →

$$F(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$F(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\xi s}{\omega_n} + 1}$$

Put $(s = j\omega)$

$$F(j\omega) = \frac{1}{\frac{(j\omega)^2}{\omega_n^2} + \frac{2\xi(j\omega)}{\omega_n} + 1}$$

$$F(j\omega) = \frac{1}{-\frac{\omega^2}{\omega_n^2} + 1 + j2\xi\frac{\omega}{\omega_n}}$$

$$|F(j\omega)| = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

Its dB value;

$$-20\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}$$

Asymptotic approximation →

$$-20\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}$$

Case(1) Low freq:

$$1 >> \left(\frac{\omega}{\omega_n}\right)^2$$

$$-20\log\sqrt{1} = 0\,db$$

Case(2) High freq.

$$\left(\frac{\omega}{\omega_n}\right)^2 >> 1$$

$$-20\log\sqrt{\left[\left(\frac{\omega}{\omega_n}\right)^2\right]^2}$$

$$-40\log\left(\frac{\omega}{\omega_n}\right) \quad \text{---} \quad (i)$$

$$-40\log\omega + 40\log\omega_n$$

$$m \quad x \quad + \quad c$$

$$\boxed{\text{slope}(m) = -40\,\frac{db}{dec}}$$

Corner freq. (ω_{cf})

$$0 = -40\log\left(\frac{\omega}{\omega_n}\right)$$

$$\log\left(\frac{\omega}{\omega_n}\right) = 0$$

$$\left(\frac{\omega}{\omega_n}\right) = \log^{-1}(0) = 1$$

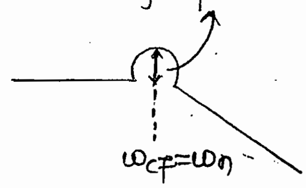
$$\omega = \omega_{cf} = \omega_n$$

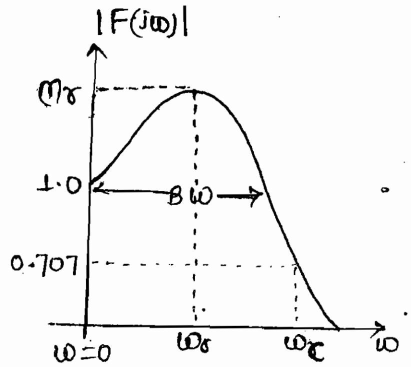
Error at ω_{cf} →

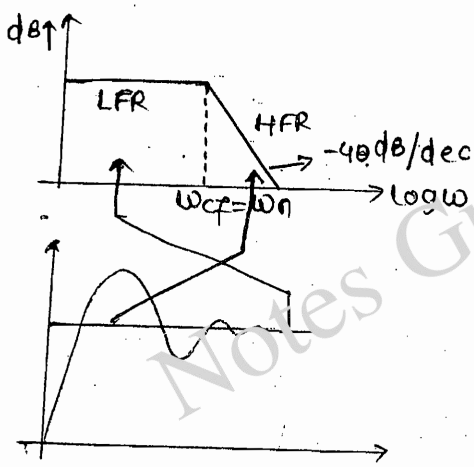
at $\omega = \omega_{cf} = \omega_n$

$$-20\log\sqrt{(1-1)^2 + (2\xi)^2}$$

$$-20\log 2\xi$$







* Freq. domain specification →

(1.) Resonant freq. (ω_r) → It is defined as the freq. at which the magnitude has max^m value.

$$|F(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}}$$

where; $u = \frac{\omega}{\omega_n}$

at $\omega = \omega_r$ = resonant freq. ; $u = u_r = \frac{\omega_r}{\omega_n}$

$$\frac{d}{du_r}\left[(1-u_r^2)^2 + (2\xi u_r)^2\right]^{-1/2} = 0$$

$$-\frac{1}{2}\left[(1-u_r^2)^2 + (2\xi u_r)^2\right]^{-3/2} \frac{d}{du_r}\left[(1-u_r^2)^2 + (2\xi u_r)^2\right] = 0.$$

$$2(1-u_r^2)(-2u_r) + 4\xi^2 2u_r = 0$$

$$(2-2u_r^2)(-2u_r) + 8\xi^2 u_r = 0$$

$$-4u_r + 4u_r^3 + 8\xi^2 u_r = 0$$

$$-1 + u_r^2 + 2\xi^2 = 0$$

$$u_r^2 = 1 - 2\xi^2$$

$$u_r = \sqrt{1-2\xi^2}$$

$$\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}\ r/s}$$

For 'ω_r' to be real & +ve

$$2\xi^2 < 1 \Rightarrow \boxed{\xi < \frac{1}{\sqrt{2}}}$$

It is corelerated with $\omega_d = \omega_n\sqrt{1-\xi^2}\ r/s$

For 'ω_d' to be real & +ve

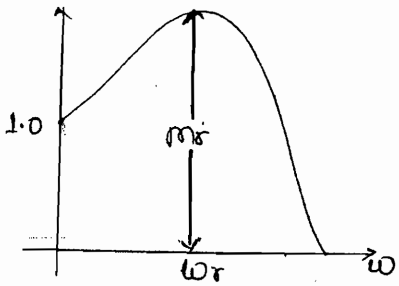
$$\xi^2 < 1 \Rightarrow \boxed{\xi < 1}$$

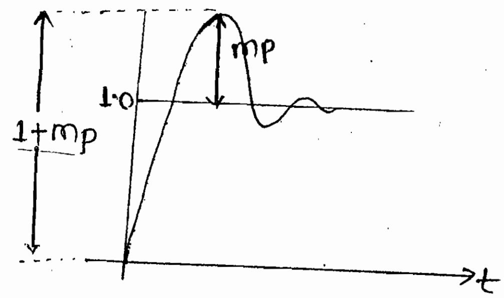
2.) Resonant peak (OR) peak magnitude (M_r) → It is the max^m value of magnitude occuring at resonant freq. ω_r.

$$|F(j\omega)|_{\omega=\omega_r} = M_r$$

$$M_r = \frac{1}{\sqrt{\left[1-\left(\frac{\omega_n\sqrt{1-2\xi^2}}{\omega_n}\right)^2\right]^2 + \left(\frac{2\xi\omega_n\sqrt{1-2\xi^2}}{\omega_n}\right)^2}}$$

$$\boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$$





$\xi < \frac{1}{\sqrt{2}} \Rightarrow M_r > 1$

$\xi = \frac{1}{\sqrt{2}} \Rightarrow M_r = 1$

$\xi > \frac{1}{\sqrt{2}} \Rightarrow$ No Mr

Q.10
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$$M(j\omega) = \frac{10}{100-\omega^2+10\sqrt{2}j\omega} = \frac{1}{1-\frac{\omega^2}{100}+\frac{10\sqrt{2}j\omega}{100}}$$

$$\frac{1}{1-\left(\frac{\omega}{10}\right)^2+j\frac{\sqrt{2}\omega}{10}} = \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+2j\xi\frac{\omega}{\omega_n}}$$

$$2\xi\frac{\omega}{\omega_n} = \frac{\sqrt{2}\omega}{10} \quad ; \quad \boxed{M_r = 1}$$

(3.) Band width (BW) → It is the range of freq. over which the magnitude has a value of $1/\sqrt{2}$.

* It indicates the speed of responce of the sys.
* wider BW implies faster responce.

$$\boxed{BW \propto \frac{1}{t_r}}$$

where; tr = Rise time

(4.) Cut off freq → It is the freq. at which the magnitude has a value of $\frac{1}{\sqrt{2}}$

* It indicates the ability of a sys. to distinguish signal from noise.

$$|F(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2+(2\xi u)^2}}$$

where; $u = \frac{\omega}{\omega_n}$

at $\omega = \omega_c$ = cutoff freq.

$$u = u_c = \frac{\omega_c}{\omega_n}$$

$$\frac{1}{\sqrt{(1-u_c^2)^2+(2\xi u_c)^2}} = \frac{1}{\sqrt{2}}$$

$$(1-u_c^2)^2+(2\xi u_c)^2 = 2$$

$$u_c^4 + 1 - 2u_c^2 + 4\xi^2 u_c^2 - 2 = 0$$

$$\Rightarrow u_c^4 + u_c^2(4\xi^2-2) - 1 = 0$$

$$\Rightarrow \frac{-(4\xi^2-2) \pm \sqrt{(4\xi^2-2)^2+4}}{2}$$

$$\Rightarrow 1-2\xi^2 \pm \frac{\sqrt{16\xi^4-16\xi^2+8}}{2}$$

$$= 1-2\xi^2 \pm \sqrt{4\xi^4-4\xi^2+2}$$

$$u_c^2 = 1-2\xi^2 + \sqrt{4\xi^4-4\xi^2+2}$$

$$u_c = \sqrt{1-2\xi^2+\sqrt{4\xi^4-4\xi^2+2}}$$

$$\boxed{\omega_c \text{ (or) } BW = \omega_n\sqrt{1-2\xi^2+\sqrt{4\xi^4-4\xi^2+2}} \text{ r/s}}$$

linear approximation

$$\omega_c \text{ (or) } BW = \omega_n(-1.19\xi + 1.85)$$

* Freq. Responce analysis of dead time (or) transportation lag →

$$F(s) = \frac{Y(s)}{X(s)} = e^{-Ts}$$

$$F(j\omega) = e^{-j\omega T} = \cos\omega T - j\sin\omega T$$

$$|F(j\omega)| = \sqrt{\cos^2\omega T + \sin^2\omega T} = 1$$

$$\angle F(j\omega) = \tan^{-1}\left(\frac{-\sin\omega T}{\cos\omega T}\right)$$

$$= \tan^{-1}(-\tan\omega T)$$

$$= -\omega T \text{ (radians)}$$

$$\boxed{e^{-j\omega T} = 1 \angle -\omega T \text{ (radians)}}$$

π rad $= 180^\circ$

$-\omega T$ rad $= ?$

$$\frac{-\omega T}{\pi} \times 180^\circ = -57.3\,\omega T \text{ (degree)}$$

$$\boxed{e^{-j\omega T} = 1 \angle -57.3\,\omega T \text{ (degree)}}$$

* Stability from freq. responce plot →

$$1 + G(s)H(s) = 0$$

$$G(s) \cdot H(s) = -1$$

Put $(s = j\omega)$

$$G(j\omega) \cdot H(j\omega) = -1 + j0 \quad \text{(critical point)}$$

Stability criteria →

(1.) Gain cross over freq. (ω_{gc})

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 1 \text{ (or) } 0\,db$$

(2.) Phase cross over freq. (ω_{pc})

$$\angle G(j\omega) \cdot H(j\omega)|_{\omega=\omega_{pc}} = -180^\circ$$

(3.) Gain margin (Gm):- It is the "allowable Gain"

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = X \quad ; \quad Gm(db) = 20\log\left(\frac{1}{X}\right)$$

$$\cancel{Gm = \frac{1}{X}}$$

(4) Phase margin (pm) → It is the "Allowable phase lag"

$$\angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}} = \phi^\circ \; ; \quad pm \Rightarrow 180+\phi^\circ$$

Stable ⇒ gm & pm = +ve ⇒ $\omega_{gc} < \omega_{pc}$
Unstable ⇒ gm & pm = −ve ⇒ $\omega_{gc} > \omega_{pc}$
marignaly stable ⇒ Gm = Pm = 0 ; $\omega_{gc} = \omega_{pc}$

* GM & pm for 2nd order sys →

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s} = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

$$G(j\omega) = \frac{\omega_n^2}{j\omega(j\omega+2\xi\omega_n)} = \frac{\omega_n^2+j0}{(0+j\omega)(j\omega+2\xi\omega_n)}$$

$$|G(j\omega)| = \frac{\omega_n^2}{\omega\sqrt{\omega^2+4\xi^2\omega_n^2}}$$

$$\angle G(j\omega) = \frac{0^\circ}{(90^\circ)\left(\tan^{-1}\frac{\omega}{2\xi\omega_n}\right)}$$

$$= -90^\circ - \tan^{-1}\left(\frac{\omega}{2\xi\omega_n}\right)$$

$$G(j\omega) = \frac{\omega_n^2+j0}{-\omega^2+2\xi\omega\cdot\omega_n j}$$

$$\angle G(j\omega) = \frac{0^\circ}{180-\tan^{-1}\left(\frac{2\xi\omega_n}{\omega}\right)}$$

$$= -180^\circ + \tan^{-1}\left(\frac{2\xi\omega_n}{\omega}\right)$$

at $\omega = \omega_{pc} = \infty$ r/s.

$$\angle G(j\omega) = -180^\circ$$

$$|G(j\omega)|_{\omega=\omega_{pc}=\infty} = X$$

$$X = 0 \Rightarrow Gm = \frac{1}{0} = \infty$$

$$\boxed{\begin{matrix}\omega_{pc} = \infty \\ Gm = \infty\end{matrix}}$$

at $\omega = \omega_{gc}$

$$\frac{\omega_n^2}{\omega\sqrt{\omega^2 + 4\xi^2\omega_n^2}} = 1$$

$$\omega_n^4 = \omega^2(\omega^2 + 4\xi^2\omega_n^2)$$

$$\omega^4 + \omega^2 4\xi^2\omega_n - \omega_n^4 = 0$$

$$\omega^2 = \frac{-4\xi^2\omega_n^2 \pm \sqrt{16\xi^4\omega_n^4 + 4\omega_n^4}}{2}$$

$$= -2\xi^2\omega_n^2 \pm \omega_n^2\sqrt{4\xi^4+1}$$

$$\omega^2 = -2\xi^2\omega_n^2 + \omega_n^2\sqrt{4\xi^4+1}$$

$$\boxed{\omega = \omega_{gc} = \omega_n\sqrt{-2\xi^2 + \sqrt{4\xi^4+1}} \ \ r/s}$$

$$\angle G(j\omega)\big|_{\omega=\omega_{gc}} = \phi = -90^\circ - \tan^{-1}\left(\frac{\omega_n\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}{2\xi\omega_n}\right)$$

$$pm = 180^\circ + \phi$$

$$\boxed{pm = 90^\circ - \tan^{-1}\left[\frac{\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}{2\xi}\right]}$$

$$\angle G(j\omega)\big|_{\omega=\omega_{gc}} = \phi = -180^\circ + \tan^{-1}\left[\frac{2\xi\omega_n}{\omega_n\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}\right]$$

$$pm = 180^\circ + \phi$$

$$\boxed{pm = \tan^{-1}\left[\frac{2\xi}{\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}\right]}$$

linear approximation

$$\boxed{pm = 100\xi}$$

3/70

$\% mp = 50\%$

$mp = 0.5$

$e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.5$

$\xi = 0.215$

Period of oscillations = 0.2s.

$T = \frac{1}{fd} = 0.2s$

$fd = 5Hz$

$\omega_d = 2\pi fd$

$\omega_d = 2\pi \times 5 = 31.41\ r/s$

$\omega_n\sqrt{1-\xi^2} = 31.41$

$\omega_n\sqrt{1-(0.215)^2} = 31.41$

$\omega_n = 32.16\ r/s$

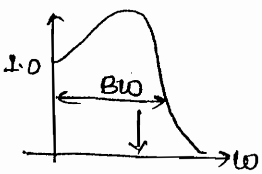
$\omega_r = \omega_n\sqrt{1-2\xi^2}$

$\omega_r = 32.16\sqrt{1-2(0.215)^2}$

$\boxed{\omega_r = 30.63}\ r/s.$

4/70 (c) 11/71 (a.)

4/70



Sharp cut off c/s ⇒ less BW ⇒ more t_r → less stable

BW↓, Mr↑, stability↓, t_r↑

5/70

$1 + \frac{100}{s(s+10)} = 0$

$s^2 + 10s + 100 = 0$

$\omega_n = 10\ r/s$

$2\xi \times 10 = 10$

$\xi = 0.5$

$\omega_r = 10\sqrt{10 - 2\times 0.5^2} = 7.07\ r/s$

$BW = \omega_n(-1.19\xi + 1.85)$

$= 10(-1.19\xi \times 0.5 + 1.85)$

$\boxed{BW = 12.7\ r/s}$

Conv(2) 74

$1 + \frac{K}{s(s+a)} = 0$

$s^2 + as + K = 0$

$K = \omega_n^2$

$a = 2\xi\omega_n$

Given;

$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.04$

$\xi = 0.6\ ;\ \cancel{0.8}$

$\omega_r = \omega_n\sqrt{1-2(0.6)^2} = 11.55$

$\omega_n = 22.2\ r/s$

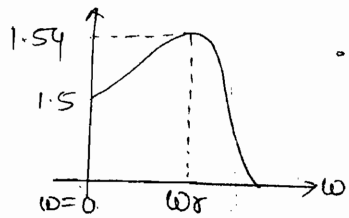
$K = (22.2)^2 = 492.84$

$a = 2\times 0.6\times 22.2 = 26.4$

Que.→ The TF of 2nd order sys. is

$$\frac{k\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Its freq. responce is shown in fig.



The value of 'k' is (a) 1 (b) 1.54 (c) 1.5 (d) 1.04

Soln→

$$TF = \frac{k\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$= \frac{k}{\frac{s^2}{\omega_n^2}+\frac{2\xi s}{\omega_n}+1}$$

(put $s=j\omega$);

$$TF = \frac{k}{1-\left(\frac{\omega}{\omega_n}\right)^2+j2\xi\frac{\omega}{\omega_n}}$$

$$= \frac{k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\boxed{\begin{array}{l} \text{at } \omega=0; \\ |F(j\omega)| = k = 1.5 \end{array}}$$

6/70

$$G(s) = \frac{2\sqrt{3}}{s(s+1)}$$

$$1+\frac{2\sqrt{3}}{s(s+1)}=0$$

$$s^2+s+3.46=0$$

$$\omega_n=\sqrt{3.46}=1.86 \text{ r/s}$$

$$2\xi\times1.86=1$$

$$\xi=0.27$$

$$PM=100\xi=100\times0.27=27^\circ \simeq 30^\circ$$

7/70

$$G(s)=\frac{as+1}{s^2}$$

$$1+\frac{as+1}{s^2}=0$$

$$s^2+as+1=0$$

$$\omega_n=1 \text{ r/s}$$

$$2\xi\times1=a$$

Given; $PM=45^\circ$, $\xi=\frac{PM}{100}=\frac{45}{100}=0.45$

$$\therefore a=2\times0.45=0.9$$

ans(c)

8/70

$$G(s) = \frac{1}{s(s^2+s+1)}$$

$$G(j\omega) = \frac{1}{j\omega(j^2\omega^2+j\omega+1)}$$

$$G(j\omega) = \frac{1}{j\omega(1-\omega^2+j\omega)}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{(1-\omega^2)^2+\omega^2}}$$

$$\underline{|G(j\omega)} = -90^\circ - \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right)$$

at $\omega = \omega_{pc}$

$$-90 - \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) = -180^\circ$$

$$-\tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) = -90^\circ$$

$$1-\omega^2 = 0$$

$$\omega = \omega_{pc} = 1\, r/s$$

$$|G(j\omega)|_{\omega=\omega_{pc}=1} = X$$

$$X = \frac{1}{1\sqrt{(1-1)^2+1^2}} = 1$$

$$Gm = \frac{1}{X} = \frac{1}{1} = 1$$

$$Gm(db) = 20\log 1 = 0\,db$$

$$\boxed{Gm(db) = 0\,db}$$

9/71

$$G(s) = \frac{3e^{-2s}}{s(s+2)}$$

$$G(j\omega) = \frac{3e^{-2j\omega}}{j\omega(j\omega+2)}$$

$$|G(j\omega)| = \frac{3\times 1}{\omega\sqrt{\omega^2+4}}$$

$$\underline{|G(j\omega)} = \frac{0^\circ(-57.3^\circ\times 2\omega)}{(90^\circ)(\tan^{-1}0.5\omega)}$$

$$= (-90^\circ) - 114.6\omega - \tan^{-1}(0.5\omega)$$

at $\omega = \omega_{gc}$

$$\frac{3}{\omega\sqrt{\omega^2+4}} = 1$$

$$9 = \omega^2(\omega^2+4)$$

$$\omega^4 + 4\omega^2 - 9 = 0$$

$$\omega^2 = \frac{-4 \pm \sqrt{16+36}}{2}$$

$$\omega^2 = -2 \pm 3.6$$

$$\omega^2 = 1.6, -5.6$$

$$\omega^2 = 1.6$$

$$\omega_{gc} = \sqrt{1.6} = 1.26\, r/s.$$

$$\underline{|G(j\omega)}|_{\omega=\omega_{gc}=1.26\, r/s} = \phi$$

$$\phi = -90^\circ - 114.6\times 1.26 - \tan^{-1}0.5\times 1.26$$

$$\phi = -267.5^\circ$$

$$Pm = 180^\circ - 267.5^\circ$$

$$Pm = -87.5^\circ$$

By observation
$\omega_{pc} < \omega_{gc}$
$Gm = Pm = -ve$

12/71

$$G(s) = \frac{s+5}{s^2+4s+9}$$

$$G(j\omega) = \frac{j\omega+5}{9-\omega^2+4j\omega}$$

$$\angle G(j\omega)\Big|_{\omega<3} = \frac{\tan^{-1}(\omega/5)}{\tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)} = \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)$$

$$\angle G(j\omega)\Big|_{\omega>3} = \frac{\tan^{-1}\omega/5}{180^\circ - \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)} = \tan^{-1}\left(\frac{\omega}{5}\right) - 180^\circ + \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)$$

$\omega = 0$, $\angle G(j\omega) = 0^\circ$

$\omega = \infty$; $\angle G(j\omega) = 90^\circ - 180^\circ + 0^\circ = -90^\circ$

$$\boxed{0^\circ \longrightarrow 90^\circ}$$

16/72

correction

$\cancel{X}$ $6 \rightarrow 8$ ans.(a)

20/73

$$G(s) = \frac{e^{-Ts}}{s(s+1)}$$

$$G(j\omega) = \frac{e^{-j\omega T}}{j\omega(j\omega+1)}$$

$$\angle G(j\omega) = \frac{-\omega T}{\left(\frac{\pi}{2}\right)\left(\tan^{-1}\omega\right)}$$

$$= -\frac{\pi}{2} - \omega T - \tan^{-1}\omega$$

At $\omega = \omega_1$

$\angle G(j\omega) = 0$

$-\frac{\pi}{2} - \omega_1 T - \tan^{-1}\omega_1 = 0$

$-\tan^{-1}\omega_1 = \frac{\pi}{2} + \omega_1 T$

$-\omega_1 = \tan\left(\frac{\pi}{2} + \omega_1 T\right)$

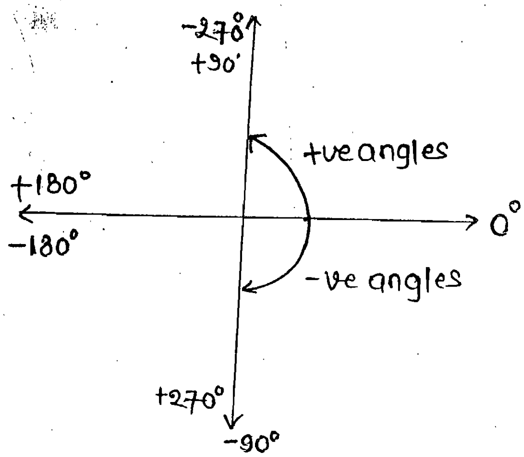
$-\omega_1 = -\cot(\omega_1 T)$

$$\boxed{\omega_1 = \cot(\omega_1 T)}$$

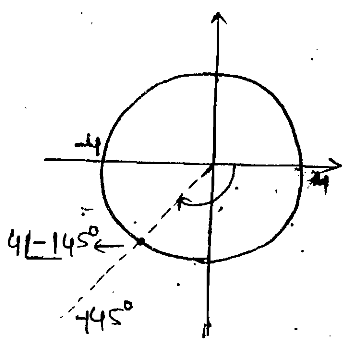
POLAR PLOT

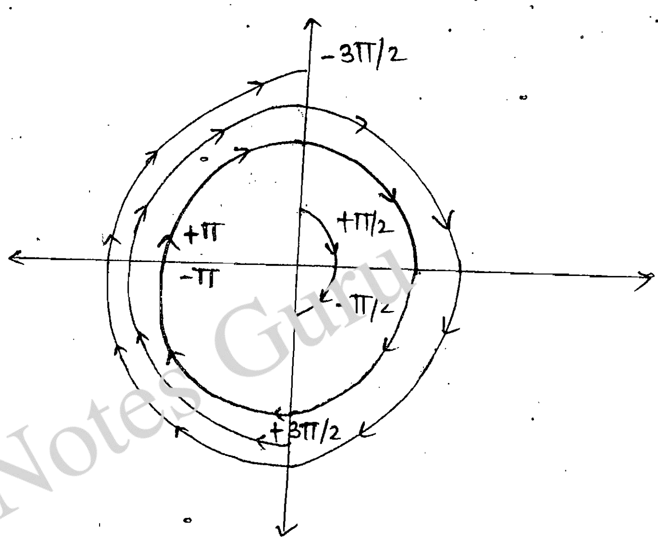
* It is a plot of absolute values of magnitude & phase angle in degrees of open loop TF $G(j\omega).H(j\omega)$ vs ω drawn on polar co-ordinates.

<u>Polar co-ordinates :-</u>



<u>Eg:-</u> $4\angle -45°$

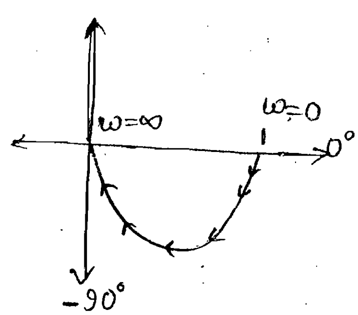


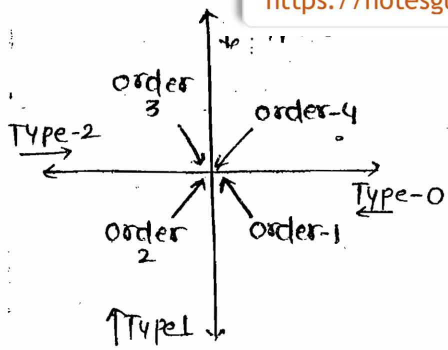


$$G(s) = \frac{1}{(s+1)} = \frac{1}{(1+s)} = \frac{1}{(1+j\omega)}$$

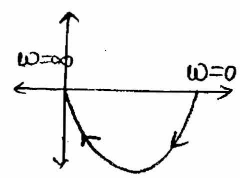
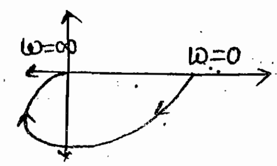
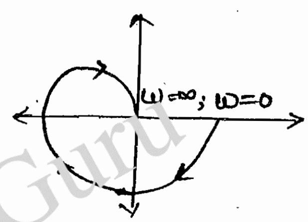
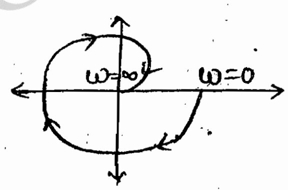
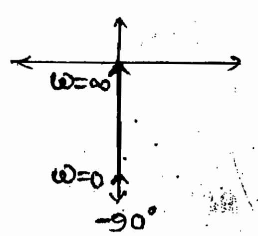
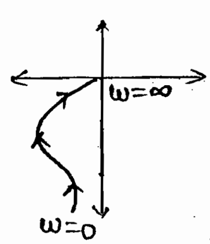
$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}} \; ; \; \angle G(j\omega) = -\tan^{-1}\omega$$

ω	0	∞
$\lvert G(j\omega)\rvert$	1	0
$\angle G(j\omega)$	0°	-90°



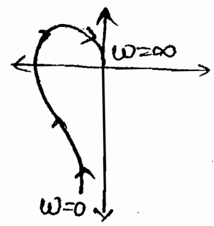


* General shapes of polar plot →

Type/order	Polar plot
(1.) Type-0/ order-1 $$G(s) = \frac{1}{(1+Ts)}$$	
(2.) Type-0/ order-2 $$G(s) = \frac{1}{(1+T_1s)(1+T_2s)}$$	
(3.) Type-0/order-3 $$G(s) = \frac{1}{(1+T_1s)(1+T_2s)(1+T_3s)}$$	
(4.) Type-0/ order-4 $$G(s) = \frac{1}{(1+T_1s)(1+T_2s)(1+T_3s)(1+T_4s)}$$	
(5) Type-1/ order-1 $$G(s) = \frac{1}{s} = \frac{1}{j\omega} = \frac{1}{\omega}\angle -90^\circ$$	
(6.) Type-1/ order-2 $$G(s) = \frac{1}{s(1+Ts)}$$	

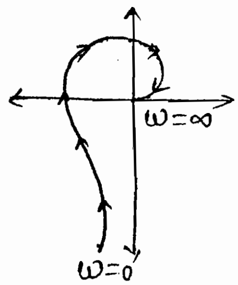
(7.) Type-1 / order-3

$$G(s) = \frac{1}{s(1+T_1 s)(1+T_2 s)}$$



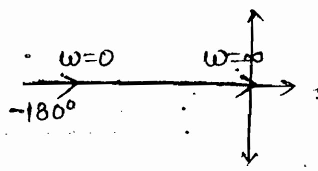
(8.) Type-1 / order-4

$$G(s) = \frac{1}{s(1+T_1 s)(1+T_2 s)(1+T_3 s)}$$



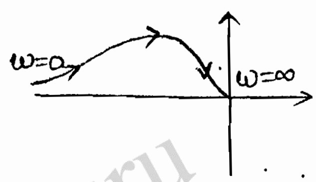
(9.) Type-2 / order-2

$$G(s) = \frac{1}{s^2} = \frac{1}{(j\omega)^2} = \frac{1}{\omega^2}\angle -180^\circ$$



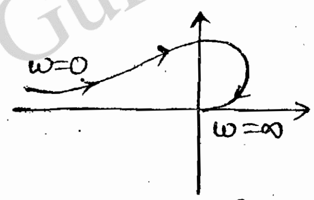
(10.) Type-2 / order-3

$$G(s) = \frac{1}{s^2(1+Ts)}$$



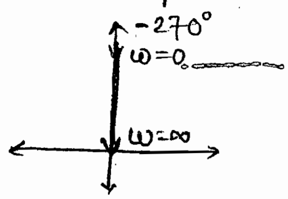
(11.) Type-2 / order-4

$$G(s) = \frac{1}{s^2(1+T_1 s)(1+T_2 s)}$$



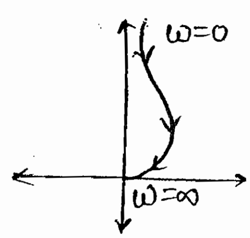
(12.) Type-3 / order-3

$$G(s) = \frac{1}{s^3} = \frac{1}{(j\omega)^3} = \frac{1}{\omega^3}\angle -270^\circ$$



(13.) Type-3 / order-4

$$G(s) = \frac{1}{s^3(1+Ts)}$$



DATE-23/11/14

*** Effect of adding zeros ~~on the~~ of the polar plot →** For min^m phase (or) Non-min^m phase f^n when zeros are added it is necessary to check the intersection of polar plot with −ve real axis b/n $\omega = 0$ & $\omega = \infty$.

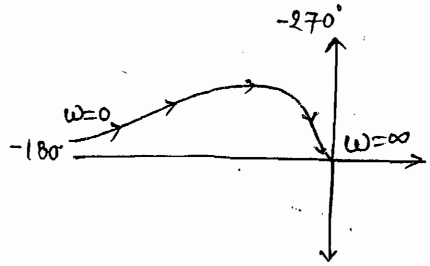
(4/75)

$$G(s) = \frac{1}{s^2(1+s)}$$

$$\angle G(j\omega) = -180^\circ - \tan^{-1}(\omega)$$

$\omega = 0$, $|G(j\omega)| = \infty$; $\angle G(j\omega) = -180^\circ$

$\omega = \infty$, $|G(j\omega)| = 0$; $\angle G(j\omega) = -270^\circ$

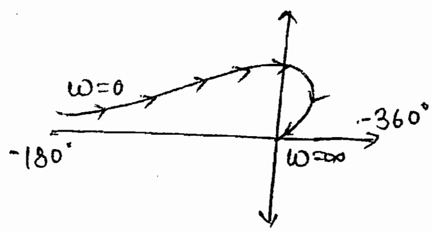


$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

$$\angle G(j\omega) = -180 - \tan^{-1}\omega - \tan^{-1}2\omega$$

$\omega = 0$; $|G(j\omega)| = \infty$; $\angle G(j\omega) = -180^\circ$

$\omega = \infty$; $|G(j\omega)| = 0$; $\angle G(j\omega) = -360^\circ$



$$G(s) = \frac{1+4s}{s^2(1+s)(1+2s)}$$

$$\angle G(j\omega) = -180 - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}4\omega$$

$\omega = 0$, $|G(j\omega)| = \infty$; $\angle G(j\omega) = -180^\circ$

$\omega = \infty$, $|G(j\omega)| = 0$; $\angle G(j\omega) = -270^\circ$

Because of present of one zero, the polar plot will cut the −ve real axis at some value. So for finding that value,

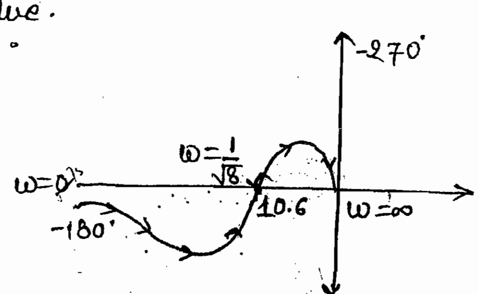
$$-180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}4\omega = -180^\circ$$

$$\tan^{-1}4\omega = \tan^{-1}\omega + \tan^{-1}2\omega$$

$$4\cancel{\omega} = \frac{\cancel{\omega} + 2\cancel{\omega}}{1 - 2\omega^2}$$

$$4 - 8\omega^2 = 3;$$

$$\omega = \omega_{pc} = \frac{1}{\sqrt{8}} \text{ r/s}$$



$$|G(j\omega)|_{\left(\omega = \omega_{pc} = \frac{1}{\sqrt{8}}\right)} = X$$

$$X = \frac{\sqrt{1 + \left(\frac{4}{\sqrt{8}}\right)^2}}{\sqrt{1 + \left(\frac{1}{\sqrt{8}}\right)^2}\sqrt{1 + \left(\frac{2}{\sqrt{8}}\right)^2}} = 10.6$$

Conv(4)
75

$$G(s) = \frac{s+2}{(s+1)(s-1)}$$

$$= \frac{2(1+0.5s)}{1(1+s)(-1)(1-s)}$$

$$G(s) = \frac{-2(1+0.5s)}{(1+s)(1-s)}$$

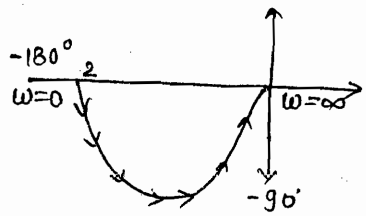
$$G(j\omega) = \frac{-2(1+0.5j\omega)}{(1+j\omega)(1-j\omega)}$$

$$|G(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{(1+\omega^2)}$$

$$\angle G(j\omega) = \frac{(-180^\circ)(\tan^{-1}0.5\omega)}{(\tan^{-1}\omega)(-\tan^{-1}\omega)}$$

$$= -180^\circ + \tan^{-1}0.5\omega$$

ω	0	∞
$\lvert G(j\omega)\rvert$	2	0
$\angle G(j\omega)$	-180°	-90°



Note:- -ve gain $(-K+j0)$ contributes -180° for all 'ω'

$$G(s) = \frac{(s+2)}{(s+1)(s-4)} = \frac{-0.5(1+0.5s)}{(1+s)(1-0.25s)}$$

$$G(j\omega) = \frac{-0.5(1+0.5j\omega)}{(1+j\omega)(1-0.25j\omega)}$$

$$|G(j\omega)| = \frac{0.5\sqrt{1+(0.5\omega)^2}}{\sqrt{1+\omega^2}\sqrt{1+(0.25\omega)^2}}$$

$$\angle G(j\omega) = \frac{(-180^\circ)(\tan^{-1}0.5\omega)}{(\tan^{-1}\omega)(-\tan^{-1}0.25\omega)}$$

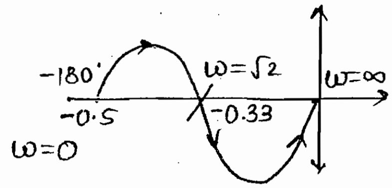
$$-180^\circ + \tan^{-1}0.5\omega + \tan^{-1}0.25\omega - \tan^{-1}\omega$$

ω	0	$\sqrt{2}$	∞
$\lvert G(j\omega)\rvert$	0.5	0.33	0
$\angle G(j\omega)$	-180°	-180°	-90°

$$-180^\circ + \tan^{-1}0.5\omega + \tan^{-1}0.25\omega - \tan^{-1}\omega = -180^\circ$$

$$\tan^{-1}\omega = \tan^{-1}0.5\omega + \tan^{-1}0.25\omega$$

$$\omega = \frac{0.5\omega + 0.25\omega}{1 - 0.125\omega}$$



$$1 - 0.125\omega^2 = 0.75$$

$$\omega = \omega_{pc} = \sqrt{2}\ r/s$$

$$X = \frac{0.5\sqrt{1+(0.5\times\sqrt{2})^2}}{\sqrt{1+(\sqrt{2})^2}\sqrt{1+(0.25\times\sqrt{2})^2}} = 0.33$$

* Find GM & PM from polar plots →

$GM = \frac{1}{X}$: $GM(db) = 20\log\left(\frac{1}{X}\right) = +ve$

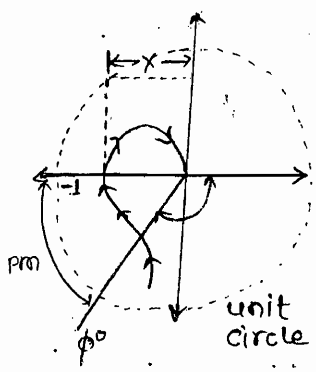
$PM = \phi - (-180°) = 180° + \phi = +ve$

$X = 1$; $GM = \frac{1}{X} = \frac{1}{1} = 1$, $GM(db) = 20\log 1 = 0\,db$

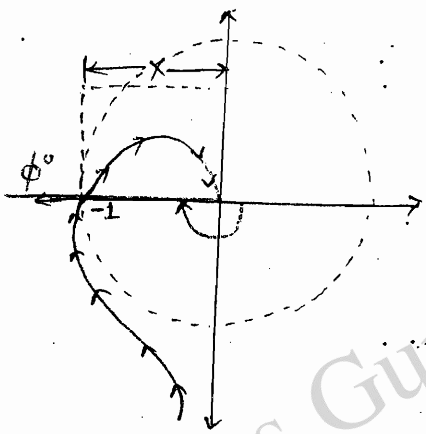
$\phi = -180°$; $PM = 180° - 180° = 0$

$GM = \frac{1}{X} = GM(db) = 20\log\left(\frac{1}{X}\right) = -ve$

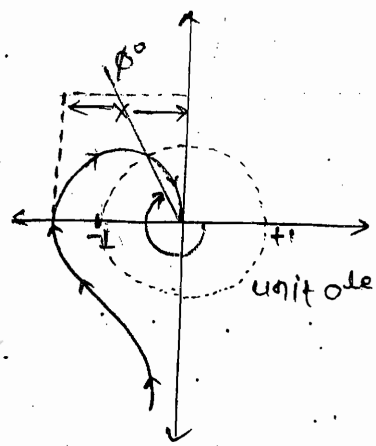
$PM = 180° + \phi = -ve$



stable

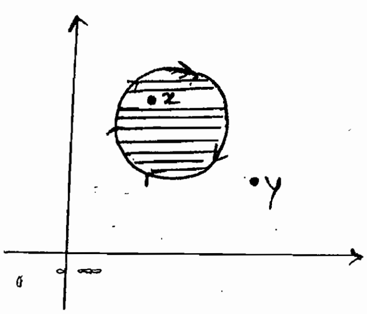


marginally stable

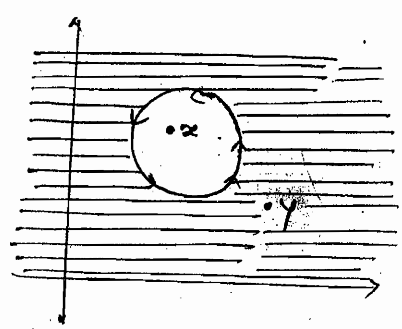


Unstable

Concept of enclosement & encirculement →



Fig(1)



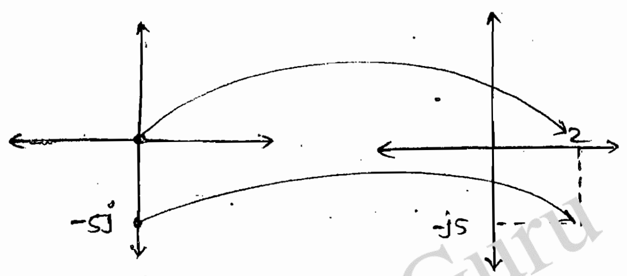
Fig(2)

* A point is said to be enclosed by a contour if it lies to the Right side of the dirn of contour.
* A point is said to be encircule if the contour is a closed path.
* In fig (2) point y is said to be enclosed where as point x is said to be
- only encircule in Acw dirn
* In polar plots if the critical point $-1+j0$ is not enclosed than the sys. is said to be stable.

* Theory of Nyquist plots →

* The mapping theorem states that every point of s-plane will get mapped onto a corresponding point in p(s) plane where p(s) is any fn of s.

Principle of mapping →

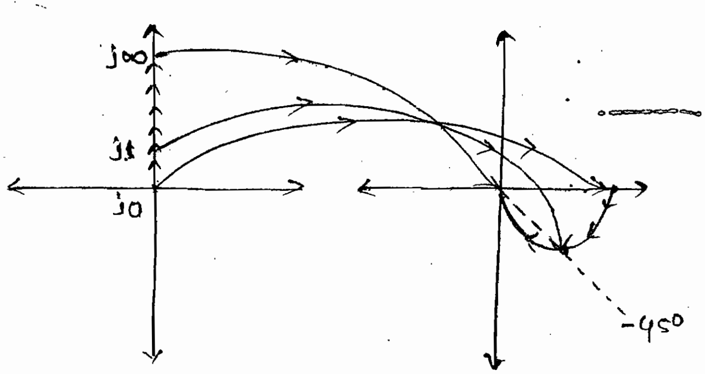


$P(S) = S+2$

$P(0) = 0+2 = 2$

$P(-jS) = 2-5j$

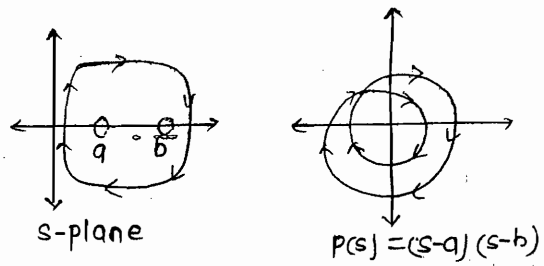
Eg:- Polar plot.



$P(S) = G(S).H(S) = \frac{1}{S+1} = \frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1}\omega$

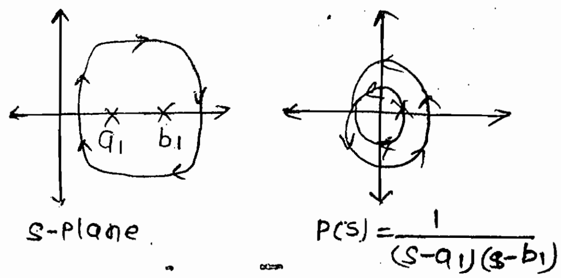
Principle of argument →

Case(i) →

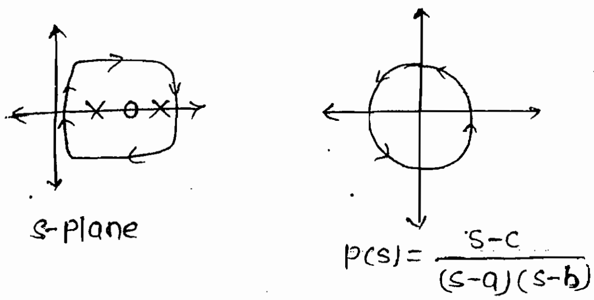


PA → Pole ACW

Case(ii) →



Case(iii) →



$$\boxed{N = P - Z}$$

where; N = no. of encirclement
= +ve (ACW dirn)
= −ve (CW dirn)
P = No. of poles in RHS of s-plane
Z = No. of zeros in RHS of s-plane.

* The principle of argument may be stated as if the s-plane closed contour encloses p poles & z- zeros (p>z) in RHS of s-plane then the origin of P(s) plane in encircled (p-z) times in ACW dirn.

* Nyquist stability criteria →

$$G(s)H(s) = \frac{K(s \pm Z_1)}{s(s \pm P_1)}$$

$$1 + G(s)H(s)$$

$$1 + \frac{K(s \pm Z_1)}{s(s \pm P_1)}$$

$$\frac{s(s \pm P_1) + K(s + Z_1) \longrightarrow \text{CL poles}}{s(s \pm P_1) \longrightarrow \text{OL poles}} \quad \text{------- (i)}$$

UC → Upper closed loop pole

LO → Lower open loop zeros

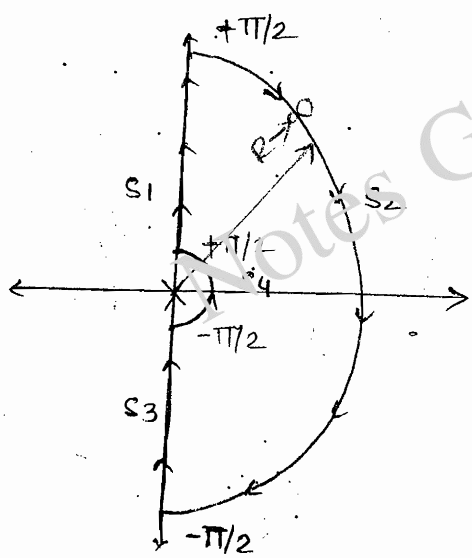
Applying $N = P - Z$ to eqn (i)

P = No. of OL poles in RHS of s-plane

Z = No. of CL poles in RHS of s-plane

(i) $Z = 0$, $N = P$

Nyquist Path →



To map S_1 ⇒ Polar plot

To map S_2 ⇒ put $s = \lim_{R \to \infty} Re^{j\theta}$

$$\left(\theta = +\frac{\pi}{2} \rightarrow -\frac{\pi}{2}\right)$$

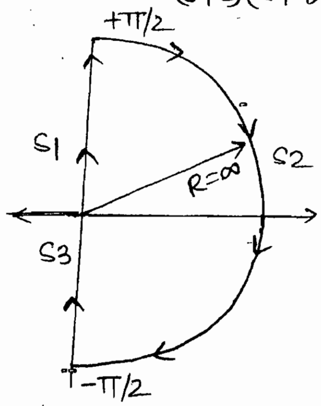
To map S_3 ⇒ Inverse polar plot

To map S_4 ⇒ put $s = \lim_{r \to 0} re^{j\theta}$

$$\left(\theta = -\frac{\pi}{2} \rightarrow +\frac{\pi}{2}\right)$$

Eg:- $G(s) = \frac{10}{(s+2)(s+4)}$

P = RHS pole
N = Incirclement

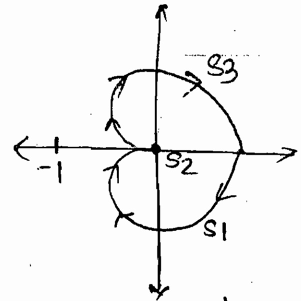


S-plane

$N = P - Z$

$0 = 0 - Z$

$\boxed{Z=0}$ Stable



$G(s) = \frac{10}{(s+2)(s+4)}$

To map $S_2 \rightarrow$

$$G(s) = \frac{10}{s^2} = \frac{10}{\lim_{R\to\infty} (Re^{j\theta})^2} = \frac{10}{\lim_{R\to\infty} R^2 e^{j2\theta}} = 0e^{2j\theta}$$

Que:- w/o constructing Nquist plot find the No. of encirclement about $-1+j0$?

Soln →

$$1 + \frac{10}{(s+2)(s+4)} = 0$$

$$s^2 + 6s + 18 = 0$$

s^2	1	18
s	6	0
s^0	18	0

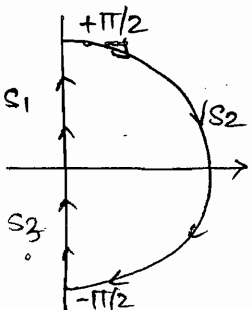
$N = P - Z$
$\downarrow$ $\downarrow$
0 0

$\boxed{N=0}$

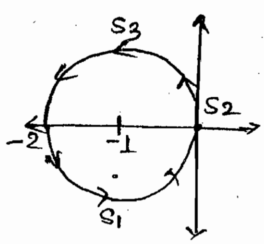
Z = sign change
P = open loop pole

Que:- $G(s) = \frac{(s+2)}{(s+1)(s-1)}$

Soln → for also this que section will 3, because of No. poles in origin.



S-plane



$G(s) = \frac{s+2}{(s+1)(s-1)}$

$N = P - Z$

$1 = 1 - Z$

$\boxed{Z=0}$

$$1 + \frac{s+2}{s^2-1} = 0$$

$$s^2 - 1 + s + 2 = 0$$

$$s^2 + s + 1 = 0$$

s^2	1	1
s^1	1	0
s^0	1	0

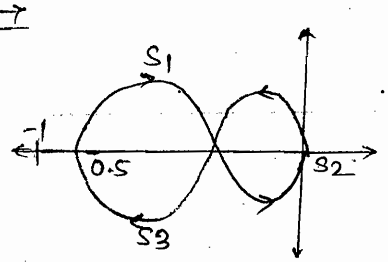
$N = P - Z$

$P \downarrow 1$, $Z \downarrow 0$

$\boxed{N = 1}$

Q.→ $G(s) = \frac{(s+2)}{(s+1)(s+4)}$

Sol^n→



$$1 + \frac{s+2}{(s+1)(s-4)} = 0$$

$$s^2 + 4s + s - 4 + s + 2 = 0$$

$$s^2 - 2s - 2 = 0$$

s^2	1	-2
s^1	-2	0
s^0	-2	0

$N = P - Z$

$N = 1 - 1$

$\boxed{N = 0}$

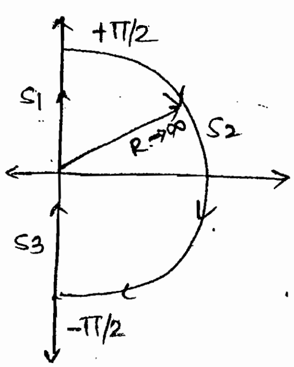
$N = P - Z$

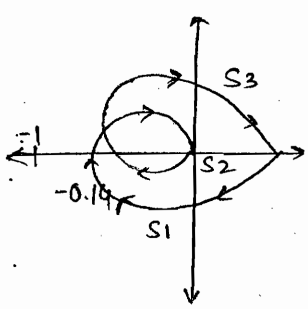
$0 = 1 - Z$

$\boxed{Z = 1}$ Unstable.

Q→ $G(s) = \frac{100}{(s+2)(s+4)(s+8)}$

Sol^n→





$$g(s) = \frac{100}{(s+2)(s+4)(s+8)}$$

$N = P - Z$

$0 \quad 0 - Z$

$\boxed{Z = 0}$ Stable

shortcut method →

$$\frac{100}{\cancel{(-\omega^2+6)j}} \qquad \frac{100}{(-\omega^2+6j\omega+8)(j\omega+8)}$$

$$\frac{100}{-j\omega^3-8\omega^2-6\omega^2+48j\omega+8j\omega+64}$$

$$\frac{100}{64-14\omega^2+j(56\omega-\omega^3)}$$

$$56\omega-\omega^3=0$$

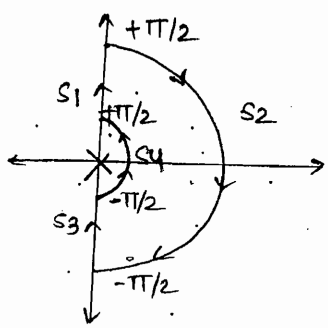
$$\omega^2=56$$

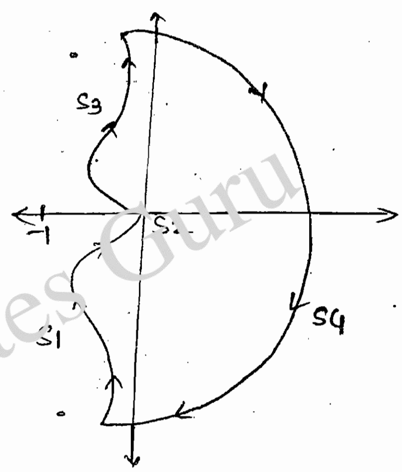
$$\omega=\omega_{pc}=\sqrt{56}\ r/s$$

$$X=\frac{100}{64-14(7.4)^2}=-0.14$$

Que→ $G(s)=\frac{10}{s(1+5s)}$

Soln→





$N=P-Z$

$0=0-Z$

$\boxed{Z=0}$ stable

$$G(s)=\frac{10}{s(1+5s)}$$

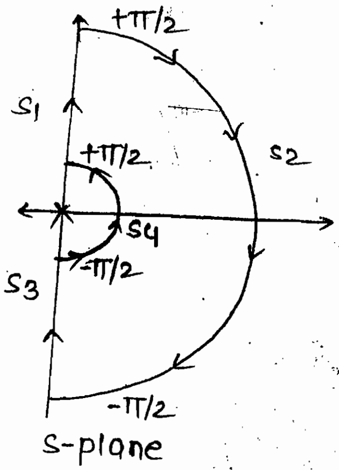
To map S4→

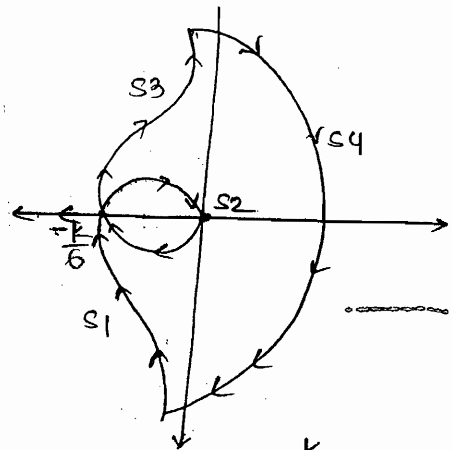
$$G(s)=\frac{10}{s(1)}=\frac{10}{\lim\limits_{r\to 0} re^{j\theta}}=\infty e^{-j\theta}\ \left(\theta=-\frac{\pi}{2}\to+\frac{\pi}{2}\right)$$

$$=\infty e^{j\pi/2}\to\infty e^{-j\pi/2}$$

Que:- $G(s) = \dfrac{K}{s(s+1)(s+2)}$

Sol^n →





$G(s) = \dfrac{K}{s(s+1)(s+2)}$

To map s_4 →

$$G(s) = \frac{K}{s(1)(2)} = \frac{0.5K}{\lim\limits_{r \to 0} re^{j\theta}} = \infty e^{-j\theta} \quad \left(\theta = -\frac{\pi}{2} \to +\frac{\pi}{2}\right)$$

$$= \infty e^{j\pi/2} \to \infty e^{-j\pi/2}$$

$$\frac{K}{j\omega(-\omega^2 + 3j\omega + 2)}$$

$$\frac{K}{-3\omega^2 + j(2\omega - \omega^3)}$$

$$2\omega - \omega^3 = 0$$

$$\omega = \omega_{pc} = \sqrt{2} \text{ r/s.}$$

$$\frac{K}{-3(\sqrt{2})^2} = \frac{-K}{6}$$

$$\boxed{\begin{aligned} X &= \frac{K}{6} \\ GM &= \frac{1}{X} = \frac{6}{K} \\ GM &\propto \frac{1}{K} \end{aligned}}$$

$\frac{-K}{6} = -1$; $K_{mar} = 6$

Case (1) $K > 6$

$-2 = 0 - Z$

$Z = 2$

Unstable

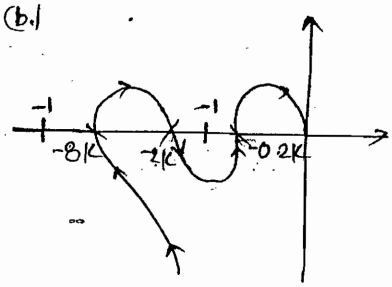
Case (2) $K < 6$

$0 = 0 - Z$

$Z = 0$

stable

22/74 (b.)



(1) $-0.2K=-1$

$K_{mar}=5$

$K<5 \rightarrow$ stable

(2.) $-2K=-1$

$K_{mar}=\frac{1}{2}$

$K>\frac{1}{2} \rightarrow$ stable

(3) $-8K=-1$

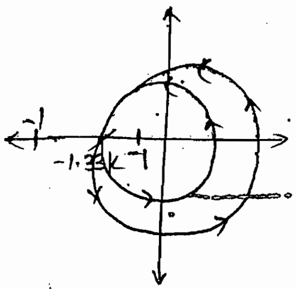
$K_{mar}=\frac{1}{8}$

$K<\frac{1}{8} \rightarrow$ stab

$$\boxed{\begin{array}{c}\frac{1}{2}<K<5 \\ \& \\ K<\frac{1}{8}\end{array}}$$

19/73

$$G(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)}$$



$-1.33K=-1$

$K_{mar}=\frac{1}{1.33}=(0.75)$

(i) $K<\frac{1}{1.33}$

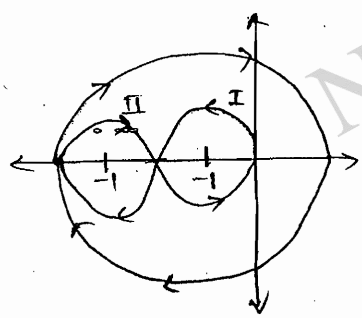
$0=2-Z$

$Z=2$ (Unstable)

(ii) $K>\frac{1}{1.33}$

$2=2-Z$; $Z=0$ (stable)

18/72



given $P=0$

Region (I)

$1-1=0-Z$

$Z=0$

Stable

Region (II)

$-2=0-Z$

$Z=2$

unstable

15/71

$$G(s)=\frac{1}{s(1+sT_1)(1+sT_2)}$$

$$G(j\omega)=\frac{1}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

$$\cancel{= \frac{1}{-\omega^2(}}$$

$$= \frac{1}{j\omega\left[1+j\omega(T_1-T_2)-\omega^2 T_1T_2\right]}$$

$$= \frac{1}{-\omega^2(T_1+T_2)+j(\omega-\omega^3 T_1T_2)}$$

$$\omega-\omega^3 T_1T_2=0$$

$$1-\omega^2 T_1T_2=0$$

$$\boxed{\omega=\omega_{pc}=\frac{1}{\sqrt{T_1T_2}}\ r/s}$$

$$\frac{1}{-\left(\frac{1}{\sqrt{T_1T_2}}\right)^2(T_1+T_2)}$$

$$= \frac{-T_1T_2}{T_1+T_2}$$

$$X = \frac{T_1T_2}{T_1+T_2}$$

$$\boxed{G_m = \frac{T_1+T_2}{T_1T_2}}$$

Date:-24/11/14

14/71 (a)

21/73 If the sys. is unstable then its pole will be RHS & its magnitude is not affected but the phase will be affected.

(A.) $K=K_1$ $\quad K\downarrow \Rightarrow \xi\uparrow,\ M_r\downarrow \longrightarrow 4$

(B.) $K=K_2$ $\quad K\uparrow \Rightarrow \xi\downarrow,\ M_r\uparrow \longrightarrow 3$

$K_2>K_1$

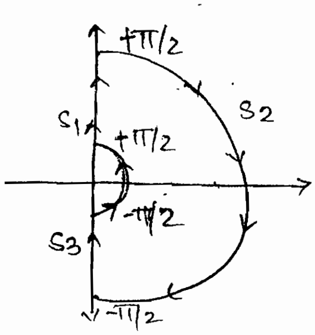
ans(a)

(C.) marginaly stable
(Responce is oscillatory $\Rightarrow \xi=0,\ M_r=\infty$) $\longrightarrow 2$

(d.) Unstable sys.
Phase c/s will not ~~tend to~~ tend to -270° $\longrightarrow 1$

Que.$\rightarrow$ $\dfrac{10}{S(S-5)}$

Soln$\rightarrow$



To map S1
Polar plot$\rightarrow$

$$G(s) = \frac{-2}{S(1-0.2S)}$$

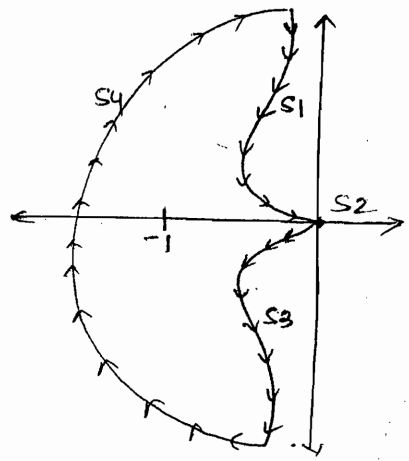
$$G(j\omega) = \frac{-2}{j\omega(1-0.2j\omega)}$$

$$|G(j\omega)| = \frac{2}{\omega\sqrt{1+(0.2\omega)^2}}$$

$$\underline{|G(j\omega)} = \frac{-180^\circ}{90(-\tan^{-1}0.2\omega)} = -180-90+\tan^{-1}0.2\omega = -270^\circ+\tan^{-1}0.2\omega$$

ω	0	∞
$\lvert G(j\omega)\rvert$	∞	0
$\underline{\lvert G(j\omega)}$	-270°	-180

To map S_4 →



$G(s) = \frac{10}{s(s-5)}$

$$G(s) = \frac{10}{s(-5)} = \frac{2}{s(-1)}$$

$$= \frac{2}{\lim_{r\to 0} re^{j\theta}e^{j\pi}} \quad (\because -1 = e^{j\pi})$$

$$= \infty e^{-j(\theta+\pi)}$$

$$\theta = -\frac{\pi}{2} \text{ to } +\frac{\pi}{2}$$

$$\infty e^{-j(-\frac{\pi}{2}+\pi)} = \infty e^{-j\pi/2}$$

$$\infty e^{-j(\pi/2+\pi)} = \infty e^{-j3\pi/2}$$

$$\infty e^{-j\pi/2} \longrightarrow \infty e^{-j3\pi/2}$$

$N = P - Z$

$-1 = 1 - Z$

$Z = 2$ (Unstable)

Que → $G(s) = \frac{s+2}{s^2(s-1)}$

Sol^n →

$$G(s) = \frac{s+2}{s^2(s-1)}$$

$$G(s) = \frac{2(1+0.5s)}{s^2(-1)(1-s)}$$

$$G(j\omega) = \frac{-2(1+0.5j\omega)}{(j\omega)^2(1-j\omega)}$$

$$|G(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{\omega^2\sqrt{1+\omega^2}}$$

$$\underline{|G(j\omega)} = \frac{-180(\tan^{-1}0.5\omega)}{(180)(-\tan^{-1}\omega)}$$

$$= -180-180+\tan^{-1}(0.5\omega)+\tan^{-1}(\omega)$$

	0	∞
$\lvert G(j\omega)\rvert$	∞	0
$\underline{\lvert G(j\omega)}$	-360	-180

Because of the presence of zero

$$-180-180+\tan^{-1}(0.5\omega)+\tan^{-1}\omega = -180$$

$$\tan^{-1}(0.2\omega)+\tan^{-1}\omega = 180$$

$$\tan^{-1}\left(\frac{0.2\omega+\omega}{1-0.2\omega^2}\right) = 180$$

$$0.2\omega+\omega = 0$$

To map $S_4 \rightarrow$

$$G(s) = \frac{2}{s^2(-1)}$$

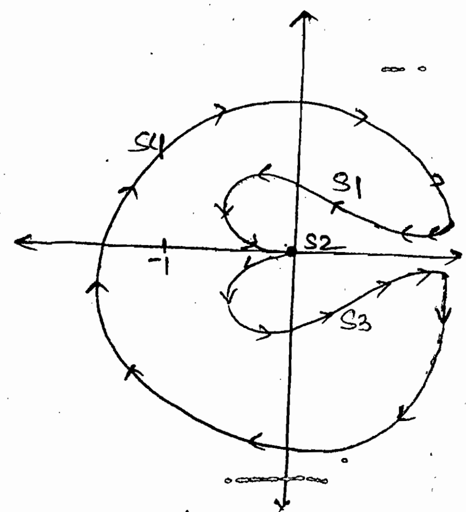
$$= \frac{2}{\lim\limits_{r \to 0} (re^{j\theta})^2 e^{j\pi}}$$

$$= \frac{2}{\lim\limits_{r \to 0} r^2 e^{j2\theta} e^{j\pi}}$$

$$\infty e^{-j(2\theta+\pi)}$$

$$\theta = -\frac{\pi}{2} \rightarrow +\frac{\pi}{2}$$

$$\infty e^{j0} \longrightarrow \infty e^{-j(2\pi)}$$



$N = P - Z$

$-1 = 1 - Z$ $\boxed{Z = 2}$ (unstable)

$N = -1$ (Because of acw dirn encirclement)

Check $\rightarrow$

$$1 + \frac{s+2}{s^2(s-1)} = 0$$

$$s^3 - s^2 + s + 2 = 0$$

s^3	1	0
s^2	-1	2
s^1	3	0
s^0	2	1

$N = P - Z$, $P \rightarrow 1$, $Z \rightarrow -2$

$\boxed{N = -1}$

Que $\rightarrow$ $G(s) = \dfrac{1}{s^2(1+s)(1+2s)}$

Soln $\rightarrow$ To map $S_4 \rightarrow$

$$G(s) = \frac{1}{s^2}$$

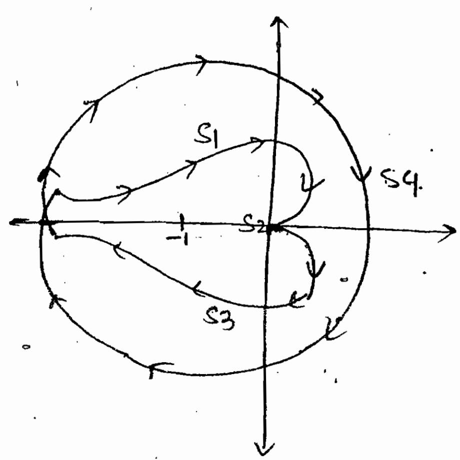
$$= \frac{1}{\lim\limits_{r \to 0} (re^{j\theta})^2}$$

$$= \frac{1}{\lim\limits_{r \to 0} r^2 e^{j2\theta}}$$

$$= \infty e^{-j2\theta}$$

$$\theta = -\pi/2 \rightarrow +\pi/2$$

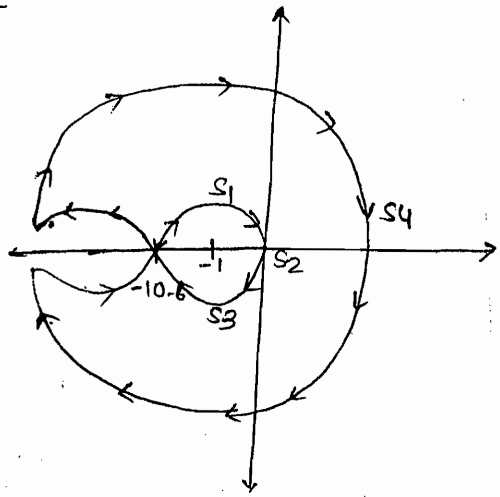
$$= \infty e^{j\pi} \rightarrow \infty e^{-j\pi}$$



$N = P - Z$

Q→ $G(s) = \frac{1+4s}{s^2(1+s)(1+2s)}$

Sol^n→



$N = P - Z$

$-2 = 0 - Z$

$Z = 2$

Stable

N = -2 (No. of encirclement of -1 are 2 times in cw dirn)

Con u(4) 75

$G(s) = \frac{s}{1-0.2s}$

To map of s_1→

$G(j\omega) = \frac{(j\omega)}{1-0.2(j\omega)}$

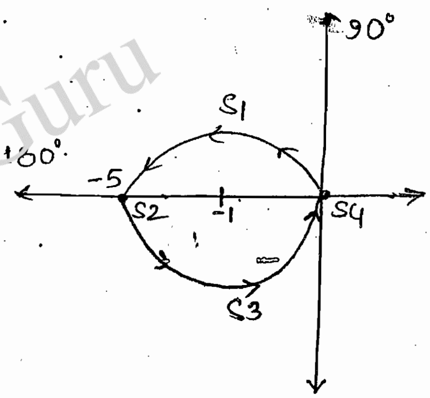
$|G(j\omega)| = \frac{\omega}{\sqrt{1+(0.2\omega)^2}}$

$\angle G(j\omega) = 90 + \tan^{-1}(0.2\omega)$

ω	0	∞
$\lvert G(j\omega)\rvert$	0	5
$\angle G(j\omega)$	90°	180°

To map s_2→

$G(s) = \frac{\cancel{s}}{-0.2\cancel{s}} = -5$



To map s_4→

$G(s) = s$

$= \lim_{r \to 0} (re^{j\theta})$

$= 0e^{j\theta}$

$$\boxed{\begin{aligned} N &= P - Z \\ 1 &= 1 - Z \\ Z &= 0 \end{aligned}}$$

(Stable)

Que→ $G(s) = \frac{K(1+s)^2}{s^3}$

Soln→ To map s_1→

$G(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3}$

$|G(j\omega)| = \frac{K(1+\omega^2)}{\omega^3}$

$\angle G(j\omega) = -270^\circ + 2\tan^{-1}\omega$

ω	0	1	∞
$\lvert G(j\omega)\rvert$	∞	$2K$	0
$\angle G(j\omega)$	-270°	-180°	-90

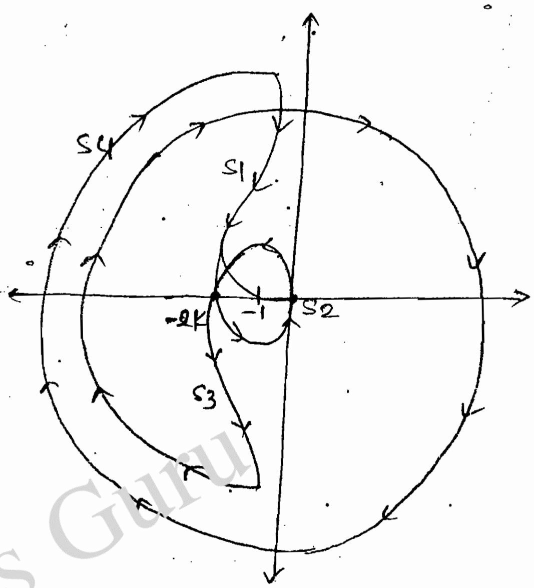
$-270^\circ + 2\tan^{-1}\omega = -180^\circ$

$2\tan^{-1}\omega = 90^\circ$

$\tan^{-1}\omega = 45$

$\omega = \omega_{pc} = 1\ r/s$

$x = \frac{K(1+1^2)}{1^3} = 2K$



To map s_4→

$G(s) = \frac{K}{s^3}$

$= \frac{K}{\lim_{r\to 0} r^3 e^{j3\theta}}$

$\infty e^{-j3\theta} \left(\theta = -\frac{\pi}{2} \to +\frac{\pi}{2}\right)$

$\infty e^{j3\pi/2} \to \infty e^{-j3\pi/2}$

$-2K = -1$, $K_{mar} = 0.5$

$K > 0.5$

$1 - 1 = 0 - Z$

$\boxed{Z = 0}$ stable

Que→ $G(s) = \dfrac{10e^{-s}}{s(s+5)}$

Soln→ To map s_1→

polar plot

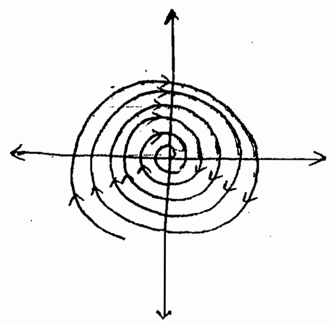
$$G(s) = \frac{2e^{-s}}{s(1+0.2s)}$$

$$G(j\omega) = \frac{2e^{-j\omega}}{(j\omega)(1+0.2j\omega)}$$

$$|G(j\omega)| = \frac{2\times 1}{\omega\sqrt{1+(0.2\omega)^2}}$$

$$\angle G(j\omega) = -90^\circ - 57.3\omega - \tan^{-1}0.2\omega$$

ω	0	∞
$\lvert G(j\omega)\rvert$	∞	0
$\angle G(j\omega)$	-90	-90°



Sagar Sen
8871453536

BODE PLOT

* It is a plot of the dB value of magnitude & phase angle in degrees vs $\log\omega$.

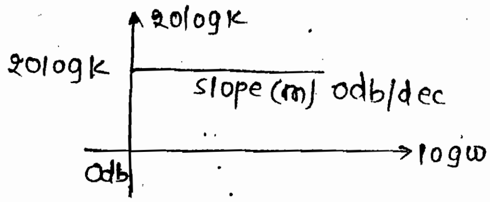
(1) System Gain $(\pm K)$ →

$F(s) = \pm K$.

$F(j\omega) = \pm K + j0$

$|F(j\omega)| = \sqrt{(\pm K)^2 + 0^2} = K$

Its dB value is



$\underline{|F(j\omega)} = K + j0 \rightarrow 0°$ for all 'ω'

$= -K + j0 \rightarrow -180°$ for all 'ω'

(2) Integral & derivative Factors $(s)^{\pm n}$ → (poles/zeros at origin)

$F(j\omega) = (j\omega)^{\pm n} = (0 + j\omega)^{\pm n}$

$|F(j\omega)| = \left[\sqrt{0 + \omega^2}\right]^{\pm n} = (\omega)^{\pm n}$

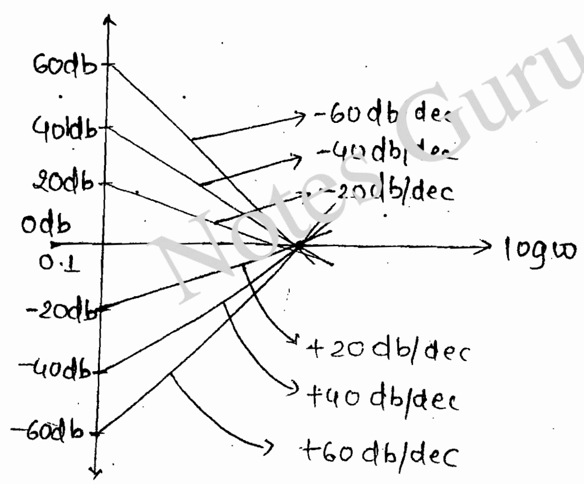
Its db value is

$20\log(\omega)^{\pm n} \Rightarrow \pm 20\cancel{\log} \times n \log\omega$ —(i)

$\boxed{\text{Slope}(m) = \pm 20 \times n \text{ db/dec}}$

$\cancel{\pm 20\log} \cdot \pm 20 \times n \log\omega = 0$

$\log\omega = 0, \ \omega = \log^{-1}(0) = 1 \text{ r/s}$



*** First order factors →** $(1 \pm Ts)^{\pm 1}$

$F(j\omega) = (1 \pm j\omega T)^{\pm 1}$

$|F(j\omega)| = \left[\sqrt{1 + (\omega T)^2}\right]^{\pm 1}$

Its dB value

$$20\log\left[\sqrt{1+(\omega T)^2}\right]^{\pm1}$$

$$\pm 20\log\sqrt{1+(\omega T)^2} \longrightarrow (i)$$

Asymptotic approximations →

Case(i) → Low Freq.

$$1 >> (\omega T)^2$$

$$\pm 20\log\sqrt{1} = 0\,db$$

Case(ii) High Freq.

$$(\omega T)^2 >> 1$$

$$\pm 20\log\sqrt{(\omega T)^2}$$

$$\pm 20\log(\omega T) \text{ ----- } (2)$$

$$\pm 20\log\omega \pm 20\log T$$

$$(mx + c)$$

$$\boxed{\text{Slope}(m) = \pm 20\,db/dec}$$

Corner Freq. (ω_{cf}) →

$$\pm 20\log\omega T = 0$$

$$\log\omega T = 0$$

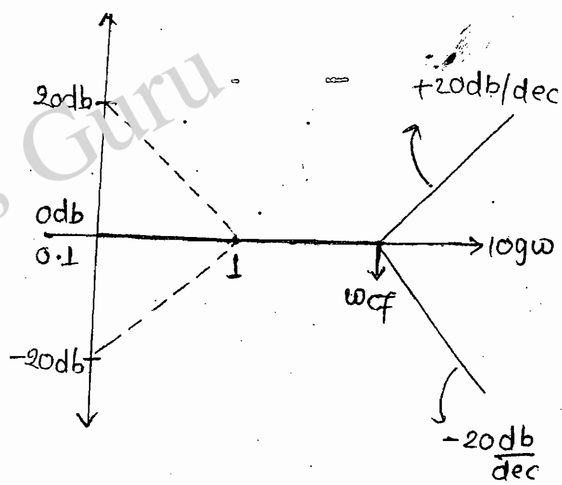
$$\omega T = \log^{-1}(0) = 1$$

$$\boxed{\omega = \omega_{cf} = \frac{1}{T}\ r/s}$$

Ex:- $(s \pm 2)^{\pm1}$

$$\left(1 \pm \frac{s}{2}\right)^{\pm1}$$

$$T = \frac{1}{2} \Rightarrow \omega_{cf} = 2\ r/s$$



Error at ω_{cf} →

$$\text{At } \omega = \omega_{cf} = \frac{1}{T}$$

$$\pm 20\log\sqrt{1+\left(\frac{1}{T}\times T\right)^2} = \pm 20\log\sqrt{2} = \pm 3\,db$$

* **Quadratic factors** → $(s^2+2\xi\omega_n s+\omega_n^2)^{\pm 1}$

$$\left(\frac{s^2}{\omega_n^2}+\frac{2\xi s}{\omega_n}+1\right)^{\pm 1}$$

(put $s=j\omega$)

$$\left[1-\left(\frac{\omega}{\omega_n}\right)^2+j2\xi\left(\frac{\omega}{\omega_n}\right)\right]^{\pm 1}$$

$$\pm 20\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+2\xi\left(\frac{\omega}{\omega_n}\right)^2}$$

LFR
0db/dec

HFR
$\pm 40\log\left(\frac{\omega}{\omega_n}\right)$

$$\boxed{\text{slope (m)} = \pm 40\,\text{db/dec}}$$

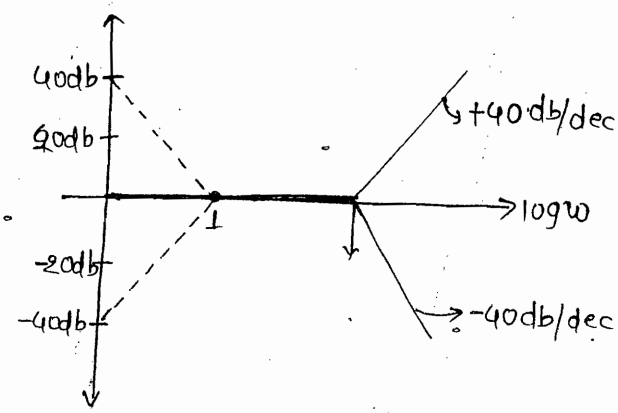
Corner freq. ($\omega_{cf}=\omega_n$ r/s)

$(s^2+4s+25)^{\pm 1}$

$\omega_n^2=25$, $\omega_n=\omega_{cf}=5$ r/s.

error at ω_{cf} →

$\pm 20\log 2\xi$

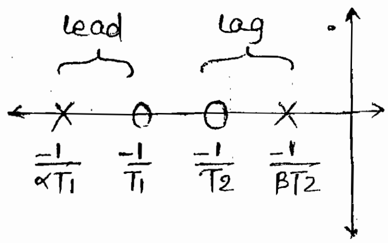


factors	corner freq.	magnitude	slope
k	–	20logk	0db/dec
$(j\omega)^{\pm n}$	–	$\pm 20\times n\log\omega$	$\pm 20\times n\frac{db}{dec}$
$(1\pm j\omega T)^{\pm 1}$	$\frac{1}{T}$	$\pm 20\log\omega T$	±20db/dec
$\left[1-\left(\frac{\omega}{\omega_n}\right)^2+2j\xi\left(\frac{\omega}{\omega_n}\right)\right]^{\pm 1}$	ω_n	$\pm 40\log\left(\frac{\omega}{\omega_n}\right)$	±40db/dec

* Bode plot for lag-lead compensator →

$$F(s) = \frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+T_1 s)(1+T_2 s)}{(1+\alpha T_1 s)(1+\beta T_2 s)}$$

$$F(j\omega) = \frac{\alpha(1+j\omega T_1)(1+j\omega T_2)}{(1+j\omega T_1)(1+j\omega \beta T_2)}$$



$$\angle F(j\omega) = \tan^{-1}\omega T_1 - \tan^{-1}\omega\alpha T_1 + \tan^{-1}\omega T_2 - \tan^{-1}\omega\beta T_2$$

magnitude table →

	Factor	Corner freq.	magnitude
(1.)	$K=\alpha$	—	$20\log\alpha$
(2.)	$(j\omega)^{\pm n}$	—	Nill
(3.)	$\frac{1}{1+j\omega\beta T_2}$	$\frac{1}{\beta T_2}$	-20 db/dec
(4.)	$1+j\omega T_2$	$\frac{1}{T_2}$	$+20$ db/dec
(5.)	$1+j\omega T_1$	$\frac{1}{T_1}$	$+20$ db/dec
(6.)	$\frac{1}{1+j\omega\alpha T_1}$	$\frac{1}{\alpha T_1}$	-20 db/dec

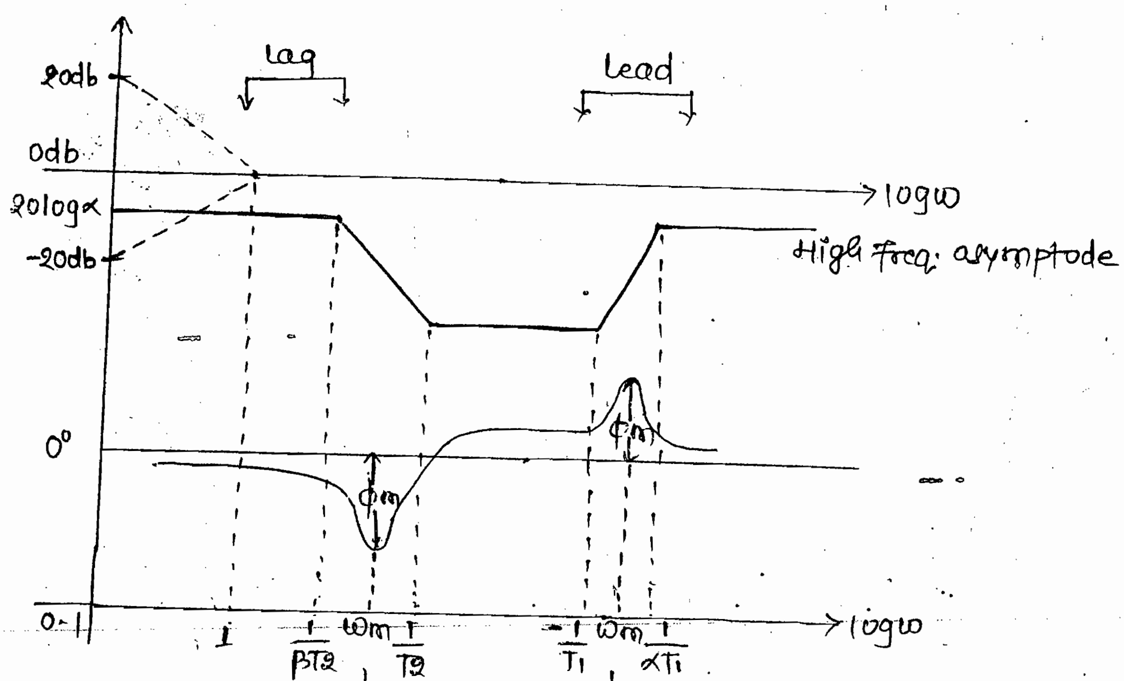
0

(-20)

0

$+20$

0



$$\omega_m = \sqrt{\frac{1}{\beta T_2} \times \frac{1}{T_2}} = \frac{1}{T_2\sqrt{\beta}} \qquad \omega_m = \sqrt{\frac{1}{T_1} \times \frac{1}{\alpha T_1}} = \frac{1}{T_1\sqrt{\alpha}}$$

For calculate High freq. asymptode

* C/s of phase lead compensator :- The phase lead compensator shifts the gain cross over freq. to higher values where the desired phase margin is acceptable.
Hence it is effective when the slope of uncompensated sys. near the gain cross over freq. is low.

* The max^m phase lead occurs at the geometric mean of 2 corner freq.

$$\angle F(j\omega) = \phi = \tan^{-1}\omega T_1 - \tan^{-1}\omega\alpha T_1$$

$$\tan\phi = \tan(\tan^{-1}\omega T_1 - \tan^{-1}\omega\alpha T_1)$$

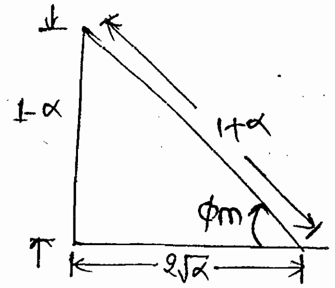
$$\tan\phi = \frac{\omega T_1 - \omega\alpha T_1}{1+(\omega T_1)^2\alpha} = \frac{\omega T_1(1-\alpha)}{1+(\omega T_1)^2\alpha}$$

At $\omega = \omega_m = \frac{1}{T_1\sqrt{\alpha}}$; $\phi = \phi_m$

$$\tan\phi_m = \frac{\frac{1}{T_1\sqrt{\alpha}} \cdot T_1(1-\alpha)}{1+\left(\frac{1}{T_1\sqrt{\alpha}} \cdot T_1\right)^2 \cdot \alpha} = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\tan\phi_m = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\boxed{\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right)}$$



$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\boxed{\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)}$$

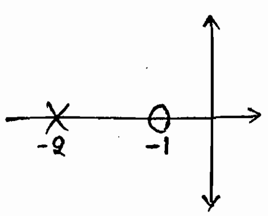
$$\sin\phi_m(1+\alpha) = 1-\alpha$$

$$\alpha\sin\phi_m + \alpha = 1 - \sin\phi_m$$

$$\alpha(1+\sin\phi_m) = 1-\sin\phi_m$$

$$\boxed{\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}}$$

(3.) Polar plot →



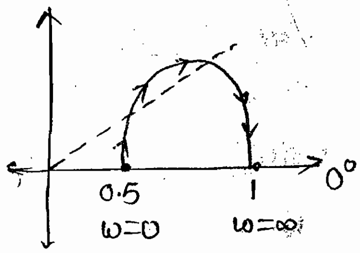
$$F(s) = \frac{s+1}{s+2} = \frac{0.5(1+s)}{(1+0.5s)}$$

$$F(j\omega) = \frac{0.5(1+j\omega)}{(1+0.5j\omega)}$$

$$|F(j\omega)| = \frac{0.5\sqrt{1+\omega^2}}{\sqrt{1+(0.5\omega)^2}}$$

$$\angle F(j\omega) = \tan^{-1}\omega - \tan^{-1}0.5\omega$$

ω	0	1	∞
$\lvert F(j\omega)\rvert$	0.5		1
$\angle F(j\omega)$	0°	18.4°	0°



* **c/s of phase lag compensator** → * phase lag compensator shifts the gain cross over freq. to lower values where the desired ϕ margin is acceptable.

Hence it is effective when the slope of the uncompensated sys. near gain cross over freq. is high.

* The max^m phase lag occurs at the geometric mean of the 2 corner freq.

$$\underline{|F(j\omega)} = \phi = \tan^{-1}\omega T_2 - \tan^{-1}\omega\beta T_2$$

$$\tan\phi = \tan\left(\tan^{-1}\omega T_2 - \tan^{-1}\omega\beta T_2\right)$$

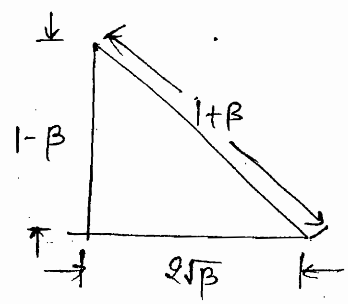
$$\tan\phi = \frac{\omega T_2 - \omega T_2\beta}{1+(\omega T_2)^2\cdot\beta} = \frac{\omega T_2(1-\beta)}{1+(\omega T_2)^2\beta}$$

$$\text{At } \omega = \omega_m = \frac{1}{T_2\sqrt{\beta}} \;;\; \phi = \phi_m$$

$$\tan\phi_m = \frac{\frac{1}{T_2\sqrt{\beta}} \times T_2(1-\beta)}{1+\left[\frac{1}{T_2\sqrt{\beta}}\cdot T_2\right]^2\cdot\beta} = \frac{1-\beta}{2\sqrt{\beta}}$$

$$\tan\phi_m = \frac{1-\beta}{2\sqrt{\beta}}$$

$$\boxed{\phi_m = \tan^{-1}\left(\frac{1-\beta}{2\sqrt{\beta}}\right)}$$



$$\sin\phi_m = \frac{1-\beta}{1+\beta}$$

$$\boxed{\phi_m = \sin^{-1}\left(\frac{1-\beta}{1+\beta}\right)}$$

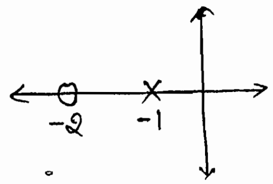
$$\sin\phi_m(1+\beta) = 1-\beta$$

$$\beta\sin\phi_m + \beta = 1-\sin\phi_m$$

$$\beta(1+\sin\phi_m) = 1-\sin\phi_m$$

$$\boxed{\beta = \frac{1-\sin\phi_m}{1+\sin\phi_m}}$$

(3.) Polar plot →



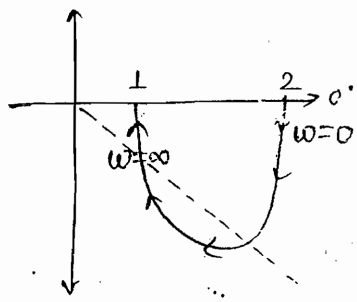
ω	0	1	∞
$\lvert F(j\omega)\rvert$	2		1
$\angle F(j\omega)$	0°	-18.4°	0°

$$F(s) = \frac{s+2}{s+1} = \frac{2(1+0.5s)}{(1+s)}$$

$$F(j\omega) = \frac{2(1+0.5j\omega)}{(1+j\omega)}$$

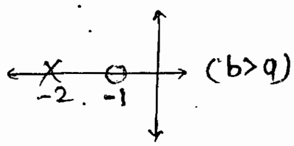
$$|F(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{\sqrt{1+\omega^2}}$$

$$\angle F(j\omega) = \tan^{-1}0.5\omega - \tan^{-1}\omega$$



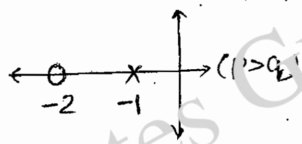
(1/76) lead :-

$$\frac{s+a}{s+b} = \frac{s+1}{s+2}$$



lag:-

$$\frac{s+p}{s+q} = \frac{s+2}{s+1}$$



(3/76)

$$\alpha = \frac{R_2}{R_1+R_2}$$

$$\phi_m = \sin^{-1}\left(\frac{1-\frac{R_2}{R_1+R_2}}{1+\frac{R_2}{R_1+R_2}}\right)$$

$$\phi_m = \sin^{-1}\left(\frac{R_1}{R_1+2R_2}\right)$$

(2/76)

$$G_c(s) = \frac{5(1+0.3s)}{(1+0.1s)}$$

$$\frac{1+0.3s}{1+0.1s} = \frac{1+Ts}{1+KTs}$$

$T=0.3$, $KT=0.1$

$K\times 0.3 = 0.1$; $K=\alpha=\frac{1}{3}$

lead compensator

$$\phi_m = \sin^{-1}\left(\frac{1-1/3}{1+1/3}\right)$$

$$\phi_m = \sin^{-1}(1/2) = 30^\circ$$

ans(c)

(4/76)

$$\frac{1+aTs}{1+Ts} = \frac{1+T_1s}{1+\alpha T_1 s}$$

$T_1 = aT$; $\alpha T_1 = T$

$\alpha \times aT = T$; $\alpha = \frac{1}{a}$

$$\phi_m = \sin^{-1}\left(\frac{1-1/a}{1+1/a}\right) = \sin^{-1}\left(\frac{a-1}{a+1}\right)$$

6/76

$$\frac{K(1+0.3S)}{(1+0.17S)} = \frac{1+TS}{1+\alpha TS}$$

$T = 0.3$; $\alpha T = 0.17$

$\alpha \times 0.3 = 0.17$

$\alpha = 0.56$

$T = R_1 C$

$R_1 \times 10^{-6} = 0.3$

$\boxed{R_1 = 300k\Omega}$

$$\alpha = \frac{R_2}{R_1 + R_2}$$

$$0.56 = \frac{R_2}{300 + R_2}$$

$\boxed{R_2 = 400k\Omega}$

10/77

$$\frac{1+2S}{1+0.2S} = \frac{1+TS}{1+KTS}$$

$T = 2$; $KT = 0.2$

$K \times 2 = 0.2$, $K = 0.1$

(Lead controller)

11/74 (b.) $\frac{S+9.9}{S+3} \Rightarrow$ lag.

(d) $\frac{S+6}{S} = 1 + \frac{6}{S} \longrightarrow$ PI

$K_P + \frac{K_I}{3}$

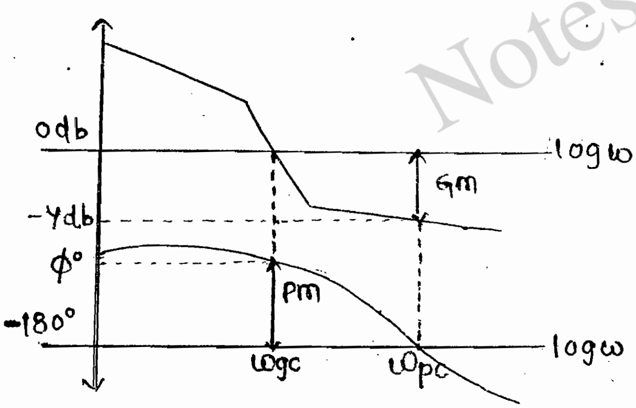
(c.) Can't be ans.

~~(a)~~ To improved transient state lead compensator must be used

5/76 (d.) **7/77** d **10/77** (c) **12/77** (a)

DATE-25/11/14

* To find Gm & Pm from BODE plots →

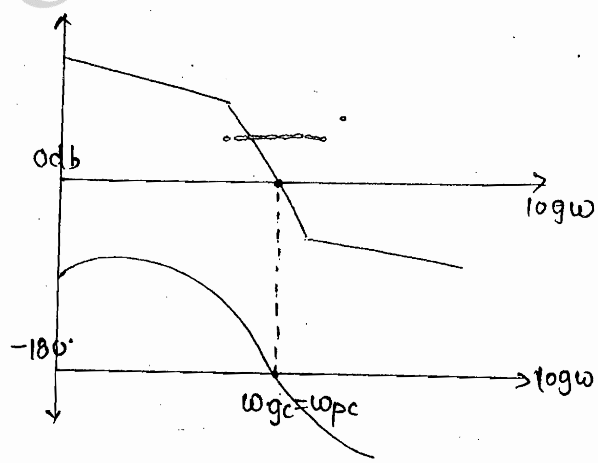


stable

$Pm = \phi - (-180°) = 180° + \phi = +ve$

$Gm = 0 - (-y) = +y\,db = +ve$

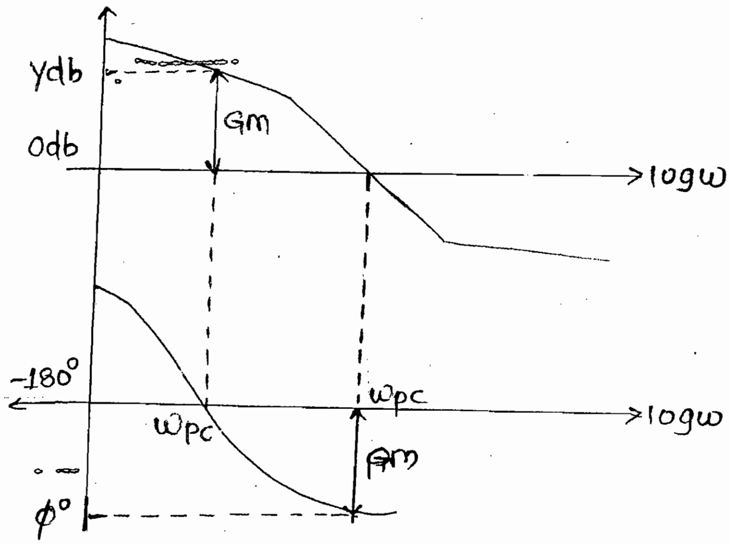
$\boxed{\omega_{gc} < \omega_{pc}}$



$Gm = Pm = 0$

marginaly stable

$\boxed{\omega_{gc} = \omega_{pc}}$



$$pm = 180° + \phi = -ve$$

$$Gm = 0 - y = -ydb = -ve$$

$$\boxed{\omega_{gc} > \omega_{pc}}$$

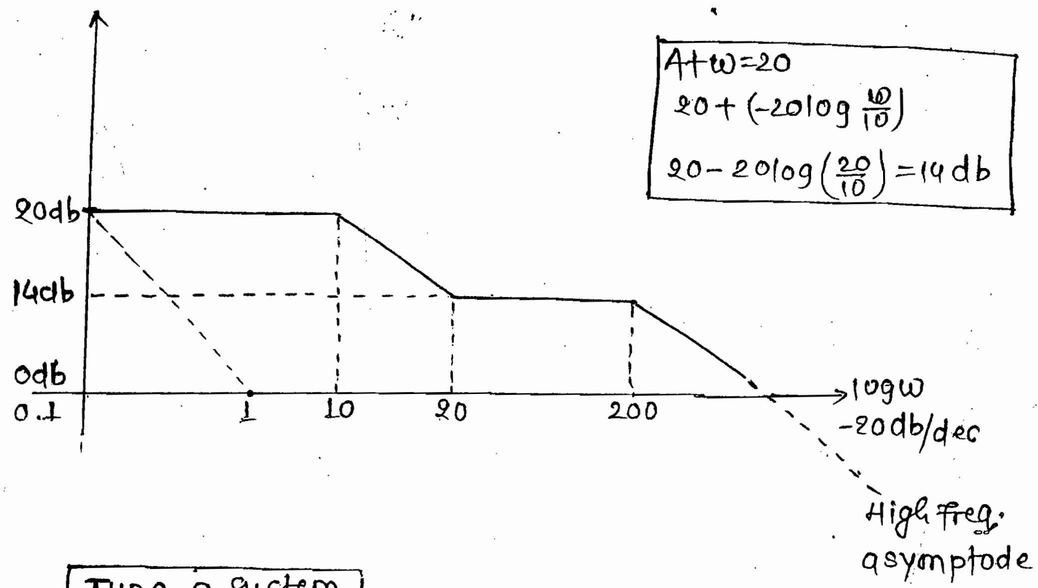
Unstable

Que.→ $G(s) = \dfrac{10^3(s+20)}{(s+10)(s+200)}$

Soln→ $G(s) = \dfrac{\frac{10^3}{\cancel{20}}(20)\left(\frac{s}{20}+1\right)}{10 \times 200\left(\frac{s}{10}+1\right)\left(\frac{s}{200}+1\right)}$

$$= \frac{10\left(1 + j\omega/20\right)}{\left(1+j\omega/10\right)\left(1+j\omega/200\right)}$$

	Factors	CF	magnitude
(1.)	$k=10$	–	$20\log 10 = 20db$
(2.)	$(j\omega)^{\pm n}$	–	Nil
(3.)	$\dfrac{1}{1+\frac{j\omega}{10}}$	0 10 -20	$-20\log\left(\frac{\omega}{10}\right)$ $(-20db/dec)$
(4.)	$1+\left(\frac{j\omega}{20}\right)$	20 0	$+20\log\left(\frac{\omega}{20}\right)$ $(+20db/dec)$
(5.)	$\dfrac{1}{1+\frac{j\omega}{200}}$	200 -20	$-20\log\left(\omega/200\right)$ $(-20db/dec)$



At ω=20
$20 + \left(-20\log\frac{\omega}{10}\right)$
$20 - 20\log\left(\frac{20}{10}\right) = 14\,db$

Type-0 system
$K_V = K_A = 0$
$20\log K_p = 20$
$K_p = \log^{-1}(1) = 10$

<u>Que.</u>→ $G(s) = \dfrac{20(s+1)}{s(s+2)(s+5)}$

<u>Soln</u>→ $G(s) = \dfrac{20(1+s)}{10\cdot s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{5}\right)}$

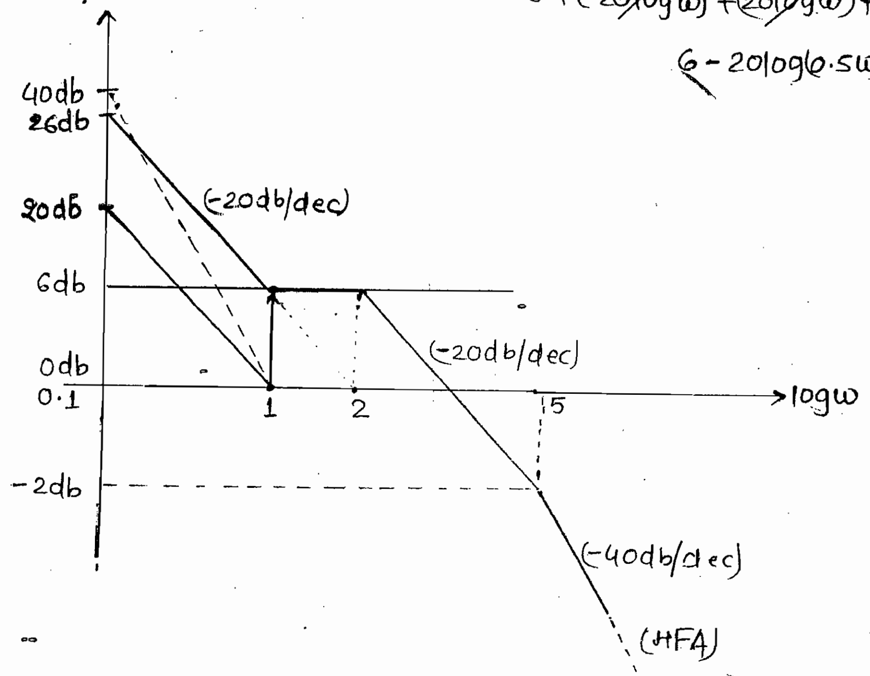
$G(j\omega) = \dfrac{2(1+j\omega)}{(j\omega)(1+0.5j\omega)(1+0.2j\omega)}$

factor	CF	magnitude
(i) $K=2$	-	$20\log 2 = 6db$
(ii) $\frac{1}{j\omega}$	-	$-20\log\omega$ ($-20db/dec$)
	-20	
(iii) $1+j\omega$	1	$+20\log\omega$ ($+20db/dec$)
	0	
(iv) $\frac{1}{1+0.5j\omega}$	2	$-20\log 0.5\omega$ ($-20db/dec$)
	-20	
(v) $\frac{1}{1+0.2j\omega}$	5	$-20\log 0.2\omega$ ($-20db/dec$)
	-40	

At $\omega=5$

$$6+(-20\cancel{\log\omega})+(\cancel{20\log\omega})+(-20\log 0.5\omega)$$

$$6-20\log(0.5\omega) = 6-20\log(0.5\times5)$$
$$= -2db$$



y m x c $\boxed{C=20\log k}$

$y = -20\log\omega + 6$

At $\omega = 0.1$

$y = -20\log 0.1 + 6 = 20+6 = 26db$

$\boxed{\begin{array}{l}\text{Type-1 system} \\ k_p=\infty,\ k_A=0 \\ k_v=\omega=2 \\ 20\log k - 20\log\omega = 0 \\ k=k_v=\omega\end{array}}$

Con-que.(1)→ (74)

$$G(s) = \frac{3(s+1)(s+700)}{s^2(s^2+18s+400)}$$

sol^n→ quadratic factors must be considered as quadratic factor.

$$G(s) = \frac{3(1+s)\left(1+\frac{s}{700}\right)700}{s^2\times400\left[1+\frac{18s}{400}+\frac{s^2}{400}\right]}$$

$$G(j\omega) = \frac{3(1+j\omega)\left(1+\frac{j\omega}{700}\right)700}{-\omega^2\times400\left[1-\frac{\omega^2}{400}+\frac{j18\omega}{400}\right]}$$

$$\left.\angle G(j\omega)\right|_{\omega<20} = \frac{(0^\circ)(\tan^{-1}\omega)\left(\tan^{-1}\frac{\omega}{700}\right)}{(180^\circ)\left[\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)\right]}$$

$$\left.\angle G(j\omega)\right|_{\omega>20} = \frac{(0^\circ)(\tan^{-1}\omega)\left(\tan^{-1}\frac{\omega}{700}\right)}{(180^\circ)\left[180^\circ-\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)\right]}$$

Ex:- cal^n of phase angle for quadratic term

$\omega = $

$$-180+\left[-\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)\right]$$

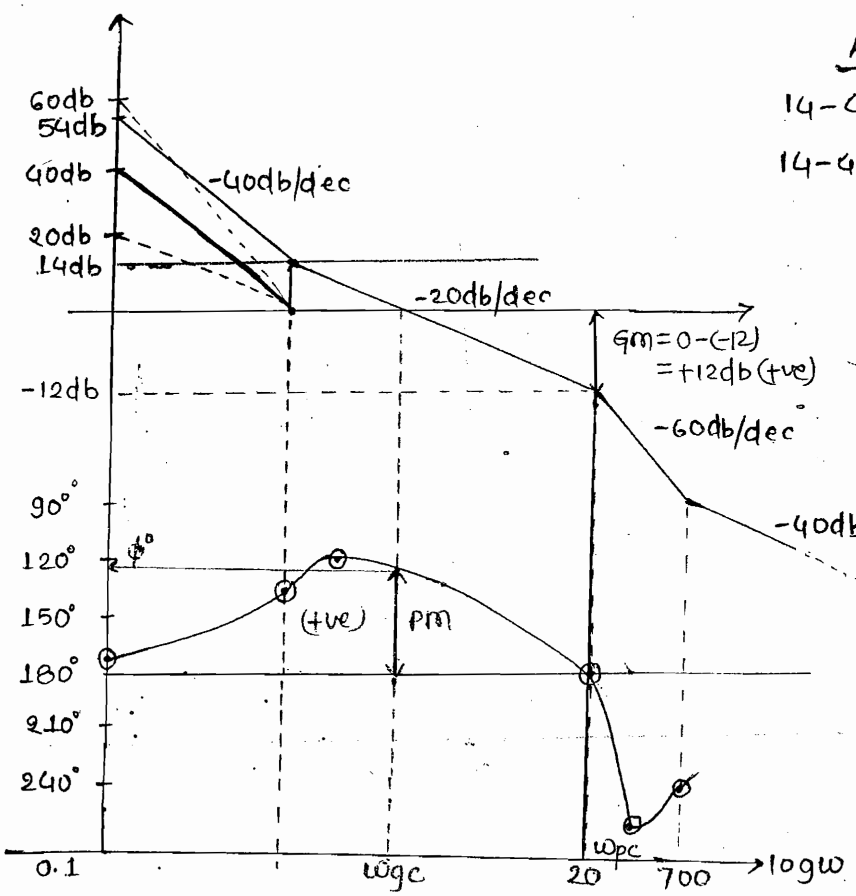
$$(-180^\circ+10.6^\circ) = -169.4^\circ$$

Phase angle table →

ω	$-180°$	$+\tan^{-1}\omega$	$+\tan^{-1}\frac{\omega}{700}$	$+\tan^{-1}\frac{18\omega}{400-\omega^2}$	ϕ_R
0.1	$-180°$	$5.7°$	$0.008°$	$-0.25°$	$-174°$
1	$-180°$	$45°$	$0.08°$	$-2.5°$	$-137°$
5	$-180°$	$78.6°$	$0.4°$	$-13.5°$	$-114°$
20	$-180°$	$87°$	$1.6°$	$-90°$	$-181°$
100	$-180°$	$89.4°$	$8°$	$+10.6-180° = -169.4°$	$-251°$
700	$-180°$	$89.9°$	$45°$	$+1.5-180° = -178.5°$	$-223°$

magnitude table →

Factor	CF	magnitude
(1) $k = 5.2$	–	$20\log 5.2 = 14\,db$
(2) $\frac{1}{(j\omega)^2}$	–	$+$ $-40\log\omega$ (-40 db/dec)
	-40	
(3) $(1+j\omega)$	1	$20\log\omega$ ($+20$ db/dec)
	-20	
(4) $\frac{1}{1-\frac{\omega^2}{400}+j\frac{18\omega}{400}}$	20	$-40\log\left(\frac{\omega}{20}\right)$ (-40 db/dec)
	-60	
(5) $1+\frac{j\omega}{700}$	700	$20\log\left(\frac{\omega}{700}\right)$ ($+20$ db/dec)
	-40	



At ω=20

$14 - 40\log\omega + 20\log\omega$

$14 - 40\log 20 + 20\log 20 = -12\text{db}$

At ω=0.1

$y = -40\log 0.1 + 14$

$y = 40 + 14 = 54\text{db}$

Type-2 system

$K_p = K_v = \infty$

$K_A = \omega^2$

$20\log K - 20\log\omega^2 = 0$

$K = K_A = \omega^2$

$\omega_{pc} > \omega_{gc}$

Stable system

$GM = PM = +ve$

13/71

$$G(s) = \frac{K(s+10)(s+20)}{s^3(s+100)(s+200)}$$

Sol^n→

$$G(s) = \frac{0.01K(1+0.1s)(1+0.05s)}{s^3\left(1+\frac{s}{100}\right)\left(1+\frac{s}{200}\right)}$$

$$G(j\omega) = \frac{\frac{K}{100}\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{20}\right)}{(j\omega)^3\left(1+\frac{j\omega}{100}\right)\left(1+\frac{j\omega}{200}\right)}$$

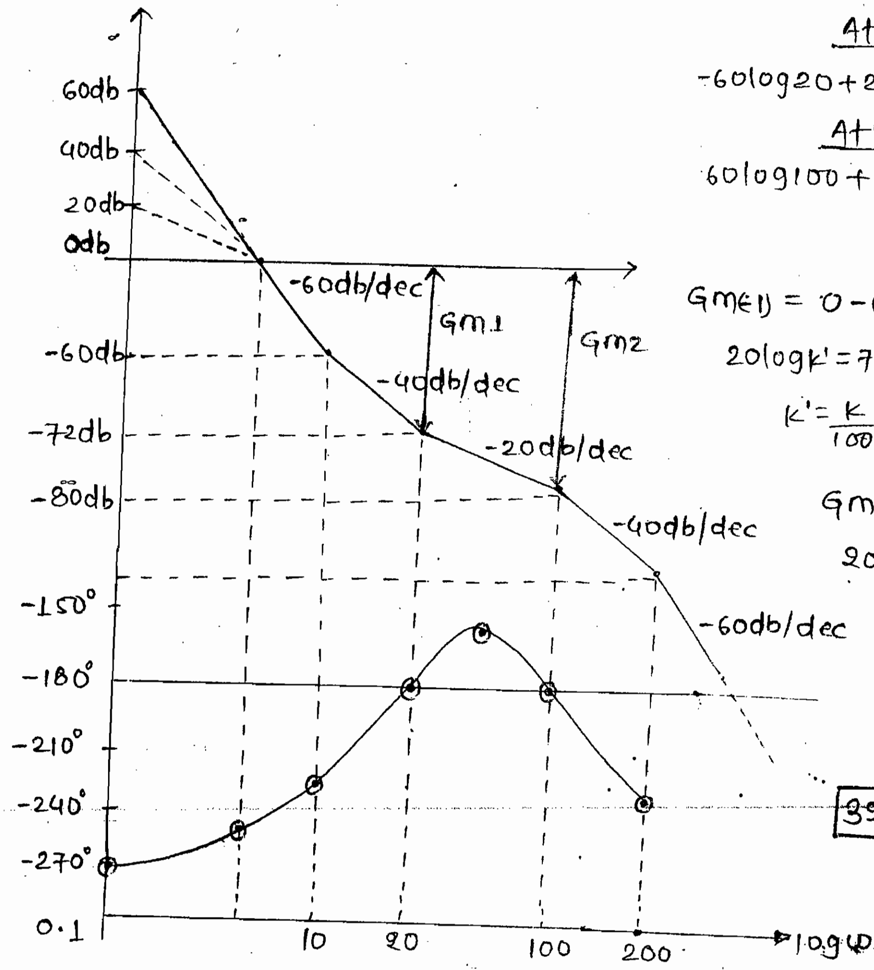
$$\angle G(j\omega) = \frac{(0^\circ)\left(\tan^{-1}\frac{\omega}{10}\right)\left(\tan^{-1}\frac{\omega}{20}\right)}{270^\circ\left(\tan^{-1}\frac{\omega}{100}\right)\left(\tan^{-1}\frac{\omega}{200}\right)}$$

$$\phi_R = -270^\circ + \tan^{-1}\frac{\omega}{10} + \tan^{-1}\frac{\omega}{20} - \tan^{-1}\frac{\omega}{100} - \tan^{-1}\frac{\omega}{200}$$

ω	0.1	1	10	20	50	100	200
ϕ_R	-269°	-262°	-207°	-179°	-163°	-179°	-207°

mag. table →

	Factor	CF	magnitude
(1.)	$k' = \frac{k}{100}$	-	$20\log k'$
(2.)	$\frac{1}{(j\omega)^3}$	-	$-60\log\omega$ $(-60\,db/dec)$
		-60	
(3.)	$1+\frac{j\omega}{10}$	10	$20\log\frac{\omega}{10}$ $(+20\,db/dec)$
		-40	
(4.)	$1+\frac{j\omega}{20}$	20	$20\log\omega/20$ $(+20\,db/dec)$
		-20	
(5.)	$\frac{1}{1+\frac{j\omega}{100}}$	100	$-20\log\frac{\omega}{100}$ $(-20\,db/dec)$
		-40	
(6.)	$\frac{1}{1+\frac{j\omega}{200}}$	200	$-20\log\frac{\omega}{200}$ $(-20\,db/dec)$
		-60	



At $\omega = 20$

$-60\log 20 + 20\log 2 = -72\,db$

At $\omega = 100$

$60\log 100 + 20\log 10 + 20\log 5 = -86\,db$

$Gm(1) = 0 - (-72) = 72\,db$

$20\log k' = 72 \Rightarrow k' = 3981$

$k' = \frac{k}{100} = 3981\,;\ k = 398100$

$Gm(2) = 0 - (-86) = 86\,db$

$20\log k' = 86 \Rightarrow k' = 19952$

$k' = \frac{k}{100} = 19952$

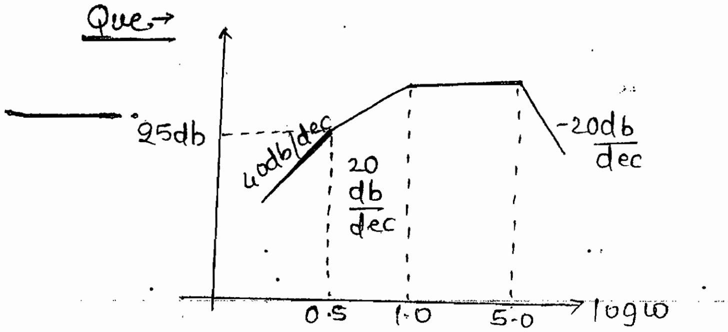
$k = 1995200$

$\boxed{398100 < k < 1995200}$

Inverse Bode Plot

* Observe the starting slope. This will give the information of poles (or) zeros at origin.
* For the starting slope write the eqn $y = mx + c$.
  $c = 20\log K$.
* At every corner freq. observe the change in slopes. This will give the information of 1st (or) higher order factors.

Que→



Soln→

$$G(s) = \frac{70 s^2}{(1+2s)(1+s)\left(1+\frac{s}{5}\right)}$$

At $\omega = 0.5$

$y = mx + c$

$25 = 40\log 0.5 + c$

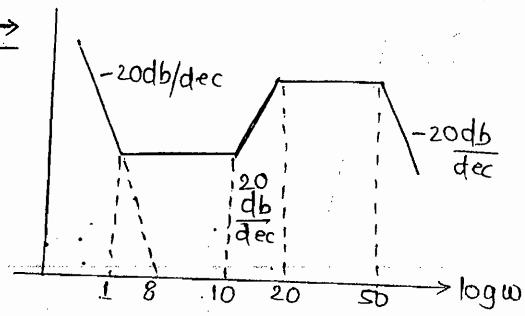
$c = 37$

$20\log K = 37$

$K = \log^{-1}\left(\frac{37}{20}\right)$

$K = 70$

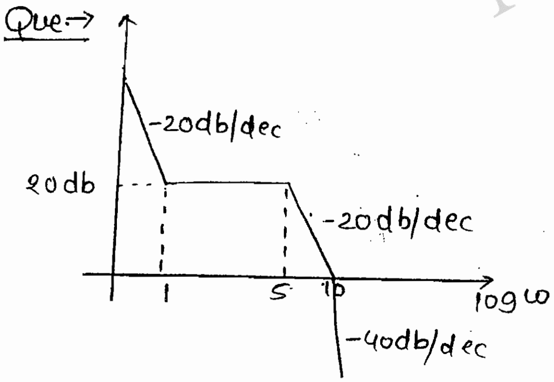
Que→



Soln→

$$G(s) = \frac{8(1+s)\left(1+\frac{s}{10}\right)}{s\left(1+\frac{s}{20}\right)\left(1+\frac{s}{50}\right)}$$

Que→



Soln→

$$G(s) = \frac{10(1+s)}{s\left(1+\frac{s}{5}\right)\left(1+\frac{s}{10}\right)}$$

At $\omega = 1$

$y = mx + c$

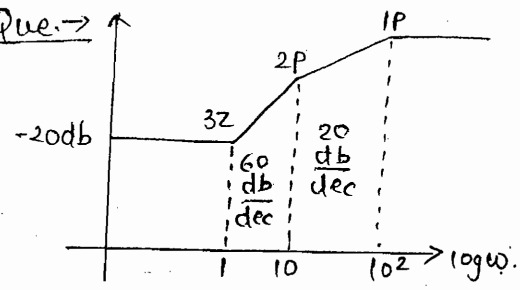
$20 = -20\log 1 + c$

$c = 20$

$20\log K = 20$

$K = \log^{-1}(1) = 0$

Que→



Soln→

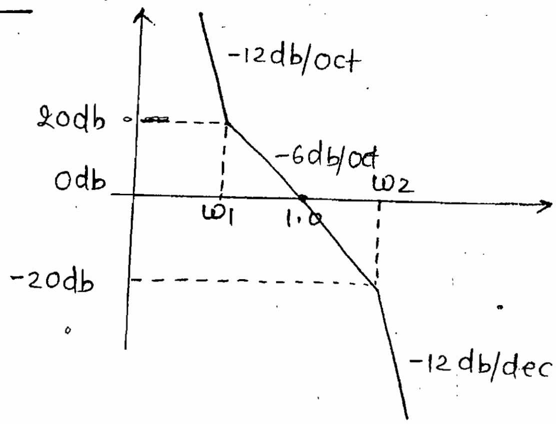
$$G(s) = \frac{0.1(1+s)^3}{\left(1+\frac{s}{10}\right)^2\left(1+\frac{s}{100}\right)}$$

$20\log K = -20$

$K = \log^{-1}(-1)$

$K = 0.1$

Que.→



Soln→

decade scale

$\omega_2 = 10\omega_1$

db value magnitude $= \pm 20 \times n \log\omega$

slope (m) $= \pm 20 \times n \log 10$

$= \pm 20 \times n$ db/dec

Octave scale

$\omega_2 = 2\omega_1$

db value magnitude $= \pm 20 \times n \log$

slope (m) $= \pm 20 \times n \log 2$

$= \pm 6 \times n$ db/oct

$$\boxed{\pm 6 \times n \frac{db}{oct} \simeq \pm 20 \times n \frac{db}{dec}}$$

2nd line→

At $\omega = 1$

$y = mx + c$

$0 = -20\log 1 + c$

$c = 0$

At $\omega = \omega_1$

$y = mx + c$

$20 = -20\log\omega_1 + 0$

$\omega_1 = \log^{-1}(-1) = 0.1 r/s$

At $\omega = \omega_2$

$y = mx + c$

$-20 = -20\log\omega_2 + 0$

$\omega_2 = \log^{-1}(1) = 10 r/s$

To Find k→

1st line

At $\omega = \omega_1 = 0.1$

$y = mx + c$

$20 = -40\log 0.1 + c$

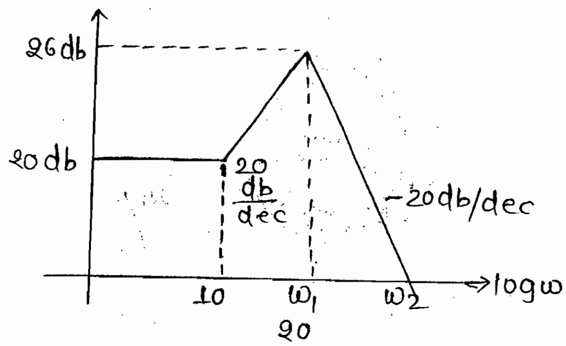
$c = -20$

$20\log k = -20$

$\boxed{k = 0.1}$

$$\boxed{G(s) = \frac{0.1(1+10s)}{s^2(1+0.1s)}}$$

Que.→



Soln→

To find ω_1

2nd line

At $\omega = 10$

$y = mx + c$

$20 = 20\log 10 + c$

$c = 0$

At $\omega = \omega_1$

$y = mx + c$

$26 = 20\log\omega_1 + 0$

$\omega_1 = \log^{-1}\left(\frac{26}{20}\right) = 20\, r/s$

To find ω_2

3rd line

At $\omega = \omega_1 = 20$

$y = mx + c$

$26 = -20\log 20 + c$

$c = 52$

At $\omega = \omega_2$

$y = mx + c$

$0 = -20\log\omega_2 + 52$

$-52 = -20\log\omega_2$

$\omega_2 = \log^{-1}\left(\frac{52}{20}\right) = 400\, r/s$

To find K

$20\log K = 20$

$K = \log^{-1}(1)$

$= 10$

$$\boxed{G(s) = \frac{10\left(1 + \frac{s}{10}\right)}{\left(1 + \frac{s}{20}\right)^2}}$$

DATE- 26/11/14

M & N circles

* Nicol charts →

* The Nicol chart consist of magnitudes & phase angles of a closed loop system represented as a family of circles known as M & N circles.

* This chart gives information about closed loop freq. responce.

*

* M-circles →

Let the CLTF

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

Let $G(s) = x + jy$

$$\frac{C(s)}{R(s)} = \frac{x+jy}{1+x+jy}$$

The magnitude (M)

$$M = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}}$$

$$M^2\left[(1+x)^2+y^2\right] = x^2+y^2$$

$$M^2x^2 - x^2 + M^2y^2 - y^2 + 2xM^2 + M^2 = 0$$

$$(M^2-1)x^2 + y^2(M^2-1) + 2xM^2 + M^2 = 0 \text{ ------ (i)}$$

In eq^n (i) $M = 1$

$2x+1=0$, It reprents a straight line passing through $-\frac{1}{2}, 0$

If $M \neq 1$ eq^n (i) represents a family of circles .

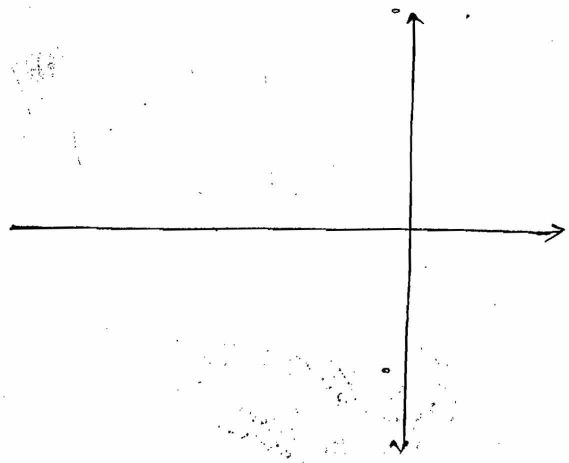
$$x^2+y^2+2x\frac{M^2}{M^2-1} + \frac{M^2}{M^2-1} = 0 \text{ ----- (ii)}$$

$$x^2+y^2+2x\frac{M^2}{M^2-1} + \frac{M^2}{M^2-1} + \frac{M^2}{(M^2-1)^2} = \frac{M^2}{(M^2-1)^2}$$

$$x^2 + 2x\frac{M^2}{M^2-1} + \frac{M^4}{(M^2-1)^2} + y^2 = \frac{M^2}{(M^2-1)^2}$$

$$\left[x + \frac{M^2}{M^2-1}\right]^2 + y^2 = \frac{M^2}{(M^2-1)^2}$$

$$\text{Center} = \frac{-m^2}{m^2-1}, 0 \qquad \text{Radius} = \frac{m}{m^2-1}$$



(4.) N-circles →

Let α = phase angles of CL system

$N = \tan\alpha$ represents a family of circles

$$\alpha = \tan^{-1}\frac{y}{x} - \tan^{-1}\left(\frac{y}{1-x}\right)$$

$$N = \tan\alpha = \tan\left[\tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{1+x}\right]$$

$$N = \frac{\frac{y}{x} - \frac{y}{1+x}}{1 + \frac{y^2}{x(1+x)}}$$

$$x^2 + x + y^2 - \frac{y}{N} = 0 \quad \text{------} (i)$$

adding term $\frac{1}{4} + \left(\frac{1}{2N}\right)^2$ on both side.

$$x^2 + x + \frac{1}{4} + y^2 - \frac{y}{N} + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

Center; $= \frac{-1}{2}, \frac{1}{2N}$ Radius $= \sqrt{\frac{1}{4}+\left(\frac{1}{2N}\right)^2}$

All N-circles intersects the real axis b/n -1 & origin only.

23/74 (b) 3 is wrong because geometric rules are not applicable for log scale.

24/74

$$x^2+y^2+2x\frac{m^2}{m^2-1}+\frac{m^2}{m^2-1}=0 \quad ----(1)$$

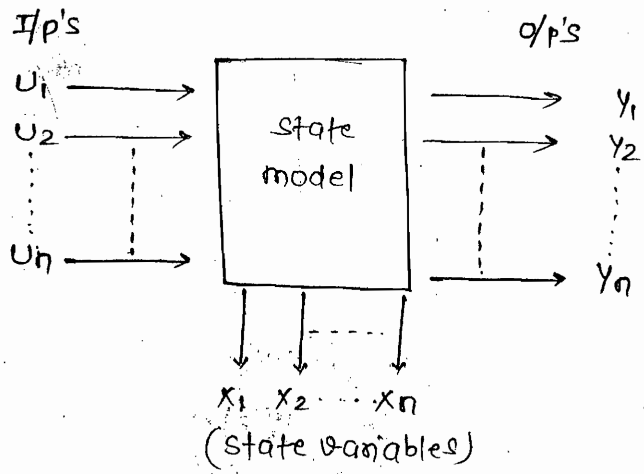
$$x^2+y^2+2.25x+1.125=0$$

$$\frac{m^2}{m^2-1}=1.125$$

$$\boxed{m=3}$$

25/74 (a)

Chaper-(05) State space variables



(1) State eqn →

$$\dot{X}(t) = A x(t) + B U(t)$$

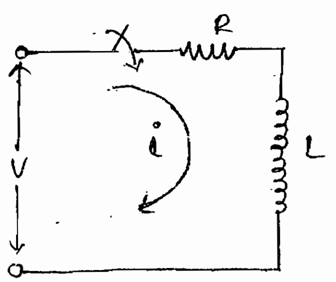
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix}$$

(2) O/p eqn →

$$Y(t) = CX(t) + DU(t)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix}$$

Ex:-



at $t = 0^-$

$i(0)^- = i_L(0)^- = 0$ Amps

At $t = 0^+$

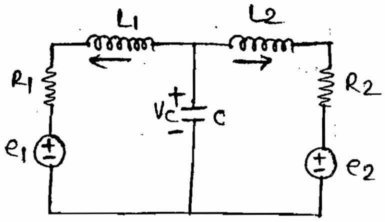
$$i_L(0)^+ = \frac{1}{L}\int_{t=0}^{0^+} V\,dt = 0 \text{ Amps}$$

$$V = iR + L\frac{di}{dt}$$

$$\boxed{\frac{di}{dt} = \frac{-R}{L}i + \frac{V}{L} \quad (A/s)}$$

State eq^n of N/ω.

Conv(1)
80



Loop-(1)

$$L_1\frac{di_1}{dt} + i_1R_1 + e_1 - V_c = 0$$

$$\frac{di_1}{dt} = \frac{-R_1}{L_1}i_1 + \frac{V_c}{L_1} - \frac{e_1}{L_1} \text{ ------(i)}$$

KCL at Node V_c →

$$i_1 + i_2 + C\frac{dV_c}{dt} = 0$$

$$\frac{dV_c}{dt} = \frac{-i_1}{C} - \frac{i_2}{C} \text{ -----(ii)}$$

Loop(2)

$$L_2\frac{di_2}{dt} + i_2R_2 + e_2 - V_c = 0$$

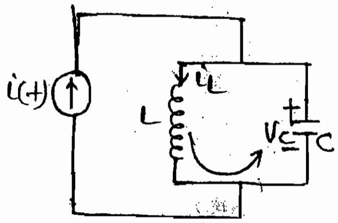
$$\frac{di_2}{dt} = \frac{-R_2}{L_2}i_2 + \frac{V_c}{L_2} - \frac{e_2}{L_2} \text{ -------(ii)}$$

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R_1}{L_1} & 0 & \frac{1}{L_1} \\ 0 & \frac{-R_2}{L_2} & \frac{1}{L_2} \\ \frac{-1}{C} & \frac{-1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{-1}{L_1} & 0 \\ 0 & \frac{-1}{L_2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

Let o/p $y = i_1$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} + 0$$

(b.)



$$i(t) = i_L + i_C$$

$$i(t) = i_L + \frac{C\,dV_c}{dt}$$

$$\frac{dV_c}{dt} = \frac{-i_L}{C} + \frac{i(t)}{C} \text{ ----- (i)}$$

$$\frac{L\,di_L}{dt} - V_c = 0 \quad \Rightarrow \quad \frac{di_L}{dt} = \frac{V_c}{L} \text{ --- (ii)}$$

* Type-(1.) Problem →

To obtain state model from differential eqn.

$$\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 10y = 20u$$

$$[s^3 + 4s^2 + 6s + 10]\, Y(s) = 20\,U(s)$$

Let $y = x_1$

$$\frac{dy}{dt} = \dot{x}_1 = x_2 \; ; \quad \frac{d^2y}{dt^2} = \dot{x}_2 = x_3 \; ; \quad \frac{d^3y}{dt^3} = \dot{x}_3$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = 20u - 4x_3 - 6x_2 - 10x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -6 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0$$

BUSH / COMPANION FORM

* Shortcut method for BUSH form →

(7/80)

$$\frac{Y(s)}{U(s)} = \frac{1}{s^4 + 5s^3 + 8s^2 + 6s + 3}$$

← Reverse order with rev. sign

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & -6 & -8 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

constant numerator.

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Que.→ $\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 9y = 2\frac{du}{dt} + u$

Sol^n→ $(s^2 + 7s + 9)\, Y(s) = (2s+1)\, U(s)$

$$\frac{Y(s)}{U(s)} = \frac{(2s+1)}{s^2 + 7s + 9}$$

Que. (8.) phase variable method →

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2 + 7s + 9} \quad (2s+1)$$

← only rev order *

* rev order with rev sign

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Ex:-

$$\frac{Y(s)}{U(s)} = \frac{10(s^2+2s+4)}{s^3+6s^2+8s+12}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -12 & -8 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u$$

$$y = [4 \quad 2 \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0$$

* Type (2) problem →

To obtain TF from state model.

$\dot{X}(t) = AX(t) + BU(t)$ ---- (i)

$Y(t) = CX(t) + DU(t)$

Applying LT on eqn (i)

$sX(s) - X(0) = AX(s) + BU(s)$

$Y(s) = CX(s) + DU(s)$

For TF $X(0) = 0$

$sX(s) - AX(s) = BU(s)$

$(sI - A) X(s) = BU(s)$

$X(s) = (sI - A)^{-1} B \cdot U(s)$

$Y(s) = [C(sI - A)^{-1}B + D] \, U(s)$

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1} B + D$$

(Transfer matrix)

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Q.(9) → $\dot{X} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u.$

$Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T \cdot X$

Solⁿ →

$SI - A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$

$= \begin{bmatrix} (S+1) & 0 \\ 0 & (S+2) \end{bmatrix}$

$adj\,(SI-A) = \begin{bmatrix} (S+2) & 0 \\ 0 & (S+1) \end{bmatrix}$

$|(SI-A)| = (S+1)(S+2)$

$(SI-A)^{-1} = \begin{bmatrix} \frac{1}{S+1} & 0 \\ 0 & \frac{1}{S+2} \end{bmatrix}$

$(SI-A)^{-1} \cdot B = \begin{bmatrix} \frac{1}{S+1} & 0 \\ 0 & \frac{1}{S+2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{S+1} \\ 0 \end{bmatrix}$

$C(SI-A)^{-1} \cdot B \Rightarrow \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{S+1} \\ 0 \end{bmatrix}$

$$\boxed{\left[\frac{Y(S)}{U(S)}\right] = \frac{1}{S+1}}$$

Q.(2) $\dot{X}(t) = -2X(t) + 2U(t)$

$Y(t) = 0.5X(t)$

Solⁿ → $SX(S) = -2X(S) + 2U(S)$

$(S+2)X(S) = 2U(S)$

$X(S) = \frac{2}{S+2} U(S)$

$$Y(s) = 0.5X(s)$$

$$Y(s) = 0.5 \times \frac{2}{s+2} U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{1}{s+2}}$$

* Type(3.) Problem → Stability for sm →

$$\frac{Y(s)}{U(s)} = C[SI-A]^{-1}B + D$$

$$= C \cdot \frac{Adj(SI-A) \cdot B}{|SI-A|} + D$$

$$\frac{Y(s)}{U(s)} = \frac{C \cdot Adj(SI-A) \cdot B + |SI-A| \cdot D}{|SI-A|}$$

Zeros → $C \cdot Adj(SI-A) \cdot B + |SI-A| \cdot D$

Poles → $(1+G(s) \cdot H(s)=0) = |SI-A| = 0$

$$\boxed{\text{Eigen values of sys. matrix } [A] = \text{CL poles}}$$

Conv(2) 80

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$C = \begin{bmatrix} -17 & -5 \end{bmatrix} X + [1] r$$

Soln →

$$(SI-A) = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} S & -1 \\ 20 & S+9 \end{bmatrix}$$

$$Adj(SI-A) = \begin{bmatrix} S+9 & 1 \\ -20 & S \end{bmatrix}$$

$$Adj(SI-A)B \Rightarrow \begin{bmatrix} s+9 & 1 \\ -20 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$C \cdot Adj(SI-A) \cdot B \Rightarrow \begin{bmatrix} -17 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ s \end{bmatrix} = -17-5s$$

$$|SI-A| = s(s+9)+20$$

$$|SI-A| \cancel{D} = (s^2+9s+20)(1)$$

$$\text{zeros} \Rightarrow -17-5s+s^2+9s+20=0$$

$$s^2+4s+3=0$$

$$s=-1,-3$$

$$\text{poles} \Rightarrow |SI-A|=0$$

$$s^2+9s+20=0$$

$s=-4,-5$ (sys. is stable because of -ve eigen values)

$\boxed{\text{Que.(6)} \; 80}$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \qquad u = \begin{bmatrix} -0.5 & -3 & -5 \end{bmatrix} X + V$$

Sol $\Rightarrow$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times 1} \begin{bmatrix} -0.5 & -3 & -5 \end{bmatrix}_{1\times 3} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.5 & -3 & -5 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V$$

$\boxed{\text{The sum of principle diagonal elements} = \text{Sum of eigen values}}$

ans (9)

* Type (04) Diagonalization →

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

Let $X = PZ$ ⇒ transformation matrix

$$P\dot{Z} = APZ + BU$$
$$Y = CPZ + DU$$

$$\boxed{\begin{aligned}\dot{Z} &= [P^{-1}AP]Z + [P^{-1}B]U \\ Y &= CPZ + DU\end{aligned}}$$

$$P = \text{Vander mode matrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \cdots & \lambda_n^2 \end{bmatrix}$$

$P^{-1}AP$ ⇒ diagonal matrix with diagonal elements as eigen values.

Ex:→ Diagonalize

$$A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} S & -1 \\ 6 & S+5 \end{bmatrix}$$

$$|SI - A| = S(S+5) + 6 = 0$$
$$S^2 + 5S + 6 = 0$$
$$S = -2, -3$$

$$P = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$$

$$P^{-1} = \frac{Adj(P)}{|P|} = \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$$

* TYPE (05) Controllability & observability →

Controllability → To control the state variables

observability → To measure the state variables.

KALMAN'S Test →

$$Q_c = [B \quad AB \quad A^2B \quad A^{n-1}B] = |Q_c| \neq 0$$

$$Q_o = [C^T \quad A^TC^T \quad (A^T)^2C^T \ldots (A^T)^{n-1}C^T] = |Q_o| \neq 0$$

Q.1 → $\dot{X} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = [1 \quad 1] X$

Soln → $Q_c = [B \quad AB]$ $\qquad Q_o = [C^T \quad A^TC^T]$

$AB = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ $\qquad A^TC^T = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$Q_c = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} = -1$ $\qquad Q_o = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} = -4$

Q.(10) $\dot{X} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ $\qquad y = [b \quad 0] x$

Soln → $A^TC^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} b \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 2b \end{bmatrix}$

$Q_o = \begin{bmatrix} b & b \\ 0 & 2b \end{bmatrix} = 2b^2 \neq 0$

ans.(c)

* Type (6) Solution of State eqn →

$$\dot{X}(t) = AX(t) + Bu(t)$$

(a) Free Responce → $U(t) = 0$

$$\dot{X}(t) = AX(t)$$

$$\dot{X}(t) - AX(t) = 0 \quad \text{-----(i)}$$

$$X(t) = ke^{At} \quad \text{-----(ii)}$$

Applying LT to eqn (i)

$$sX(s) - X(0) - AX(s) = 0$$

$$sX(s) - AX(s) = X(0)$$

$$(SI - A)\, X(s) = X(0)$$

$$X(s) = (SI - A)^{-1} \cdot X(0)$$

$\phi(s) = (SI - A)^{-1}$ = Resolvant matrix

$$X(t) = \left[\mathcal{L}^{-1}(SI - A)^{-1}\right] X(0)$$

$$X(t) = e^{At} \quad k$$

$$\boxed{\phi(t) = e^{At} = \mathcal{L}^{-1}(SI - A)^{-1}}$$

State transition matrix

(b) Forced Responce →

$$sX(s) - X(0) = AX(s) + BU(s)$$

$$sX(s) - AX(s) = X(0) + BU(s)$$

$$(SI - A)\, X(s) = X(0) + BU(s)$$

$$X(s) = (SI - A)^{-1} X(0) + (SI - A)^{-1} B \cdot U(s)$$

$$X(t) = \left[\mathcal{L}^{-1}(SI - A)^{-1}\right] X(0) + \mathcal{L}^{-1}\left[(SI - A)^{-1} B\, U(s)\right]$$

Que.(3) → $\dot{X} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Soln →

$$\boxed{\begin{array}{l} \text{ZIR (zero impulse responce)} \\ \text{short cut method} \\ \text{At } t=0 \\ X(t) = X(0) \end{array}}$$

$$SI-A \Rightarrow \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (S-1) & 0 \\ -1 & (S-1) \end{bmatrix}$$

$$(SI-A)^{-1} = \begin{bmatrix} \frac{1}{(S-1)} & 0 \\ \frac{1}{(S-1)^2} & \frac{1}{(S-1)} \end{bmatrix}$$

$$e^{At} = \phi(t) = \mathcal{L}^{-1}(SI-A)^{-1}$$

at $t=0$

$X(t) = X(0)$, $e^{A(t)} = I$

$$S.T.M. \Rightarrow \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

$$\boxed{\begin{array}{l} \text{Property of STM} \\ \text{At } t=0,\ e^{A0} = I \end{array}}$$

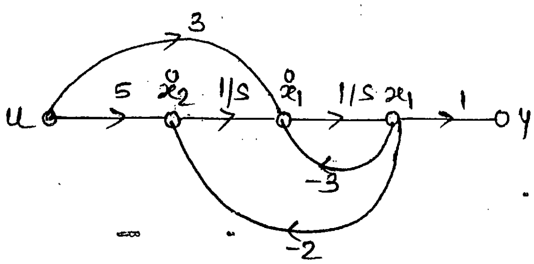
$$X(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ te^t \end{bmatrix}$$

$\boxed{\text{Ans. (C)}}$

* TYPE(7) State diagrams →

(1). Physical variable form →

$$\frac{Y(s)}{U(s)} = \frac{3s+5}{s^2+3s+2} = \frac{\frac{3}{s}+\frac{5}{s^2}}{1-\left[\frac{-3}{s}-\frac{2}{s^2}\right]}$$



$$\dot{x}_1 = -3x_1 + x_2 + 3u$$

$$\dot{x}_2 = -2x_1 + 5u$$

$$y = x_1$$

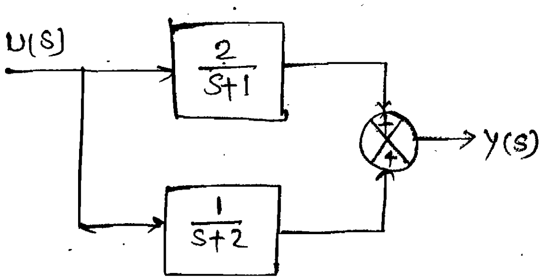
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}3\\ 5\end{bmatrix}u$$

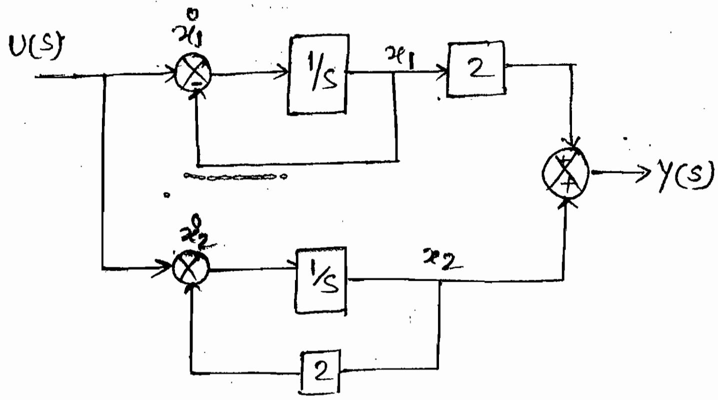
$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

(2.) Canonical variable form →

$$\frac{Y(s)}{U(s)} = \frac{3s+5}{(s+1)(s+2)} = \frac{2}{s+1} + \frac{1}{s+2}$$

$$Y(s) = \frac{2U(s)}{s+1} + \frac{U(s)}{s+2}$$





$$\dot{x}_1 = -x_1 + u$$

$$\dot{x}_2 = -2x_2 + u$$

$$y = 2x_1 + x_2$$

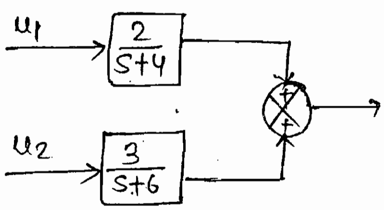
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

Unit matrix

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Residues of PF's

Ex:-



$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$Y = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

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