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1. Static Electro-Magnetic Fields.	→ 40% IES
2. EM Waves	(9/10 others)
3. VI Waves - Transmission lines	80%
4. Guided Waves	(Guide)
5. Radiated Waves - Antennas] 10%

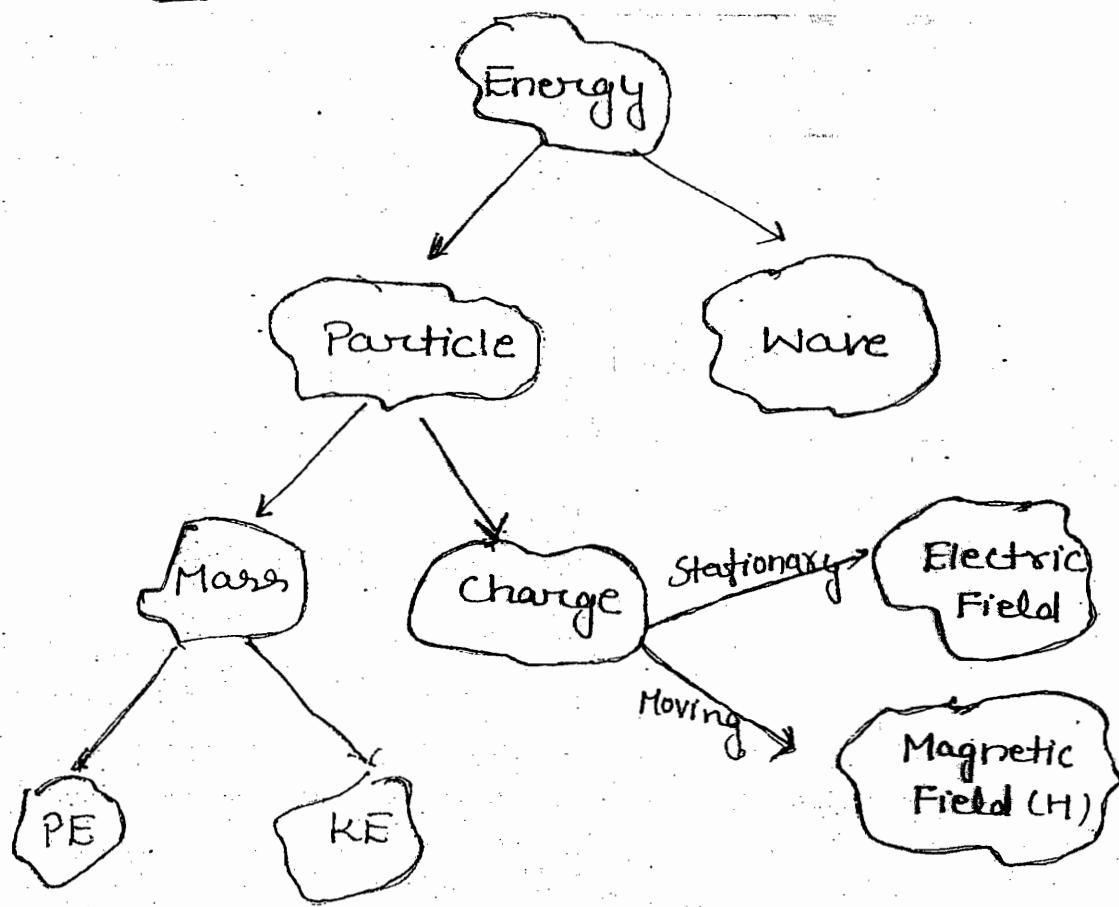
Text Books:-

1. William Hayt (For theory of first chapter)
2. For vector calculus → N.D. Sardikar
3. For problems → Schaum Series
4. Jorden → For EM Waves & Guided Waves
5. John A. Ryder → For VI Waves
6. K.B. Prasad & Balanis → Antenna
7. Nathan Ida → For whole EMT

→ VSRS 22@GMAIL.COM

→ VSR Swadesh

Static Electromagnetic Fields



Electric Field :-

It is a form of energy that is all around a charge & influences other charges nearby.

Magnetic Field (H) :-

It is a form of energy that is all around a moving charge & influences other moving charges nearby.

NOTE :-

The word influence in electric fields is a linear accelerating attractive or repulsive force on a charge resulting in straight line path. This is Coulomb's law and hence electric

$$\vec{F} = q(\vec{v} \times \vec{B}) = qvB \hat{i} \rightarrow \text{Linear path}$$

↓ energised

The word influence in magnetic field is only on moving charges such that forces perpendicular to velocity and the field.

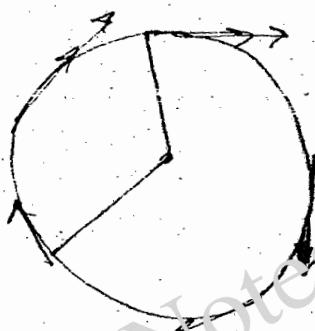
$\vec{F} \perp \text{displacement}$

So that workdone = 0

This is Lorentz law and particle acquire circular path

$$\vec{F} = q(v \times B) = \frac{qv^2}{r} \rightarrow \text{Circular path}$$

deflection



Summary :-

i) Charge is stationary then \rightarrow Electric Field (Static)



\rightarrow Time invariant

e.g:- (i) accumulated charge

\rightarrow Space Varying

(ii) D.C voltage

Electric field is energy in the above discussion

2. Charge is moving without acceleration or with constant velocity linearly with time

$$Q = kt$$

$$\frac{dQ}{dt} = k = I \quad (\text{dc current})$$

and DC current cause magnetic field which is static in nature

→ Magnetic field is the energy in above discussion.

3. Charge is moving with acceleration which creates Electric field ~~energy~~ $E(t)$ as well as magnetic field $H(t)$ which are power.

Basic Terms & Definitions:-

There is a measure of the field strength at any point in the field

(I) \vec{E} → Electric field Intensity (V/m or N/C)

for unit length

(II) \vec{H} → Magnetic flux density (Amp/m) [field intensity]

(III) \vec{B} → Magnetic flux density (Weber/m^2)
[or
Tesla]

(IV) $\vec{\delta}$ → Electric flux density ($\text{Coulombs}/m^2$)
[per unit area]

(V) ϵ → Permittivity of the medium

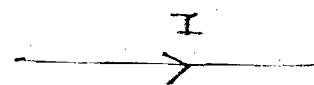
The ability to permit/allow/hold electric field in that medium

(III) current carry

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→ Scalar

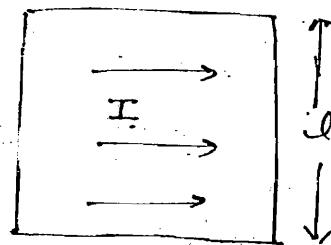
→ Amp



(III) surface current :-

$$\rightarrow \vec{K} = \frac{dI}{ds} = \text{Amp/m}$$

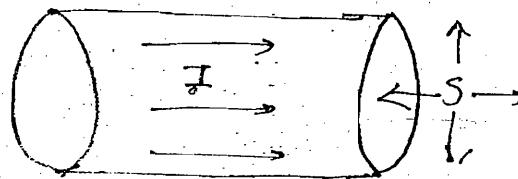
→ vector



(IV) Solid conductor (J) :-

→ vector

$$\rightarrow J = \frac{dI}{ds} = \text{Amp/m}^2$$



Vector Calculus :-

It is a study of directional integrations & directional derivatives.

Directional Integration :-

It is the study of the total effects or cumulative effects of a phenomena in a specific direction in a specific region.

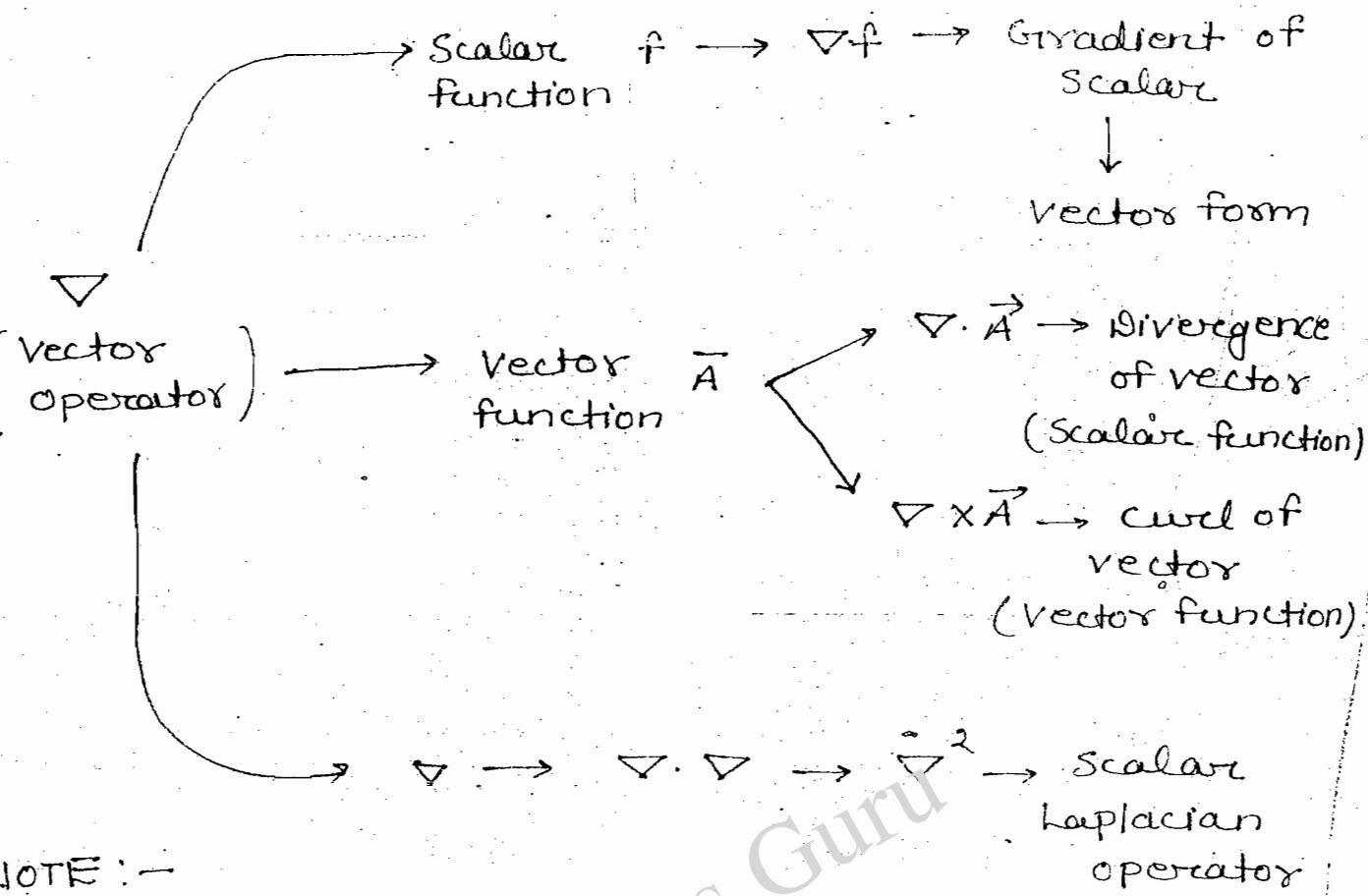
Directional derivative :-

It is the study of the instantaneous or rate of change analysis of a phenomenon in a specific direction in a specific region.

e.g:- Del - operator

$$\nabla \rightarrow \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z}$$

- It is used to get <https://notesguru.info> of change of various space varying quantities in 3d-space
- Del is called as vector spatial derivative operators



NOTE :-

Vector Identities :-

$$\nabla \times \nabla f = 0$$

$$\nabla \times (\nabla f) = 0$$

$$\text{curl (Grad. of scalar)} = 0$$

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

$$\text{Div. (curl of vector)} = 0$$

$$\nabla \times (\nabla \times \vec{A}) = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

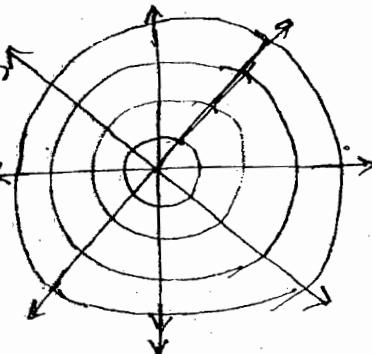
Consider a cause or source which has effects spread outward from the cause.

Eg:- (i) Light from a bulb

(ii) Air velocity from a punctured tyre

For all such phenomenon the strength decreases as the area of expansion inc. such that total outflow is same.

The total outflow through any enclosing surface is always the same (constant) and this constant depends on central cause.



$$\begin{aligned}\text{Total Outflow} &= \uparrow \text{Strength} \times \text{Area} \\ &= \text{constant.} \propto \text{Cause}\end{aligned}$$

The strength for all such phenomenon can be expressed as the constant per unit area & cause/area i.e. called as density.

NOTE:-

If the cause is a coulomb charge then the repulsive force or attractive force is called as electric flux and the strength is called as flux density (ψ_e) such that

$$\oint \psi_e \cdot dS = \psi_e \text{ total} \propto Q$$

\Rightarrow

$$\oint \psi_e \cdot dS = Q$$

If proportionality constant is 1 then this called Gauss law in integral form

NOTE 1-

- If the surface is closed then the effects are the partial flux crossing i.e.

$$\int \mathbf{g} \cdot d\mathbf{s} = \Psi_e \neq \Psi_{e\text{ total}}$$

This is not a Gauss law

vector

- Every closed surface is identified by a finite volume

eg:- $4\pi r^2$ sphere $\rightarrow \frac{4}{3}\pi r^3$

$2\pi r h$ cylinder $\rightarrow \pi r^2 h$

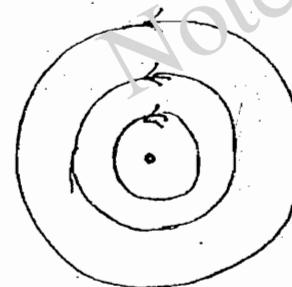
ba^2 cube $\rightarrow a^3$

- Closed surface having direction while finite volume doesn't having direction

Circulation and curl :-

Cause or Source \rightarrow

Effects \rightarrow around the cause



→ The total circulation in any closed length is always a constant and this constant depends on the central cause.

eg:- air velocity under the fan

$$\begin{aligned} \text{Total circulation} &= \text{strength} \downarrow \times \text{length} \uparrow \\ &= \text{constant} \propto \text{cause} \end{aligned}$$

Strength = $\frac{\text{constant}}{\text{length}}$ or $\frac{\text{cause}}{\text{length}}$

→ Intensity

Cause or Source → I ampere current

Effects → Magnetic line field

Strength → Magnetic field intensity (H^I)

$$\oint H \cdot d\ell = I$$

closed

This is amperes law
in integral form

Note :-

Every closed line is identified with a finite area enclosed

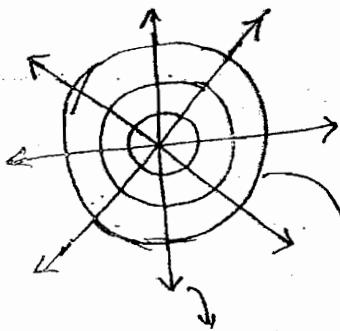
vector

vector

$$2\pi r \xrightarrow{\text{circle}} \pi r^2$$

$$4a \xrightarrow{\text{square}} a^2$$

Summary 1 :-



$$\Psi_e = \Phi \quad \text{i.e. outflow} = \text{Strength} \times \text{total area}$$

= Cause

Φ is constant

Electric flux has the units

of coulomb's

Φ 's direction & direction of divergence

closed surface element

strength around
the cause

$$\stackrel{\rightarrow}{\mathbf{H}} = \frac{d\mathbf{Q}}{ds} = \frac{\mathbf{C}}{m^2}$$

where ds = any closed surface element

$$\text{strength at the cause} = \frac{d\mathbf{Q}}{dv} = \frac{d}{dl} \left(\frac{d\mathbf{Q}}{ds} \right)$$

$$= \nabla \cdot \mathbf{H} = \text{divergence of } \mathbf{H}$$

$$= \frac{\text{outflow}}{\text{volume}} = \rho_v$$

$$\nabla \cdot \mathbf{H} = \rho_v$$

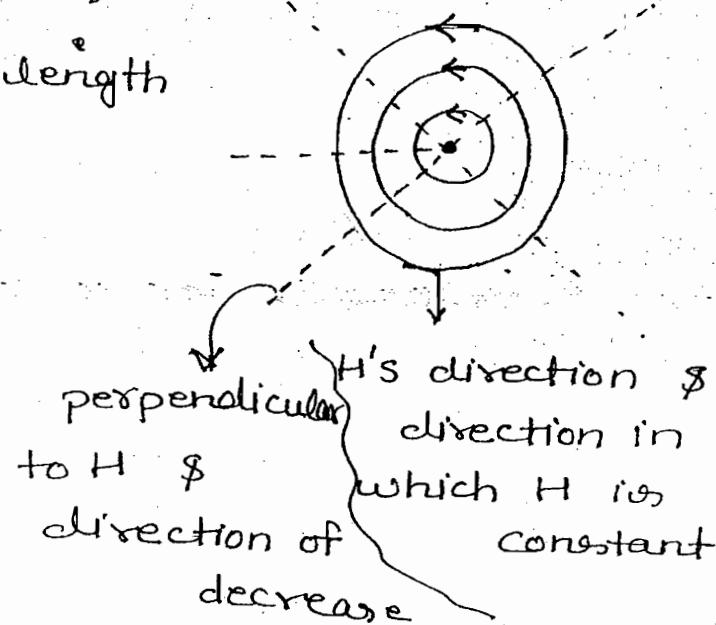
→ Gauss law in point form

- The (.) dot product signifies that \mathbf{H} 's change or derivative exists only when we move in direction of \mathbf{H} . This is called as directional derivative
- The (.) dot product indicates a surface (vector) derivative resulting in volume (scalar) derivative

Summary 2:-

$$\text{Total circulation} = \text{Strength} \times \text{length}$$

$$= \text{current}$$



$$\text{Total Circulation} = \frac{\text{Strength}}{\text{length}} = \text{current}$$

$$H = \frac{\text{Strength around}}{\text{the current}} = \frac{dI}{dl} = \frac{\text{Amp}}{m}$$

where dl = any close line element

$$\text{Strength at the} = \frac{d}{dl} \left(\frac{dI}{dl} \right) = \nabla \times H$$

= curl of H

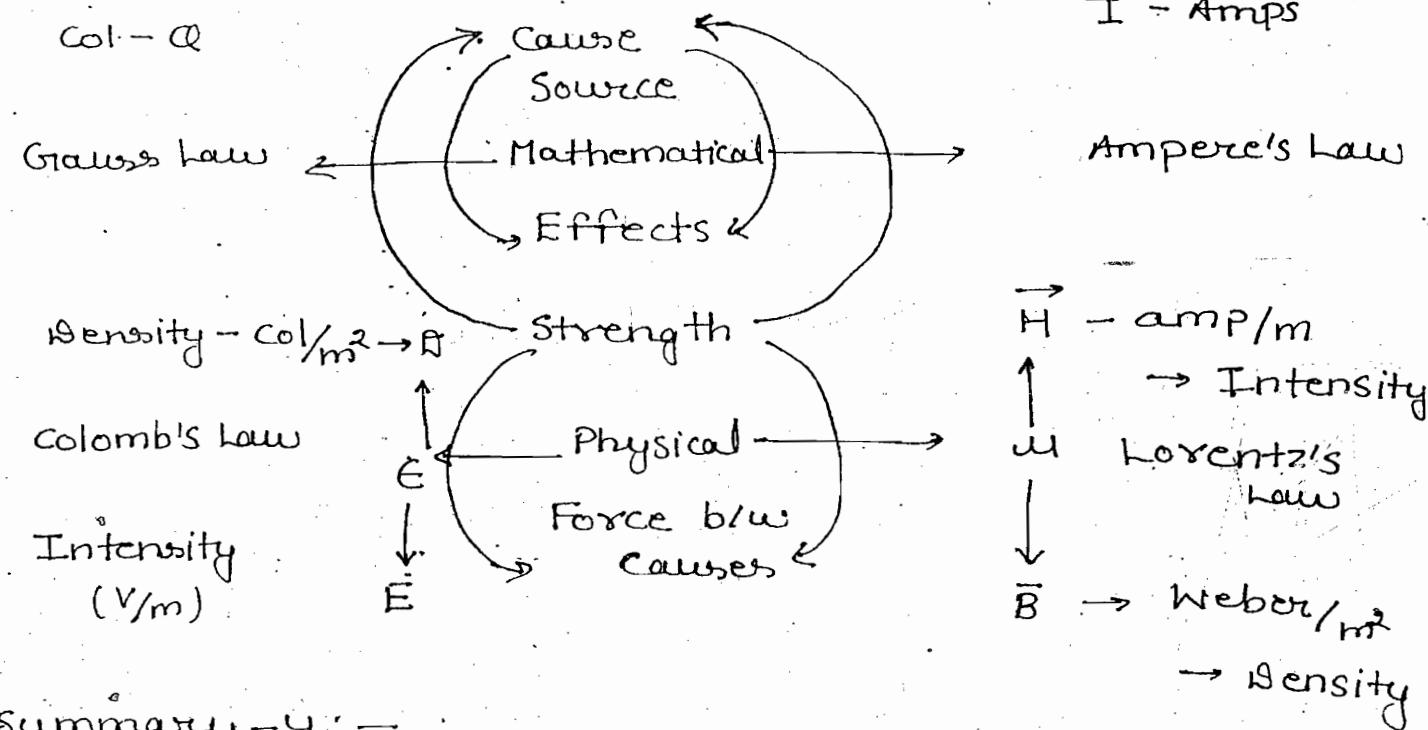
$$= \frac{\text{Circulation}}{\text{Area}} = J$$

$$\boxed{\nabla \times H = J}$$

→ This is Ampere's law in point form

→ The (\times) cross product signifies that H changes only when we move perpendicular to H .

→ The (\times) cross product implies a vector (line) derivative resulting a vector (surface) derivative

Summary - 3 :-Summary - 4:-

Divergence & Stoke's theorem:-

$$\oint \mathbf{B} \cdot d\mathbf{s} = \alpha = \int \mathbf{P}_V dv = \int (\nabla \cdot \mathbf{B}) dv$$

This is Gauss divergence theorem

$$\oint \mathbf{H} \cdot d\mathbf{l} = I = \int \mathbf{J} \cdot d\mathbf{s} = \int (\nabla \times \mathbf{H}) \cdot d\mathbf{s}$$

Stoke's theorem

Note:-

Wrong Notations:-

$$\oint \mathbf{H} \cdot d\mathbf{l} \neq \int (\nabla \times \mathbf{H}) \cdot d\mathbf{s} \quad \rightarrow \quad \oint \mathbf{B} \cdot d\mathbf{s} = \int (\nabla \cdot \mathbf{B}) dv$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int (\nabla \cdot \mathbf{H}) ds \quad \rightarrow \quad \oint \mathbf{B} ds = \int (\nabla \times \mathbf{B}) dv$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = \oint (\nabla \times \mathbf{H}) \cdot d\mathbf{s}$$

Note:-

Maxwell equations having various formats, 2 field, 2 (.) dot, 2 (x) cross, 2 line, 2 surface, 4 eqn

summary 5:-

Maxwell's Equation

Integral form

1. $\oint \mathbf{A} \cdot d\mathbf{s} = 0$ [Gauss law]

2. $\oint \mathbf{E} \cdot d\mathbf{l} = 0$ [Irrotational vector]

3. $\oint \mathbf{B} \cdot d\mathbf{s} = 0$ [solenoidal]

4. $\oint \mathbf{H} \cdot d\mathbf{l} = I$ [Ampere's law]

Point form

1. $\nabla \cdot \mathbf{B} = \rho_v$

2. $\nabla \times \mathbf{E} = 0$

3. $\nabla \cdot \mathbf{B} = 0$

4. $\nabla \times \mathbf{H} = \mathbf{J}$

Note:-

→ $\nabla \times \mathbf{E} = 0$. (Always)

Curl is generally applied with intensity vector

$$\nabla \times \left(\frac{\mathbf{B}}{\epsilon} \right) = 0$$

If $\epsilon = \text{constant}$ i.e. medium is homogeneous & isotropic

$$\Rightarrow \frac{1}{\epsilon} \nabla \times \mathbf{B} = 0 \Rightarrow \boxed{\nabla \times \mathbf{B} = 0}$$

→ Acc. to stoke's theorem (For integral form)

$$\oint \mathbf{E} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{E}) \cdot d\mathbf{s} = 0$$

$$\nabla \cdot B = 0 \quad (\text{Always})$$

Divergence is generally applied with density vector

$$\nabla \cdot (\mu H) = 0$$

If μ is constant i.e. medium is homogeneous and isotropic

$$\mu (\nabla \cdot H) = 0$$

$$\nabla \cdot H = 0 \rightarrow \text{Also correct}$$

For integral form, apply divergence theorem

$$\oint B \cdot dS = \int (\nabla \cdot B) dV = 0$$

II) $\oint E \cdot dl = 0 \rightarrow \text{KVL}$

III) $\oint B \cdot ds = 0 \rightarrow \text{KCL}$

Coordinate Systems

It is a way of addressing/locating points in 3d-space from a known reference

Reference:-

3 infinite mutually orthogonal planes
 \rightarrow XY, YZ & ZX planes

Cartesian Coordinate System:-

e.g:- 1. Uniform plane waves

2. Rectangular wave guides

3. Capacitor plates

\rightarrow Parameters (x, y, z)

\rightarrow Unit vector (a_x, a_y, a_z)

1 infinite line

axial

z-axis

Cylindrical coordinate system

- eg:- (I) Line charges
 (II) I carrying waves wires
 (III) cylindrical waveguide

Parameter $\rightarrow \rho, \phi, z$

Unit vectors $\rightarrow a_\rho, a_\phi, a_z$

Reference :-

1 Single point

point

origin

Spherical coordinate systems

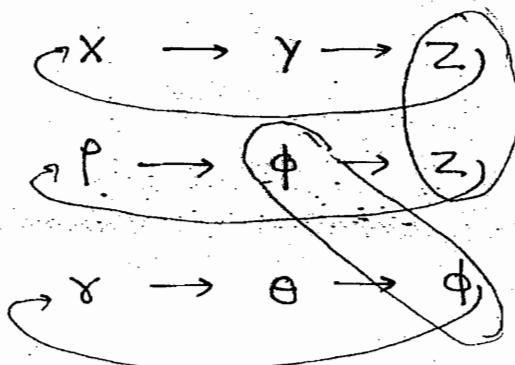
- eg:- (I) point charges

- (II) antennas

Parameter $\rightarrow r, \theta, \phi$

Unit vectors $\rightarrow a_r, a_\theta, a_\phi$

Relation b/w coordinate systems :-



All the three co-ordinate systems are unit

orthogonal

orthonormal

right handed system

Orthogonal :-

The (.) dot product of any two unit vectors of the same coordinate system is always zero

$$\rightarrow \mathbf{a}_x \cdot \mathbf{a}_y = \mathbf{a}_y \cdot \mathbf{a}_z = \mathbf{a}_z \cdot \mathbf{a}_x = 0$$

$$\rightarrow \mathbf{a}_x \cdot \mathbf{a}_x = \mathbf{a}_y \cdot \mathbf{a}_y = \mathbf{a}_z \cdot \mathbf{a}_z = 1$$

Orthonormal :-

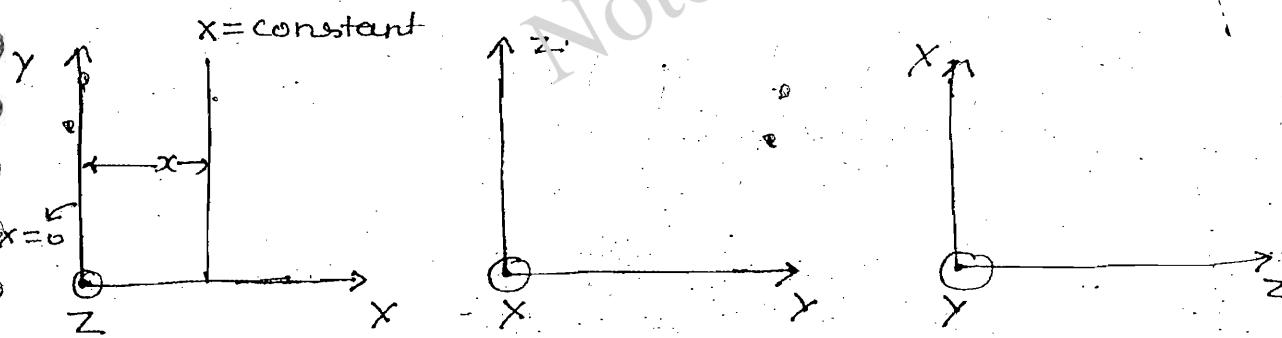
The (x) cross product of any two unit vectors of the same co-ordinate system is always the third unit vector

$$\mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$

$$\mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x$$

$$\mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y$$

Cartesian Coordinate Systems :-



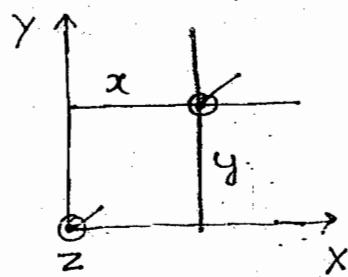
x → It is the shortest distance or perpendicular distance of the point from YZ plane

Note :-

The locus of all the points with $x = \text{constant}$ is an infinite plane parallel to YZ plane

Note! -

The locus of all the points with $x, y = \text{constant}$
is an infinite line parallel to z-axis

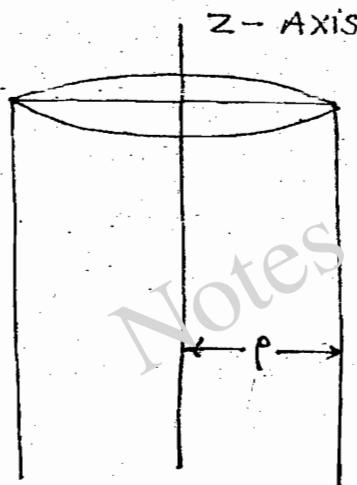


Cylindrical Coordinate System:

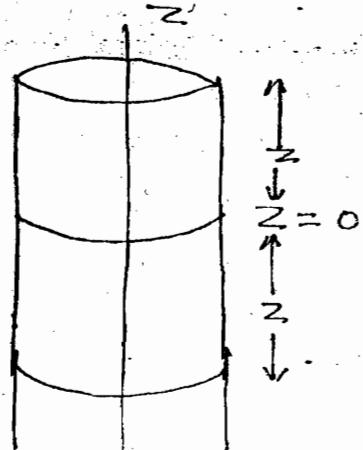
$\rho \rightarrow$ It is the shortest distance / perpendicular distance / radial off distance of the point from reference line

Note! -

The locus of all the points with $\rho = \text{constant}$.
is a concentric cylinder around a reference line

Range $\rightarrow [0, \infty)$ $z \rightarrow$

It is the height of the point along the reference line

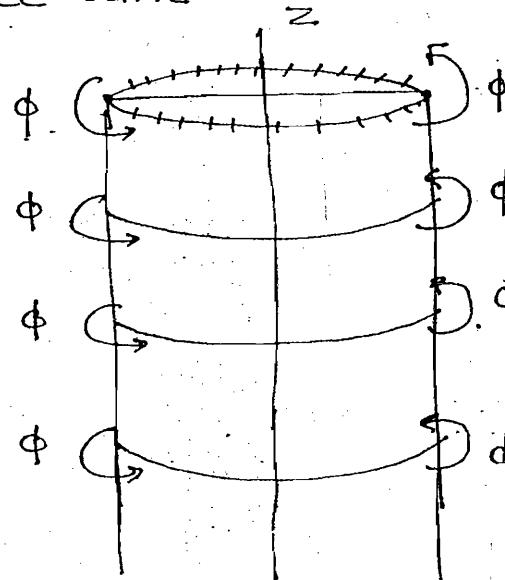
Range: $(-\infty, \infty)$ 

Note:-

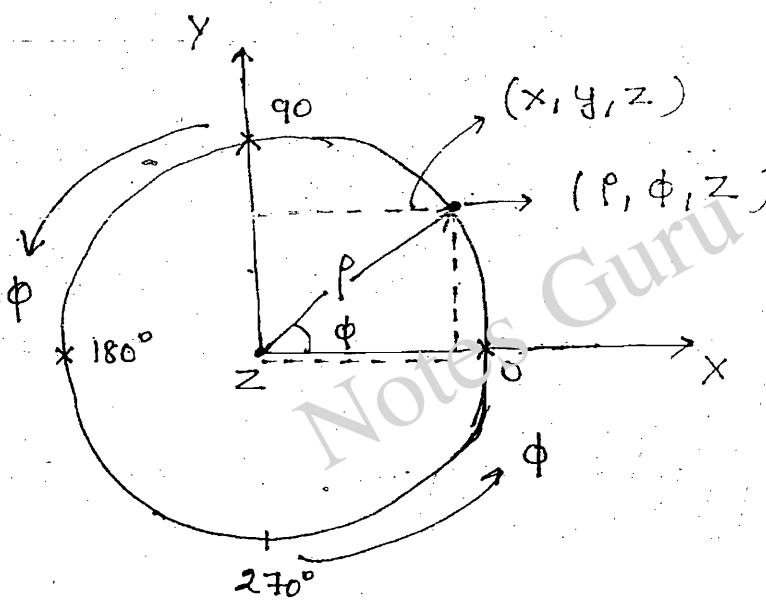
The locus of all points such that $\rho, z = \text{constant}$ is a concentric circle around the line.

ϕ :-

It is the orientation of the point around the reference line



Range $\leftarrow [0, 2\pi\right)$



Point Transformation:-

Cartesian \longleftrightarrow cylindrical

$$x = \rho \cos \phi$$

$$y = \rho \sin \phi$$

$$z = z$$

$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}(y/x)$$

Spherical Coordinate System

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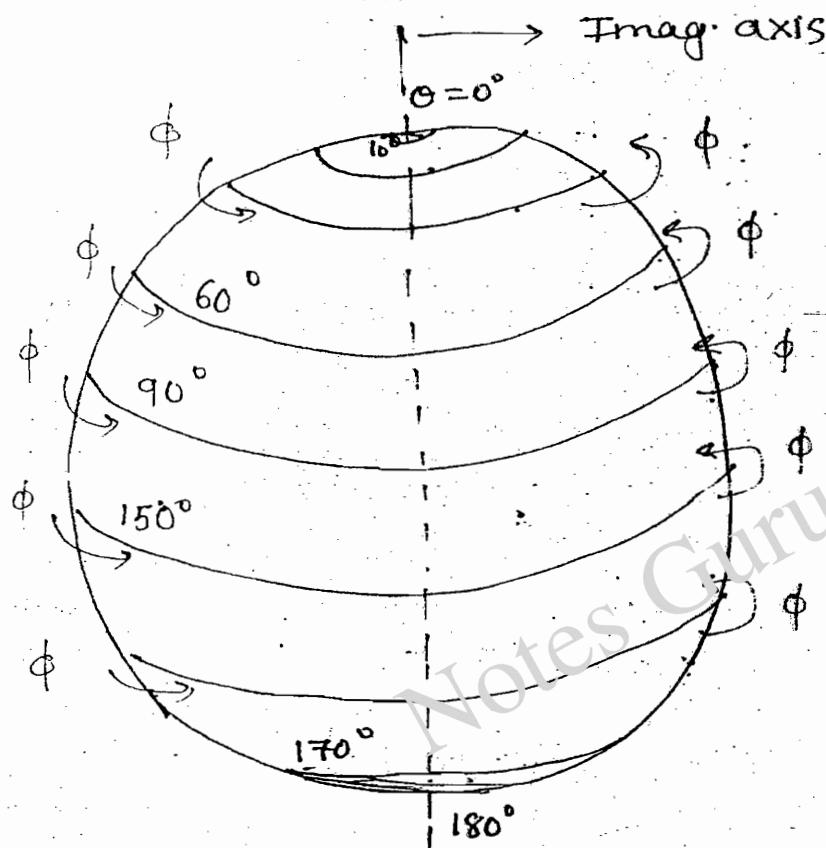
$r \rightarrow$

It is the radial distance or shortest distance of the point from reference point.

Note:-

The locus of all the points with r is constant is its concentric sphere around the origin.

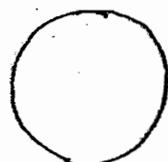
Range $\rightarrow [0, \infty)$



$\theta \rightarrow$

It is the angle of cone whose base lies a circle on the $r = \text{constant}$ sphere.

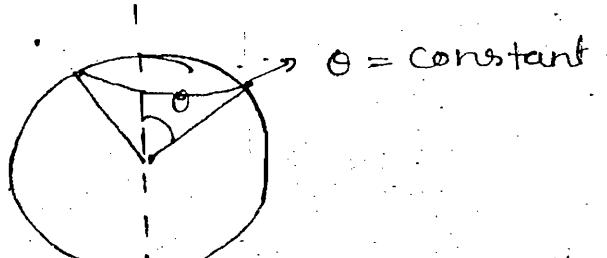
Range $\rightarrow [0, \pi]$



Identification of θ

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Consider an imaginary axis through the centre of the sphere and take a radial segment inclined by θ with imaginary axis. Rotate the radial segment which results in a cone whose base is a circle on the $r = \text{constant}$ sphere.



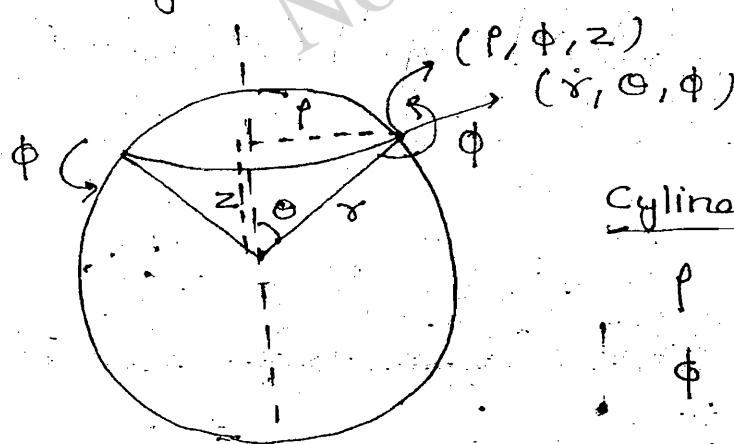
ϕ :-

It is the orientation angle around the imaginary axis.

Range of $\phi \rightarrow [0, 2\pi]$

Point transformation :-

If z-axis of cylindrical coordinates coincide with imaginary axis of spherical coordinates then ϕ is same in both the coordinate system.



Cylindrical \leftrightarrow Spherical

$$\rho = r \sin \theta$$

$$\phi = \phi$$

$$z = r \cos \theta$$

Point Transformation :-

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Cartesian \longleftrightarrow Spherical

$$x = \rho \sin\theta \cos\phi$$

$$y = \rho \sin\theta \cdot \sin\phi$$

$$z = \rho \cos\theta$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

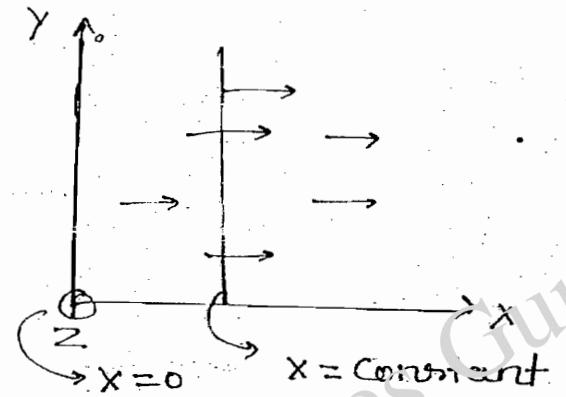
Unit Vectors & Orthogonality in coordinate system:-

Unit vector of Parameter :-

It has unit magnitude & a direction in which the parameter increases.

i) Cartesian coordinate system :-

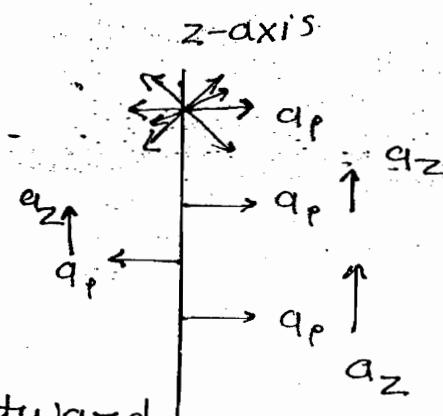
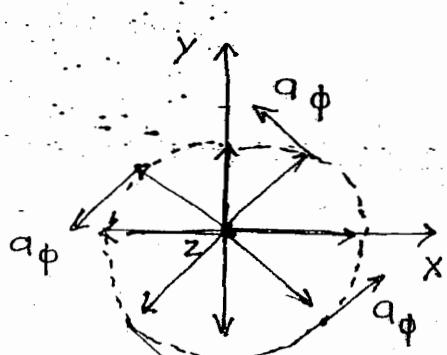
a_x :-



It is always normal to yz plane or any plane which has $x = \text{constant}$.

ii) Cylindrical coordinate system :-

a_ρ :-



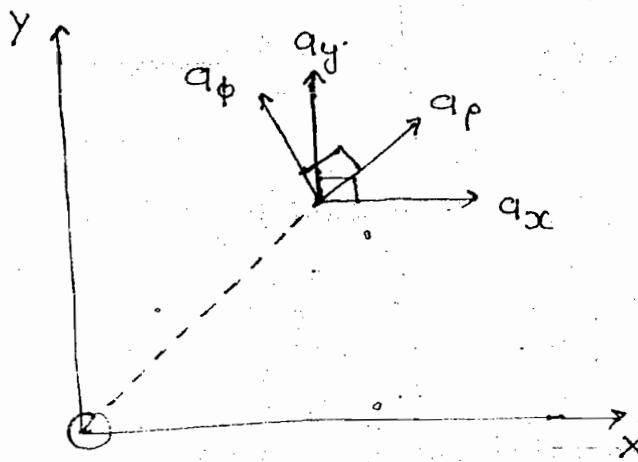
It is radially outward from the reference line

It is tangentially to the reference line

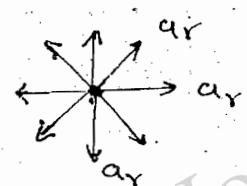
$$\boxed{a_r \perp a_\phi \perp a_z}$$

It is linearly along the reference line

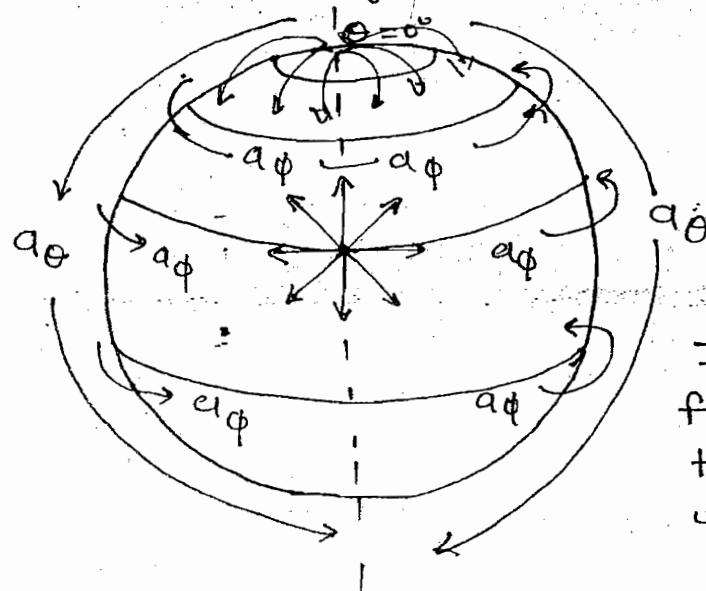
Summary:-



Spherical Coordinate System:-



It is radially outward from the origin



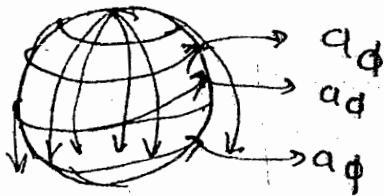
It is vertically
from the top to
the bottom of the
sphere

It is horizontally
axis.

maginacy

$\phi = \text{constant}, \theta \uparrow$

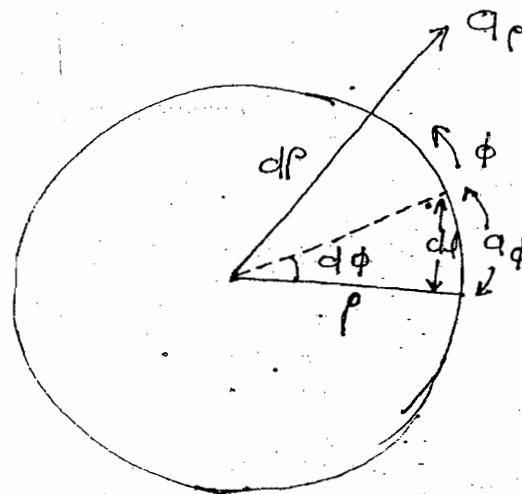
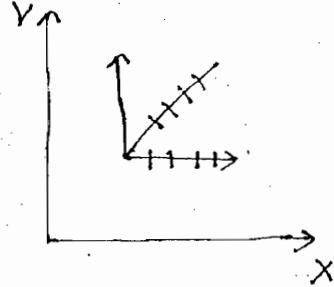
$\phi = \text{constant}; \theta \uparrow$



$$a_r \perp a_\theta + a_\phi$$

Line as a Vector:-

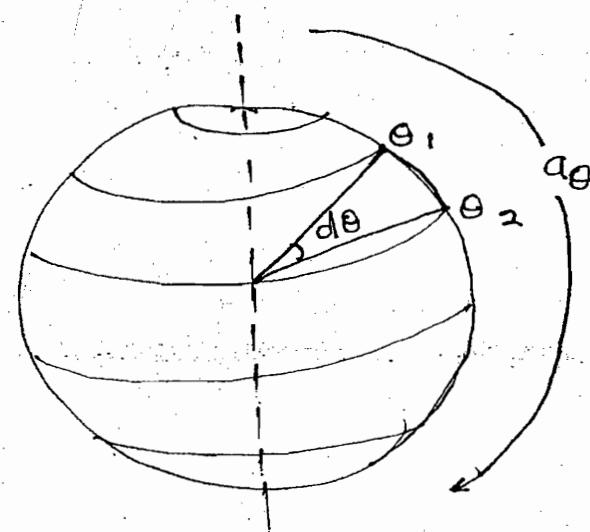
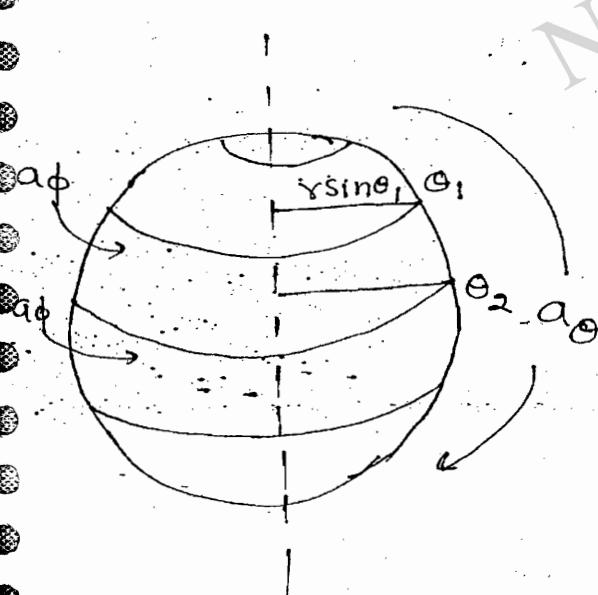
It has a magnitude equal to its length and a direction in which the length parameter increase

Cartesian :-

$$d\vec{r} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$$

Spherical :-

$$d\vec{r} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin\theta \cdot d\phi \hat{a}_\phi$$



In angular direction length is curvature

Note :-

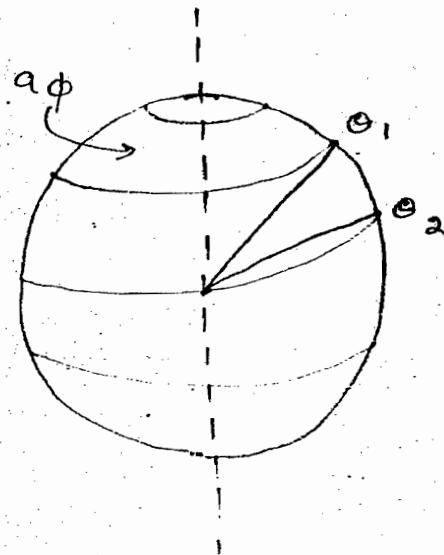
In angular directions length is a curvature
not straight line or linear

$$\text{Curvature length} = \text{Radius} \times \text{angle}$$

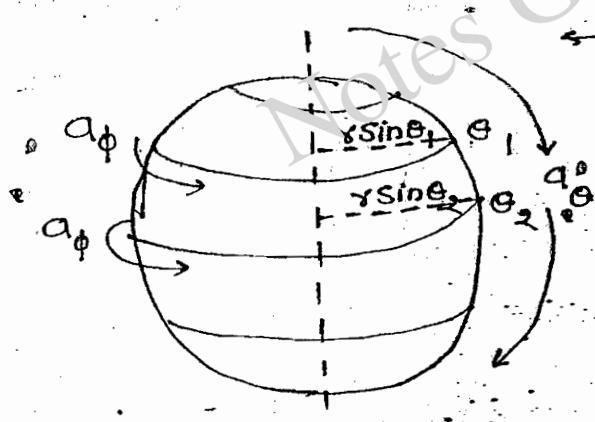
$$\text{Cartesian} \rightarrow d\vec{l} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$$

$$\text{Cylindrical} \rightarrow d\vec{l} = dp \hat{a}_p + p d\phi \hat{a}_\phi + dz \hat{a}_z$$

$$\text{Spherical} \rightarrow d\vec{l} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin\theta d\phi \hat{a}_\phi$$



Top View



Note:-

In ϕ direction in spherical coordinating system curvature length is height on the sphere dependent i.e. it depends on the θ value of the circle.

Parameters :-

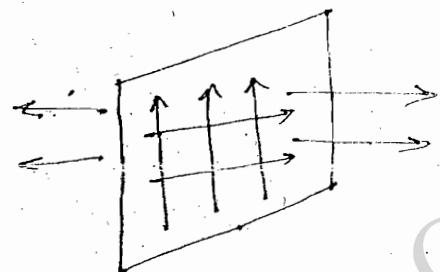
Scaling Factors

x	y	z	1	1	1
r	φ	z	1	r	1
r	θ	φ	1	r	rsinθ
u	v	w	h ₁	h ₂	h ₃

$$d\vec{u} = h_1 du \hat{a}_x + h_2 dv \hat{a}_y + h_3 dw \hat{a}_w$$

Surface as a vector :-

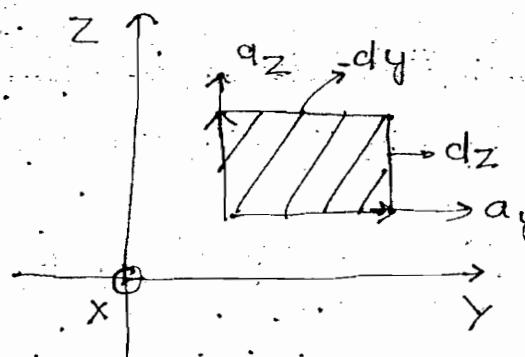
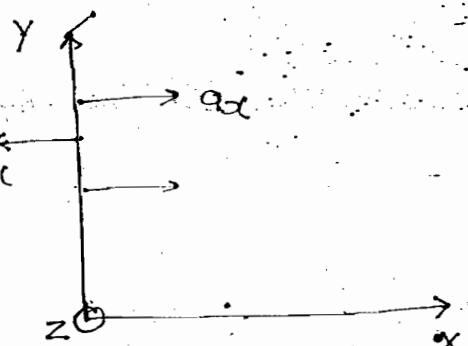
It has a magnitude equal to its area and a direction normal to the plane of surface.



Direction is unique only when we take normal to the surface as there can be two tangential directions.

x = constant surface

x = o plane, YZ plane

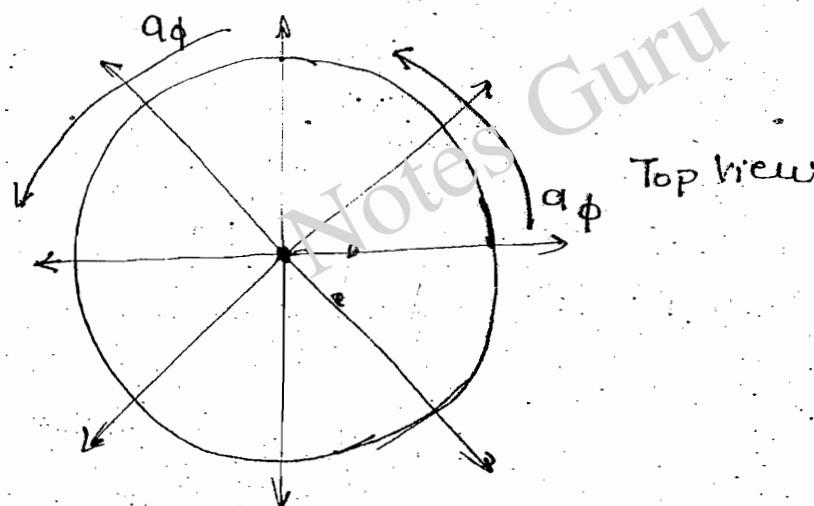
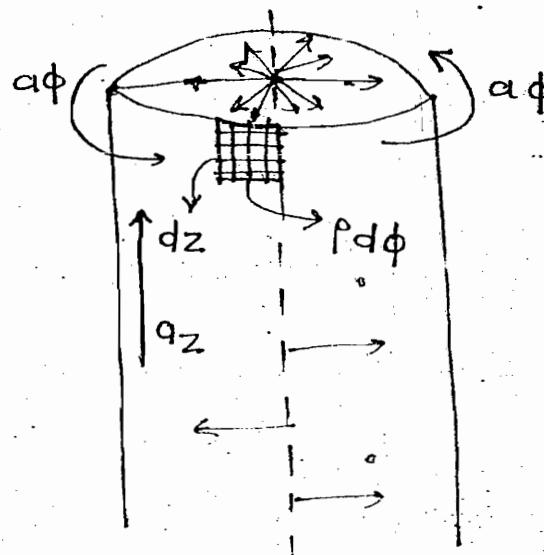


Cartesian :-

$$d\vec{s} = dy dz \hat{a}_x + dz dx \hat{a}_y + dx dy \hat{a}_z$$

↓ ↓
mag. direction

Cylindrical Coordinate system :-

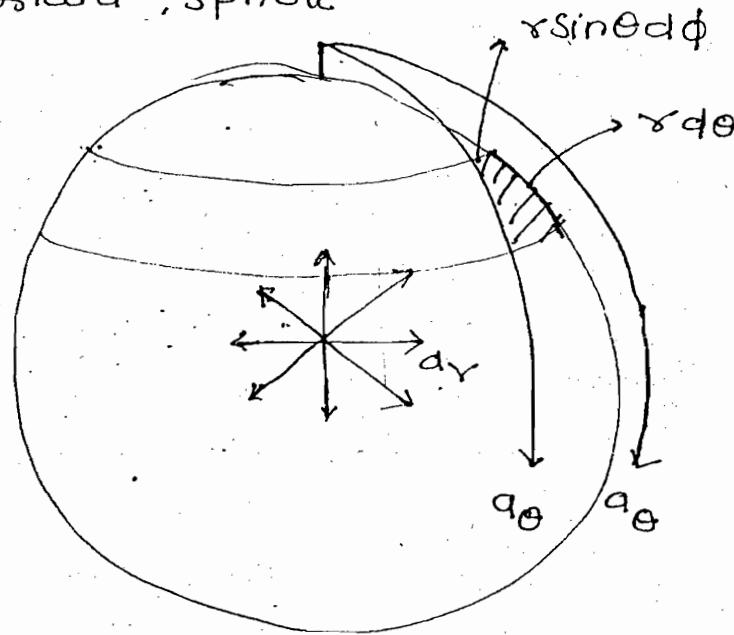


\hat{a}_ρ = normal i.e. direction

\hat{a}_ϕ, \hat{a}_z = tangential \rightarrow II to surface

$$d\vec{s} = \rho d\phi dz \hat{a}_\rho + dz d\rho \hat{a}_\phi + \rho d\rho d\phi \hat{a}_z$$

$\gamma = \text{constant}$, sphere



$a_r = \text{normal}$
↓
direction

$a_\theta, a_\phi = \text{tangential}$

$$d\vec{s} = r^2 \sin\theta d\theta d\phi a_r + r \sin\theta d\phi dr a_\theta + r dr d\theta a_\phi$$

$$ds = h_2 h_3 dv dw a_u + h_3 h_1 dw du a_v + h_1 h_2 du dv a_w$$

Summary:-

1 parameter = constant] → surface
2 parameter = variable	

Surface direction = Constant direction
= UNIQUE

2 Parameter = Constant] → line
1 Parameter = Variable	

Lines direction = Variable's direction
= UNIQUE

3 Parameters = <https://notesguru.info> → Volume

3 Parameters = constant] → Point

Volume as a scalar :-

It has no unique direction & it is the scalar triple product of lengths in all three dimensions

$$\begin{aligned} dv &= dx dy dz \\ &= r dr d\theta d\phi \\ &= r^2 \sin\theta dr d\theta d\phi \\ &= h_1 h_2 h_3 du dv dw \end{aligned}$$

Workbook -1

2. At Point A

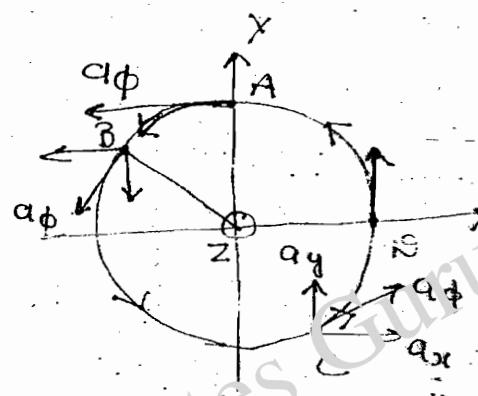
$$a_\phi = -q_x$$

At Point B

$$a_\phi = \frac{-q_x - q_y}{\sqrt{2}}$$

At Point C

$$a_\phi = \frac{q_x + q_y}{\sqrt{2}}$$



divided by $\sqrt{2}$
to make
magnitude = 1

At point D

$$a_\phi = q_y$$

3.

$$\vec{B} = xy q_x + yz q_y + zx q_z \text{ A/m}^2$$

$$y = 2 \quad 0 < x < 4$$

$$0 < z < 2$$

Since only 2 variable are present. Hence it is not a closed surface

$$ds = dx dz q_y$$

$$\int \mathbf{B} \cdot d\mathbf{s} = \Psi_e$$

$$\Rightarrow \Psi_e = \int_{x=0}^4 \int_{z=0}^2 yz dx dz \Big|_{y=2}$$

$$= 2 \frac{z^2}{2} \Big|_0^2 \cdot x \Big|_0^4 = 16 C \rightarrow \text{flux}$$

4. $\vec{D} = 5(r-3)^2 a_r$
 $r=4, 0 < \phi < \pi$
 $-5 < z < 5$

$$ds = r d\phi dz a_r$$

$$\Psi_e = \int_{\phi=0}^{\pi} \int_{z=-5}^5 5(r-3)^2 a_r \cdot r d\phi dz a_r \Big|_{r=4}$$

$$= 20 \phi \Big|_0^{\pi} z \Big|_{-5}^5$$

$$= 200\pi$$

Divergence and curl of vector :-

$$\vec{A} = A_u a_u + A_v a_v + A_w a_w$$

$$\nabla \cdot \vec{A} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u} (h_2 h_3 A_u) + \frac{\partial}{\partial v} (h_3 h_1 A_v) + \frac{\partial}{\partial w} (h_1 h_2 A_w) \right]$$

Cartesian :-

$$\nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Cylindrical :-

$$\nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial}{\partial \phi} (r A_\rho) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Spherical :-

$$\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

Workbook - I

$$\vec{v} = \rho \cdot z \cos^2 \phi \hat{a}_z$$

$$\rho_v = ? \quad \text{at } (1, \pi/4, 3)$$

$$\nabla \cdot \vec{v} = \rho_v$$

$$\nabla \cdot \vec{v} = \frac{\partial (\rho z \cos^2 \phi)}{\partial z}$$

$$= \rho \cos^2 \phi = 1 \cdot \frac{1}{2} = 0.5 \text{ c/m}^3$$

$$\vec{v} = \rho \cdot z \cdot \cos^2 \phi \hat{a}_p$$

$$\nabla \cdot \vec{v} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \cdot \rho z \cos^2 \phi)$$

$$= \frac{1}{\rho} z \cos^2 \phi \cdot 2\rho = 2 \cdot 3 \cdot \frac{1}{2} = 3 \text{ c/m}^3$$

$$\vec{F} = \rho \hat{a}_p + \rho \sin^2 \phi \hat{a}_\phi - z \hat{a}_z$$

$$\nabla \cdot \vec{F} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \cdot \rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} (\rho \cdot \sin^2 \phi) + \frac{\partial (-z)}{\partial z}$$

$$= 2 + 2 \sin \phi \cos \phi - 1$$

$$= 1 + \sin 2\phi$$

Ans - D

Ques - GATE 2012

$$\vec{A} = k \gamma^n \hat{a}_x \quad n=?$$

$$\text{If } \nabla \cdot \vec{A} = 0$$

SOLN:-

$$\nabla \cdot \vec{A} = \frac{1}{\gamma^2} \frac{\partial}{\partial r} \quad \text{https://notesguru.info}$$

$$= \frac{k}{\gamma^2} (n+2) \gamma^{n+1} = 0$$

$$\Rightarrow n = -2, \text{ Ans.}$$

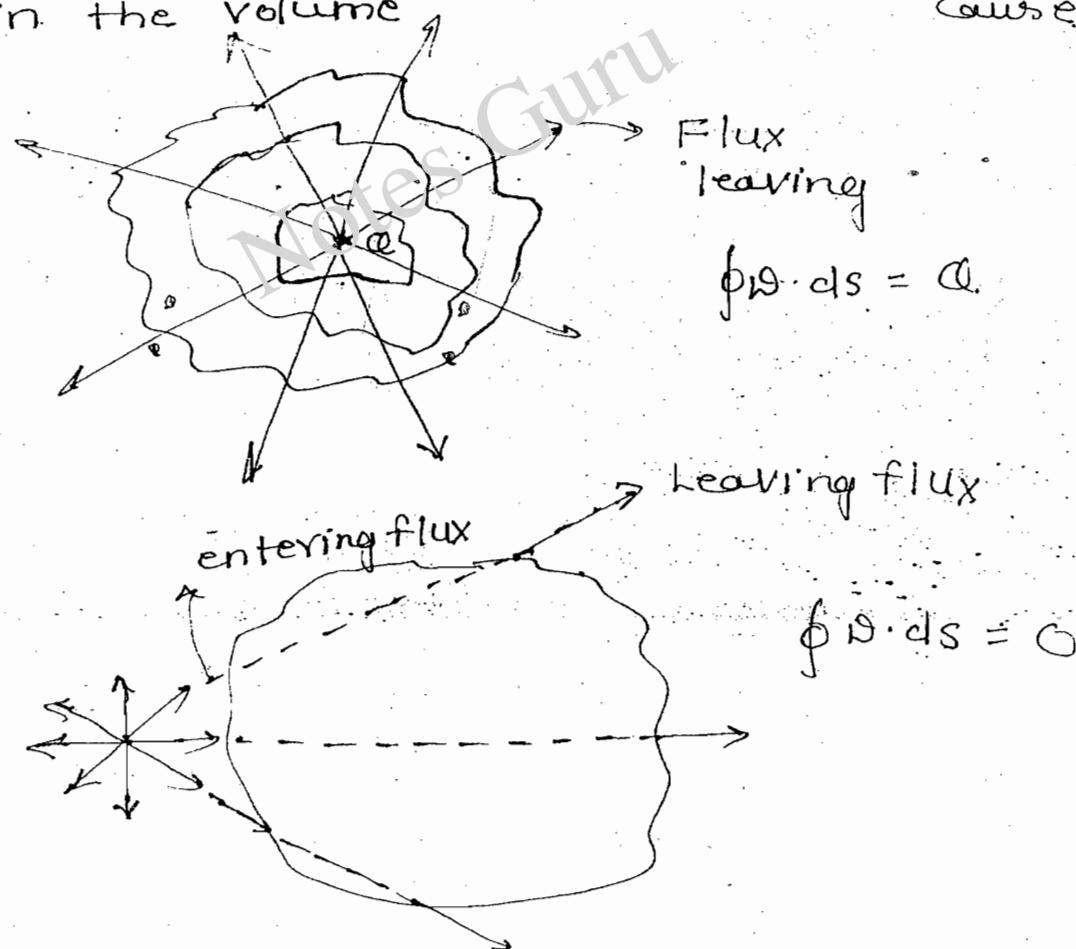
Curl of a vector \vec{A} :-

$$\nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 q_u & h_2 q_v & h_3 q_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$$

Static Electric Fields :-

Gauss's Law :- total effects

The net electric flux leaving any closed surface is always equal to the charge enclosed in the volume cause



The word electric field physically represent the attractive or repulsive force on any the test charge placed in the electric field. Hence electric field or electric flux or lines of force physically represent the direction in which a test charge moves away when placed in the field

- When the surface is closed the total outflow or flux leaving is independent of density and area i.e.

$$\oint \mathbf{D} \cdot d\mathbf{s} = 0$$

- If the charge is outside the effects still exists but net effects is zero

$$\begin{matrix} \text{entering} \\ \text{flux} \end{matrix} = \begin{matrix} \text{leaving} \\ \text{flux} \end{matrix}$$

$$\oint \mathbf{D} \cdot d\mathbf{s} = 0$$

- If the surface is not completely enclosing the flux crossing depends on the density of the flux and the surface area of consideration.

This is crossing flux

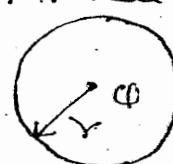
$$\oint \mathbf{D} \cdot d\mathbf{s} = \Phi_e$$

↓
Not Gauss law

Application of Gauss law :-

- Electric field strength of a point charge:-

Consider a concentric symmetrical surface such that σ is same everywhere



$$\oint \vec{B} \cdot d\vec{s} = q$$

constant

or non-directed

$$\Rightarrow \vec{B} \cdot \vec{S} = Q$$

$$\Rightarrow B = \frac{Q}{S}$$

$$\Rightarrow B = \frac{Q}{4\pi r^2} a_r \quad C/m^2$$

→ Coulomb had a different measure of field strength i.e. the force measured & he related this with charge measure using ϵ . This is called as intensity.

$$\vec{E} = \frac{\vec{F}}{q_1} = \frac{\vec{B}}{\epsilon} = \frac{Q}{4\pi\epsilon r^2} a_r$$

Intensity
(N/C)

$$\Rightarrow F = \frac{q_1 q_2}{4\pi\epsilon r^2}$$

This is Coulomb's law

Static Magnetic Fields

Biot - Savart's Law (Ampere's law for current elements) :-

It is derived from Ampere's law & consider a $d\vec{l}$ length I (dc current) carrying as a basic cause of H field

$I d\vec{l}$ — (Amp-m) is basic cause of H field

$$\vec{H} = \frac{I d\vec{l} \times a_r}{4\pi r^2}$$

Intensity
(Amp/m)

M M

→ \vec{H} direction = <https://notesguru.info> Radial vector
direction [cross] from current

→ Lorentz had a different measure of field strength i.e. density measure which is physically force per basic cause

$$\vec{B} = \mu \vec{H} = \frac{\mu I dl \times \hat{a}_x}{4\pi r^2} = \frac{\vec{F}}{Idl} = \frac{\text{Newton}}{\text{Amp-m}}$$

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\Rightarrow dF = dq \left(\frac{dl}{dt} \cdot B \cos\theta \right) = Idl B$$

$$\Rightarrow B = \frac{F}{Idl}$$

Note:-

$$D.E = \frac{col}{m^2} \times \frac{\text{Newton}}{col} = \frac{\text{Newton}}{m^2}$$

= Electric pressure at that point

$$B.H = \frac{\text{Newton}}{\text{Amp-m}} \times \frac{\text{Amp}}{m} = \frac{\text{Newton}}{m^2}$$

= Magnetic Pressure.

$$\frac{\text{Newton} \times m}{m^2 \times m} = \frac{\text{Joules}}{m^3} = \text{Energy density}$$

$$\frac{1}{2} D.E = \frac{1}{2} \epsilon E^2 = \text{Energy density in electric field}$$

$$\frac{1}{2} B.H = \frac{1}{2} \mu H^2 = \text{Energy density in } H \text{ field}$$

Workbook

$$\varrho \rightarrow (0, 0, 3)$$

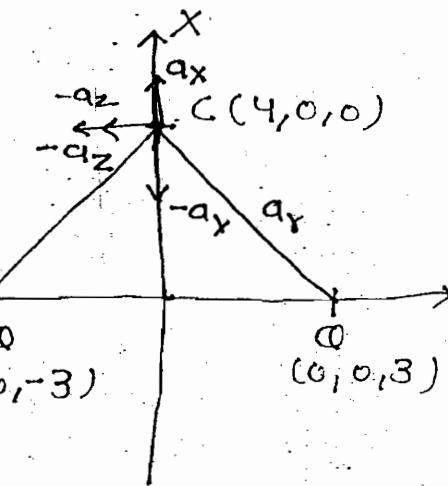
$$-\varrho \rightarrow (0, 0, -3)$$

E at C (4, 0, 0)

$$E = \frac{\varrho}{4\pi\epsilon_0 r^2} \hat{a}_r$$

$$E_1 = (\alpha_x, -\alpha_z)$$

$$(\varrho) = \frac{4\alpha_x - 3\alpha_z}{5} (0, 0, -3)$$

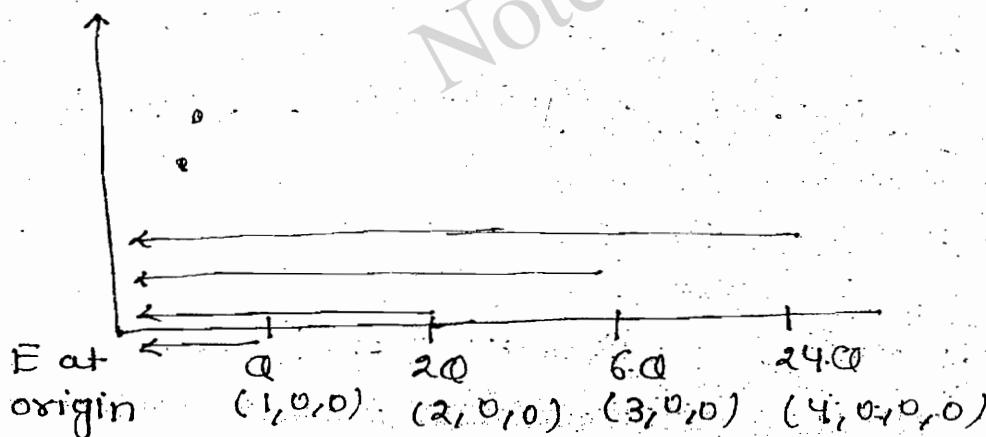


$$E_2 = (-\alpha_x, -\alpha_z)$$

$$(-\varrho) = \left(-\frac{4\alpha_x + 3\alpha_z}{5} \right)$$

$$E_{\text{total}} = -\alpha_z$$

(direction)



$$E_T = E_1 + E_2 + E_3 + E_4$$

$$E = \frac{\varrho}{4\pi\epsilon_0 r^2} \hat{a}_r$$

$$\alpha_x = -\alpha_x$$

$$E_T = \left(\frac{Q}{4\pi\epsilon(1)^2} + \frac{2Q}{4\pi\epsilon(2)^2} + \frac{6Q}{4\pi\epsilon(3)^2} + \frac{24Q}{4\pi\epsilon(4)^2} \right) \hat{a}_x$$

$$= \frac{Q}{4\pi\epsilon} \left(1 + \frac{1}{2} + \frac{2}{3} + \frac{3}{2} \right) (-\hat{a}_x)$$

- Net flux leaving = $\oint \mathbf{D} \cdot d\mathbf{s} = Q = \text{charge enclosed}$

Sphere \rightarrow centre = origin

radius = 6m

$$2C \rightarrow (4, 8, 3) \rightarrow d_1 = \sqrt{4^2 + 8^2 + 3^2} > 6$$

$$8C \rightarrow (2, -1, -3) \rightarrow d_2 = \sqrt{2^2 + 1^2 + 3^2} < 6$$

$$-12C \rightarrow (-4, 0, 1) \rightarrow d_3 = \sqrt{4^2 + 1^2} < 6$$

Net flux leaving = $-4C$

Centre = $(2, -3, 2)$

$$d_1 = \sqrt{2^2 + 1^2 + 1^2} > 6 \rightarrow \text{out}$$

$$d_2 = \sqrt{0^2 + 2^2 + 5^2} < 6 \rightarrow \text{in}$$

$$d_3 = \sqrt{6^2 + 3^2 + 1^2} > 6 \rightarrow \text{out}$$

Net flux leaving = $8C$

Net flux leaving = $Q = -\rho_L V$

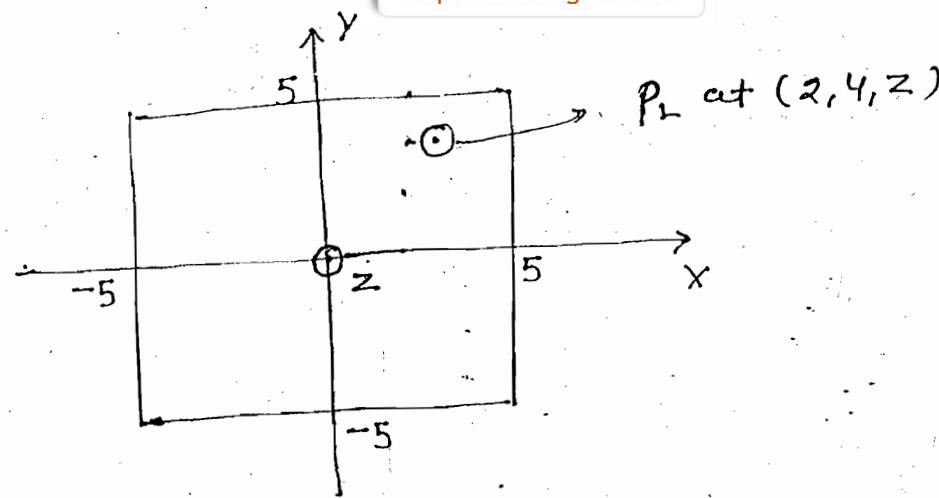
$$\rho_L = 15 \text{ nC/m}$$

at $x = 2, y = 4$ for all z

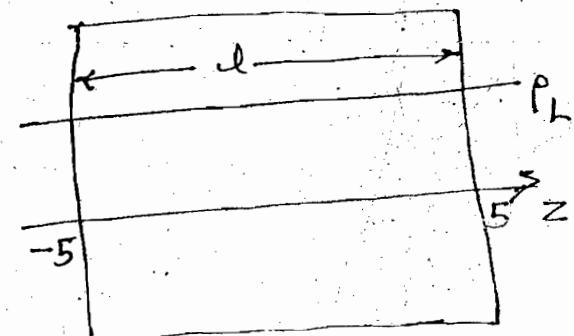
Closed surface = $-5 < x < 5$

$-5 < y < 5$

$-5 < z < 5$

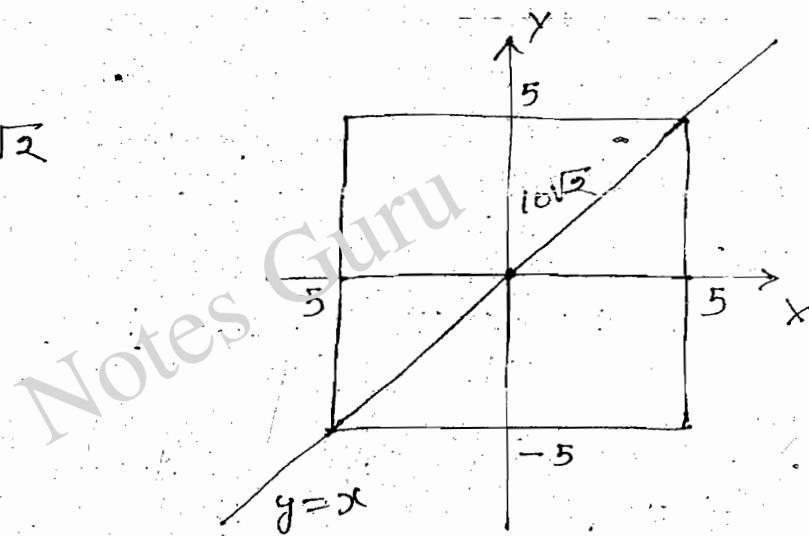


$$\begin{aligned} Q &= 15 \frac{\text{nC}}{\text{m}} \times 10 \\ &= 150 \text{ nC}, \text{ Ans} \end{aligned}$$



13:-

$$\begin{aligned} Q &= 15 \times 10\sqrt{2} \\ &= 150\sqrt{2} \end{aligned}$$



14:-

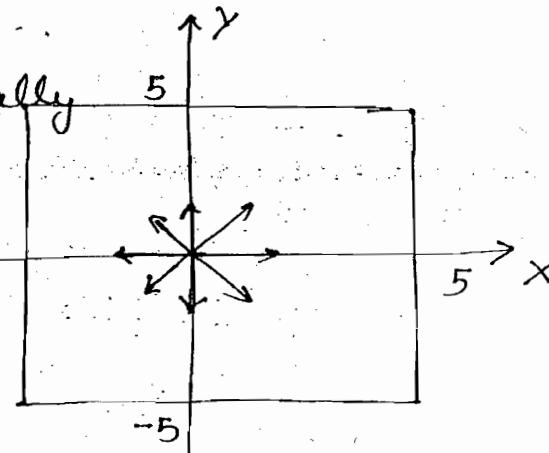
$$Q = 6 \text{ nC}$$

6 nC \rightarrow flux symmetrically

crosses 6 sides of

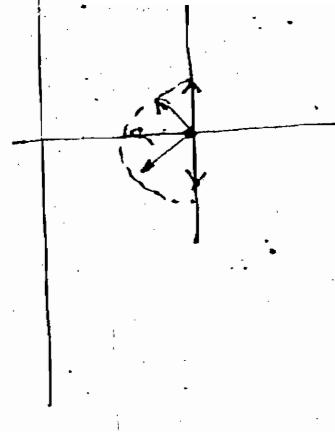
cube 1 nC-flux

Cross per 1 side



$z = 5\text{m-plane}$

3nc, 14rs



Note:-

A point charge and a infinite sheet near by has half the flux crossing through

$$\oint \mathbf{D} \cdot d\mathbf{s} = Q$$

constant

$$\mathbf{D} \cdot \mathbf{S} = Q$$

$$\Rightarrow D = \frac{Q}{S}$$

$$= \rho_V \frac{\frac{4}{3}\pi r^3}{4\pi r^2} \frac{\rho_V r}{3}$$

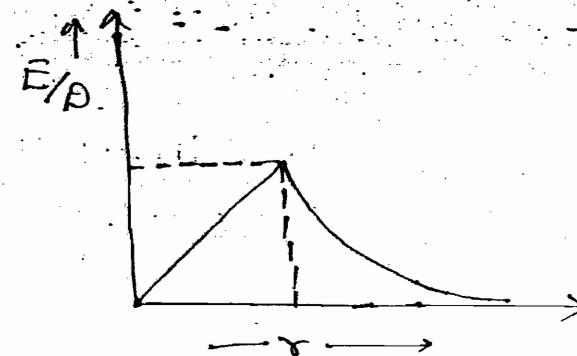
$$E = \frac{\rho_V r}{3\epsilon_0}$$

$$E(R) = \frac{\rho_V R}{6\epsilon_0}$$

$$\mathbf{D} \cdot \mathbf{S} = Q$$

$$\Rightarrow D = \frac{Q}{S}$$

$$= \rho_V \frac{\frac{4}{3}\pi R^3}{4\pi r^2}$$



$$D = \frac{\rho_V R^3}{3\epsilon_0}$$

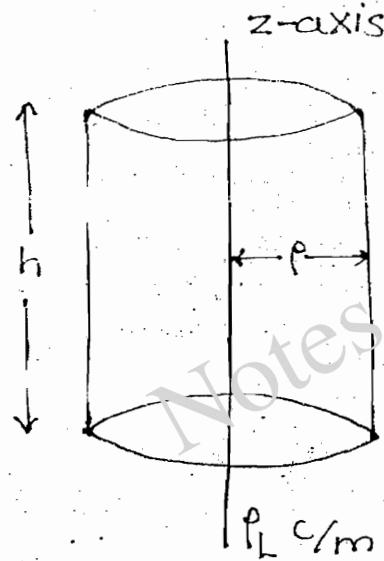
$$E = \frac{\rho_V R^3}{3\epsilon_0 r^2}$$

$$E(2R) = \frac{\rho_V R}{12\epsilon_0}$$

Gauss Law Application - 2 :-

Electric field strength of an infinite length ρ_L C/m line charge :-

Consider a concentric axisymmetric Gaussian surface which is a cylinder around the charge



$$\oint \mathbf{D} \cdot d\mathbf{S} = q_p$$

$\rightarrow \varphi = \text{constant}$
 $\downarrow \rightarrow q_p \text{ directed -}$
 constant

$$\mathbf{D} \cdot \mathbf{S} = q_p$$

$$\Rightarrow D = \frac{q_p}{S}$$

$$\Rightarrow D = \frac{\rho_L \cdot L}{2\pi r \epsilon_0}$$

$$\Rightarrow E = \frac{\rho_L}{2\pi r \epsilon_0} q_p$$

$$\Rightarrow E = \frac{\rho_L}{2\pi r \epsilon_0} q_p$$

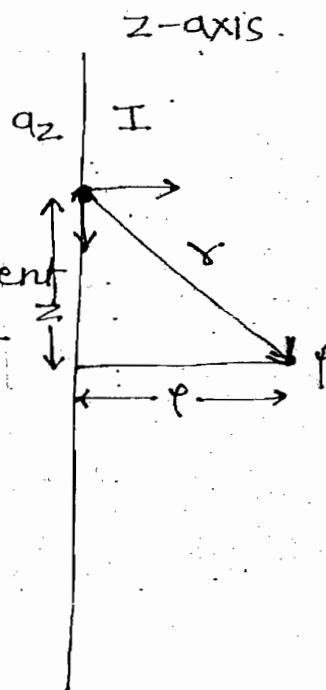
Magnetic field strength

<https://notesguru.info>

carrying wire :-

$$H = \frac{Idl \times a_r}{4\pi r^2}$$

consider a small dl length I current element cut the z point height above the point



$$dl = dz a_z$$

$$\gamma = \sqrt{\rho^2 + z^2}$$

$$a_r = \frac{\gamma}{|\gamma|}$$

$$= \frac{\rho a_\rho - z a_z}{\sqrt{\rho^2 + z^2}}$$

$$dH = \frac{Idz a_z}{4\pi(\rho^2 + z^2)} \times \left(\frac{\rho a_\rho - z a_z}{\sqrt{\rho^2 + z^2}} \right)$$

$$H = \int_{z=-\infty}^{\infty} dH$$

$$\Rightarrow H = \frac{Ip}{4\pi} \int_{z=-\infty}^{\infty} \frac{dz (a_z \times a_\rho)}{(\rho^2 + z^2)^{3/2}}$$

$$\text{Put } z = \rho \tan \theta$$

$$dz = \rho \sec^2 \theta d\theta$$

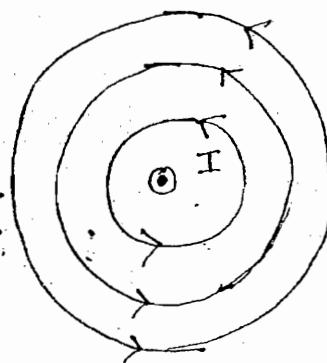
$$(\rho^2 + z^2)^{3/2} = \rho^3 \sec^3 \theta$$

$$H = \frac{I \rho}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2 \theta}{\rho^3 \sec^3 \theta} - \frac{1}{4\pi \rho} \int_{-\pi/2}^{\pi/2} \cos \theta \cdot d\theta$$

$\theta = -\frac{\pi}{2}$

$$\Rightarrow H = \frac{I}{2\pi\rho} (a_z \times a_p) = \frac{I}{2\pi\rho} a_\phi$$

Note :-



$$\oint H \cdot dL = I$$

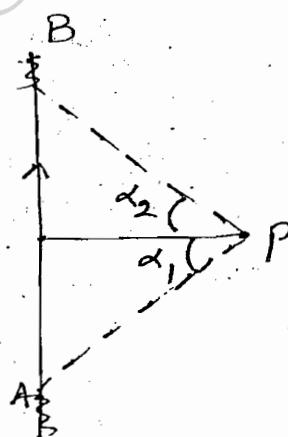
$H \cdot \text{Length} = \text{current}$

$$H = \frac{I}{2\pi\rho} a_\phi$$

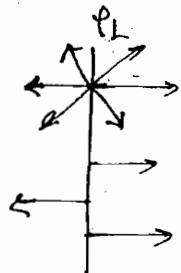
Extension :-

H-field of finite length I wire :-

$$H = \frac{I}{4\pi\rho} (\sin \alpha_1 + \sin \alpha_2) a_\phi$$



Summary :-



$\rho_L \rightarrow$ Line charge

$E \rightarrow$ Field

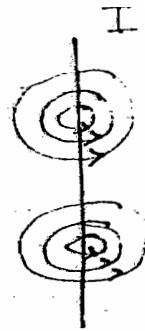
$$\theta = \frac{\rho_L}{2\pi\rho} a_p$$

$$E = \frac{\rho_L}{2\pi\epsilon\rho} a_p$$

I carrying H-field

$$H = \frac{I}{2\pi r} a_\phi$$

$$B = \frac{\mu I}{2\pi r} a_\phi$$



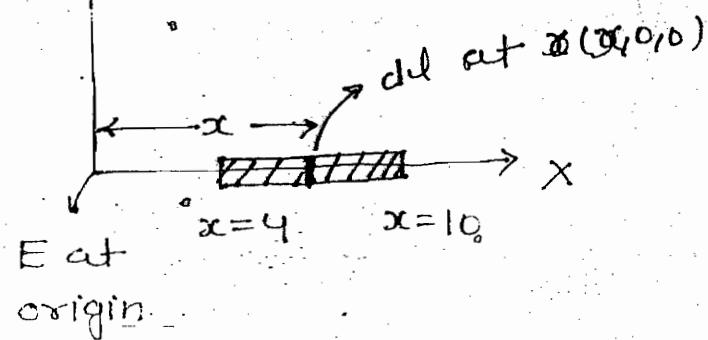
7:- consider a dl length
on the charge and
apply the $\frac{Q}{4\pi\epsilon_0 r^2} dr$

Field calculation

$$Q = \rho_L l dx$$

$$r = x$$

$$dr = -dx$$



$$dE = \frac{\rho_L dx}{4\pi\epsilon_0 x^2} (-ax)$$

$$E = \frac{\rho_L}{4\pi\epsilon_0} \int_{x=4}^{10} \frac{dx}{x^2} (-ax)$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \left(-\frac{1}{x} \right) \Big|_4^{10} (-ax)$$

$$H = \frac{8}{4\pi 3} \left(1 + \frac{4}{5}\right) a_z$$

(Y-axis)

$$a_\phi = a_z \times a_p$$

H-direction = $I \times$ Radial direction

$$= \frac{8}{4\pi 3} \times \frac{9}{5} (a_z)$$

$$H = \frac{8}{4\pi \cdot 4} \left(\frac{3}{5} + 1\right) (a_x \times a_y)$$

$$(X\text{-axis}) = \frac{8}{16\pi} \times \frac{8}{5} (a_z)$$

$$H = \frac{2}{\pi} a_z$$

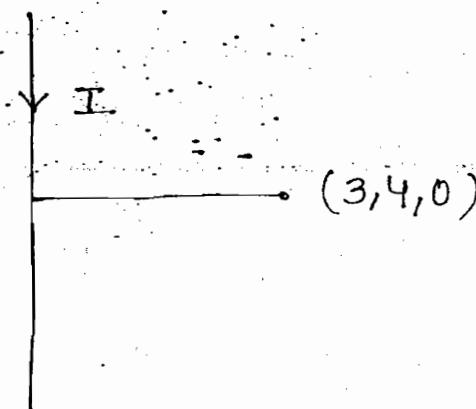
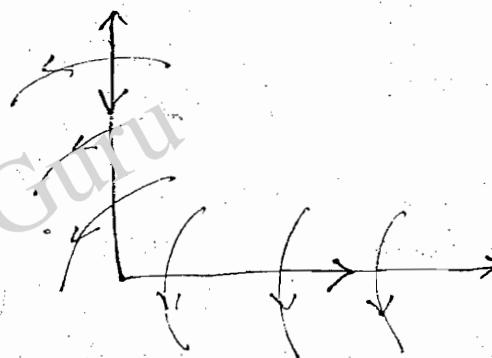
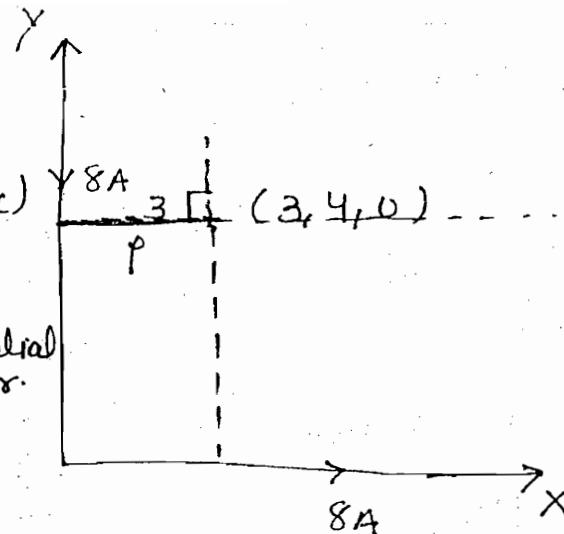
ques:- Repeat the above problem if current is

only entire y-axis

Soln:-

$$H = \frac{8}{2\pi \cdot 3} a_z$$

$$= \frac{1.33}{\pi} a_z$$



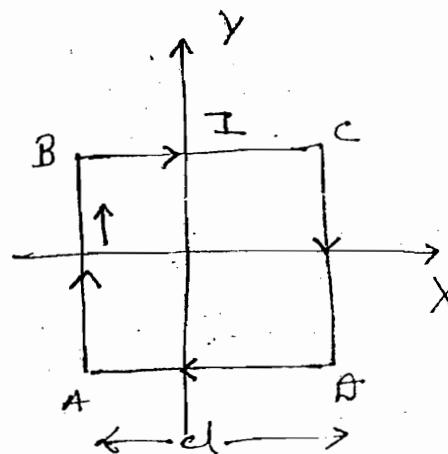
19

$$H_{AB} = a_y \times a_{zc} = -a_z$$

$$H_{BC} = a_x \times -a_y = -a_z$$

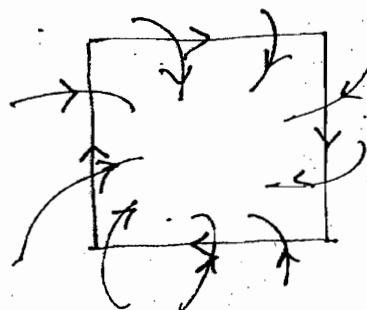
Note:-

→ For any symmetric current distribution H



field at geometric centre is always max

→ For any asymmetric charge distribution



E field at the geometric centre is always zero

$$H = \frac{I}{4\pi \cdot \frac{d}{2}} \cdot \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \times (-a_z)$$

$$= \frac{2\sqrt{2} I}{\pi d} (-a_z) = 0.9 \frac{I}{d} (-a_z)$$

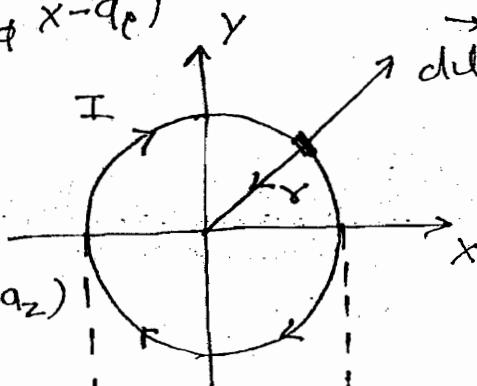
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$$dH = \frac{I d\vec{l}}{4\pi r^2} (-a_\theta \times -a_r)$$

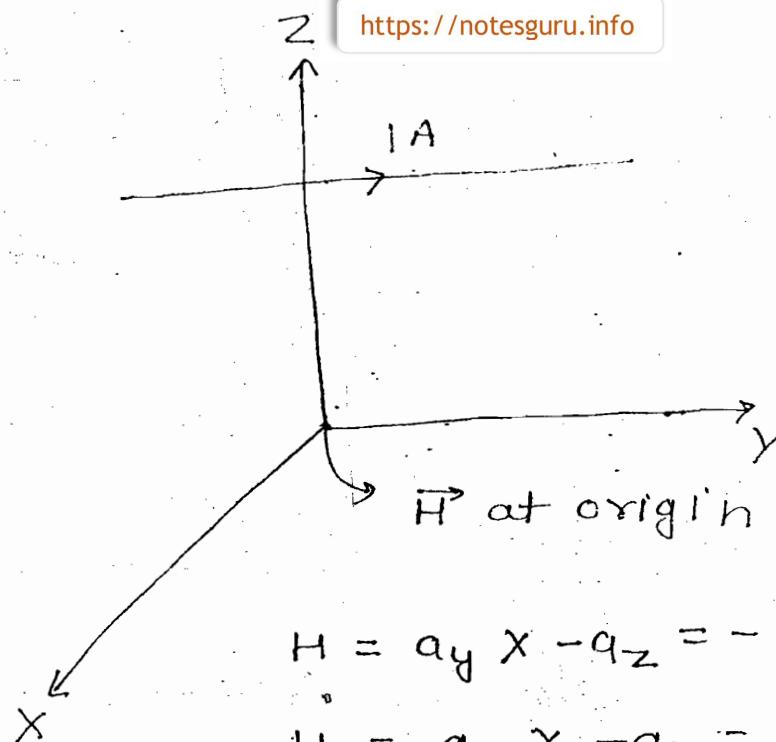
$$H = \int dH$$

$$= \frac{I 2\pi r}{4\pi r^2} (-a_z)$$

$$= \frac{I}{2r} = \frac{I}{d} (-a_z)$$



22 :-



$$H = a_y x - a_z = -a_x$$

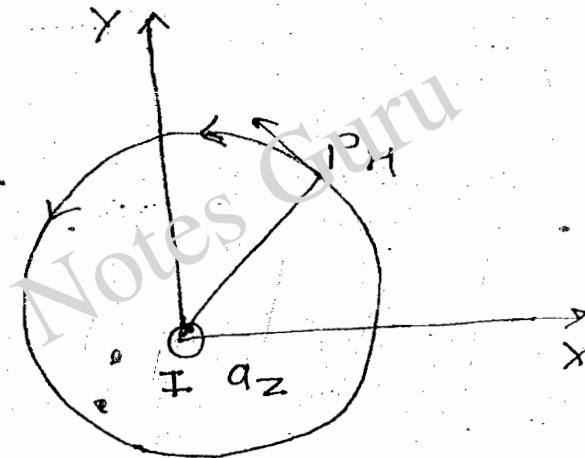
$$H = a_x x - a_y = -a_z$$

Ans - d

23.

$$H_q = a_z \times a_p$$

Ans - C)



ρ_L at $y=3, z=5$ for all x

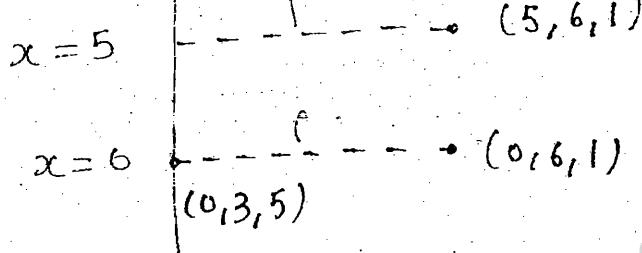
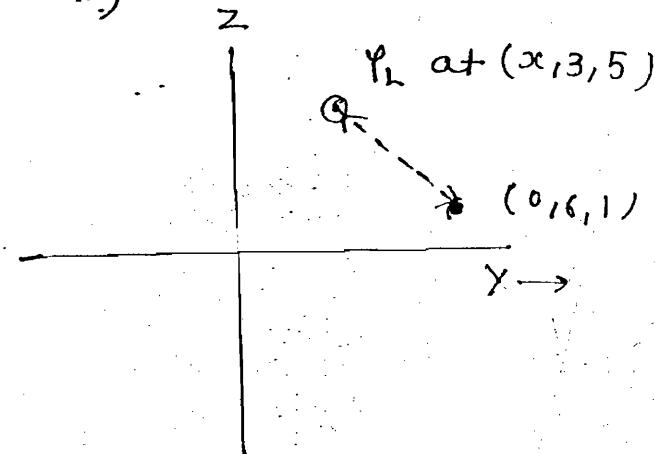
Given \vec{E} at $(0, 6, 1)$

$\vec{E} = ?$ at $(5, 6, 1)$

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0} q_p$$

$$E \propto \frac{1}{r}$$

ρ_L at $(x, 3, 5)$



For both points r is same

$$r = \sqrt{3^2 + 4^2} = 5$$

Note:-

In general z -axis $\rightarrow (0, 0, z)$

Any point (x, y, z)

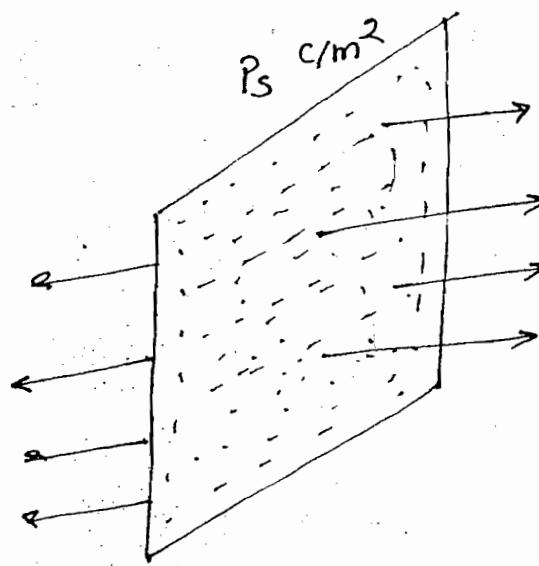
$$r = \sqrt{x^2 + y^2}$$

Any line parallel to x -axis $\rightarrow (x, a, b)$

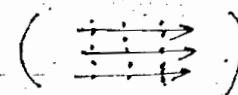
Any point $\rightarrow (x, y, z)$

$$r = \sqrt{(y-a)^2 + (z-b)^2}$$

ρ_s C/m² charge density :-



→ The electric field is completely normal to the sheet and the field lines are parallel to themselves



→ The flux density is same everywhere. The field is uniform.

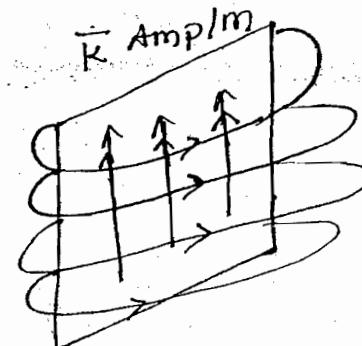
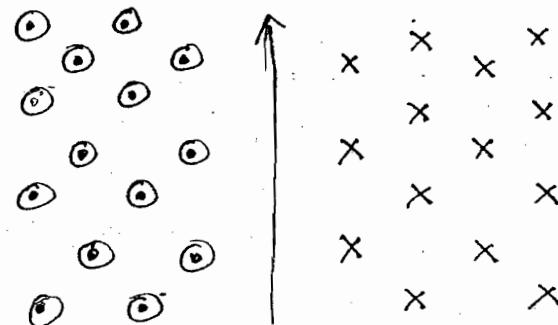
$$\sigma = \frac{\rho_s a_N}{2}$$

$$\rightarrow E = \frac{\sigma}{\epsilon} = \frac{\rho_s a_N}{2\epsilon} \Rightarrow E = \frac{\rho_s a_N}{2\epsilon}$$

**

$$E = \frac{\rho_s a_N}{2\epsilon}$$

Magnetic field strength of an infinite sheet of ρ_s C/m² charge density :-



- The magnetic field is normal to the current and the field lines are parallel to themselves and to the sheet
- The flux density is same everywhere i.e. field is uniform

$$\rightarrow H = \frac{\mu_0 K}{2} \times a_N \text{ Amp/m}$$

and

$$B = \frac{\mu_0 K}{2} \times a_N$$

24.

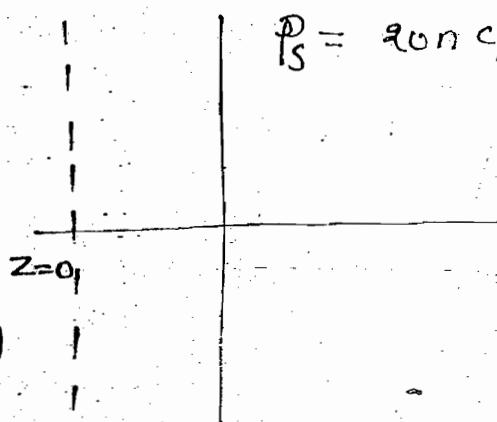
$$a_N = \pm a_z$$

$$\text{At } z=0, a_N = -a_z$$

$$\begin{aligned} E &= \frac{P_s}{2\epsilon} a_N \\ &= \frac{20 \times 10^{-9}}{2 \times \frac{1}{36\pi \times 10^9}} (-a_z) \end{aligned}$$

$z=10$

$$P_s = 20 \text{ nC/m}^2$$

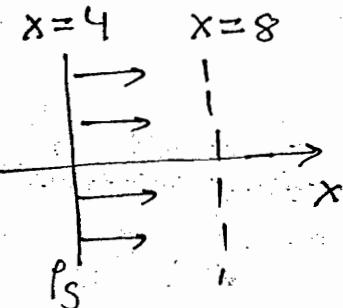
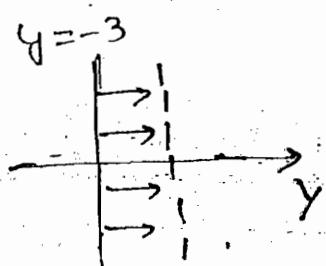
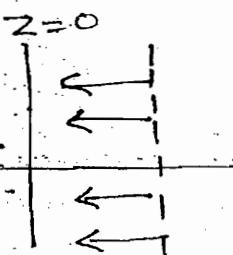


25.

$$x = 4 \quad P_{s1} = 18 \text{ nC/m}^2 \rightarrow E_1 = \pm q_x$$

$$y = -3 \quad P_{s2} = 9 \text{ nC/m}^2 \rightarrow E_2 = \pm a_y$$

$$z = 0 \quad P_{s3} = -24 \text{ nC/m}^2 \rightarrow E_3 = \pm a_z$$

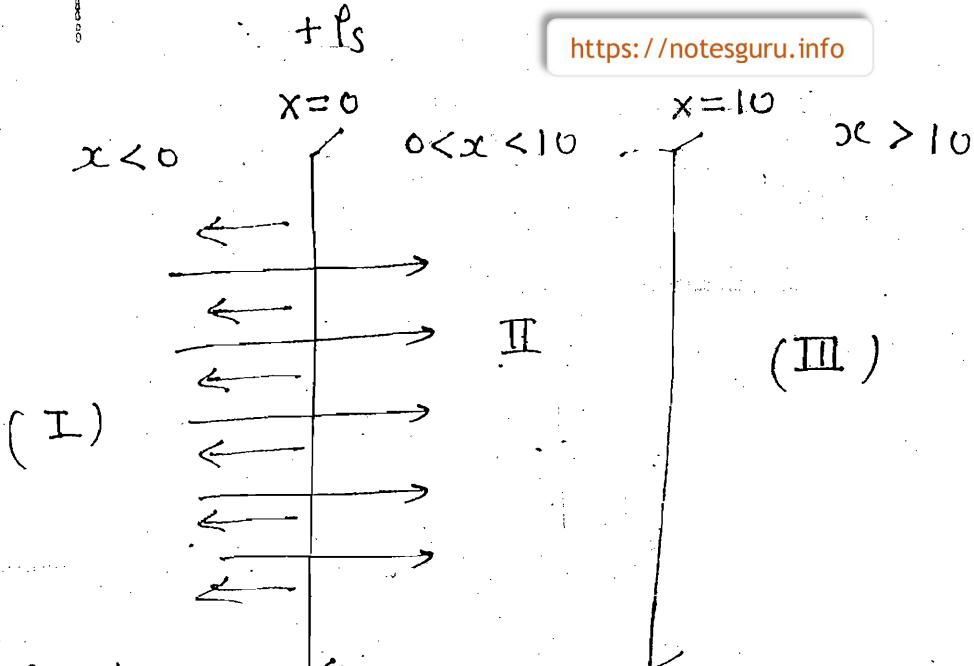


$$E_3 = \frac{24}{2\epsilon} (-a_z)$$

$$E_2 = \frac{9}{2\epsilon} a_y n N C$$

$$E_1 = \frac{18}{2\epsilon} a_x n N C$$

$$E_T = \frac{3}{2\epsilon} (6a_x + 3a_y - 8a_z) n N C$$



In first region :-

$$E_1 = \frac{p_s}{2\epsilon} (-a_x)$$

$$E_2 = \frac{p_s}{2\epsilon} (a_x)$$

$$\vec{E}_{\text{Net}} = 0$$

In third region :-

$$\vec{E}_{\text{Net}} = 0$$

In second region :-

$$E_1 = \frac{p_s}{2\epsilon} a_x$$

$$E_2 = \frac{p_s}{2\epsilon} a_x$$

$$E = \frac{p_s}{\epsilon} a_x \quad 0 < x < 10$$

$$= 0$$

elsewhere

$$\vec{K} = 30 a_z \text{ mA/m} \rightarrow y=0 \text{ plane} \Rightarrow z-x \text{ plane}$$

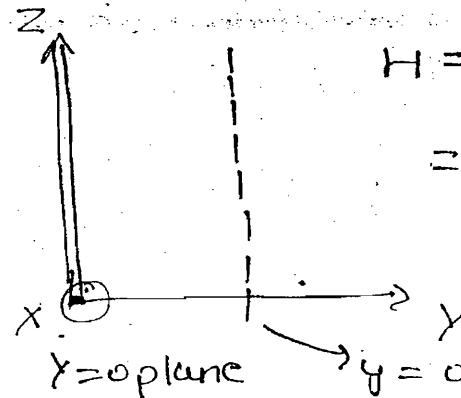
$$H = \frac{\vec{K}}{2} \times a_N$$

at $y=2$

$$a_N = \pm a_y$$

for $y=20$,

$$a_N = a_y$$

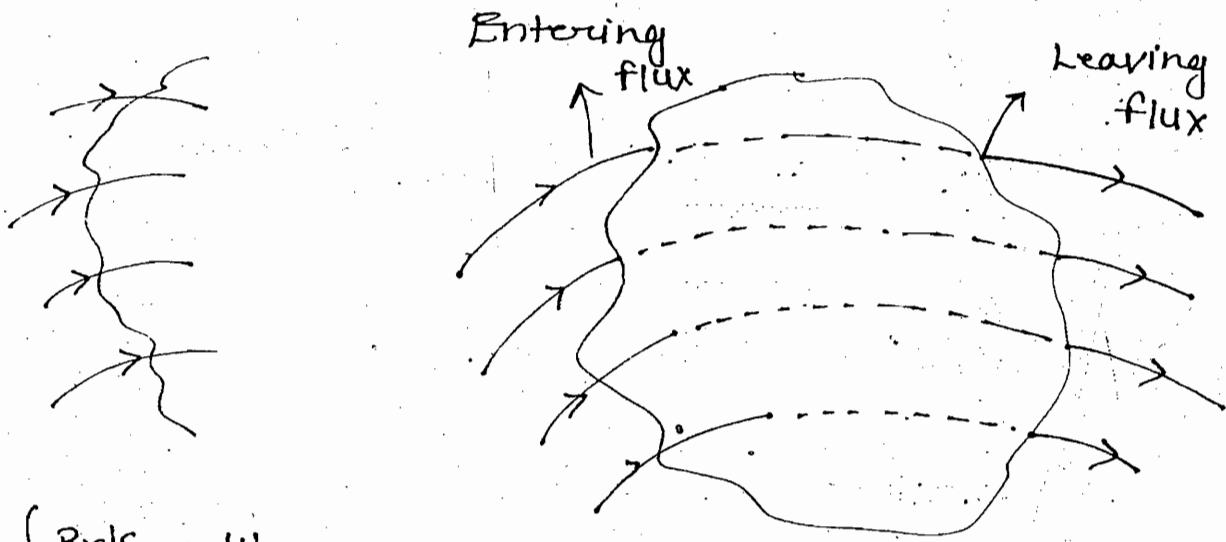


$$H = \frac{30}{2} a_z \times a_y$$

$$= -15 a_x \text{ mA/m}$$

Ans-(A)

- Magnetic field always forms closed lines around the current and has no starting / ending point for flux lines i.e. No sources/sinks exists for B flux lines



$$\int \mathbf{B} \cdot d\mathbf{s} = \Psi_m$$

This is not Maxwell's equation

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0$$

This is Maxwell III equation

- For any closed surface

Entering flux = Leaving flux

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0$$

B field is solenoidal (No divergence)

$$\nabla \cdot \mathbf{B} = 0$$

Note :-

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0 \quad \begin{cases} \alpha = 0 \\ +\alpha, -\alpha \end{cases} \quad \Rightarrow \quad \boxed{\oint \mathbf{B} \cdot d\mathbf{s} = 0}$$

Always

- By comparison, equals of charge in E fields do not exist in B fields i.e. every cause of B field is a dipole, i.e. Magnetic monopoles don't exist.

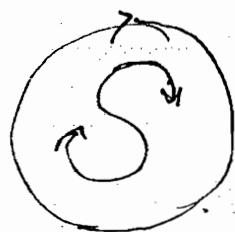
eg:- Bar M - Dipole

<https://notesguru.info>

best example:- I. current carrying wire

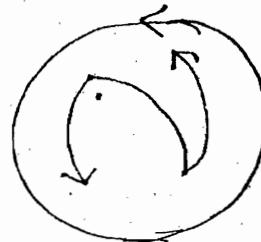
→ I always flows only when both the polarities exist and are connected hence it is always treated as magnetic dipoles.

→ I always flows in closed circuits and every closed I wire is a magnetic dipole



Clockwise

→ South pole



Anticlockwise

→ North pole

→ Cause → current → closed

Entering current = Leaving current
(Junction)

Effect - B field - closed around the cause

Entering flux = Leaving flux (closed surface)

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0$$

→ KCL in Magnetic Field

Potential, Gradient, closed line Integral in E field:-

Potential is a ^{scalar} measure of E field

strength in terms of energy at any point in the field

V (Volts)

Potential at any point

= Work done by the charge to reach the point
charge

$$V = \frac{W}{Q} = \frac{\text{Work}}{\text{Coulomb}}$$

\Rightarrow Work = Force. displacement

$$\Rightarrow dW = \vec{F} \cdot d\vec{u} = Q \vec{E} \cdot d\vec{u}$$

= Workdone in field / force direction,

= Workdone on the charge

→ Work is done on the charge when it moves in the field direction and hence the charge acquires energy

$$V = \frac{W}{Q} = - \int \vec{E} \cdot d\vec{u} \rightarrow \text{scalar potential function of space}$$

$$V_{AB} = - \int_{\text{Ref B}}^{\text{Ref A}} \vec{E} \cdot d\vec{u} = \text{Potential diff b/w A \& B}$$

If ref. B has $V_B = 0$, then V_A is called as absolute potential at A.

Potential function of a point charge :-

$$V = - \int \vec{E} \cdot d\vec{u}$$

Spherical coord.

$$\Rightarrow - \int \frac{Q}{4\pi\epsilon_0 r^2} dr \hat{r} = \frac{Q}{4\pi\epsilon_0 r}$$

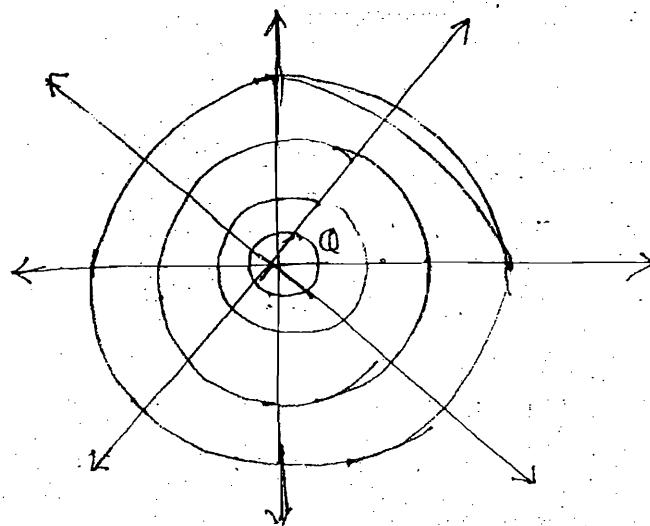
$$\Rightarrow V = \frac{Q}{4\pi\epsilon_0 r}$$

or directed vector

$V \rightarrow$ Potential $= \frac{1}{r}$ decrease
 → scalar →

If $r = \text{constant}$ then $V = \text{constant}$

↳ concentric sphere → surface



→ The family of concentric equipotential surfaces represent the potential distribution and variation of potential in the region and similar to vector intensity line.

Potential function of a line charge:-

$$V = - \int \vec{E} \cdot d\vec{r}$$

$$= - \int \frac{\rho_L}{2\pi\epsilon_0 r^2} dr$$

$$\Rightarrow V = \frac{\rho_L}{2\pi\epsilon_0} \ln \left(\frac{r_2}{r_1} \right)$$

If $r = \text{constant}$ $\Rightarrow V = \text{constant}$

Equipotential surfaces are concentric cylinders

\vec{q}_p directed vector

$V \rightarrow$ Potential - logarithmic decrease
- scalar

Potential function of a sheet charge (Uniform) :-

$$V = - \int \vec{E} \cdot d\vec{l}$$

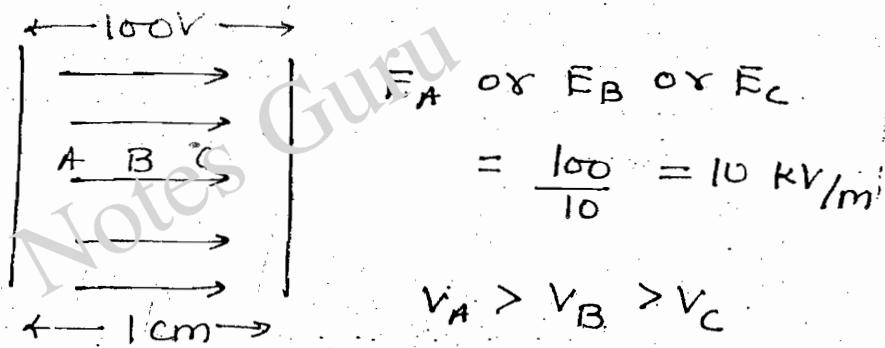
$$\Rightarrow V = -Ed$$

$$\Rightarrow E = -\frac{V}{d}$$

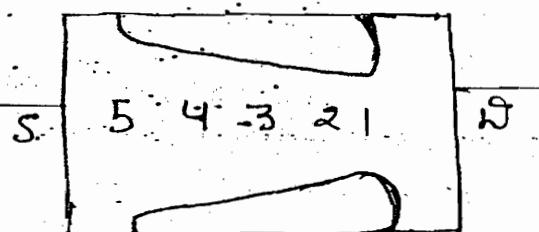
$\vec{E} \rightarrow$ Intensity - Uniform

$V \rightarrow$ Potential - linear decrease

eg:- Capacitor Plates



FET channel



$$V = - \int \vec{E} \cdot d\vec{l} \quad \text{https://notesguru.info} \quad (ay) \cdot (dx a_x + dy ay)$$

$$V = - \int_{x=0}^4 y dx - \int_{y=0}^1 x dy$$

(0, 0, 0)

$$= - \int_0^4 \frac{2x}{4} dx - \int_0^1 4y dy$$

$$= -\frac{1}{8} (16) - \frac{4}{2} \times 1$$

$$= -4 V, \text{ Ans.}$$

Using the straight line path or equation

$$\frac{y-1}{1-0} = \frac{x-4}{4-0}$$

$$x = 4y$$

Note:

In line integral,

$$\int f(x, y, z) dx$$

We have to transform y & z in terms of x using the known relationship

i.e. path of integration

$$\int f(x, y, z) dx = \int g(x) dx$$

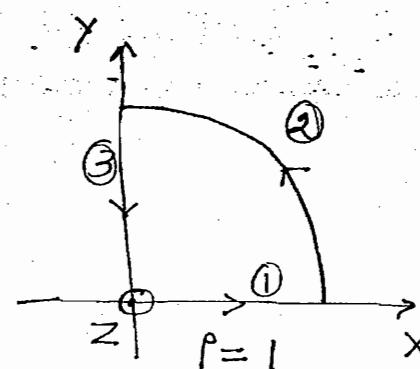
$$\vec{A} = 2\rho \cos\phi \hat{a}_\rho$$

$$\oint A \cdot d\vec{l} = 3 \rightarrow \int A \cdot d\vec{l}$$

$$(1) d\vec{l} = d\rho \hat{a}_\rho$$

$$\phi = 0$$

$$A = 2\rho a_\rho$$



$$\int A \cdot d\ell = \int_{\rho=0}^1 2 \cdot q_p \cdot q_p \cdot q_p$$

$$= \rho^2 |_0^1 = 1$$

$$2 \rightarrow d\ell = \rho d\phi \underline{a_\phi} \quad (\because q_\phi \cdot q_\rho = 0)$$

$$\int A \cdot d\ell = 0$$

$$3 \rightarrow d\ell = d\rho \underline{a_\rho}$$

ρ from 1 to 0

$$\phi = 90^\circ \Rightarrow A = 0$$

$$3 \rightarrow \vec{E} = 4a_x - 3a_y + 5a_z \rightarrow \text{Uniform}$$

$$W = \sigma \int \vec{E} \cdot d\vec{\ell} = \sigma E \cdot l$$

Unit length vector = $\frac{3a_x + 2a_y + 4a_z}{\sqrt{3^2 + 2^2 + 4^2}}$

(9, 4, 1)
(6, 2, -3)

(. 3m length vector)

$$W = 5 (4a_x - 3a_y + 5a_z) \cdot \frac{(3a_x + 2a_y + 4a_z) \times 2}{\sqrt{29}}$$

$$= \frac{260}{\sqrt{29}} J$$

Potential Gradient :-

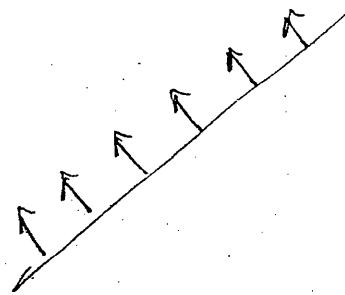
<https://notesguru.info>

Scalar surface equation f $\xrightarrow{\nabla f}$ Vector (Normal) to the surface (direction)

eg:- Linear surface

$$f = 4x + 7y - 15z = 18$$

$$\nabla f = 4\hat{a}_x + 7\hat{a}_y - 15\hat{a}_z$$



eg:- Non-linear surface

$$f = 4xy - 15x^2yz$$

$$\begin{aligned}\nabla f = & (4y - 30xyz)\hat{a}_x + (4x - 15xz^2)\hat{a}_y \\ & + 15x^2y\hat{a}_z\end{aligned}$$



$$V = - \int \vec{E} \cdot d\vec{l}$$

$$dV = - \vec{E} \cdot d\vec{l} \cos\theta$$

$$\Rightarrow \boxed{\frac{dV}{dl} = - E \cos\theta}$$

Case-(1) :-

$$\text{If } \theta = 90^\circ \Rightarrow V = \text{constant}$$

→ The direction of E field is always normal to equi-potential surfaces.

$$\text{If } \theta = 0^\circ / 180^\circ \Rightarrow \left| \frac{dV}{dl} \right|_{\max} = |E|$$

→ The magnitude of E intensity is always the maximum rate of change of potential per unit length
If

Case-(III) :-

$$\text{If } \theta = 90^\circ \Rightarrow |E| = \left| \frac{dV}{dl} \right|_{\max}$$

→ The direction of E intensity is always the direction in which potential decreases by maximum

Note:-

The operation is physically called as gradient.
Hence $E = -\nabla V$ = potential gradient is field intensity.

→ Gradient signifies the maximum rate of change of scalar along with the direction of change.

e.g:- Given $V = 4(x^2 - y^2)$ for all Z

Find the equation of equipotential surface passing through $(3, 1, 9)$

$$\text{Soln:- } V \text{ at } (3, 1, 9) = 4(9-1) = 32V$$

All x & y when $V=32V$ is the equipotential surface

$$\Rightarrow x^2 - y^2 = 8$$

summary:-

Equation of equipotential surface

OR

Voltage function (V)

$$\rightarrow \nabla V \rightarrow$$

vector Intensity function (E)
which is normal to equi-potential surfaces

$$\nabla V = \frac{1}{h_1} \frac{\partial V}{\partial u} \hat{a}_u + \frac{1}{h_2} \frac{\partial V}{\partial v} \hat{a}_v + \frac{1}{h_3} \frac{\partial V}{\partial w} \hat{a}_w$$

eg:- If $V = \frac{4 \cos \theta}{r^2}$

Find \vec{E} at $(2, \pi/4, \pi)$

Soln:- $E = -\nabla V$

$$\nabla V = \frac{1}{1} \frac{\partial}{\partial r} \left(\frac{4 \cos \theta}{r^2} \right) \hat{a}_r + \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{4 \cos \theta}{r^2} \right) \hat{a}_\theta$$

$$= 4 \cos \theta \left(-\frac{2}{r^3} \right) \hat{a}_r + \frac{4}{r^3} (-\sin \theta) \hat{a}_\theta$$

$$E = \frac{4}{r^3} (2 \cos \theta \hat{a}_r + \sin \theta \hat{a}_\theta) \Big|_{(2, \pi/4, \pi)}$$

$$= \frac{1}{2} (\sqrt{2} \hat{a}_r + \frac{1}{\sqrt{2}} \hat{a}_\theta)$$

(a) $V = 0$ $\nabla V = 6x \hat{a}_x + (3x^2 - z) \hat{a}_y - 4z \hat{a}_z$

$\neq 0$ at $(1, 0, -1)$

(b) $x^2y = 1$ in xy plane

$$V = 3(1) - 4(0)$$

$V = 3$ \rightarrow equipotential surface

(c) At $(2, -1, 4) \Rightarrow V = -8$

$$V = 3(4)(-1) - (-1)4 = -8$$

(d) Normal vector at P $= \frac{\nabla V}{|\nabla V|} \Big|_{(2, -1, 4)} = \frac{-12 \hat{a}_x + 8 \hat{a}_y - 4 \hat{a}_z}{\sqrt{12^2 + 8^2 + 4^2}}$

$$= (-0.83 \hat{x} + 0.55 \hat{y} + 0.07 \hat{z})$$

Closed line integral of electric field = Maxwell's II Eqn:-

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

- Potential difference exists b/w 2 different points but b/w the same point potential difference is zero
- Potential is unique at a point at a time
- Workdone in moving a charge in any closed path is always zero
- In any closed path energy acquired = energy lost i.e Energy acquired in field direction = energy lost opposite to field direction.
- Electric field is a conservative field & never forms closed loops
i.e. Irrotational vector ($\nabla \times \mathbf{E} = 0$)
- Workdone in moving a charge b/w two points is independent of path of consideration.
- This is KVL in electric fields.

Potential, Vector Potential and Ampere's Law in H field :-

- Potential is called as MMF or Magneto Motive force and is a scalar measure of magnetic field strength at any point in the field.

$$V_m = MMF = \int \mathbf{H} \cdot d\mathbf{l} = \text{Amp}$$

$$\mathbf{H} = \nabla V_m$$

$$\boxed{\oint \mathbf{H} \cdot d\mathbf{l} = I}$$

This is Ampere's law in Integral form

$$\nabla \times H = J$$

This is Ampere's law in point form

Statement:- effects

The net circulation of magnetic field in any closed line is always equal to the current crossing the surface enclosed.

$$\text{Circulation} = \text{Strength} \times \text{length}$$

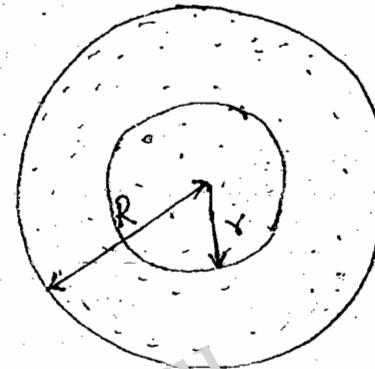
$$= \text{current}$$

Consider a closed ring line symmetrical & concentric with the I flow direction such that $H = \text{constant}$ everywhere

$$\oint H \cdot d\ell = I$$



constant



$$H \cdot \text{Length} = \text{current crossing}$$

$$H = \frac{\frac{I}{2\pi r^2} \cdot 2\pi r}{\frac{I}{2\pi r^2}} = \frac{I}{2\pi r}$$

$$r > R$$

$$H = \frac{I}{2\pi r}$$

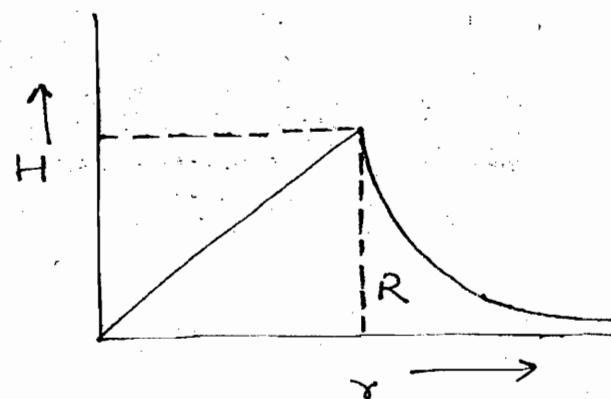
Note:-

$$\text{Point} \rightarrow \frac{1}{r^2}$$

$$\text{Line} \rightarrow \frac{1}{r}$$

$$\text{Sheet} \rightarrow \text{Uniform}$$

$$\text{Solid} \rightarrow r$$



Limitation of Scalar Potential (MMF) :-

→ The definition of potential in electric field is consistent with its nature i.e. irrotational nature

$$\mathbf{E} = -\nabla V$$

$$\nabla \times \mathbf{E} = 0$$

$$\nabla \times (-\nabla V) = 0$$

$$\text{curl (Grad of Scalar)} = 0$$

→ A similar definition in magnetic field for MMF is not consistent with Ampere's law

$$\mathbf{H} = \nabla V_m$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \times (\nabla V_m) = 0 = \mathbf{J}$$

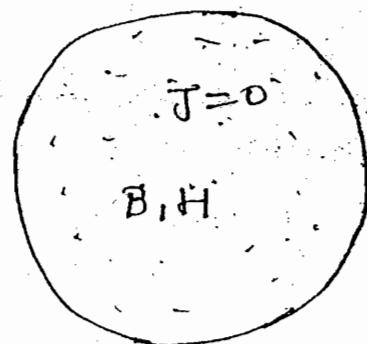
Hence MMF is defined and exists in only those regions where $\mathbf{J} = 0$

i.e. outside current flowing regions

i.e. free space conditions and outside conductor

e.g. :- (i) MMF b/w windings of solenoids

(ii) MMF in air gaps in machines



$\mathbf{B}, \mathbf{H}, \mathbf{J}$

Vector Potential A' :-

<https://notesguru.info>

→ The basic definition of vector potential is in accordance to nature of magnetic field i.e. solenoidal nature. Hence if curl of $A = B$ then A is called as vector potential

$$\vec{B} = \nabla \times \vec{A}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

$$\text{Div. (curl of vector)} = 0$$

→ The term vector potential has a units weber/m which physically signifies the work for current element

$$\vec{A} = \text{Weber/m} = \frac{\text{Joules}}{\text{Amp-m}}$$

$$= \frac{W}{I \vec{dl}}$$

and as $I \vec{dl}$ is a vector, \vec{A} is a vector quantity.

Note:-

$$\frac{\text{Weber}}{\text{Second}} = \text{Volts}$$

$$\frac{\text{Weber}}{\text{second}} = \frac{\text{Joules}}{\text{Coulomb}}$$

$$\frac{\text{Weber}}{\text{m}} = \frac{\text{Joules}}{\text{Amp-m}}$$

$$V = \frac{W}{Q} = \frac{4\pi\epsilon_0}{\mu I d l}$$

$$\vec{A} = \frac{\vec{W}}{I d l} = \frac{\mu I d l}{4\pi r^2}$$

A 's direction is along I flow direction & is a solenoidal vector.

Summary 1:—

- Q — coulomb — point charge — Basic cause (scalar) → E field
- $I d l$ — Amp-m — current element — H field (vector)
- $Q \rightarrow \rho_L d l = \rho_s ds = \rho_v dv$
- $I d l \rightarrow \vec{k} ds = \vec{j} dv$
- $\vec{E} \rightarrow$ Intensity — Strength — Force — $\frac{\vec{F}}{Q} = \frac{1}{\epsilon}$
- $\vec{B} \rightarrow$ Density — Strength — Force — $\frac{\vec{F}}{I d l}$ dependent
- $\vec{B} \rightarrow$ Density — Strength — $\frac{d\phi}{ds} = c/m^2$ — μ dependent
- $\vec{H} \rightarrow$ Intensity — Strength — $\frac{dI}{dl} = \text{Amp/m}$
- $V \rightarrow$ Potential — Strength — Work = $\frac{W}{Q}$ → Scalar (Volts)
- $\vec{A} \rightarrow$ Potential — Strength — Work = $\frac{W}{I d l}$ (Weber/m) → Vector.

Summary :-

Points

Scalar function $\xrightarrow{\nabla}$ Vector f^n
 Gradient Intensity
 (perm)

$$\text{eg: } V \text{ volts} \rightarrow \nabla V \rightarrow E \text{ Volts/m}$$

Line

Vector function $\xrightarrow{\nabla \times}$ Vector f^n
 Intensity curl Intensity
 (perm) (perm²)

$$\text{eg: } \text{Amp/m } \vec{H} \rightarrow \nabla \times \vec{H} \rightarrow J \text{ Amp/m}^2$$

$$\text{Web/m } \vec{A} \rightarrow \nabla \times \vec{A} \rightarrow \vec{B} \text{ Web/m}^2$$

Surface

Vector function $\xrightarrow{\nabla}$ Scalar f^n
 Density volume volume
 (perm²) (perm³)

$$\text{eg: } N \rightarrow \nabla \cdot \vec{n} \rightarrow \rho_v \text{ C/m}^3$$

(C/m²)

$$\oint A \cdot d\ell = \frac{\text{Weber}}{m} \times m = \text{Webers}$$

OR

flux

$$\oint \vec{A} \cdot \vec{d\ell} = \int (\nabla \times \vec{A}) \cdot ds = \int \vec{B} \cdot ds = \psi_m$$

34-

Stoke's theorem

$$\vec{V} = \nabla \times \vec{A}$$

C \rightarrow closed line

S_c \rightarrow surface

B

$$\oint A \cdot d\ell = \int (\nabla \times \vec{A}) \cdot ds = \int \vec{V} \cdot \vec{ds}$$

→ They are second order differential equations relating volume charge and potential developed inside / outside the charge

$$\nabla \cdot \mathbf{D} = \rho_V$$

$$\nabla \cdot [\epsilon (-\nabla V)] = \rho_V$$

If $\epsilon = \text{constant}$ then

$$\boxed{\nabla^2 V = -\frac{\rho_V}{\epsilon}} \rightarrow \text{Poisson's Equation}$$

For $\rho_V = 0$, i.e. outside the charge, charge free region

$$\boxed{\nabla^2 V = 0} \rightarrow \text{Laplace Equation.}$$

Note:-

Inspite of being II order differential equation they always have a unique solution i.e. voltage is a single valued function of space

This is called as Uniqueness theorem

Mathematically

$$\nabla^2 V = \nabla \cdot (\nabla V)$$

$$\begin{aligned} \nabla \cdot (\nabla V) &= \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u} \left(\frac{h_2 h_3}{h_1} \frac{\partial V}{\partial u} \right) + \frac{\partial}{\partial v} \left(\frac{h_3 h_1}{h_2} \frac{\partial V}{\partial v} \right) \right. \\ &\quad \left. + \frac{\partial}{\partial w} \left(\frac{h_1 h_2}{h_3} \frac{\partial V}{\partial w} \right) \right] \end{aligned}$$

\vec{A} vs \vec{J} in Magnetic Fields

$$\nabla \times H = \vec{J}$$

$$\Rightarrow \nabla \times \left(\frac{B}{\mu} \right) = \vec{J}$$

$$\Rightarrow \nabla \times (\nabla \times A) = \mu \vec{J}$$

$$\Rightarrow \nabla (\nabla \cdot A) + \nabla^2 A = \mu \vec{J}$$

$$\Rightarrow \boxed{\nabla^2 A = -\mu \vec{J}}$$

For current free region

$$\boxed{\nabla^2 A = 0}$$

$$V = -\frac{6r^5}{\epsilon_0}$$

$$\nabla^2 V = -\rho_V / \epsilon \quad Q = \int \rho_V dr$$

$$\nabla^2 V = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} \left(-\frac{6r^5}{\epsilon} \right) \right) = -\frac{\rho_V}{\epsilon}$$

$$\Rightarrow \frac{1}{r^2} \frac{d}{dr} (r^2 \cdot 6 \cdot r^4) = \rho_V$$

$$\frac{30}{r^2} \cdot 6 \cdot r^5 = \rho_V$$

$$\Rightarrow \rho_V = 180r^3$$

$$Q = \int_{r=0}^1 \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} 180r^3 \cdot r^2 \sin\theta dr d\theta d\phi$$

$$= 180 \frac{\epsilon_0}{6} \int_0^1 r^6 dr = 120\pi C, \text{ Ans.}$$

36. $P_V = -10^{-8} (1 + 10\rho)$

V on $\rho = 5\text{cm}$

Given $\rho = 2\text{cm}$ $V \neq E = 0$

$$\nabla^2 V = -\frac{P_V}{\epsilon}$$

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) = \frac{10^{-8} (1 + 10\rho)}{\frac{1}{36\pi \times 10^9}}$$

$$= 360\pi (1 + 10\rho)$$

$$\rho \frac{\partial V}{\partial \rho} = 360\pi \int (\rho + 10\rho^2) d\rho$$

$$\rho \frac{\partial V}{\partial \rho} = 360\pi \left(\frac{\rho^2}{2} + \frac{10\rho^3}{3} \right)$$

$$\Rightarrow V = 360\pi \int \left(\frac{\rho^2}{2} + \frac{10\rho^2}{3} \right) d\rho$$

$$= 360\pi \left(\frac{\rho^3}{4} + \frac{10\rho^3}{9} \right) \Big|_{2 \times 10^{-3}}^{5 \times 10^{-2}}$$

$$\Rightarrow V = 0.5V$$

37.

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -\frac{P_V}{\epsilon}$$

$$= 20 \cdot 3 \cdot 2 \cdot x + 10 \cdot 4 \cdot 3 \cdot y^2 \Big|_{(2,0)} = -\frac{P_V}{\epsilon}$$

$$\Rightarrow P_V = -240\epsilon_0$$

38 $\phi \rightarrow 1\text{ dimension}$ satisfying - Laplace

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} = 0$$

$\frac{\partial \phi}{\partial x} = c_1$ = Rate of change is constant

$$\frac{\phi_2 - \phi_1}{d} = \frac{\phi_3 - \phi_2}{2d}$$

$$\phi = c_1 x + c_2 \rightarrow \text{Linear equation}$$

Ans - B

39. $v = \sinhx \cdot \cosky e^{pz}$

$$\nabla^2 v = 0$$

$$\frac{e^x + e^{-x}}{2} = \sinhx$$

$$\nabla^2 v = Iv - k^2 v + p^2 v = 0$$

$$\Rightarrow k = \sqrt{1+p^2}$$

- If a field is spread out into two different medium and known the field in one region. The field in the adjacent region can be calculated under two conditions

Case-(I) :-

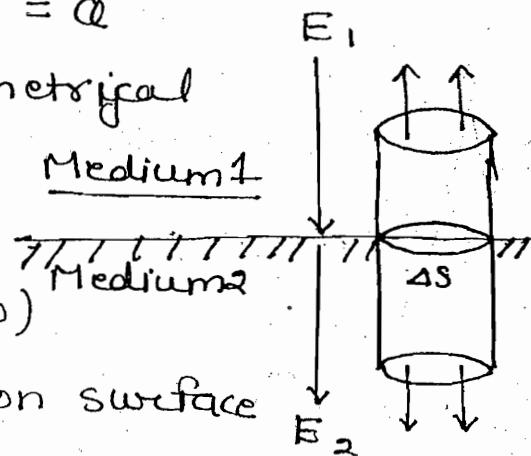
Electric field is normal to the boundary.

Using Gauss law $\oint \mathbf{E} \cdot d\mathbf{s} = Q$

consider a surface symmetrical in both the medium

$$\epsilon_2 \Delta S - \epsilon_1 \Delta S = 0$$

$\therefore Q=0$



If charge is present on surface then

$$\epsilon_2 \Delta S - \epsilon_1 \Delta S = \rho_s \Delta S$$

$$\theta_{n_1} = \theta_{n_2}$$

$$\theta_{n_2} - \theta_{n_1} = \rho_s$$

Statement:-

The normal components of electric flux density is same on either side but otherwise discontinuous when a surface charge density exists on the boundary

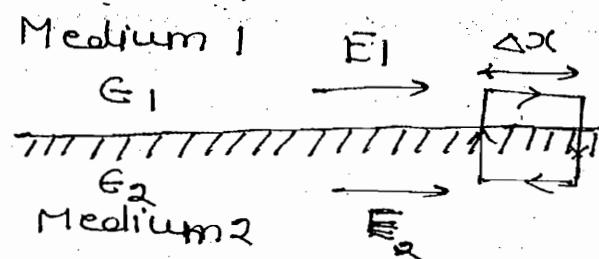
Case-(II) :-

Electric field is parallel to boundary.

Using $\oint \mathbf{E} \cdot d\mathbf{l} = 0$.

$$E_1 \Delta x - E_2 \Delta x = 0$$

$$\Rightarrow E_1 = E_2$$



$$E_{t1} = E_{t2}$$

The tangent components of electric field intensity is always continuous

Q.

$$E_{n_1} = 2a_x$$

$$E_{t_1} = -3a_y + a_z$$

$x < 0$

$$\epsilon_{R_1} = 1.5$$

$$E_1 = 2a_x - 3a_y + a_z$$

$$E_{t_1} = E_{t_2} = -3a_y + a_z$$

$x = 0$ Plane

YZ Plane

$x > 0$

$$\epsilon_{R_2} = 2.5$$



$$D_{n_1} = 1.5 \times 2a_x \times \epsilon_0 \quad (D_{n_1} = \epsilon_0 \epsilon_R \times E_n)$$

$$= 3a_x = D_{n_2}$$

$$\therefore 3\epsilon_0 a_x \rightarrow D_{n_2} = 3\epsilon_0 a_x$$

$$E_{n_2} = \frac{3\epsilon_0 a_x}{2.5 \epsilon_0} = 1.2 a_x$$

$$D_1 = \epsilon_0 (3a_x - 4.5a_y + 1.5a_z)$$

$$D_2 = \epsilon_0 (3a_x - 7.5a_y + 2.5a_z)$$

$E_2 = 1.2a_x - 3a_y + a_z$

Ans

Note:-

→ The value of vector dec. then its projection dec. i.e. moves away from normal and moves towards the surface

→ The projection of normal component is dec. in E_2 when compare to E_1 . Hence the field is shifting away from the normal. It can be verified with inc. tangential component when Comparing D_1 & D_2

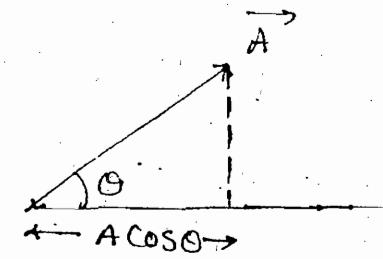
Sur

A's scalar:

$$= A \cos \theta$$

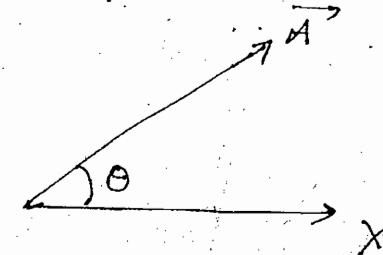
$$= \frac{A \cdot B}{|A| |B|}$$

$$= \frac{A \cdot B}{|B|}$$



A's projection in x

$$= \frac{A \cdot a_x}{|a_x|}$$



A's vector projection on B

$$= (A \cos \theta) \hat{B}$$

$$= \frac{(A \cdot B)}{|B|} \hat{B}$$

$$= \frac{(A \cdot B)}{|B|^2} \vec{B}$$

41

$$\omega_2 - \omega_1 = f_s$$

$$1 \cdot \epsilon_0 - 2 \cdot 2 \epsilon_0 = p_s$$

$$p_s = -3\epsilon_0$$

$$E_2 = \epsilon_0 \quad E_2 = a_x$$

$$E_1 = 2\epsilon_0 \quad E_1 = 2a_x$$

42

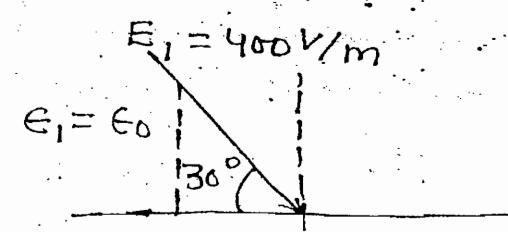
$$E_{t_1} = E_{t_2}$$

$$E_1 \cos 30^\circ = E_{t_2}$$

$$\Rightarrow E_{t_2} = 200\sqrt{3}$$

$$\epsilon_0 E_1 \sin 30^\circ = 20 \epsilon_0 E_{n_2}$$

$$\Rightarrow E_{n_2} = 10$$



ru

Ohm's law and Continuity Equation:

$$\int (\nabla \cdot J) dv = \oint J \cdot ds = I = \frac{d\phi}{dt} = \frac{d}{dt} \int \rho_v dv = \int \frac{\partial \rho_v}{\partial t} dv$$

i.e. $I = \frac{d\phi}{dt} = \frac{d}{dt} \int \rho_v dv = \int \frac{\partial \rho_v}{\partial t} dv$

$$\Rightarrow I = \oint J \cdot ds = \int (\nabla \cdot J) dv$$

Divergence theorem

$$\boxed{\nabla \cdot J = \frac{\partial \rho_v}{\partial t}}$$

→ Current has outflow and divergence at a rate depending on decrease of volume charges with time,

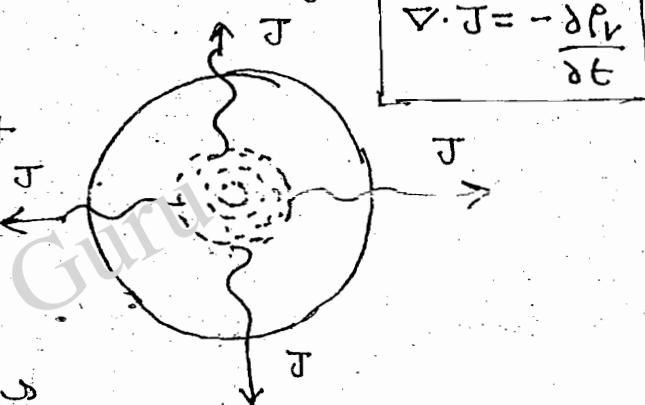
→ Practically

Entering current = leaving current

charge can neither be created nor be destroyed

$$\oint J \cdot ds = 0 \quad] \rightarrow \text{Always}$$

$$\nabla \cdot J = 0$$



→ For a uni-directional current flow

$$\frac{\partial J}{\partial x_c} = \frac{\partial \rho_v}{\partial t}$$

$$\partial J = \partial \rho_v \frac{\partial x_c}{\partial t}$$

Integrating on both sides

$$J = \rho_v V_d$$

where $V_d = \frac{\partial x_c}{\partial t} = \text{drift velocity}$

$v_f = \text{free vel}$

m

= vacuum tubes, CRO

= few $10^6 - 10^7$ m/s

$$v_f \propto \sqrt{E}$$

where $v_d = \frac{\delta x}{\delta t}$ = drift velocity

= charge in a material (bulk)

= few cm/sec

$$v_d \propto E \Rightarrow$$

$$v_d = \mu E$$

↓
mobility of the particle

$$J = \rho_v v_d$$

$$J = \rho_v \mu E$$

$$\Rightarrow J = \sigma E \rightarrow \text{Ohm's law in point form.}$$

where σ = conductivity (mho/m) = $\rho_v \mu$

= ability to allow current

= available free \times mobility of carriers

Case-(1) :-

$\sigma = \infty \rightarrow \text{very good conductor}$

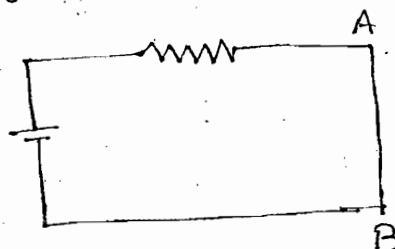
$$\frac{J}{\sigma} = E = 0$$

\rightarrow Electric field cannot exist in a very good conductor

$$\rightarrow V = \int \sigma \cdot d\ell = \text{constant}$$

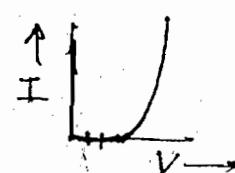
conductor is always a equi-potential region

→ Potential difference or voltage cannot exist in a very good conductor only current can flow but voltage difference good conductor cannot exist



$V_{AB} = 0$ but current flows

$$V \uparrow V_{AB} = 0 \quad I_{AB} \uparrow$$



eg:- (i) diode current after cutting voltage

(ii) Accumulation → does not exist — only flow exists

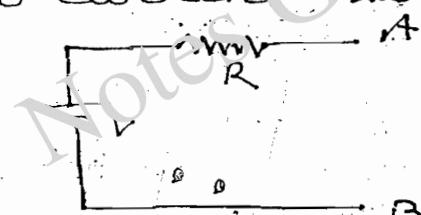
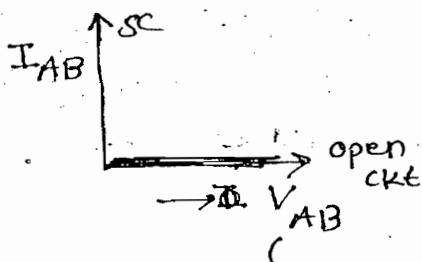
Case - (ii) :-

$\sigma = 0$ very good dielectric.

$$J = 0$$

→ current cannot exist in a very good dielect

→ Voltage or potential difference or Electric field can exist but flow current exist



→ Accumulation can exist but flow can't exist.

$$\nabla \cdot J = - \frac{\partial \rho_v}{\partial t}$$

$$J = \sigma E$$

$$\nabla \cdot (\sigma E) = - \frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \sigma (\nabla \cdot E) = - \frac{\partial \rho_v}{\partial t}$$

The τ_V is time &

$$f_V(t) = f_{V_0} \cdot e^{-\frac{t}{\tau_V}}$$

Note:-

Every charge density exponentially spread on the medium at a rate depending on $\frac{1}{\tau_V}$ that called the relaxation time

$$\frac{1}{\tau_V} = \text{Relaxation time} = \frac{\text{Farad}}{m \times \frac{\text{mho}}{m}}$$

= ohms \times farad

Boundary conditions at conductor surfaces: → $= \text{second}$

(I) $E_{t_1} = E_{t_2}$ (General)

As $\sigma = \infty$ along the conductor

$$\frac{J}{\sigma} = E = 0 \text{ along the surface}$$

$$\Rightarrow E_{\text{tang.}} = 0$$

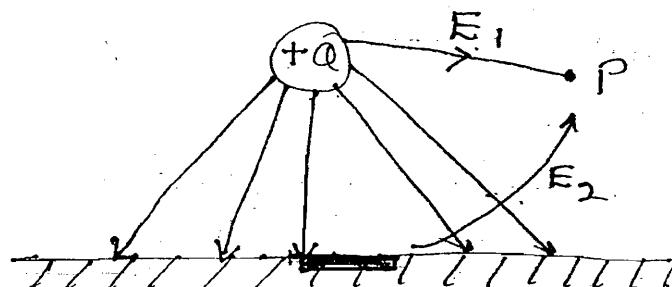
(II) $\partial n_2 - \partial n_1 = p_s$ (General)

Electric field can be normal to a conductor which depends on p_s on the surface

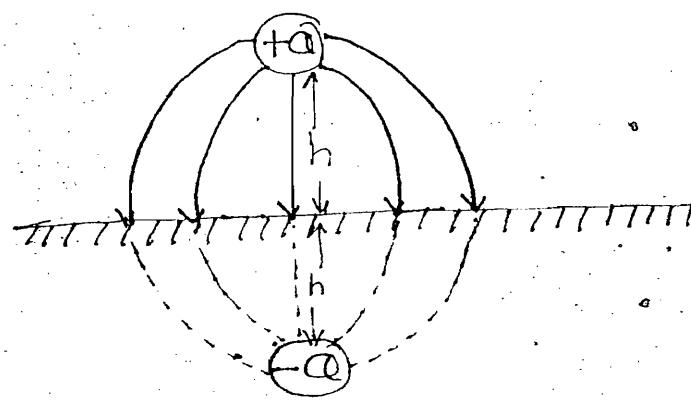
$$\partial_{\text{normal}} = p_s$$

Conductors, Induction, Method of Images:-

→ When a charge is placed near a conductor surface the field is as shown below



- The field has tan component which displaces the free charges along the conductor surface. Since they are periodically accumulated all the other along the conductor surface. This is called as induced charge.
- The resultant field at any point is the vector sum of field due to actual charge and induced charge.
- This field is such that the tangential components are removed and has only normal components as shown below.



- The field appears to be a dipole field with negative charge called as image charge below the conductor surface.

Summary:

- Every charge and its induction effects are represented by a image charge below the conductor obeying all rules of light and optics.

44. $D_1 = 2(a_x - \sqrt{3}a_z) C/m^2 = P_s$

$$P_s = |D_n| = 2\sqrt{1+3} = 4$$

45. $E_n = 2 V/m$

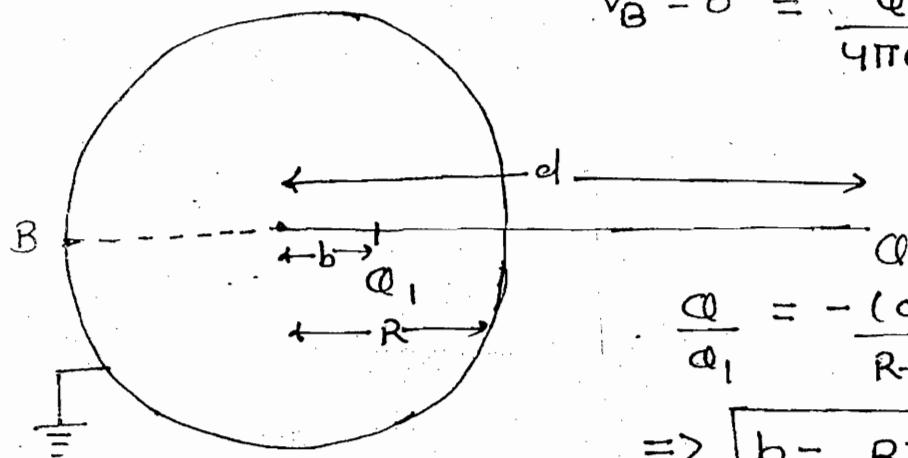
$$D_n = 80 \epsilon_0 E_n = P_s$$

$$\Rightarrow P_s = 80 \times 8 \times 8 \times 10^{-12} \times 2$$

$$= 1.41 \times 10^{-9} C/m^2$$

$$\frac{Q}{4\pi\epsilon(d-R)} + \frac{Q_1}{4\pi\epsilon(R-b)}$$

$$V_B = 0 = \frac{Q}{4\pi\epsilon(d+R)} + \frac{Q_1}{4\pi\epsilon(R+b)}$$

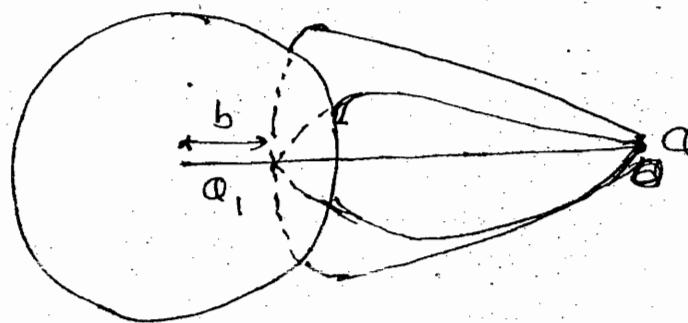


$$\frac{Q}{Q_1} = -\frac{(d-R)}{R-b} = -\frac{(d+R)}{(R+b)}$$

$$\Rightarrow b = \frac{R^2}{d}$$

$$Q_1 = -\frac{R}{d}$$

Note:-



F. 4MC, Ans.

Energy density in Electric fields ($\frac{dWE}{dv} = \frac{1}{2}\epsilon E^2$) :-

Consider $Q_1, Q_2, Q_3, \dots, Q_n$ point charges assemble in a region

Total energy of the Electric field = Total Energy expended in assembling the charges

$$W_1 = 0$$

V_{21} = Potential at 2nd

$$W_2 = -Q_2 V_{21}$$

due to 1st charge

$$W_3 = -Q_3 V_{31} - Q_3 V_{32}$$

$$W_4 = -Q_4 V_{41} - Q_4 V_{43}$$

$$W_n = -Q_n V_{n1} - Q_n V_{n2} - \dots - Q_n V_{n,n-1}$$

$$\alpha_2 v_{21} = \alpha_2 \frac{q_1}{4\pi\epsilon r_{21}}$$

Substitute Subscripts can be interchange without change in meaning of value

$$w_1 = 0$$

$$w_2 = -\alpha_1 v_2$$

$$w_3 = -\alpha_1 v_{13} - \alpha_2 v_{23}$$

$$w_4 = -\alpha_1 v_{14} - \alpha_2 v_{24} - \alpha_3 v_{34}$$

⋮

$$w_n = -\alpha_1 v_{1n} - \alpha_2 v_{2n} - \dots - \alpha_{n-1} v_{n-1 \cdot n}$$

$$W_E = w_1 + w_2 + w_3 + \dots + w_n$$

total energy

$$2W_E = -\alpha_1 v_1 - \alpha_2 v_2 - \alpha_3 v_3 - \dots - \alpha_n v_n$$

$$\Rightarrow W_E = -\frac{1}{2} \sum_{i=1}^n \alpha_i v_i$$

For a continuous charge distribution

$$W_E = -\frac{1}{2} \int \rho_v v dv = -\frac{1}{2} \int (\nabla \cdot \mathbf{v}) v dv = \int \frac{1}{2} \mathbf{v} \cdot (-\nabla v) dv$$

$$\Rightarrow W_E = \int \frac{1}{2} (\mathbf{v} \cdot \mathbf{v}) dv$$

$\frac{dW_E}{dv}$ = Energy density

= Strength of energy at any point

$$= \frac{1}{2} (\mathbf{v} \cdot \mathbf{v}) = \frac{1}{2} \mathbf{v}^2$$

Extension:-

$$\frac{dW_H}{dv} = \text{Magnetic Energy density}$$

$$= \frac{1}{2} \mathbf{B} \cdot \mathbf{H} = \frac{1}{2} \mathbf{H}^2$$

→ Capacitance is the ability to confine Electric field in a finitely small region:

$$C = \text{Farad} = \frac{\oint S \cdot dS}{\int E \cdot dL} = \epsilon \frac{\oint E \cdot dS}{\int E \cdot dL} = \frac{Q}{V}$$

→ It is the ratio with charge utilized to the potential developed by the charge.

Specific Geometries:-

- Parallel Plates
- concentric cylinders
- concentric sphere

Inductance:-

Inductance is the ability to confine H field in a finitely small region

$$L = \text{Henry} = \frac{\int B \cdot dS}{\oint H \cdot dL} = \mu_0 \frac{\int H \cdot dS}{\oint H \cdot dL} = \frac{\Psi_m}{I}$$

→ It is the ratio of the flux developed to the current utilized by the flux

Specific Geometries:-

- Solenoids
- concentric cylinders
- Toroids

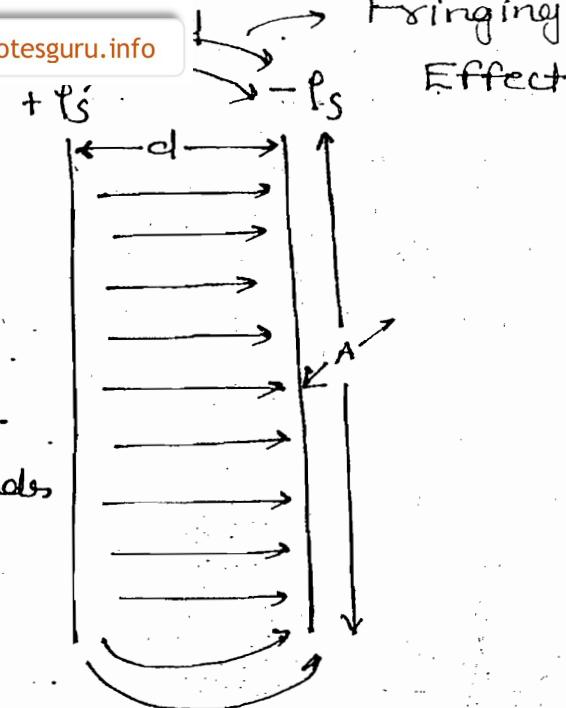
Parallel Plate Capacitors:-

Parallel Plate Capacitor

<https://notesguru.info>

$$E = \frac{P_s}{\epsilon}$$

$$C = \frac{Q}{V} = \frac{P_s A}{V} = \frac{\epsilon A}{d}$$



Capacitance is independent of Q or V and always depends on Area and distance or physical dimensions

$$\begin{aligned} W_E &= \frac{1}{2} \epsilon E^2 (Ad) \\ &= \frac{1}{2} \frac{\epsilon A}{d} (Ed)^2 \\ &= \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV \end{aligned}$$

Multiple Dielectrics in Capacitors!

Case-(1) :-

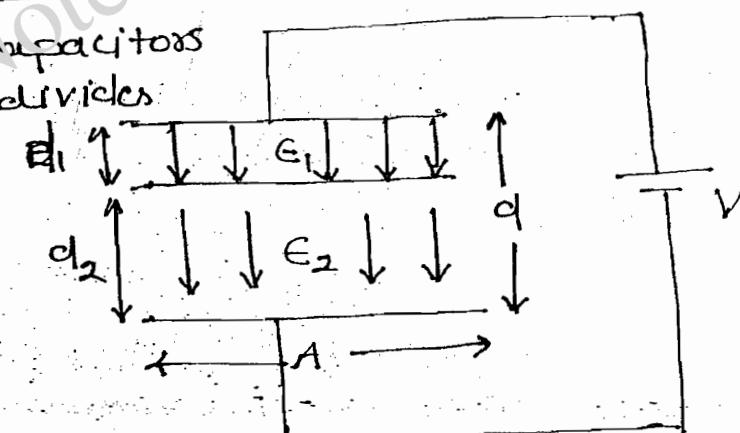
Equal areas of cross-section unequal width of the dielectrics.

They are two capacitors in series as voltage dividers

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{\epsilon_1 A}{d_1} \cdot \frac{\epsilon_2 A}{d_2}$$

$$\frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2}$$



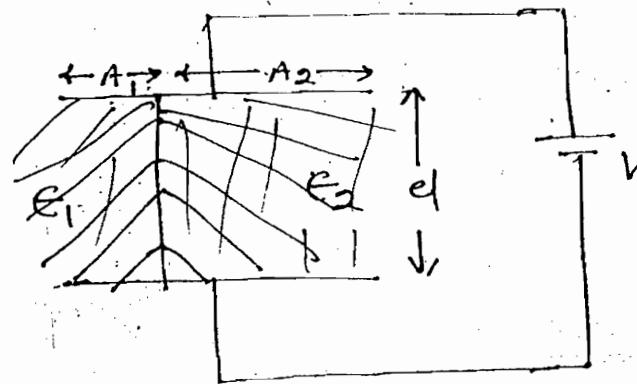
$$\theta_1 = \theta_2 \Rightarrow \epsilon_1 E_1 = \epsilon_2 E_2 \Rightarrow \frac{\epsilon_1 V_1}{d_1} = \frac{\epsilon_2 V_2}{d_2}$$

$$\Rightarrow \frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1} \right) \left(\frac{d_1}{d_2} \right)$$

Unequal areas of cross section and equal width of the dielectrics :-

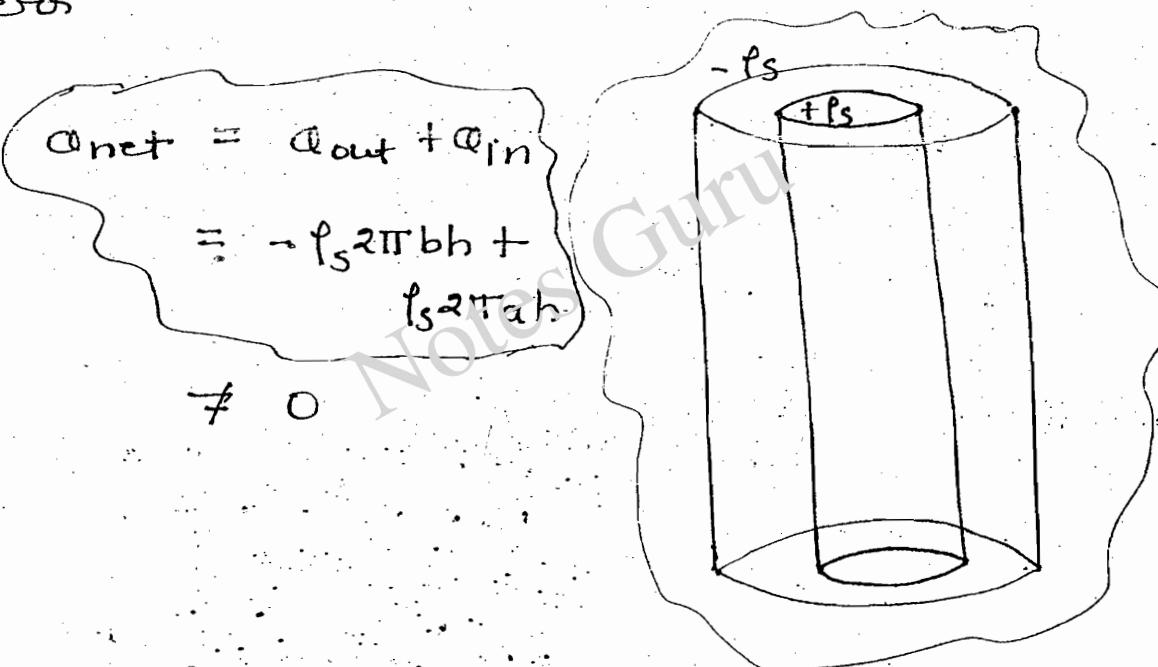
$$C_{eq} = C_1 + C_2$$

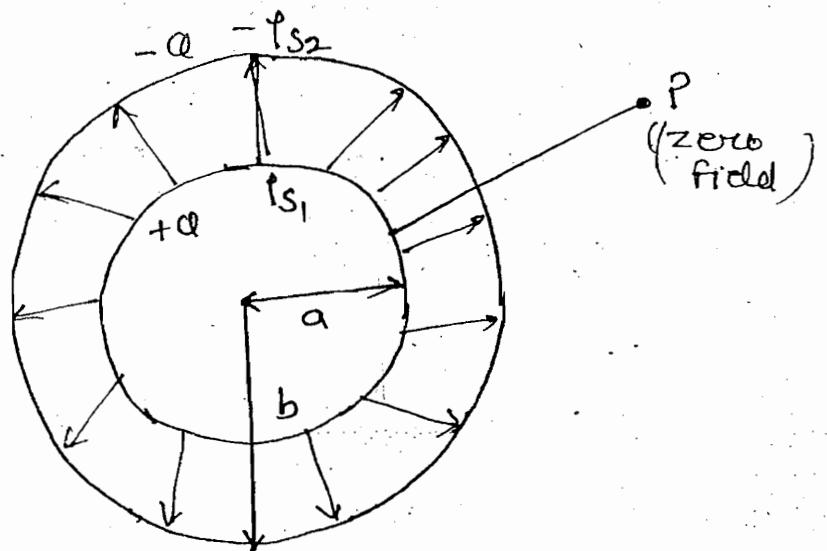
$$= \frac{\epsilon_1 A_1 + \epsilon_2 A_2}{d}$$



Cocentric Cylinder Capacitance :-

Two concentric cylinders with equal and opposite charge density may not have confined flux but two equal and opposite charges on concentric cylinders has flux confined only b/w the cylinders.





$$\mathcal{D} \propto \rho_s \propto \frac{1}{r}$$

→ The field of a sheet of cylindrical charge obeys same geometry as a line charge

$$C = \frac{Q}{V} = \frac{\rho_s h}{\frac{\rho_s h}{2\pi\epsilon_0} \ln(\frac{b}{a})}$$

⇒

$$C = \frac{2\pi\epsilon_0 h}{\ln(b/a)}$$

Extension:-

Cocentric sphere capacitance.

$$C = \frac{Q}{V}$$

$$C = \frac{a}{\frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)} = \frac{4\pi\epsilon_0}{\left(\frac{1}{a} - \frac{1}{b} \right)}$$

$$W_{E1} = 0 + 0 \cdot \frac{Q}{4\pi\epsilon_0 \frac{1}{2}} + 0 \cdot 0 = 0$$

$$+ Q \cdot \frac{Q}{4\pi\epsilon_0 \frac{1}{2}} + 0 \cdot Q = \frac{5Q^2}{4\pi\epsilon_0}$$

$$W_{E2} = \frac{5Q^2}{8\pi\epsilon_0} = \frac{W_{E1}}{2}$$

Note:-

$$W_E \propto QV \propto \frac{1}{r}$$

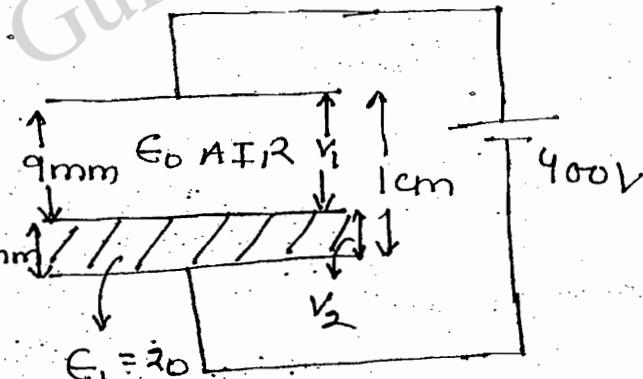
$$\frac{1}{2} (J \cdot A) = \frac{\text{Amp}}{\text{m}^2} \times \frac{\text{Joules}}{\text{Amp} \cdot \text{m}} = \frac{\text{Joules}}{\text{m}^3}$$

$$\Rightarrow \frac{1}{2} (J \cdot A) = \frac{1}{2} ((\nabla \times H) \cdot A) = \frac{1}{2} (H (\nabla \times A)) = \frac{1}{2} B \cdot H$$

$$E_{AIR} = \frac{V_{AIR}}{9\text{mm}}$$

$$V_1 + V_2 = 400 - CII$$

$$\frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1} \right) \left(\frac{C_1}{C_2} \right) \rightarrow CII$$



$$\frac{V_1}{V_2} = \frac{20\epsilon_0}{\epsilon_0} \times \frac{9\text{mm}}{1\text{mm}} = 180$$

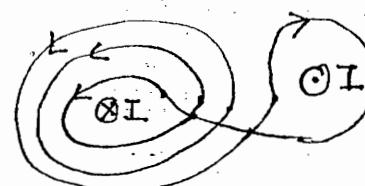
$$V_1 = 180V_2$$

$$\text{Ans} \rightarrow V_1 = 44 \text{ kV/m}$$

$$\text{Circulation} = \oint H \cdot dL = I$$

$$= I + I + I - (-I)$$

$$= 4I, \text{ Ans.}$$



Time Varying Fields and Maxwell's Equations

Static Fields - Maxwell's Equations:-

Integral Form

$$\text{I) } \oint \mathbf{B} \cdot d\mathbf{s} = 0$$

$$\text{II) } \oint \mathbf{E} \cdot d\mathbf{l} = 0$$

$$\text{III) } \oint \mathbf{H} \cdot d\mathbf{l} = I$$

$$\text{IV) } \oint \mathbf{B} \cdot d\mathbf{s} = 0$$

Point form

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = J$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Note:-

- The surface ~~and~~ Integral and Divergence expressions are consistently the same in static / time varying fields.
- The line integral and curl expression are modified in time varying fields to explain AC voltages and currents.

Open Integral's (Not Maxwell Equations): -

$$1. \int \mathbf{B} \cdot d\mathbf{s} = \Psi_{e,0} = \text{Coulombs}$$

$$2. \int \mathbf{E} \cdot d\mathbf{l} = V^e = \text{EMF} = \text{Volts}$$

← time

$$3. \int \mathbf{H} \cdot d\mathbf{l} = \Psi_{m,0} = \text{Webers}$$

$$4. \int \mathbf{B} \cdot d\mathbf{s} = \Psi_m = \text{MMF} = \text{Amps}$$

Maxwell 2nd Equation and Faraday's Law:

Faraday's Law Statement:

EMF or voltage is induced even in a closed conductor when the magnetic flux crossing the surface changes with time.

i.e. Rate of change of induced EMF

flux is equal to

$$\oint \mathbf{E} \cdot d\mathbf{l} = V = - \frac{d\Phi_m}{dt} \xrightarrow{\text{due to Lenz's Law}} = - \frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{s}$$
$$= \int - \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

Lenz's Law:—

The induced EMF ^{always} opposes the basic changing flux (Cause)

$$\oint \mathbf{E} \cdot d\mathbf{l} = \int - \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = V = - \frac{d\Phi_m}{dt} = - \frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{s} = \int - \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} \leftarrow$$

Apply Stoke's theorem = $\int \nabla \times \mathbf{E} \cdot d\mathbf{s}$

$$\boxed{\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}}$$

Modified Maxwell's Second Equation

Note:—

Potential is unique at a time at a point in a space but changes with time and Hence the modification

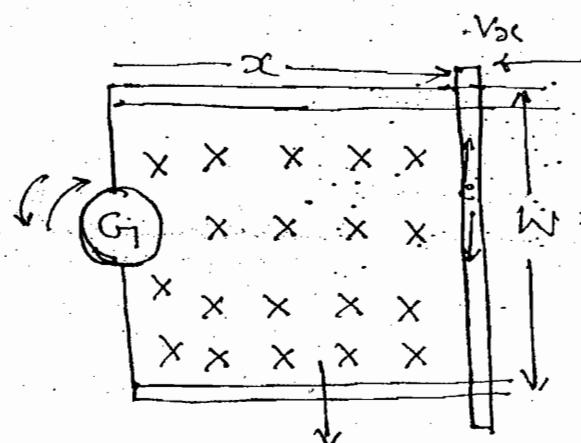
Sliding Rail Experiment:—

$$V = - \frac{d\Phi_m}{dt}$$

$$\Rightarrow V = - \frac{d}{dt} (B \cdot A)$$

$$\Rightarrow \boxed{V = - B \cdot W \frac{dx}{dt}}$$

$$\Rightarrow \boxed{V = - B \cdot W \cdot V_{oc}}$$



Static Uniform
B field

Note:-

- When a conducting rod moves in a magnetic field a force exists on the electron obeying Lorentz law. This displaces the electrons towards one side.
- Thus accumulation is called as induced voltage.
- As the rod moves to and fro the voltage polarity changes. This is called as AC voltage.
- The already displaced \vec{E} opposes another further coming \vec{E} 's. This is called as Lenz's law.

$$F_y = q(V_x \times B_z) = qE$$

$$\Rightarrow \frac{d\phi}{dt} B \sin \theta = \frac{V}{W} \quad F_C$$

$$\Rightarrow V = \frac{d}{dt} (\alpha BW) = \frac{d}{dt} (BA) \quad (\theta = 90^\circ)$$

$$\Rightarrow V = \frac{d\Phi_m}{dt}$$

Maxwell's IV Equation and Inconsistency of Ampere's Law!

- When a capacitor is connected with AC harmonic voltage there is a current flowing in the wire and plates obeying Ohm's law like any linear element.

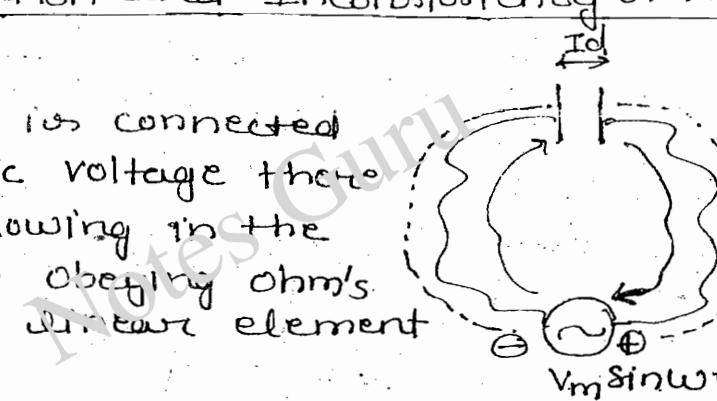
$$I = C \frac{dv}{dt}$$

$$I = C \cdot W \cdot V_m \sin(\omega t + 90^\circ)$$

$$(e^{j90^\circ} = j) \rightarrow \text{Phase} \dots \text{Phase}$$

$$I = j \omega C V_m \sin \omega t$$

$$V = \left(\frac{1}{j \omega C} \right) I$$



- There is no current flowing b/w plates as they are dielectric. Hence the circuit is not closed. But Ampere's law state that current flows only

closed circuits. Hence
law as

difficult Ampere's

$$\oint H \cdot dl = I_c + I_d$$

$$\nabla \times H = J_c + J_d$$

Using equation of continuity

$$\nabla \cdot J_d = \frac{\partial \rho}{\partial t} = \frac{\partial (\nabla \cdot A)}{\partial t} = \nabla \cdot \frac{\partial A}{\partial t} \Rightarrow$$

$$\Rightarrow J_d = \frac{\partial A}{\partial t} \quad \& \quad I_d = \int \frac{\partial A}{\partial t} \cdot ds$$

Maxwell IV Equation is,

$$\oint H \cdot dl = I_c + \int \frac{\partial A}{\partial t} \cdot ds$$

$$\Rightarrow \nabla \times H = J_c + \frac{\partial A}{\partial t}$$

where I_c = conduction current

OR

= Moving E's

→ When an AC voltage applied to the capacitor plate, the plates are alternatively charge and discharge. This continues takes place as the polarity changes. Hence this self establish continuity.

→ As ρ_s on the plates changes with time, A b/w the plates changes with time. Hence $\frac{\partial A}{\partial t}$ is

$$\frac{C/m^2}{\text{second}} = \text{Amp}/m^2 = J_d$$

This is also a format of current.
This is called as Displacement current density.

Summary 1:—

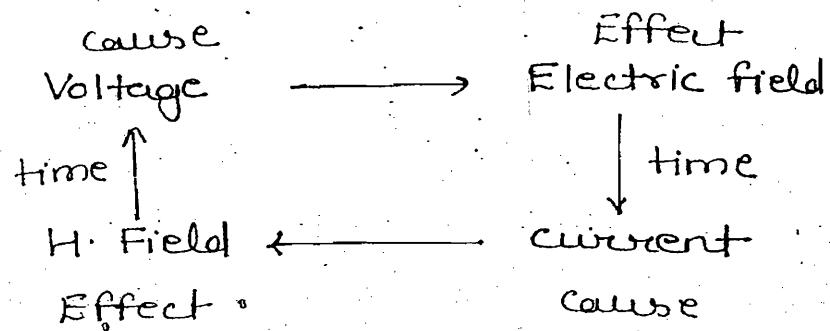
$$\oint E \cdot dl = - \frac{\partial B}{\partial t} \cdot ds$$

$$\oint H \cdot dl = \int \frac{\partial A}{\partial t} \cdot ds + J_c$$

- A time varying magnetic field is a cause of voltage
- A time varying electric flux is a format of current

$$\frac{\text{Weber}}{\text{second}} = \frac{\text{Volts}}{\text{second}}$$

$$\frac{\text{Coulomb}}{\text{sec}} = \frac{\text{Amp}}{\text{sec}}$$

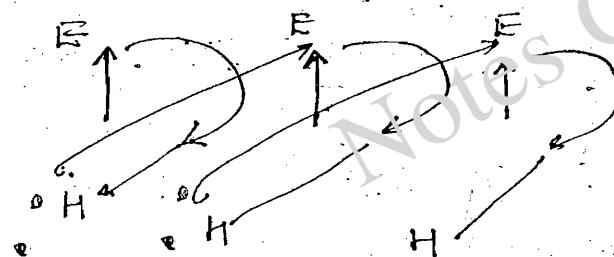


Summary 2:-

Space varying electric field $\nabla \times E = -\frac{\partial B}{\partial t}$ → time varying B field

Space varying magnetic field $\nabla \times H = \frac{\partial B}{\partial t} + J_c$

→ A time varying E field produces a space varying orthogonal H field and vice-versa



H field $\xrightleftharpoons[\text{space}]{\text{time}} E \cdot \text{Field}$

→ Accumulation leads to flow and flow leads to accumulation sustaining each other.

Summary 3:-

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t} + \text{J}_c$$

$$\rightarrow \mu = \text{Henry/m}, \epsilon \quad \text{https://notesguru.info} \quad \sigma = \text{mho/m} \rightarrow$$

For E to transform to H and vice-versa the material and its permitting abilities are also important.

i.e. Material constants decides the E/H dynamics in the medium

$$\rightarrow \vec{E} - \text{Volts/m} , \vec{H} - \text{amps/m} , \nabla - \text{perm}$$

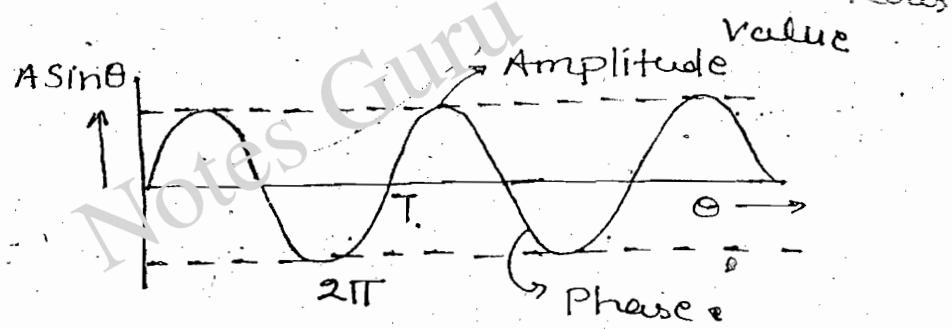
Note:-

→ For sustain oscillation or ever existing phenomenon are produce when the time derivative of E and H is always back the same function and such a function called as Harmonic function.

Harmonic Functions:-

They are 2 dimension quantities and having 3 formats

$$\begin{array}{l} \rightarrow A \sin \theta \\ \rightarrow A \cos \theta \\ \rightarrow A e^{j\theta} \end{array} \left. \begin{array}{l} \text{Amplitude} \rightarrow \text{Domain 1} \rightarrow \text{Peak value} \\ \text{Phase} \rightarrow \text{Domain 2} \rightarrow \text{Instantaneous value} \end{array} \right\}$$



Property 1:-

The phase should be a linear function of the variable

$$(1) \quad \theta \propto t \quad t \rightarrow \text{time Harmonic}$$

$$\Rightarrow \theta = \omega t \quad \omega = \text{Phase shift constant per unit time}$$

$$= \frac{2\pi}{T} \text{ rad/second}$$

$$(11) \quad \theta \propto z \rightarrow \text{Space Harmonic}$$

$$\Rightarrow \theta = \beta z \quad \beta = \text{Phase shift constant per unit length} = \frac{2\pi}{L} \text{ rad/m}$$

The derivative back the same function shifted orthogonally by 90°

$$A \sin \omega t \xrightarrow{\text{I-derivative}} \omega A \sin(\omega t + 90^\circ) \xrightarrow[\text{der.}]{\text{II}} \omega^2 A \sin(\omega t + 180^\circ)$$

$$\xrightarrow[\text{III-derivative}]{\text{III}} \omega^3 A \sin(\omega t + 270^\circ)$$

$$A e^{j\omega t} \xrightarrow[\text{der.}]{\text{I}} \omega A e^{j(\omega t + 90^\circ)} \xrightarrow[\text{der.}]{\text{II}} \omega^2 A e^{j(\omega t + 180^\circ)}$$

$$(\because j^3 = -j = e^{j270^\circ})$$

$$(-1 = e^{j180^\circ})$$

$$\downarrow \text{III-der.}$$

$$\omega^3 A e^{j(\omega t + 270^\circ)}$$

$$(\because j = e^{j90^\circ})$$

→ All harmonics obey the basic property the second order derivative is back the same function

→ They unsatisfied the differential equation

$$\nabla^2 - M^2 = 0$$

$$\nabla^2 + M^2 = 0$$

e.g:- E/H field equation in free space

Electro magnetic wave equation in free space

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$$

Taking ∇ on both sides

$$\nabla \times (\nabla \times \vec{H}) = \epsilon \frac{\partial}{\partial t} (\nabla \times \vec{E})$$

$$-\nabla^2 \vec{H} = \epsilon \frac{\partial}{\partial t} \left(-\mu \frac{\partial \vec{H}}{\partial t} \right)$$

$$\boxed{\nabla^2 H = \epsilon \mu \frac{\partial^2 H}{\partial t^2}}$$

Space Harmonic

$$\nabla^2 E = \epsilon \mu \frac{\partial^2 E}{\partial t^2}$$

Time Harmonic

These are called as E/H wave equations in free space

Eg-(2) :- V/I Equations in LC circuits

$$I = C \frac{dV}{dt}$$

$$V = -L \frac{dI}{dt} = -L \frac{d}{dt} \left(C \frac{dV}{dt} \right)$$

$$\Rightarrow V = -LC \frac{d^2 V}{dt^2}$$

$$\Rightarrow \boxed{\frac{d^2 V}{dt^2} = -\frac{1}{LC} V}$$

and

$$\boxed{\frac{d^2 I}{dt^2} = -\frac{1}{LC} I}$$

By comparison

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

Types of Exponential Functions:-

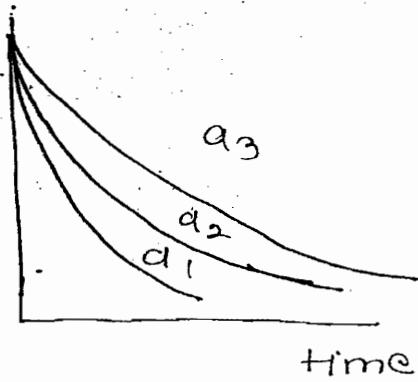
$\rightarrow e^{-kt}$ vs t

(1) case - (1) :-

$$K = a = \text{true real no. } e^{-kt}$$

$$a_1 > a_2 > a_3$$

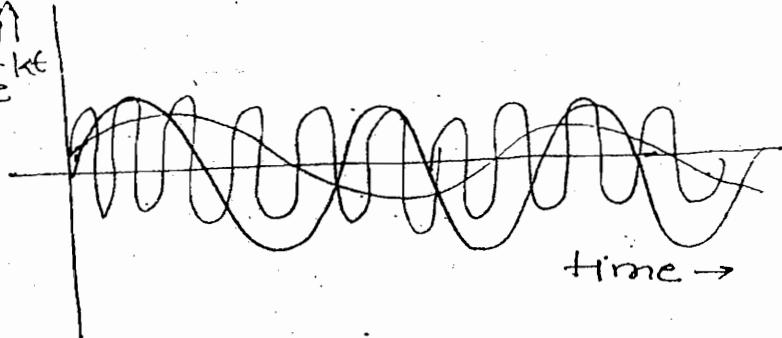
\rightarrow Real Exponential.



II) Case - (II) :-

$$\kappa = j\omega = \text{purely imaginary}$$

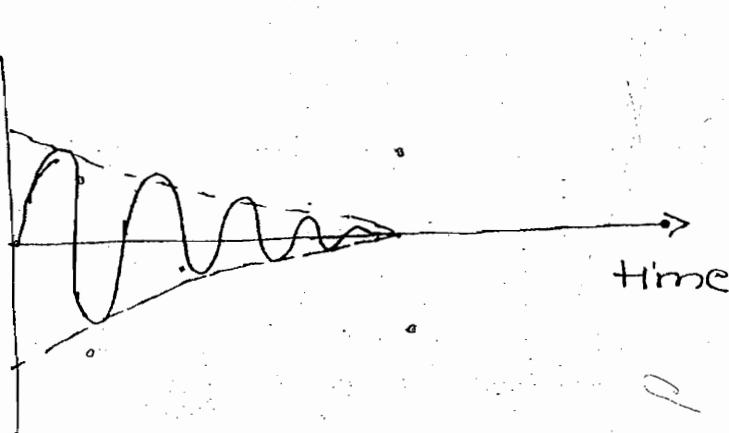
ω = Phase Shift
constant



III) Case - (III) :-

$$\kappa = a + j\omega$$

$$(e^{-at})(e^{-j\omega t}) = e^{-(a+j\omega)t} = A \cdot e^{-j\theta}$$



Note:-

→ j stands for an orthogonal shift from domain 1 to a second independent domain 2

eg:- (i) east and north displacements

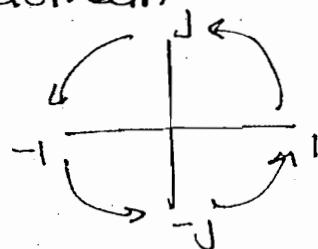
(ii) Resistance and reactance of a impedance

(iii) Amplitude and phase of a harmonic

→ A scaling of j in domain 1 is a shift of 90° in domain 2

$$\text{eg:- (i) } j\omega t e^{-j\omega t} \rightarrow \omega e^{j(\omega t + 90^\circ)}$$

(ii) S-domain



$$1 \times j = j = 1 \angle 90^\circ$$

$$j \times j = -1 = 1 \angle 180^\circ$$

$$-1 \times j = -j = 1 \angle 270^\circ$$

$$-j \times j = 1 = 1 \angle 360^\circ$$

EM Propagation in Gic

<https://notesguru.info>

b :- (σ , ϵ , μ)

Using Maxwell's Equation

$$\nabla \times H = -E + \epsilon \frac{\partial E}{\partial t}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

→ Every EM wave needs a time harmonic source at one end

$$\left. \begin{aligned} E_s &= E_0 e^{j\omega t} \\ H_s &= H_0 e^{j\omega t} \end{aligned} \right\} \quad \omega = \text{frequency in rad/s}$$

Range of ω :-

$f/\omega \rightarrow \text{Hz/kHz} \rightarrow \text{AF Wave}$

$\rightarrow \text{MHz} \rightarrow \text{Radio frequency Wave}$

$\rightarrow \text{GHz}/10^{12} \text{Hz} \rightarrow \text{Microwave}$

$\rightarrow 10^{15} - 10^{18} \text{Hz} \rightarrow \text{Light wave}$

$\rightarrow 10^{20} - 10^{22} \text{Hz} \rightarrow \text{X-rays / Gamma rays}$

$\rightarrow 10^{25} \text{Hz} \rightarrow \text{Cosmic Rays}$

⇒ EM Wave Spectrum

$$\nabla \times H = -E + \epsilon \frac{\partial E}{\partial t} = -\underline{E_0 e^{j\omega t}} + \epsilon j\omega \underline{E_0 e^{j\omega t}}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} = (\sigma + j\omega \epsilon) E$$

$$\Rightarrow \nabla \times H = (\sigma + j\omega \epsilon) E \quad \text{---(I)}$$

$$\nabla \times E = -j\omega \mu H \quad \text{---(II)}$$

$$\nabla \times E = \frac{\mu}{\epsilon} \omega$$

$$= -\mu j \omega H_0 e^{j\omega t} = -j\omega \mu H$$

$$\nabla \times E = -j\omega \mu H \quad \text{--- (ii)}$$

Eq-(i) & (ii) are called as Maxwell Equations for time Harmonic source

Take H from (ii) & put in (i)

$$H = \frac{\nabla \times E}{-j\omega \mu}$$

$$\Rightarrow \nabla \times \left(\frac{\nabla \times E}{-j\omega \mu} \right) = (\omega + j\omega \epsilon) E$$

$$\Rightarrow \nabla \times \left(\nabla \cdot E \right) - \nabla^2 E = -j\omega \mu (\omega + j\omega \epsilon) E$$

For a charge free region

$$\nabla^2 E = j\omega \mu (\omega + j\omega \epsilon) E \quad \text{--- (iii)}$$

Similarly

$$\nabla^2 H = j\omega \mu (\omega + j\omega \epsilon) H \quad \text{--- (iv)}$$

Space \swarrow Time

These are called as Helmholtz's Propogation Equations.

Note:-

- If the source is the time harmonic then the effect is space harmonic propagation in the medium.

$$\nabla^2 E = j\omega \mu (\sigma + j\omega \epsilon) E \quad (\text{III})$$

$$\nabla^2 H = j\omega \mu (\sigma + j\omega \epsilon) H \quad (\text{IV})$$

Consider ∇ in one direction dimension of Z only
 E is in x direction, H is in y -direction

$$\text{consider } \sqrt{j\omega \mu (\sigma + j\omega \epsilon)} = \gamma$$

$$\frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x \quad (\text{III})$$

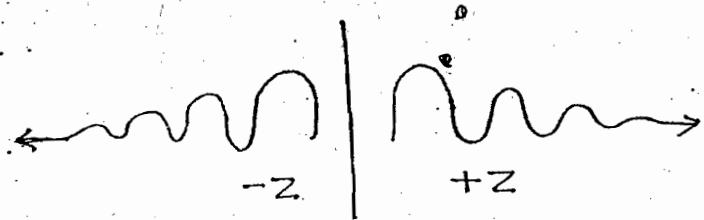
$$\frac{\partial^2 H_y}{\partial z^2} = \gamma^2 H_y \quad (\text{IV})$$

The $E(z)$ and $H(z)$ solution is.

$$E(z)_x = (c_1 e^{-\gamma z} + c_2 e^{\gamma z}) a_x \quad (\text{V})$$

$$H(z)_y = (c_3 e^{-\gamma z} + c_4 e^{\gamma z}) a_y \quad (\text{VI})$$

The solution physically represent two waves travelling on either sides from a planar source and both decaying. This is called as uniform plane wave



Considering only the $+z$ solution and c_1 & c_3 being the initial source values. The final E/H wave solution is

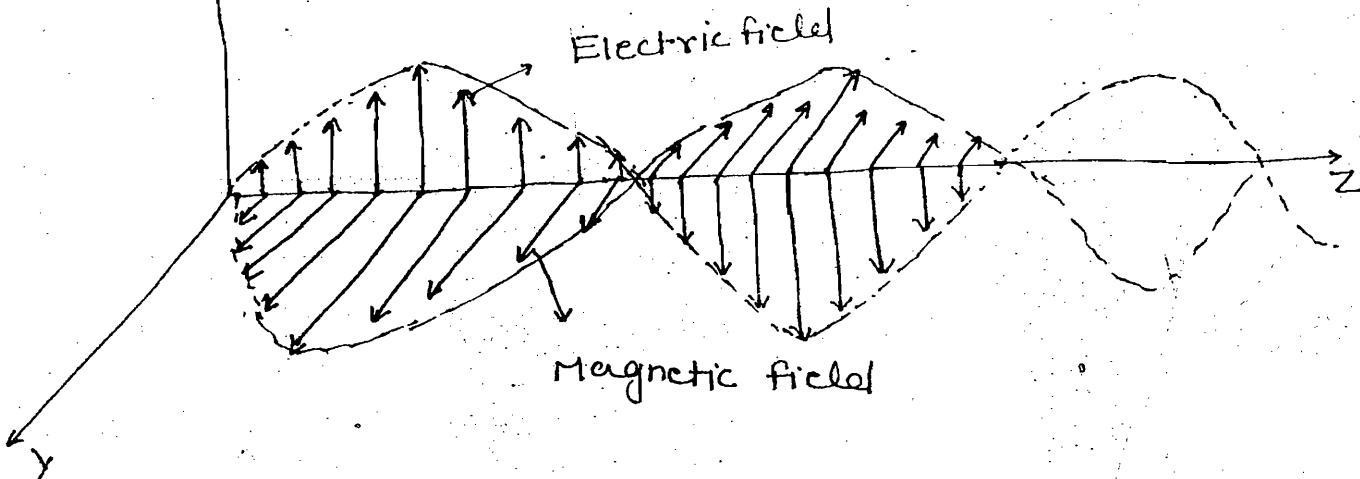
$$E(z, t)_x = (E_0 e^{j\omega t} \cdot e^{-\gamma z}) a_x$$

$$H(z, t)_y = (H_0 e^{j\omega t} \cdot e^{-\gamma z}) a_y$$

The solution is a product of time and space harmonics.

$$E_0 e^{j\omega t} \rightarrow \text{electric field shift with time}$$

$$H_0 e^{j\omega t} \rightarrow H \text{ field } //$$



- Electric and H field perpendicular and shifts simultaneously with time
- Cross product result in transverse and dot product result in longitudinal.

Propagation constant (γ) :-

γ is the constant of the exponential with z that decides the course of propagation i.e. γ is called as propagation constant.

$$\gamma = \sqrt{j\omega \mu (\epsilon + j\omega \epsilon)} = \alpha + j\beta$$

$$E(z, t) = (E_0 e^{-\alpha z} \cdot e^{j\omega t} e^{-j\beta z}) a_x$$

$E(z, t)_x = (E_0 e^{-\alpha z}) e^{j(\omega t - \beta z)} a_x$	field direction
$H(z, t)_y = ((H_0 e^{-\alpha z}) e^{j(\omega t - \beta z)}) j a_y$	

Amplitude Phase

Note-1:-

Every EM wave is harmonic whose amplitude exponentially decays at α rate

α = attenuation constant

Note-2:-

Every EM wave is a harmonic whose phase independently but linearly changes with time and space.

Note-3:-

The harmonic E & H have direction and obeying the basic transverse nature with propagation

$E \times H$	= Propogation constant
(V/m)	(A/m)
	(N/Ay)

Intrinsic Wave Impedance (η):-

$$E \rightleftharpoons H$$

$$\eta = \frac{E}{H} = \frac{\text{Volts/m}}{\text{Amp/m}} = \text{ohm}$$

When E is transformed to H & vice-versa the rate of transformation is called as intrinsic impedance of the medium.

$$\nabla \times E = -j\omega \mu H$$

$$\frac{\partial}{\partial z} a_z \times E_{az} = -j\omega \mu H a_y$$

$$\frac{\partial E}{\partial z} = -j\omega \mu H$$

$$\text{As } E = E_0 e^{-Yz}$$

$$\Rightarrow -YE = -j\omega \mu H$$

$$\Rightarrow \frac{E}{H} = \frac{j\omega \mu}{\sqrt{j\omega \mu (\tau + j\omega \epsilon)}}$$

\Rightarrow

$$\frac{E}{H} = \sqrt{\frac{j\omega}{\sigma + j\omega\epsilon}}$$

$$n = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \text{complex} = R + jX$$

The transformation has a resistance which has a loss (α) in transformation and a reactance which leads to phase shift (β)

$$V = \alpha + j\beta$$

$$n = R + jX$$

Workbook - 2

1) $V = -\frac{d\psi_m}{dt} = -\frac{d}{dt} \left(\frac{1}{3} t^3 \right) = -\frac{1}{3} \cdot 3t^2 \Big|_{t=3s} = 9V$

$$\Rightarrow \boxed{\lambda = -1}$$

(2) $V = -\frac{d\psi_m}{dt} = -\frac{d}{dt} (BA) = -\pi r^2 \frac{dB}{dt}$

$$= -\pi \times (0.1)^2 \cdot 0 \{ -\sin(120\pi t) \cdot 120\pi \}$$

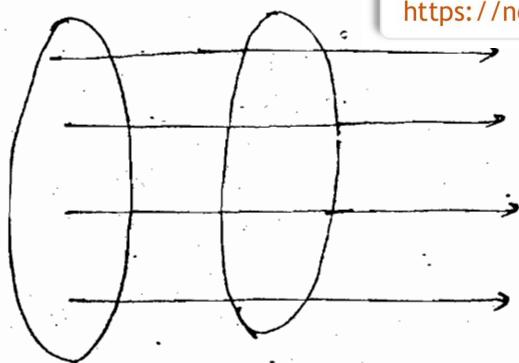
$$= 118 \sin(120\pi t) \text{ Ans}$$

3) $B \rightarrow$ Linearly decrease with time $B(t) = -kt$
 \rightarrow Uniform decrease with space

$$V = -\frac{d\psi_m}{dt} = -\frac{d}{dt} (BA) = -A(-k) = \text{constant}$$

$$= \text{DC voltage}$$

- \rightarrow Up to open circuit E flows and emf is induced
- \rightarrow Induced emf converts into current in coil 1



Ans - D

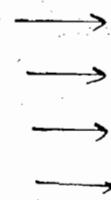
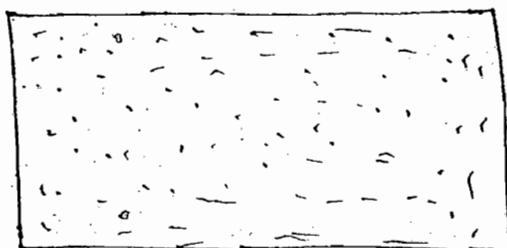
$$V = 0$$

$$V \neq 0$$

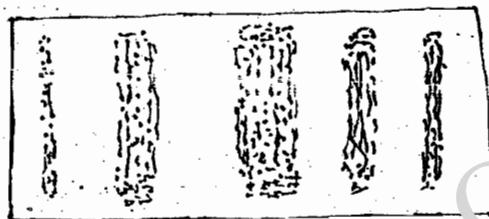
$$I \neq 0$$

$$I = 0$$

Note:-



DC current



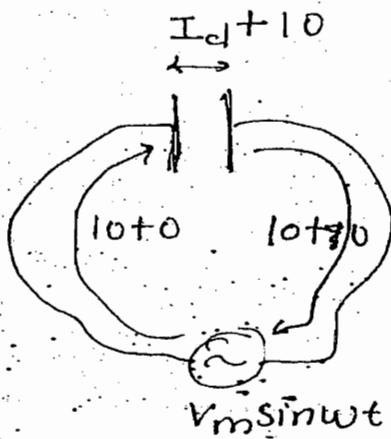
AC current

$$I_c = I_d$$

$$\oint H \cdot dL = I_c + I_d \neq 2I_d$$

or
2I_c

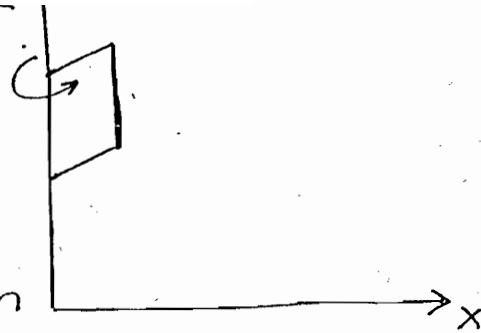
$$J_d = \epsilon \frac{\delta E}{\delta t}$$



$$I_d = \epsilon A \cdot j \omega E = j \omega \frac{\epsilon A}{d} V$$

Ans - (a)

$$\overrightarrow{B(t)x}$$



Uniform and Harmonic
in time

↓
Transformer Action

AC Voltages and harmonic $B(t)$ being involved.

$$V = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}(BA) = -B \underbrace{\frac{dA}{dt}}_{\text{Rotation}} - A \underbrace{\frac{dB}{dt}}$$

$$B(t) = \cos(\omega t) \rightarrow \text{AC voltage}$$

Ans - Ce)

Radiation → EM Waves → generate due to cross product

$$Y = \alpha + j\beta$$

Amplitude = $|E|$

$$|E| = E_0 e^{-\alpha z} = \frac{1}{c} E_0 \quad \text{when } z=20m$$

$$\Rightarrow \alpha z = 1 \Rightarrow \alpha = \frac{1}{20}$$

$$\theta \propto z \Rightarrow \theta = \beta z$$

$$\Rightarrow \beta = \frac{\pi/6}{20m} = \frac{\pi}{120}$$

$$Y = \frac{1}{20} + j \frac{\pi}{120}$$

$$|E| \rightarrow 20\% \circ$$

$$|E| \rightarrow 20 \rightarrow 100 \xrightarrow{E_0} z = 5m$$

$$|E| \rightarrow 40 \rightarrow 100 \rightarrow z = ?$$

It is not a linear function

$$|E| = E_0 e^{-\alpha z}$$

$$\Rightarrow 20 = 100 e^{-\alpha 5} \Rightarrow e^{5\alpha} = 5$$

$$\Rightarrow 5\alpha = \ln 5$$

$$\Rightarrow \alpha = \frac{\ln 5}{5}$$

$$40 = 100 e^{-\alpha z}$$

$$\Rightarrow e^{\alpha z} = \frac{100}{40} = 2.5$$

$$\Rightarrow \alpha z = \ln 2.5$$

$$\Rightarrow z = \frac{5 \ln 2.5}{\ln 5} = 2.8, \text{ Ans.}$$

Note:-

Change is depending on function value

Cause-(1) :-

EM Wave
Propagation
in free
space

$\frac{3}{2}$

$$|E| = E_0 e^{-\alpha z}$$

$$E_0 = 100$$

$$z = \frac{1}{\alpha}$$

$$\frac{100}{e} = 37$$

$$z = \frac{1}{\alpha}$$

$$\frac{100}{e^2} = 13$$

$$z = \frac{1}{\alpha}$$

$$\frac{100}{e^3} = 4$$

EM Wave Propagation in free space

$$\left[\omega = 0, \epsilon = \epsilon_0 \right]$$

$$\mu = \mu_0$$

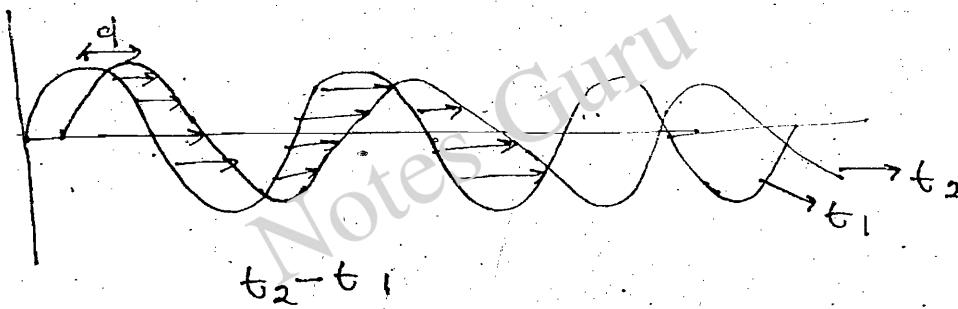
$$Y = \sqrt{j\omega\mu(\omega + j\omega\epsilon)} = j\omega\sqrt{\mu_0\epsilon_0} = \text{purely imaginary}$$

$\Rightarrow \alpha = 0$ No attenuation of EM wave in free space

$\beta = \omega\sqrt{\mu_0\epsilon_0}$ = only phase shift and propagation constant exists

$$n = \sqrt{\frac{j\omega\mu}{\omega + j\omega\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \text{real}$$

$$= \sqrt{4\pi \times 10^{-7} \times 36\pi \times 10^9} = 120\pi = 377$$

Phase Velocity (v_p) :-

The distance travelled by any in-phase point per unit time is called as phase velocity.

$$v_p = \frac{d}{t}$$

$$= \frac{1}{T} = df = \frac{2\pi d}{2\pi T}$$

$$\Rightarrow v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\Rightarrow v_p = 3 \times 10^8 \text{ m/s}$$

→ Ideal dielectrics = Perfect / lossless dielectrics

$$\rightarrow \epsilon = \epsilon_0, \mu = \mu_0$$

$$V = \sqrt{j\omega\mu(\epsilon + j\omega\epsilon)} = j\omega\sqrt{\mu_0\epsilon_0\epsilon_R}$$

= purely imaginary

$\Rightarrow \alpha = 0$ i.e. No attenuation of EM wave in ideal dielectrics

$\beta = \omega\sqrt{\mu_0\epsilon_0\epsilon_R}$ = only phase shift and property exist.

$$h = \sqrt{\frac{j\omega\mu}{\epsilon + j\omega\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_R}} = \text{real}$$

$$= \sqrt{\frac{4\pi \times 10^{-7} \times 36\pi \times 10^9}{\epsilon_R}} = \frac{120\pi}{\sqrt{\epsilon_R}} = \frac{377}{\sqrt{\epsilon_R}}$$

Phase Velocity (v_p) :-

$$v_p = \frac{\lambda}{T} = \lambda f = \frac{2\pi\lambda}{2\pi T}$$

$$\Rightarrow v_p = \frac{\omega}{\beta_0} = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_R}}$$

$$\Rightarrow v_p = \frac{3 \times 10^8}{\sqrt{\epsilon_R}} \text{ m/s}$$

$$E(x, t) = 25 \sin(\omega t + 4x) a_y$$

By comparison, propagation direction $\rightarrow -a_x$

$$\beta = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{\pi}{2}$$

$$f = \frac{3 \times 10^8}{\frac{\pi}{2}} = \frac{600}{\pi} \text{ MHz}$$

Method - 1 :-

$$\nabla \times E = -j\omega \mu H = -\mu \frac{\partial H}{\partial t}$$

$$H = -\frac{1}{\mu} \int (\nabla \times E) dt$$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix}$$

Method 2 :-

$$E(x, t) = 25 \sin(\omega t + 4x) a_y$$

$$\underline{\text{Step-1:-}} \quad \frac{|E|}{|H|} = \text{const}$$

Step-2:- when n is real E & H have the same harmonic and phase

$$\underline{\text{Step-3:-}} \quad \frac{E}{H} = \frac{\text{const}}{c \nu_p} \quad (\text{Basic Transverse Nature})$$

$$H(x, t) = \left(\frac{25}{120\pi} \right) \sin(\omega t + 4x) (-a_x \times a_y)$$

$$= \left(\frac{-25}{120\pi} \right) \sin(\omega t + 4x) a_z$$

$$E(y, t) = 25 \sin(10^8 t + y) a_z$$

free space lossless dielectrics

$$\omega = 10^8$$

$$\beta = 1$$

$$V_p = \frac{3 \times 10^8}{\sqrt{\epsilon_R}} = 10^8$$

$$V_p = \frac{\omega}{\beta} = 10^8 \neq 3 \times 10^8$$

$$\Rightarrow \epsilon_R = 9$$

$$f = \frac{10^8}{2\pi}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{2\pi}$$

$$H(y, t) = \left(\frac{25}{120\pi} \right) \sin(10^8 t - y) (a_y \times a_z)$$

$$= \left(\frac{5}{8\pi} \right) \sin(10^8 t - y) a_x$$

$$H = (0.5 e^{-0.1x}) \cos(10^6 t - 2x) a_z$$

$$\omega = 10^6 \text{ rad/second}$$

$$\beta = \omega = \frac{2\pi}{\lambda} \Rightarrow \lambda = 3.14$$

Propagation \rightarrow t_{disc}

\rightarrow Polarization means E field direction.

Note:-

Polarization direction is always E field orientation and direction

2. Free Space

$$E = 50 \sin(10^7 t + \beta z) a_y$$

$$\rightarrow \beta = \frac{10^7}{3 \times 10^8} = \frac{1}{30} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = 60\pi = 188 \text{ m}$$

$$\rightarrow K = \beta = \frac{1}{30} = 0.033$$

\rightarrow Free space, no attenuation

$$13. H = 0.5 e^{-j\omega t} \sin$$

$$V_p = \frac{10^6}{2} = 5 \times 10^5$$

$$V = 0.1 + j2$$

direction $\rightarrow +q_x$

$$14. \tau = 0 \quad \epsilon_R = 81$$

$$\omega = 6\pi \times 10^8, \beta = ?$$

$$V = \frac{3 \times 10^8}{\sqrt{81}} = \frac{6\pi \times 10^8}{\beta}$$

$$\Rightarrow \beta = 18\pi$$

$$\tau = 0 \quad \mu_R = 2 \quad \epsilon_R = 8$$

$$n = \sqrt{\frac{\mu_0 \mu_R}{\epsilon_0 \epsilon_R}} = 120\pi \sqrt{\frac{2}{8}} = 188\Omega$$

$$15. H = 0.1 \cos(\omega t - \beta z) q_x$$

$$E = (0.1 \times 377) \cos(\omega t - \beta z) (q_x \times q_z)$$

$$\approx -37.7 \cos(\omega t - \beta z) q_y$$

Aries - 8

$$16. \beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_R} = \frac{2\pi}{\lambda}$$

$$\lambda \propto \frac{1}{\sqrt{\epsilon}}$$

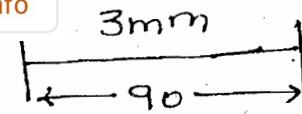
$$\frac{\lambda_0}{\lambda} = \sqrt{\frac{\epsilon_0 \epsilon_R}{\epsilon_0}} = \frac{2}{1} \Rightarrow \epsilon_R = 4$$

$$17. E = 20 e^{-j5z} q_x - 25 e^{-j5z} q_y$$

$$H = \left(\frac{20}{120\pi} \right) e^{-j5z} q_y - 25 e^{-j5z} (-q_x)$$

$$= \left(\frac{25}{120\pi} \right) e^{-j5z} q_x + \left(\frac{20}{120\pi} \right) e^{-j5z} q_y$$

$$19 = \beta = \frac{\theta}{z} = \frac{2\pi}{\lambda}$$



$$= \frac{\pi/2}{3 \text{ mm}} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = 12 \text{ mm}$$

$$\lambda f = 12 \times 10^{-3} \times 16 \times 10^9 = \frac{3 \times 10^8}{\sqrt{\epsilon_R}}$$

$$\Rightarrow \epsilon_R = 6.25$$

$$20 E = A \cos \left(\omega t - \frac{\omega}{c} z \right) a_y$$

$$\frac{\omega}{c} = \beta$$

$$H = \left[\frac{A}{\sqrt{\mu/\epsilon}} \right] \cos \left(\omega t - \frac{\omega}{c} z \right) (-a_x)$$

$$= -A \sqrt{\frac{\epsilon}{\mu}} \cos \left(\omega t - \frac{\omega}{c} z \right) a_x$$

Ans - 19

$$\nabla^2 E + k^2 E = 0$$

$$\nabla^2 E = \gamma^2 E \quad (\gamma^2 - k^2 = 0)$$

$$e^{-rz} + c^{rz} \quad \leftarrow$$

$$\nabla^2 E - \gamma^2 E = 0$$

$$\text{if } \gamma = j\beta$$

$$e^{-j\beta z} + e^{j\beta z}$$

$$\leftarrow \nabla^2 E + \beta^2 E = 0$$

$$(+z)$$

$$(\beta^2 + k^2 = 0)$$

Ans - C

$$21 E(x, z, t)_y = 25 \sin(\omega t - 3zc + 4z) a_y$$

$$\text{Unit Prop. direction} = \frac{3a_x - 4a_z}{5}$$

$$\beta = \sqrt{\beta_x^2 + \beta_z^2} = 5 = \frac{2\pi}{\lambda}$$

$$\nabla^2 E = \mu \epsilon \frac{\partial^2}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial z^2} = \mu \epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

$$E(z, t) = E_0 e^{j\omega t} \cdot e^{-jBz}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} - \mu \epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

$$E(x, z, t) = E_0 e^{j\omega t} \cdot e^{-jB_x x} \cdot e^{-jB_z z}$$

It is a product solution of time, x & z harmonics.

$$E(x, z, t) = 25 \sin(\omega t - 3x + 4z) a_y$$

$$H(x, z, t) = \left(\frac{25}{120\pi} \right) \sin(\omega t - 3x + 4z)$$

$$- \frac{(3a_x - 4a_z)}{5} \times a_y$$

$$= \left(\frac{25}{120\pi} \right) \sin(\omega t - 3x + 4z) \left(\frac{3a_z + 4a_x}{5} \right).$$

$$E(x, z, t)_y$$

$$H(x, z, t)(x, z)$$

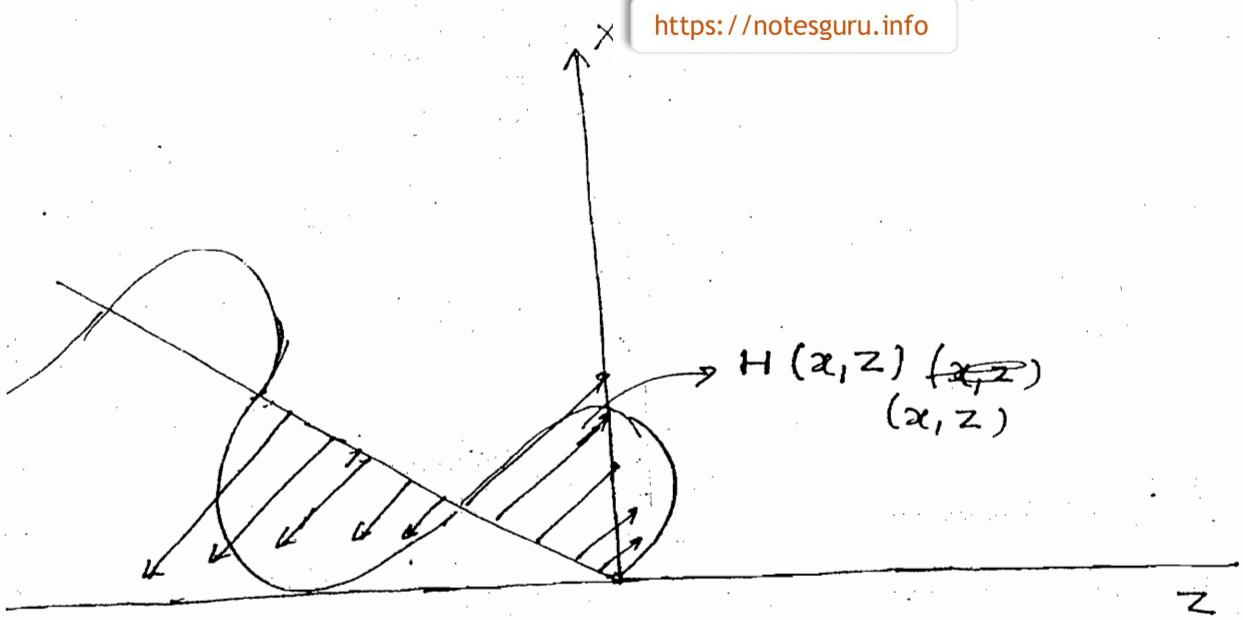
$$\text{Prop. direction} \rightarrow 3a_x - 4a_z$$

$$H \text{ direction} \rightarrow 4a_x + 3a_z$$

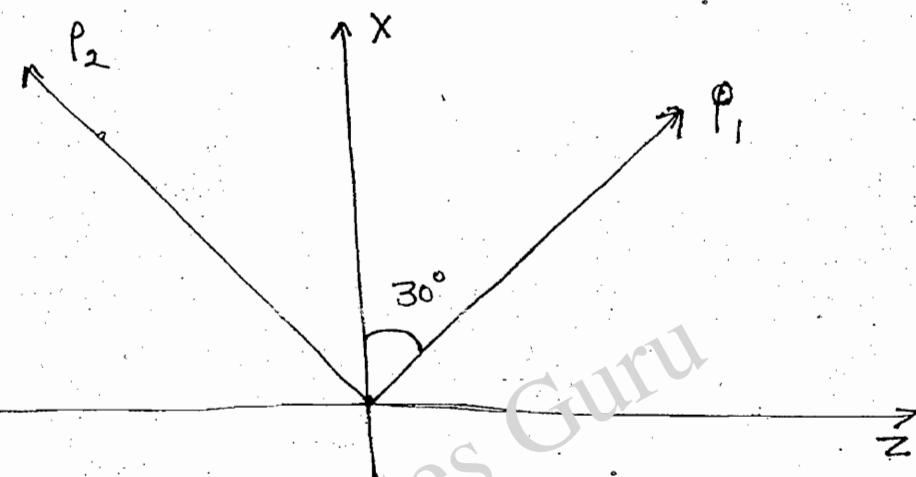
$$E \text{ direction} \rightarrow a_y \quad (\because \text{dot product} = 0)$$

E ⊥ H ⊥ P

→ Basic Transverse Nature



Ans



90° to Y-axis \rightarrow z-x Plane

30° to the X-axis

$$E(x, z, t) = E_0 e^{j\omega t} \cdot e^{-j\beta_x \cdot x} \cdot e^{\pm j\beta_z \cdot z}$$

$$\tan 30^\circ = \frac{\beta_z}{\beta_x} = \frac{1}{\sqrt{3}}$$

$$\beta_x = \sqrt{3} \beta_z$$

$$\beta = \frac{2\pi}{\lambda} = \sqrt{\beta_x^2 + \beta_z^2} = \sqrt{(\sqrt{3} \beta_z)^2 + \beta_z^2}$$

Ans - (a)

$$\Rightarrow \beta_z = \frac{\pi}{\lambda} \quad \beta_x = \frac{\sqrt{3}\pi}{\lambda}$$

$$24. \quad \beta = 280\pi = \therefore$$

$$\lambda = \frac{1}{140}$$

$$f = 14 \times 10^9$$

$$\lambda f = 10^8 = \frac{3 \times 10^8}{\sqrt{\epsilon_R}} \Rightarrow \epsilon_R = 9$$

$$\frac{E_p}{3} = \frac{120\pi}{\sqrt{9}} \Rightarrow E_p = 120\pi, \text{ Ans}$$

Note:-

$R, L, G_1, C \rightarrow$ Primary constant — $\frac{V}{I}$ Waves

Transmission
line

$\mu, \sigma, \epsilon \rightarrow$ Material constant — E/H Waves

Materials

$$\gamma = \sqrt{(R+j\omega L)(G_1+j\omega C)}$$

$$Y = \sqrt{j\omega \mu (\sigma + j\omega \epsilon)}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G_1+j\omega C}}$$

$$n = \sqrt{\frac{j\omega \mu}{\sigma + j\omega \epsilon}}$$

Note:- (1)

$$\mu = \frac{|Y_n|}{\omega}$$

$$\sigma = \text{Real} [Y/n]$$

$$\epsilon = \frac{\text{Imag.}[Y/n]}{\omega}$$

Note 2:-

$$n = \sqrt{\frac{j\omega \mu}{\sigma + j\omega \epsilon}} = \text{complex} = \frac{E_x}{H_y} = |n| e^{j\theta}$$

$$= \left| \frac{E_x}{H_y} \right| e^{j\theta}$$

OL

- In general material directions but have a phase shift or time delay.

i.e. orthogonal in

which is η 's phase

Example :- $E \rightarrow \sin(\omega t - \beta z) a_x$

$$H \rightarrow \sin(\omega t - \beta z - \theta) a_y$$

Note 3:-

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\alpha^2 - \beta^2 + j2\alpha\beta = j\omega\mu\sigma - \omega^2\mu\epsilon$$

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1 \right)$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1 \right)$$

Loss Tangent or Dissipation Factor :-

$\frac{\sigma}{\omega\epsilon}$ decides the propagation aspects α or β of the EM Waves

If $\frac{\sigma}{\omega\epsilon} \gg 1 \Rightarrow$ very good conductor

α is very large \Rightarrow EM Wave propagation is very tough

→ EM waves cannot easily propagate in good conductors

→ The propagation aspects depends on $\frac{\sigma}{\omega\epsilon}$ and hence on frequency - ω also

e.g:- $\frac{\sigma}{\omega\epsilon} \gg 1$ For human body upto light frequency

$\frac{\sigma}{\omega\epsilon} \ll 1$ For human body beyond x-ray frequency

eg:- $\frac{\sigma}{\omega\epsilon} \gg 1$ For e <https://notesguru.info> Hz only

$\frac{\sigma}{\omega\epsilon} \ll 1$ for earth beyond microwaves

summary:-

The term $\frac{\sigma}{\omega\epsilon} = \left| \frac{J_c}{J_d} \right|$ = loss tangent and hence decides the behaviour of the material

$J_c \gg J_d \Rightarrow$ good conductor

$J_d \gg J_c \Rightarrow$ good dielectric

$$\frac{J_c}{J_d} = \frac{-E}{\epsilon \frac{dE}{dt}} = \frac{-E}{j\omega\epsilon E} = \frac{\sigma}{\omega\epsilon}$$

Case - (III) :-

EM Wave Propagation in very good conductors
($\sigma \gg \omega\epsilon$)

$$\begin{aligned} Y &= \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \approx \sqrt{j\omega\mu\sigma} \\ &= \sqrt{\omega\mu\sigma} [90^\circ] = \sqrt{\omega\mu\sigma} [45^\circ] \\ &\quad = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}} \end{aligned}$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\begin{aligned} n &= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} [90^\circ] \\ &= \sqrt{\frac{\omega\mu}{\sigma}} [45^\circ] = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}} \end{aligned}$$

$$R = X = \sqrt{\frac{\omega\mu}{2\sigma}}$$

→ η 's real part (R) = cause of attenuation (α)

→ η 's Imaginary part (X) = cause of phase shift (β)

η 's phase = 45° means E_x & H_y are delayed in time by 45°

Skin Depth or Depth of penetration: -

$$|E| = E_0 e^{-\alpha z}$$

$$E_0 = 100$$

$$\begin{aligned} z &= \frac{1}{\alpha} \\ &\downarrow \\ &37 \\ z &= \frac{2}{\alpha} \\ &\downarrow \\ &13 \\ \frac{z}{\lambda} & \\ &\downarrow \\ &4 \end{aligned}$$

→ The first $\frac{1}{2}$ distance travelled by the wave is where majority of the wave amplitude is attenuated. This distance is called as skin depth. Hence skin depth is the distance travelled by the wave where the wave leakage to $\frac{1}{e}$ times (37%) of the initial value.

$$S = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \mu}} = \sqrt{\frac{2\pi}{\omega \mu}} \cdot \frac{1}{\pi} = \frac{1}{\omega R_s}$$

where $R_s = \eta$'s real part = skin resistance

$$25. \quad V_p = \frac{\omega}{\beta} = \frac{\omega}{\lambda} = \omega \delta = 2\pi \times 2 \times 10^5 \times 4 \times 10^{-6}$$

$$= 1.6\pi \text{ m/s} \approx 5 \text{ m/s}$$

$$26. \quad \omega' = 4\omega \quad \delta = \sqrt{\frac{2}{\omega \mu_0}} \Rightarrow \delta' = \frac{\delta}{2}$$

$$V_p' = \omega' \delta' = 4\omega \frac{\delta}{2} = 2V_p = 10 \text{ m/s}$$

Note:-

$$\frac{\omega}{\omega_0} = 0 \quad \text{Free space} \rightarrow 3 \times 10^8 \text{ m/s}$$

$$\text{Ideal dielectric} \rightarrow \frac{3 \times 10^8}{\sqrt{\epsilon_R}}$$

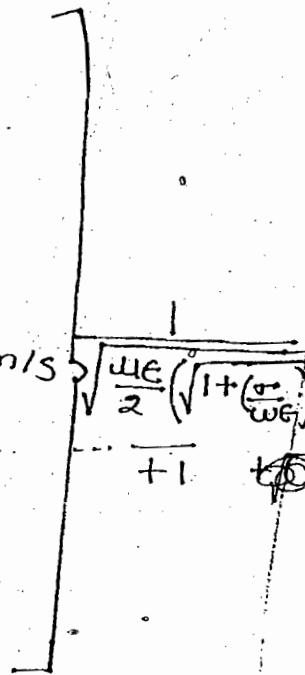
$$\frac{\omega}{\omega_0} \uparrow$$

$$\text{General Material} \rightarrow 10^5 - 10^6 \text{ m/s}$$

$$\frac{\omega}{\omega_0} \gg 1$$

$$\text{Good conductor} \rightarrow \text{few m/s}$$

$$V_p = \frac{\omega}{\sqrt{\frac{\omega \mu_0}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega_0} \right)^2} + 1 \right)}}$$



$$\frac{\omega}{\omega_0} = 1.73$$

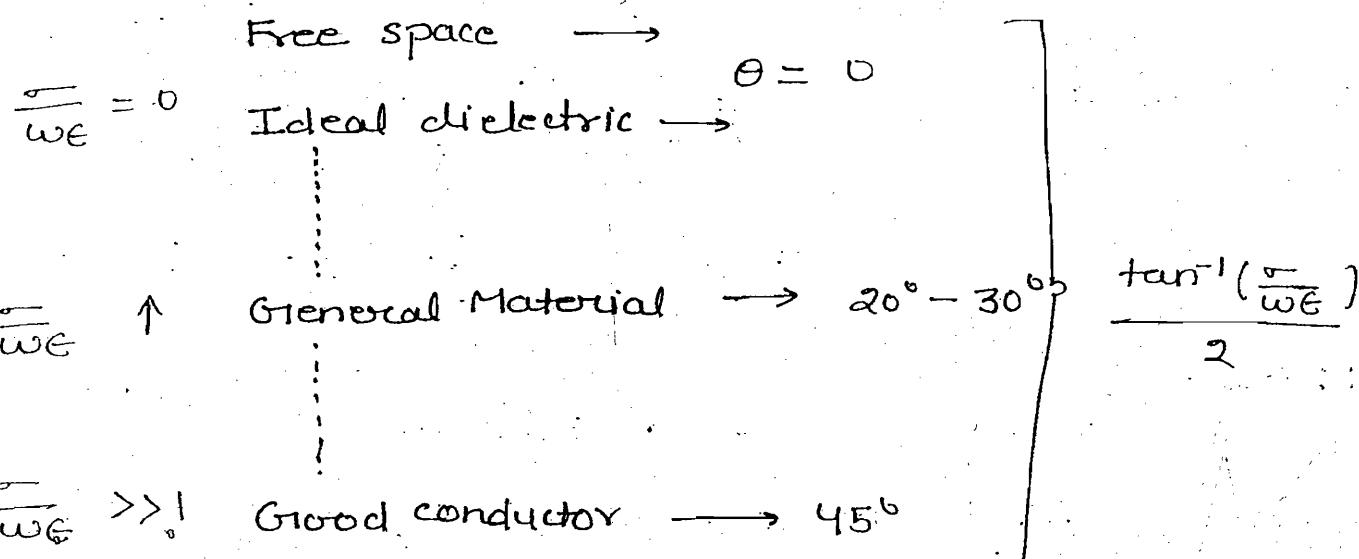
$$\eta' \text{ s phase} = \sqrt{\frac{j\omega \mu_0}{\sigma + j\omega \epsilon_0}} = \sqrt{\frac{\omega \mu_0 90}{K \tan^{-1}(\frac{\omega \epsilon_0}{\sigma})}}$$

$$= \sqrt{K 1 \frac{90 - \tan^{-1}(\frac{\omega \epsilon_0}{\sigma})}{2}}$$

$$+ \frac{\tan^{-1}(\sigma/\omega_0)}{2}$$

$$27. \quad n' \text{ s phase} = \frac{90 - \tan^{-1}(\frac{\omega \epsilon_0}{\sigma})}{2} = \frac{\tan^{-1}\sqrt{3}}{2} = \frac{60}{2} = 30^\circ$$

Note:- n 's phase = θ



28.

$$J_c = J_d$$

$$\frac{\omega}{\omega_E} = 1$$

$$\Rightarrow \frac{\omega}{2\pi f E} = 1$$

$$f = \frac{10^{-2}}{2\pi \times 4 \times 1} = 45 \text{ MHz}$$

29.

$$\frac{I_c}{I_d} = \frac{J_c}{J_d} = \frac{\omega}{\omega_E}$$

$$\Rightarrow I_d = \frac{I_c \omega_E}{58} = \frac{1 \times 2\pi \times 50 \times 8.8 \times 10^{-12}}{58}$$

Ans - (B)

$$n = \sqrt{j\omega\mu} \equiv \sqrt{\omega\mu} 145^\circ$$

$$= R + jX \quad \underline{\text{Ans - 9}}$$

31:

$$\eta = 0.02 \quad 145^\circ$$

32. Lossy dielectric, $\frac{\sigma}{\omega\epsilon} \ll 1$

$$\begin{aligned} Y &= \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{j\omega\mu - j\omega\epsilon \left(1 + \frac{\sigma}{j\omega\epsilon}\right)} \\ &= j\omega\sqrt{\mu\epsilon} \left(1 + \frac{\sigma}{j\omega\epsilon}\right) = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} + j\omega\sqrt{\mu\epsilon} \end{aligned}$$

Power, Power density and Poynting Vector :-

→ Energy that can be transformed into other formats is said to be a power format.

$$\frac{\text{Joules}}{\text{second}} = \text{Watts}$$

Using Maxwell's IV Equation

$$\nabla \times H = -\sigma E + \epsilon \frac{\partial E}{\partial t}$$

Take $\cdot E$ on both sides,

$$(\nabla \times H) \cdot E = -E^2 + \epsilon \frac{\partial E}{\partial t} \cdot E$$

$$\Rightarrow -\nabla \cdot (E \times H) + H \cdot (\nabla \times E) = -E^2 + \epsilon \frac{\partial E}{\partial t} \cdot E$$

$$\Rightarrow -\nabla \cdot (E \times H) - \sigma E^2 = \epsilon \frac{\partial E}{\partial t} \cdot E * \omega \frac{\partial H}{\partial t} \cdot H$$

$$= \frac{\gamma}{\sigma t} \left(\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right)^2$$

Take $\int dv$ on both sides

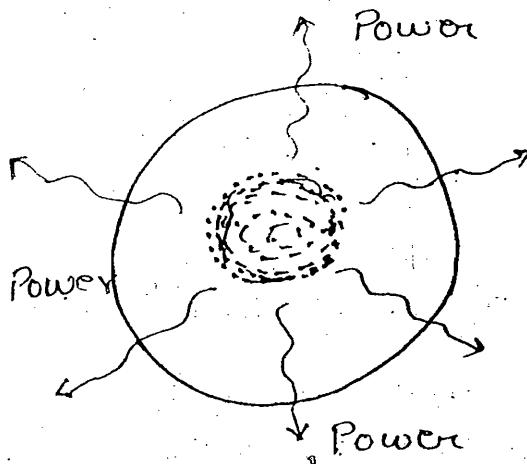
$$\int \nabla \cdot (E \times H) dv + \int -\sigma E^2 dv = -\frac{\gamma}{\sigma t} \int \left(\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right) dv$$

Using Divergence

<https://notesguru.info>

$$\oint (E \times H) \cdot dS + \int -E^2 dv = \text{Rate of decrease of } E/H$$

Energy inside a volume



The LHS for a EM wave power crossing the surface

- Hence $E \times H =$
 - = Power density of EM Wave
 - = Strength of power of EM Wave
 - = Poynting Vector of the EM Wave
 - = It has the direction of propagation

$$E(z, t)_x \times H(z, t)_y = P(z, t)_z$$

- = Power density at a time at a point in space
- = Instantaneous Poynting Vector

$P(z)_{\text{avg.}}$ = Time Average Poynting Vector

$$= E_{\text{RMS}} \cdot H_{\text{RMS}} = \frac{E_0 e^{-\alpha z}}{\sqrt{2}} \cdot \frac{H_0 e^{-\alpha z}}{\sqrt{2}}$$

$$P(z)_{\text{avg.}} = \frac{1}{2} E_0 H_0 e^{-2\alpha z} = \frac{1}{2} \frac{E_0^2}{\eta} e^{-2\alpha z} = \frac{1}{2} \eta H_0^2 e^{-2\alpha z}$$

Note 1:-

The average power decreases at 2α rate exponentially in the medium

→ This power lost in the wave is acquired by the medium as ohmic power

$$\text{J.E. or } -E^2 \text{ Watts/m}^3$$

Note 2:-

Alternatively

$$P(z)_{\text{avg.}} = \frac{1}{2} (E \times H^*)$$

* conjugate operation means average power is independent of harmonic and its phase and depends only on amplitude.

> In free space

$$P_{\text{avg.}} = \frac{1}{2} E_0 H_0 = \frac{1}{2} \frac{E_0^2}{\eta} = \frac{1}{2} \eta H_0^2$$

Workbook! :-

3. $P_{\text{avg.}} = \frac{1}{2} \eta H_0^2 (-ay)$

$$H_0 = 0.1$$

$$= \frac{1}{2} \cdot 120\pi \cdot (0.1)^2 (-ay) \rightarrow B$$

34. $W = \int P_{\text{avg.}} ds$

$$= \int \frac{1}{2} \frac{E_0^2}{\eta} a_x \cdot ds \quad a_x$$

$$= \frac{1}{2} \cdot \frac{60 \cdot 60}{120\pi} \pi \times 4^2 = 240 \text{ W}$$

$a_x = \text{constant}$

35. C

36. (1) $H = 50 \sin(\omega t - \beta z) a_x + 150 \sin(\omega t - \beta z) a_y$

$$P_{\text{avg.}} = \frac{1}{2} \eta H_0^2 (a_z)$$

$$H = 50 \sin(\omega t - \beta z) \left(\frac{a_x + 3a_y}{\sqrt{10}} \right) \sqrt{10}$$

$$= (50\sqrt{10}) \sin(\omega t - \beta z)$$

$$P_{avg} = \frac{1}{2} \cdot 120\pi \cdot (50\sqrt{10})^2 a_z$$

$$H_0 = 50\sqrt{10}$$

$$\begin{aligned} \text{(ii)} \quad H &= 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_x \\ &= 50 \sin(\omega t - \beta z) a_x (1 + 3j) \\ &= 50 (1 + 3j) \sin(\omega t - \beta z) a_x \end{aligned}$$

Note:-

Orthogonal field components in time ^{or} space have same effect

$$\text{(iii)} \quad H = 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_y$$

$$H = 50 \sin(\omega t - \beta z) (a_x + 3j a_y)$$

$$P_{avg} = \frac{1}{2} (E \times H^*)$$

$$E = (50\eta) \sin(\omega t - \beta z) (-a_y) + (150\eta) \cos(\omega t - \beta z) (a_x)$$

$$= (50\eta) \sin(\omega t - \beta z) (3j a_x - a_y)$$

$$P_{avg} = \frac{1}{2} \cdot 50 \cdot 120\pi (3j a_x - a_y) \times 50 (a_x - 3j a_y)$$

$$= \frac{1}{2} \cdot 120\pi \cdot 50^2 (a_z + 3j a_z)$$

$$= \frac{1}{2} \cdot 120\pi (50\sqrt{10})^2 a_z$$

$$E = (a_x + ja) \quad \text{https://notesguru.info}$$

$$H = \frac{k}{\omega u} (a_y + ja_x) e^{jkz - j\omega t}$$

$$\begin{aligned} P_{avg.} &= \frac{1}{2} (a_x + ja_y) \times \frac{k}{\omega u} (a_y - ja_x) \\ &= \frac{k}{2\omega u} (a_z - a_z) = 0 \rightarrow \text{Null vector} \end{aligned}$$

Conventional :-

$$E = 100 (ja_x + 2a_y - ja_z) e^{j\omega t}$$

$$H = (-a_x + ja_y + a_z) e^{j\omega t}$$

$$\begin{aligned} P_{avg.} &= \frac{1}{2} (E \times H^*) = \frac{1}{2} 100 (ja_x + 2a_y - ja_z) \times \\ &\quad (-a_x - ja_y + a_z) \\ &= 50 \begin{vmatrix} a_x & a_y & -a_z \\ j & 2 & -j \\ -j & -j & 1 \end{vmatrix} \\ &= 50 (3a_x + 3a_z) \end{aligned}$$

$$|P_{avg.}| = 150\sqrt{2}$$

$$\text{Pavg. direction} = (a_x + a_z)$$

$$\text{Unit Pavg. direction} = \frac{a_x + a_z}{\sqrt{2}}$$

Wave Polarization:-

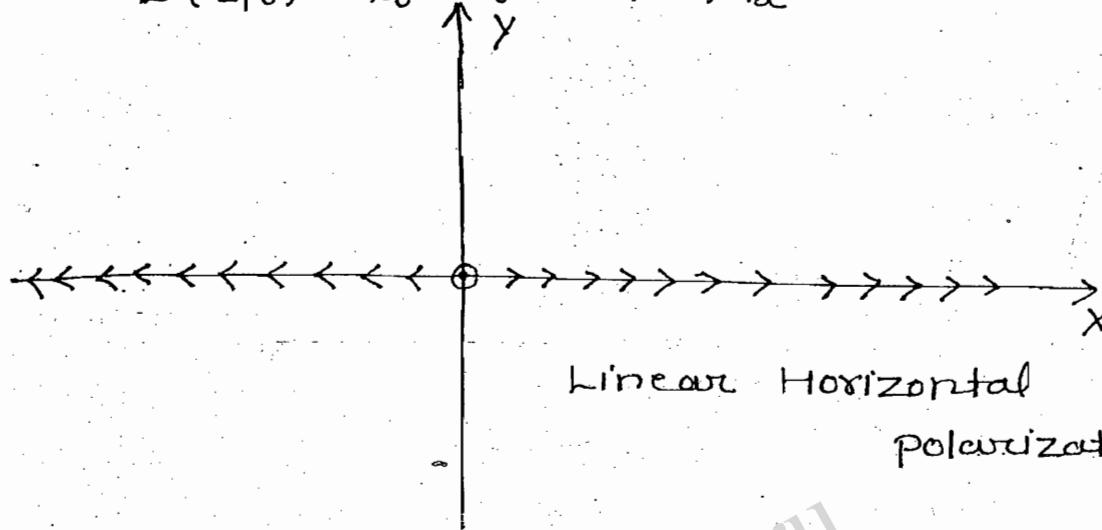
It is the electric field orientation & the possible planar components of the E field satisfying the basic transverse nature

EM Wave is propagating along z-axis in the following cases:-

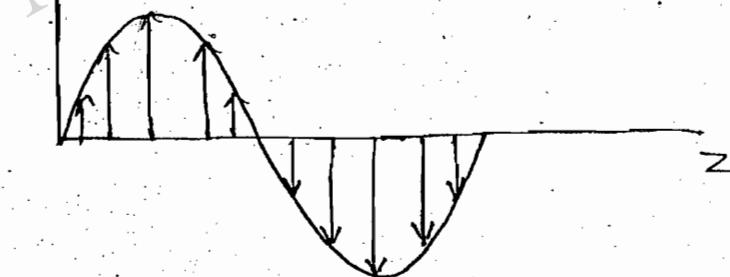
Case (I) :-

E field is oriented in x-direction

$$E(z,t) = E_0 \sin(\omega t - \beta z) a_x$$



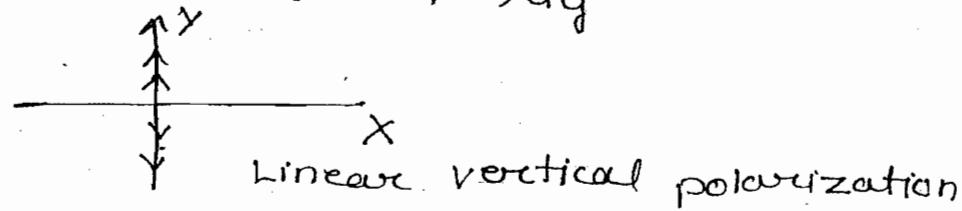
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Case -(II) :-

E field is oriented in y-direction

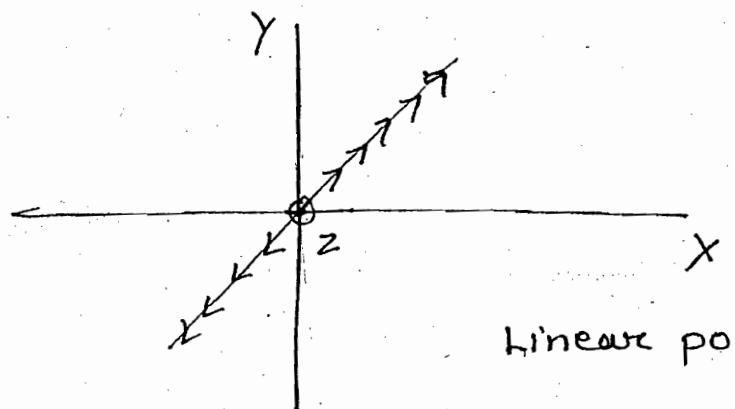
$$E(z,t) = E_0 \sin(\omega t - \beta z) a_y$$



Case-(III) :-

E field is oriented in x & y directions.

$$\mathbf{E}(z,t) = E_1 \sin(\omega t - \beta z) \mathbf{a}_x + E_2 \sin(\omega t - \beta z) \mathbf{a}_y$$



Linear polarization

Note:-

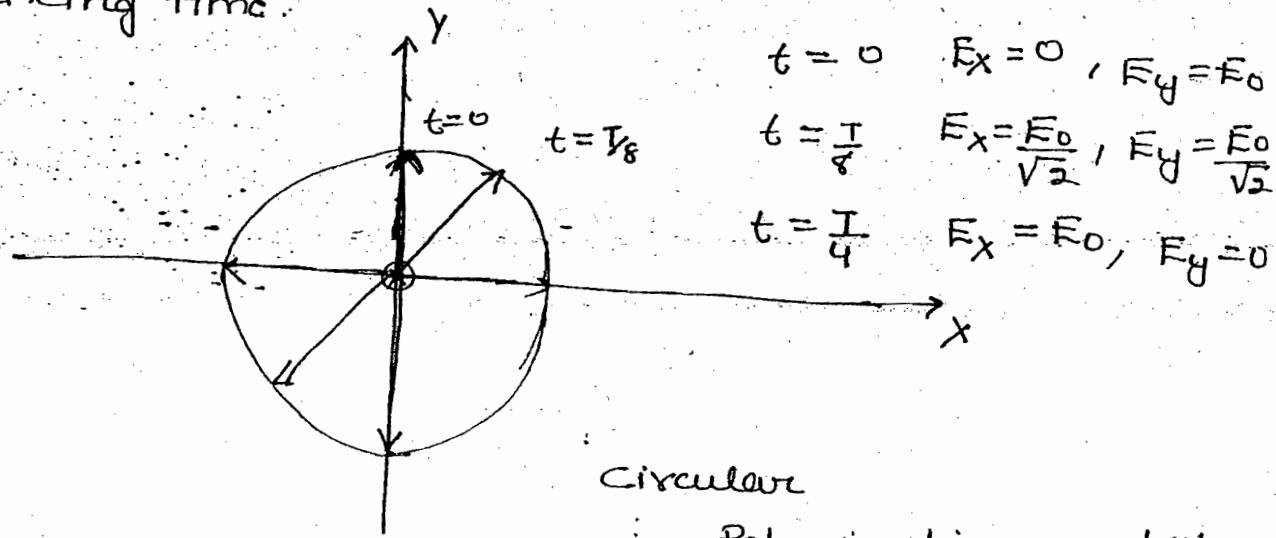
If the EM waves has single E field components or two planar components both inphase the wave is said to be linearly polarized

Case-(IV) :-

E field is oriented in x & y directions

$$\mathbf{E}(z,t) = E_0 \sin(\omega t - \beta z) \mathbf{a}_x + E_0 \cos(\omega t - \beta z) \mathbf{a}_y$$

→ Polarization is identified by studying the trace of the E field on the $z=0$ plane for various advancing time.



summary:-

If the EM Waves have components of E field both out of phase at 90° and equal amplitude, the wave is circularly polarised.

Note:-

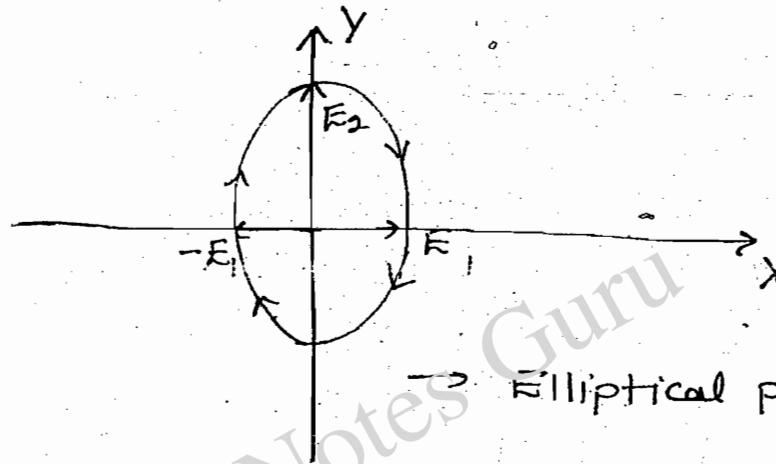
Sense of rotation left or right, if the left hand thumb points towards propagation direction and the closed finger along advancing time, the wave is left circularly polarised

e.g.:- out of paper - propagation clockwise time advancement - left circular

case-(V) :-

E field is oriented in x & y directions

$$\mathbf{E}(z,t) = E_1 \sin(\omega t - \beta z) \mathbf{a}_x + E_2 \cos(\omega t - \beta z) \mathbf{a}_y$$

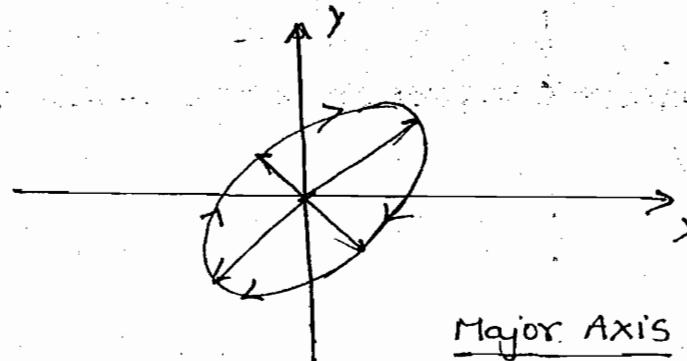


Elliptical polarization

case-(VI) :-

E field is oriented in x & y direction

$$\mathbf{E}(z,t) = E_1 \sin(\omega t - \beta z) \mathbf{a}_x + E_2 \sin(\omega t - \beta z + \theta) \mathbf{a}_y$$



Major Axis = AR = [1, ∞)

Minor Axis

Circle
Linear

Workbook :-

3 (i) $\vec{E} = 25 \sin(\omega t + 4x) (a_y + 6a_z)$
 \rightarrow Linear

ii) $\vec{E} = 25 \sin(\omega t + 4x) a_y + 25 \cos(\omega t + 4x) a_z$
 \rightarrow Circular

iii) $\vec{E} = 25 \sin(\omega t + 4x + 60^\circ) a_y + 25 \cos(\omega t + 4x + 60^\circ) a_z$
 \rightarrow Circular

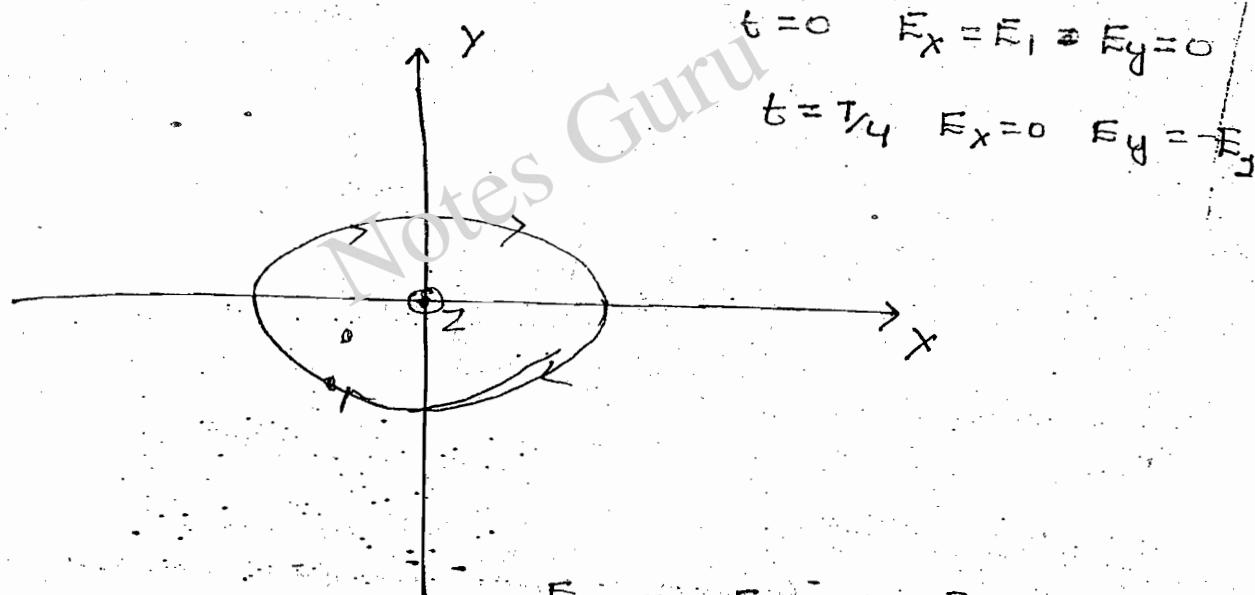
iv) $\vec{E} = 25 \sin(\omega t + 4x) a_y + (25\sqrt{2}) \sin(\omega t + 4x + 45^\circ) a_z$
 \rightarrow Elliptical

v) Elliptical

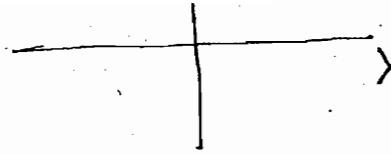
vi) Circular

g. $\theta = 60^\circ \rightarrow$ elliptical

o. $\theta = 90^\circ$



$$(ay + jaz) e^{j\theta}$$
$$\downarrow$$
$$sin$$
$$\downarrow$$
$$cos$$
$$\downarrow$$
$$cos$$
$$-sin$$

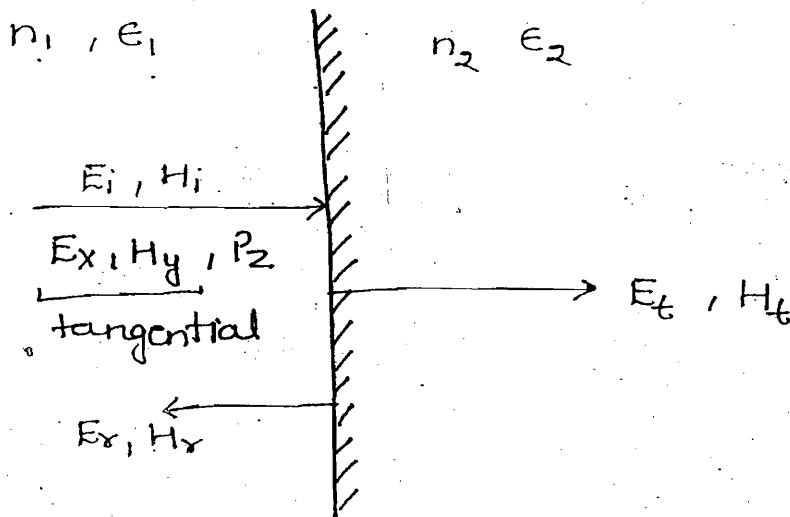


Reflections / Transmissions of EM Waves.

(I) Normal Incidence (II) Dielectric - Dielectric

$$z < 0$$

$$z = 0 \text{ (XY Plane)}$$



- (I) $E_t > E_i$ or $E_t < E_i \rightarrow$ It depends on ϵ_1 & ϵ_2
- (II) $H_t > H_i$ if $E_t < E_i$
- (III) $H_t < H_i$ if $E_t > E_i$
- (IV) $P_t < P_i \rightarrow$ Always

E_x	H_y	Prop. Z	
a_x	a_y	a_z	\rightarrow Incidence
a_x	$-a_y$	$-a_z$	\rightarrow Reflection
$-a_x$	a_y	$-a_z$	

→ When reflections occur either E or H negates along with propagation direction but not both

$$E_i = n_1 H_i$$

$$E_t = n_2 H_t$$

$$E_x = -n_1 H_y$$

→ For normal incidence the E/H fields are 90° tangential.

$$E_{t_1} = E_{t_2}$$

$$E_t$$

$$H_{t_1} = H_{t_2}$$

$$E_i + E_r = E_t$$

$$1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$$\Rightarrow 1 + \Gamma = \tau \quad \text{--- (1)}$$

$$\text{where } \Gamma = \frac{E_r}{E_i}$$

τ = Reflection coefficient
for the E fields

$\tau = \frac{E_t}{E_i}$ = Transmission coefficient
for the E fields

$$H_i + H_r = H_t$$

$$\frac{E_i}{n_1} - \frac{E_r}{n_1} = \frac{H_t}{n_2}$$

$$E_i - E_r = \frac{n_1}{n_2} E_t$$

$$\Gamma = \frac{n_2 - n_1}{n_2 + n_1}$$

Note :-

$$\Gamma = \frac{n_1 - n_2}{n_1 + n_2} = -\Gamma_E$$

$$\Gamma_P = \Gamma_E \cdot \Gamma_H = -\Gamma_E^2 = -\Gamma_H^2$$

$$1 + \Gamma = \tau$$

(iv) If $n_1 = \sqrt{\frac{\mu_0}{\epsilon_1}}$ & $n_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$

$$\Gamma_E = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

If $\epsilon_1 > \epsilon_2 \Rightarrow \Gamma_E = +ve \quad \tau_E > 1$

$$E_t > E_i$$

$$\Rightarrow \Gamma_H = -ve, \quad \tau_H < 1$$

$$H_t < H_i$$

$$\Gamma_P = -ve, \quad \tau_P < 1 \rightarrow \text{Always}$$

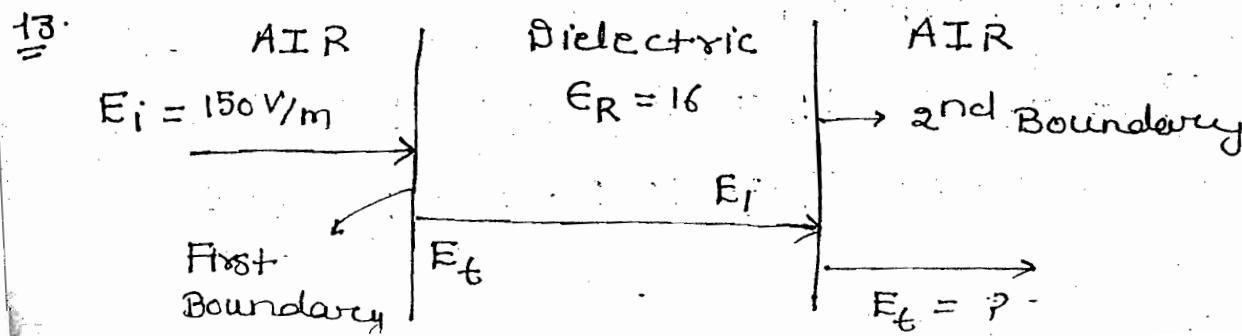
Workbook:-

12. Air - Dielectric $\frac{\epsilon_0}{4\epsilon_0} \quad \tau_P = ?$

$$\Gamma_E = \frac{\sqrt{\epsilon_0} - \sqrt{4\epsilon_0}}{\sqrt{\epsilon_0} + \sqrt{4\epsilon_0}} = \frac{1-2}{1+2} = -\frac{1}{3}$$

$$\Gamma_H = +\frac{1}{3} \quad \Gamma_P = -\frac{1}{9}$$

$$\Rightarrow \tau_P = 1 - \frac{1}{9} = \frac{8}{9}$$



$$\Gamma_E = \frac{1 - \sqrt{16}}{1 + \sqrt{16}} = \frac{-3}{5}$$

$$\tau_E = 1 - \frac{3}{5} = \frac{2}{5} = \frac{E_t}{E_i} \Rightarrow E_t = \frac{2}{5} \times 150 \text{ V/m}$$

$$= 60 \text{ V/m}$$

E_i at the 2nd Boundary = 60 V/m

$$\Gamma_E = \frac{\sqrt{16} - \sqrt{1}}{\sqrt{16} + \sqrt{1}} = \frac{3}{5}$$

$$\tau_E = 1 + \frac{3}{5} = \frac{8}{5} = \frac{E_t}{E_i}$$

$$E_t = \frac{8}{5} \times 60 = 96 \text{ V/m}$$

$$\Gamma_E = \frac{1 - \sqrt{4}}{1 + \sqrt{4}} = \frac{-1}{3} = -\frac{1}{3}$$

$$\tau_E = 1 - \frac{1}{3} = \frac{2}{3} = \frac{E_t}{E_i}$$

$$E_t = \frac{2}{3} E_0 \cos(\omega t - 2\beta z) a_y$$

$$(\beta' = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_R} = 2\beta)$$

Note:-

$\alpha, \beta, \gamma, n, V_p, d \rightarrow$ Propogation aspects

& material aspects and change as ϵ, μ changes

$\omega \rightarrow$ source aspect / time aspect and is same for every material.

$$\Gamma_E = -\frac{1}{3} \quad H_i = \cos(10^8 t - \beta z) a_y$$

$$E_i = 120\pi \cdot \cos(10^8 t - \beta z) a_x$$

$$\frac{10^8}{\beta} = 3 \times 10^8$$

$$E_Y = \left(-\frac{1}{3} \cdot 120\pi\right) \cos \left(10^8 t + \frac{\pi}{3}\right) a_x$$

16.

$$\frac{E_t}{E_Y} = \frac{E_t}{E_i - E_Y} = \frac{\Gamma}{\Gamma} = \frac{1 + \Gamma}{\Gamma} = -2$$

$$\Gamma = -\frac{1}{3} = \frac{1 - \sqrt{\epsilon_2/\epsilon_1}}{1 + \sqrt{\epsilon_2/\epsilon_1}} \Rightarrow \frac{\epsilon_2}{\epsilon_1} = 4$$

48. EM Wave reflections at conductors! —

$$\Gamma_E = \frac{n_2 - n_1}{n_2 + n_1} \quad n_1 = 120\pi = 377$$

$$n_2 = \sqrt{j\omega\mu} = 0$$

→ EM wave power is completely reflected at conductors

$$\Gamma_E = \frac{E_Y}{E_i} = 1$$

$$E_i + E_Y = E_t = 0$$

$$\tau_E = 0$$

$$\Gamma_H = \frac{H_Y}{H_i} = 1 \quad \text{Ans-cc}$$

$$H_i + H_Y = H_t = H_{\max}$$

$$\tau_H = 2$$

$$n = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} \quad (\text{if } \sigma = 0) \text{ real}$$

$$n = \sqrt{\frac{\mu^*}{\epsilon^*}} \quad (\text{if } \sigma \neq 0) \text{ complex}$$

Aos $\mu^* = \epsilon$

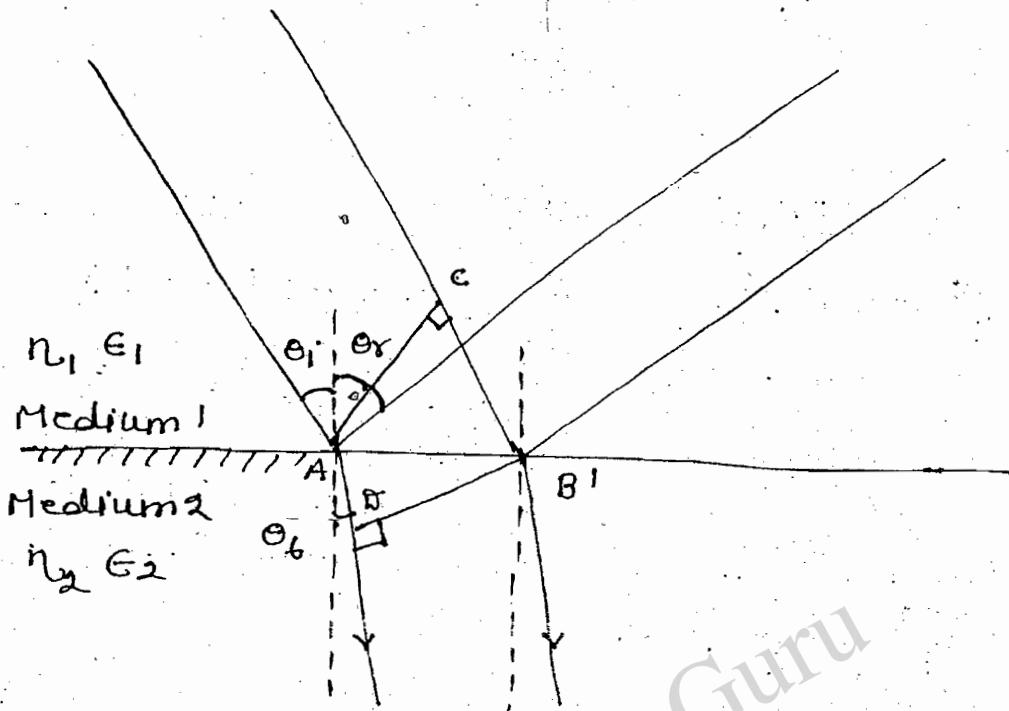
<https://notesguru.info>

$$n = \sqrt{\frac{\mu^*}{\epsilon}} = 1$$

$$\Gamma = \frac{1 - 377}{1 + 377} \Rightarrow \Gamma \approx -1$$

Aos - (c)

Oblique (Inclined) Incidence:



- A/C are inphase points in medium 1
- CB is the extra distance travelled in medium 1
- B/S are inphase points in medium 2
- AB is the extra distance travelled in medium 2

$$\frac{CB}{AB} = \frac{v_1}{v_2} = \text{Refracting Index of med. 2 w.r.t med. 1}$$

$$\frac{AB \cos(90^\circ - \theta_i)}{AB \cos(90^\circ - \theta_r)} = \frac{1}{\sqrt{\mu_0 \epsilon_2}}$$

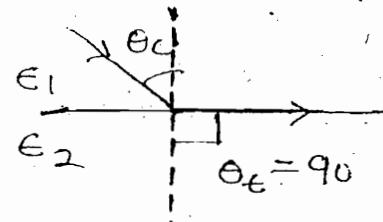
$$\boxed{\frac{\sin \theta_i}{\sin \theta_r} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

→ Refracting Index

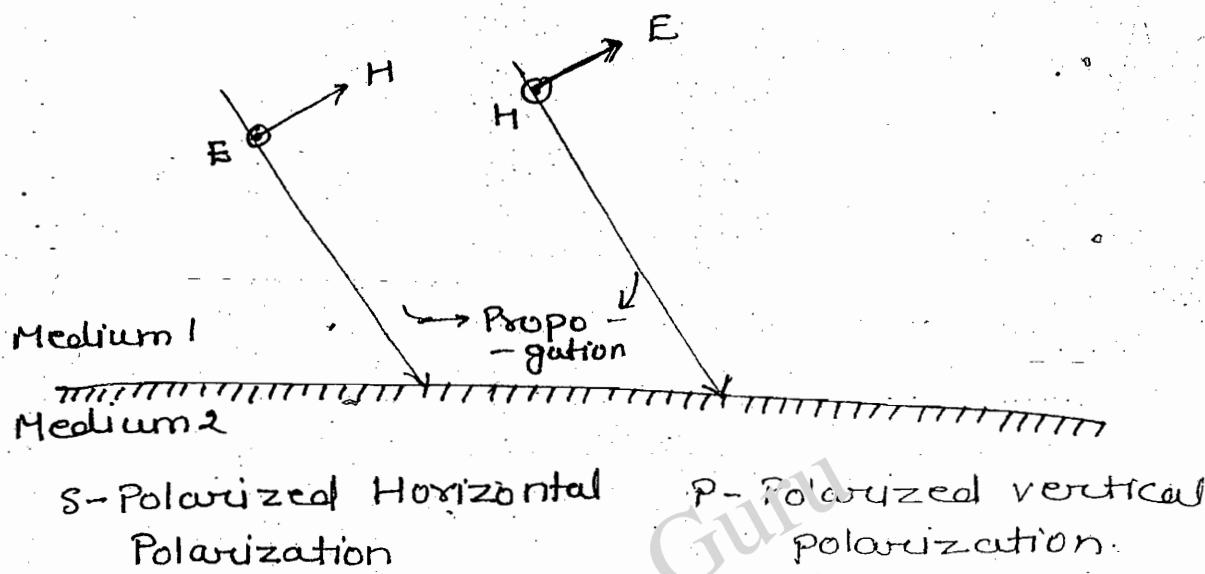
If $\theta_t = 90^\circ$ then $\theta_i = \theta_c$ = critical angle

For all $\theta_i > \theta_c$ complete reflections and zero transmission takes place

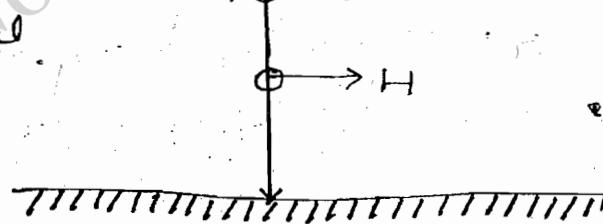
$$\sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$



S & P Polarized Waves:-



In normal incidence both fields are horizontal



Applying the Boundary conditions,

$$E_{t1} = E_{t2}$$

For s-polarized waves

$$E_i + E_r = E_t$$

$$1 + \tau_s = \tau_s$$

— (1s)

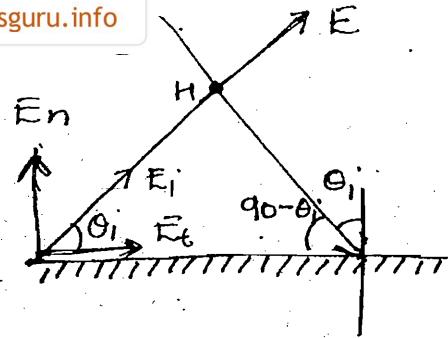
Note! -

The angle b/w pr normals is the same angle b/w fields and surface

For p-polarised waves

$$E_i \cos \theta_i + E_y \cos \theta_y = E_t \cos \theta_t$$

$$E_i + E_y = E_t \frac{\cos \theta_t}{\cos \theta_i}$$



$$I + \Gamma = \tau \frac{\cos \theta_t}{\cos \theta_i} \quad \text{--- } I_p$$

Similarly using $H_{t1} = H_{t2}$

$$\Gamma_s = \frac{n_2 \sec \theta_t - n_1 \sec \theta_i}{n_2 \sec \theta_t + n_1 \sec \theta_i}$$

$$\Gamma_p = \frac{n_2 \cos \theta_t - n_1 \cos \theta_i}{n_2 \cos \theta_t + n_1 \cos \theta_i}$$

Note! -

$\Gamma_s \neq \Gamma_p$ in oblique incidence

but when $\theta_i = \theta_y = \theta_t = 0$ then

$$\Gamma_s = \Gamma_p = \frac{n_2 - n_1}{n_2 + n_1}$$

$$\text{As } n_1 = \sqrt{\frac{\mu_0}{\epsilon_1}} \quad n_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$$

$$\Gamma_s = \frac{\sqrt{\epsilon_1} \cos \theta_i - \sqrt{\epsilon_2} \cos \theta_t}{\sqrt{\epsilon_1} \cos \theta_i + \sqrt{\epsilon_2} \cos \theta_t}$$

$$\frac{\sin \theta_i}{\sin \theta_t} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \quad \text{https://notesguru.info}$$

$$\sin \theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_i$$

$$\cos \theta_t = \sqrt{1 - \frac{\epsilon_1 \sin^2 \theta_i}{\epsilon_2}}$$

$$T_s = \frac{\cos \theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}}$$

$$T_p = \frac{\frac{\epsilon_2}{\epsilon_1} \cos \theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}}{\frac{\epsilon_2}{\epsilon_1} \cos \theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}}$$

If $T_p = 0$ i.e. zero reflections & complete transmission

$$\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 (1 - \sin^2 \theta_i) = \frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i$$

$$\sin^2 \theta_i \left(1 - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2\right) = \frac{\epsilon_2}{\epsilon_1} - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2$$

$$\sin^2 \theta_i = \frac{\frac{\epsilon_2}{\epsilon_1} \left(1 - \frac{\epsilon_2}{\epsilon_1}\right)}{\left(1 + \frac{\epsilon_2}{\epsilon_1}\right) \left(1 - \frac{\epsilon_2}{\epsilon_1}\right)}$$

$$\sin \theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}}$$

$$\tan \theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

This $\theta_i = \theta_B$ is a Brewster angle
for p-polarised wave

If $R_s = 0$

$$\cos^2 \theta_i = \frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i$$

$$\Rightarrow \epsilon_2 = \epsilon_1$$

Brewster angle and zero reflections cannot occur for s-polarized waves.

Summary:-

zero transmission, complete reflections

θ_c = critical angle

zero reflections, complete transmission

θ_B = Brewster angle

$$\sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$\epsilon_1 > \epsilon_2$ is a required condition

No such polarization restriction

For all $\theta_i > \theta_c$

→ the same effect

$$2. \tan \theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

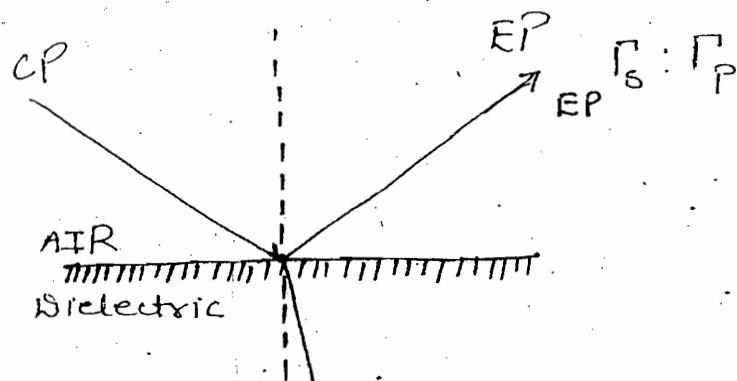
3. No such medium restriction

4. Exists only for p-polarized waves

5. At $\theta_i = \theta_B$ only this occurs

RCP \rightarrow S + P

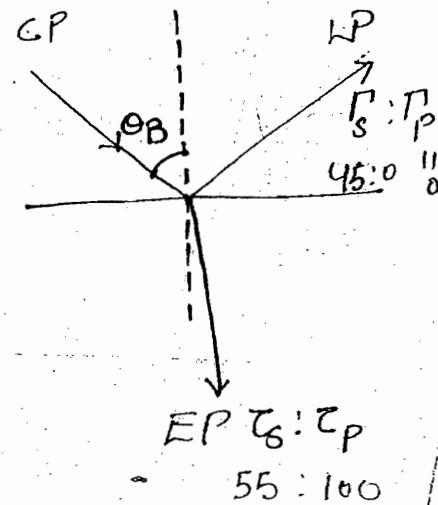
49.



$$\tan \theta_B = \tan 60^\circ = \sqrt{\frac{\epsilon_R \epsilon_0}{\epsilon_0}}$$

$$\Rightarrow \epsilon_R = 3$$

$$EP \quad \Gamma_s : \Gamma_p$$



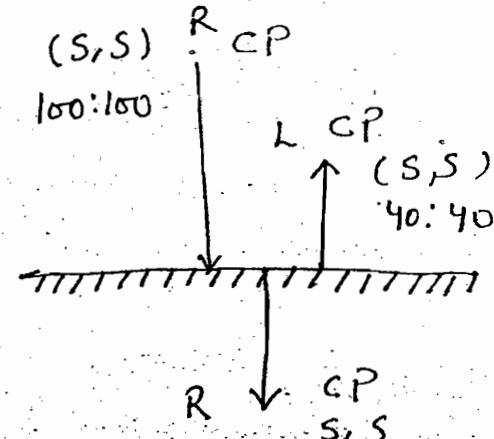
50.

Linearly Polarised

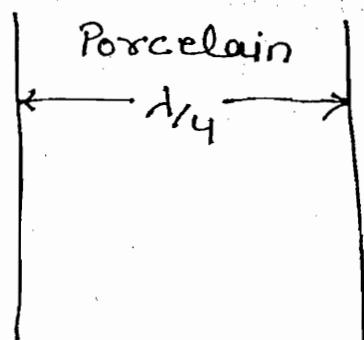
51.

a b

Notes Guru



52.



$$\text{Thickness} = \frac{\lambda}{4} = \left(\frac{3 \times 10^8}{\sqrt{\epsilon_R} \times 10 \times 10^9 \times 4} \right)$$

Lo

$J_b \rightarrow$ Bond cur

- μ_R
- $J_b \rightarrow$ due to spinning/ revolving electrons constituting a magnetic dipole.
 - $J_c \rightarrow$ conduction current - moving electron
 - $J_d \rightarrow$ displacement current → changing Electric flux

$$\underline{E} = 3 \sin(\omega t - \beta z) a_x + 6 \sin(\omega t - \beta z + 75^\circ) a_y$$

$$E_0 = \sqrt{3^2 + 6^2} = 3\sqrt{5}$$

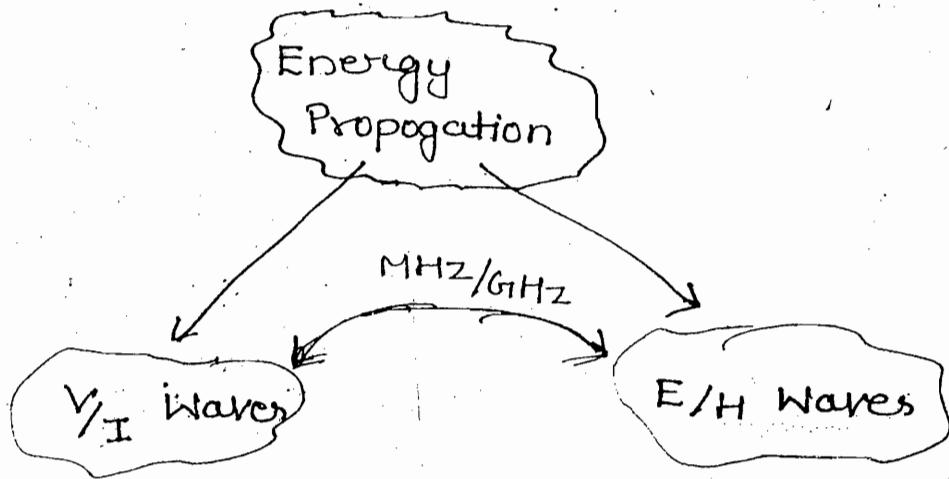
$$\begin{aligned} P_{avg} &= \frac{1}{2} \frac{E_0^2}{\eta} a_z \\ &= \frac{1}{2} \frac{(3\sqrt{5})^2}{120\pi} \approx 58 \text{ mW} \end{aligned}$$

OR

$$\underline{E} = 3 \sin(\omega t - \beta z) (a_x + 2 e^{j75^\circ} a_y)$$

$$\underline{H} = \left(\frac{3}{120\pi} \right) \sin(\omega t - \beta z) (a_y - 2 e^{-j75^\circ} a_x)$$

$$\begin{aligned} P_{avg} &= \frac{1}{2} \cdot 3 (a_x + 2 e^{j75^\circ} a_y) \times \left(\frac{3}{120\pi} (a_y - 2 e^{-j75^\circ} a_x) \right) \\ &= \frac{1}{2} \cdot \frac{3 \cdot 3}{120\pi} (a_z + 4 a_{az}) \\ &= \frac{1}{2} \cdot \frac{(3\sqrt{5})^2}{120\pi} a_z \end{aligned}$$



- $\epsilon = \infty$
 - Conductors
 - Wired or point to point
 - Extremely low frequency $f < \text{kHz}$
 - $\epsilon = 0$
 - Free space
 - Broadcast wireless
 - Extremely high frequency $f > 10^2 \text{ Hz}$

```

graph LR
    Source["Source: Vs, Is"] --> Line[Transmission line]
    Line --> Load["Load: R, L, G, C"]
    Load --> Source
    
```

The diagram illustrates a transmission system. It starts with a 'Source' block containing voltage V_s and current I_s . An arrow points from the source to a 'Transmission line' block. From the transmission line, another arrow points to a 'Load' block containing resistance R , inductance L , conductance G , and capacitance C . A feedback arrow originates from the load and points back to the source.

$$V = \sqrt{(R+j\omega L)(G_1+j\omega C)}$$

$$Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad \leftarrow \text{Propagation}$$

$$E_S \rightarrow \omega \text{ source} \rightarrow M_r \text{ Material constant} \rightarrow Z_m \rightarrow E(z) H(z)$$

$$Y = \sqrt{j\omega M_r(\sigma + j\omega\epsilon)}$$

$$\gamma = \sqrt{\frac{j\omega M_r}{\sigma + j\omega\epsilon}}$$

Primary Constants:-

Resistance :-

- It is the resistance of the conducting material all along the length of the line.

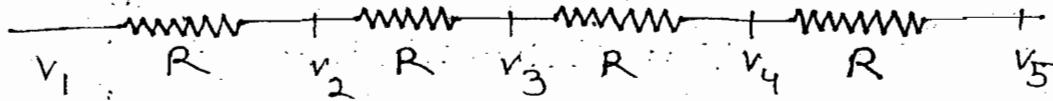
$$\rightarrow R = \rho \frac{l}{A} \quad R \propto l$$

It is distributed all along the length

$$R = \frac{R}{l} = \text{ohm/m}$$

\downarrow
Primary Constant

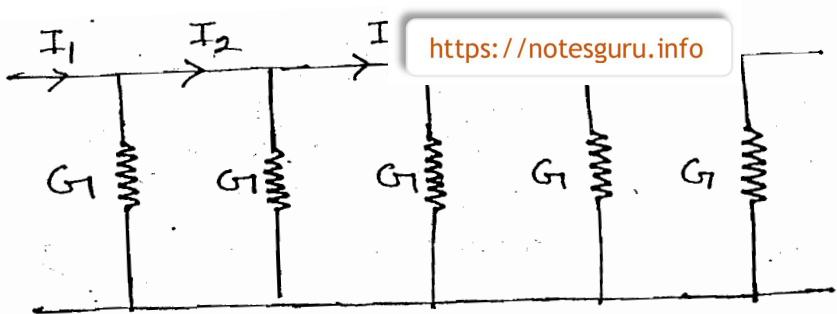
- It is the resistance at any instance on the line
- It appears in series along the line



- It causes a voltage decay all along the line

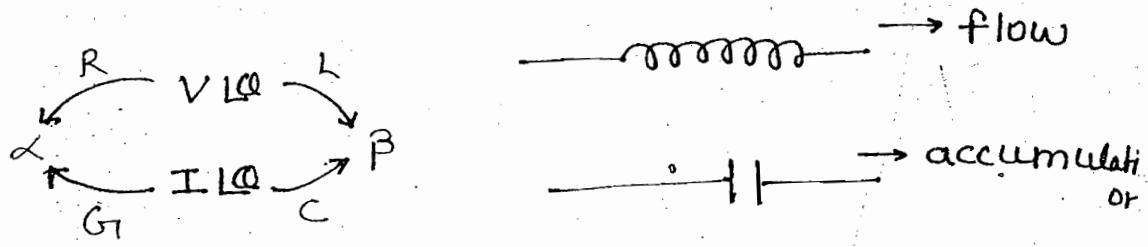
Conductance:-

- It is the conductance of the insulating material b/w the lines
- It appears in shunt b/w the lines



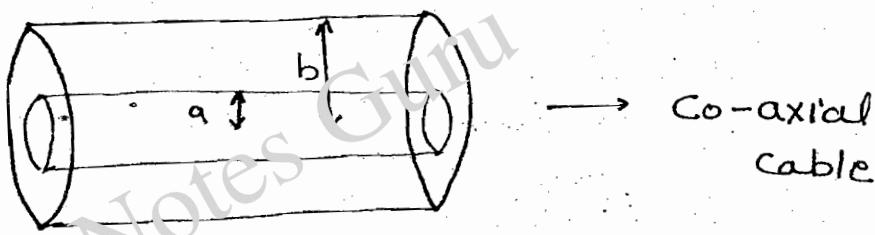
- I_t causes current decay
- I_t is distributed all along the length

$$G_l = \frac{G_1}{l} = \frac{mho}{m}$$

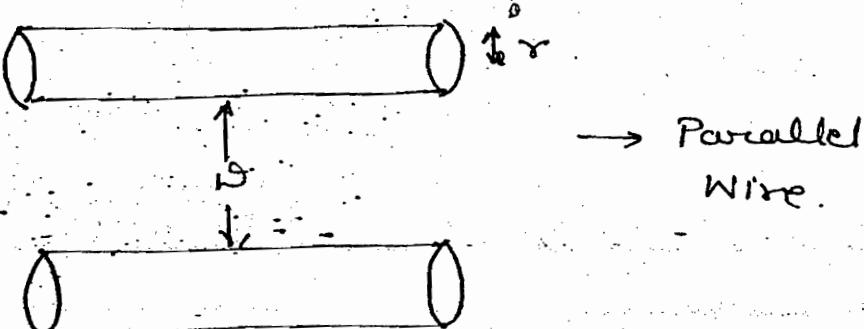


Capacitance:-

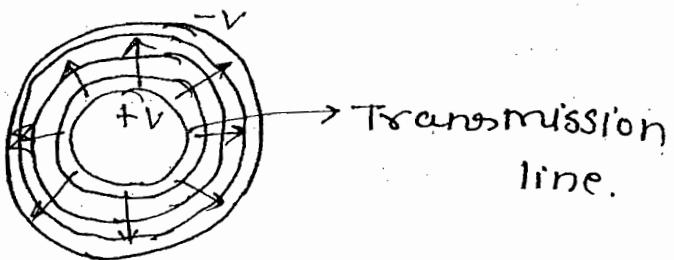
- It is due to the voltages on the line there is a surrounded E field and hence capacitance



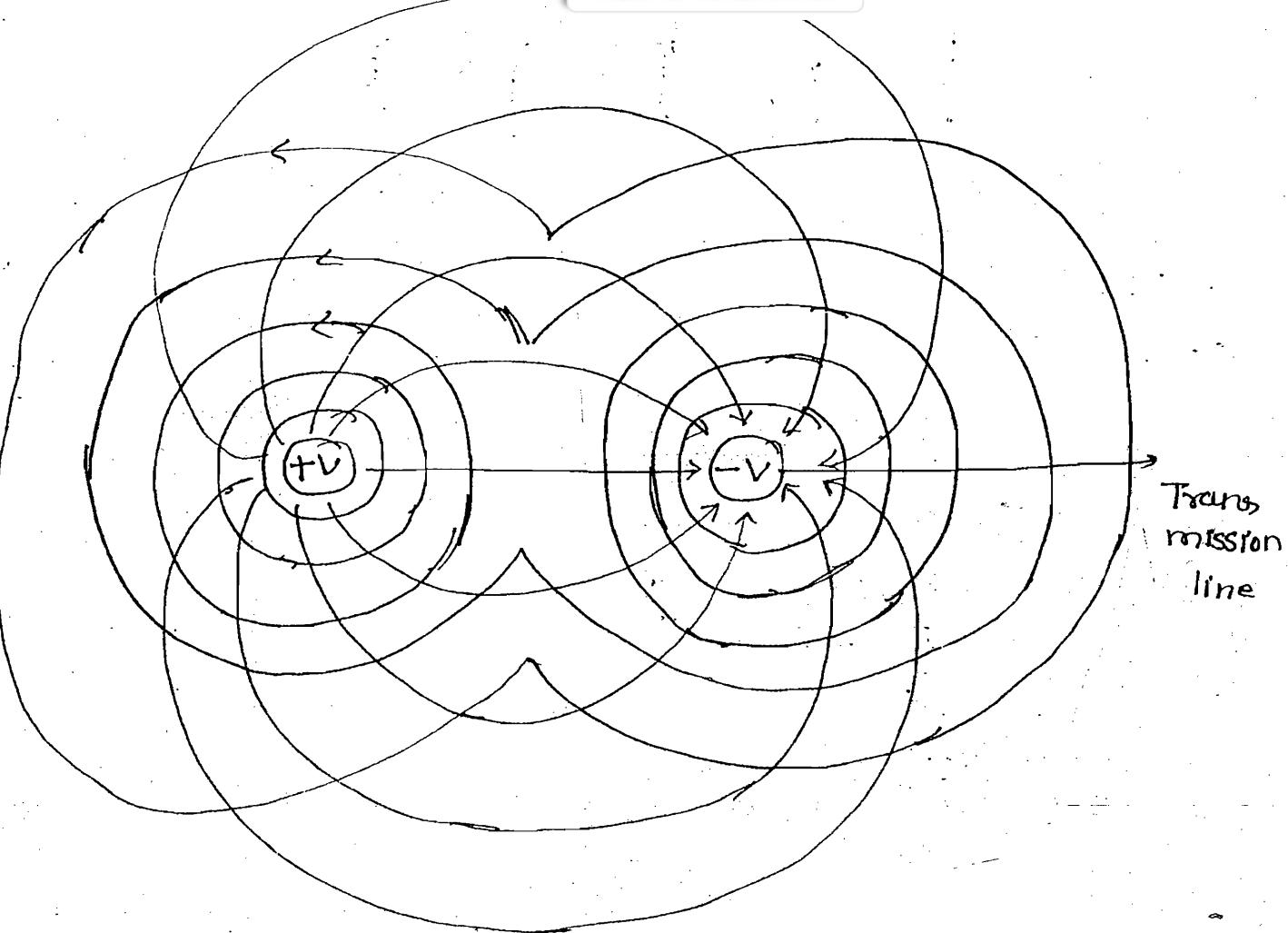
→ Co-axial cable



→ Parallel Wire.



→ Transmission line.



$$C = \frac{2\pi \epsilon_0 l}{\ln(b/a)} \rightarrow \text{Co-axial cable}$$

$$C = \frac{\pi \epsilon_0 l}{\ln(\beta/r)} \rightarrow \text{Parallel wire}$$

→ C axial → capacitance is distributed

$$\rightarrow C = \frac{C}{l} = \frac{\text{Farads}}{\text{m}}$$

→ It appears in shunt b/w the cables

Inductance:-

→ It is due to the currents on the line there is a surrounded H-field, and hence inductance

→ $L = \frac{\mu_0 I \ln(b/a)}{2\pi} \rightarrow \text{co-axial cable}$

• $L = \frac{\mu_0 I \cdot \ln(D/r)}{\pi} \rightarrow \text{parallel wire}$

→ L is distributed i.e. $L \propto \lambda$

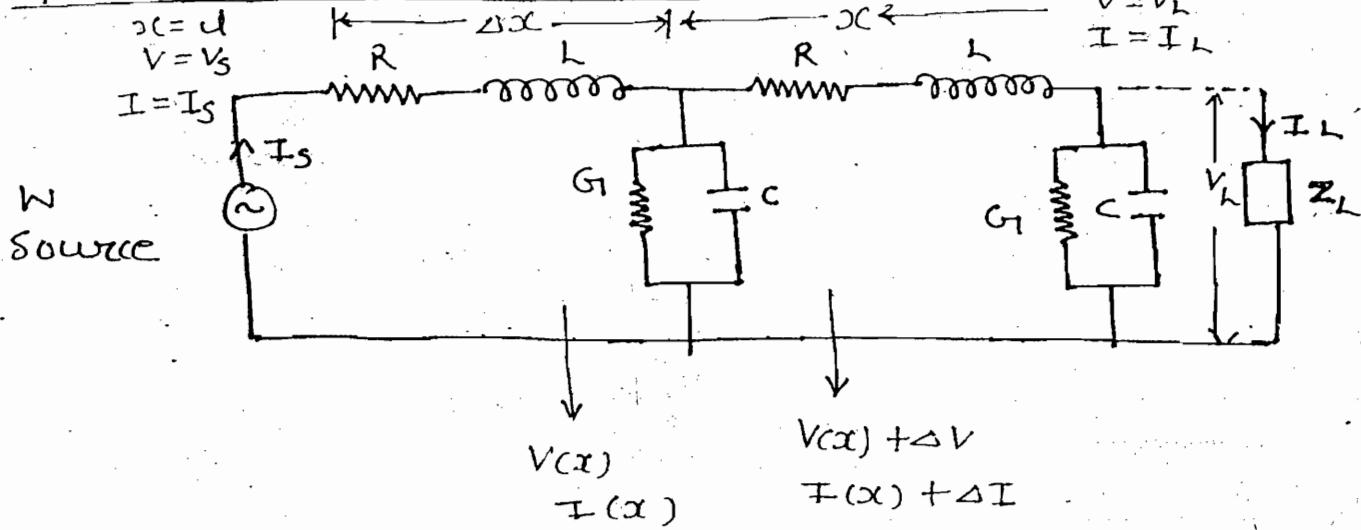
$$L' = \frac{L}{\lambda} \text{ Henry/m}$$

Note:-

$$LC = \frac{L}{C} = \text{distributed} \times \text{distributed} = \mu \epsilon$$

For any transmission line and for any geometry.

V/I Equations on the line:-



$$Z_{\text{source}} \text{ of } \Delta x \text{ length} = (R + j\omega L) \cdot \Delta x$$

$$Y_{\text{shunt}} \text{ of } \Delta x \text{ length} = (G_1 + j\omega C) \cdot \Delta x$$

$$\Delta V = I (R + j\omega L) \cdot \Delta x$$

$$\frac{\Delta V}{\Delta x} = \frac{dV}{dx} = (R + j\omega L) I \quad (I)$$

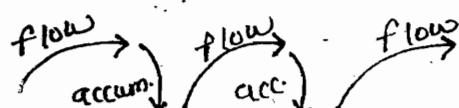
$$\frac{\Delta I}{\Delta x} = \frac{dI}{dx} = (G_1 + j\omega C) V \quad (II)$$

Take I from (I) & put in (II)

$$\frac{d}{dx} \left[\left(\frac{1}{R + j\omega L} \right) \frac{dV}{dx} \right] = (G_1 + j\omega C) V$$

$$\frac{d^2V}{dx^2} = (R + j\omega L)(G_1 + j\omega C) \quad (III)$$

$$\frac{d^2I}{dx^2} = (R + j\omega L)(G_1 + j\omega C) \quad (IV)$$



→ Meaning of Harmonic

Note:-

If the source is time harmonic at one end the effect is space harmonic all along the length of the line.

Let $\sqrt{R+j\omega L}$ <https://notesguru.info> \downarrow Y (per m)

Eq-(III) & (IV) can be written as Unit

$$\frac{d^2V}{dx^2} - Y^2 V = 0$$

$$\frac{d^2I}{dx^2} - Y^2 I = 0$$

The V/I solution on the line is

$$V(x) = C_1 e^{-Yx} + C_2 e^{Yx} \quad (V)$$

$$I(x) = C_3 e^{-Yx} + C_4 e^{Yx} \quad (VI)$$

Applying initial conditions, ($x=0, V=V_L, I=I_L$)

$$V_L = C_1 + C_2 \quad (VII)$$

$$I_L = C_3 + C_4 \quad (VIII)$$

Using eq-(I) in eq-(V)

$$\frac{dV}{dx} = -YC_1 e^{-Yx} + YC_2 e^{Yx} = (R+j\omega L) I$$

Put $x=0$

$$\frac{C_2 - C_1}{C_1 + C_2} = \frac{(R+j\omega L) I_L}{\sqrt{(R+j\omega L)(G_1+j\omega C)}}$$

$$= \sqrt{\frac{R+j\omega L}{G_1+j\omega C}} I_L = I_L Z_0$$

$$\text{where } Z_0 = \sqrt{\frac{R+j\omega L}{G_1+j\omega C}}$$

$$I_L Z_0 = C_2 - C_1 \quad (IX)$$

$$\text{Similarly } \frac{V_L}{Z_0} = C_4 - C_3 \quad (X)$$

$$C_1 = \frac{V_L - I_L Z_0}{2}$$

$$C_2 = \frac{V_L + I_L Z_0}{2}$$

$$C_3 = \frac{I_L - \frac{V_L}{Z_0}}{2} + \frac{\frac{V_L}{Z_0}}{2}$$

$$V(x) = \frac{V_L}{2} \left[\left(1 - \frac{Z_0}{Z_L} \right) e^{-rx} + \left(1 + \frac{Z_0}{Z_L} \right) e^{rx} \right]$$

$$V(x) = \frac{V_L (Z_L + Z_0)}{2Z_L} \left[e^{rx} + \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-rx} \right]$$

$$V(x) = V_0 \left(e^{rx} + \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-rx} \right)$$

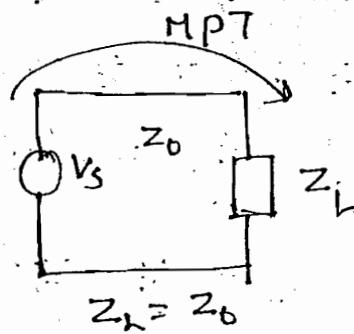
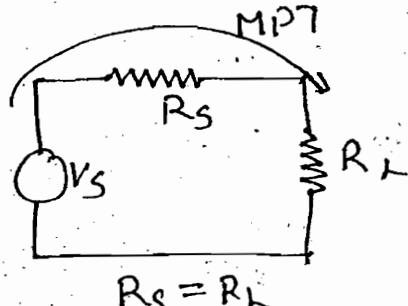
$$I(x) = \frac{I_L (Z_L + Z_0)}{2Z_0} \left[e^{rx} + \left(\frac{Z_0 - Z_L}{Z_0 + Z_L} \right) e^{-rx} \right]$$

$e^{rx} \rightarrow -x$, Source-load forward wave

$e^{-rx} \rightarrow +x$, load-source, reflected wave

→ Every transmission line has two waveforms i.e. the forward and reflected and the reflected wave does not exist in one single condition i.e. $Z_L = Z_0$. This is called as matched condition and this is a condition where complete source power is absorbed by the load.

→ This is similar to max power transfer in N/W or EM wave reflections b/w materials.



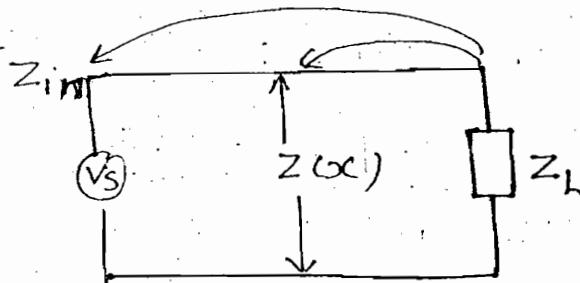
$$R_S = R_L$$

$$Z_L = Z_0$$

Note:-

- Z_0 is a unique impedance of the line with which if the line is terminated the reflections on the line and hence called as characteristic impedance

Impedance on line (z) and i/p impedance (Z_{in}):-



Note:-

Due to the loaded one end and primary constant in the line, impedance is different on different point of line such that

$$z(x) = \frac{V(x)}{I(x)} \quad Z_{in} = z(0)$$

$$V(x) = V_L \left(\frac{(Z_L + Z_0)}{2} e^{Yx} + \frac{(Z_L - Z_0)}{2} e^{-Yx} \right)$$

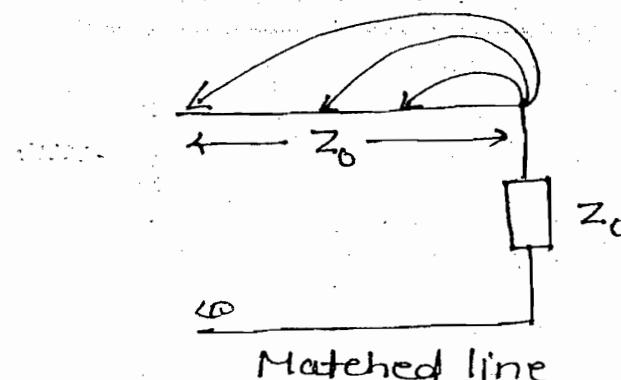
$$V(x) = V_L \cosh(Yx) + I_L Z_0 \sinh(Yx)$$

Similarly

$$I(x) = I_L \cosh(Yx) + \frac{V_L}{Z_0} \sinh(Yx)$$

$$z(x) = Z_0 \left[\frac{Z_L \cosh(Yx) + Z_0 \sinh(Yx)}{Z_0 \cosh(Yx) + Z_L \sinh(Yx)} \right]$$

If $Z_L = Z_0$ then $z(x) = Z_0 = Z_L = Z_{in}$



→ For a matched line the line loss is also zero anywhere on the line.

Note:-

Z_0 is that unique impedance with which if the line is terminated impedance on the line is also the same.

Short circuit and open circuit lines! -

→ If $Z_L = 0 \Rightarrow$ S.C. line.

$$Z_{in} = Z_{SC} = Z_0 \tanh(r\lambda_c)$$

→ If $Z_L = \infty \Rightarrow$ O.C. line

$$Z_{in} = Z_{OC} = Z_0 \coth(r\lambda_c)$$

$$\Rightarrow Z_0 = \sqrt{Z_{SC} \cdot Z_{OC}}$$

It is geometric mean of S.C. and O.C. i/p impedances.

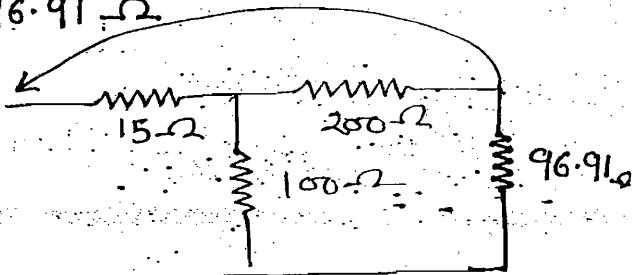
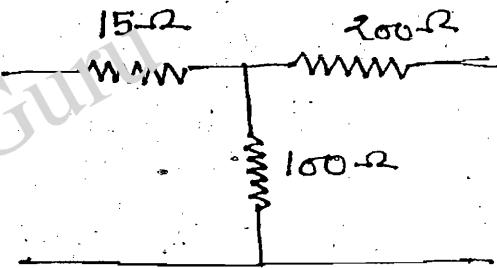
e.g:-

$$Z_{OC} = 115\Omega$$

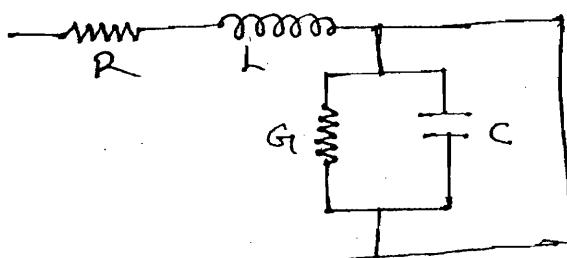
$$Z_{SC} = 15 + \frac{100 \cdot 200}{100 + 200}$$

$$= 81.67\Omega$$

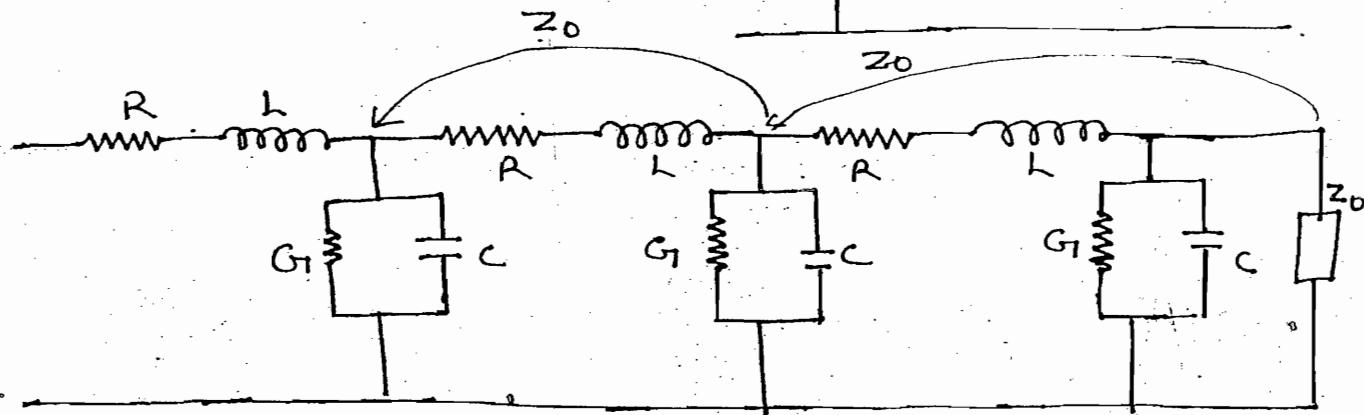
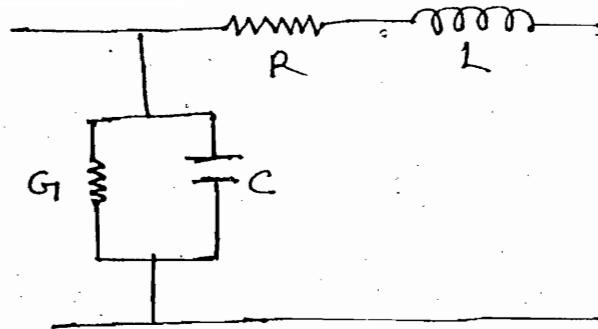
$$Z_0 = \sqrt{81.67 \times 115} = 96.91\Omega$$



$$Z_{SC} = R + j\omega L$$



$$Z_{oc} = \frac{1}{G_1 + j\omega C}$$



Note:-

- Any N/W either discrete or continuous when terminated with its characteristic impedance the same impedance appears on other end
- For a transmission line whose every RLC branch having Z_0 characteristics impedance when terminated with Z_0 at one end the loading effect appears to be the same anywhere on the line.
- Lossless line and distortionless line,

$$\gamma = \sqrt{R^2 + }$$

Lossless line and distortionless line:-

$$\begin{aligned} \gamma &= \sqrt{(R+j\omega L)(G_1+j\omega C)} = \text{propagation constant} \\ &= \alpha + j\beta \end{aligned}$$

If $\alpha = 0$ everywhere on the line, the line is said to be lossless

$$(i) R = G_1 = 0$$

$$(ii) \omega L \gg R \quad (iii) \omega C \gg G_1 \quad] \text{ High frequency lines}$$

$$Y = j\omega \sqrt{LC}$$

$$= j\beta$$

$$\beta = \omega \sqrt{LC}$$

$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_R}} = \frac{3 \times 10^8}{\sqrt{\epsilon_R}}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}} = \text{real}$$

$$= \sqrt{\frac{\mu_0 \ln(b/a)}{2\pi} \frac{\ln(b/a)}{2\pi \epsilon}}$$

$$= \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_R}} \cdot \frac{\ln(b/a)}{2\pi}$$

$$Z_0 = \frac{60 \ln(b/a)}{\sqrt{\epsilon_R}}$$

→ Co-Axial cable

→ Z_0 is a design aspect of the line and depends on the cable dimensions but not on R, L, G, C or operational frequency of the line.

$$Z_0 = \frac{120 \ln(b/a)}{\sqrt{\epsilon_R}}$$

→ Parallel wire.

Distortionless line:-

- If the phase is linear the wave is said to distortionless i.e. $\beta \propto \omega \rightarrow$ For distortionless line
- Equal Rise time = Full time is characteristic of distortionless line.

series arm

time constant

~ in time
constant

$$\frac{L}{R} = \frac{C}{G_1} \Rightarrow \boxed{LG_1 = RC}$$

$$Y = \sqrt{R \left(1 + \frac{j\omega L}{R}\right) G_1 \left(1 + \frac{j\omega C}{G_1}\right)}$$

$$= \sqrt{RG_1} \left(1 + \frac{j\omega L}{R}\right) = \text{complex}$$

$$Z_0 = \sqrt{\frac{R \left(1 + \frac{j\omega L}{R}\right)}{G_1 \left(1 + \frac{j\omega C}{G_1}\right)}} = \sqrt{\frac{R}{G_1}} = \text{Real}$$

Note:-

Every lossless line \Rightarrow distortionless \neq lossless

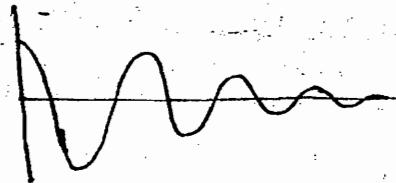
$$R = G_1 = 0 \longrightarrow LG_1 = RC$$

$$\beta = \omega \sqrt{LC} \longrightarrow \beta \propto \omega$$

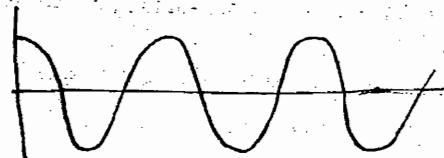
Workbook!:-

$$1) Z_{in} = Z_0 \left[\frac{Z_L \cosh(j\beta l) + Z_0 \sinh(j\beta l)}{Z_0 \cosh(j\beta l) + Z_L \sinh(j\beta l)} \right]$$

$$\cosh \theta = \frac{e^\theta + e^{-\theta}}{2} \quad \cosh(j\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$



$$j = \cos \theta$$



$$\sinh(j\theta) = j \sin \theta$$

$$Z_{in} = Z_0 \left[\frac{z_L \cos \beta d}{Z_0 \cos \beta d + j z_L \sin \beta d} \right]$$

(I) $d = \frac{\lambda}{8}$ $\beta d = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \frac{\pi}{4}$

$$Z_{in} = Z_0 \left[\frac{z_L + j z_0}{Z_0 + j z_L} \right]$$

(II) $d = \lambda/4$ $\beta d = \frac{\pi}{2}$ $Z_{in} = \frac{Z_0^2}{z_L}$

(III) $d = \frac{\lambda}{2}$ $\beta d = \pi$ $Z_{in} = z_L$

$$z_L = j50 \quad z_0 = 50\Omega$$

$$Z_{in} = 50 \left[\frac{j50 + j50}{50 + j50} \right] = \infty \rightarrow 0/C$$

$$Z_{in} (\lambda/4) = \frac{50 \times 50}{j50} = -j50\Omega$$

$$Z_{in} = z_L = j50\Omega$$

$(\lambda/2)$

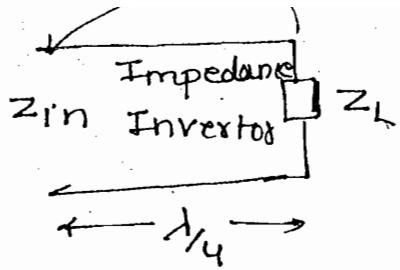
Note 1:-

→ βd is a critical aspect of the line that decides the nature and performance of the line and it is called an electrical paths line

→ Length/ λ relationship is a critical aspect.

Note 2:-

$$d = \lambda/4 \quad Z_{in} = \frac{Z_0^2}{z_L}$$



It's one end impedance has opposite nature to other end impedance hence called as impedance inverter

Note 3:-

$$d = \lambda/2 \quad Z_{in} = Z_L$$

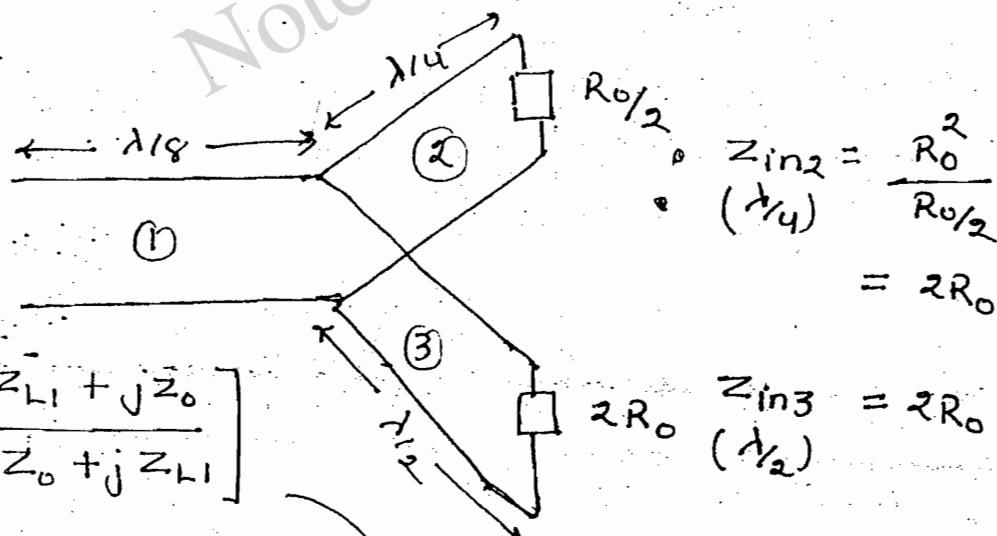
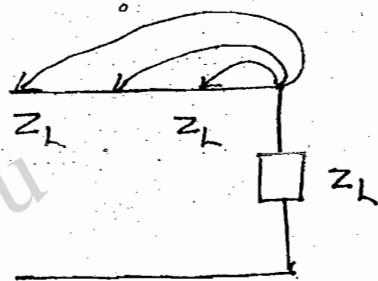
It is called as impedance reflector

Note 4:-

If, $d \rightarrow 0$ or $\lambda \rightarrow \infty$ where $\beta d \rightarrow 0$

$$Z(x) = Z_L$$

For electrically short line the wire is a simple connecting wire or a short ckt.



$$Z_{in1} (d/8) = Z_0 \left[\frac{Z_{L1} + jZ_0}{Z_0 + jZ_{L1}} \right]$$

$$\begin{aligned} Z_{in2} (d/4) &= \frac{R_0^2}{(d/4)} \\ &= 2R_0 \end{aligned}$$

$$Z_{in3} (d/2) = 2R_0$$

$$Z_{L1} = Z_{in2} \parallel Z_{in3}$$

$$= R_0 \left[\frac{R_0 + jR_0}{R_0 + jR_0} \right]$$

$$= R_0$$

$$Z_{in2} = \frac{R_o}{\frac{2}{2R_o}} \quad (1/4)$$

$$Z_{in3} = \frac{R_o}{2} \quad (1/2)$$

$$Z_{in1} = Z_0 \left[\frac{Z_L + jZ_0}{Z_0 + jZ_L} \right] = R_o \left[\frac{\frac{R_o}{4} + jR_o}{R_o + j\frac{R_o}{4}} \right]$$

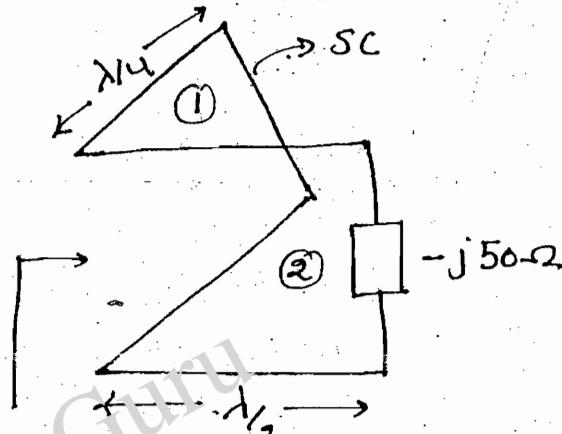
$$= R_o \left[\frac{1 + j4}{4 + j} \right]$$

$$Y_{in} = Y_{in1} + Y_{in2}$$

$$Y_{in1} = \frac{Z_L}{Z_0^2} = 0 \quad (1/4)$$

$$Y_{in2} = \frac{1}{Z_L} = \frac{1}{-j50} \quad (1/2)$$

$$= j0.02 \quad Y_{in} = ?$$



$$Z_{in} = (Z_{in1} || Z_{in2})$$

$$= (\infty || -j50) = -j50$$

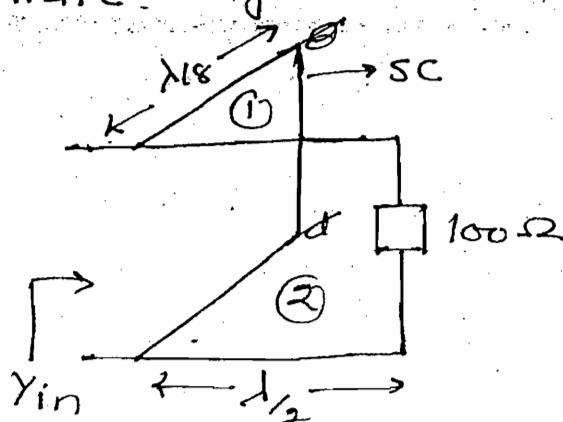
$$Y_{in} = j0.02 \text{ S}$$

Stub \Rightarrow SC line of finite length

$$Y_{in} = Y_{in1} + Y_{in2}$$

$$Z_{in1} = Z_0 \left[\frac{0 + jZ_0}{Z_0 + j0} \right] = jZ_0 = j50 \Omega$$

$$Y_{in1} = \frac{1}{j50} = -j0.02$$



$$Y_{in2} = \frac{1}{Z_L} = \frac{1}{R_0}$$

$$Y_{in} = Y_{in1} + Y_{in2} = 0.01 - j0.02 \text{ S}$$

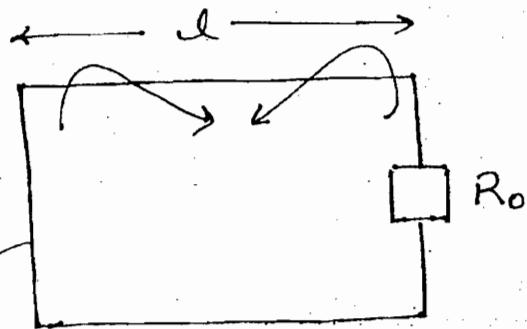
8:

$$l = \lambda/4$$

$$Z_{in1} = R_0 \left[\frac{0 + jR_0}{R_0 + j0} \right]$$

SC

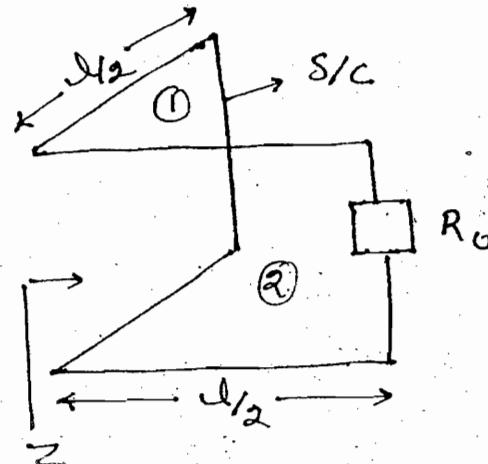
$$= jR_0$$



$$Z_{in2} = R_0$$

$$Z = \frac{jR_0 \cdot R_0}{jR_0 + R_0}$$

$$= \frac{jR_0}{j+1}$$



$$l = \lambda \quad Z = R_0$$

$$l = \lambda/8 \quad Z = SC$$

9: $V_p = 2 \times 10^8 \text{ m/s}$ $f = 10 \text{ MHz}$ $\lambda = 20 \text{ m}$ $\Rightarrow l = \lambda/2$
 $\lambda = 10 \text{ m}$

$$Z_{in} = Z_L = 30 - j40 \Omega$$

10: $f' = 5 \text{ MHz}$ $\lambda' = 40 \text{ m}$ $\Rightarrow l = \frac{\lambda'}{4}$

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{50 \times 50}{30 - j40} = 30 + j40$$

11: $Z_{in} = j60 = Z_{sc} = Z_0 \tanh(j\beta l)$
 $= jZ_0 \tan \beta l$

$$\beta l = \frac{\pi}{4}$$

$$Z_0 = 60 \Omega$$

$$= jZ_0$$

$$f' = 2f$$

$$12 \text{ MHz} \longrightarrow 24 \text{ MHz}$$

$$\lambda' = \lambda_2$$

$$\beta' d = 2\beta d = \frac{\pi}{2}$$

$$\therefore Z_{in} = Z_{sc} = jZ_0 \tan \frac{\pi}{2} = \infty$$

$$f = 12 \text{ MHz}$$

$$Z_{in} = j60 = j\omega L \Rightarrow L = \frac{60}{2\pi \times 12 \times 10^6} \\ = \frac{2.5}{\pi} \mu\text{H}$$

Summary:-

$$Z_{sc} = jZ_0 \tan \beta d$$

$$0 < \beta d < \frac{\pi}{2} \quad \left. \begin{array}{l} \\ \end{array} \right\} Z_{in} \text{ is inductive} \\ 0 < d < \frac{\lambda}{4} \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$\frac{\pi}{2} < \beta d < \pi \quad \left. \begin{array}{l} \\ \end{array} \right\} Z_{in} \text{ is capacitive} \\ \frac{\lambda}{4} < d < \frac{\lambda}{2}$$

$$Z_{oc} = -jZ_0 / \tan \beta d$$

Z_{in} inductive \longrightarrow Z_{in} is capacitive

Z_{in} is capacitive \longrightarrow Z_{in} is inductive.

→ one end of a line is s.c or o.c then the other end is purely reactive

4 (a) $Z_{in} = -jZ_0$ $\rightarrow \lambda/4 \text{ length } X$

✓ (b) $Z_{in} = \pm j\infty$ for SC line $\lambda/4$ length

$$Z_{SC} = jZ_0 \tan \beta l$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

X (c) $Z_{in} = -jZ_0 \rightarrow \text{o/c line} \rightarrow \lambda/2 \text{ length}$



$$Z_{in} = Z_L$$

✓ (d) $Z_{in} = Z_0$

15. $Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{LC}{C^2}} = \sqrt{\mu_0 \epsilon_0 \epsilon_R} = \frac{\sqrt{\epsilon_R}}{\nu_p c}$

16. (a) $\alpha = \text{even}, \beta = \text{odd} \quad X$

(b) $\alpha = \text{constant}, \beta = k\omega \quad \checkmark$

(c) $\frac{L}{C} = \text{constant} = R/G_1$

(d) Matched line ($Z_L = Z_0$) $\quad X$

17. $\beta l = \omega \sqrt{LC} l = 108^\circ$

Reflection Coefficient:-

<https://notesguru.info>

$$V(x) = \underbrace{V_f e^{j\beta x}}_{V_f} + \underbrace{V_r \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j\beta x}}_{V_r}$$

$$\frac{V_r}{V_f} = \text{Reflection coefficient for the voltages anywhere on the line} = \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j2\beta x}$$

$$= \Gamma(x)$$

$$\Gamma(x) = \frac{Z(x) - Z_0}{Z(x) + Z_0}$$

$$\text{At } x=0 \quad \Gamma \text{ at load} = \frac{Z_L - Z_0}{Z_L + Z_0} = \text{complex}$$

$$= | \Gamma | e^{j\phi} = \left| \frac{V_r}{V_f} \right| e^{j\phi}$$

$|\Gamma| \rightarrow$ It is the ratio of reflected voltage to the forward voltage amplitude

$\phi \rightarrow$ It is the time delay or phase difference b/w the reflected and forward voltage.

Γ is the measure of mismatch b/w the expected impedance and the actual impedance Z_L at the load.

$\Gamma(x)$ is the measure of mismatch b/w the expected impedance Z_0 and the actual impedance $Z(x)$ anywhere on the line.

$$(I) \Gamma = \frac{Z_0 - Z_L}{Z_0 + Z_L} = -\Gamma_r$$

If V_f and I_f they are inphase

$$V_o e^{j\beta x} + V_r \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j\beta x}$$

$$I_o e^{j\beta x} - I_r \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j\beta x}$$

then V_r and I_r are out of phase by 180°

(II)

$$0 \leq |\Gamma| \leq 1$$

Matched line Complete Reflection
(Mismatch)

Lecture - 13

Four causes of complete mismatch on a line ($|\Gamma|=1$):

Cause - (1):-

$$Z_L = jR_o = \text{Inductive load}$$

$$Z_0 = R_o = \text{Lossless line}$$

$$|\Gamma| = 1$$

$$V_f \rightarrow \sin$$

$$V_r \rightarrow \cos$$

$$\Gamma = \frac{jR_o - R_o}{jR_o + R_o} = \frac{j-1}{j+1}$$

$$= \frac{\sqrt{2} [135]}{\sqrt{2} [45^\circ]}$$

$$= 1 [90^\circ] = j$$

case-(II) :-

$$Z_L = -jR_0 = \text{c.c} \quad \text{https://notesguru.info}$$

$Z_0 = R_0$ = lossless line

$$\Gamma = \frac{-jR_0 - R_0}{-jR_0 + R_0} = \frac{-j-1}{-j+1}$$

$$= \frac{\sqrt{2} \angle 45^\circ}{\sqrt{2} \angle 135^\circ} = -j$$

Note:-

Real Power dissipation in any resistive load is always zero.

Case-(III) :-

$$Z_L = 0 \Rightarrow \text{s.c. load}$$

$Z_0 = R_0$ = lossless line

$$\Gamma = \frac{0 - Z_0}{0 + Z_0} = -1 = 1 \angle 180^\circ$$

$$V_f + V_r = 0$$

$$V_f \rightarrow \sin$$

$$V_r = -\sin$$

Voltage at s.c. is always zero

Case-(IV) :-

$$Z_L = \infty \Rightarrow \text{o.c. load}$$

$Z_0 = R_0$ = lossless line

$$\Gamma = \frac{1 - Z_0/Z_L}{1 + Z_0/Z_L} = 1/0$$

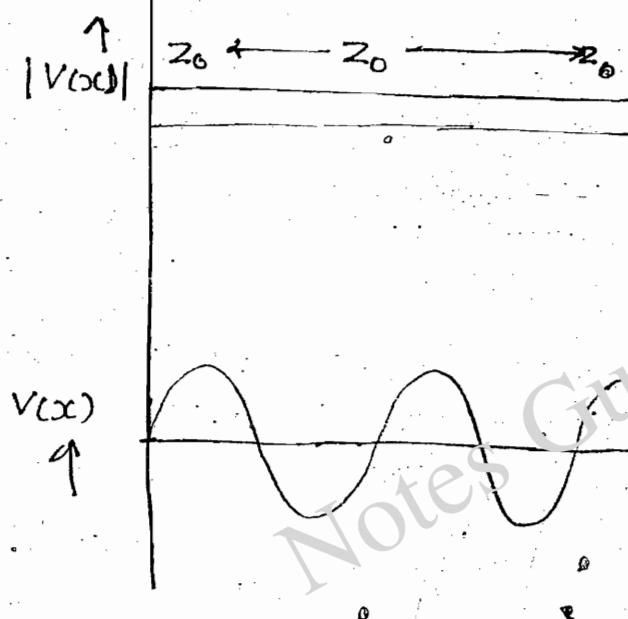
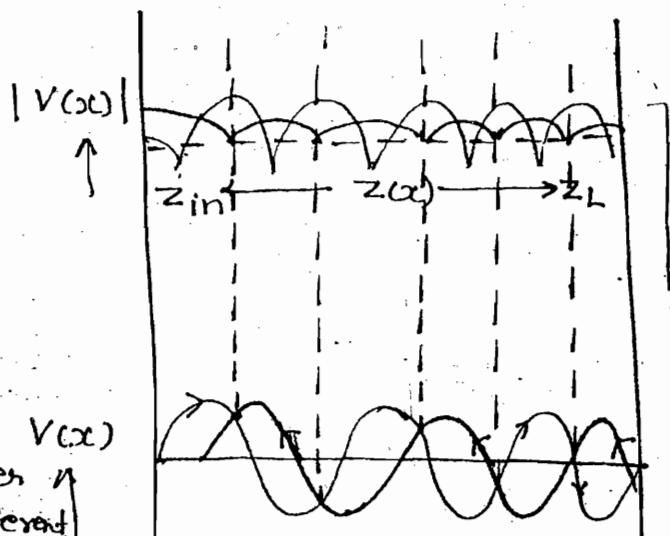
$$V_f + V_r = V_{\max}$$

Voltage at o.c. is always maximum

$$|\Gamma| = 1$$

$$V_f \rightarrow \sin$$

$$V_r = \sin$$



$$\begin{aligned} V(x) &= V_0 e^{j\beta x} + V_0 |R| e^{j\theta} e^{-j\beta x} \\ &= V_0 e^{j\beta x} + V_0 |R| e^{j(-\beta x + \theta)} \end{aligned}$$

The magnitude plot of a single harmonic matched line as a constant value straight line graph. The magnitude plot of two harmonics mismatched line as shown below:-

$$|V(x)| = V_{max} = V_0 + V_0 |R| \text{ when they are inphase}$$

$$\beta x - (-\beta x + \theta) = 2n\pi$$

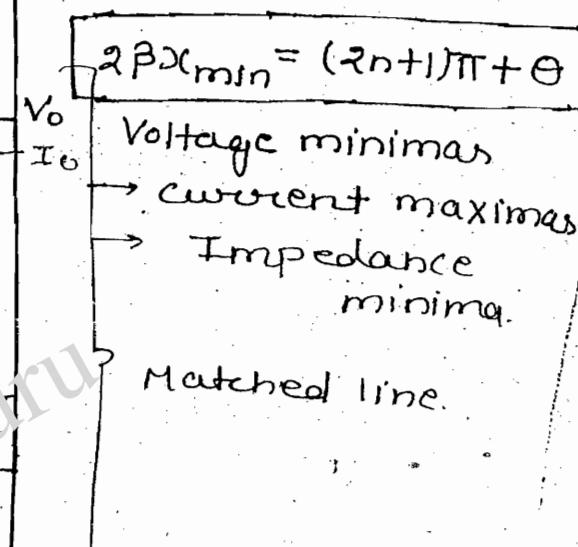
$$\begin{aligned} |V(x)| \\ n=0,1,2 \end{aligned}$$

$$\begin{aligned} \beta x - (-\beta x + \theta) &= 2n\pi \\ n = 0, 1, 2, \dots & \quad |V(x)| \end{aligned}$$

$$2\beta x_{max} = 2n\pi + \theta$$

Voltage Maxima
→ current minima
→ Impedance maxima

Mismatched line



$$2\beta x_{max} = 2n\pi$$

Voltage Maxima Positions.

$|V(x)| = V_{min} = V_0 - V_0 |\Gamma|$ when they are out of phase

$$2\beta x_{min} = (2n+1)\pi + \theta$$

→ Voltage Minima

The magnitude plot of two harmonics travelling in opposite direction having periodic interference results in addition and cancellation leading to standing wave formation

Note 1:-

The current also has identical standing wave pattern with the voltage maxima coincide with current minima

Note 2:-

As the impedance is different at different point of line voltage and current distribution is also extremely spread on the line such that

$$Z_{max} = \frac{V_{max}}{I_{min}}$$

$$Z_{min} = \frac{V_{min}}{I_{max}}$$

Note 3:-

$$\beta x - \frac{2\pi}{\lambda} x = \text{Periodic} \rightarrow 2\pi \quad \left. \begin{array}{l} \text{Harmonic} \\ x \rightarrow \text{Periodic} - \lambda \end{array} \right\}$$

$$2\beta x - \frac{2\pi}{\lambda_2} x \rightarrow \text{Periodic} - 2\pi \quad \left. \begin{array}{l} \text{Standing} \\ \text{wave} \end{array} \right\}$$

$$x \rightarrow \text{Periodic} - \lambda_2$$

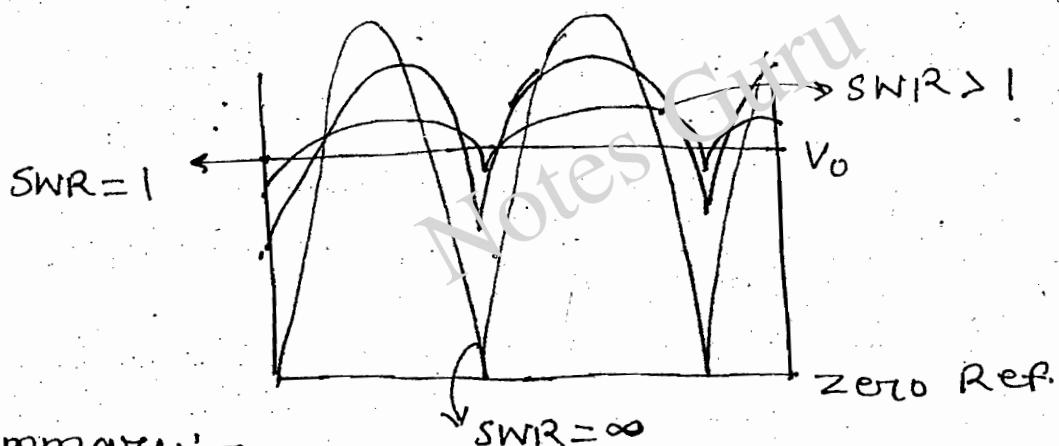
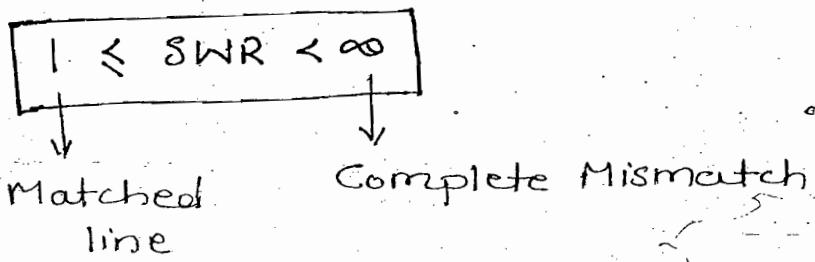
for $\lambda/2$ Standing Wave Ratio (SWR):-

It can be VSWR or CSWR

$$= \frac{V_{max}}{V_{min}} \text{ or } \frac{I_{max}}{I_{min}}$$

$$\boxed{SWR = \frac{V_0 + V_0 |\Gamma|}{V_0 - V_0 |\Gamma|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$$

$$0 \leq |\Gamma| \leq 1$$

Summary:-

$$\left. \begin{array}{l} Z_0 \uparrow \\ V_0 \uparrow \\ I_0 \uparrow \end{array} \right\} \text{SWR}$$

$$\begin{aligned} Z_{max} &= \frac{V_{max}}{I_{min}} \\ &= \frac{V_0 + V_0 |\Gamma|}{I_0 - I_0 |\Gamma|} \end{aligned}$$

SWR is a measure of mismatch on the line

$$Z_{max} = Z_0 \cdot SWR$$

$$Z_{min} = Z_0 / SWR$$

$$\text{ESWR} = 5 = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$|\Gamma| = \frac{5-1}{5+1} = \frac{2}{3}$$

$$\Gamma = \frac{n_2 - n_1}{n_2 + n_1} = \frac{2}{3} \angle 0^\circ = \pm \frac{2}{3}$$

$\rightarrow 120\pi$
 $\frac{120\pi}{\sqrt{Z_R}}$ (real)

As $n_2 < n_1$

$$\Gamma = -\frac{2}{3} = \frac{n_2 - 120\pi}{n_2 + 120\pi} \Rightarrow n_2 = 24\pi$$

$$\text{VSWR} = 4$$

$$|\Gamma| = \frac{4-1}{4+1} = \frac{3}{5}$$

$$\Gamma = \frac{Z_L - 50}{Z_L + 50} = \frac{3}{5} \angle 0^\circ = \pm \frac{3}{5}$$

$\rightarrow \text{real}$
 $\rightarrow \text{real}$

V_{\min} is exactly at the load, Z_{\min} is at the load

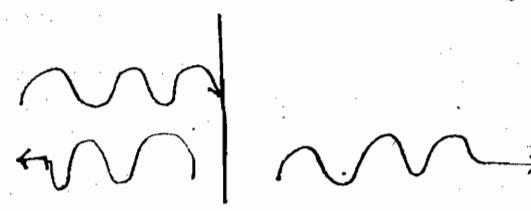
$$Z_L < Z_0$$

$\Rightarrow \Gamma$ is negative

$$\Gamma = -\frac{3}{5} = \frac{Z_L - 50}{Z_L + 50} \Rightarrow Z_L = 12.5\Omega$$

\rightarrow 1st voltage maxima occurs at $\lambda/4$

$$2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = 2n\pi + \theta \Rightarrow \boxed{\theta = \pi}$$



0 or 180°

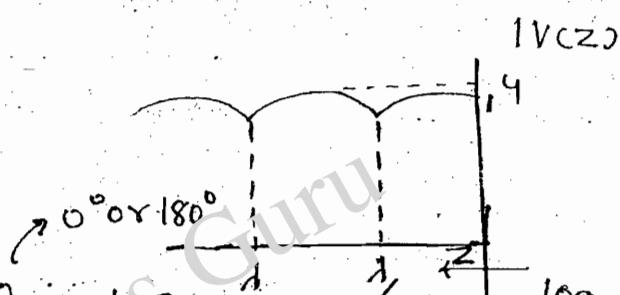
$$\Gamma = \frac{n_2 - n_1}{n_2 + n_1} = \frac{2}{3} \angle 0^\circ = \pm \frac{2}{3}$$

$\rightarrow 120\pi$ (real)

$$\frac{120\pi}{\sqrt{Z_R}}$$
 (real)

$$\text{VSWR} = 4$$

$$|\Gamma| = \frac{4-1}{4+1} = \frac{3}{5}$$



$1V(Z_0)$

$$\Gamma = \frac{Z_L - 50}{Z_L + 50} = \frac{3}{5} \angle 0^\circ = \pm \frac{3}{5}$$

$\rightarrow \text{real}$
 $\rightarrow \text{real}$

V_{\min} is exactly at the load, Z_{\min} is at the load

$$12.5\Omega$$

$\Rightarrow \Gamma$ is negative

$$\Gamma = -\frac{3}{5} = \frac{Z_L - 50}{Z_L + 50} \Rightarrow Z_L = 12.5\Omega$$

\rightarrow 1st voltage maxima occurs at $\lambda/4$

$$2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = 2n\pi + \theta \Rightarrow \boxed{\theta = \pi}$$

n Note!—

2 → If Z_L, Z_0 are real

$$Z_L > Z_0$$

Γ is real & positive

Γ 's phase is 0

18

$$2\beta x_{max} = 2n\pi + \theta \Rightarrow x_{max} = 0$$

21

3

First voltage maxima exactly occurs

4

eg:- OC line where $\Gamma = +1$

5

→ If Z_L, Z_0 are real

6

$$Z_L < Z_0$$

7

Γ is real and negative

First voltage minima exactly occurs

8

eg:- SC line where $\Gamma = -1$

9

$$\text{VSWR} = \frac{1 + \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|}{1 - \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|}$$

If Z_L, Z_0 are real & $Z_L > Z_0$

$$\text{VSWR} = \frac{1 + \frac{Z_L - Z_0}{Z_L + Z_0}}{1 - \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\boxed{\text{VSWR} = \frac{Z_L}{Z_0}}$$

10

If Z_L, Z_0 are real

$$Z_L < Z_0$$

$$\boxed{\text{VSWR} = 3}$$

15 Γ at load = $0.6 \angle 60^\circ$

$$\Gamma(x) = \frac{z(x)-z_0}{z(x)+z_0} = \frac{z_L-z_0}{z_L+z_B} e^{-j2\beta \cdot \frac{x}{\lambda}}$$

$$\Gamma(\lambda_8) = \Gamma(0) e^{-j2\beta \cdot \frac{\lambda}{8}}$$

$$= 0.6 e^{j60^\circ} e^{-j2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}}$$

$$= 0.6 e^{j60^\circ} e^{-j90^\circ}$$

$$= 0.6 \angle -30^\circ$$

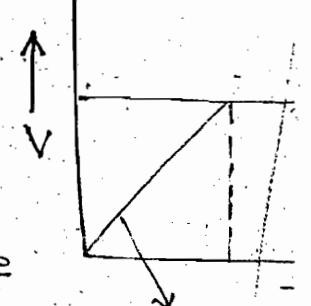
21. $18+$
← 17
7cm
 $h=0$

2 25:

30V D.C. Battery

D.C. Transients

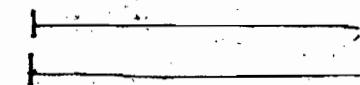
During the transient state even DC has time variations and all AC fundamentals are applicable



$$v_f + v_r = \text{Arising}$$

30V

$$\Gamma = \frac{10}{30} = \frac{z_L - 50}{z_L + 50}$$



$$\Rightarrow z_L = 100 \Omega$$

$$v_f =$$

$$I_L = \frac{30}{z_L}$$

(Steady state)

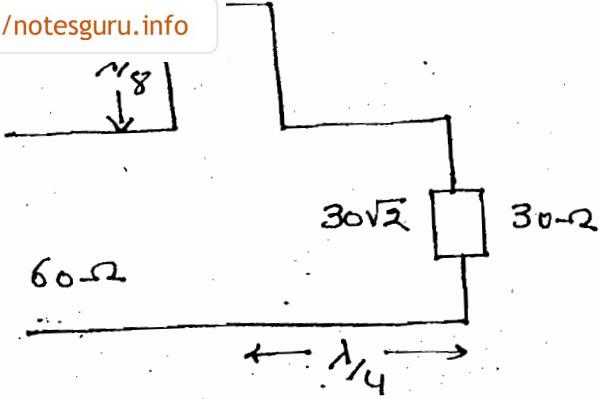
$$= \frac{30}{100}$$

$$= 0.3 \text{ Amp.}$$

$n=0$

2. 2

$$\begin{aligned} \underline{\underline{26}} \quad VSWR &= \frac{1 + |\Gamma|}{1 - |\Gamma|} \\ &= \frac{\sqrt{17} + 1}{\sqrt{17} - 1} \\ &= 1.63 \end{aligned}$$



$$\begin{aligned} Z_{L_1} &= Z_{in2} + Z_{in3} \\ &= 60 + j30\Omega \end{aligned}$$

$$Z_{in2} = j30\Omega$$

$$Z_{in3} = \frac{(30\sqrt{2})^2}{30} = 60\Omega$$

$$\Gamma = \frac{Z_{L_1} - jZ_0}{Z_{L_1} + jZ_0} = \frac{60 + j30 - 60}{60 + j80} = \frac{j}{j+4}$$

$$|\Gamma| = \frac{1}{\sqrt{17}}$$

$$\underline{\underline{27}} \quad V_f = 25 \sin(bxc - 75^\circ)$$

$$\Gamma = 0.6 \angle -30^\circ$$

$$V_r = ?$$

$$\frac{|V_r|}{|V_f|} = 0.6 \quad |V_r| = 15$$

$$|V_f|$$

$$\begin{aligned} V_r \text{ phase} &= V_f \text{ phase} + \Gamma \text{ phase} \\ &= -75^\circ - 30^\circ = -105^\circ \end{aligned}$$

$$V_r = 15 \sin(-bxc - 105^\circ)$$

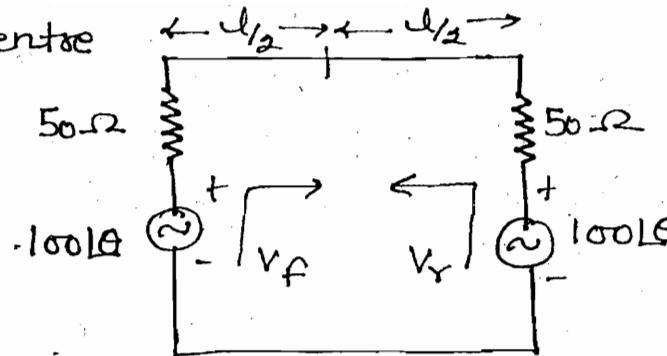
V_f & V_y are inphase <https://notesguru.info>

and add up at centre

I_f & I_y cancel 50Ω at the centre

$$|I_f| = |I_y|$$

$$\Rightarrow I_{\min} = 0$$



$$Z_{\max} = \frac{V_{\max}}{I_{\min}} = \infty$$

29.

$$VSWR = \frac{Z_L}{Z_0} = \frac{75}{50} = 1.5$$

Smith Chart (Circle Diagrams) :-

→ It is used to calculate Γ and VSWR for a known Z_L & Z_0 .

$$\begin{aligned}\Gamma &= \frac{Z_L - Z_0}{Z_L + Z_0} \\ &= \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}\end{aligned}$$

→ It is used normalised or relative ratio of Z_L/Z_0 for its calculation

→ It is a rectangular graph or polar plot of Γ_y vs Γ_x having two families of circles

→ constant R circles

where $R = \text{Real } [Z_L/Z_0]$

→ constant X circles.

$X = \text{Imag } [Z_L/Z_0]$

$$\Gamma_y + j\Gamma_i = \frac{R+jx}{R+jx+1} = \frac{R+jx}{(R+1)+jx}$$

$$\left(\Gamma_y - \frac{R}{R+1}\right)^2 + \Gamma_i^2 = \left(\frac{1}{R+1}\right)^2$$

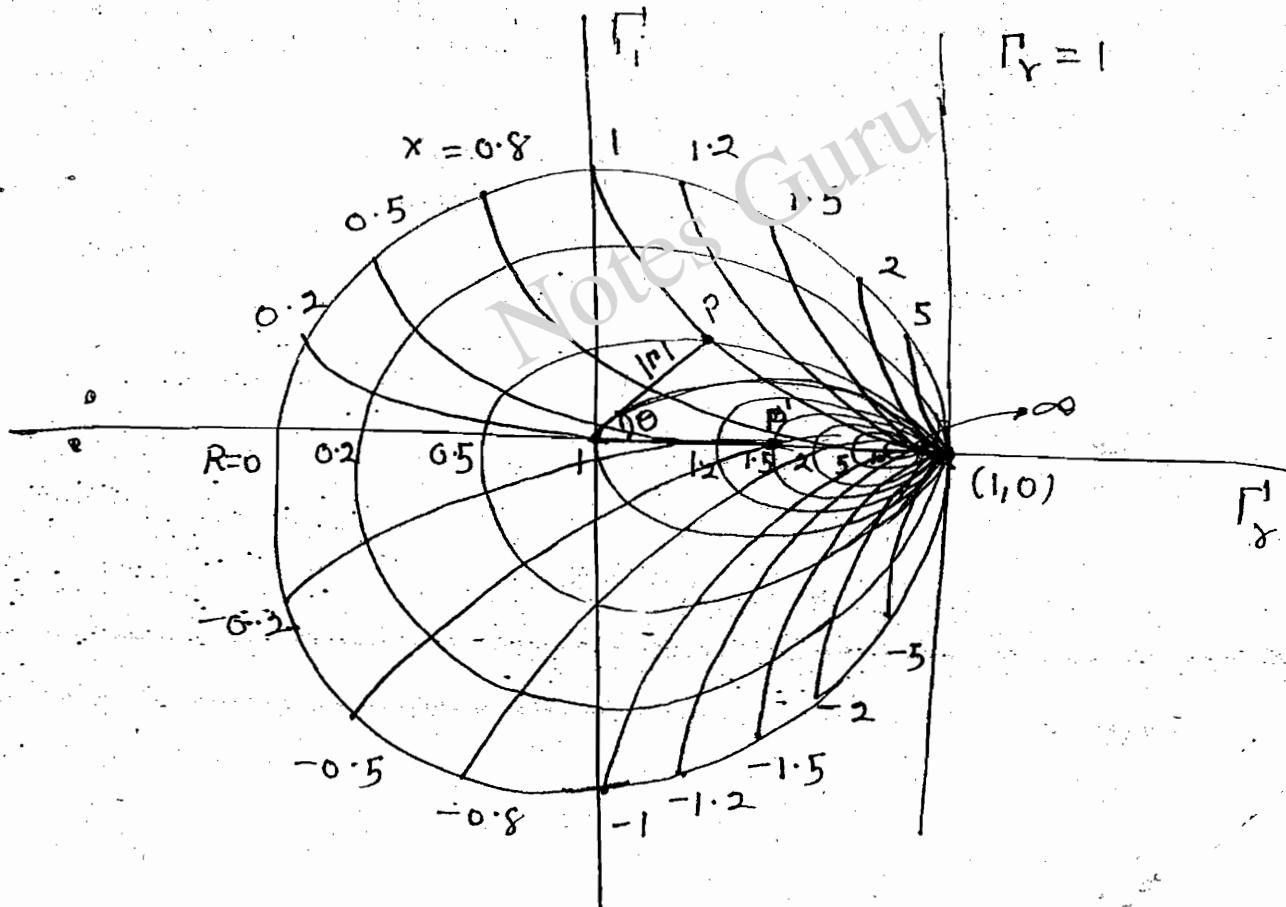
constant R circles
Equation.

$$(\Gamma_y - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

constant x circles
Equation.

Properties of constant R circles:-

- (a) Centres - $\left(\frac{R}{R+1}, 0\right)$
- (b) Radius - $\left(\frac{1}{R+1}\right)$
- (c) They all pass through $(1, 0)$
- (d) Range $(0, \infty)$



1. The $R=0$ circle is the possible R circle and it is periphery or boundary of Smith chart.
2. All circles $R=0$ to 1 extend into the four quadrants the circles with $R>1$ confined in the Ist and IV quadrants
3. All the R -circles are concurrent with their centres on a straight line and have a common tangent $\Gamma_y = 1$ axis

Properties of constant X -circles:-

- a) Centres - $(1, \frac{1}{x})$
- b) Radius - $(\frac{1}{x})$
- c) They all pass through $(1, 0)$
- d) Range $(-\infty, \infty)$
1. $x=0$ circle is the biggest possible X circle and it is Γ_x axis
2. All circles with $x=0$ to 1 extend into the first two quadrant but circles with $x>1$ confined in the Ist quadrant
3. All the X -circles are concurrent and have a common tangent with Γ_x axis

Note:-

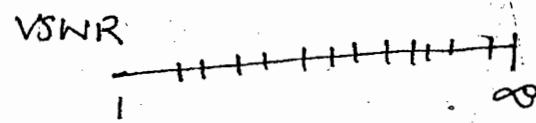
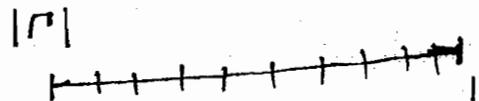
- The centres of R circles lie on the common tangent of X -circle and vice-versa. Hence R and X constitute a pair of orthogonal families of circles
- A horizontal movement on the $X=0$ circle as R value increasing and a clockwise movement on the $R=0$ circle as X increasing inductively (try)

calculation of Γ :-

For a known Z_L and Z_0 calculate

$\frac{Z_L}{Z_0} = R + jX$ and locate the corresponding point of intersection of R and X circle

- Join the point P to origin the OP length $= |\Gamma|$
- OP segments inclination with Γ_y axis is Γ 's phase
- Good



Calculation of VSWR :-

- For a known $|\Gamma|$ identify a point P' on the $|\Gamma|$ real axis such that OP' length $= |\Gamma|$. The R circle value at P' is VSWR

$$\left| \frac{R+jX}{Z_0} \right| = \frac{Z_L}{Z_0} = \text{VSWR}$$

$x_e=0$
 Γ_y axis

Identify following points on the Smith's Chart:-

(I) $Z_L = jR_0$, $Z_0 = R_0$.

$$\frac{Z_L}{Z_0} = \frac{0 + j1}{1} = -j$$

\downarrow \downarrow
 $R=0$ $X=1$

$$\Gamma = j$$

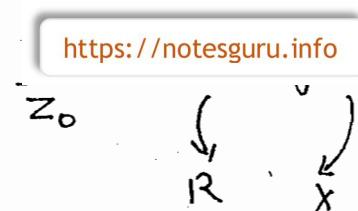
(II) $Z_L = -jR_0$
 $Z_0 = R_0$

bottom

(III)

$$Z_L = 0$$

$$Z_0 = R_0$$



$$\Gamma = -1$$

Extremely Left

(IV)

$$Z_L = \infty$$

$$Z_0 = R_0$$

Extremely Right

(V)

$$Z_L = R_0$$

$$Z_0 = R_0$$

→ centre $\Gamma = 0$

Impedance Matching Technique:-

Active
(Bias)
CB Amp
CC Amp

Passive
Series
 $\lambda/4$
Quarterwave
transformer

$$Z_0 = \frac{60 \ln(b/a)}{\sqrt{\epsilon_r}}$$

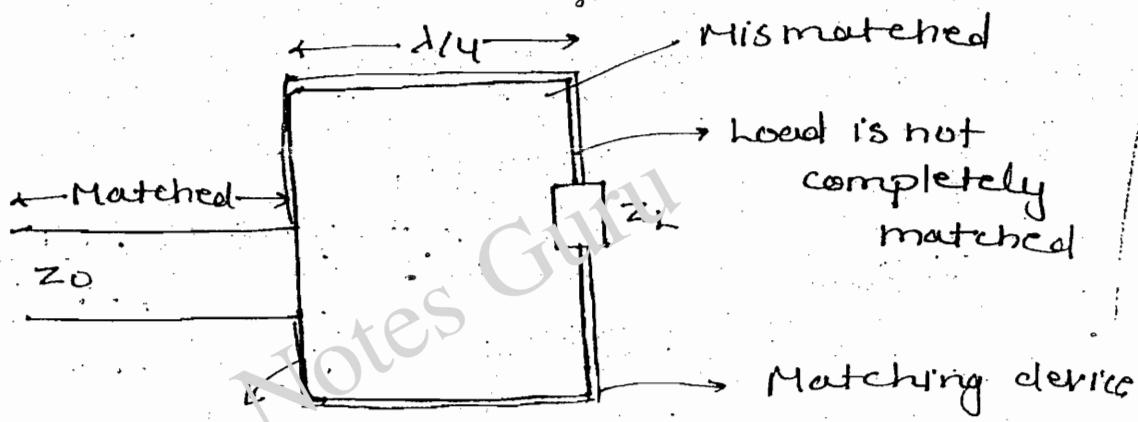
= few 10Ω to
 100Ω

$$Z_L = \text{few } \Omega - k\Omega$$

Stub
matching

$(Z_L \neq Z_0)$
Most often

Series ($\lambda/4$) Quarterwave transformer:-



The line is matched

The termination of Z_0 line is Z_{in} of $\lambda/4$ matching device

$$\therefore \frac{Z_0^2}{Z_L} = Z_0$$

Design needs

$$Z_0' = \sqrt{Z_0 Z_L}$$

e.g.: - $50\Omega \rightarrow$ line $100\Omega \rightarrow$ load

$$\left(\text{load}\right) = \frac{100-50}{100+50} = \frac{1}{3}$$

Using a $\lambda/4$ transformer

$$Z_0' = \sqrt{50 \times 100} = 70\Omega$$

$$\left(\text{at load}\right) = \frac{100-70}{100+70} < \frac{1}{3}$$

i) The technique is used for a narrow range of frequencies as the length of the transformer is frequency dependent.

ii) $Z_0' = \sqrt{Z_0 \cdot Z_L}$ → The technique is suited only for lossless line and resistive load.

Shunt - Stub matching:-

A stub is S.C or O.C line of a pre-calculated length, placed at a precalculated position such that the line is matched from the stub to the source

Design of a stub:-

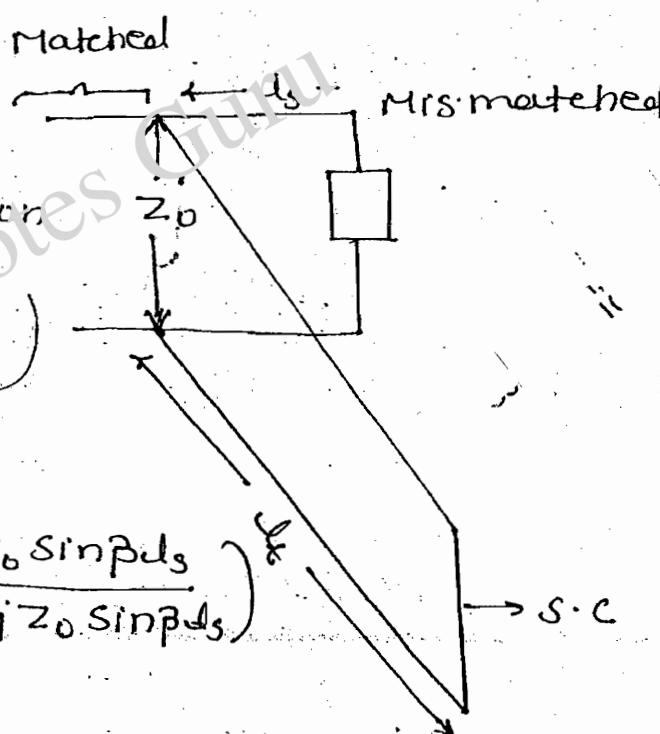
Step-(1):-

Identify a length l_s on the line from the load where the impedance has a real part Z_0

$$Z(l_s) = Z_0 \pm jX$$

l_s = stub position

$$l_s = \frac{d}{2\pi} \tan^{-1} \left(\sqrt{\frac{Z_L}{Z_0}} \right)$$



$$Z(l_s) = Z_0 \left(\frac{Z_L \cos \beta l_s + j Z_0 \sin \beta l_s}{Z_0 \cos \beta l_s + j Z_0 \sin \beta l_s} \right)$$

$$Z(l_s) = \text{Real} \left[\frac{Z_L \cos \beta l_s + j Z_0 \sin \beta l_s}{Z_0 \cos \beta l_s + j Z_0 \sin \beta l_s} \right] = 1$$

Step-(II):-

At this identified position place an equal and opposite reactance in shunt such that cancel to the existing reactance i.e.

$$z(j_s) = z_0 \pm jX \quad (+jX) \rightarrow \text{Stub}$$

$$Y(j_s) = Y_0 \pm jB \quad (+jB) \rightarrow$$

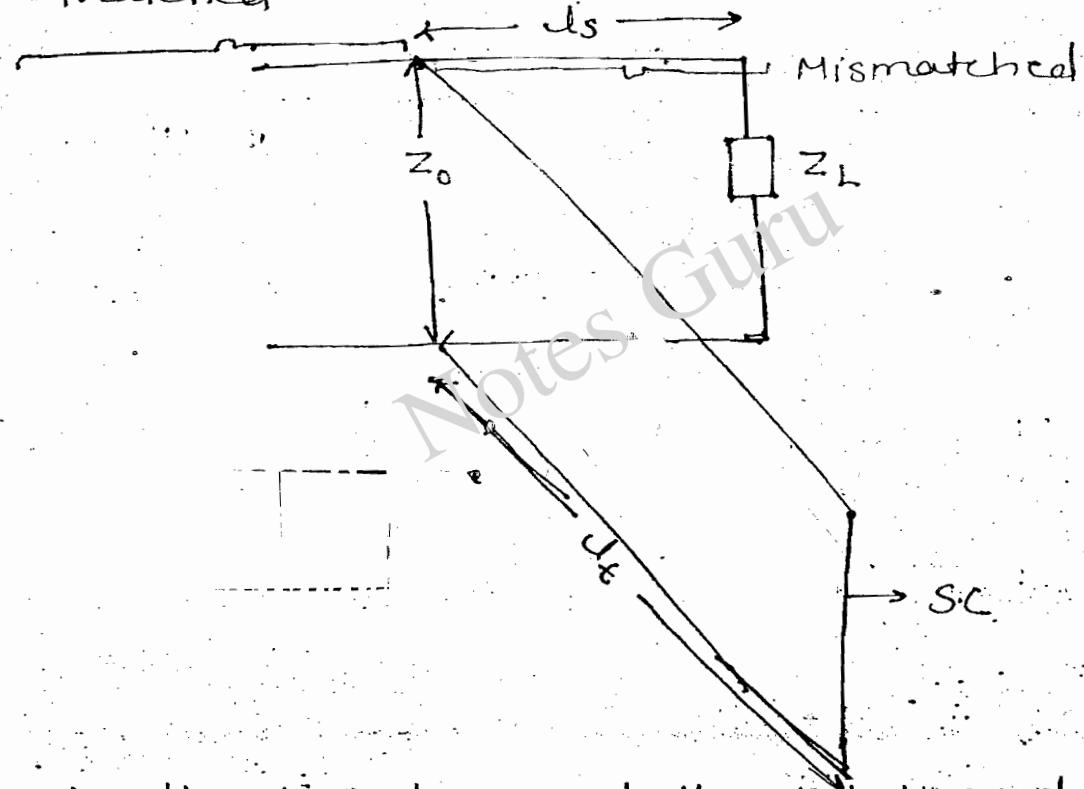
Step-(III):-

This reactance is realised from a SC or OC stub of a finite length l_f such that

$$z_{sc} = jz_0 \tan \beta l_f \approx -\text{Imag}[z(j_s)]$$

$$l_f = \text{Stub length} = \frac{1}{2\pi} \tan^{-1} \left(\frac{\sqrt{z_L z_0}}{z_L - z_0} \right)$$

matched



The junction impedance at the stub and the line purely real as Z_0 and hence matched from the stub to the source

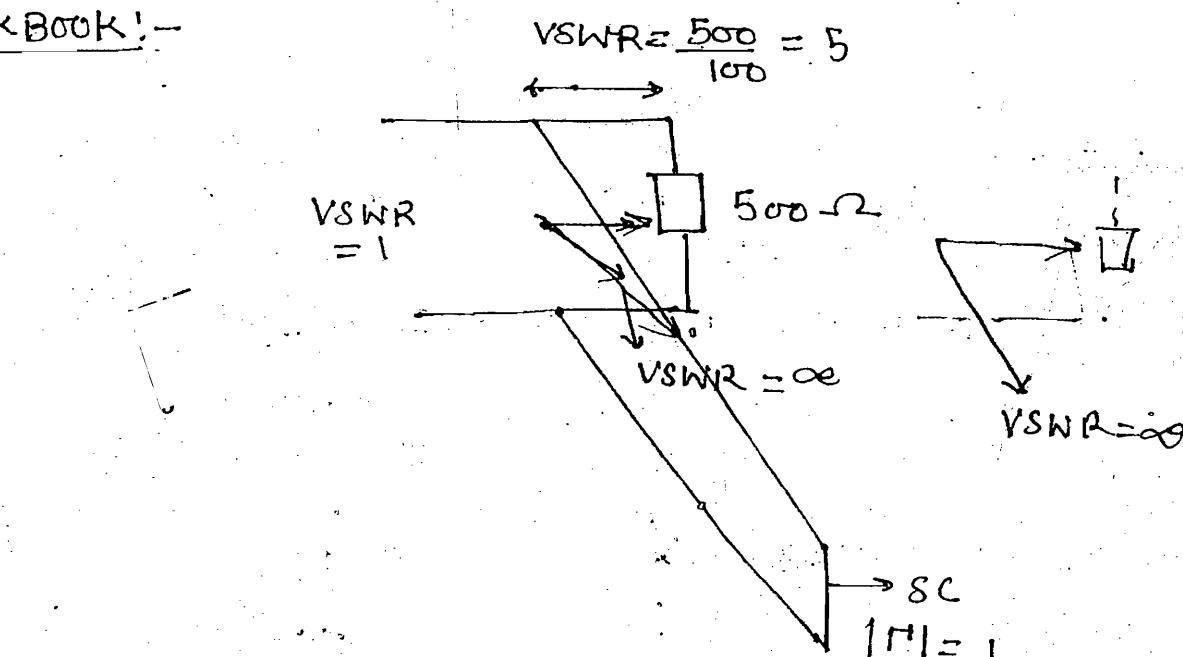
As the frequency on the line changes the stub length and stub position both have to be adjusted which is mechanically inconvenient

however in SC stub to using movable shorts

NOTE:-

For wide range of frequencies we use double stub matching where l_{s_1} and l_{s_2} are two stub position fixed and l_{t_1} and l_{t_2} are varical.

WORKBOOK:-



$$\text{VSWR} = 3$$

$$|\gamma|_v = \frac{3-1}{3+1} = \frac{1}{2}$$

$$\Gamma_p = -\frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{4} = 25$$

$$Z_0' = \sqrt{225 \times 250} = 240 \Omega$$

$$Z_L = 75 - j40$$

$$Z_0 = 75$$

Note:-

Stub can never be placed at node

OC \rightarrow Stubs \rightarrow More difficult to realize

SC \rightarrow Stubs

Ans-(a)

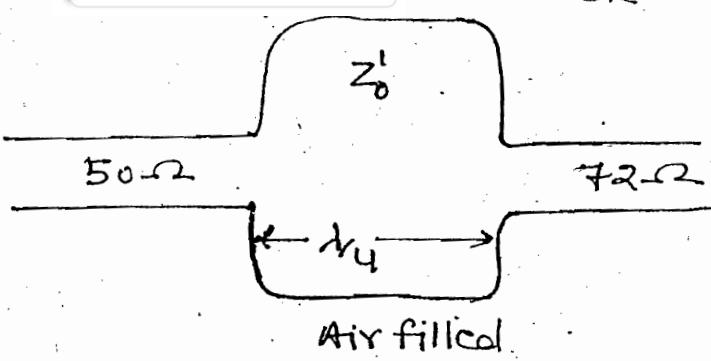
5. $\rightarrow \text{D}$
36.

$$Z_0' = \sqrt{50 \cdot 72}$$

$$= 60 \Omega$$

$$= \frac{60 \ln(b/a)}{\sqrt{\epsilon_R}}$$

$$\ln(b/a) = 1 \Rightarrow \frac{b}{a} = e = 2.71 \Rightarrow b = 27 \text{ mm}$$



37. $0.5 - j0.3 = \frac{Z_L}{Z_0} \approx \text{Normalized value}$

$$\begin{aligned} \text{Actual value} &= (0.5 + j0.3) \times 50 \\ &= 25 - j15 \end{aligned}$$

38. Clocking on
 $R = \text{constant}$

$$Z \text{ at } P = R + jX_1$$

$$Z' \text{ at } P' = R + jX_2$$

with $X_2 > X_1$

$$Z' \text{ at } P' = R + jX_1 + (jX_2)$$

Ans - (a) inductive

39. (I) $\frac{50 + j50}{50} = 1 + j1$

$\downarrow R \quad \downarrow X$

(II) $\frac{10 + j10}{50} = 0.2 + j0.2$

$\downarrow R \quad \downarrow X$

(III) $\frac{25 - j50}{50} = 0.5 - j1$

$\downarrow R \quad \downarrow X$

40. (I) one revolution, for 360° -length $= \lambda/2$
All impedances repeat for $\lambda/2$

(II) Clockwise movement

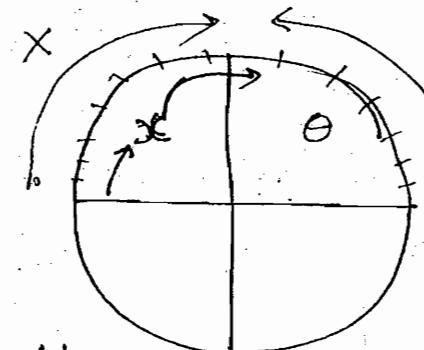
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towards generator $\propto \uparrow$

$$Z(\alpha) = Z_0 \left[\frac{z_L \cos \beta \alpha + j z_0 \sin \beta \alpha}{z_0 \cos \beta \alpha + j z_L \sin \beta \alpha} \right]$$

$$= j z_0 \tan \beta \alpha$$

$\alpha \uparrow \quad X \uparrow$



Three scales

- x
- θ
- α

(III) $\Gamma(\alpha) = \Gamma(0)$

$$e^{-j\alpha \beta \alpha}$$

Three scales on the Boundary

(I) x - Reactance of the load

(II) α - Length on the line

(III) Γ 's phase

All the three are physical scales

→ one is sufficient

(IV) Impedance (Working Statement)

Conventional :-

$$V(\alpha) = \frac{V_L (z_L + z_0)}{2z_L} \left[e^{r\alpha} + \frac{z_L - z_0}{z_L + z_0} e^{-r\alpha} \right]$$

$$= V_L \cosh r\alpha$$

$$\text{If } r = j\beta \quad V(\alpha) = V_L \cos \beta \alpha \quad \rightarrow (1)$$

OR

$$V(\alpha) = V_L \cosh r\alpha + I_L z_0 \sinh r\alpha$$

$$V(\alpha) = V_L \cosh r\alpha$$

$$I(\alpha) = I_L \cosh(r\alpha) + \frac{V_L}{z_0} \sinh(r\alpha)$$

$$I(x) = \frac{V_L}{Z_0} \sin x \quad \text{Sir} \quad \text{https://notesguru.info}$$

$$\text{If } Y = j\beta \quad I(x) = \frac{jV_L}{Z_0} \sin \beta x$$

$$V_L = 12V$$

$$Z_0 = 60\Omega$$

$$x = \lambda/4$$

$$d = \lambda/2$$

$$\text{if } Y = j\beta \quad V(x) = V_L \cos \beta x$$

$$Z_L = 100 - \frac{j}{\omega C} = 100 - j180\Omega$$

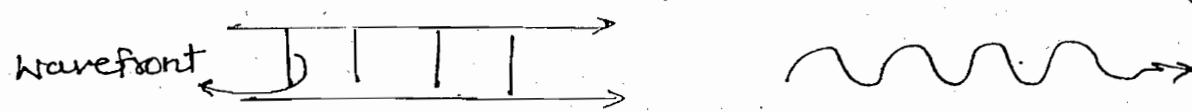
$$= 100 - \frac{j}{2\pi \times 10^8 \times \frac{1}{36\pi \times 10^9}}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \text{VSWR}$$

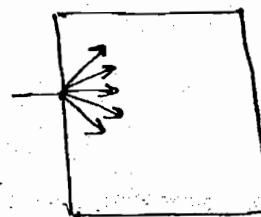
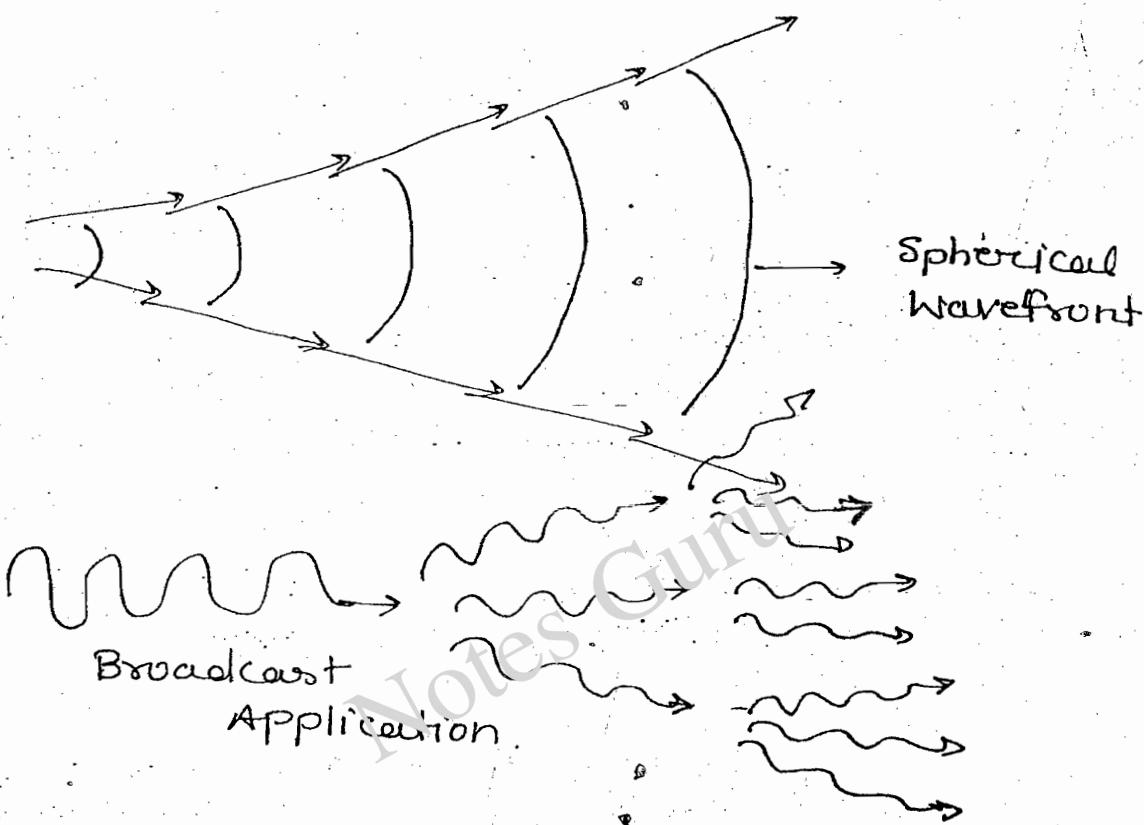
$$Z_{\max} = Z_0 (\text{SWR})$$

$$Z_{\min} = Z_0 / \text{SWR}$$

Uniform Plane Wave $\rightarrow E(z,t)x, H(z,t)y$



Dispersive Beams $\rightarrow E(x,y,z,t)_{(x,y,z)}$
 $H(x,y,z,t)_{(\omega_x,y,z)}$



Scattering
Diffraction
Diffusion

→ All practical EM waves are dispersive in nature and hence they obey Huygen's wave principle that every ray is a source of secondary emission. This is the cause of diffraction, diffraction and scattering properties of EM Waves

→ This is an active broadcast application but a serious limitation in point to point communication. Hence waveguide are used to restrict the wave with a specific bound

$$\left. \begin{array}{l} E(x, z, t) \\ H(x, z, t) \end{array} \right\} \begin{array}{l} \text{one dimension restriction in } x \\ \text{one dimension propagation in } z \end{array}$$

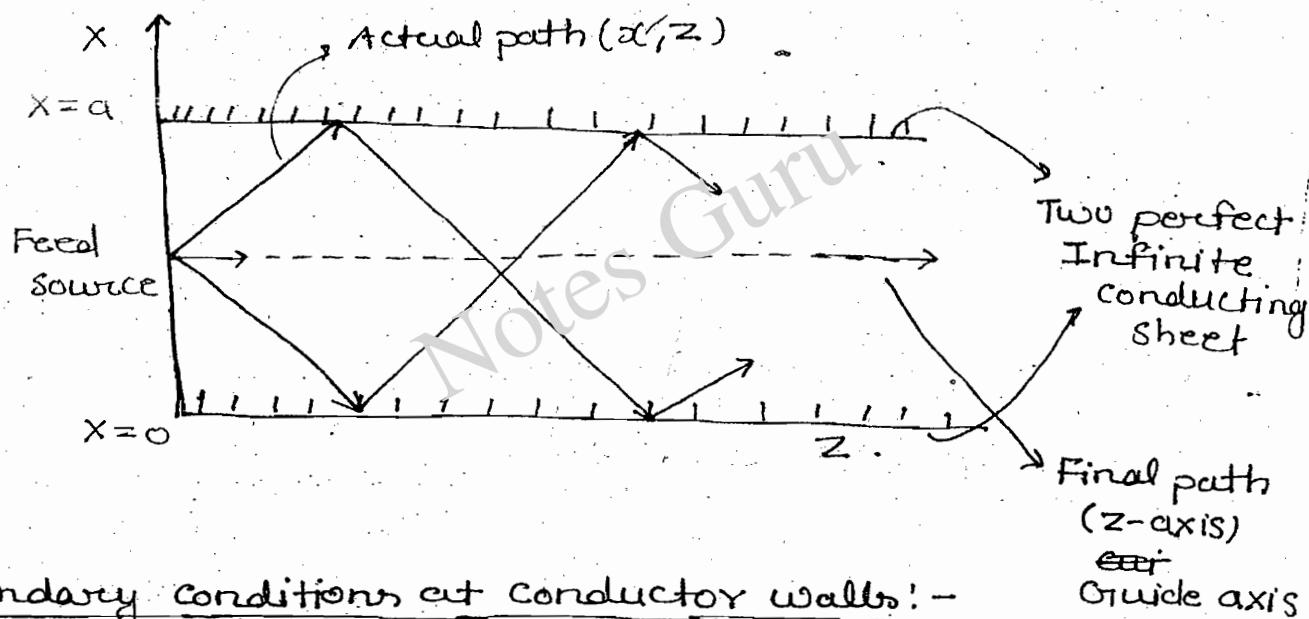
↓
Parallel plane waveguide

e.g:- earth and ionosphere guiding

$$\left. \begin{array}{l} E(x, y, z, t) \\ H(x, y, z, t) \end{array} \right\} \begin{array}{l} \text{2 dimension restriction in } x \\ \text{1 dimension propagation in } z \end{array}$$

↓
Rectangular Waveguide

Parallel Plane Waveguides :-



Boundary conditions at conductor walls:-

$$E_{tang} = 0 \text{ at } x=0, x=a$$

$$E(x)_{tang} = 0 \text{ at } x=0, x=a$$

$$E(x)_y \neq E(x)_z = 0 \text{ at Guide walls}$$

Using $\nabla^2 E = \gamma^2 E$ Helmholtz's Equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu \epsilon E$$

Let $E(z)$ be any natural harmonic $e^{-\gamma z}$ as the z side is un-restricted

where $\bar{\gamma} = \gamma_z$ = Propagation constant in guide axis.

$$\frac{\partial^2 E}{\partial x^2} + \bar{\gamma}^2 E = -\omega^2 \mu \epsilon E$$

$$\frac{\partial^2 E}{\partial x^2} = -(\gamma^2 + \omega^2 \mu \epsilon) E$$

$$\text{where } \bar{\gamma}^2 + \omega^2 \mu \epsilon = V_x^2$$

where V_x = Propagation constant in x side or restricted side

$$\frac{\partial^2 E}{\partial x^2} = -V_x^2 E$$

The $E(x)$ solution is also harmonic

$$E(x) = C_1 \sin(V_x x) + C_2 \cos(V_x x)$$

The restricted side propagation has to be trigonometric harmonic only

Applying the Boundary conditions

at $x=0$

$$E(0)_{\text{tang}} = 0 + C_2 = 0$$

$$\Rightarrow C_2 = 0$$

Note:-

The tangential E field harmonic has to be a 'sin' only in the restricted side

at $x=a$

$$E(a)_{\text{tang}} = C_1 \sin(V_x a) = 0$$

$$V_x = \frac{m\pi}{a}$$

Note:-

The restricted guide propagation constant can take only discrete solutions but not any continuous value

$$\text{Finally } \tilde{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon}$$

Concept 1:-

(f_c) cut off frequency of the guide

$$\text{If } \left(\frac{m\pi}{a}\right)^2 > \omega^2 \mu \epsilon$$

then $\tilde{\gamma} = \tilde{\alpha} + j\tilde{\beta}$. No propagation along the guide axis

$$\text{If } \omega^2 \mu \epsilon > \left(\frac{m\pi}{a}\right)^2$$

$$\omega > \frac{m\pi}{a\sqrt{\mu \epsilon}}$$

$$\text{then } \tilde{\gamma} = 0 + j\tilde{\beta}$$

The wave travels along the guide axis without attenuation.

$$\omega > \frac{m\pi c}{a}$$

Note:-

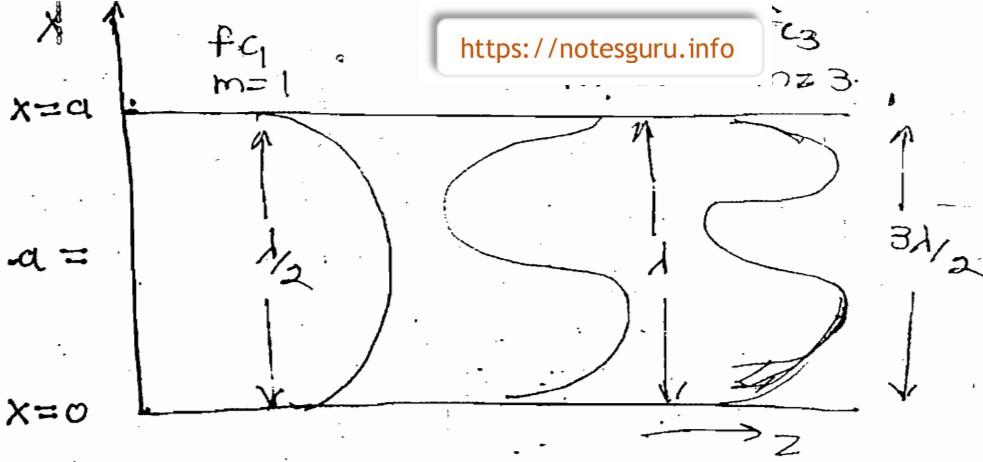
Every waveguide has a minimum cut-off frequency below which there cannot be propagation i.e. there is a max wavelength above which there cannot be propagation and this wavelength is comparable to the guide dimensions

$$\text{Hence } \omega_c > \frac{m\pi c}{a}, f_c > \frac{mc}{2a}, d_c > \frac{2a}{m}$$

$$\Rightarrow \omega_c = \frac{m\pi c}{a}$$

$$f_c = \frac{mc}{2a}$$

$$d_c = \frac{2a}{m}$$



At exact cut-off frequency where $\tilde{\gamma} = 0$, the wave resonates b/w guide walls and oscillates b/w the walls for 'm' such frequencies.

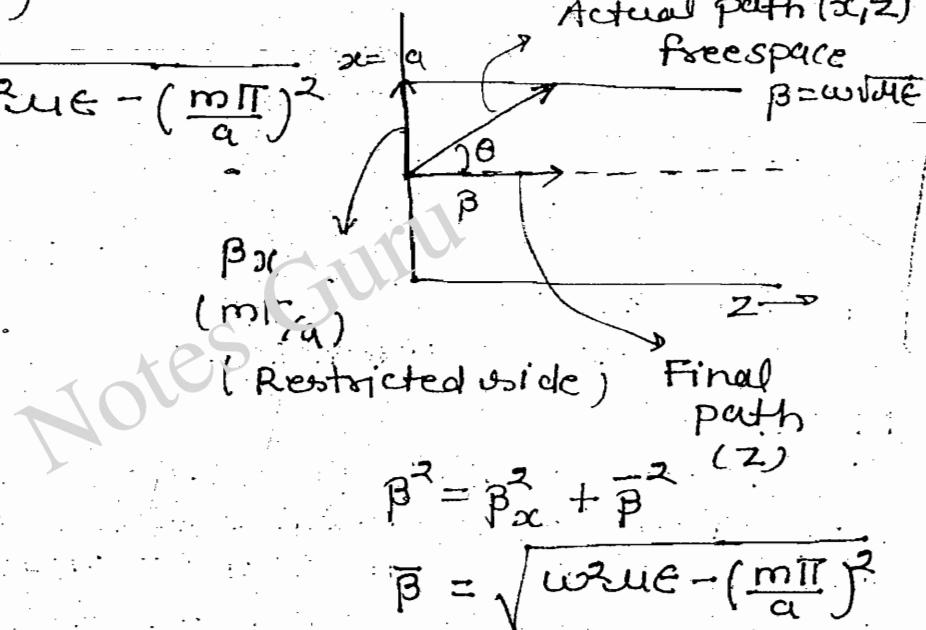
Concept 2 :-

Wave Angle or Tilt Angle :-

$$\tilde{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon}$$

$$= j \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$

$$\beta = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$



$$\beta^2 = \beta_x^2 + \beta^2$$

$$\bar{\beta} = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$

Phase Velocity

along the guide axis, $\bar{v}_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}}$

$$\bar{v}_p = \frac{1}{\sqrt{\mu \epsilon - \left(\frac{m\pi}{a\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{\mu \epsilon}} \cdot \frac{1}{\sqrt{1 - \left(\frac{m\pi}{a\omega\sqrt{\mu \epsilon}}\right)^2}}$$

$$\boxed{\bar{V}_p = \frac{c}{\sqrt{1 - (\frac{\omega_c}{\omega})^2}}}$$

$$\bar{\beta} = \beta \cos \theta$$

$$\frac{2\pi}{\lambda} = \frac{2\pi}{\lambda} \cos \theta$$

$$\Rightarrow \lambda = \frac{\lambda}{\cos \theta}$$

$$\Rightarrow \lambda f = \frac{df}{\cos \theta}$$

$$\boxed{\bar{V}_p = \frac{c}{\cos \theta}}$$

By comparison

$$\boxed{\sin \theta = \frac{f_c}{f}}$$

Note:-

Every frequency has a unique tilt angle and unique velocity inside the guide so that two different frequencies never overlap to each other

$$\bullet f_3 > f_2 > f_1 > f_c$$

Concept 3:-

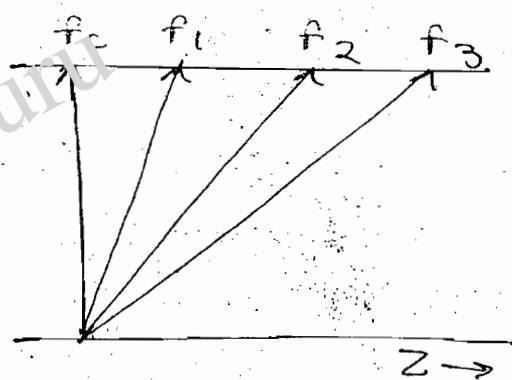
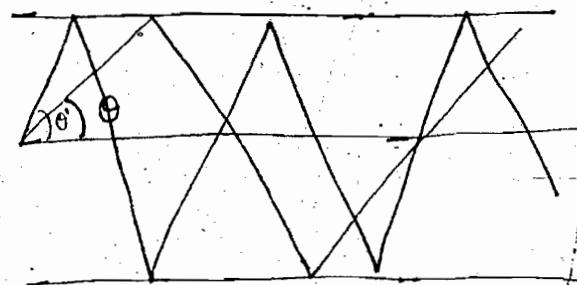
Group Velocity (\bar{V}_g)!-

$$\bar{V}_p = \frac{c}{\cos \theta} \Rightarrow \bar{V}_p > c \rightarrow \text{Always}$$

In linear conditions, $\beta \propto \omega$

velocity is phase velocity $V_p = \frac{\omega}{\beta}$

e.g!- Lossless EM Waves in free space
lossless VI Waves in transmission line



$$\beta = \omega \sqrt{\mu \epsilon}$$

$$\beta = \omega \sqrt{1/c}$$

But in dispersive condition β is not linear.

with ω

e.g:- $v_g = \frac{d\omega}{d\beta} = \text{Group velocity}$

$$\bar{\beta} = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2} \rightarrow \text{dispersive conditions}$$

β is not linear with ω

$$\begin{aligned} \frac{d\bar{\beta}}{d\omega} &= \frac{1}{\frac{2}{\omega} \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}} \cdot \cancel{2\omega \mu \epsilon} \\ &= \frac{\mu \epsilon}{\sqrt{\mu \epsilon - \left(\frac{m\pi}{a\omega}\right)^2}} \\ &= \frac{\sqrt{\mu \epsilon}}{\sqrt{1 - \left(\frac{m\pi}{a\omega \sqrt{\mu \epsilon}}\right)^2}} \\ &= \frac{1}{c \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}} \end{aligned}$$

$$\bar{v}_g = c \cos \theta$$

$$\bar{v}_g < c \rightarrow \text{Always}$$

$$\boxed{\bar{v}_g : \bar{v}_p = c^2} \rightarrow \text{Always}$$

Concept 4:-

Modes of Operation

Mode of Operation:-Note:-

Mode stands for physical connection of field which can be axial or longitudinal feed

→ The no. of field connection decides the integer m of that mode and thus m decides the cut-off freq of the mode

→ The mode can be identified from field such that m stands for no. of half cycles b/w the guide walls and no. of maxima b/w the guide walls.

Transverse Electric (TE), TM and TEM Modes: -

$$\begin{aligned} E(x, z, t) & \xrightarrow{(x, y, z)} \\ H(x, z, t) & \xrightarrow{(x, y, z)} \end{aligned}$$

Using Maxwell's equation

$$\nabla \times \vec{E} = -j\omega \mu H$$

$$\nabla \times \vec{H} = j\omega \epsilon E$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega \mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega \mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega \mu H_z$$

$$\left| \begin{array}{ccc} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{array} \right| = \nabla \times \vec{E}$$

→ All y derivatives are zero

→ All z derivatives are back the same function with $\sqrt{\epsilon}$ scaling

$$\nabla \times \vec{E} = -j\omega \mu H$$

$$\nabla \times \vec{H} = j\omega \epsilon E$$

$$\nabla E_y = -j\omega \mu H_x$$

$$\nabla H_y = j\omega \epsilon E_x$$

$$-\nabla E_x - \frac{\partial E_z}{\partial x} = -j\omega \mu H_y$$

$$-\nabla H_x - \frac{\partial H_z}{\partial x} = j\omega \epsilon E_y$$

$$\frac{\partial E_y}{\partial x} = -j\omega \mu H_z$$

$$\frac{\partial H_y}{\partial x} = j\omega \epsilon E_z$$

$$H_x = -\frac{\gamma E_y}{j\omega \mu}$$

$$-\nabla \left(-\frac{\gamma E_y}{j\omega \mu} \right) - \frac{\partial H_z}{\partial x} = j\omega t E_y$$

$$\gamma^2 E_y - j\omega \mu \frac{\partial H_z}{\partial x} = -\omega^2 \mu \epsilon E_y$$

$$(\gamma^2 + \omega^2 \mu \epsilon) E_y = j\omega \mu \frac{\partial H_z}{\partial x}$$

$$(I) \quad \frac{\partial H_z}{\partial x} = \frac{\gamma^2}{j\omega \mu} E_y$$

$$(II) \quad \frac{\partial H_z}{\partial z} = -\frac{\gamma^2}{\gamma} H_{dc}$$

$$(III) \quad \frac{\partial E_z}{\partial x} = -\frac{\gamma^2}{\gamma} H_y$$

$$(IV) \quad \frac{\partial E_z}{\partial z} = -\frac{\gamma^2}{\gamma} E_x$$

Note:-

The axially directed field components determine the complete standing of the wave.

If $E_z = 0 \rightarrow$ physical connection

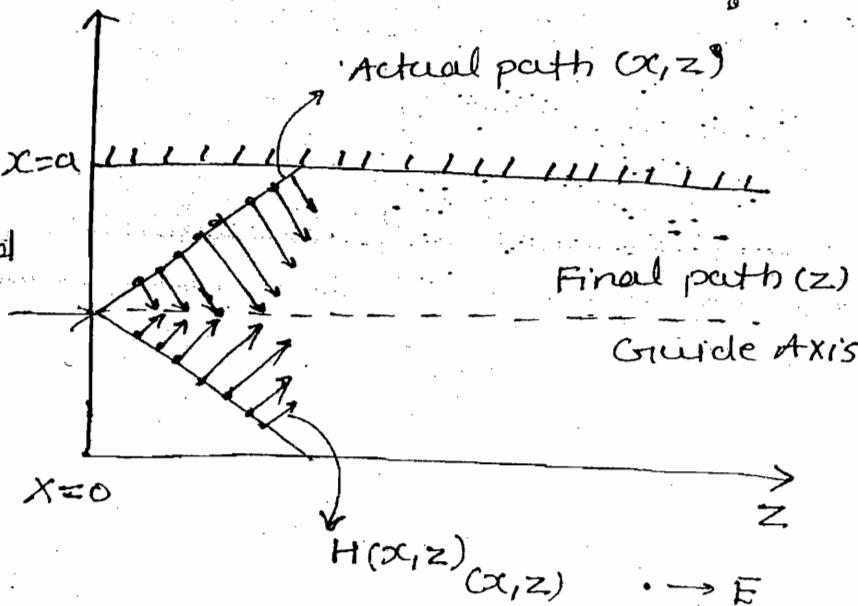
then $E_x = H_y = 0$ The waves becomes

$E(x, z, t)$

$H(x, z, t)$

$E \perp$ Guide Axis

The wave is called
as TE wave



If $H_z = 0$, then
the wave becomes $H(x, z, t)y$
 $E(x, z, t)_{x, z}$

\rightarrow H \perp Guide Axis

The wave is called as TM Waves

E \perp H \perp Propogation

TF Wave solutions in Parallel - Plane Waveguides: $\rightarrow H_y$

The waves has

$$(E_x = E_z = H_y = 0)$$

$$E(x, z, t)_y = E_{yo} \cdot \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma z} e^{j\omega t} \text{ dy}$$

$$E(x, z, t)_x = H_{xo} \cdot \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma z} e^{j\omega t} \text{ ax}$$

$$H(x, z, t)_z = H_{zo} \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z} e^{j\omega t} \text{ az}$$

It is a product (Ans) solution of time, x & z

Harmonics

$$\frac{E(t)}{H(t)} \longrightarrow e^{j\omega t} \text{ Source harmonic}$$

$$\frac{E(z)}{H(z)} \longrightarrow e^{-\gamma z} \text{ Natural Harmonic}$$

$$\frac{E(x)}{H(x)} \longrightarrow \text{Trigonometric Harmonic}$$

$$E(x)_{\text{tang}} \longrightarrow \sin' \text{ Harmonic} \quad \frac{\delta H_z}{\delta x} = () H_x$$

$$E(x)_y \text{ or } E(x)_z \quad \uparrow \quad = () E_y$$

If $m=0$ the TF wave does not exist and
hence $m=1$ is the least value required for
propogation

$$(H_z = H_x = E_y = 0)$$

$$H(x, z, t)_y = H_{y0} \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z} e^{j\omega t} e^{j\theta}$$

$$E(x, z, t)_x = E_{x0} \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z} e^{j\omega t} e^{j\theta}$$

$$E(x, z, t)_z = E_{z0} \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma z} e^{j\omega t} e^{j\theta}$$

If $m=0$, $E_z = 0$

The wave becomes $E(z, t)_x$

$$H(z, t)_y$$

This wave is called as ~~TEM waves~~ TEM Waves with $E_z = H_z = 0$

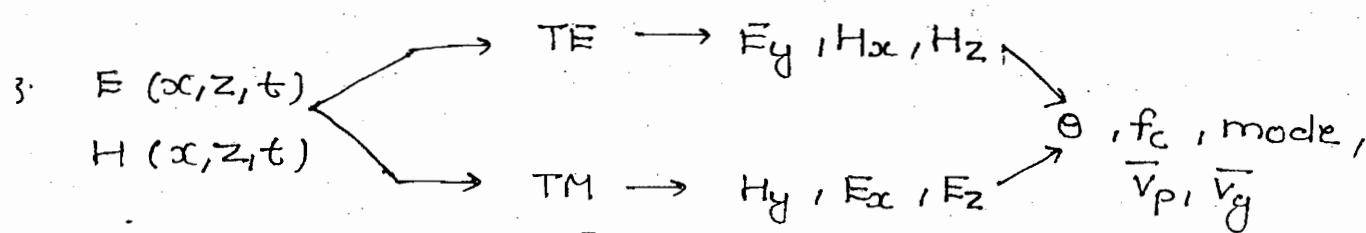
Properties of TEM Waves! -

- I) It has $m=0$ $\bar{\gamma} = \gamma = j\omega\sqrt{\mu\epsilon}$
 - i.e. Propogates only along guide axis's i.e. $\theta=0$ \rightarrow Always
- II) It has $f_c = 0$ i.e. No cut off frequency
- III) It has $m=0$ i.e. No multi-mode connection mechanism

Summary:-

$E(x, t)$ \rightarrow Wave at f_c , $\theta = 90^\circ$, $\bar{\gamma} = 0$
 $H(x, t)$ ($f_{c_1}, f_{c_2}, \dots, f_{cm}$ exists)

$E(z, t)_x$ \rightarrow TEM wave, $\theta = 0^\circ$ $\bar{\gamma} = \gamma = j\omega\sqrt{\mu\epsilon}$
 $H(z, t)_y$ $m=0$, $f_c = 0$



Waveguides (Workbook)

<https://notesguru.info>

I) $f = 22 \text{ GHz}$ $\sin\theta = \frac{f_c}{f}$ $t_c = \frac{mc}{2a}$

$$f_c = \frac{1.36 \times 10^8}{2.25 \times 10^{-2}} = 6 \text{ GHz}$$

1st Mode $6 \text{ GHz} \rightarrow \infty$

1st Mode $\sin\theta = \frac{6}{22}$

1st Mode $12 \text{ GHz} \rightarrow \infty$

III Mode $\sin\theta = \frac{18}{22}$

IIIrd Mode $18 \text{ GHz} \rightarrow \infty$

II) $f = 40 \text{ GHz}$

$$f_c = 36 \times 10^9 = \frac{3.3 \times 10^8}{2 \times a} \Rightarrow a = 1.25 \text{ cm}$$

III) $\sin\theta = \frac{36}{40} = \frac{f_{c3}}{f_3} = \frac{f_{c7}}{f_7} = \frac{84}{f_7}$

3rd Mode $\rightarrow 36$

7th Mode $\rightarrow 84$

$$f_7 = 93.3 \text{ GHz}$$

IV) $f = 20 \text{ GHz}$

$$f_c = \frac{1.3 \times 10^8}{\sqrt{9} \cdot 2 \times 3 \times 10^{-2}}$$

$$= \frac{5}{3} = 1.67 \text{ GHz}$$

1st $\rightarrow 1.67 \rightarrow \infty$
2nd $\rightarrow 3.34 \rightarrow \infty$

$$\sin\theta = \frac{5/3}{2} = \frac{5}{6}$$

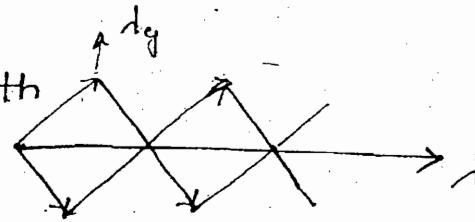
Note:-

1. $\vec{E}, \vec{B}, \vec{J}, \vec{V_p}, \vec{V_g}$, TRANSVERSE \rightarrow w.r.t guide axis

$$\eta_{TE} = \frac{E_{trans}}{H_{trans}} = \frac{E_y}{H_x} = \frac{E_{total}}{H_{total} \cos\theta} = \frac{120\pi}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$n_{TM} = \frac{E_{0c}}{H_0} = \frac{1}{\sqrt{1 + \left(\frac{f_c}{f}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{\lambda_c}{\lambda}\right)^2}}$$

$$\lambda = \frac{\lambda}{\cos\theta} = \lambda_g = \text{Guide Wavelength}$$



$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda_c}{\lambda}\right)^2}}$$

$$1 - \left(\frac{\lambda}{\lambda_c}\right)^2 = \left(\frac{\lambda}{\lambda_g}\right)^2$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$$

$$\frac{c}{f}$$

$$\frac{2\pi}{\lambda}$$

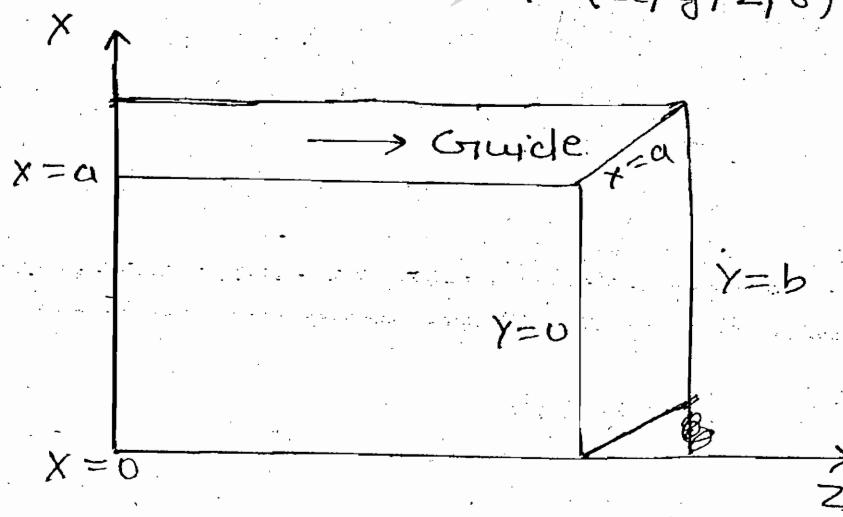
free space wavelength

cut-off wavelength

Rectangular Waveguides:-

The waves are $E(x, y; z, t)$, $H(x, y; z, t)$

$$E(x, y; z, t) \quad H(x, y; z, t)$$



→ It is a single conductor structure used to confine EM wave in 2 dimension using 4 walls

$$x=0 \quad x=a$$

$$y=0 \quad y=b$$

$$\text{It has } V_x = \frac{m\pi}{a}$$

$$V_y = \frac{n\pi}{b}$$

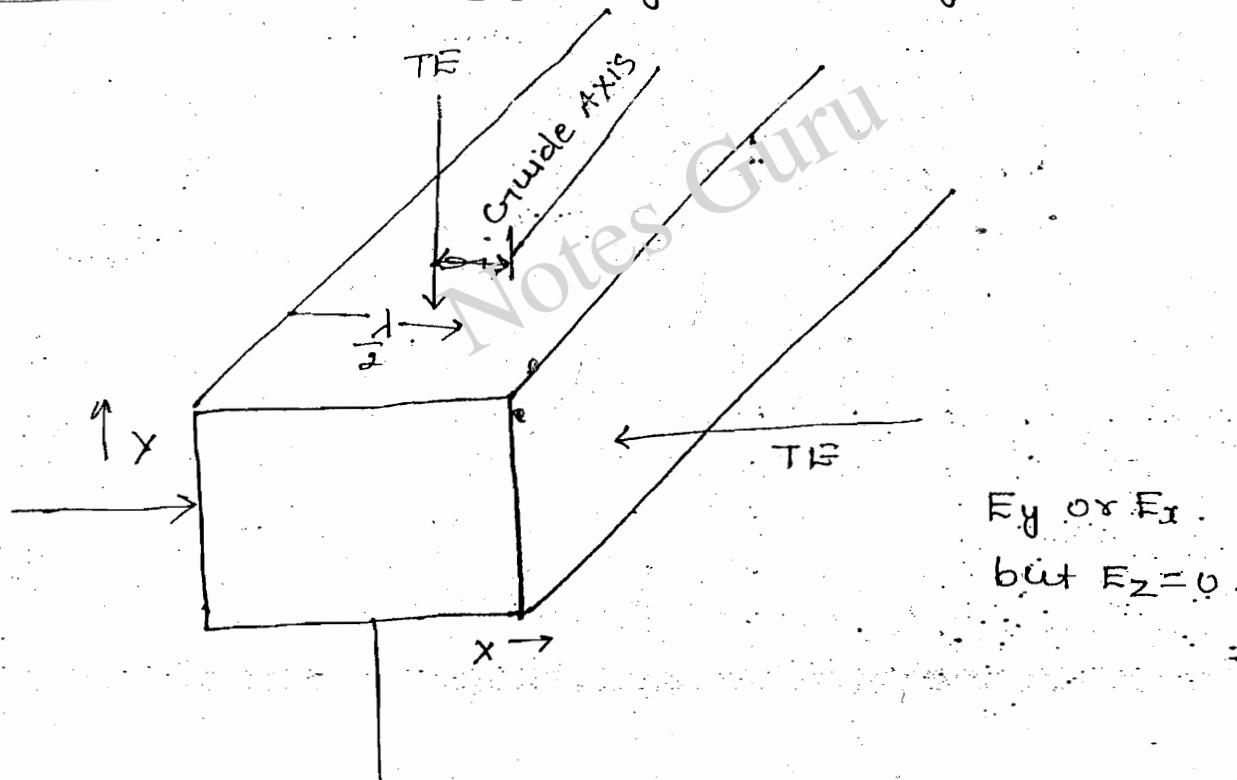
Applying $\nabla^2 E = -\omega^2 \mu \epsilon E$

$$\bar{V} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \omega \sqrt{\mu \epsilon}$$

$$\omega_c = \text{cut-off frequency} = \left(\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \right) c$$

$$f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \right) \frac{c}{2}$$

Modes and Feeds in Rectangular Waveguides:-



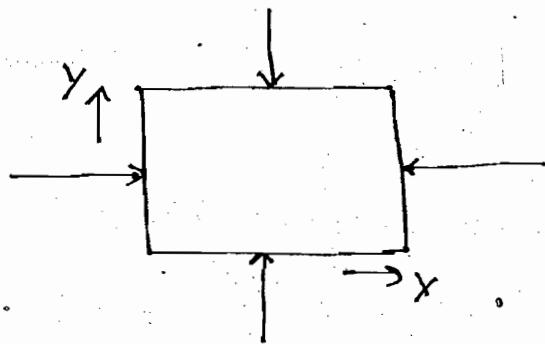
A horizontal or vertical field always gives a suitable E field but not along the guide axis

The no. of out of phase connections decides the integers m and n.

The modes are always designated as TE_{mn}
 $H_x \neq H_y$ but $H_z = 0$

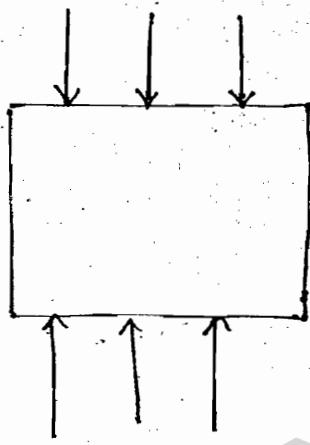
A lateral or axial feed always gives a suitable H field but not along the guide axis

(I)



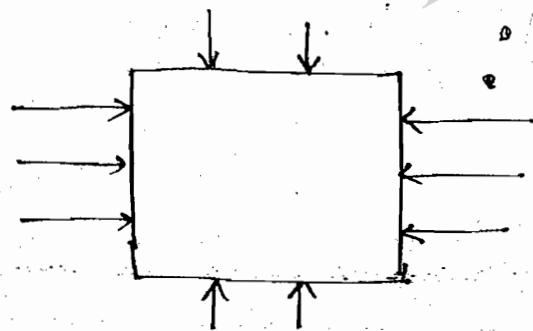
TE_{11}

(II)



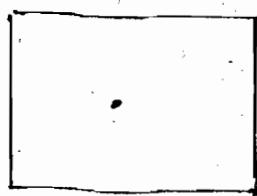
TE_{03}

(III)

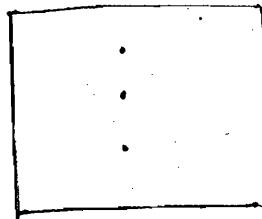


TE_{32} TE_{23}

(IV)



TM_{11}



TM₁₃

(v)



TM₄₂

Note:-

TM₀₀ or TM_{0N} modes are even overt and do not exist in rectangular waveguides.

Such a connection mechanism doesn't exist.

$$(i) f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \right) \frac{c}{2}$$

For both TE & TM are same. So TE₇₄, TM₇₄ are same cut-off called degenerate mode.

Two different modes having the same f_c but different connection mechanism are said to be degenerate modes.

(ii) For m=1, n=0

or m=0, n=1

i.e. TE₁₀ & TE₀₁ mode have the least f_c.

$$\left. \begin{aligned} f_c &= \frac{c}{2a} \\ (TE_{10}) & \\ f_c &= \frac{c}{2b} \end{aligned} \right\}$$

If a>b TE₁₀ is dominant mode

a<b TE₀₁ is dominant mode

- Broadside dimension decide dominant mode
- Narrow side dimensions decides the maximum operable voltage and power handling ability

TM Wave Solutions in Rectangular Waveguide ($\text{Hz} = 0$):-

The wave is $E(x, y, z, t)_{(x, y, z)}$

$H(x, y, z, t)_{(x, y)}$

$$E(x, y, z, t)_z = E_{z0} \cdot \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} e_z$$

$$E(x, y, z, t)_x = E_{x0} \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} e_x$$

$$E(x, y, z, t)_y = E_{y0} \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} e_y$$

$$H(x, y, z, t)_x = H_{x0} \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} e_x$$

$$H(x, y, z, t)_y = H_{y0} \left(\frac{n\pi}{a}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} e_y$$

It is a product solution of x, y, z, t harmonics

$$\frac{E(t)}{H(t)} \rightarrow e^{j\omega t} \rightarrow \text{Source harmonic}$$

$$\frac{E(z)}{H(z)} \rightarrow e^{-\gamma z} \rightarrow \text{Natural Harmonic}$$

$$\frac{H}{E(x, y)} \rightarrow \text{Trigonometric harmonic}$$

$$\left. \begin{array}{l} E(x, y) \text{ or } E(x, z) \\ E(y, x) \text{ or } E(y, z) \end{array} \right\} \text{Tangential harmonic} \left. \begin{array}{l} \sin \\ \text{Harmonics} \end{array} \right\}$$

If $m=0$ & $n \neq 0$ or $m \neq 0$ & $n=0$

TM waves do not exist i.e. they are even sheet modes

E Wave Solutions in Rect

<https://notesguru.info>

wide ($E_z = 0$): -

The wave is $E(x, y, z, t)_{(x, y)}$

$H(x, y, z, t)_{(x, y, z)}$

$$H(x, y, z, t)_z = H_{zo} \cdot \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_z$$

$$E(x, y, z, t)_x = E_{xo} \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$E(x, y, z, t)_y = E_{yo} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_y$$

$$H(x, y, z, t)_x = H_{xo} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$H(x, y, z, t)_y = H_{yo} \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_y$$

If $m \neq 0$ & $n \neq 0$

The TE_{m0} wave exists $E(x, z, t)_y$
 $H(x, z, t)_{(x, z)}$

If $m=0$ & $n \neq 0$

The TE_{0n} wave exists $E(y, z, t)_x$
 $H(y, z, t)_{(y, z)}$

If $E_z = H_z = 0$ the wave cannot exist

or $m=n=0$

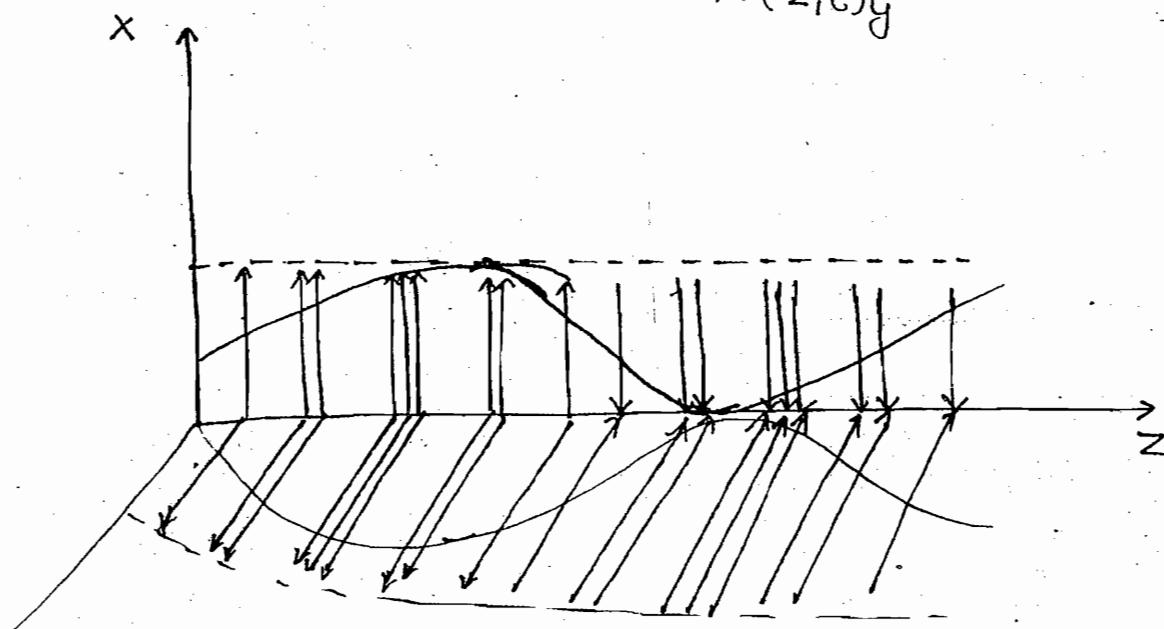
i.e. TEM waves do not exist in rectangular waveguide.

In general TEM waves do not exist in all single conductor guides

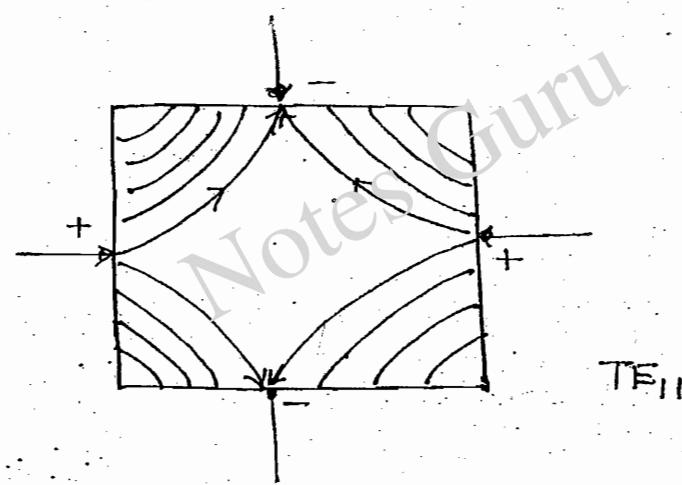
TEM needs two distinct conductor for its existence.

(i) Uniform plane Wave - $E(z, t)_x$

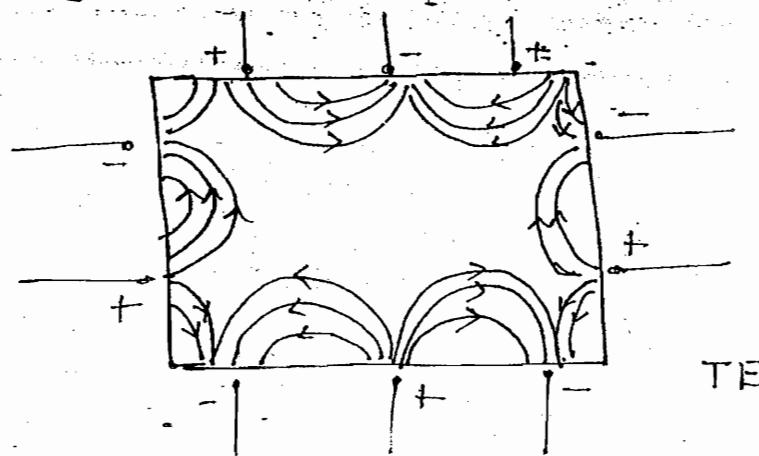
$$H(z, t)_y$$



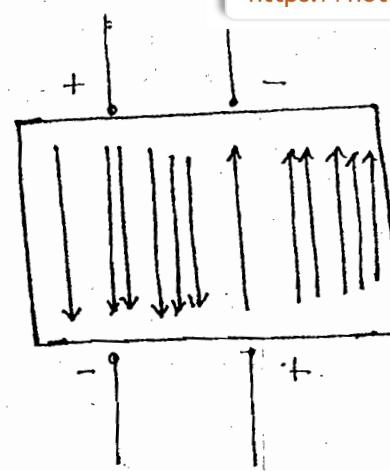
TE Waves! - (E_x, E_y)



TE₁₁

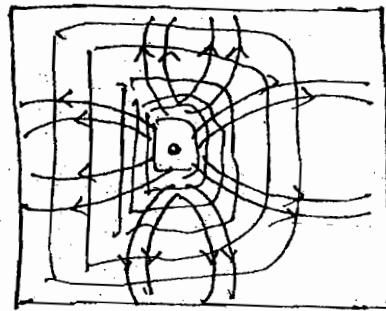


TE₃₂



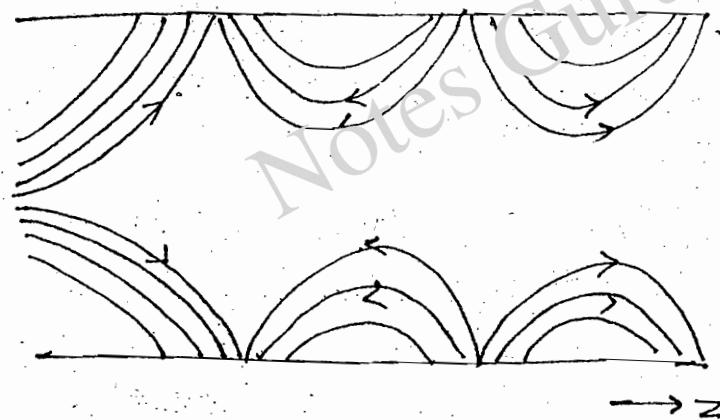
TE_{20}

TM Waves (E_x, E_y, E_z) :-

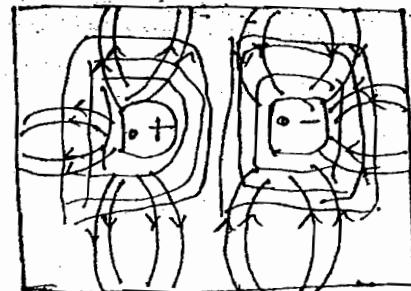


TE

TM_{11}



TM₂₁ :-



TM_{21}

With a guide axis on either side and both E & H fields traced on the board the wave is called as TEM waves else TE or TM.

e.g:- Co-axial cable and all other low frequency transmission line. So TEM waves for low frequency energy transfer format.

TM/TE \rightarrow High frequency transmission waveguide

Workbook!-

$$\frac{1}{R} = \frac{1}{d_c^2} + \frac{1}{d_g^2}$$

$$\frac{3 \times 10^8}{2.5 \times 10^9} = 12 \text{ cm}$$

TE₁₀ $\rightarrow 2a = 20 \text{ cm}$

$$d_g =$$

$$f_c = \frac{10 \times 10^9}{2 \times a} = \frac{1 \times 3 \times 10^8}{2 \times a}$$

$$a = 1.5 \text{ cm}$$

$$a \cdot b > 2$$

$$a > b$$

$$6. f_c = \frac{1 \times 3 \times 10^8}{\sqrt{4 \times 2 \times 3 \times 10^{-2}}} = 2.5 \text{ GHz}$$

$$7. v_p > c$$

$$8. n_{TE} = \frac{120\pi}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{377}{\sqrt{1 - \left(\frac{90}{30}\right)^2}} = 400 \Omega$$

$$f_c = \frac{c}{2a} = 10 \text{ GHz}$$

$$E(x, z, t)_y$$



Not depends on y

$$\Rightarrow n=0$$

$$TE_{m0}$$

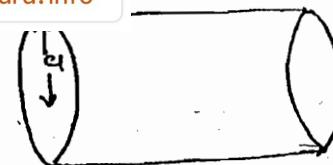
$$TE_{20}$$

$$\sin\left(\frac{2\pi}{4}x\right)$$

$$\sin(wt - \beta z)$$

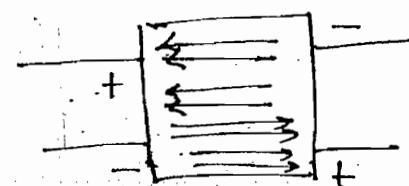
11. TEM exists \rightarrow single mode

(B.)



12. No cut off freq. \rightarrow TEM exists Ans-A

13. TE₀₂



$$14. f_c = \frac{3 \times 10^8}{2 \times 2 \times 10^{-2}} = \frac{3 \times 10^8}{2 \times \sqrt{\epsilon_r} \times 1 \times 10^{-2}} \Rightarrow \epsilon_r = 4$$

15. Note:- Parallel waveguide

TEM $\rightarrow (0 - \infty)$

TE₁₁/TM₁₁ $\rightarrow (f_1 - \infty)$

TE₂₁/TM₂₁ $\rightarrow (f_{c_2} - \infty)$

$$15. \left. \begin{array}{l} \textcircled{1} m=1 n=0 \\ \textcircled{2} m=0 n=1 \\ \textcircled{3} m=1 n=1 \end{array} \right\} f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \right) \frac{c}{2}$$

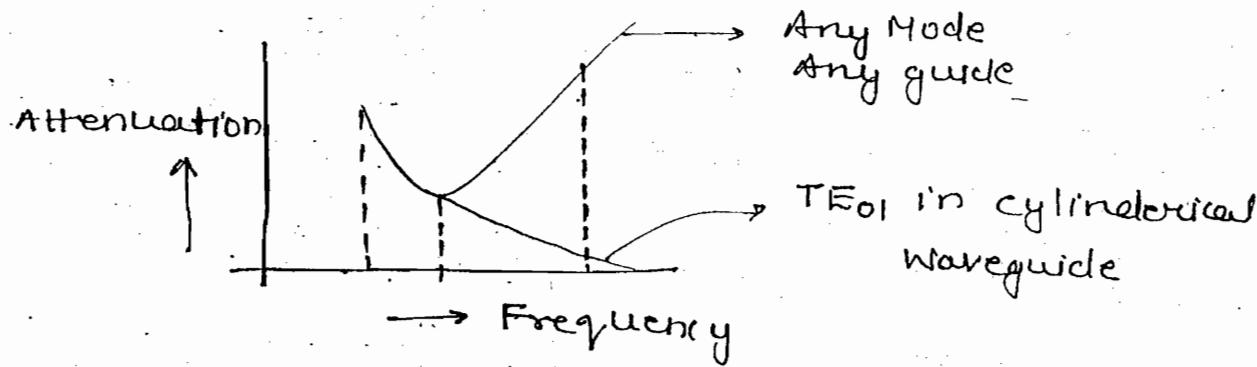
4x3 cm

$$\textcircled{1} \rightarrow TE_{10} \quad f_{c_1} = \frac{c}{2a} = 3.75 \text{ GHz} \quad (3.75 \rightarrow \infty)$$

$$TE_{01} \quad f_{c_2} = \frac{c}{2b} = 5 \text{ GHz} \quad (5 \text{ GHz} \rightarrow \infty)$$

$$\frac{TE_{11}}{TM_{11}} \quad f_{c_3} = 6.25 \text{ GHz} \quad (6.25 \text{ GHz} \rightarrow \infty)$$

From (3.75-5) GHz there is strictly single mode operation.

Practically Graph :-

With practical non-ideal conducting walls, higher freq. are not preferred in lower modes

16.

→ D

7.

→ TE₁₀ only

$$f_c < f < f_C$$

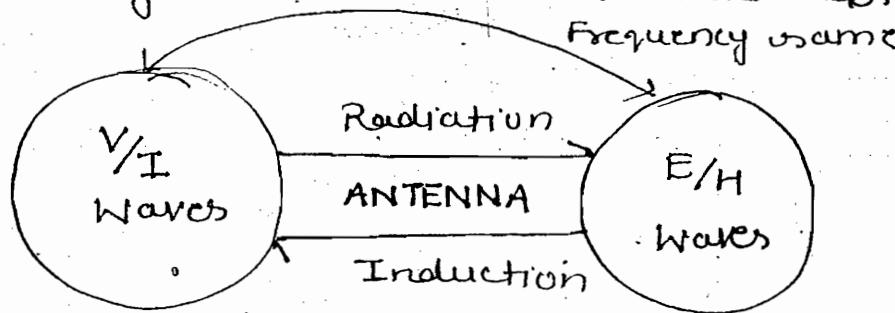
TE₁₀TE₀₁

$$\frac{c}{2a} < \frac{c}{\lambda} < \frac{c}{2b}$$

$$2a > \lambda > 2b$$

ANTENNAS

- Hertzian Dipole / Halfwave Dipole
- Basic Terms and Definitions → 50
- Antenna Arrays
- FRIIS - Free Space Propogation
- Study / classification of Antennas



Hertzian Dipole as a Basic Radiating Element :-

It is a dL length $I(t) = I_m \sin \omega t$ (oscillatory or Harmonic) current element with $dL \ll \lambda$

$$(I) A(t) = \frac{\mu I(t) dL}{4\pi r}$$

$$(II) B(t) = \nabla \times A$$

$$(III) \nabla \times H = \epsilon \frac{dE}{dt} \quad (IV) E(t) = \frac{1}{\epsilon} \int (\nabla \times H) dt$$

Every harmonic current produces a time / space harmonic E & H around it. Thus EM wave generation is called as radiation

Expression for Radiated fields of Hertzian Dipole:-

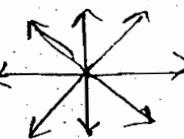
$$\left. \begin{aligned} \text{radiated} \\ \text{wave} \end{aligned} \right\} \begin{aligned} E(x, \theta, \phi, t)_\theta &= \left(\frac{I_m dL \cdot \sin \theta \cdot \omega}{4\pi \epsilon c^2 x} \right) \sin \omega t \cdot e^{-j\beta x} a_\theta \\ H(x, \theta, \phi, t)_\phi &= \left(\frac{I_m dL \sin \theta \omega}{4\pi c x} \right) \sin \omega t \cdot e^{-j\beta x} a_\phi \end{aligned}$$

$$W \rightarrow E(z, t)_x = E_x$$

Properties of Radiated Waves:-

I) They travel radially outward and hence their amplitudes decrease as $\frac{1}{r}$ due to their power density decreases as $\frac{1}{r^2}$

→ Amplitude dec. but does not attenuate.



$$E_\theta \times H_\phi = P_r$$

$$\frac{E_\theta}{H_\phi} = \frac{1}{\epsilon c} = \frac{1}{\epsilon \sqrt{\mu \epsilon}} = \sqrt{\frac{\mu}{\epsilon}} = 120\pi \Omega$$

II) The amplitude of the radiated wave is not omni-directional and depends on θ and ϕ
i.e. Radiation is directive in nature

III) The amplitude of the radiated wave always depends on $(\frac{d}{\lambda})$ ratio
i.e. frequency decides radiated power

Total Power Radiated from a Hertzian Dipole:-

$$W_r = \oint P_{avg} ds$$

$$= \oint \frac{1}{2} \frac{E_0^2}{\eta} a_r \cdot ds \cdot a_r$$

Sphere

$$W_r = \int_0^{\pi} \int_0^{2\pi} \left(\frac{Im \mu S \sin \theta \omega}{4\pi \epsilon c^2 r} \right)^2 \frac{1}{120\pi} r^2 \sin \theta d\theta d\phi$$

$\theta = 0 \quad \phi = 0$

$$\left(\omega = \frac{2\pi c}{\lambda} \right)$$

$$W_r = I_{rms}^2 \cdot 80\pi^2$$

The expression is similar to a resistor dissipating power as heat i.e. every antenna radiates power in the form of EM wave.

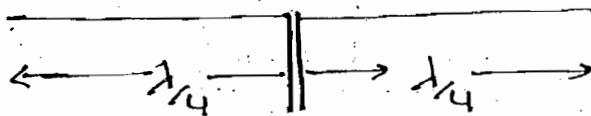
R_r = Radiation resistance of Hertzian Dipole

$$= 80\pi^2 \left(\frac{dl}{d} \right)^2$$

→ R_r is a measure of radiated power for a given input current i.e. it should be as possible for a practical antenna.

Halfwave dipole as a fundamental Antenna!

→ A centre feed $\lambda_{1/2}$ -dipole is a transmission opened out by λ_4 on either sides



→ The length of antenna strictly depends on the frequency of operation

FM

$$f = 100 \text{ MHz}$$

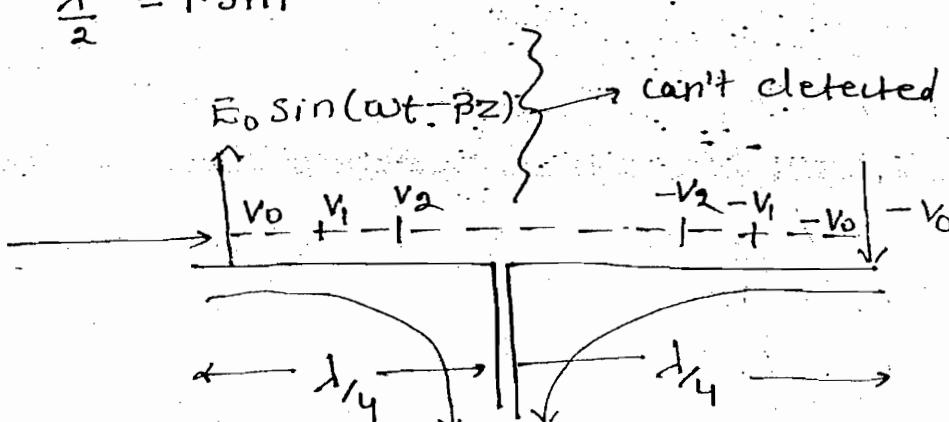
$$\lambda = 3 \text{ m}$$

$$\frac{\lambda}{2} = 1.5 \text{ m}$$

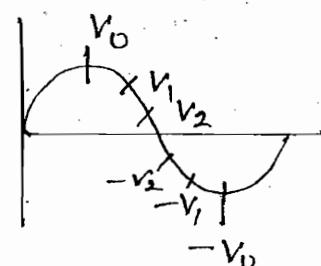
GISM

$$f = 1.800 \text{ MHz}$$

$$\lambda = \text{few cm}$$



100



When an EM waves having right frequency and right polarization travels along the axis of the antenna, it induced V_0 voltage at one end and progressively decrease voltage such that it is $-V_0$ at the other end. Hence the word half wave dipole.

These voltages drive a current towards centre which is maximum in the line such that

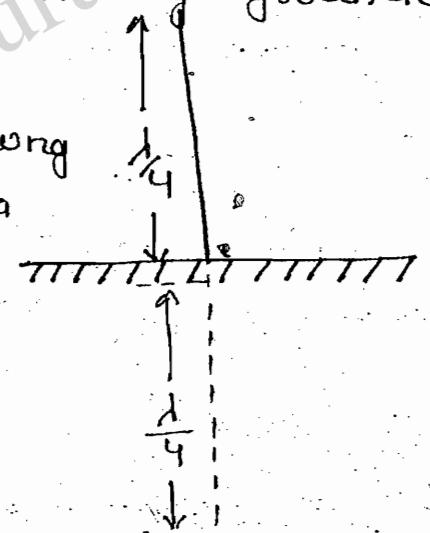
$$\text{Assymptotic} \left\{ \begin{array}{l} |V(z)| = V_0 \sin \beta z \\ |I(z)| = I_0 \cos \beta z \end{array} \right. \rightarrow z = -\frac{\lambda}{4} \text{ to } \frac{\lambda}{4}$$

V/I distribution

* $V_0 = -ve$ at other end b.c. of phase diff = 180°

Halfwave Monopole as a Practical Antenna !

- It is a low frequency vertically grounded single wire of $\lambda/4$ length
- It is a base feed and along with its image works like a harmonic dipole.



Summary !

A halfwave dipole is an array of Hertzian dipoles with dL from $-\lambda/4$ to $\lambda/4$ and

$$I_m = |I(z)| = I_0 \cos \beta z$$

Assymptotic

$$E(r, \theta, \phi, t)_\theta = \left(\frac{60 I_0}{\gamma} \frac{\cos(\frac{\pi}{2} \cos\theta)}{\sin\theta} \right) \sin\omega t \cdot e^{-j\beta r} a_\theta$$

$$H(r, \theta, \phi, t)_\phi = \left(\frac{I_0}{2\pi\gamma} \frac{\cos(\frac{\pi}{2} \cos\theta)}{\sin\theta} \right) \sin\omega t \cdot e^{-j\beta r} a_\phi$$

Total power radiated from Halfwave Dipole

$$W_r = \int \frac{1}{2} \frac{E_0^2}{n} \cdot dr$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{2} \left(\frac{60 I_0}{\gamma} \frac{\cos(\frac{\pi}{2} \cos\theta)}{\sin\theta} \right)^2 \frac{1}{120\pi} r^2 \sin\theta d\phi dr$$

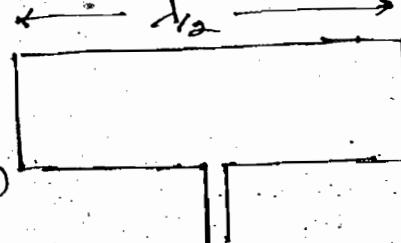
$$= I_{rms}^2 (73)$$

R_r for a Halfwave Dipole = 73Ω

R_r for Quarterwave monopole = 36.5Ω

R_r for a folded dipole = $2^2 \times 73\Omega = 292\Omega$

R_r for n-dipoles = $n^2 \times 73$



Basic Terms and Definitions:-

a) Isotropic Antenna:-

It radiates power in all directions uniformly. Its E field pattern is independent of θ and ϕ .

Eg:- Broadcast antenna

(ii) Radiation Power Density

<https://notesguru.info>

It is the strength of the radiated EM wave in any direction at any distance from the antenna

$$\frac{\text{Power}}{\text{Area}} \text{ or } \text{Watts/m}^2 = \frac{dW_r}{ds}$$

= Poynting vector of EM wave

$$= \frac{1}{2} \frac{E_0^2}{\eta} (\gamma, \theta, \phi)$$

$$= V(\gamma, \theta, \phi)$$

mp

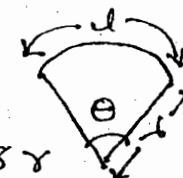
III) Radiation Power Intensity :-

It is the strength of the radiated EM wave in any direction from the antenna

$$\frac{\text{Power}}{\text{direction}} = \frac{\text{Power}/\text{solid angle}}$$

$$= \text{with solid angle} = \frac{dW_r}{ds\Omega}$$

$$\Omega = \theta r$$



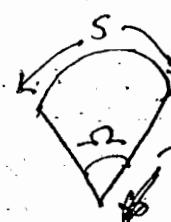
$$\text{If } \theta = 1 \text{ radian then } \Omega = r$$

$$\text{If } \theta = 6.28 \text{ radian then } \Omega = 6.28 r$$

$$\Omega = \pi r^2$$

$$\pi r^2 = 1 \text{ steradian}$$

$$r^2 = \gamma^2$$



$$\pi \gamma^2 = 12.56 \text{ steradian}$$

$$\text{Total surface area} = 12.56 \gamma^2$$

Any small, incremental

$$\frac{r^2 d\Omega}{d\phi} = r^2 \sin\theta \cdot d\phi$$

$$d\Omega = \sin\theta d\theta d\phi$$

$$\frac{dW_r}{d\Omega} = \frac{dW_r}{ds} \cdot \frac{ds}{d\Omega}$$

$$= V(r, \theta, \phi) r^2 = \Psi(\theta, \phi)$$

eg:- Isotropic Antenna

$$\frac{dW_r}{ds} = \frac{W_r}{4\pi r^2} = V_{avg}$$

$$\frac{dW_r}{d\Omega} = \frac{W_r}{4\pi} = \Psi_{avg}$$

IV) Gain of an Antenna:-

G_D → Directive gain

G_P → Power gain

β → Directivity

(a) Directive Gain (G_D):-

The radiation intensity of the antenna in a given direction to the radiation intensity of isotropic ~~no~~ antenna

$$G_D = \frac{\Psi(\theta, \phi)}{\Psi_{avg}} = \frac{4\pi \Psi(\theta, \phi)}{W_r} = \frac{4\pi \Psi(\theta, \phi)}{\int \Psi(\theta, \phi) d\Omega}$$

$G_D > 1$ or $G_D < 1 \rightarrow$ depends on direction

(b) Power Gain:-

$$G_P = \frac{4\pi \cdot \Psi(\theta, \phi)}{W_N}$$

$$W_r = \text{Total Power}$$

$$W_N = \text{Total I/p power}$$

$$G_{IP} = \frac{4\pi \Psi(\theta, \phi)}{W_r} \cdot \frac{W_r}{W_N}$$

= $G_{IP} \times \text{Efficiency of Radiation}$

$$\text{Efficiency} = \frac{W_r}{W_r + W_{NL}} = \frac{R_r}{R_r + R_{NL}}$$

$$\text{Directivity} = G_{IP}|_{\max}$$

$$\boxed{\theta \geq 1} \rightarrow \text{Always}$$

$\theta = 1$ for isotropic antenna

Radiation Pattern of an Antenna:-

It is a polar plot of radiation intensity indicating various directions around the antenna where radiation is finitely strong.

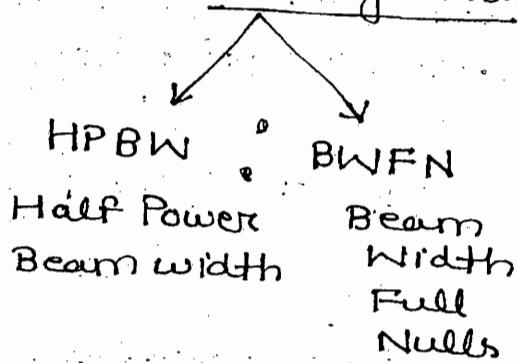
$$\text{eg:- } |E| = \frac{k \sin \theta}{r}$$

$$\Psi = \Psi_0 \sin^2 \theta$$

$$\theta = 0^\circ / 180^\circ$$

$|E| = 0 \rightarrow \text{Null points}$

$\rightarrow \Theta_{NP}$



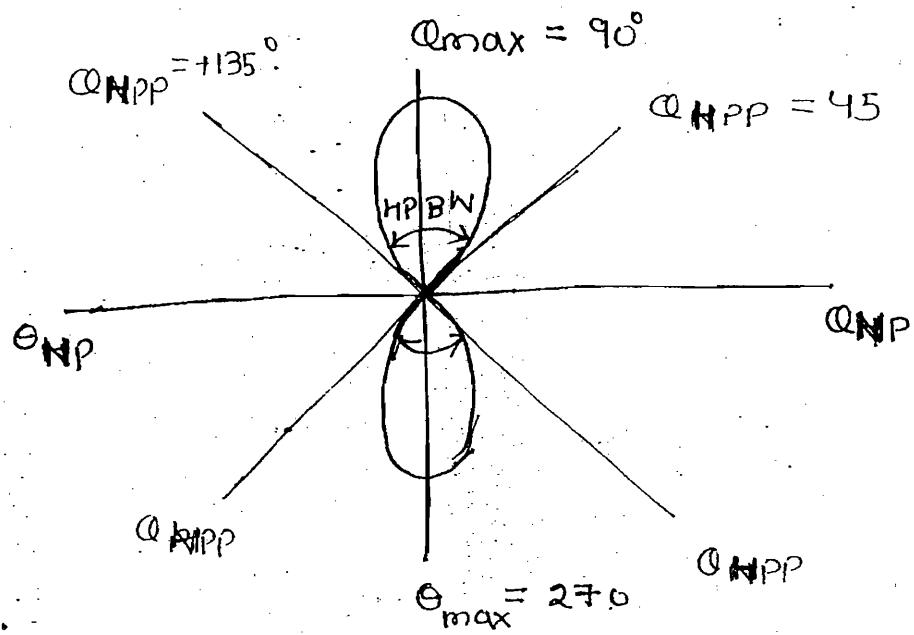
$$\theta = 90^\circ / 270^\circ$$

$$5^\circ / 135^\circ / 225^\circ / 315^\circ$$

$$|E| = E_{\max}$$

$$|E| = \frac{E_{\max}}{\sqrt{2}}$$

$\rightarrow \theta_{\max}$



$\theta_{HPBW} = HPP$ i.e. next HPP
in the maxima

(Half power pt.)

$$= 135^\circ - 45^\circ = 90^\circ$$

The antenna considered has a ϕ independent pattern and hence a circular top view of the beam (for all ϕ)

In general

$$\theta \times \phi = \Omega_A \text{ Beam solid angle}$$

$HPBW = HPBW$

$$= \text{steradian} = \text{radian}^2$$

In circular

$$\left(\frac{\theta}{HPBW}\right)^2 = \Omega_A$$

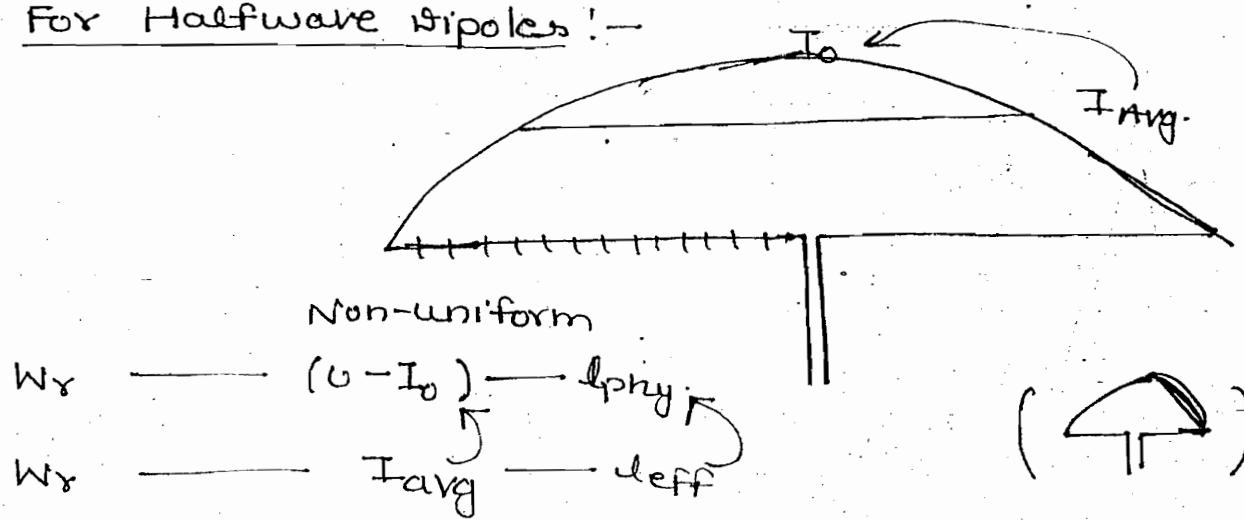
101

$$\omega \propto \frac{1}{\Omega_A}$$

$$\Rightarrow \boxed{\omega = \frac{4\pi}{\Omega_A}}$$

(VI) Effective length of an antenna:-

(a) For Halfwave dipoles:-



Note!:-

→ An antenna radiates W_R power over its physical length and non-uniform currents everywhere. Effective length is a length required to radiate same power assuming uniform currents everywhere.

$$I_{avg} = \frac{1}{\pi} \int_0^{\pi} I_0 \sin t dt = \frac{2I_0}{\pi}$$

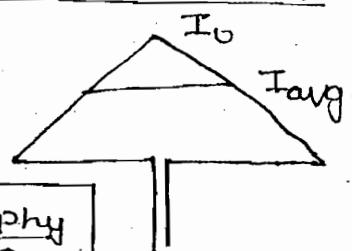
$$\boxed{I_{eff} = \frac{2I_{phy}}{\pi}}$$

(b) For electrically short dipoles ($I_{phy} < \frac{sd}{10}$):-

$|I(z)| \propto z$
Linear current

$$I_{avg} = \frac{I_0}{2}$$

$$\boxed{I_{eff} = \frac{I_{phy}}{2}}$$



c) For Hertzian Dipole

<https://notesguru.info>

$$I_{\text{phy}} = I_{\text{eff.}} = \frac{dl}{\lambda}$$

$$I_{\text{avg}} = I_m \rightarrow \text{Uniform currents}$$

$$\text{For Hertzian Dipole, } R_y = 80\pi^2 \left(\frac{dl}{\lambda} \right)^2$$

$$\text{For any Dipole, } R_y = 80\pi^2 \left(\frac{I_{\text{eff.}}}{\lambda} \right)^2$$

$$\begin{aligned} \text{For Electrically short Dipoles } R_y &= 80\pi^2 \left(\frac{l}{2\lambda} \right)^2 \\ &= 20\pi^2 \left(\frac{l}{\lambda} \right)^2 \end{aligned}$$

$$\text{ii) Monopoles } R_y = 10\pi^2 \left(\frac{l}{\lambda} \right)^2$$

$$\text{Monopoles over conducting earth } R_y = 10\pi^2 \left(\frac{2h}{\lambda} \right)^2$$

over conducting earth

$$= 40\pi^2 \left(\frac{h}{\lambda} \right)^2$$

$$\text{For halfwave dipole } = 80\pi^2 \left(\frac{2l}{\pi\lambda} \right)^2$$

$$= 320\pi^2 \left(\frac{l}{\lambda} \right)^2 = 80\Omega$$

vii) Capture Area or Effecting Area (A_c) :-

In microwave antenna $l = \text{few cm}$

$\lambda = \text{few 1000 km}$

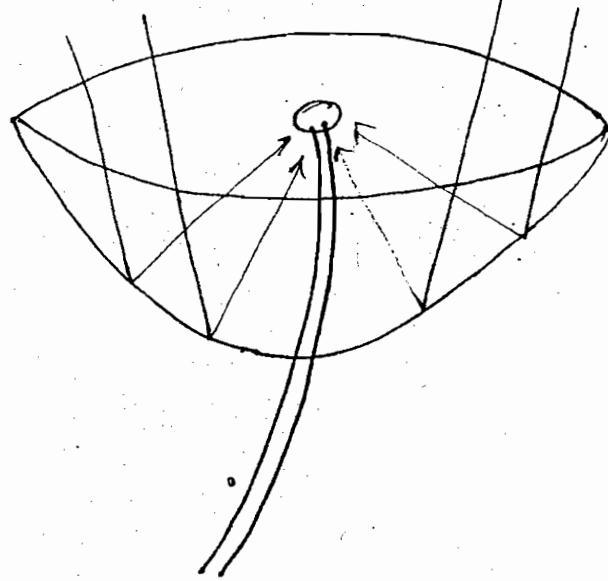
$A_c = \text{capture Area}$

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derail

Poynting Vector

$$A_c = \frac{\lambda^2}{4\pi} D$$



Workbook:

$$\theta \rightarrow G_{1,0} \Big|_{\max} \rightarrow \Psi(\theta, \phi) \rightarrow U(r, \theta, \phi) \rightarrow E(r, \theta, \phi)$$

$$\text{Hertzian Dipole, } |E| = \frac{k \sin \theta}{r}$$

$$U(r, \theta, \phi) = \frac{1}{2} \frac{k^2 \sin^2 \theta}{r^2 \cdot n}$$

$$\Psi(\theta, \phi) = \frac{1}{2} \frac{k^2 \sin^2 \theta}{n}$$

$$G_{1,0} = 4\pi^2 \frac{1}{2} \frac{k^2 \sin^2 \theta}{n}$$

$$\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{2} \frac{k^2 \sin^2 \theta}{n} \sin \theta d\theta \cdot d\phi = 2 \sin^2 \theta$$

$$\int_{\theta=0}^{\pi} \sin^3 \theta d\theta = \frac{4}{3}$$

\Leftarrow

$$= \frac{3}{2} \sin^2 \theta$$

$$\theta = \theta_{\text{HPBW}} \max$$

For Halfwave Dipole, $\theta = 1.63$

4- $\frac{\lambda}{2}$ dipoles

$$W_Y = 4 I_{\text{rms}}^2 \cdot 73 = 4 \left(\frac{0.5}{\sqrt{2}} \right)^2 \cdot 73 = 36.5 \text{ Watts}$$

$$\theta = \frac{4\pi}{R_A}$$

$$10 \log \theta = 44 \text{ dB}$$

$$\Rightarrow \theta = \frac{4\pi}{(\theta_{\text{HPBW}})^2}$$

$$\Rightarrow \theta = 10^{4.4}$$

$$10^{4.4} = \frac{4 \times 3.14 \times (57)^2}{(\theta_{\text{HPBW}})^2}$$

$$\text{Efficiency} = \frac{R_Y}{R_Y + R_d}$$

R_Y for $\lambda/8$ dipole

$$R_Y \lambda/2 = 73 \Omega$$

$$R_Y \lambda/4 = 36.5 \Omega$$

$$R_Y \lambda/8 = 18.25 \Omega$$

$$\text{Efficiency} = \frac{18.25}{19.75} = 89\%$$

$$\begin{aligned} \lambda &\rightarrow 492 \text{ m} \\ \therefore d &\rightarrow 124 \text{ m} \\ \frac{\lambda}{4} &= 123 \text{ m} \end{aligned}$$

$$R_Y = 36.5 \Omega$$

$$10 \log 4 = 6 \text{ dB}$$

$$G_t = 4$$

$$N_{IN} = 1 \text{ mW}$$

lossless

$$N_Y = N_{IN}$$

→ Ans - (B)

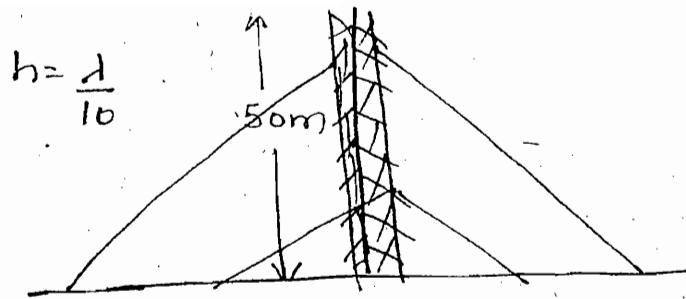
3.

(B)

$$h = 50 \text{ m}$$

$$f = 60 \text{ kHz}$$

$$\lambda = \frac{3 \times 10^8}{6 \times 10^5} = 500 \text{ m}$$



$$R_y = 40\pi^2 \left(\frac{50}{500}\right)^2$$

$$\approx 4\Omega = \frac{2\pi^2}{5}$$

$$125 - \frac{1}{4} - 36.5 \Omega$$

$$50 \text{ m} - \frac{1}{10} - 4\Omega$$

$$W_y = I_{rms}^2 R_y$$

$$\Psi_{avg.} = \frac{100}{4\pi} = \frac{25}{\pi} = 7.96$$

$$V_{avg.} = \frac{100}{4\pi \times (10 \times 10^3)^2} = 7.96 \times 10^{-8}$$

$$= 0.08 \mu \text{W}$$

$$\Psi \propto W_y \propto R_y \propto \frac{1}{f^2}$$

$$f' = f_{1/2}$$

$$\text{strongly} \quad \Psi' = \Psi_{1/4}$$

$$\theta = ?$$

$$\theta = ? \text{ dB}$$

$$\theta = \frac{4\pi \Psi(\theta, \phi)_{max}}{W_y}$$

$$= \frac{4\pi \cdot 150}{0.9 (W_{IN})} = \frac{4\pi \cdot 150}{36\pi} = 16.67 = \theta$$

$$10 \log 16.67 = 11.76 \text{ dB}$$

$$\theta = 11.76 \text{ dB}$$

1A → Im - using

→ I_{rms}

$$\lambda = \frac{3 \times 10^8}{10 \times 10^6} \times 30 \text{ m}$$

$$W_r = I_{\text{rms}}^2 R_x$$

$$R_x = 80\pi^2 \left(\frac{1}{30}\right)^2 = 0.88 \Omega$$

$$W_r = 0.88$$

$$d = \frac{\lambda}{30}$$

$$\frac{1}{2} \rightarrow 73 \Omega$$

$$\frac{1}{4} \rightarrow 36.5 \Omega$$

$$\frac{1}{10} \rightarrow 4 \Omega$$

$$\frac{1}{30} \rightarrow 0.88 \Omega$$

$$\left(\frac{dl}{\lambda}\right)$$

13. $\Theta_{\text{HPBW}} = 90^\circ$

14. A

15. Uniform currents

$$R_x = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

$$l = 5 \text{ m}$$

$$\lambda = 100 \text{ m}$$

$$= 80\pi^2 \left(\frac{1}{200}\right)^2 \approx 2 \Omega$$

16. A

17. $l = 1.5 \text{ m}$

$f = 100 \text{ MHz}$

$\lambda = 3 \text{ m}$

$$l = \frac{\lambda}{2}$$

$$l = 15 \text{ m}$$

$$f = 10 \text{ MHz}$$

$$l = \frac{\lambda}{2}$$

$$\lambda = 30 \text{ m}$$

Halfwave Dipole

2-Element Isotropic Array:-

$\gamma \gg d$ For - zone analysis

$$E_y = E_1 + E_2 = E_0 + kE_0 e^{j\psi}$$

where $k=1$ if $|I_1| = |I_2|$

ψ = Phase diff. b/w the two fields

= current having phase difference (α) + path difference, giving phase

Path difference = $d \cos \theta$

$$\text{phase diff.} = \frac{2\pi}{\lambda} d \cos \theta = \beta d \cos \theta$$

$$\boxed{\psi = \alpha + \beta d \cos \theta}$$

$$E_T = E_0 (1 + \cos \psi + j \sin \psi)$$

$$|E_T| = |E_0| \sqrt{(1 + \cos \psi)^2 + \sin^2 \psi}$$

$$= |E_0| \sqrt{2 + 2 \cos \psi}$$

$$|E_T| = 2 E_0 \cos (\psi/2)$$

Case - (1) :-

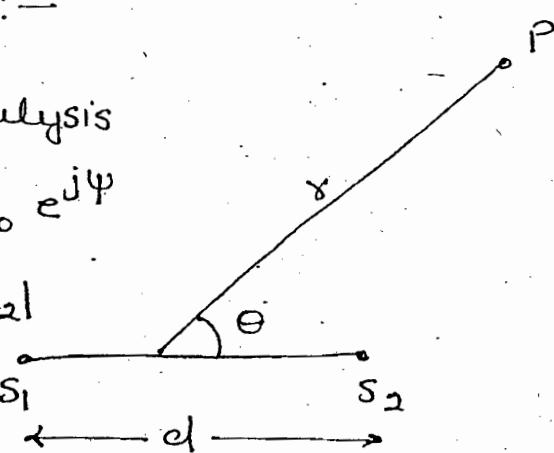
$$d = \frac{\lambda}{2}, \alpha = 0$$

$$\psi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos \theta$$

$$|E_T| = 2 E_0 \cos \left(\frac{\pi}{2} \cos \theta \right)$$

$$\text{If } \theta = 90^\circ / 270^\circ \rightarrow \theta_{\max}$$

$$|E_T| = 2 E_0 = E_{\max}$$



If $\theta = 0^\circ / 180^\circ$

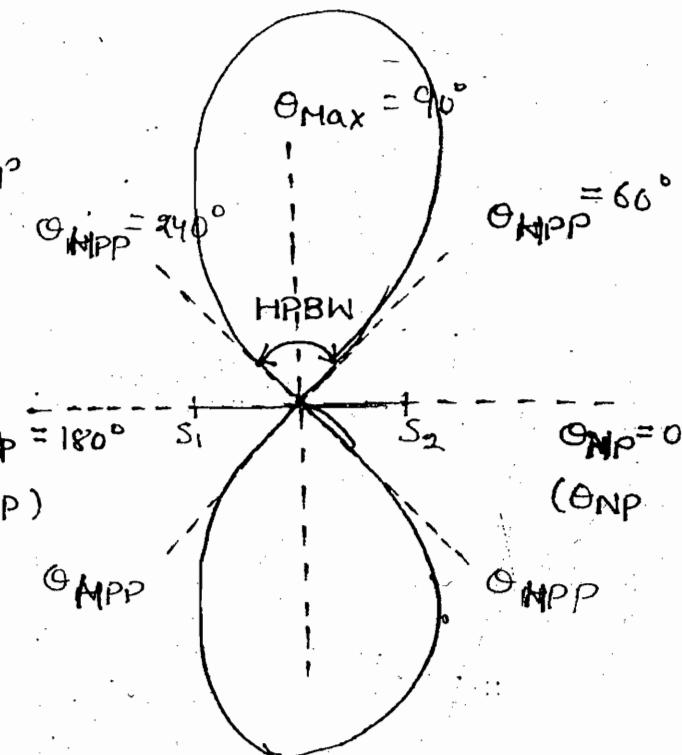
<https://notesguru.info>

$$|E_T| = 0$$

If $\theta = 60^\circ / 120^\circ / 240^\circ / 300^\circ$

$\rightarrow \theta_{MPP}$

$$|E_T| = \frac{E_{max}}{\sqrt{2}}$$



Note:-

The currents on either side inphase and max. at the centre and halfwave dipole has also broad side pattern

Case - (ii) :-

$$d = \frac{\lambda}{2}, \lambda = \pi$$

$$\Psi = \pi + \frac{2\pi}{\lambda} \cdot \frac{d}{2} \cos \theta$$

$$|E_T| = 2E_0 \sin\left(\frac{\pi}{2} \cos \theta\right)$$

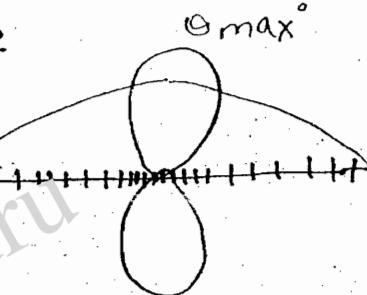
If $\theta = 90^\circ / 270^\circ$ $\rightarrow \theta_{NP}$

$$|E_T| = 0$$

If $\theta = 0^\circ / 180^\circ$ $\rightarrow \theta_{max}$

$$|E_T| = E_{max} = 2E_0$$

NP



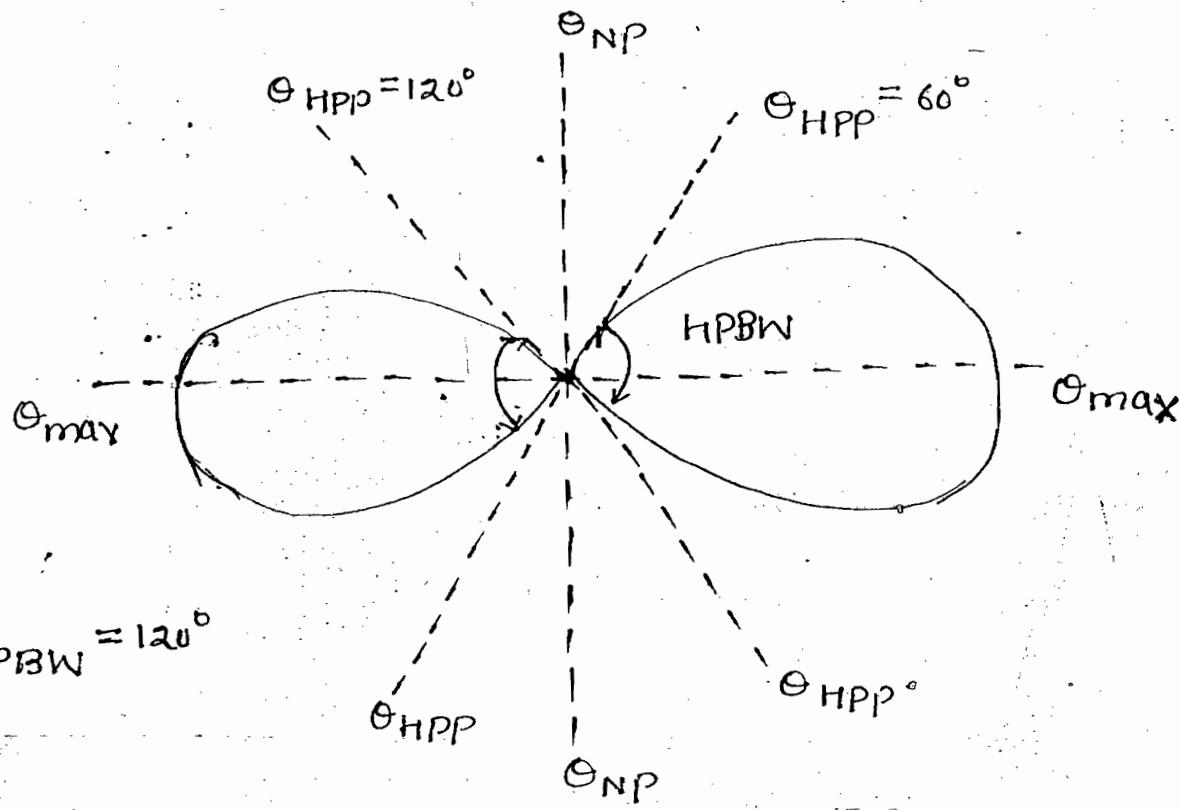
If $\theta = 60^\circ / 120^\circ / 240^\circ / 300^\circ$

$\rightarrow \theta_{HPP}$

$$|E_T| = \frac{E_{max}}{\sqrt{2}}$$

This is called as ray whose

$$\theta_{\max} = 0^\circ / 180^\circ$$



Case - (III) :-

$$d = \lambda, \alpha = 0^\circ$$

$$E_T = 2E_0 \cos(\pi \cos \theta)$$

$$\text{If } \theta = 0^\circ / 90^\circ / 180^\circ / 270^\circ$$

$$\rightarrow \theta_{\max}$$

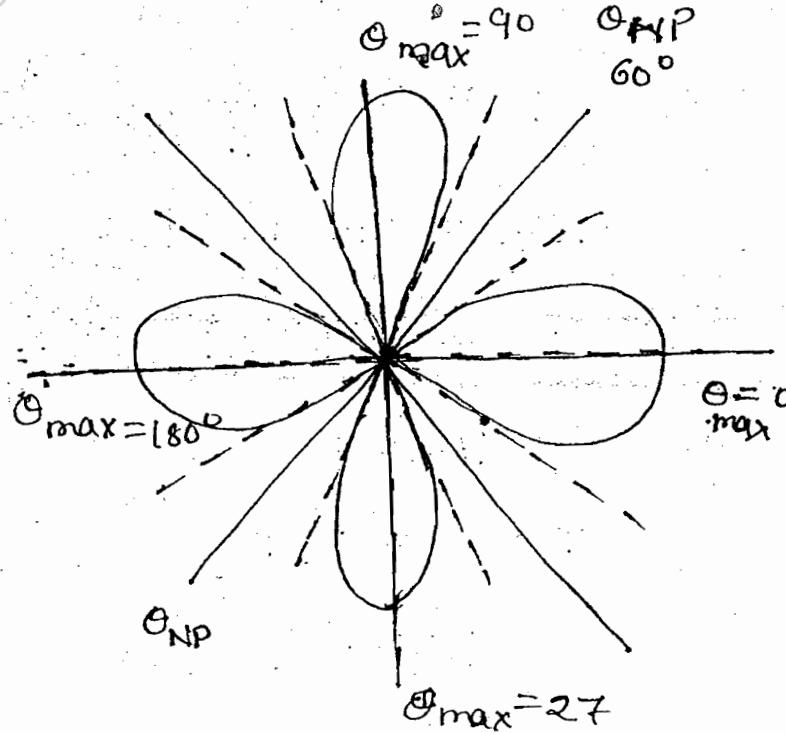
$$\text{If } \theta = 0^\circ / 90^\circ / 180^\circ / 270^\circ$$

$$\rightarrow \theta_{\max}$$

$$\text{If } \theta = 60^\circ / 120^\circ / 240^\circ / 300^\circ$$

$$\rightarrow \theta_{NP}$$

$$\cos \theta = \pm \frac{1}{4} \text{ or } \pm \frac{3}{4}$$



In general θ_{\max} can

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towards a specific direction

With $\psi \rightarrow 0$

$$\alpha + \beta d \cos \theta_{\max} = 0$$

$$\cos \theta_{\max} = -\frac{\alpha}{\beta d}$$

For broadside array,

$$\theta_{\max} = 90^\circ / 270^\circ$$

$$\Rightarrow \alpha = 0$$

Inphase currents

For end-fire array

$$\theta_{\max} = 0^\circ / 180^\circ \Rightarrow \alpha = \pm \beta d$$

$$\theta_{\max} = \alpha \neq 0$$

Extension:

For n elements, uniform, linear isotropic sources.

$$E_T = E_0 (1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}) \quad \begin{array}{c} d \quad d \quad d \\ \downarrow \quad \downarrow \quad \downarrow \\ s_1 \quad s_2 \end{array} \quad \dots \quad s_n$$

G.P. with $a = 1$ $\gamma = e^{j\psi}$

$$E_T = \frac{E_0 (1 - e^{jn\psi})}{(1 - e^{j\psi})}$$

$$|E_T| = \frac{E_0 \sin(n\psi/2)}{\sin(\psi/2)}$$

If $\psi \rightarrow 0$

$$|E_T| = E_0 \frac{n(\psi/2)}{\sin(\psi/2)} = nE_0$$

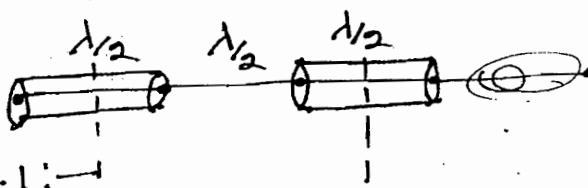
$$= E_{\max}$$

If $n=2$

$$|E_T| = 2E_0 \cos(\psi/2)$$

Workbooks

Theorem of Multiplication of Patterns (Symm. Arrays) 2



Step 1: →

Identify the two points that symm. makes the group and Identify Eu unit pattern.

UNIT:-

$$d = \frac{1}{2}, \lambda = 0$$

2-Element Array

$$E_u = \cos\left(\frac{\pi}{2} \cos\theta\right)$$

Step 2: →

Identify the group formed from the two points and its pattern

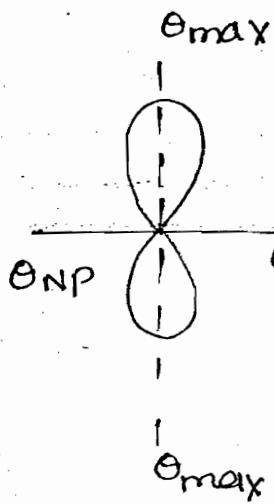
GROUP:- $d = 1, \lambda = 0$

$$E_g = \cos(\pi \cos\theta)$$

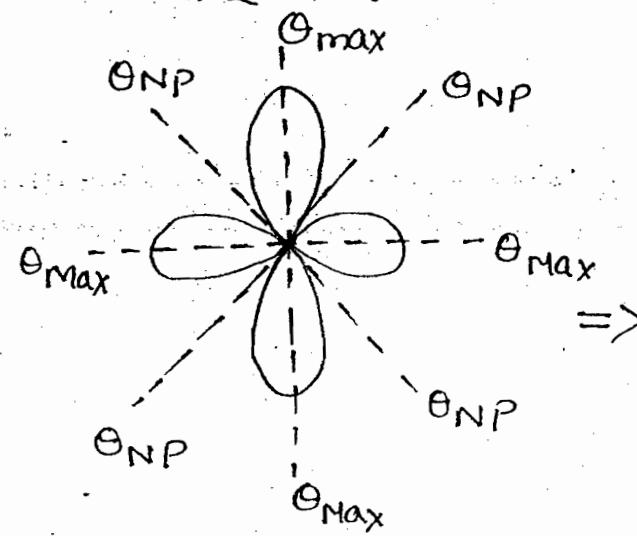
Step 3: →

Resultant Pattern = Unit Pattern \times Group Pattern

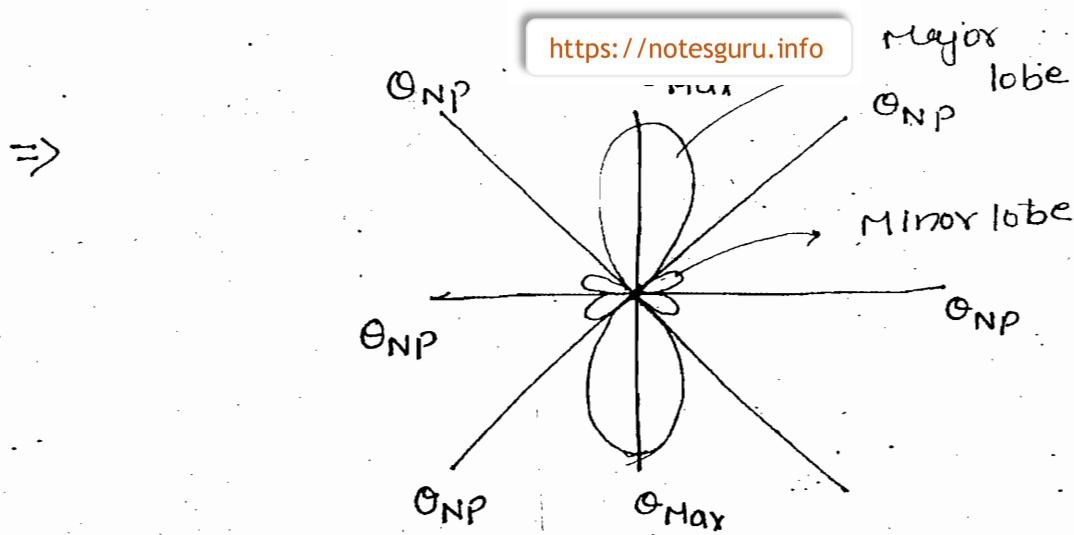
$$= \cos\left(\frac{\pi}{2} \cos\theta\right) \cos(\pi \cos\theta)$$



X



=>



Workbook! —

19. C

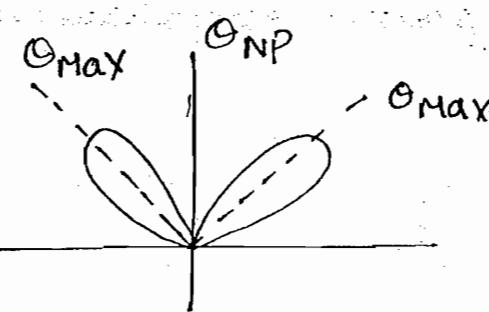
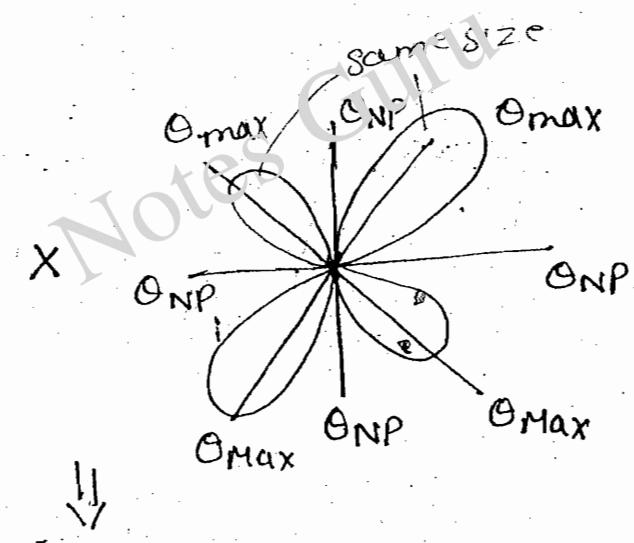
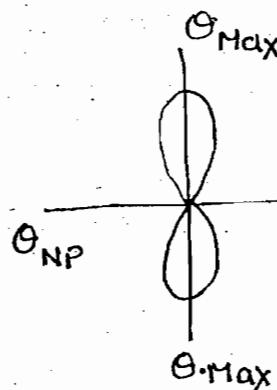
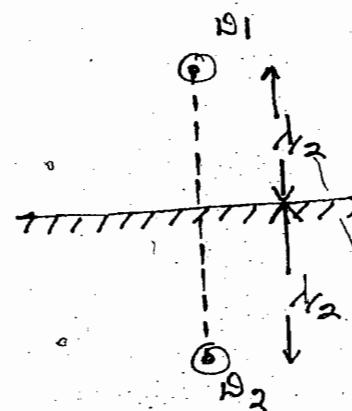
20. Unit \rightarrow Dipole Antenna

$$E_u = \cos\left(\frac{\pi}{2} \cos\theta\right)$$

GROUP \Rightarrow $d=1$

$$\lambda = \pi - 2 \text{ Dipoles}$$

$$E_g = \sin(\pi \cos\theta)$$



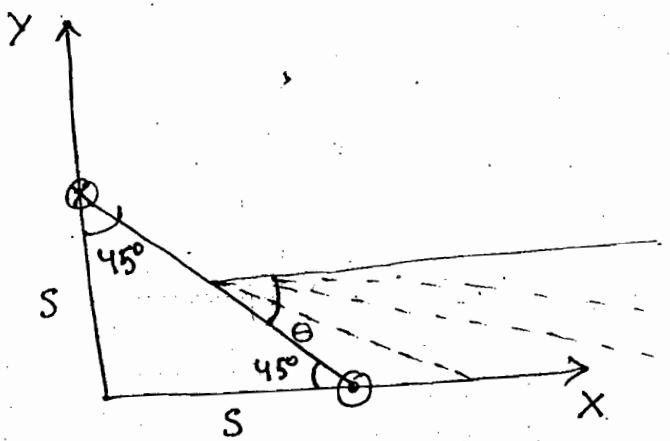
Ans-B

$$\theta = \frac{\pi}{2} \text{ Plane}$$

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$$z=0$$

XY Plane



$$d = \sqrt{2}s, \alpha = \pi$$

$$\theta = 45^\circ$$

$$\begin{aligned}\frac{E_T}{E_0} &= 2 \cos \left(\frac{\alpha + \beta d \cos \theta}{2} \right) \\ &= 2 \cos \left(\pi + \frac{2\pi}{\lambda} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \right) \\ &= 2 \sin \left(\frac{\pi s}{\lambda} \right)\end{aligned}$$

2. $\theta_{\max} = 60^\circ$ off end-fire

$$\alpha + \beta d \cos 60^\circ = 0$$

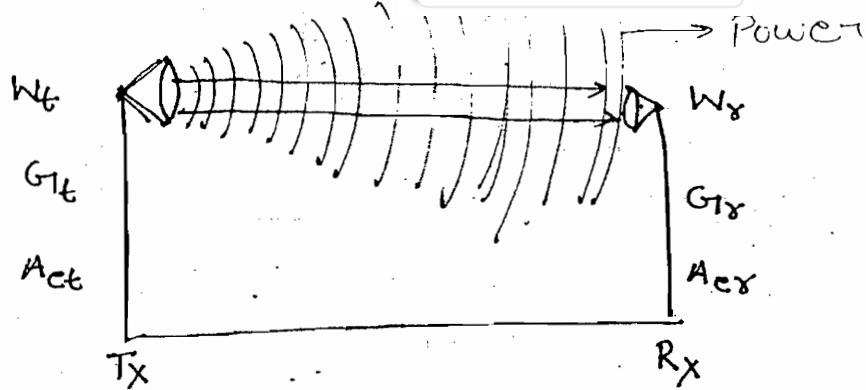
$$\alpha = -\frac{2\pi}{\lambda} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = -\frac{\pi}{4}$$

23. Unambiguous

Direction finding

→ 4 lobe pattern

$$d = \lambda \quad \alpha = 0^\circ$$



Power density at Rx = $\frac{W_t \cdot G_{t,0}}{4\pi d^2}$

$W_r = \frac{W_t \cdot G_{t,0} \cdot A_{crx}}{4\pi d^2}$

$$A_{crx} = \frac{\lambda^2 G_{r,0}}{4\pi}$$

$$W_r = \frac{W_t \cdot G_{t,0} \cdot G_{r,0}}{\left(\frac{4\pi d}{\lambda}\right)^2}$$

$\left(\frac{4\pi d}{\lambda}\right)^2 = L_s$ = Loss due to spatial dispersion.

$$W_r (\text{dBW}) = W_t (\text{dBW}) + G_{t,0} (\text{dB}) + G_{r,0} (\text{dB}) - L_s (\text{dB})$$

$$\frac{W_t G_{t,0}}{4\pi d^2} = \frac{1}{2} \frac{E_0^2}{n} = \frac{E_{rms}^2}{120\pi}$$

$$E_{rms} = \sqrt{\frac{30 W_t \cdot G_{t,0}}{c}}$$

Workbook! —

17. $W_r (\text{dBW}) = 10 \text{ dBW} + 10 \text{ dB} + G_{1r} (\text{dB}) - 100 \text{ dB}$

Tx — circularly polarized

Rx — Linearly polarised.

$\frac{1}{2}$ — Power received

due to polarization mismatch

$$G_{1r} = \frac{1}{2} \quad 10 \log \frac{1}{2} = -3 \text{ dB}$$

$$G_1 = 2 = 3 \text{ dB}$$

→ doubled

24. $W_r = \frac{W_t \cdot G_{1t} \cdot A_{cr}}{4\pi d^2} = 0.8 \text{ W}$

25. $W_r = \frac{W_t \cdot (G_1)^2}{\left(\frac{4\pi d}{\lambda}\right)^2} \quad d = 30 \text{ km}$

$$\lambda = \frac{3 \times 10^8}{1 \times 10^9} = 0.3 \text{ m}$$

$$W_r = -30 \text{ dBm} \quad \text{for } W_t = 1 \text{ W}$$

$$\left(\frac{W_r}{W_t} = -30 \text{ dBm} \right)$$

$$\left(\frac{W_r}{W_t} = -30 \text{ dB} \right) \times 10^{-3}$$

$$\left(10 \log \frac{W_r}{W_t} = -30 \right) \times 10^{-3}$$

$$\left(\frac{W_r}{W_t} = 10^{-3} \right) \cdot 10^3$$

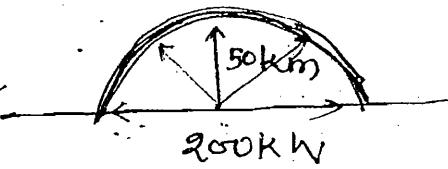
Ans - 9

$$\frac{W_r}{W_t} = 10^{-6}$$

$$E_{rms} = \frac{\sqrt{30 \cdot 1c}}{10 \times 10^3}$$

$$P_{avg} = \frac{W_t}{2\pi c d^2} a_s$$

$$= \frac{200 \times 10^3}{2\pi \times (50 \times 10^3)^2} .48$$



Exms — 5kms — c

$$E_{rms}^1 = \frac{E_{rms}}{\sqrt{2}} = d' = ?$$

Further more P

to detect 3dB decrease

$$3dB \text{ in power} = \frac{1}{2} = \text{power}$$

$$-\frac{1}{\sqrt{3}} = \text{field}$$

$$\pi = \frac{2}{\sqrt{3}}$$

$$\pi_1 = \sqrt{\lambda} \pi$$

$$d' = \sqrt{2d} = \sqrt{2 \times 5} \text{ km/s} = 7 \text{ km}$$

Further move = 2 km

Ex-
d

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