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POLYPHASE AC CKT ANALYSIS

Phase → "A current carrying condn" (or) Live wire (or) Hot wire.

Poly → More than one.

Introduction →

* Y Connected gen:

* Δ Connected gen, / mesh

* Power Calculation,

* Concept of P, Q, S & PF

* Measurement of 3-φ power

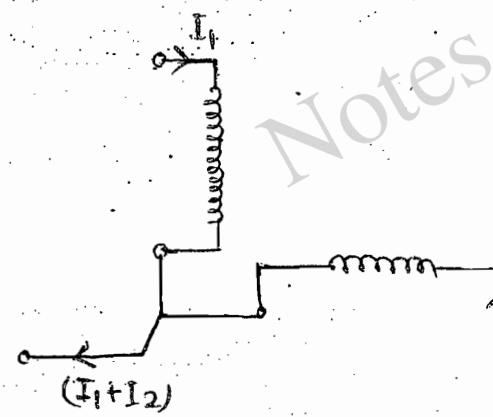
* Analysis of 3-φ balanced load.

* Analysis of 3-φ Unbalanced load.

* Floating point Neutral. (Millman's theorem).

* Application of 3-φ

2 phase wire →



* The 3rd condn gets more amount of current, so the cross section area will be more for 3rd condn

* So 2phase wire sys. is not used because of uneconomical

3 phase system →

(1) 3 wires are used of same thickness.

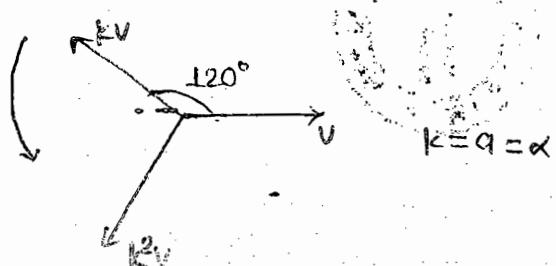
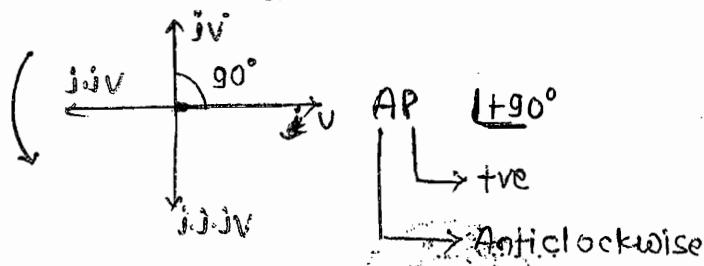
(2) Power handling capacity increases with little amount.

$$P = \sqrt{3} VI \cos \phi$$

$$= 1.732 VI \cos \phi$$

Mathematical Operators →

(1.) "j" operator (2) "k" operators.



* 2φ electric power was an earlier polyphase AC power distribution.

Two cond^rs were used in this type.

* The advantage of 2φ wire over 1φ wire is its self starting electric motors.

* Two-φ clt typically uses 2 pairs of cond^r, sometimes they are required to be interconnected but the common cond^r carries the vector sum of φ currents which requires a larger cond^r.

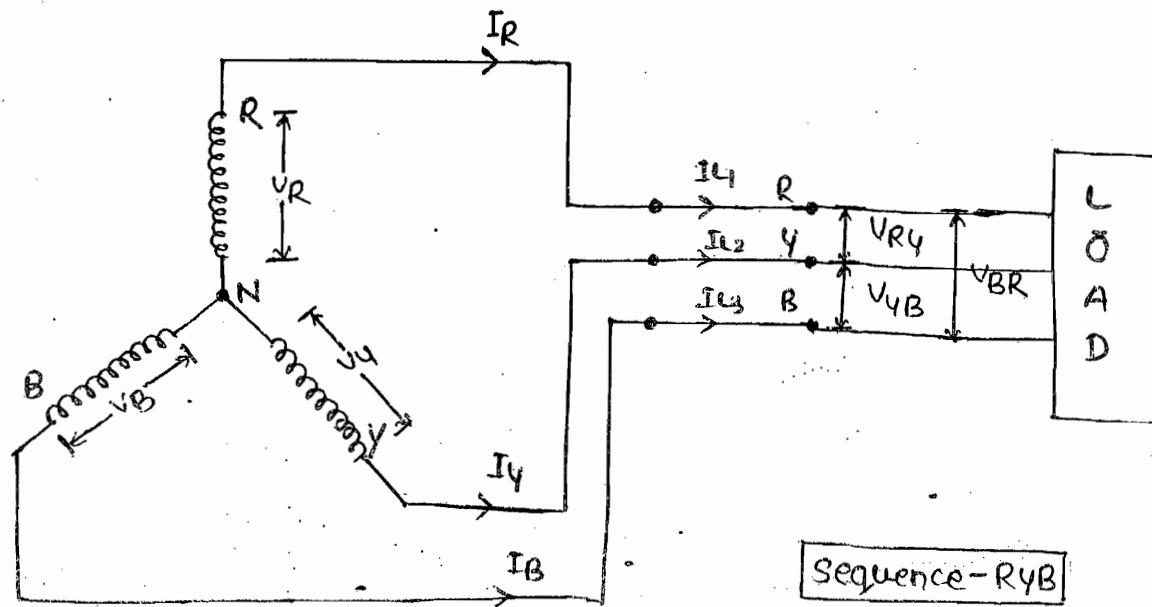
* But in 3φ 3 cond^rs are of same size, the most obv. ad. of 3φ power ×^{1.73} using 3 wires as compare to 1φ-2 wires is the power transmitted in 3φ is 73% more but uses only 50% of additional wire.

* Earlier 2φ is also implemented by 3 wire but it introduces a symmetry & the voltage drop in a neutral does not make the phases 90° apart.

* For a given frame size of a m/c a 3φ m/c will have larger capacity than a 1φ m/c. The torque produced in a 3φ motor will be more uniform whereas in 1φ motor it is pulsating.

* The amt of cond^r material needed to transmit same amt of power is lesser for 3φ sys. Therefore it is economical.

* Star Connected Genⁿ →



Conclusions →

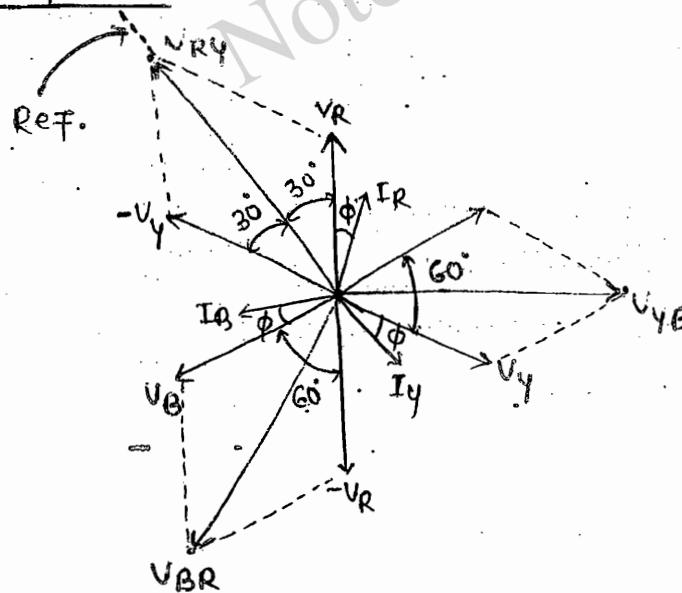
$$(1) \quad V_R = V_Y = V_B = V_{PH}, \quad V_{RY} = V_{YB} = V_{BR} = V_L$$

$$I_R = I_Y = I_B = I_{PH}, \quad I_L = I_{L2} = I_{L3} = I_L$$

(2) Line current = Phase current

$$I_L = I_{ph}$$

(3.) Phasor diagram →



$$V_{RY} = V_R + V_Y$$

$$= V_R + (-V_Y)$$

$$V_{YB} = V_Y - V_B$$

$$= V_Y + (-V_B)$$

$$V_{BR} = V_B - V_R$$

$$= V_B + (-V_R)$$

$$V_{RY} = \sqrt{|V_R|^2 + |V_Y|^2 + 2|V_R||V_Y|\cos 60^\circ}$$

$$V_{RY} = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}^2 \times \frac{1}{2}}$$

$$V_L = \sqrt{3} V_{ph}$$

4.) Angle between V_L & $I_L \rightarrow$

$$\text{For load R-L load} = 30^\circ + \phi$$

$$\begin{aligned} \text{R load} &= 30^\circ & \left\{ \begin{array}{l} \text{From phasor diagram} \\ V_{RY} = V_L \end{array} \right. \\ \text{RC load} &= 30 - \phi \end{aligned}$$

5.) In λ Connection b/w any 2 adjacent phase vol. the angles always 60° .

(6) w.r.t Ref V_{RY}, V_{YB}, V_{BR}

$$V_{RY} = V_L 10^\circ$$

$$V_{YB} = V_L (-120^\circ)$$

$$V_{BR} = V_L 1 - 240^\circ (\text{or}) 120^\circ$$

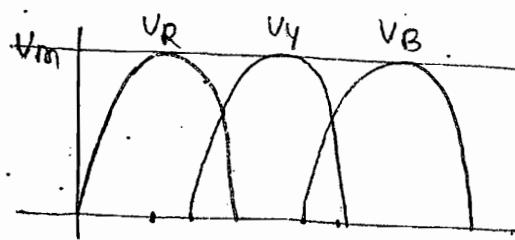
(7) w.r.t Ref V_R, V_Y, V_B

$$V_R = V_{ph} 1 - 30^\circ$$

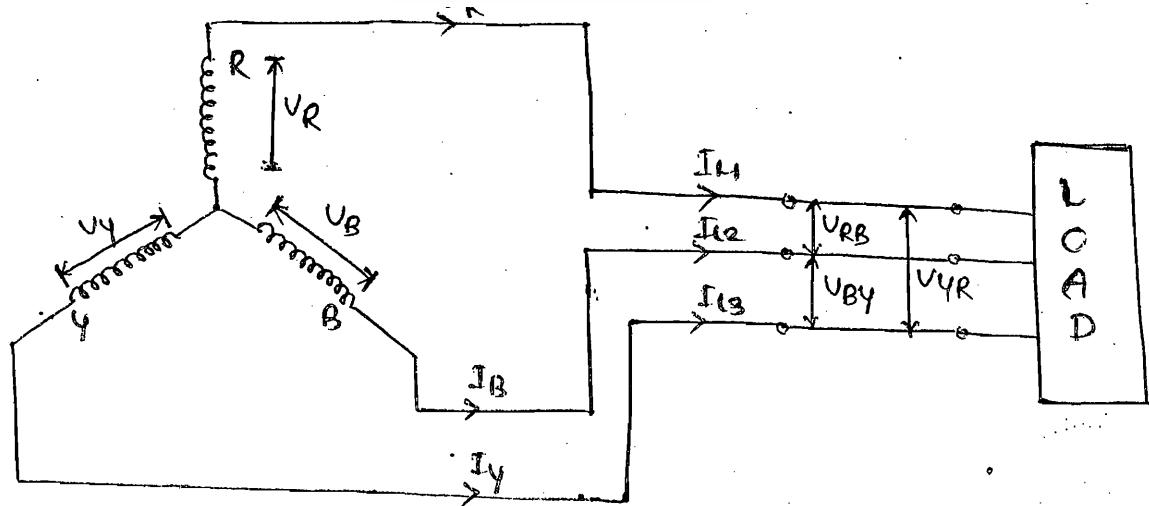
$$V_Y = V_{ph} 1 - 150^\circ$$

$$V_B = V_{ph} 1 - 270^\circ (\text{or}) 90^\circ$$

F Phase Sequence \rightarrow * It is the order by which the individual voltages attains there peak value.



Analysis for -ve ϕ seq; \rightarrow

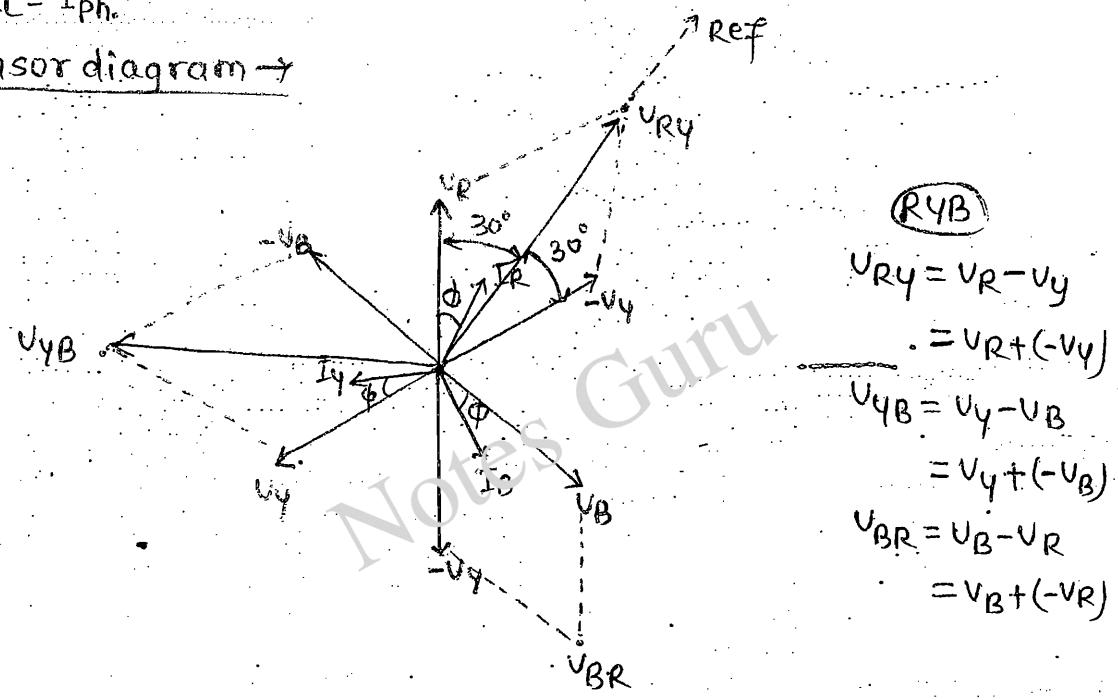


Conclusions →

(1) No change

(2) $I_L = I_{ph}$

(3) Phasor diagram →



$$V_{RY} = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos 60^\circ}$$

$$V_L = \sqrt{3} V_{ph}$$

(4) Angle b/w V_L & I_L →

$$\theta_L = 30^\circ - \phi$$

$$\theta = 30^\circ$$

$$\theta_C = 30 + \phi$$

(5) WRT Ref V_{RY}, V_{YB}, V_{BR}

$$V_{RY} = V_L \text{ } 0^\circ$$

$$V_{YB} = V_L \text{ } +120^\circ$$

$$V_{BR} = V_L \text{ } (+240 \text{ (or)} -120^\circ)$$

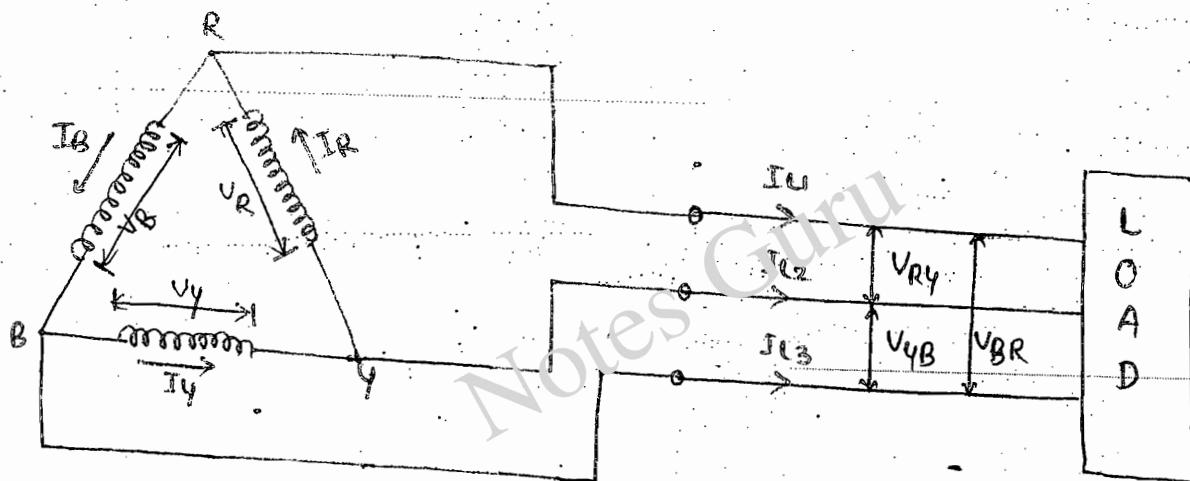
(6) WRT Ref V_R, V_Y, V_B

$$V_R = V_{ph} \text{ } (+30^\circ)$$

$$V_Y = V_{ph} \text{ } (+150^\circ)$$

$$V_B = V_{ph} \text{ } (-90^\circ)$$

Δ Connected gen^r (or) mesh connected gen^r →



Conclusions →

$$1) V_R = V_Y = V_B = V_{ph}$$

$$I_R = I_Y = I_B = I_{ph}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

$$I_L = I_U = I_V = I_W$$

$$2) V_L = V_{ph}$$

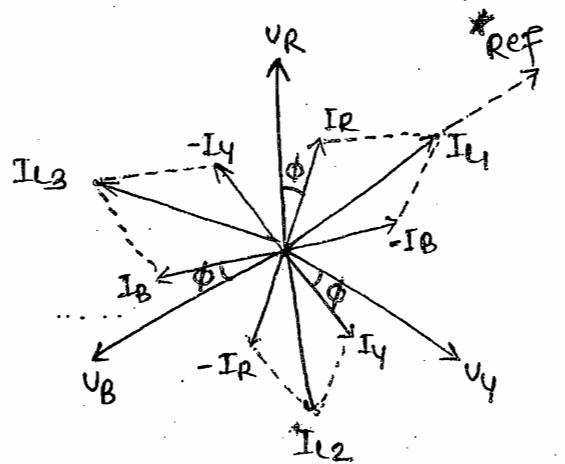
line Vol. = ph. voltage

3) Applying KCL at R, Y & B

$$I_R = I_U + I_B$$

$$I_B = I_Y + I_V$$

$$I_Y = I_B + I_W$$



$$I_L1 = I_R - I_B$$

$$I_L2 = I_Y - I_R$$

$$I_L3 = I_B - I_Y$$

$$I_L = \sqrt{I_R^2 + I_B^2 + 2|I_R||I_B|\cos 60^\circ}$$

$$I_L = \sqrt{3} I_{ph}$$

* In Δ connection between any 2 adjacent phase currents the angle is always 60° .

(4.) Angle between V_L & I_L →

$$\angle V_L I_L = -30 + \phi$$

$$\angle V_L R = -30^\circ$$

$$\angle V_L C = 30 - \phi$$

(5.) WRT Ref [I_L1, I_L2, I_L3]

$$I_L1 = I_{ph} 10^\circ$$

$$I_L2 = I_{ph} (-120^\circ)$$

$$I_L3 = I_{ph} (-240^\circ) + 120^\circ$$

(6.) WRT Ref [I_R, I_Y, I_B]

$$I_R = I_{ph} |+30^\circ$$

$$I_Y = I_{ph} |+270^\circ (or) -90^\circ$$

$$I_B = I_{ph} |+150^\circ$$

Analysis for -ve phase seq; →

- (1)
- (2)
- (3.) *No change*

(4.) Angle b/w V_L & I_L →

$$R_L = 30 - \phi$$

$$R = 30^\circ$$

$$R_C = 30 + \phi$$

(5.) WRT I_L (I_4, I_{L2}, I_{L3})

$$I_4 = I_L 0^\circ$$

$$I_{L2} = I_L L + 120^\circ$$

$$I_{L3} = I_L L - 120^\circ$$

(6.) WRT Ref (I_R, I_y, I_B)

$$I_R = I_{ph} L + 30^\circ$$

$$I_y = I_{ph} L + 270^\circ (\text{or}) + 90^\circ$$

$$I_B = I_{ph} L - 150^\circ$$

* Power Calculation →

$$1-\phi \therefore P = V I \cos \phi$$

$$3-\phi \rightarrow$$

$$P_{3-\phi} = 3 V_{ph} I_{ph} \cos \phi$$

$$P_L = 3 V_{ph} I_{ph} \cos \phi$$

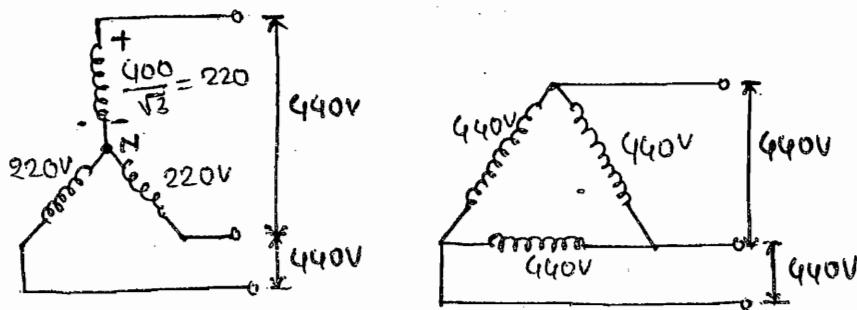
$$= 3 \frac{V_L}{\sqrt{3}} I_L \cos \phi$$

$$P_L = \sqrt{3} V_L I_L \cos \phi$$

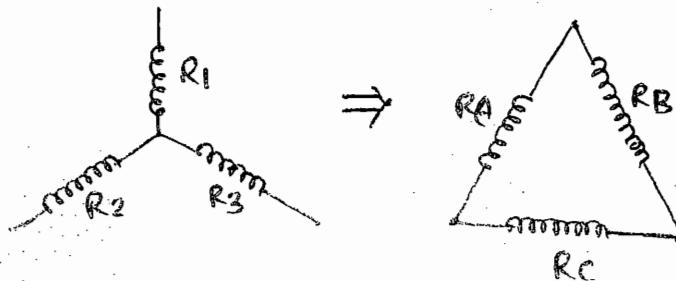
$$P_\Delta = 3 V_{ph} \cdot I_{ph} \cdot \cos \phi$$

$$= 3 V_L \frac{I_L}{\sqrt{3}} \cos \phi$$

$$P_\Delta = \sqrt{3} V_L I_L \cos \phi$$



Formula →

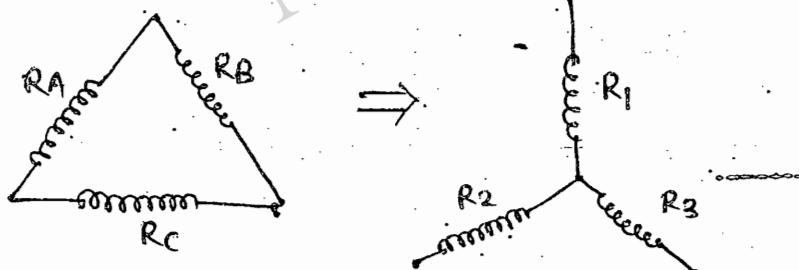


$$S \xrightarrow{\text{OR}} \Delta \text{ Remain}$$

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$



$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

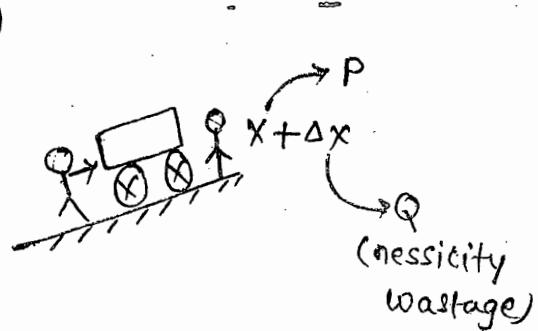
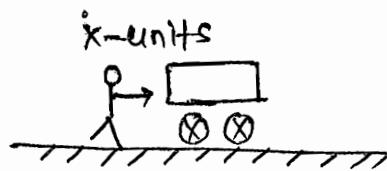
$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_B R_C}{R_A + R_B + R_C}$$

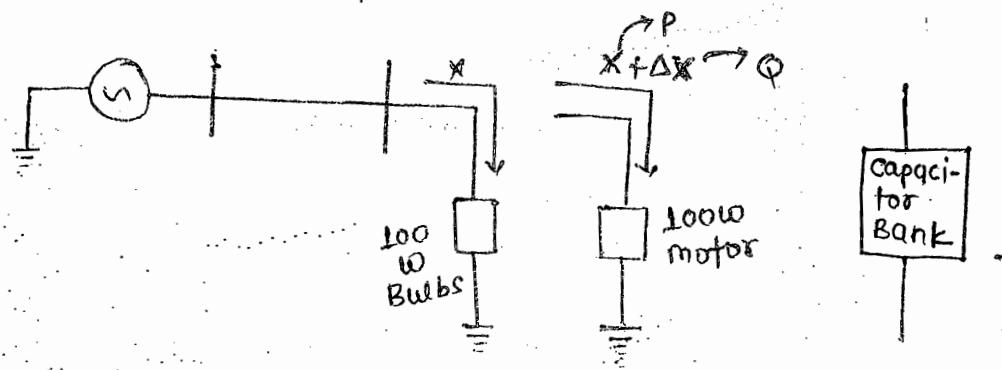
$$\Delta \xrightarrow{\text{PUS}} S \text{ Remain}$$

$$\frac{\text{Product}}{\text{Sum}}$$

STAR $\xrightarrow{(3)} \Delta$ DELTA
 (4) (5.)



$P \rightarrow$ Active Power
 $Q \rightarrow$ Reactive Power



(1) Overload

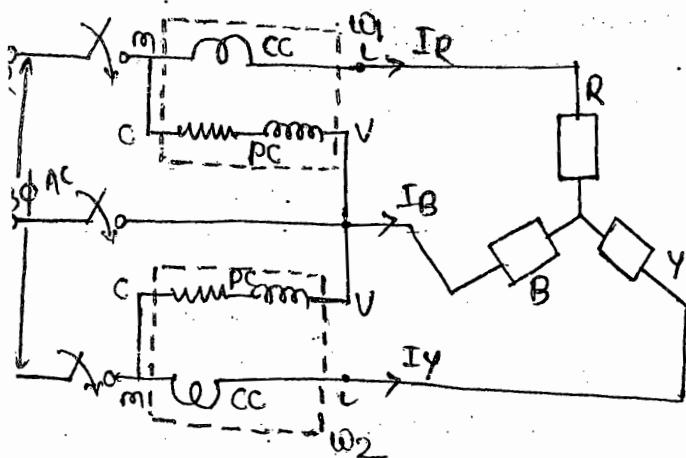
(2) $I^2R \uparrow \eta \downarrow$

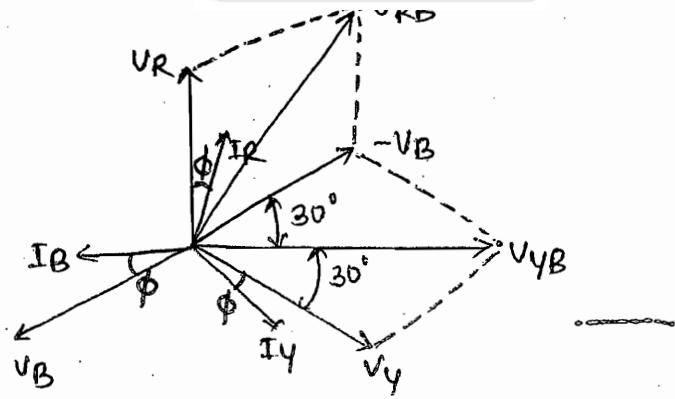
(3) Poor Vol. Regn

- * A capacitor bank is required for the removing of extra wastage.
- * The value of load increases use of capacitor Bank.

DATE-26/07/14

* Measurement of 3φ-power using 2-Wattmeter method \rightarrow
 (can be used for bal & unbalanced load)





$$\omega_1 = V_{RB} I_R \cos(30^\circ - \phi), \quad \omega_2 = V_{YB} I_y \cos(90^\circ + \phi)$$

For active power consumption

$$\begin{aligned}\omega_1 + \omega_2 &= V_L I_L [\cos(30^\circ - \phi) + \cos(90^\circ + \phi)] \\ &= V_L I_L (\sqrt{3} \cos \phi)\end{aligned}$$

$$\therefore \omega_1 + \omega_2 = \sqrt{3} V_L I_L \cos \phi = P$$

$$P = \omega_1 + \omega_2$$

For Reactive power consumption

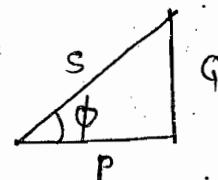
$$\omega_1 - \omega_2 = V_L I_L \sin \phi$$

$$Q = \sqrt{3} (\omega_1 - \omega_2)$$

From power triangle:-

$$\tan \phi = \frac{Q}{P}$$

$$\phi = \tan^{-1} \left(\frac{Q}{P} \right)$$



$$\phi = \tan^{-1} \left[\frac{\sqrt{3} (\omega_1 - \omega_2)}{\omega_1 + \omega_2} \right]$$

* Effect of PE on wattmeter readings →

(i) $\phi = 0^\circ$

$$\omega_1 = V_L I_L \cos 30^\circ, \quad \omega_2 = V_L I_L \cos 30^\circ$$

$$\boxed{\omega_1 = \omega_2}$$

(ii) $\phi = 30^\circ$

$$\omega_1 = V_L I_L [\cos(30 - 30)] = V_L I_L$$

$$\omega_2 = V_L I_L \cos(30 + 30) = \frac{V_L I_L}{2}$$

$$\boxed{\frac{\omega_1 - \omega_2}{2}}$$

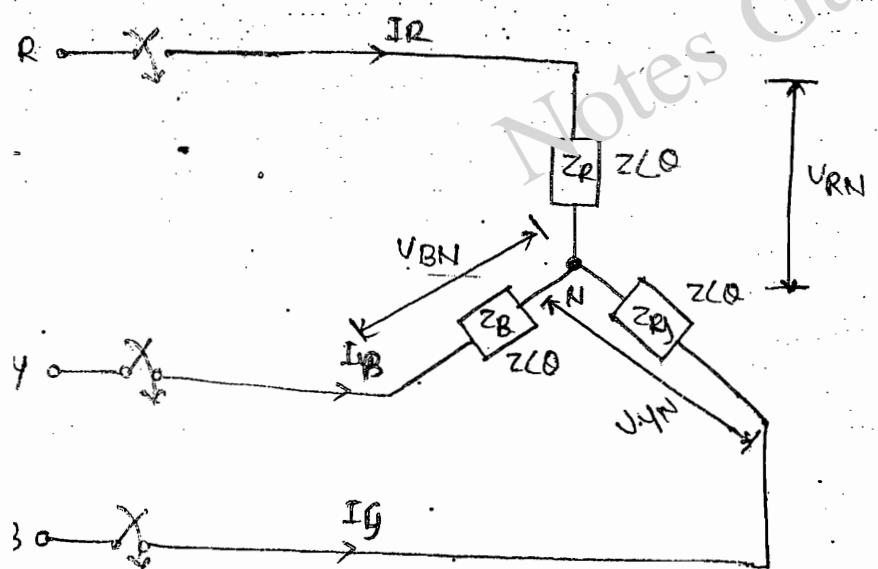
(iii) $\phi = 60^\circ$

$$\omega_1 = V_L I_L \cos(80 - 60) = \frac{\sqrt{3}}{2} V_L I_L$$

$$\omega_2 = V_L I_L \cos(30 + 60) = 0$$

Analysis of 3φ balanced ckt →

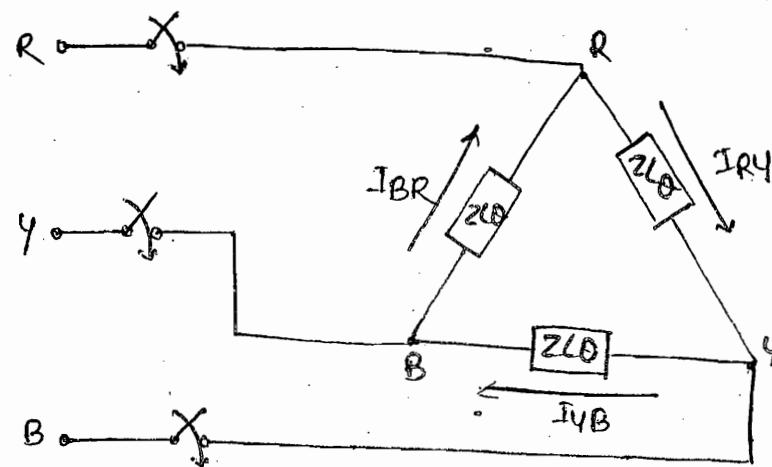
used $\rightarrow Y_s$ & Y_L • 3φ 3-wire balanced load:



$$I_R = \frac{V_{RN}}{ZL} \quad I_Y = \frac{V_{YN}}{ZL} \quad I_B = \frac{V_{BN}}{ZL}$$

$$V_{RN} = V_{YN} = V_{BN} = V_{ph} = \frac{V_L}{\sqrt{3}}$$

(ii) case(ii) $\text{Y}_S - \Delta_L$ (3φ, 3win)



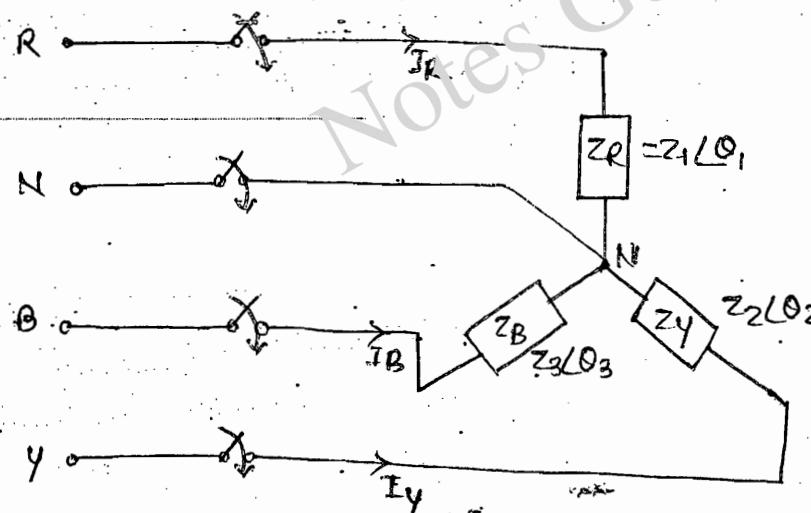
$$I_{RY} = \frac{V_{RN} L^0^\circ}{Z_L}$$

$$I_{YB} = \frac{V_{YN} L^{-120^\circ}}{Z_L}$$

$$I_{BR} = \frac{V_{BN} L^{+120^\circ}}{Z_L}$$

* Analysis of 3φ Unbalanced ckt \rightarrow

(i) 3φ ; 4wire ; Unbalanced Y-load. ($\text{Y}_S - \text{Y}_L$)



$$I_R = \frac{V_{RN} L^0^\circ}{Z_R}$$

$$I_Y = \frac{V_{YN} L^{-120^\circ}}{Z_Y}$$

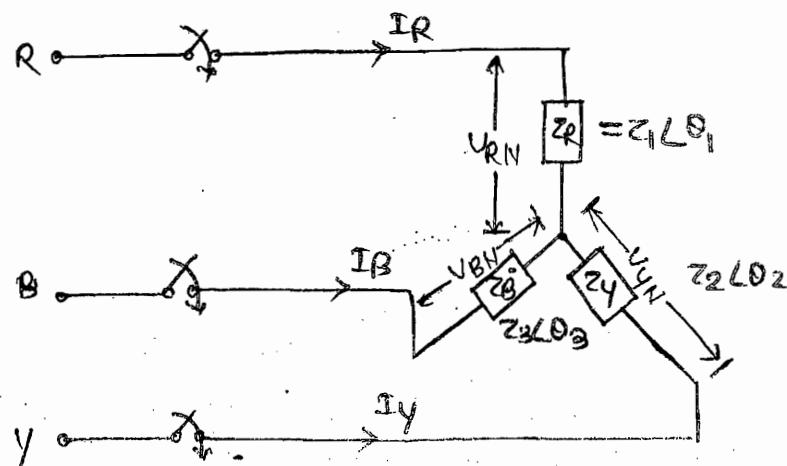
$$I_B = \frac{V_{BN} L^{+120^\circ}}{Z_B}$$

$$I_R = \frac{V_{RN}}{Z_R} = \frac{V_{RN}}{Z_1 L^0} L^0$$

$$I_Y = \frac{V_{YN}}{Z_Y} = \frac{V_{YN}}{Z_2 L^0} L^{-120^\circ}$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{V_{BN}}{Z_3 L^0} L^{+120^\circ}$$

(ii) 3φ, 3wire Unbalanced Y-load



* This is most problematic case because the vol. distribution at every impedance is different which is not acceptable case.

$$V_{RB} = I_1 Z_R + (I_1 - I_2) Z_B$$

$$= I_1 (Z_R + Z_B) - I_2 Z_B \quad \text{--- (1)}$$

$$V_{BY} = (I_2 - I_1) Z_B + I_2 Z_Y$$

$$= -I_1 Z_B + I_2 (Z_Y + Z_B) \quad \text{--- (2)}$$

$$\begin{bmatrix} V_{RB} \\ V_{BY} \end{bmatrix} = \begin{bmatrix} Z_R + Z_B & -Z_B \\ -Z_B & Z_Y + Z_B \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

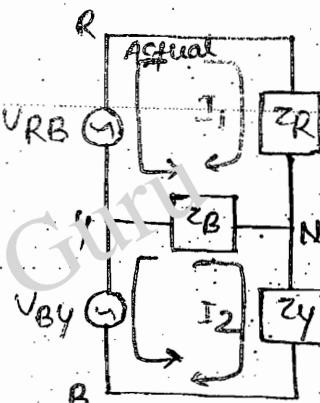
$$\Delta Z = [(Z_R + Z_B)(Z_Y + Z_B) - Z_B^2]$$

$$\Delta Z = [Z_R Z_Y + Z_R Z_B + Z_B Z_Y + Z_B^2 - Z_B^2]$$

$$\Delta Z = [Z_R Z_Y + Z_R Z_B + Z_B Z_Y]$$

$$I_2 = \frac{1}{\Delta Z} \begin{bmatrix} Z_R + Z_B & V_{RB} \\ -Z_B & V_{BY} \end{bmatrix}$$

$$I_2 = \frac{V_{BY}(Z_R + Z_B) + V_{RB}Z_B}{Z_R Z_Y + Z_R Z_B + Z_B Z_Y}$$



$$I_1 = -\frac{1}{\Delta Z} \begin{bmatrix} V_{RE} \\ V_{BY} \end{bmatrix} \begin{bmatrix} z_4 + z_B \\ z_4 + z_B \end{bmatrix}$$

$$I_1 = \frac{V_{RB}(z_4 + z_B) + V_{BY}z_B}{z_R z_4 + z_R z_B + z_B z_4}$$

From the eqn of I_1 :-

Our given Ref. Vol. are V_{R4} , V_{YB} , V_{BR}

$$I_R = I_1 = \frac{-V_{BR}(z_4 + z_B) - V_{YB}z_B}{z_R z_4 + z_R z_B + z_B z_R}$$

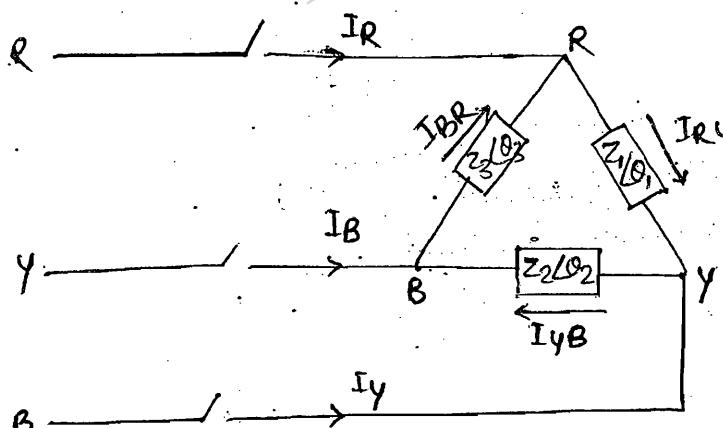
$$I_R = I_1 = -\frac{[V_{BR}(z_4 + z_B) + V_{YB}z_B]}{z_R z_4 + z_R z_B + z_B z_R}$$

From the eqn of I_2 :-

$$I_Y = I_2 = \frac{-V_{YB}(z_R + z_B) - V_{BR}z_B}{z_R z_4 + z_R z_B + z_B z_R}$$

$$I_Y = I_2 = -\frac{[V_{BR}(z_R + z_B) + V_{YB}z_B]}{z_R z_4 + z_R z_B + z_B z_R}$$

(iii) 3φ, 3 wire Unbalanced load ($Y_S - \Delta_L$) →

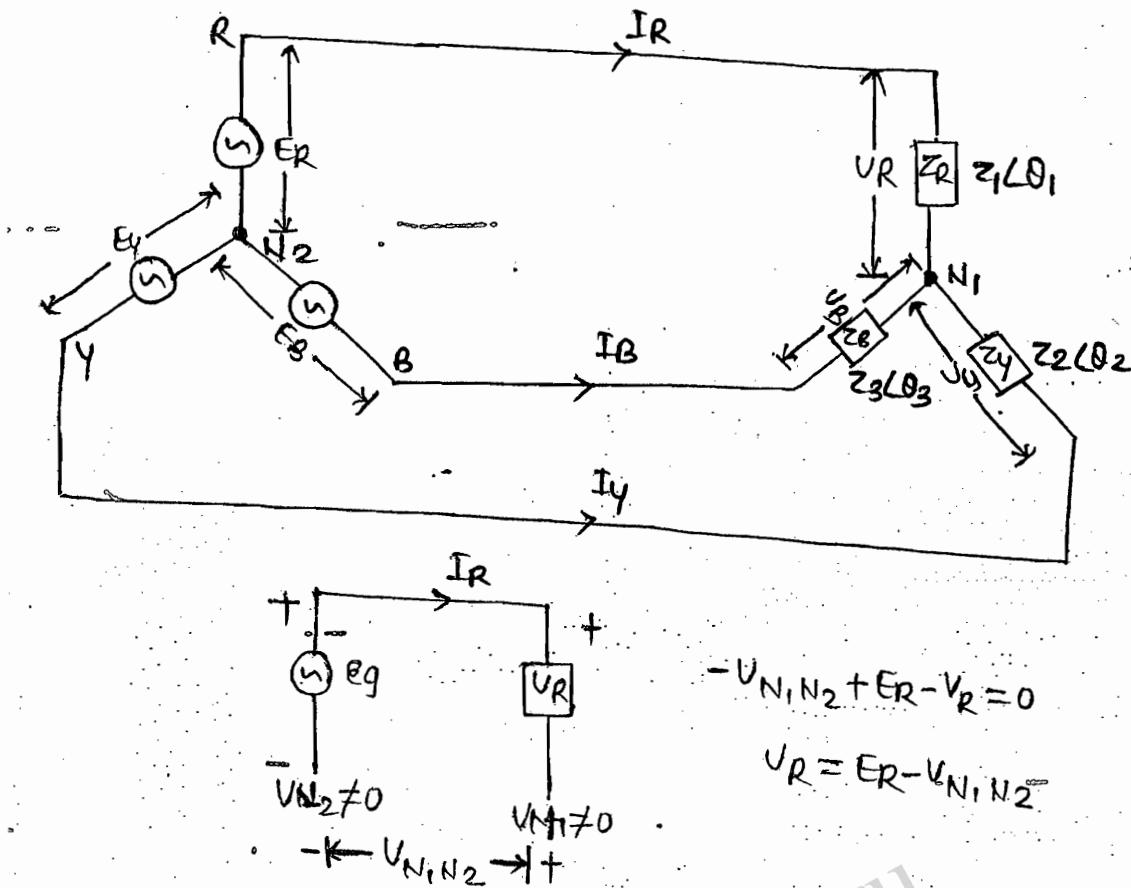


$$I_{RY} = \frac{V_{R4}}{z_L z_O} \angle 0^\circ$$

$$I_{YB} = \frac{V_{YB}}{z_L z_O} \angle -120^\circ$$

$$I_{BR} = \frac{V_{BR}}{z_L z_O} \angle +120^\circ$$

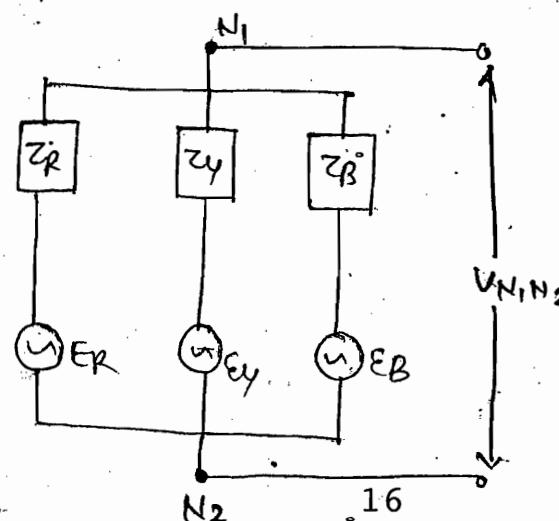
* Floating Point Neutral → When the load neutral is at N_1 & the source neutral is at N_2



$$-V_{N_1 N_2} + E_R - V_R = 0$$

$$V_R = E_R - V_{N_1 N_2}$$

In 3 wire 1 Connected load the neutral is not connected to source neutral. Hence when the load is unbalanced, the load will not be at zero volt potential. Hence the vol. of load neutral wrt source neutral is known neutral shift voltage or neutral displacement voltage & the respective load neutral is called as floating point neutral.



$$V_{N_1 N_2} = \frac{E_R Y_R + E_Y Y_Y + E_B Y_B}{Y_R + Y_Y + Y_B}$$

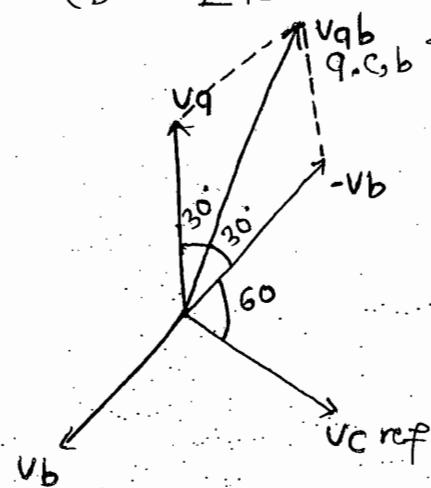
where $Y_R = \frac{1}{Z_R}$, $Y_Y = \frac{1}{Z_Y}$, $Y_B = \frac{1}{Z_B}$

- Q. → The C-φ voltage of a balanced Δ connected Supply system is $230 \angle -25^\circ$. What is the value of V_{AB} . If the phase seq. is ACB.
- (a) $398 \angle -25^\circ$ (b) $398 \angle 65^\circ$ (c) $398 \angle -5^\circ$ (d) $398 \angle -120^\circ$

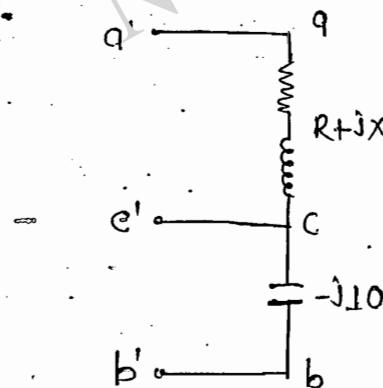
Soln → $V_C = 230 \angle -25^\circ$

$$V_{AB} = \sqrt{3} 230 \angle -25^\circ + 90^\circ$$

V_{qb} = 398 ∠ 65°



- Q. → The ckt shown in the fig is supplied by a symmetrical 3φ sys. of voltages. Find the value of R & X ; if the current flowing Cφ is 0. Assume abc phase seq.



Soln → Phase seq. is abc, so

$$V_{ab} = V_{bc} = V_{ca}$$

Current flowing through c is 0

$$I = \frac{V_{ca}}{R+jX} \angle +120^\circ$$

$$I = \frac{V_{bc} L - 120}{-j10} = \frac{V_{bc} L - 120}{10L90}$$

$$\frac{V_{bc} L - 120}{10L90} = \frac{V_{ca} L - 120}{R+jX}$$

$$\frac{L - 120 + 90}{10} = \frac{1/L120}{R+jX}$$

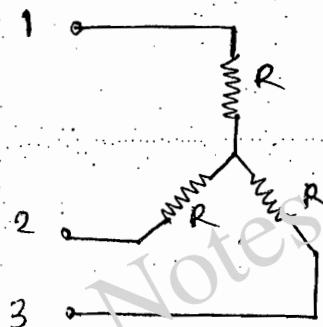
$$\frac{1}{10L30} = \frac{1/L120}{R+jX}$$

$$R+jX = (1/L120)(10L30)$$

$$R+jX = -8.66 + 5j$$

$$|R| = 8.66 \Omega, |X| = 5 \Omega$$

P → In the ckt shown below find the vol. across each branch as v.
OF V_L. Assume 1 2 3 phase seq.



Soln →

Voltage seq: is V₁₂ V₂₃ V₃₁

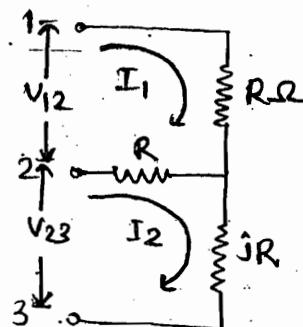
$$V_{12} = I_1 R + (I_1 - I_2) R$$

$$V_{12} L0' = I_1 (2R) - I_2 R \quad \dots \text{(i)}$$

$$V_{23} L - 120 = (I_2 - I_1) R + I_2 jR$$

$$V_{23} L - 120 = -I_1 R + I_2 (R + jR) \quad \dots \text{(ii)}$$

$$\begin{bmatrix} V_{12} L0' \\ V_{23} L - 120 \end{bmatrix} = \begin{bmatrix} 2R & -R \\ -R & R + jR \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



$$\Delta R = 2R(R+jR) = R$$

$$= 2R^2 + 2jR^2 - R^2$$

$$\Delta R = R^2 + 2jR^2$$

$$\boxed{\Delta R = R^2(1+2j)}$$

$$I_1 = \frac{1}{\Delta R} \begin{bmatrix} V_{12}L_0 & -R \\ V_{23}L-120^\circ & R+jR \end{bmatrix}$$

$$= \frac{V_{12}L_0(R+jR) + V_{23}L-120^\circ(R)}{R^2(1+2j)}$$

$$= \frac{V_{12}L_0(1+j) + V_{23}L-120^\circ}{R(1+2j)}$$

$$I_1 = \frac{V_L}{R} \left[\frac{1L_0(1+j) + 1L-120^\circ}{1+2j} \right]$$

$$I_1 = (0.231L-48^\circ) \frac{V_L}{R}$$

$$I_2 = \frac{1}{\Delta R} \begin{bmatrix} 2R & V_L L_0 \\ -R & V_L L-120^\circ \end{bmatrix}$$

$$= \frac{1}{\Delta R} [V_L L-120^\circ(2R) + V_L L_0 \cdot R]$$

$$I_2 = \frac{V_L L-120^\circ \times 2 + V_L L_0}{R(1+2j)}$$

$$= \frac{V_L}{R} \left[\frac{2L-120^\circ + 1L_0}{1+2j} \right]$$

$$I_2 = (0.774L-153^\circ) \frac{V_L}{R}$$

$$(I_1 - I_2) = \frac{V_L}{R} (0.866L12.01^\circ)$$

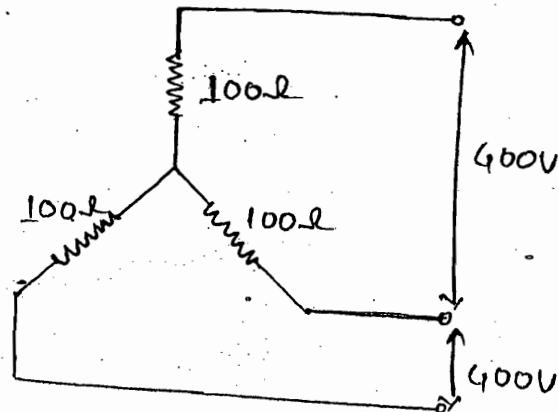
$$V_{1st \text{ ph.}} = 0.23V_L = 23\%$$

$$V_{3rd \text{ ph.}} = 86\%$$

$$V_{2nd \text{ ph.}} = 77.4 \times 19$$

- Q. → 3, 100Ω resistors are connected to 3ϕ , $400V$ AC supply. Find Power consumed when resistors are connected in
 (i) Y-configuration (ii) A-configuration
 (iii) Open Y (iv) open A.

SOLN → (i) Y-configuration



$$P = U_L I_L = \frac{400 \times 400}{100} = 1600W$$

(OR)

$$P = 3U_{ph} I_{ph} \cos\theta$$

$$= 3 \times \frac{400}{\sqrt{3}} \times \frac{4}{\sqrt{3}} \times 1$$

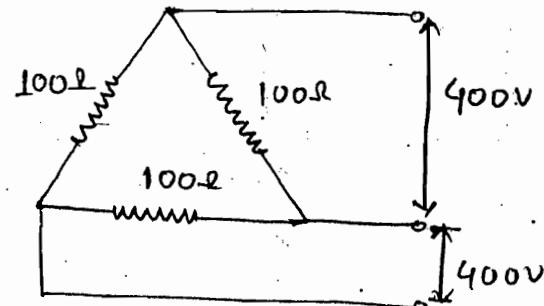
$$\boxed{P = 1600W}$$

(ii)

$$P = 3U_{ph} I_{ph} \cos\phi$$

$$= 3 \times 400 \times 4$$

$$\boxed{P = 4800W}$$

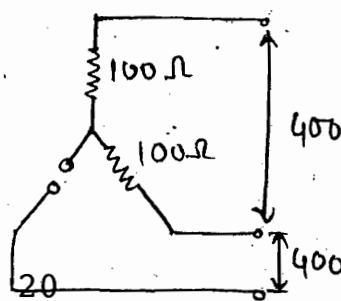


(iii)

$$P = \frac{400^2}{200}$$

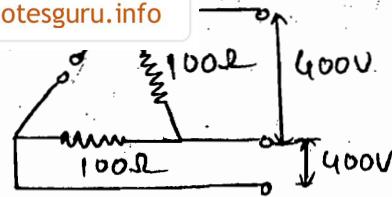
$$= \frac{1600}{200}$$

$$\boxed{P = 800W}$$

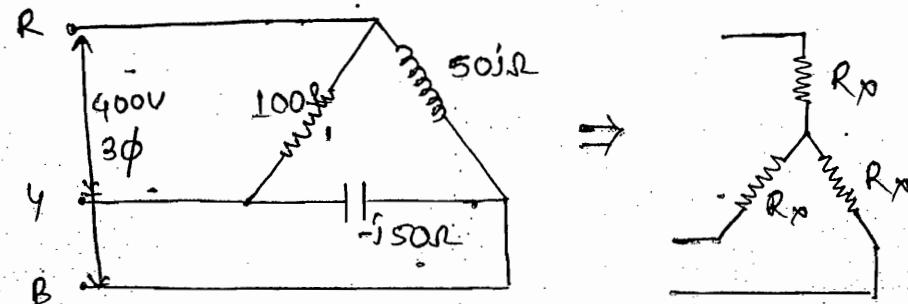


$$(iv) P_{\Delta} = \frac{800^2}{200}$$

$$P_{\Delta} = 3200W$$



Q → A set of 3 equal resistor of value R_x are connected in λ across RYB. Consumes the same power as the unbalanced Δ connected load as shown in the Fig. Find the value of R_x .



Sol² →

$$P_{\Delta} = \frac{400^2}{100} = 1600W$$

$$P_{\lambda} = \frac{400^2}{3R_x}$$

$$1600 = \frac{400^2}{3R_x}$$

$$R_x = 100\Omega$$

Q → 3 identical λ connected resistors at 1pu are connected to an unbalanced 3φ supply. The load neutral is isolated. The symmetrical component of the line vol. in pu are

$$V_{ab_1} = XLO_1, \quad V_{ab_2} = YLO_2$$

If pu val. are with the respective base values, the phase to neutral seq. voltages are:-

$$(a) V_{an_1} = XLO_1 + 30^\circ \quad (b) V_{an_1} = XLO_1 - 30^\circ$$

$$V_{an_2} = YLO_2 - 30^\circ \quad V_{an_2} = YLO_2 + 30^\circ$$

$$(c) V_{an_1} = \frac{X}{\sqrt{3}} LO_1 - 30^\circ \quad (d) V_{an_1} = \frac{X}{\sqrt{3}} LO_1 - 60^\circ$$

$$V_{an_2} = \frac{Y}{\sqrt{3}} LO_2 + 30^\circ$$

$$V_{an_2} = \frac{Y}{\sqrt{3}} LO_2 - 60^\circ$$

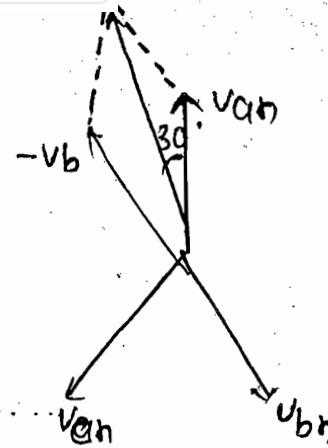
Solⁿ → +ve phase seq.

$$V_{ab1} = \sqrt{3} \cdot V_{an1}$$

$$V_{an1} = \frac{X \angle 0^\circ - 30^\circ}{\sqrt{3}}$$

-ve phase seq.

$$V_{an2} = \frac{Y}{\sqrt{3}} \angle 0^\circ + 30^\circ$$



Q → A 3φ balanced Δ connected vol. source with freq: ω rad/sec. is connected to a Δ connected balanced load which is purely inductive. The instantaneous line current & phase to neutral voltages are denoted by i_a, i_b, i_c & V_{an}, V_{bn}, V_{cn} respectively & their RMS values are denoted by v & I .

$$\text{If } R = [V_{ab}, V_{bn}, V_{cn}] \begin{bmatrix} 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Solⁿ →

$$V_{an} = V_{ph} \angle 0^\circ$$

$$V_{bn} = V_{ph} \angle -120^\circ$$

$$V_{cn} = V_{ph} \angle 120^\circ$$

$$I_a = I \angle 90^\circ$$

$$I_b = I \angle -120^\circ$$

$$I_c = I \angle 120^\circ$$

$$R = \left[\frac{-V_{bn}}{\sqrt{3}} + \frac{V_{cn}}{\sqrt{3}}, \frac{V_{an}}{\sqrt{3}} - \frac{V_{cn}}{\sqrt{3}}, \frac{-V_{an}}{\sqrt{3}} + \frac{V_{bn}}{\sqrt{3}} \right] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$R = \left[\frac{-V_{bn}}{\sqrt{3}} i_a + \frac{V_{cn}}{\sqrt{3}} i_a + \frac{V_{an}}{\sqrt{3}} i_b - \frac{V_{cn}}{\sqrt{3}} i_b - \frac{V_{an}}{\sqrt{3}} i_c + \frac{V_{bn}}{\sqrt{3}} i_c \right]$$

Notes Guru

$$R = \frac{1}{\sqrt{3}} \left[v_{bn} (i_c - i_a) + v_{an} (i_a - i_b) + v_{an} (i_b - i_c) \right]$$

$$R = \frac{Iv}{\sqrt{3}} \left\{ \begin{aligned} & [(1L-120)(1L30-1L-90)] + [(1L120)(1L-90-1L-210)] \\ & + [(1L0)(1L-210-1L30)] \end{aligned} \right\}$$

$$R = \frac{vI}{\sqrt{3}} (0)$$

$$\boxed{R=0}$$

Q. → A 3φ, 4 wire CBA system with an effective line voltage of 169.7 V has 3 impedances of $20L-30^\circ$ in a Δ connection. Determine the line current.

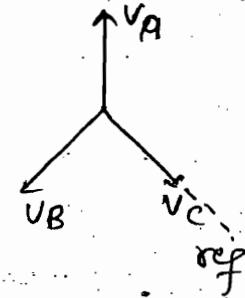
Soln → $v_L = 169.7 \text{ V}$

$$Z = 20L-30^\circ$$

$$I_A = I_4 = \frac{v_L / \sqrt{3}}{Z} = \frac{169.7 / \sqrt{3} / 120}{20L-30^\circ} = 4.9L150^\circ$$

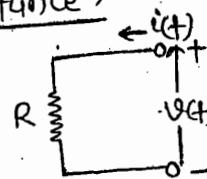
$$I_B = I_2 = \frac{98L-120^\circ}{20L-30^\circ} = 4.9L-90^\circ$$

$$I_C = I_3 = \frac{98L0}{20L-30^\circ} = 4.9L30^\circ$$



* Basic components of any ele. N/W →

(1) Resistance →



* Linear

* Bilateral.

$$V(t) = R \cdot i(t)$$

$$i(t) = \frac{1}{R} \cdot V(t)$$

... in time domain

$$V = R \cdot I$$

$$I = \frac{1}{R} \cdot V$$

... using phasors

$$V(s) = R \cdot I(s)$$

$$I(s) = \frac{V(s)}{R}$$

... in s-domain

$$s = \sigma + j\omega$$

complex freq.

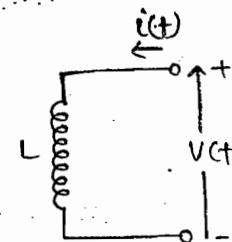
$$I = |I| \angle \theta$$

$$V = |V| \angle \theta$$

... for sinusoidal excitation.

* If the supply is given sinusoidal; then analysis is done only in the phasors.

(2) Inductor →



$$\Phi_m \propto i$$

$$\Phi_m = L \cdot i$$

... linear inductor.

$$V(t) = L \frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L} \int_0^t V(t) dt$$

$$V(s) = sL \cdot I(s)$$

$$I(s) = \frac{1}{sL} V(s)$$

... in s-domain

assuming zero initial condition.

$$Z_L = sL$$

$$Y_L = \frac{1}{sL}$$

$$V = j\omega L \cdot I$$

$$I = \frac{1}{j\omega L} V$$

... for sinusoidal excitation

... using phasors ($s = j\omega$)

$$Z_L = j\omega L$$

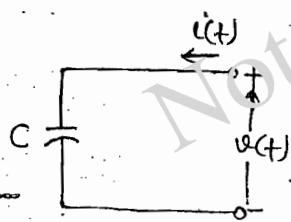
$$Y_L = \frac{1}{j\omega L}$$

* Inductor; the voltage lead current by 90° ($V(s) = j\omega I(s)$).

*some points→

- * In a linear element the variation b/n the terminal Vol. & the terminal current is linear either in the time domain (or) in s-domain (or) in both domains.
- * In a bilateral element the terminal current $i(t)$ flows in either dirn irrespective of the type of polarity of the vol. source applied b/n the 2 terminals of the element.
- * The excitation to the ele. n/w may be non-sinusoidal (or) sinusoidal in nature.
- : When the excitation is not sinusoidal any given n/w can be analysed either in the time domain (or) in the Laplace domain.
- When the excitation is sinusoidal then any given n/w can be analysed only by using phasors by substituting $s=j\omega$.
- When the ele. n/w is analysed in the s-domain the advantages are-
 - i) Any differential (or) integral eqn. is xformed to a linear eqn & the mathematical manipulation becomes simpler
 - ii) The initial condn if any present in the ckt elements are automatically taken care of.

1 Capacitor →



$$q \propto v$$

$q = Cv$ linear element.

$$i(t) = C \frac{dv(t)}{dt}$$

$$v(t) = \frac{1}{C} \int_0^t i(t) dt$$

.... in time domain

$$I(s) = Cs V(s)$$

$$V(s) = \frac{1}{sC} I(s)$$

.... in s-domain

..... assumin zero
initial condn

$$I = j\omega C V$$

$$V = \frac{1}{j\omega C} I$$

.... using phasors

.... for sinusoidal
excitation ($s=j\omega$)

$$\bar{Z}_c = \frac{1}{sC}$$

$$Y_c = sC$$

$$Z_C = \frac{1}{j\omega C}$$

$$Y_C = j\omega C$$

* Voltage leads current by 90° .

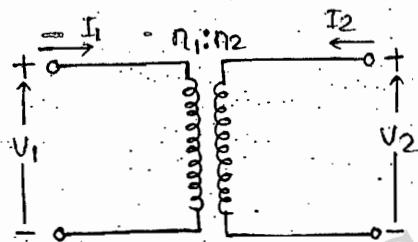
ω	L	C
$\omega = \omega$	$j\omega L$	$\frac{1}{j\omega C}$
$\omega = 0$	SC	OC
$\omega = \infty$	OC	SC

* Electrolytic capacitor is ^{Non} linear capacitor.

In the n/w we will talk about other than electrolytic capacitor.

* The advantage of this capacitor is that small size.

(4.) Transformer →



$$\frac{V_1}{V_2} = \frac{n_1}{n_2}; \quad \frac{I_1}{I_2} = \frac{n_2}{n_1}$$

$n_2 > n_1$... step up

$n_2 < n_1$... step down

If $n_1=1$, $n_2=10$

$$\frac{V_2}{V_1} = \frac{n_2}{n_1} = \frac{10}{1} > 1$$

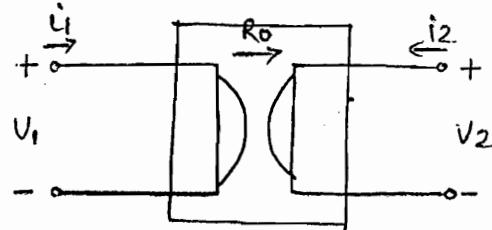
$$\frac{I_2}{I_1} = \frac{n_1}{n_2} = \frac{1}{10}$$

$$\frac{V_2}{V_1} \cdot \frac{I_2}{I_1} = 1$$

$$V_2 I_2 = V_1 I_1$$

- * Any step up Xmer does not work as an amp^r since the power at the o/p terminal is same as the power at the i/p terminal.
- * for any device to work as a amp^r the power at the o/p terminal must always be more than the power at the i/p terminal.

(5) Generator →



R_0 = Coefficient of gyrator

$$\boxed{V_1 = +R_0 i_2}$$

$$\boxed{V_2 = -R_0 i_1}$$

* Impedance inversion.

* A gyrator is a 4 terminal (or) a 2 port device.

* It can be made by using an OPAMP along with some externally connected R & C components.

* The coefficient of the gyrator depends upon the parameters of the OPAMP & numerical values of externally connected R & C components.

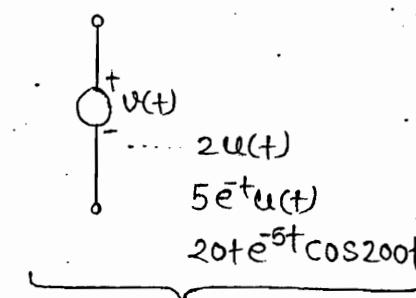
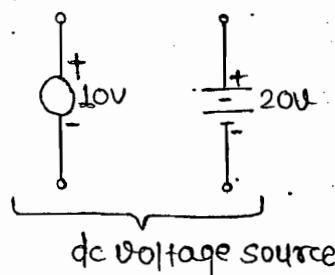
This coefficient has a fixed value for a given gyrator.

* It can be used to simulate eq. value of inductance L.

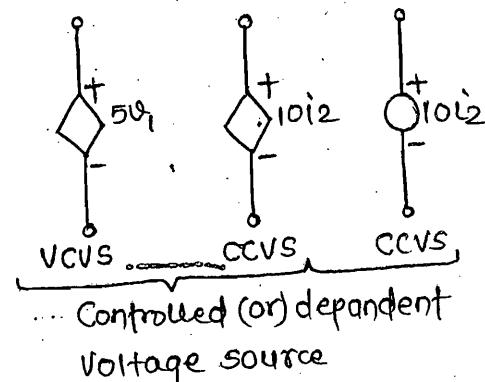
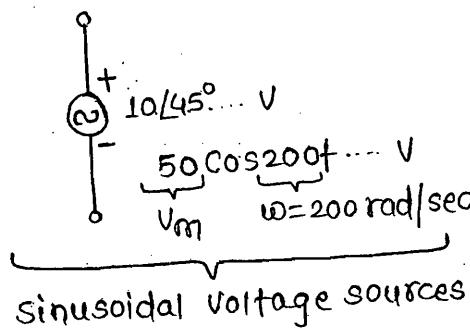
* The gyrator shows an impedance inversion.

Therefore for a capacitive load the i/o impedance is inductive & vice-versa.

(6) Voltage Sources →



time-varying non-sinusoidal
voltage sources.



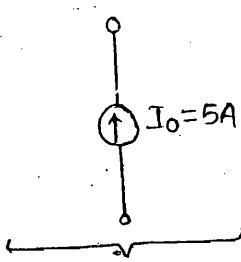
* While analysing any ele. n/w using KVL & KCL eqn the independant & dependant sources are handled exactly in the same manner except in the following 2 cases:-

- Application of superposition theorem.
- Application of Thevenin's & Nortons theorem.

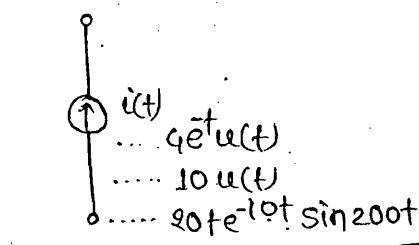
In such cases:-

- All independant vol. sources are SC (or) replaced by there internal impedances.
- All independant current sources are OC (or) replaced by there internal impedances.
- All dependant vol. & current sources remain as they are & therefore this sources are neither SC nor OC.

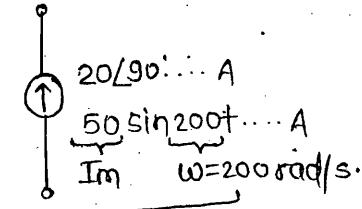
(7) Current sources →



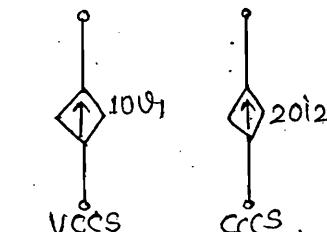
dc current source



time varying non-sinusoidal current sources



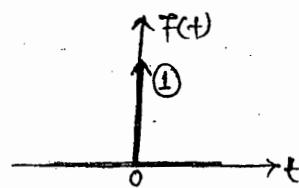
sinusoidal current sources



Controlled (or) dependant current Sources

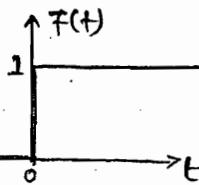
* Basic fns →

(1) Unit impulse $f^n \rightarrow$



$$f(t) = \begin{cases} \infty & ; t=0 \\ 0 & ; t \neq 0 \end{cases}$$

(2) Unit step $f^n \rightarrow$



$$f(t) = \begin{cases} 0 & ; t < 0 \\ 1 & ; t \geq 0 \end{cases}$$

* $\int_{-\infty}^{\infty} \delta(t) dt = 1$

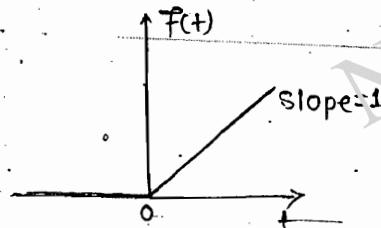
* $\mathcal{L}[u(t)] = \frac{1}{s}$

* $\int_{-\infty}^{\infty} g(t) \cdot \underbrace{\delta(t)}_{\text{exists at } t=0} dt = \overline{g(t)} \Big|_{t=0} = g(0)$

* $\int_{-\infty}^{\infty} g(t) \cdot \underbrace{\delta(t-T)}_{\text{exists at } t=T} dt = g(t) \Big|_{t=T} = g(T)$

* $\mathcal{L}[\delta(t)] = 1$

(3) Unit Ramp $f^n \rightarrow$

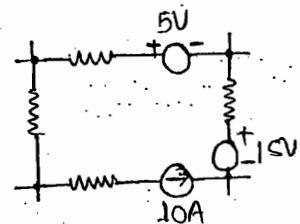
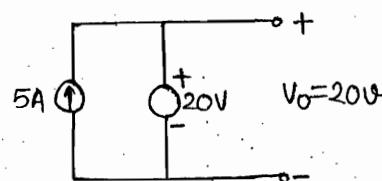
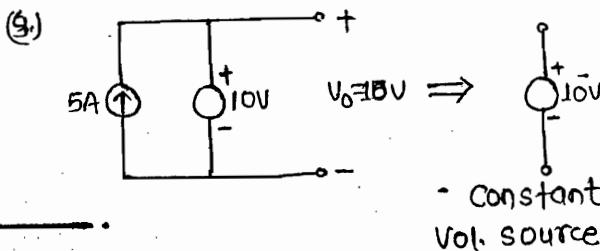
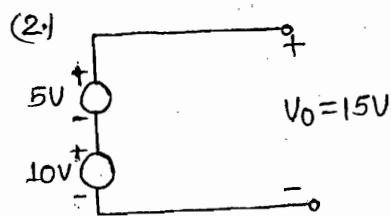
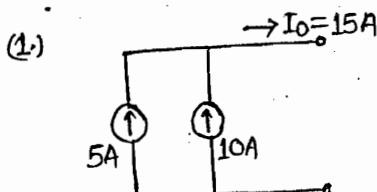


$$f(t) = \begin{cases} 0 & ; t < 0 \\ t & ; t \geq 0 \end{cases}$$

* $f(t) = t \cdot u(t) \text{ (OR) } \tau(t)$

* $\mathcal{L}[\tau(t)] = \frac{1}{s^2}$

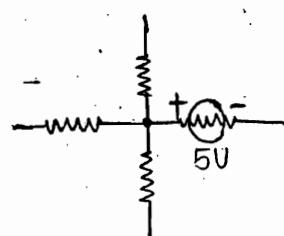
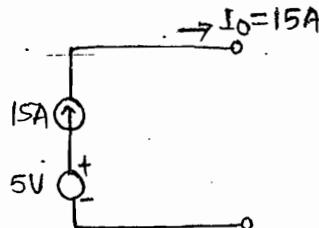
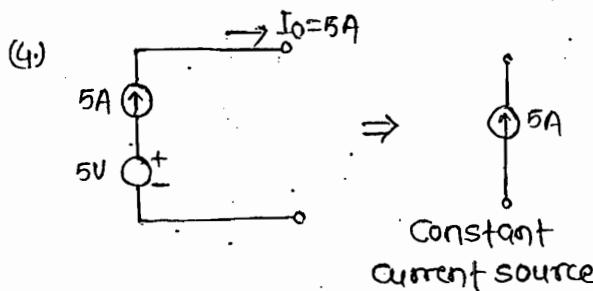
* Basic Concepts →



* A parallel combination of a vol. source & a current source behave as a const vol. source.

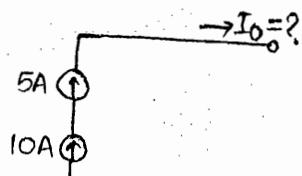
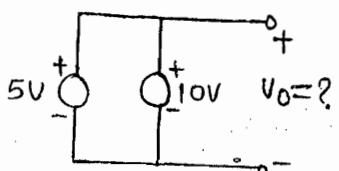
* The vol. across any current source is purely arbitrary & depends only upon externally connected vol. source (or) externally connected elements.

* We can't write a KVL eqⁿ in a closed loop which includes a const. current source since the vol. across it is purely arbitrary & is unknown.

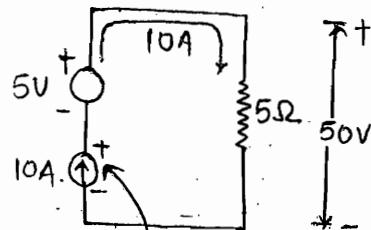
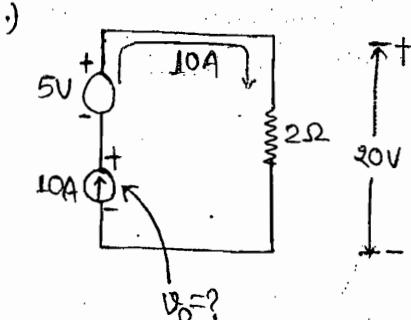
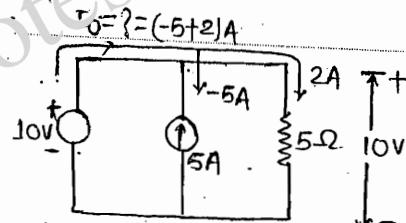
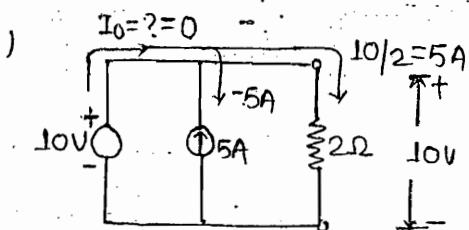
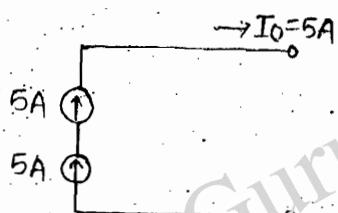
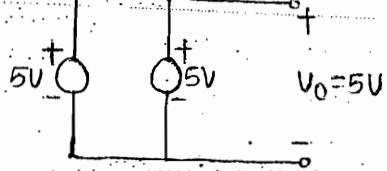


- * A series combination of a current source & a vol. source behave as const current source.
- * The current through any vol. source is purely arbitrary & depends only upon externally connected current source (or) externally connected elements.
- * We can't write KCL eqn at a node with which a const vol. source is connected.
since current through it is arbitrary & is unknown.

5.)



NOT allowed.

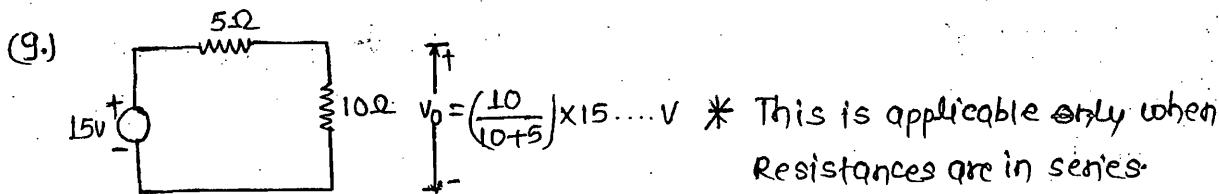


$$\text{By KVL: } -V_o - 5V + 20 = 0$$

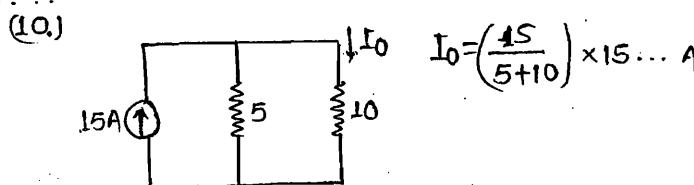
$$V_o = +15V$$

$$\text{By KVL: } -V_o - 5 + 50 = 0$$

$$V_o = 45V$$

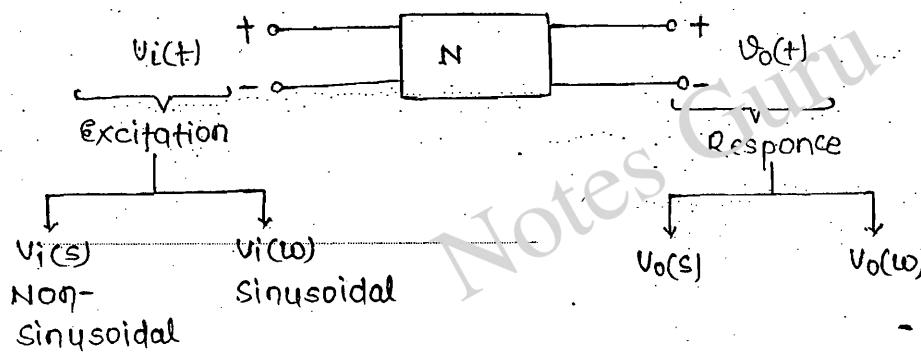


$V = IR$ Potential dividing
 $V \propto R$ Theorem



$\text{V} = IR$ Current dividing
 $I \propto \frac{1}{R}$ theorem.

* Impulse response of any elect. N/W \rightarrow



* Transfer Fⁿ \rightarrow

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = H(s) \cdot V_i(s)$$

$$\text{If; } V_i(t) = \delta(t)$$

$$V_i(s) = 1$$

$$V_o(s) = H(s)$$

$$V_o(t) = \mathcal{L}^{-1}[H(s)]$$

= h(t) Impulse Response of N/W

- * The TF of any η/ω represents the ratio of the o/p vol. to the i/p vol. each in the s-domain assuming all the initial condn in various element of the η/ω to be 0.
- * The TF helps to cal. the corresponding response of η/ω for specified excitation. For a given η/ω the TF has a fixed value.
- * The impulse response f^h represents the o/p vol. of the η/ω when the i/p vol. represents a unit impulse f^1 .
- * The impulse response also represents inverse laplace X-form of the TF of the η/ω .

Ex:- If $v_i(t) = e^{-t}u(t)$; $v_o(t) = e^{-2t}u(t)$

To find $v_o(t)$ if $v_i(t) = e^{-3t}u(t)$

$$\text{Soln. } v_i(t) = e^{-t}u(t); v_o(t) = e^{-2t}u(t)$$

$$v_i(s) = \frac{1}{s+1}; v_o(s) = \frac{1}{s+2}$$

$$\frac{v_o(s)}{v_i(s)} = H(s) = \frac{s+1}{s+2}$$

$$H(s) = \frac{s+1}{s+2} = 1 - \frac{1}{s+2}$$

$$h(t) = \mathcal{L}^{-1}H(s)$$

$$h(t) = s(t) - e^{-2t}u(t)$$

$$\text{Now; } H(s) = \frac{s+1}{s+2}$$

$$v_i(t) = e^{-3t}u(t)$$

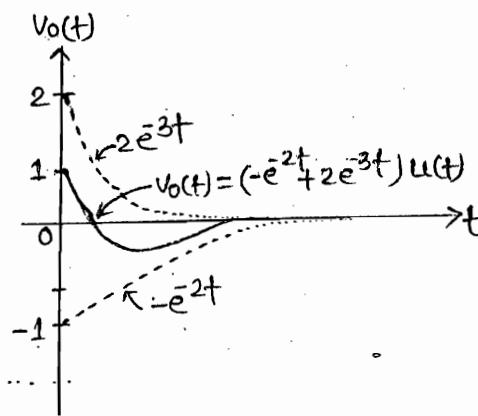
$$v_i(s) = \frac{1}{(s+3)}$$

$$v_o(s) = H(s) \cdot v_i(s)$$

$$= \frac{(s+1)}{(s+2)(s+3)}$$

$$v_o(s) = \frac{-1}{s+2} + \frac{2}{s+3}$$

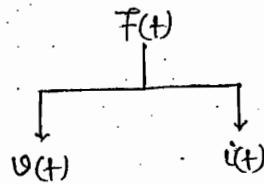
$$v_o(t) = -e^{-2t}u(t) + 2e^{-3t}u(t) \quad \boxed{\frac{1}{34}(-e^{-2t} + 2e^{-3t})u(t)}$$



$V_o(0^+) = 1$ Initial value
..... Initial condⁿ

$V_o(\infty) = 0$ Final value
..... Steady state value
..... Final condⁿ

* Initial & Final value theorem →



Initial value →

$$f(0^+) = f(t) \Big|_{t=0^+}$$

$$f(0^+) = \lim_{s \rightarrow \infty} [sF(s)] \quad \dots \text{Initial value theorem.}$$

Final value →

$$f(\infty) = f(t) \Big|_{t=\infty}$$

$$f(\infty) = \lim_{s \rightarrow 0} [sF(s)] \quad \dots \text{Final value theorem.}$$

* This is valid for other than oscillatory F !

* Laplace Xform of some basic $F^n \rightarrow$

(1.) $\delta(t) = 1$

(6.) $\cos bt = \frac{s}{s^2 + b^2}$

(2.) $u(t) = \frac{1}{s}$

(3.) $\tau(t) = \frac{1}{s^2}$

(4.) $t^n u(t) = \frac{n!}{s^{n+1}}$

(5.) $e^{-qt} = \frac{1}{s+q}$

(6.) $\tilde{F}(t-T) = F(s)e^{-sT}$

(7.) $\sin bt = \frac{b}{s^2 + b^2}$

(9.) $\frac{dF(t)}{ds} = SF(s) - f(0^+)$

$f(0^+) = f(t)|_{t=0^+}$

(10.) $\int_0^t f(t) dt = \frac{F(s)}{s} + \frac{f(0^+)}{s}$

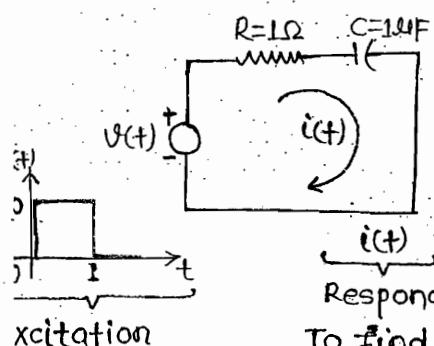
$f'(0^+) = \left[\int_0^t f(t) dt \right]_{t=0^+}$

(11.) $\bar{e}^{-at} \sin bt = \frac{b}{(s+a)^2 + b^2}$

(12.) $\bar{e}^{-at} \cos bt = \frac{s+a}{(s+a)^2 + b^2}$

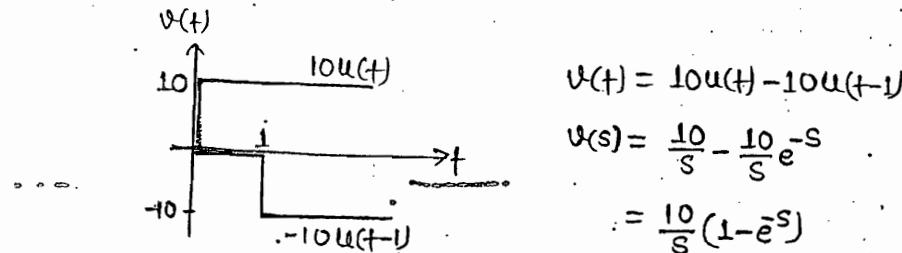
(13.) $t e^{-at} = \frac{1}{(s+a)^2}$

Example \rightarrow Series RC ckt



xcitation To find $i(t)$ for $t > 0$

assume: zero IC.



Applying KVL on the given ckt

$$R i(t) + \frac{1}{C} \int_0^t i(t) dt = V(t)$$

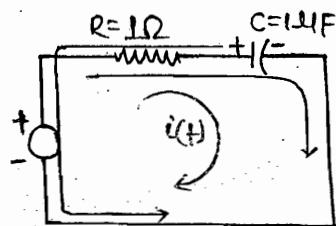
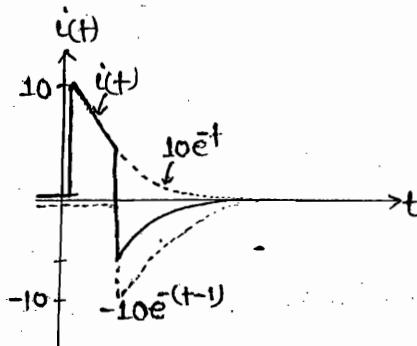
$$\Rightarrow I(s) + \frac{I(s)}{s} = \frac{10}{s}(1 - e^{-s})$$

$$+ \frac{i(0^+)}{s} \rightarrow \left(\int_0^t i(t) dt \right) \Big|_{t=0^+} = Q(t) \Big|_{t=0^+} = Q(0^+) = 0$$

$$I(s) \neq \frac{s+1}{s} = \frac{10}{s}(1-e^{-s})$$

$$I(s) = \frac{10}{(s+1)} - \frac{10}{(s+1)} e^{-s}$$

$$i(t) = 10e^t u(t) - 10e^{-(t-1)} \cdot u(t-1)$$



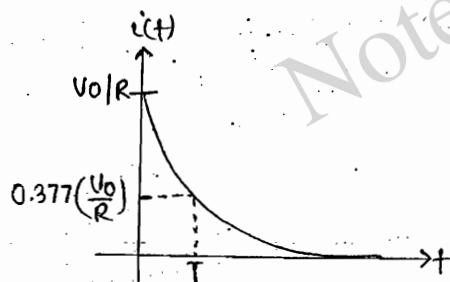
* If $v(t) = V_0 u(t)$, then $i(t) = \frac{V_0}{R} e^{-t/T} u(t)$

$T = RC$ Time constant

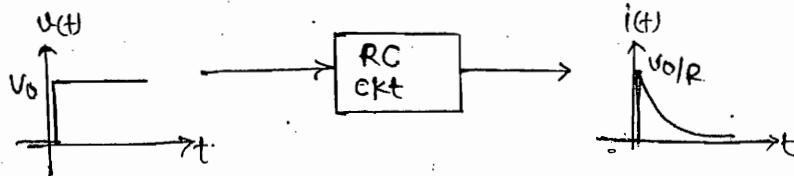
Time constant \rightarrow

$$i(t) = \frac{V_0}{R} e^{-t/T} u(t)$$

$$at t=T; i(t) = \frac{V_0}{R} e^{-1} \cong 0.377 \frac{V_0}{R}$$



Summary \rightarrow Series RC Ckt



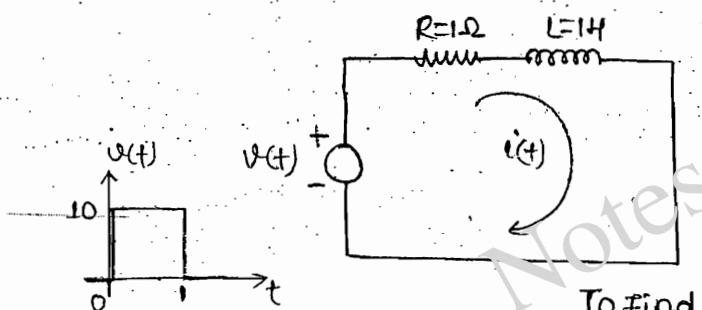
$$i(t) = \frac{V_0}{R} e^{-t/T} u(t)$$

$$T = RC$$

- * Initially at $t=0$ any capacitor acts as a SC, so that the current through the RC n/w is max^m.
- At $t=\infty$ (or) in the steady state cond'n any capacitor acts as an OC; so that the current through the n/w is 0.
- * For pulse excitation if reversal of current takes place in series RC n/w.
- * For step excitation the current through RC n/w decreases exponentially whose rate is controlled by the time const. of n/w.
- * The time cons. of RC n/w represents the time at which the current through the n/w is decreased to 37.7% of its initial value.
This time const depends only upon the numerical values of R & C components only.

$$T = \text{Req. Ceq} \quad \begin{array}{l} \text{Voltage source} \Rightarrow \text{SC} \\ \text{Current source} \Rightarrow \text{OC} \end{array}$$

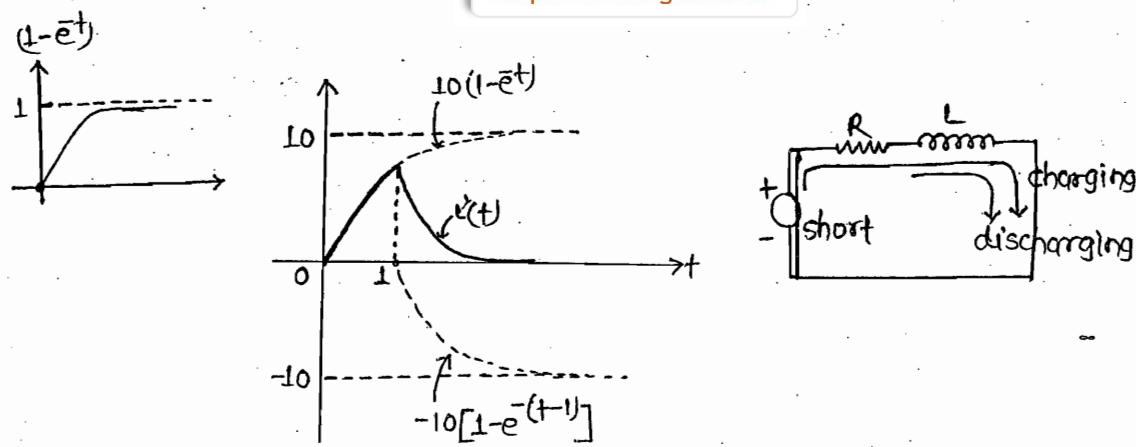
Example \rightarrow Series RL Ckt



To find
 $i(t)$ for $t > 0$
assume: zero initial cond'n

applying KVL;

$$\begin{aligned} R i(t) + L \frac{di(t)}{dt} &= v(t) \\ \Rightarrow I(s) + L s I(s) - i(0^+) &= \frac{10}{s} (1 - e^{-s}) \\ \Rightarrow I(s) &= \frac{10}{s(s+1)} (1 - e^{-s}) = 10 \left(\frac{1}{s} - \frac{1}{s+1} \right) (1 - e^{-s}) \\ \Rightarrow I(s) &= 10 \left[\frac{1}{s} - \frac{1}{s+1} \right] - 10 \left[\frac{1}{s} - \frac{1}{s+1} \right] e^{-s} \\ i(t) &= 10(1 - e^t) u(t) - 10(1 - e^{-(t+1)}) 3e^{-(t+1)} \end{aligned}$$

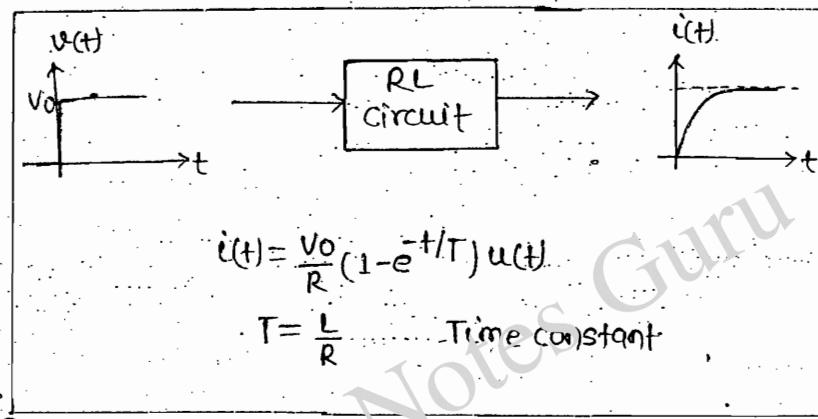


* If $V(t) = V_0 u(t)$; $R=R$ & $L=L$

$$i(t) = \frac{V_0}{R} (1 - e^{-t/T}) u(t)$$

$$T = \frac{L}{R} \quad \text{Time constant}$$

* Summary →



* Initially at $t=0$ an inductor acts as ∞ so that the current through series $RL \eta/w$ is 0.

* For pulse excitation reversal of current takes place in series $RC \eta/w$ whereas no such reversal takes place in a series $RL \eta/w$.

* For step excitation the current through the inductor increases exponentially with time constant T .

This time constant T controls the rate of increment of the current through the $RL \eta/w$ & depends only upon numerical values of $R-L$ components.

* Analysis of an electrical net/w using KVL & KCL eqn →

KVL →

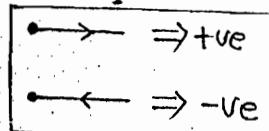
Voltage drop $\Rightarrow +ve$ Convention
Voltage Rise $\Rightarrow -ve$

$$\text{no. of KVL eqn} = b - (n - 1)$$

where; $b = \text{no. of branches}$

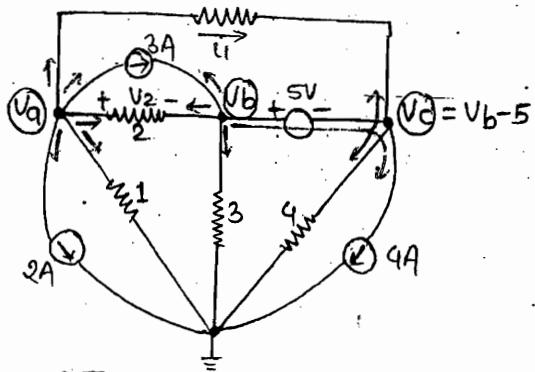
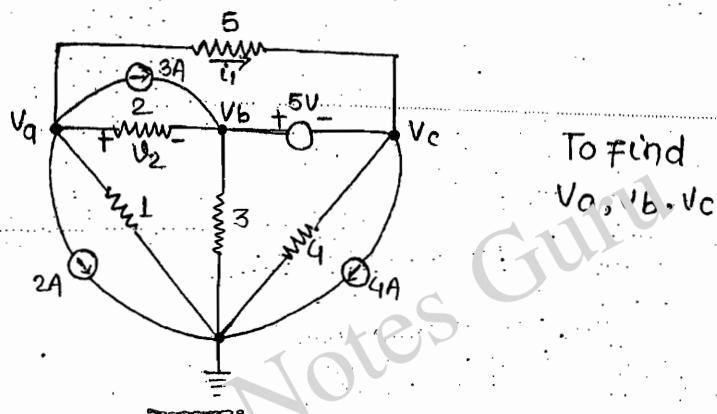
$n = \text{no. of nodes.}$

KCL →

 convention

$$\text{no. of KCL eqn} = n - 1$$

Example:-



dependant
OR
Redundant
OR
Super-node

$$\begin{aligned} V_b - V_c &= 5 \\ V_b - 5 &= V_c \end{aligned}$$

KCL (V_a)

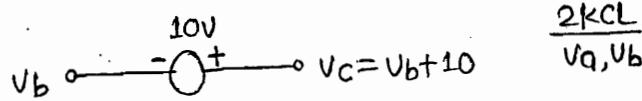
$$+2 + \frac{V_a - 0}{1} + \frac{V_a - V_b}{2} + 3 + \frac{V_a - V_c}{5} = 0 \quad \text{--- (i)}$$

KCL (V_b)

$$-3 + \frac{V_b - V_a}{2} + \frac{V_b - 0}{3} + \left[\frac{V_c - 0}{4} + 4 + \frac{V_c - V_a}{5} \right] = 0 \quad \text{--- (ii)}$$

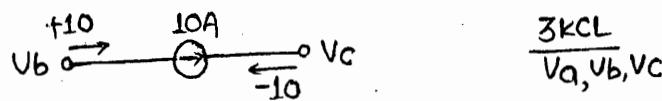
$$\frac{V_a}{1}$$

Case(i) →



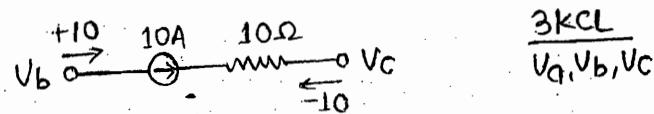
$$\frac{2KCL}{V_a, V_b}$$

Case(ii) →



$$\frac{3KCL}{V_a, V_b, V_c}$$

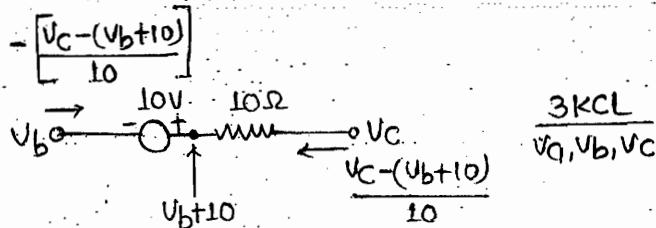
Case(iii) →



$$\frac{3KCL}{V_a, V_b, V_c}$$

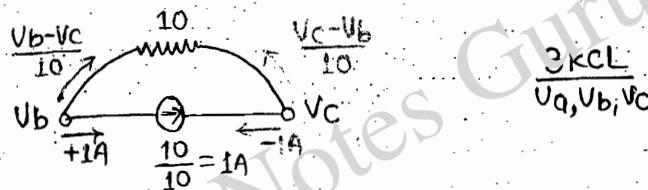
----- same as case(ii) -----

Case(iv) →



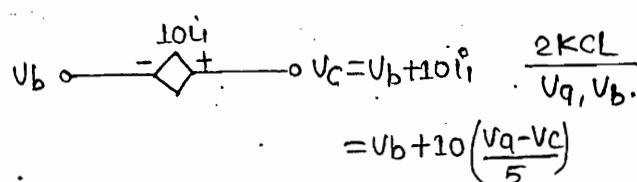
$$\frac{3KCL}{V_a, V_b, V_c}$$

Case(iv)(a) →



$$\frac{3KCL}{V_a, V_b, V_c}$$

Case(v) →

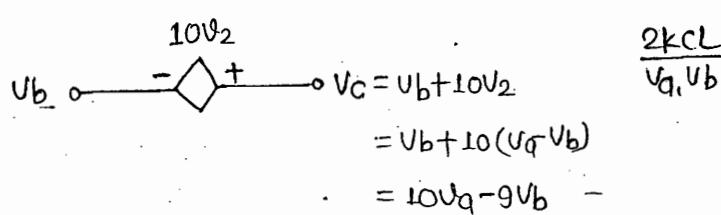


$$= Vb + 10 \left(\frac{Vq - Vc}{5} \right)$$

$$3Vc = 2Vq + Vb$$

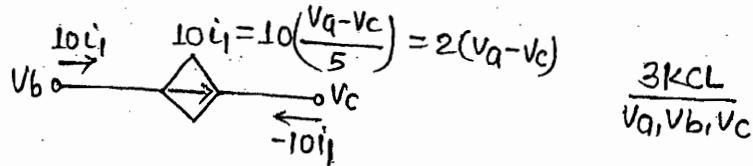
$$Vc = \frac{1}{3}(2Vq + Vb)$$

Case(vi) →

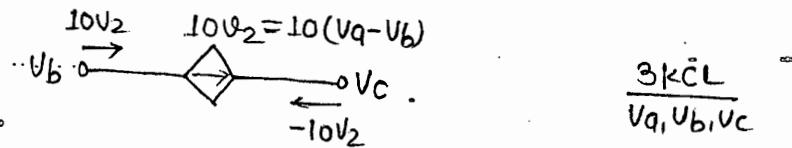


$$\frac{2KCL}{V_a, V_b}$$

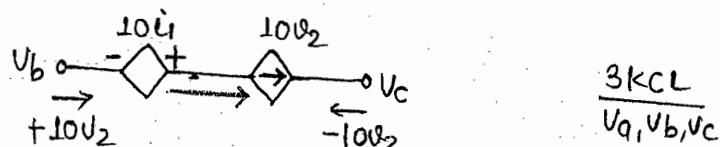
Case(7) →



Case(8) →

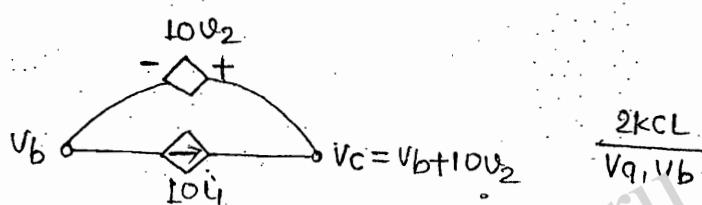


Case(9) →



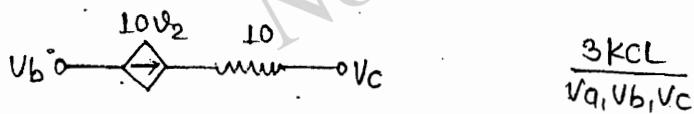
..... same as
Case(8)

Case(10) →



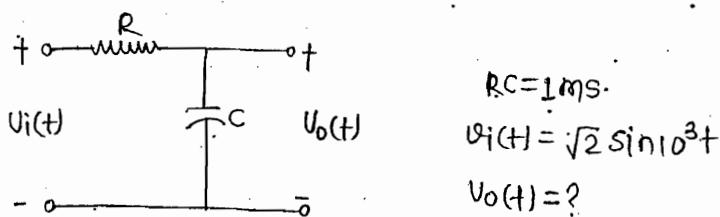
..... same as
Case(6)

Case(11) →



..... same as
Case(8)

Ques(34)
chap.(1)



Soln →

$$V_i(t) = \sqrt{2} \sin 10^3 t$$

V_C $\omega = 10^3 \text{ rad/s}$

$$V_o(t) = V_0 \sin 10^3 t$$

= ?

$$\begin{aligned}
 \frac{V_o}{V_i} = H(\omega) &= \frac{1/j\omega C}{R + 1/j\omega C} \\
 &= \frac{1}{1 + j\omega RC} \\
 &= \frac{1}{1 + j10^3 \times 10^{-3}} \\
 &= \frac{1}{1 + j1} \\
 &= \frac{1}{\sqrt{2} \angle 45^\circ}
 \end{aligned}$$

$$V_o = \frac{1}{\sqrt{2} \angle 45^\circ} \times V_i$$

$$= \frac{1}{\sqrt{2} \angle 45^\circ} \times \sqrt{2}$$

$$V_o = \frac{1}{\angle 45^\circ}$$

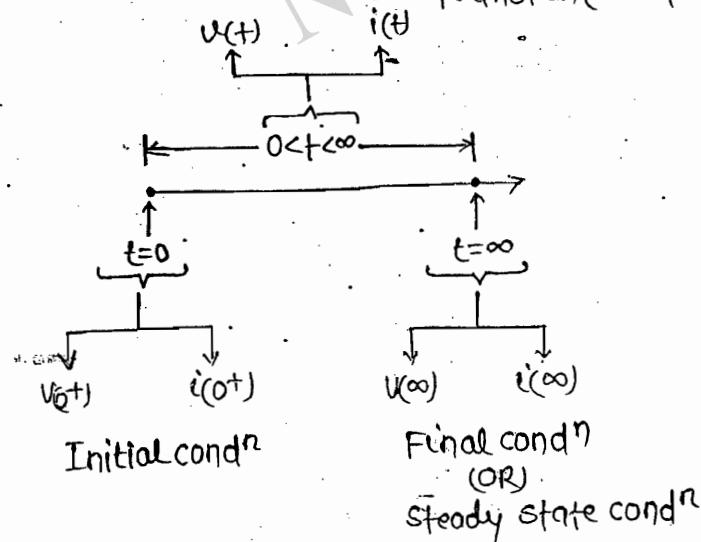
$$V_o(t) = V_o \sin 10^3 t$$

$$= \frac{1}{\angle 45^\circ} \sin 10^3 t$$

$$V_o(t) = \sin(10^3 t - 45^\circ)$$

* Transient Response of elect. N/w \rightarrow

Transient Response



* Behaviors of L & C in any ckt →

Case(1) → In time domain;

Element	$t=0^+$	$t=\infty$
C	i_C	$\frac{d}{dt}i_C = 0$
L	v_L	$\frac{d}{dt}v_L = 0$
V_0	$+V_0 -$	$+V_0 -$
I_0	I_0	I_0

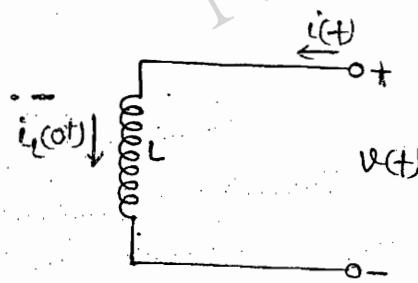
* Initially at $t=0$ any capacitor acts as a sc whereas any inductor acts as an open ckt.

* At $t=\infty$ (or) in the steady state condn any capacitor acts as an oc & an inductor acts as a sc.

* Initially at $t=0$ any capacitor with initial vol. acts as a constant vol. source whereas any inductor with initial current acts as a constant current source.

Case(2) → In s-domain;

(a) Inductor →



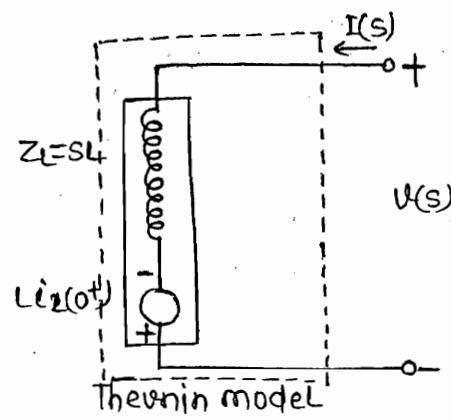
* Current $i(t)$ & $i(0^+)$ are different with each other.

$$V(t) = L \frac{di(t)}{dt}$$

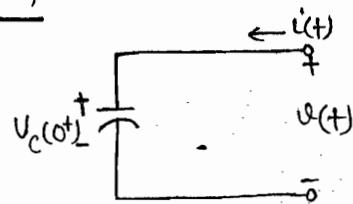
$$V(s) = [s I(s) - i(0^+)]$$

$$V(s) = sLI(s) - Li(0^+) \quad \text{... KVL}$$

... Series ckt



(b) Capacitor \rightarrow



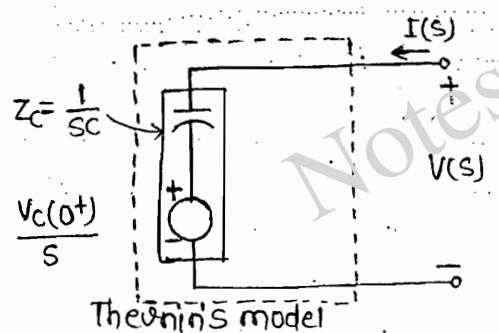
$$i(t) = C \frac{dV(t)}{dt}$$

$$I(s) = -C [sV(s) - V_C(0+)]$$

$$V(s) = \frac{1}{sC} \cdot I(s) + \frac{V_C(0+)}{s}$$

KVL

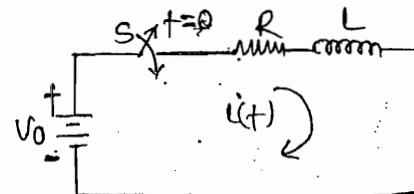
series ckt



Que \rightarrow In the ckt shown the switch is initially open. The switch is closed at $t=0$. Find the current & the rate of change of current through the n/w just after the switch has been closed?

To find $\rightarrow i(0+)$

$$\frac{di(0+)}{dt}$$



SOLN \rightarrow at $t=0^-$

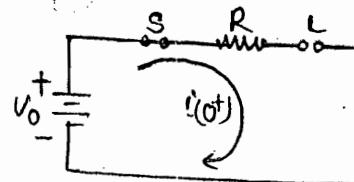
S.... open

$$i(0^-) = 0$$

at $t=0^+$

S.... closed.

L.... OC



$$i(0^+) = 0 \dots A$$

$$i(0^+) = i(0^-)$$

$$V_C(0^+) = V_C(0^-)$$

* The current through inductor & vol. across a capacitor can't change instantaneously.

Therefore these value remain same just before & just after the switch has been closed.

(b) at $t>0$ (because it is not at 0^- & not at ∞ then not OC & not SC)
S.... closed

general eqn:-

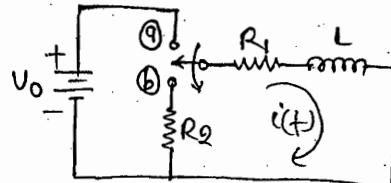
$$i(t)R + L \frac{di(t)}{dt} = -V_0$$

$t=0^+$

$$i(0^+)R + L \frac{di(0^+)}{dt} = V_0 \\ = 0$$

$$\frac{di(0^+)}{dt} = + \frac{V_0}{L} \dots A/S$$

Ques. \rightarrow In the ckt shown the switch is initially at position a for a long time till the steady state is reached. At $t=0$ the switch is moved to posn b. Find a general exp'n of the current $i(t)$ through the n/w for $t>0$.



To find
 $i(t)$ for $t>0$

DATE-30/11/14

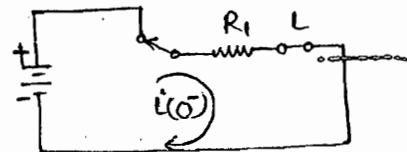
SOLⁿ \rightarrow $t=0^-$

S....at ①

Ckt is in steady-state (FC)

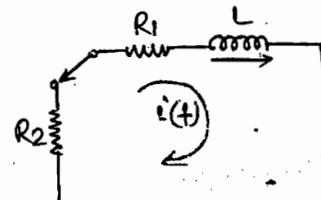
$L \rightarrow SC$

$$i(0^-) = \boxed{\frac{V_0}{R_1} = i(0^+)}$$

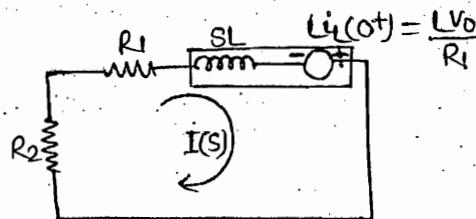


$t > 0 \rightarrow$

S....at ②



$$i(0^+) = i_L(0^+) = \frac{V_0}{R_1}$$



$$i_L(0^+) = \frac{L V_0}{R_1}$$

By KVL \rightarrow

$$I(s)(R_1 + R_2 + sL) - \frac{V_0 L}{R_1} = 0$$

$$I(s) = \frac{V_0 L}{R_1} \cdot \frac{1}{L(s+q)}$$

$$\therefore q = \frac{R_1 + R_2}{L} = \frac{R_{eq}}{L}$$

$$i(t) = \frac{V_0}{R_1} e^{-qt} u(t)$$

$$i(t) = i(0^+) e^{-t/T} \cdot u(t)$$

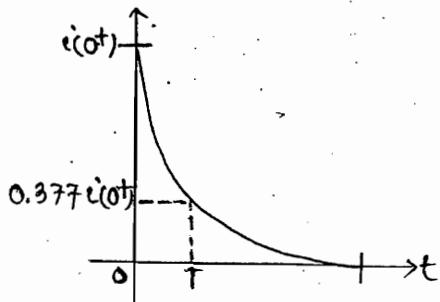
$$i(0^+) = \frac{V_0}{R_1}$$

$$T = \frac{1}{q} = \frac{1}{R_{eq}} \quad \text{time constant.}$$

$$q + t = T; \quad i(t) = i(0^+) e^{-t/T}$$

$$\cong 0.377 i(0^+)$$

i(t)



(7)
13

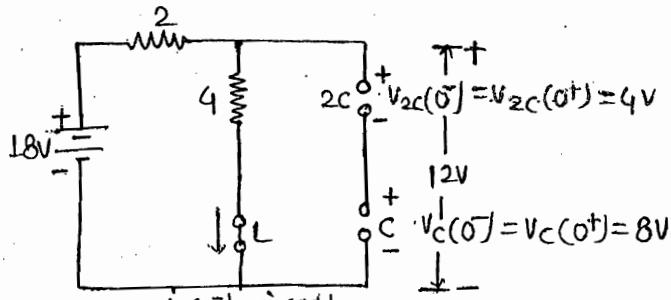
at $t=0^+$

S... open

Ckt is in steady state FC.

$2C \neq C \rightarrow OC$.

L $\rightarrow SC$.



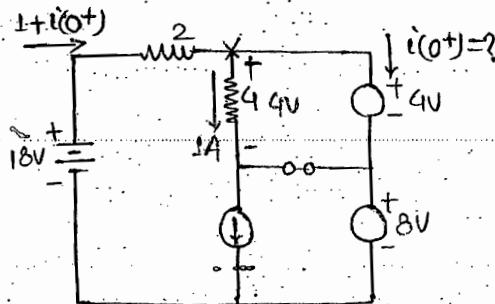
$$Q = CV$$

$$V \propto \frac{1}{C}$$

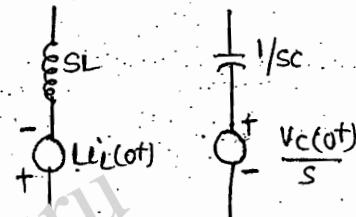
$$= \frac{18}{6} = 3A$$

at $t=0^+$

S... closed



$i(t)$ for $t > 0$



$$-18 + 2[1 + i(0^+)] + 4 + 8 = 0$$

$$i(0^+) = 2A$$

(22)
15

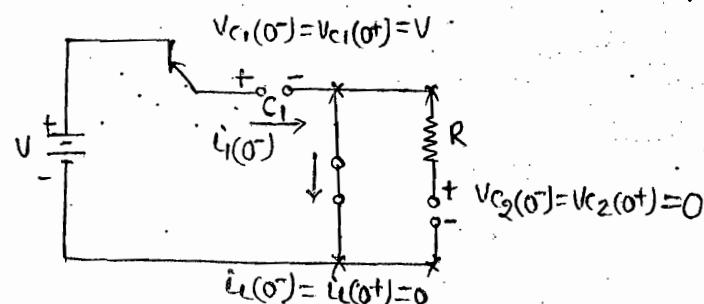
at $t=0^- \rightarrow$

S..... at (1)

Steady state/FC

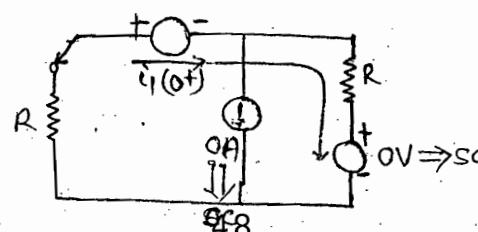
E... OC

L... SC



at $t=0^+ \rightarrow$

S... at (2)



$i_L(0+) \neq i_L(0)$ Because of capacitor current.

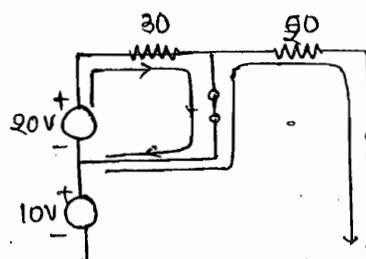
KVL

$$(R+R) i_L(0+) + V = 0$$

$$i_L(0+) = -\frac{V}{2R}$$

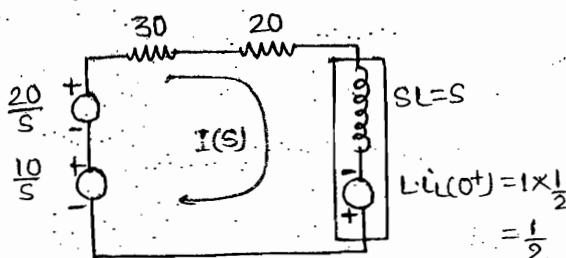
(25)
16

at $t=0 \rightarrow$
S...closed
SS/FC
 $L \Rightarrow SC$



$$i_L(0-) = i_L(0+) = \frac{10}{20} = 0.5A$$

at $t>0 \rightarrow$
S...open



$$-\frac{10}{s} - \frac{20}{s} - \frac{1}{2} + I(s)[30 + 20 + s] = 0$$

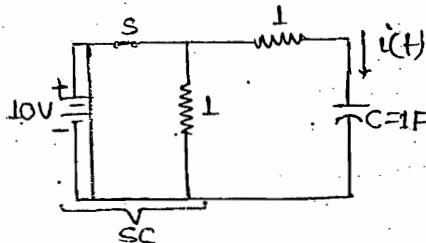
$$I(s)(s+50) = \frac{s+60}{2s}$$

$$I(s) = \frac{1}{2} \cdot \frac{(s+60)}{s(s+50)}$$

$$i(t) = 0.6 - 0.1e^{-50t}$$

(19)
15

at $t>0 \rightarrow$
S...closed



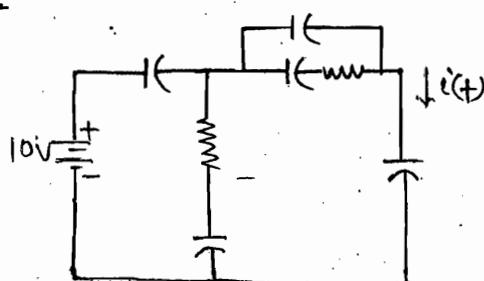
T = Req. Ceq. | 10V \Rightarrow SC

$$= 1 \times 1$$

$$T = 1s$$

* Because of TC does not depend on the vol. then so it will be shorted.

Example →



T = Req. Ceq. | 10V \Rightarrow SC

Because of many C & R the above method is not applicable for cal'ng of TC.

$T = RC$	for RC ckt
$= \frac{L}{R}$	for RL ckt
$= ?$	for RLC ckt

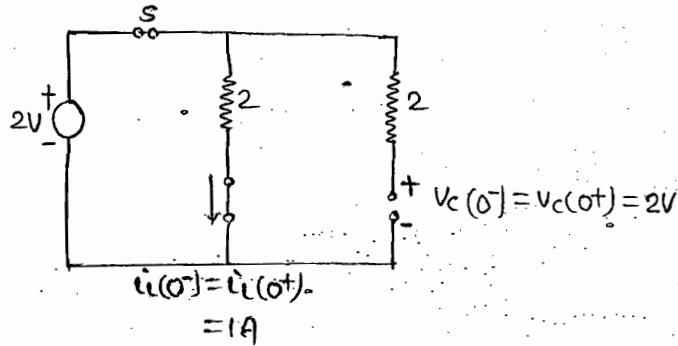
Q
12

at $t=0^- \rightarrow$

S....closed

SS/FC

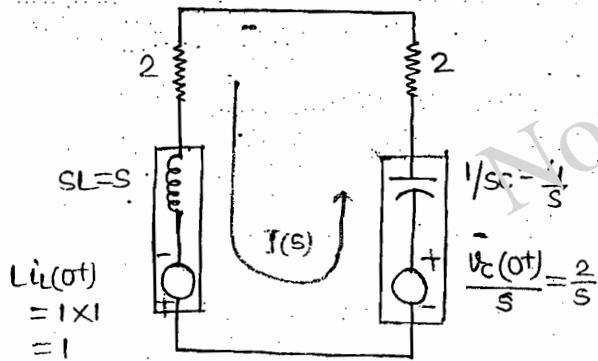
$L \Rightarrow SC, C \Rightarrow OC$



$$V_C(0^-) = V_C(0^+) = 2V$$

at $t > 0 \rightarrow$

S...open



By KVL \rightarrow

$$I(s)(2 + s + \frac{4}{s} + 2) - 1 - \frac{2}{s} = 0$$

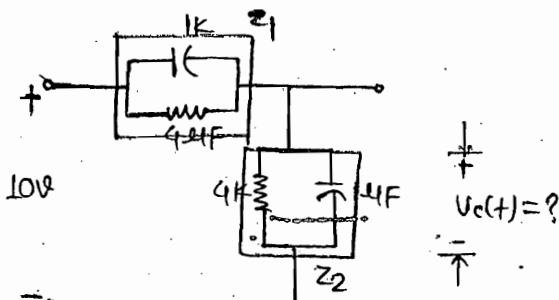
$$I(s) \cdot \frac{(s+2)^2}{s} = \frac{s+2}{s}$$

$$I(s) = \frac{1}{s+2}$$

$$i(t) = e^{-2t} = e^{-t/T}$$

$$T = 1/2 \text{ sec}$$

Q
17



$$V_C(0^+) = 0 \text{ (Given)}$$

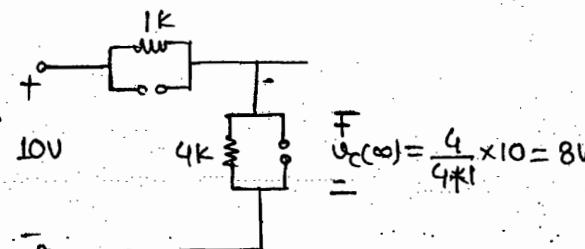
$$\frac{V_C(s)}{10/s} = H(s) = \frac{Z_2}{Z_1+Z_2} \dots \text{NOT applicable}$$

* Because of value of C will be erased & conclusion is that we can't use above method (And also value of $R_1C_1 = R_2C_2$)

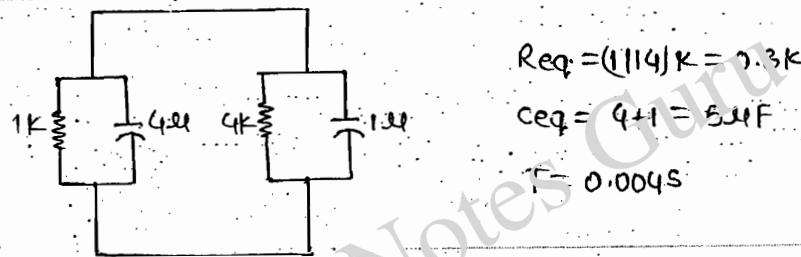
* If RC-type of n/w is given then;

$$V_C(t) = V_C(\infty) - [V_C(\infty) - V_C(0^+)] e^{-t/T} \dots \text{only for RC ckt}$$

$V_C(\infty)$ → at steady state capacitor acts as open circuit.



$$T = R_{eq} \cdot C_{eq} \quad |_{10 \rightarrow SC}$$



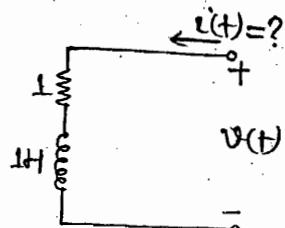
$$V_C(t) = 8 - (8-0) e^{-t/0.004}$$

$$V_C(t) = 8[1 - e^{-t/0.004}]$$

* If RL ckt is given;

$$i_L(t) = i_L(\infty) - [i_L(\infty) - i_L(0^+)] e^{-t/T}$$

(23)
15



$$V(t) = 10\sqrt{2} \cos(t+10^\circ) + 10\sqrt{5} \cos(2t+10^\circ)$$

$\omega = 1 \text{ rad/s.}$ $\omega = 2 \text{ rad/s.}$

$$Z = R + j\omega L = 1 + j1 = \sqrt{2}(45^\circ) \dots \omega = 1$$

$$= 1 + j2 = \sqrt{5}(0^\circ) \dots \omega = 2$$

$$\theta = \tan^{-1}\left(\frac{2}{1}\right)$$

$$i(t) = \frac{10\sqrt{2}}{\sqrt{2}(45^\circ)} \cos(t+10^\circ) + \frac{10\sqrt{5}}{\sqrt{5}(0^\circ)} \cos(2t+10^\circ)$$

$$i(t) = 10 \cos(t+10^\circ - 45^\circ) + 10 \cos(2t+10^\circ - 0)$$

$$i(t) = 10 \cos(t+10^\circ - 45^\circ) + 10 \cos(2t+10^\circ - 90^\circ)$$

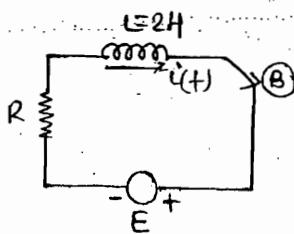
(20)
15

$$i(0) = -8A$$

$$\frac{di(0)}{dt} = 3A/s$$

$$i(\infty) = 4A$$

B :-



$$RL \rightarrow$$

$$i(t) = i(\infty) - [i(0) - \frac{i(0)}{e^{t/T}}] e^{-t/T}$$

$$i(t) = 4 - 12e^{-t/T}$$

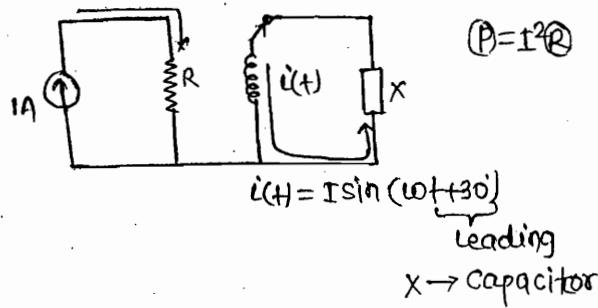
$$\frac{di(t)}{dt} = 0 + 12\left(\frac{1}{T}\right)e^{-t/T}$$

$$t=0; \quad \frac{di(0)}{dt} = 3 = \frac{12}{T}$$

$$T = 4 = \frac{L}{R} = 2$$

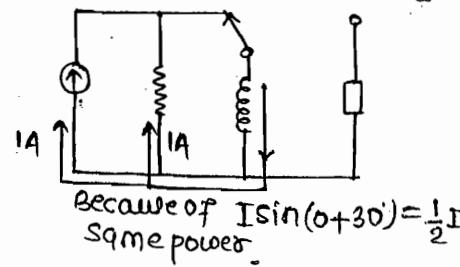
$$R = 1/2$$

13
14 at position A →



$$(P = I^2 R)$$

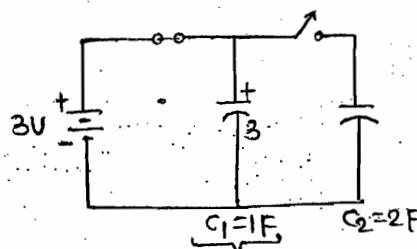
at position B → (t=0)



$$\frac{1}{2} I = 1 + 1$$

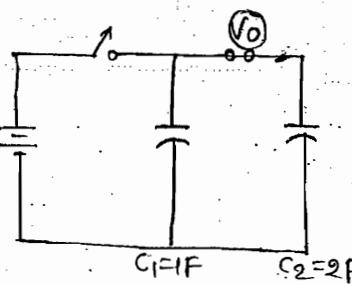
$$I = 4A$$

39
18 Case(1)



$$Q_{\text{total}} = C_1 V = 1 \times 3 = 3C$$

Case(2) →



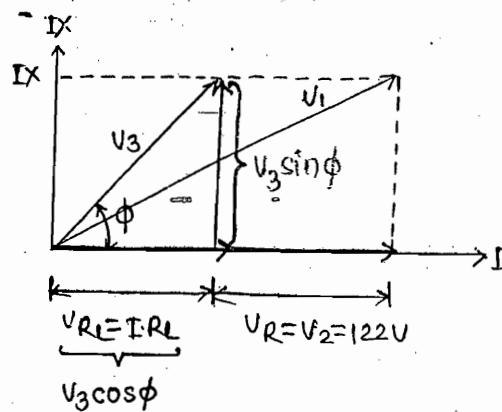
$$Q_{\text{total}} = Q_1 + Q_2$$

$$3 = C_1 V_0 + C_2 V_0$$

$$= 1 \times V_0 + 2 \times V_0$$

$$V_0 = 1V$$

52
11

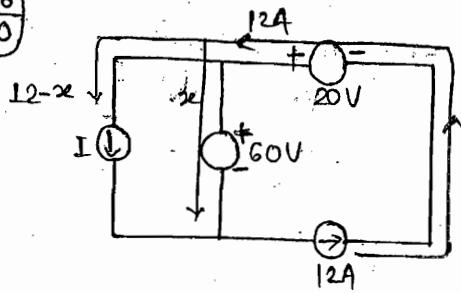
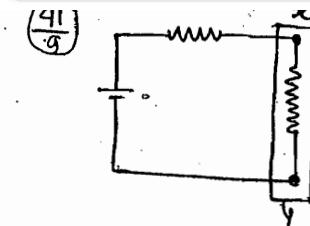


$$V_1^2 = (V_3 \cos \phi + V_2)^2 + (V_3 \sin \phi)^2$$

$$\cos \phi = PF = 0.45$$

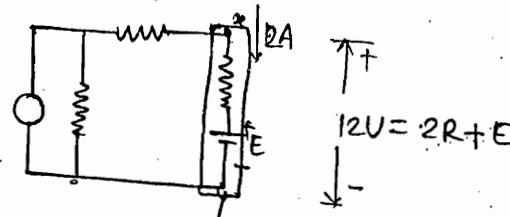
$$P = \frac{V_{RL}^2}{R_L} = \frac{(V_3 \cos \phi)^2}{R_L} = 750W$$

53
11

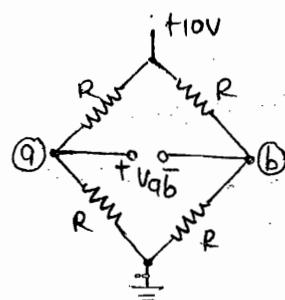
(46)
10(41)
9

$$I = \frac{18}{6} = 3A$$

$$V_{xy} = 6I = 18V$$

↑
↓

$$12V = 2R + E$$

(40)
9

$$V_{ab} = V_a - V_b$$

$$= \left(\frac{R}{R+R} \right) \times 10 - \frac{1.1R}{1.1R+R} \times 10$$

$$= 5 - \frac{1.1R}{1.1R+R} \times 10$$

(23)
640W 60W

$$P = \frac{V^2}{R} \leftarrow 220V$$

$$P \propto \frac{1}{R}$$

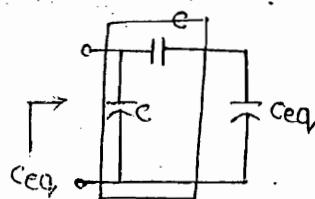
R ↑ Power Rating ↓

$$\xrightarrow{I} \quad \xrightarrow{I}$$

$$P = I^2 R$$

40W is brighter because R is ↑

(11)



$$C_{eq} = C + \frac{C \cdot C_{eq}}{C+C_{eq}}$$

$$(C+C_{eq})(C_{eq}) = C^2 + C \cdot C_{eq} + C \cdot C_{eq}$$

$$C \cdot C_{eq} + C_{eq}^2 = C^2 + C \cdot C_{eq} + C \cdot C_{eq}$$

$$C_{eq}^2 - C \cdot C_{eq} - C^2 = 0$$

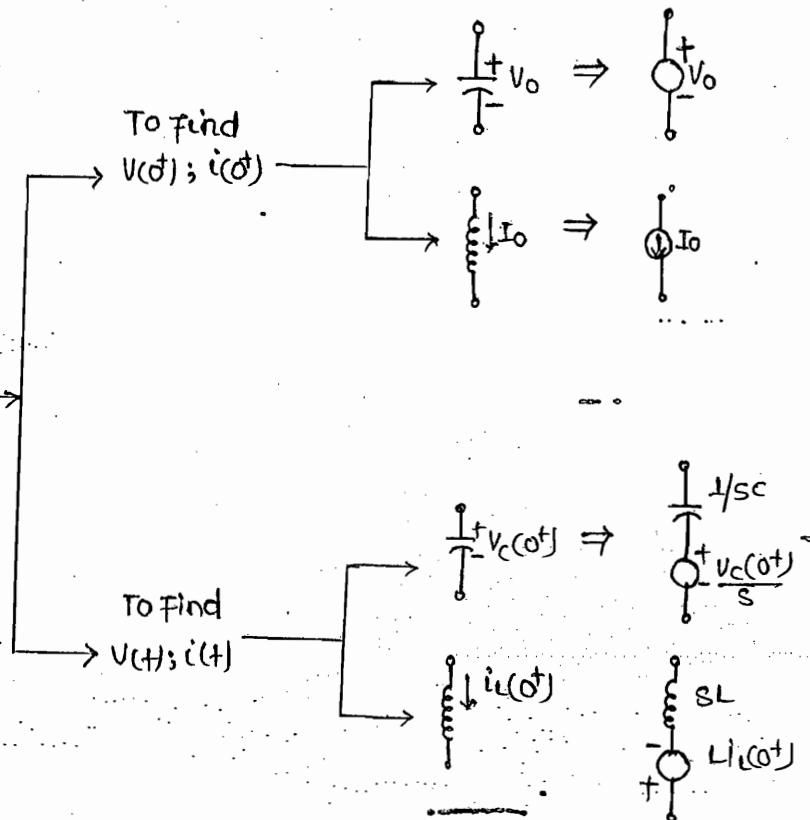
$$C_{eq} = \frac{t(C) \pm \sqrt{C^2 - 4 \times 1 \times (C^2)}}{2}$$

$$= \frac{C + \sqrt{5C^2}}{2}$$

$$= \frac{C(1 + \sqrt{5})}{2}$$

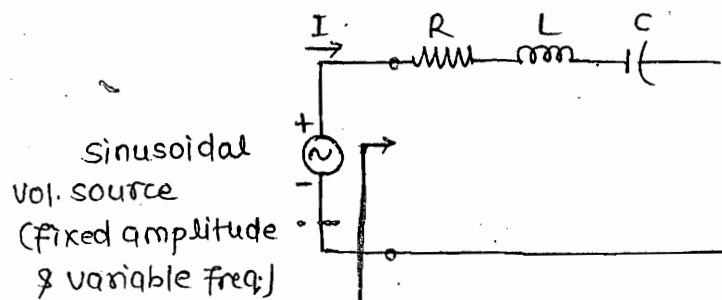
* Summary → Transient Response.

Transient Response → Find ICS in all Reactive elements
 $V_C(0^+); i_L(0^+)$



* Series RLC Ckt →

..... Series Resonance



$$Z_{in} = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j(\omega L - \frac{1}{\omega C})$$

time	C	L	
$t=t$	C	L Transient Response
$t=0$	SC	OC Effect of variation of $v(t), i(t)$
$t=\infty$	OC	SC fixed freq.
$\omega=\omega_0$	$1/j\omega_0 C$	$j\omega_0 L$ Resonance
$\omega=0$	OC	SC Effect of variation of parameters w.r.t. freq.
$\omega=\infty$	SC	OC at fix a time instant

* At $\omega = \omega_0$

$$j\text{term} = 0$$

$$\Rightarrow (\omega_0 i - \frac{1}{\omega_0 C}) j = 0$$

$$\Rightarrow \omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$\Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

freq. of resonance

$$Z_{in}|_{\omega=\omega_0} = R + j0 \dots \text{min.}$$

$$I_0 = \frac{V}{Z_0} \dots \text{max}$$

* At $\omega = \omega_0$

$$|V_L| = |V_C|$$

& are out of phase by 180°

$$Z_{in} = R + j(\omega L - \frac{1}{\omega C})$$

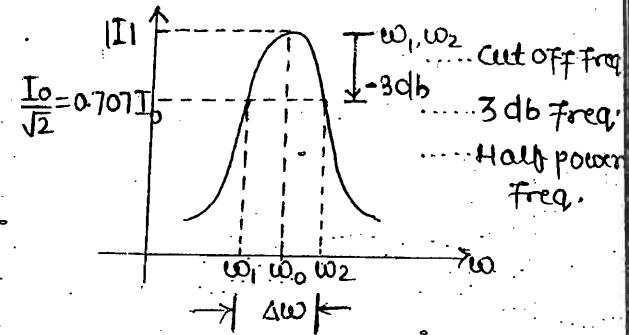
- $\omega = \omega_0$; Resistance
- $\omega > \omega_0$; Inductive
- $\omega < \omega_0$; Capacitive

* Freq. Response →

$|I|$ vs ω

$$|I| = \frac{V}{|Z_{in}|} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$I_0 = \frac{V}{R} \dots \text{max} ; \omega = \omega_0$$



* Features →

* At Resonance the Imag. part of the i/p impedance is 0.

* At resonance the I/p impedance is purely resistive has a min^m value, P/p vol. & current are in phase so that the PF is unity.

* At resonance the current drawn by the η/ω from the i/p vol. source has a max^m value & therefore any series RLC η/ω behaves as band pass filter.

* At resonance the vol. across the ind^r & across the capacitor are equal in magnitude & are phase shifted by 180° so that net vol. across LC combination is 0.

* The ckt behaves differently at different freq. of operation:-

(a) At ω_0 , η/ω is resistive & has a UPF.

(b) Above ω_0 , η/ω is inductive & PF is lagging.

(c) Below ω_0 , η/ω is capacitive & PF is leading.

* The freq. of resonance represents a rate at which ele. energy stored in the capacitor is transformed to the magnetic energy stored in the inductor & vice-versa.

The phenomena of resonance then represents the transformation b/w ele. & magnetic energy at the freq. of resonance.

* Quality Factor (Q_0) →

$$- Q_0 = \frac{|V_L|}{V} = \frac{|V_C|}{V} \dots \text{Voltage amplification factor.}$$

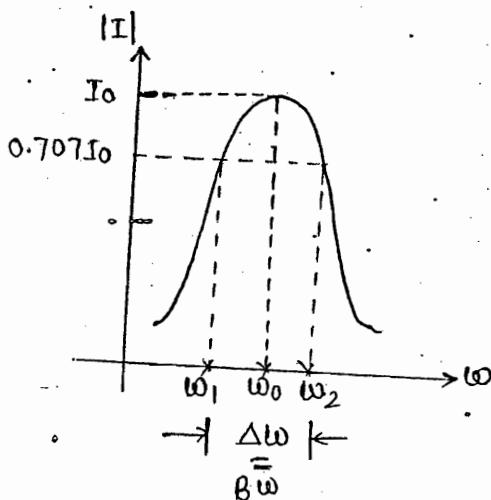
$$|V_L| = |I_0 \cdot j\omega_0 L| = |I_0 \cdot \omega_0 L| = I_0 \cdot \omega_0 L$$

$$|V_C| = |I_0 \cdot \frac{1}{j\omega_0 C}| = I_0 \frac{1}{\omega_0 C}$$

$$V = I_0 R$$

$$\begin{aligned} Q_0 &= \frac{|V_L|}{V} = \frac{I_0 \omega_0 L}{I_0 R} = \frac{\omega_0 L}{R} \\ &= \frac{\omega_0 L}{R} \\ &= \frac{1}{\omega_0 R C} \\ &= \frac{1}{R \sqrt{\frac{L}{C}}} \end{aligned}$$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 R C} = \frac{1}{\sqrt{\frac{L}{C}}} \quad Q_0 \gg 1$$



$$\Delta\omega = \omega_2 - \omega_1$$

$$Q_0 = \frac{\omega_0}{\Delta\omega} ; (\gg 1)$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\omega_2 \approx \omega_0 + \frac{1}{2}\Delta\omega$$

$$\omega_1 \approx \omega_0 - \frac{1}{2}\Delta\omega$$

$$\Delta\omega = \omega_2 - \omega_1$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$Q_0 \uparrow \rightarrow \Delta\omega \downarrow$$

* Bandwidth →

$$Q_0 = \frac{\omega_0}{\Delta\omega}$$

$$\Delta\omega = \frac{\omega_0}{Q_0} = \frac{1/\sqrt{LC}}{\frac{1}{R}\sqrt{LC}}$$

$$\boxed{\Delta\omega = \frac{R}{L} \quad \dots \text{rad/sec}}$$

$$\Delta f = \frac{R}{2\pi L} \quad \dots \text{Hz}$$

$Q_0 \uparrow \rightarrow \Delta\omega \downarrow$ $\begin{array}{l} R \downarrow \\ L \uparrow \end{array}$ Not advisable (Because of large size of ind?)

* For any tuned n/w the QF must be high & therefore the n/w should have small BW.

High QF of any tuned n/w has a the ability to select a particular signal thereby rejecting all other unwanted signal.

* Series RLC ckt →

* $\omega > \omega_0$; Inductive

Vol. across the L is max

* $\omega < \omega_0$; Capacitive

Vol. across the C is max

$$I = \frac{V}{|Z_{in}|} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$|V_L| = |I| \omega L$$

$$|V_L|^2 = \frac{(\omega L V)^2}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

For max V_L ;

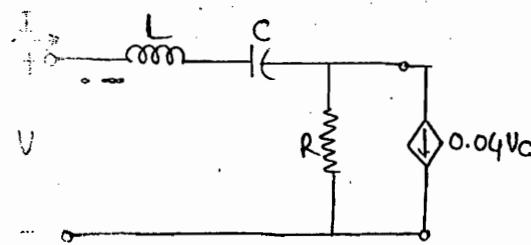
$$\frac{\partial}{\partial \omega} [|V_L|^2] = 0$$

Find ω



$$\omega > \omega_0$$

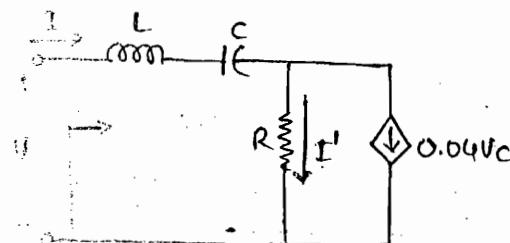
Example



$$R = 20\Omega, L = 0.1H, C = 0.1MF$$

To find; Z_{in} , ω_0 , Q_0 , $\Delta\omega$, ω_1 , ω_2

Soln



$$I' = I - 0.04Vc$$

$$\therefore Z_{in} = I - 0.04 \times I \times \frac{1}{j\omega C} = I \left(1 - \frac{0.04}{j\omega C} \right)$$

By Nodal Analysis

$$\therefore V = j\omega L \cdot I + \frac{1}{j\omega C} \cdot I + I \left(1 - \frac{0.04}{j\omega C} \right) R$$

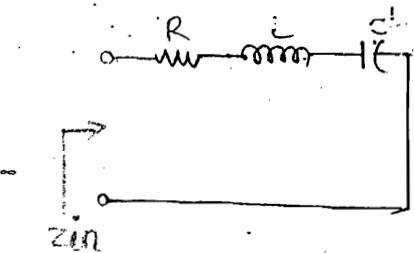
$$\therefore Z_{in} = R + j\omega L + \frac{1}{j\omega C} - \frac{0.04R}{j\omega C}$$

$$\frac{1}{j\omega C} \left[1 - \frac{0.04R}{j\omega C} \right]$$

$$\frac{1}{j\omega C \left(1 - \frac{0.04R}{j\omega C} \right)} = \frac{1}{j\omega C} \quad \left(C' = \frac{C}{1 - 0.04R} \right)$$

$$Z_{in} = R + j\omega L + \frac{1}{j\omega C}$$

series RLC ckt



$$(1) \omega_0 = \frac{1}{\sqrt{LC}} = 4470 \text{ rad/s}$$

$$(2) Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 R C} = \frac{1}{R} \sqrt{\frac{L}{C}} = 22.4$$

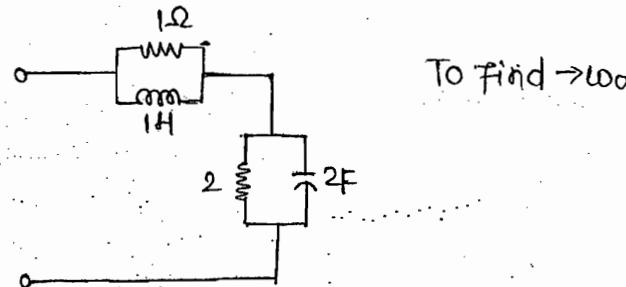
$$(3) BW = \frac{\omega_0}{Q_0} = \frac{R}{L} \text{ rad/s.}$$

$$= 200 \text{ rad/s.}$$

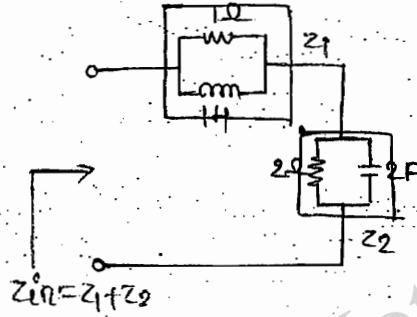
$$(4) \omega_2 \approx \omega_0 + \frac{1}{2} \Delta \omega = 4570 \text{ rad/s}$$

$$\omega_1 \approx \omega_0 - \frac{1}{2} \Delta \omega = 4370 \text{ rad/s.}$$

Example →



Soln →



$$Z_1 = \frac{R \times j\omega L}{R + j\omega L} = \frac{j\omega}{1 + j\omega} \times \frac{1 - j\omega}{1 - j\omega} = \frac{j\omega}{1 + \omega^2} + \frac{\omega^2}{1 + \omega^2}$$

---(i)

$$Z_2 = \frac{R \times j\omega C}{R + j\omega C}$$

$$= \frac{R}{1 + j\omega C} = \frac{2}{1 + j\omega 4} \times \frac{1 - j\omega 4}{1 - j\omega 4}$$

$$= \frac{2}{1 + 16\omega^2} - \frac{j\omega 8}{1 + 16\omega^2} \quad \text{---(ii)}$$

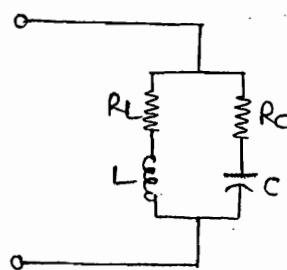
at $\omega = \omega_0$; j term = 0

then from eqn (i) & (ii)

$$\frac{\omega_0}{1 + \omega_0^2} - \frac{\omega_0 8}{1 + 16\omega_0^2} = 0$$

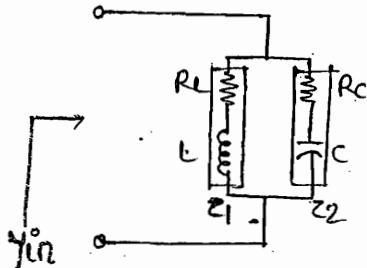
$$\boxed{\omega_0 = \sqrt{\frac{7}{8}}} \text{ rad/s.}$$

Example →



To find: ω_0

Sol ↴



$$\begin{aligned} Y_{in} &= \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{R_L + jXL} + \frac{1}{R_C - jXC} \\ &= \frac{R_L - jXL}{R_L^2 + X_L^2} + \frac{R_C + jXC}{R_C^2 + X_C^2} \end{aligned}$$

--- (i)

at $\omega = \omega_0$; $j\omega \text{ term} = 0$

$$\Rightarrow \frac{-XL}{R_L^2 + X_L^2} + \frac{XC}{R_C^2 + X_C^2} = 0$$

$$\Rightarrow \frac{XL}{R_L^2 + X_L^2} = \frac{XC}{R_C^2 + X_C^2}$$

$$\Rightarrow XL(R_C^2 + X_C^2) = XC(R_L^2 + X_L^2)$$

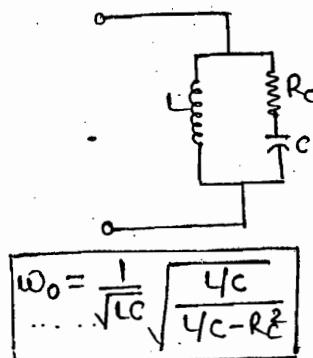
$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - XC}{R_C^2 - XC}}}$$

$$Z_{in} = () + j() \quad \equiv 0 \text{ at } \omega = \omega_0$$

$$Y_{in} = () + j() \quad \equiv 0 \text{ at } \omega = \omega_0$$

Because i/p vol. & i/p current both are in phase so we put imag. of upper part as 0. not ∞ .

Case(1) $\rightarrow R_L = 0$



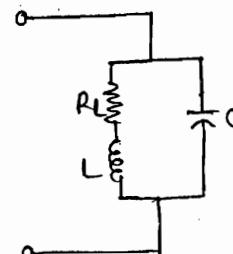
$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{4C}{4C - R_c^2}}$$

$$\frac{L}{C} - R_c^2 > 0$$

$$R_c < \sqrt{\frac{L}{C}}$$

..... cond'n of Resonance

Case(2) $\rightarrow R_C = 0$



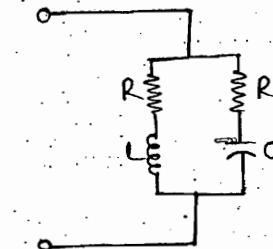
$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{R_L^2 C}{L}}$$

$$1 - \frac{R_L^2 C}{L} > 0$$

$$R_L < \sqrt{\frac{L}{C}}$$

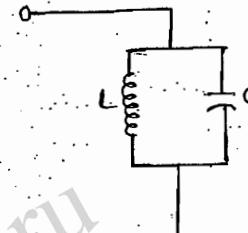
..... cond'n of Resonance

Case(3) $\rightarrow R_L = R_C = R$



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Case(4) $\rightarrow R_L = R_C = 0$



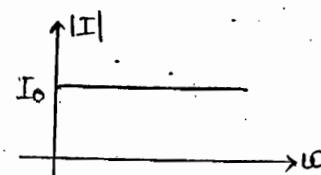
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

* The freq. of resonance does not depend upon type of connection b/w L & C.

Therefore the freq. of resonance will remain same for purely series LC & parallel LC n/w.

Case(5) $\rightarrow R_L = R_C = \sqrt{\frac{L}{C}}$

$$Y_{in} = (\text{Real}) + j(0)$$



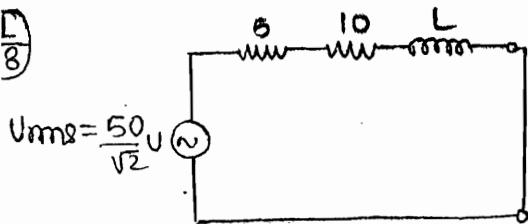
* For the specified values of R_L & R_C the i/p admittance has only the real part & the imag. part is always 0.

Therefore irrespective of the freq. of operation the i/p vol. & the i/p current are always in phase. Then the current drawn by the n/w is independant of

the freq. of operation.

Therefore no specific freq. of resonance will exist & η/ω is resonating at all freq.

(8)



$$U_{rms} = \frac{50}{\sqrt{2}} V$$

$$P_5 = 10W$$

$$PF = \cos \phi = \frac{Rt}{|Z|} = \frac{Im_{rs} \cdot Rt}{Im_{rs} \cdot |Z|} = \frac{Im_{rs} \cdot Rt}{V_{rms}}$$

$$P_5 = Im_{rs}^2 \times S$$

$$10 = Im_{rs}^2 \times S$$

$$Im_{rs} = \sqrt{2}$$

$$\boxed{PF = \cos \phi = 0.6}$$

(9)

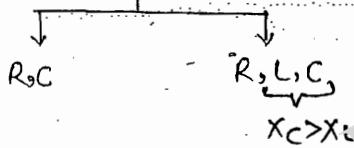
$$v = 200 \sin(2000t + 50^\circ)$$

$$= 200 \sin(2000t + 90^\circ - 90^\circ + 50^\circ) \\ = 200 \sin(2000t - 40^\circ)$$

$$i = 4 \cos(2000t + 13.2^\circ)$$

i leads v

capacitor



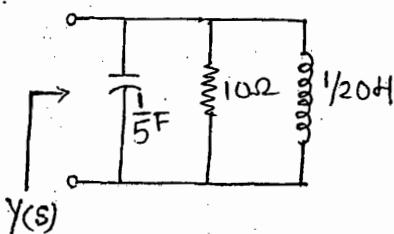
Parallel RLC ckt:-

$$\frac{1}{Z_s} = Y(s) = \frac{s^2 + 0.5s + 100}{5s^2}$$

$$= \frac{s}{5} + 0.1 + \frac{20}{s}$$

$$= Y_1 + Y_2 + Y_3$$

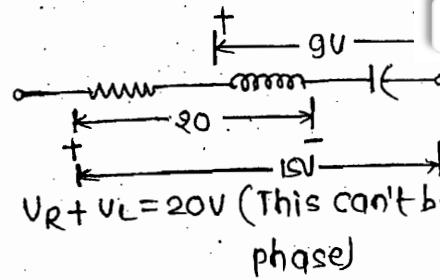
$$= BC_1 + \frac{1}{R_2} + \frac{1}{sL_3}$$



$$C_1 = \frac{1}{5} F$$

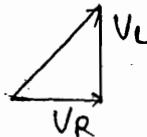
$$R_2 = 10 \Omega$$

$$L_3 = \frac{1}{20} H$$

11
49

$V_R + V_L = 20V$ (This can't be written because both are not in same phase)

$$\sqrt{V_R^2 + V_L^2} = 20 \quad \text{(i)}$$



* $V_L - V_C = 9$ --- (ii) (same reason as above) (180° phase diff)

$$\sqrt{V_R^2 + (V_L - V_C)^2} = 15 \quad \text{(iii)}$$

$$V_C = 7V$$

* Reason for written of $(V_L - V_C)$ not $(V_C - V_L)$ \rightarrow

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + jX_L - jX_C$$

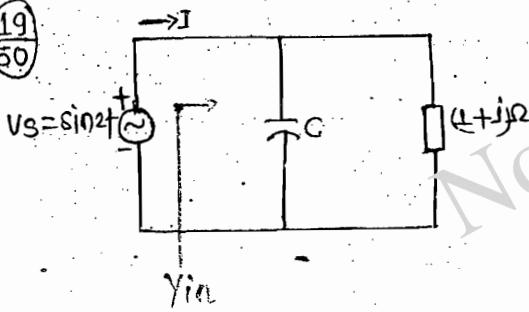
$$= R + j(X_L - X_C)$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$V = Iz = IR + j(IX_L - IX_C)$$

$$V = V_R + (V_L - V_C)j$$

19
50

V_S & RI are in phase

* CKT is in resonance at $\omega = 2 \text{ rad/s}$.

$$y_{in} = y_1 + y_2$$

$$= j\omega C + \frac{1}{1+j} \times \frac{1-j}{1-j}$$

$$= j\omega C + \frac{1}{2} - \frac{j}{2}$$

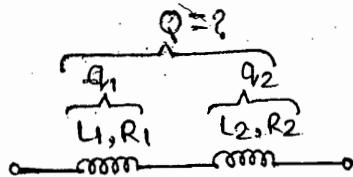
$$= j2C + \frac{1}{2} - \frac{j}{2}$$

$$\text{at reso.} \rightarrow 2C - \frac{1}{2} = 0$$

$$2C = \frac{1}{2}$$

$$C = \frac{1}{4} F$$

1)



$$q_1 = \frac{\omega L_1}{R_1}, \quad q_2 = \frac{\omega L_2}{R_2}$$

where; ω = freq. of operation
(operating freq. or) self-resonating freq.

* If 2 inductors are present then there will be no resonance.

$$Q = \frac{\omega L_{\text{eq}}}{R_{\text{eq}}} = \frac{1}{R_1 + R_2} \cdot \omega (L_1 + L_2) \quad (\text{Because of series combination})$$

$$Q = \frac{1}{R_1 + R_2} \left[\frac{\omega L_1 R_1}{R_1} + \frac{\omega L_2 R_2}{R_2} \right]$$

$$Q = \left(\frac{1}{R_1 + R_2} \right) [q_1 R_1 + q_2 R_2]$$

3) Series RLC ckt

$$R = 20\Omega, \phi = -45^\circ \quad V_L = 2V_C$$

$$IX_L = 2IX_C$$

$$X_L = 2X_C$$

$$I = \frac{V}{Z} = \frac{V}{R + j(X_L - X_C)}$$

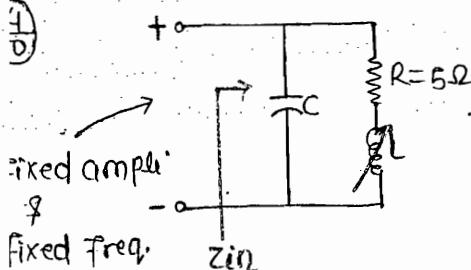
$$\phi = -\tan^{-1}\left(\frac{X_L - X_C}{R}\right) = -45^\circ$$

$$\frac{X_L - X_C}{R} = 1$$

$$X_L - \frac{1}{2}X_L = 20$$

$$X_L = 40\Omega$$

4)



$$Z_{\text{in}} = \frac{(-jX_C)(R + jX_L)}{-jX_C + (R + jX_L)}$$

$$= \frac{(-jX_C)(R + jX_L)}{R + j(X_L - X_C)} \times \frac{R - j(X_L - X_C)}{R - j(X_L - X_C)}$$

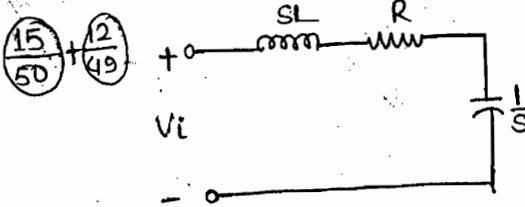
$$Z_{\text{in}} = () + j()$$

$\equiv 0$

$$X_L = \frac{1}{2} \left[X_C \pm \sqrt{X_C^2 - 4R^2} \right] \rightarrow X_C^2 = 4R^2; \text{ One value of } X_L \text{ X } \rightarrow \text{One resonance occurs}$$

$$\begin{aligned} X_C^2 &> 4R^2 \quad 66 \\ X_C &> 2R \end{aligned} \rightarrow X_C^2 < 4R^2; X_L = a + jb \quad \text{Inductive Reactance}$$

$(Z = jX_L)$ is always \neq imag.



* If series RLC is given then o/p can be taken from any elements.

$$\begin{aligned}\frac{V_o(s)}{V_i(s)} &= H(s) = \frac{1/sC}{sL + R + 1/sC} \\ &= \frac{1}{s^2LC + RCS + 1} \\ &= \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}\end{aligned}$$

$$\begin{aligned}D(s) &= s^2 + \frac{R}{L}s + \frac{1}{LC} \\ &= s^2 + 2\zeta\omega_n s + \omega_n^2\end{aligned}$$

(15) $2\zeta\omega_n = \frac{R}{L} \quad \& \quad \omega_n = \frac{1}{\sqrt{LC}}$ (12)

$$2\zeta = \frac{R}{2\sqrt{LC}}$$

for no oscillations;

$$\zeta \geq 1$$

$$\frac{R}{2\sqrt{LC}} \geq 1$$

$$R \geq 2\sqrt{\frac{L}{C}}$$

$$H(s) = \frac{10^6}{s^2 + 20s + 10^6} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

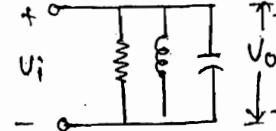
$$\frac{R}{L} = 2\zeta\omega_n = \Delta\omega (\text{BW})$$

$$\frac{1}{L\zeta} = \omega_n = \omega_0$$

$$Q_0 = \frac{\omega_0}{\Delta\omega} \quad \Delta\omega = 20^\circ \quad \omega_0 = 10^3$$

$$Q_0 = \frac{\omega_0}{\Delta\omega} = 50$$

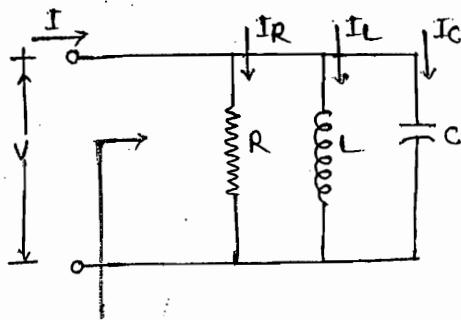
$$Q_0 = 50$$



$$\frac{V_o(s)}{V_i(s)} = 1 = H(s)$$

* Parallel RLC Ckt →

..... Parallel Resonance.



$$Y_{in} = Y_1 + Y_2 + Y_3$$

$$= \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$= \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$

at $\omega = \omega_0$; j term = 0

$$(\omega C - \frac{1}{\omega L}) = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

at $\omega = \omega_0$

$$Y_{in} = Y_0 = \frac{1}{R} + j(0) \dots \text{min}$$

$$Z_0 = R \dots \text{max}$$

$$I_0 = V \underbrace{Y_0}_{\text{min}}$$

$$|I_L| = |I_C|$$

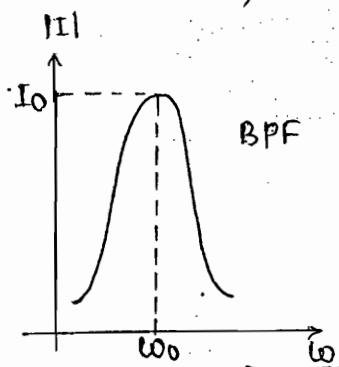
& are out of phase by 180°

$$Y_{in} = \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$

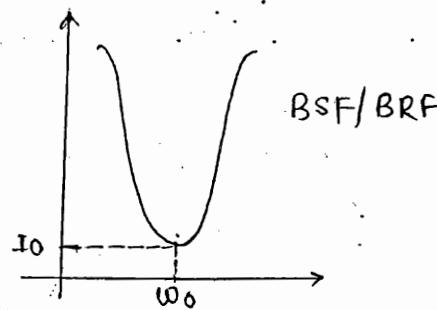
→ $\omega = \omega_0$, Resistive.

→ $\omega > \omega_0$; Capacitive.

→ $\omega < \omega_0$; Inductive.



series RLC



parallel RLC

F Features →

At resonance the Imag. part of the i/p admittance is 0.

At resonance i/p admittance is purely real; has min^m value, i/p impedance is max^m & therefore the i/p vol. & current are in phase.

At resonance the current drawn by the η/ω is min^m & therefore a parallel LCL η/ω behaves as a band stop (or) band reject filter.

At resonance the current through inductor L & current through cap. (C) are equal in magnitude & are phase shifted by 180° so that net current drawn by the C combination is 0.

The η/ω behaves differently at different freq. of operation:-

1) At $\omega = \omega_0$, the η/ω is resistive & has a UPF.

2) At ω Above ω_0 η/ω is capacitive & has a leading PF.

3) Below ω_0 η/ω is inductive & has a lagging PF.

The QF represents the current amplification factor & for any tuned η/ω it must have a high value.

Quality Factor (Q_0) →

$$Q_0 = \frac{|I_L|}{I_0} = \frac{|I_C|}{I_0} ; (\gg 1)$$

$$|I_L| = \left| \frac{V}{j\omega_0 L} \right| = \frac{V}{\omega_0 L}$$

$$|I_C| = \left| \frac{V}{1/j\omega_0 C} \right| = \omega_0 C V$$

$$I_0 = \frac{V}{R}$$

$$Q_0 = \frac{R}{\omega_0 L} = \omega_0 R C = R \sqrt{\frac{C}{L}}$$

General Results →

$$Q_0 = \frac{\omega_0}{\Delta\omega}$$

$$\Delta\omega = \omega_2 - \omega_1 \quad \omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\omega_2 \cong \omega_0 + \frac{1}{2} \Delta\omega$$

$$\omega_1 \cong \omega_0 - \frac{1}{2} \Delta\omega$$

Bandwidth \rightarrow

$$\Delta\omega = \frac{\omega_0}{Q_0}$$

$$\Delta\omega = \frac{\omega_0}{Q_0}$$

$$= \frac{4\sqrt{LC}}{R\sqrt{C}}$$

$$\Delta\omega = \frac{1}{RC} \quad \dots \text{rad/s}$$

$$\Delta f = \frac{1}{2\pi RC} \quad \dots \text{Hz}$$

$Q_0 \uparrow \rightarrow \Delta\omega \downarrow$ $\rightarrow R \uparrow \dots \text{Not desirable.}$

$C \uparrow$

$$\Delta\omega = \frac{R}{L}$$

$$\Delta f = \frac{R}{2\pi L}$$

Series RLC

$$\Delta\omega = \frac{1}{RC}$$

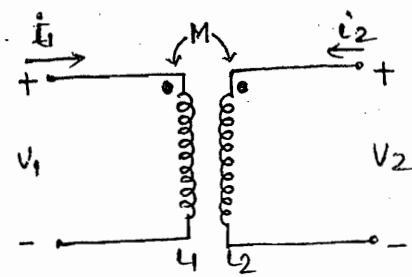
$$\Delta f = \frac{1}{2\pi RC}$$

Parallel RLC

Coupled

..... Effect of mutual inductance (M)

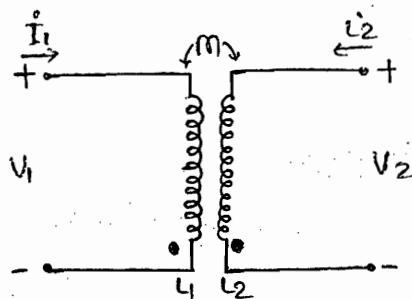
Case(1) \rightarrow



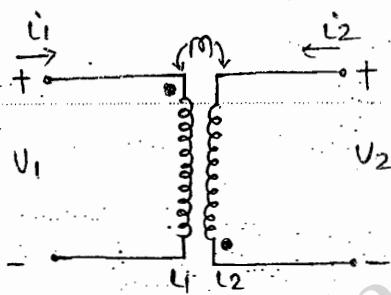
EFFECT OF M IS +ve
Einduced is additive

..... Case(1), Case(2)

Case(2) \rightarrow



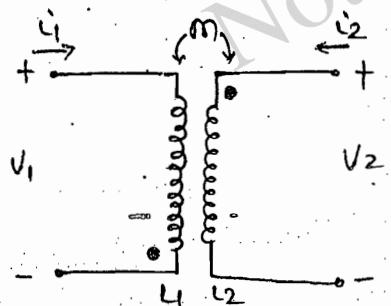
Case(3) \rightarrow



EFFECT OF M IS -ve
Einduced is subtractive

..... Case(3), Case(4)

Case(4) \rightarrow



$$V_1(t) = L_1 \frac{di_1(t)}{dt} \pm M \frac{di_2(t)}{dt}$$

$$V_2(t) = \mp M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

..... In time domain

..... For non-sinusoidal excitations

$$V_1(s) = SL_1 I_1(s) \pm SM I_2(s)$$

$$V_2(s) = \pm SM I_1(s) + SL_2 I_2(s)$$

} In s-domain assuming zero IC's

} for non-sinusoidal excitation.

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} SL_1 & \pm SM \\ \pm SM & SL_2 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} j\omega L_1 & \pm j\omega M \\ \pm j\omega M & j\omega L_2 \end{bmatrix}}_{\text{Impedance matrix}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Impedance matrix

.... for sinusoidal excitation

.... Using phasors.

* Effect of Mutual Impedances →

(1) In terms of M.

(2) In terms of K

... K is coeff. of coupling. ($K \leq 1$)

(3)

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

$$; M = K \sqrt{L_1 L_2}$$

Case(i) →

$$K < 1, M < \sqrt{L_1 L_2}$$

.... Loose coupling

.... Under coupled ckts.

Case(ii) →

$$K=1, M=\sqrt{L_1 L_2}$$

.... Critical coupling

.... Critically coupled.

iii) In terms of turns Ratio

$$\frac{V_1}{V_2} = \frac{n_1}{n_2}$$

$$\frac{I_1}{I_2} = \frac{n_2}{n_1}$$

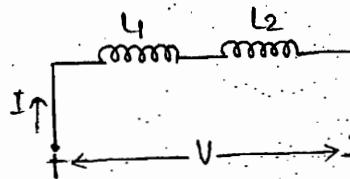
- * The effect of mutual inductance is considered whenever 2 inductors are placed physically close to each other.
- * When the current flows in the 2nd inductor some time varying magnetic flux is produced. Part of this magnetic flux will link with the 1st inductor.
- Due to Faraday's law of electromagnetic induction (or) due to Rate of change of magnetic flux, some emf is induced b/w 2 terminals of 1st inductor.

The polarity of this induced Vol. depends upon the dot convention & therefore depends upon relative sense of wdg of 2 inductors.

Then all the KVL eqns are modified depending upon the excitation.

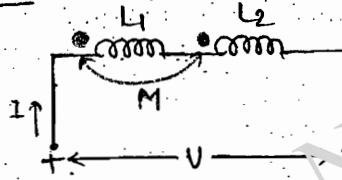
Example +

Case(1) →



$$L_{eq} = L_1 + L_2$$

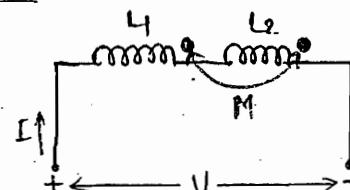
Case(2) →



$$\begin{aligned} V &= j\omega L_1 I + j\omega L_2 I + j\omega M I + j\omega M I \\ \frac{V}{I} &= Z_{eq} = j\omega(L_1 + L_2 + 2M) = j\omega L_{eq} \end{aligned}$$

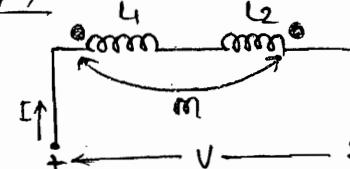
$$L_{eq} = L_1 + L_2 + 2M$$

Case(3) →



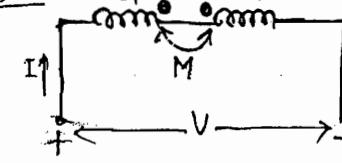
$$L_{eq} = L_1 + L_2 + 2M$$

Case(4) →



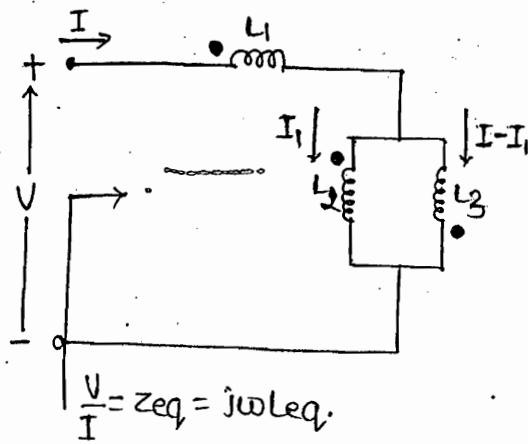
$$L_{eq} = L_1 + L_2 - 2M$$

Case(5) →



$$L_{eq} = L_1 + L_2 - 2M$$

Example →



$$\frac{V}{I} = z_{eq} = j\omega L_{eq}$$

$$L_1 = L_2 = L_3 = 10H$$

$$M_{12} = M_{21} = 1H$$

$$M_{13} = M_{31} = 2H$$

$$M_{23} = M_{32} = 3H$$

Find : L_{eq}

KVL-(1) →

$$\begin{aligned} V &= j\omega L_1 I + j\omega L_2 I_1 + \\ &\quad + j\omega M_{21} I_1 + j\omega (I - I_1) M_{31} \\ &\quad + j\omega M_{12} I - j\omega M_{32} (I - I_1) \end{aligned}$$

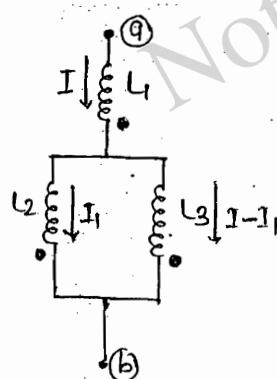
$$\left. \begin{aligned} V &= j\omega C_1 I + j\omega C_2 I_1 \dots (i) \\ &+ j\omega M_{21} I_1 - j\omega M_{31} (I - I_1) \end{aligned} \right\} V = j\omega C_1 I + j\omega C_2 I_1 \dots (ii)$$

$$\left. \begin{aligned} V &= j\omega L_1 I + j\omega L_3 (I - I_1) \\ &+ j\omega M_{21} I_1 - j\omega M_{31} (I - I_1) \\ &+ j\omega M_{13} I_1 - j\omega M_{23} I_1 \end{aligned} \right\} V = j\omega C_1 I + j\omega C_2 I_1 \dots (iii)$$

Eliminate I_1 , find $\frac{V}{I} = z_{eq} = j\omega L_{eq}$,

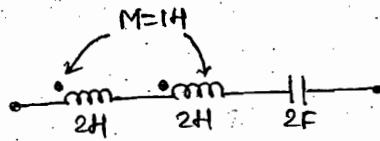
find L_{eq} .

Example →



To find L_{eq} ; b/n a & b.

(6)
49



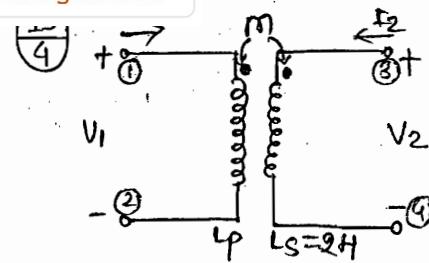
$$f_0 = \frac{1}{2\pi\sqrt{L_{eq}C}}$$

$$L_{eq} = L_1 + L_2 + 2M$$

$$= 2 + 2 + 2(1)$$

$$= 6H$$

$$f_0 = \frac{1}{2\pi\sqrt{6 \times 2}} = \frac{1}{8\pi\sqrt{3}} \text{ Hz}$$



$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$\underline{(3)-\underline{(4)}} \Rightarrow OC \quad I_2 = 0$$

$$\frac{V_1}{I_1} = Z_{eq} = j\omega L_{eq} = j\omega 4 \quad \dots \text{(i)}$$

$$V_1 = j\omega L_p I_1 + 0 \quad \dots \text{(ii)}$$

$$\frac{V_1}{I_1} = j\omega L_p = j\omega 4$$

$$L_p = 4$$

$$\underline{(3)-\underline{(4)}} \Rightarrow SC \quad V_2 = 0$$

$$\frac{V_1}{I_1} = Z_{eq} = j\omega L_{eq} = j\omega 3 \quad \dots \text{(a)}$$

$$V_1 = j\omega L_p I_1 + j\omega M I_2 \quad \dots \text{(b)}$$

$$V_2 = 0 = j\omega L_p I_2 + j\omega M I_1 \quad \dots \text{(c)}$$

$$M = \sqrt{3}$$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{M}{\sqrt{3 \times 4}} = 1/2 < 1$$

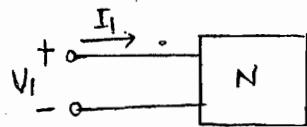
..... loose coupling.

..... Under coupled ckt.

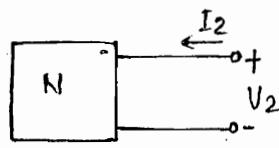
Network
Functions

..... Used to represent any elec n/w mathematically.

(1) Case (i) → One port networks



$$\frac{V_1}{I_1} = z_{11} \dots \text{driving pt. o/p impedance fn}$$



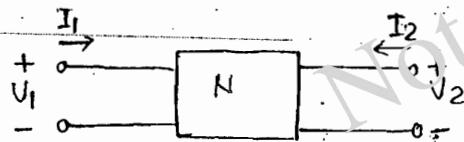
$$\frac{I_1}{V_1} = y_{11} \dots \text{driving pt. i/p admittance fn}$$

$$\frac{V_2}{I_2} = z_{22} \dots \text{driving pt. o/p impedance fn}$$

$$\frac{I_2}{V_2} = y_{22} \dots \text{driving pt. i/p admittance fn}$$

driving point
immitance fns.
↑ ↓
admittance
Impedance

(2) Case (ii) → Two port networks



(i) Driving pt. immitance fns →

$$Z_{11} = \frac{V_1}{I_1} ; Z_{22} = \frac{V_2}{I_2}$$

$$Y_{11} = \frac{I_1}{V_1} ; Y_{22} = \frac{I_2}{V_2}$$

(ii) Transfer impedance Ratio →

$$Z_{12} = \frac{V_1}{I_2} ; Z_{21} = \frac{V_2}{I_1}$$

(iii) Transfer admittance Ratio \rightarrow

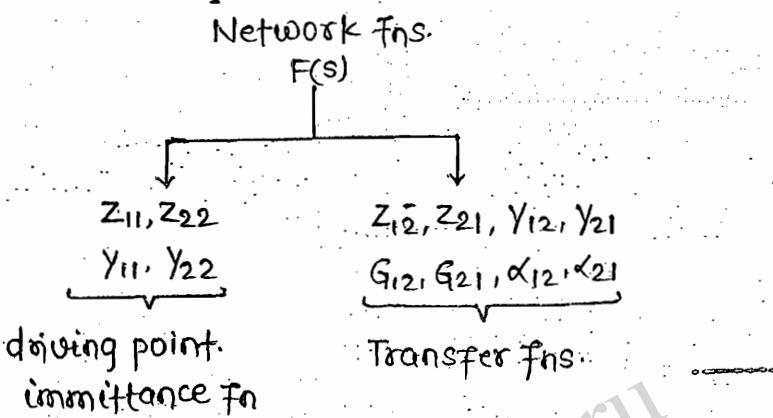
$$Y_{12} = \frac{I_1}{V_2}; Y_{21} = \frac{I_2}{V_1}$$

(iv) Transfer Vol. Ratio \rightarrow

$$G_{12} = \frac{V_1}{V_2}; G_{21} = \frac{V_2}{V_1}$$

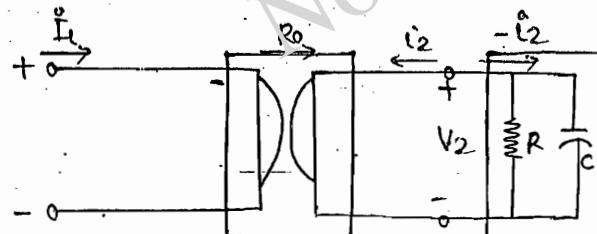
(v.) Transfer Current Ratio \rightarrow

$$\alpha_{12} = \frac{I_1}{I_2}; \alpha_{21} = \frac{I_2}{I_1}$$



* For immittance fn all the poles & zeros lie in LHP. For Tfn all the poles still lie in LHP where as some of the zeros may also lie in RHP.

Example \rightarrow



$$Z_{in} = Z_{11} = \frac{V_1}{I_1} = ?$$

$$Z = \frac{R \times \frac{1}{sc}}{R + \frac{1}{sc}} = \frac{R}{1 + SRC}$$

SOLN \rightarrow

$$V_1 = R_0 i_2 \dots \text{(i)}$$

$$V_2 = -R_0 i_1 \dots \text{(ii)}$$

$$V_2 = -i_2 Z = -i_2 \cdot \frac{R}{1 + SRC} \dots \text{(iii)}$$

From eqn (i) & (ii)

$$i_2 \cdot \frac{R}{1+SRC} = R_0 i_1$$

$$i_2 = \frac{R_0(1+SRC) \cdot i_1}{R}$$

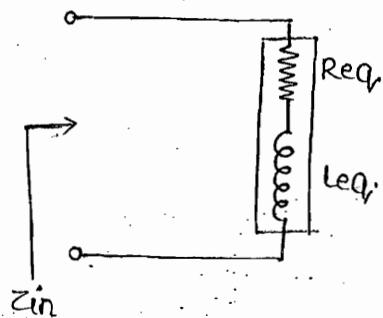
$$V_1 = \frac{R_0^2(1+SRC) \cdot i_1}{R}$$

$$\frac{V_1}{i_1} = Z_{in} = Z_{11} = \frac{R_0^2 + SR_0^2 C}{R}$$

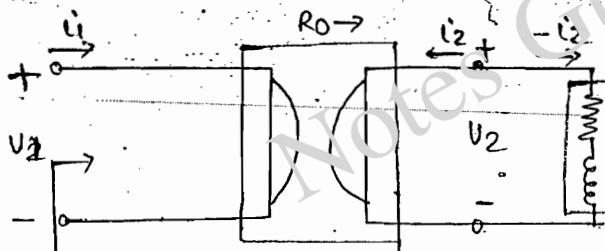
$$= Z_1 + Z_2$$

$$= R_{eq} + S L_{eq}$$

$$R_{eq} = \frac{R_0^2}{R}; L_{eq} = R_0^2 C$$



example →



$$Z_{in} = Z_{11} = \frac{V_1}{i_1}$$

SOL →

$$V_1 = +R_0 i_1 \quad \text{(i)}$$

$$V_2 = -R_0 i_2 \quad \text{(ii)}$$

$$V_2 = -i_2 \cdot Z = -i_2(R + SL) \quad \text{(iii)}$$

from eqn (i), (ii) & (iii)

$$-R_0 i_1 = -i_2(R + SL)$$

$$i_2 = \frac{R_0}{R + SL} i_1$$

$$V_1 = \frac{R_0^2}{R+SL} \cdot i_1$$

$$\frac{V_1}{i_1} = Z_{in} = \frac{R_0^2}{R+SL} = \frac{1}{Y_{in}}$$

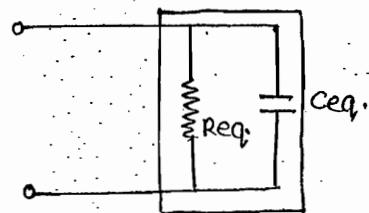
$$Y_{in} = \frac{R+SL}{R_0^2}$$

$$= \frac{R}{R_0^2} + \frac{SL}{R_0^2}$$

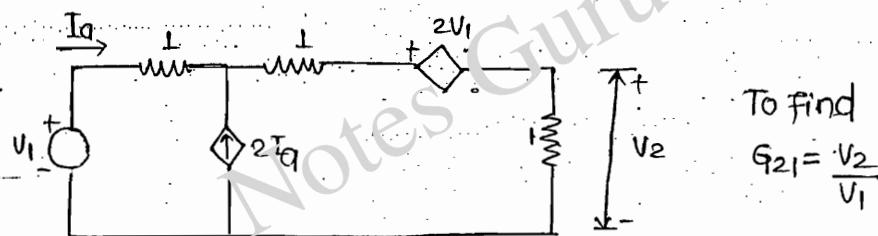
$$= Y_1 + Y_2$$

$$= \frac{1}{R_{eq}} + sC_{eq}$$

$$R_{eq} = \frac{R_0^2}{R}; C_{eq} = \frac{L}{R_0^2}$$



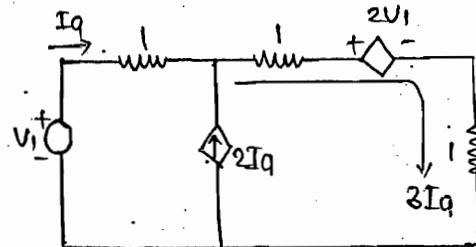
Example →



To find

$$G_{21} = \frac{V_2}{V_1}$$

Sol'n →



$$-V_1 + I_q + 1 \times 3I_q + 2U_1 + 1 \times 3I_q = 0$$

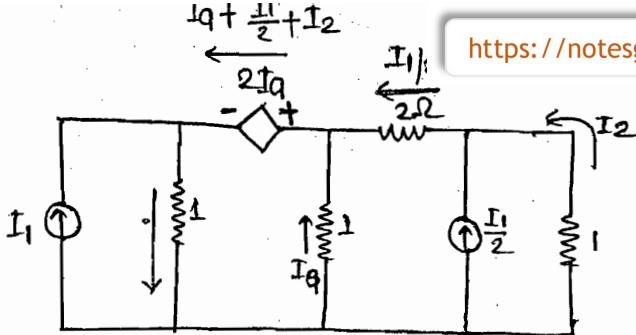
$$V_1 = -7I_q \dots\dots (i)$$

$$V_2 = 1 \times 3I_q \dots\dots (ii)$$

$$\frac{V_2}{V_1} = G_{21} = \frac{-3}{7}$$

(3)
41

$$I_q + \frac{I_1}{2} + I_2$$



To find →

$$\alpha_{21} = \frac{I_2}{I_1}$$

$$I_q + I_1 + \frac{I_1}{2} + I_2 = \frac{3}{2}I_1 + I_q + I_2$$

KVL-(1)

$$1 \times I_2 + 2 \times \left(\frac{I_1}{2} + I_2 \right) - 1 \times I_q = 0$$

$$I_q = 3I_2 + I_1 \dots \dots \text{(i)}$$

KVL-(2)

$$I_q \times 1 + 2I_q + 1 \times \left(\frac{3}{2}I_1 + I_q + I_2 \right) = 0$$

$$4I_q + \frac{3}{2}I_1 + I_2 = 0 \dots \dots \text{(ii)}$$

From eqn (i) & (ii)

$$4(3I_2 + I_1) + \frac{3}{2}I_1 + I_2 = 0$$

$$\boxed{\alpha_{21} = \frac{I_2}{I_1} = -\frac{11}{26}}$$

Ans.

Ques. → A η/ω fn is given by the following exp. Draw the pole-zero pattern & explain the significance of poles & zeros of the η/ω .

$$F(s) = \frac{(s+1)(s+3)}{s(s+2)(s+4)}$$

SOL

$$F(s) = \frac{(s+1)(s+3)}{s(s+2)(s+4)} = \frac{\text{Num. poly}}{\text{den. poly}}$$

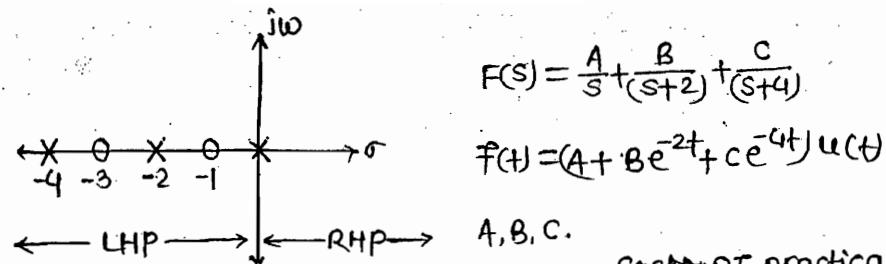
$$\begin{array}{ll} Z_{11}, Z_{22} & Z_{12}, Z_{21}, Y_{21}, Y_{12} \\ Y_{11}, Y_{22} & G_{12}, G_{21}, \alpha_{12}, \alpha_{21} \end{array}$$

Zeros

$$\begin{array}{l} \text{Num. poly} = 0 \\ F(s) = 0 \end{array} \} \quad s = -1, -3$$

$$\begin{array}{l} \text{Poles} \\ \text{deno. poly} = 0 \end{array} \} \quad F(s) = \infty \quad \} \quad s = 0, -2, -4$$

- ... poles & zeros
- ... complex freq.
- ... singularities
- ... natural freq.



$$F(s) = \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+4)}$$

$$f(t) = (A + Be^{-2t} + Ce^{-4t}) u(t)$$

A, B, C.

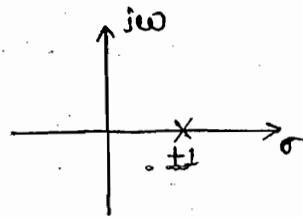
..... coef of practical fraction expansion

..... Residues.

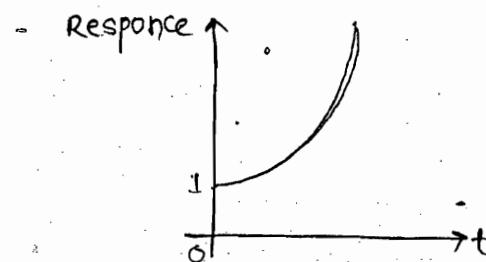
- * The zeros represent complex freq. when the numerator poly. of given n/w fⁿ becomes zero.
- Therefore at this freq. the n/w fⁿ becomes zero. The zeros control the magnitude of the response of any given ele. n/w.
- * Poles represent the complex freq. at which the denominator poly. of n/w fⁿ becomes zero.
- Therefore at this freq. the n/w fⁿ becomes ∞. The poles control the nature or shape of response of the ele. n/w.
- Therefore these poles than control the overall stability of the given n/w.
- * In any n/w no. of zeros & no. of poles are always equal.
- * for stable n/w :-
 (1) No pole should exist in RHP.
 (2) No multiple poles should exist along the jω axis including the origin.
- * The locⁿ of poles & zeros control the type of elements contain in the given n/w.
- * for driving pt. immitance fⁿ all the poles & zeros lie in LHP. For TFs all the poles lie in LHP whereas some of zeros may also lie in RHP.

EFFECT OF locⁿ OF poles on the nature of response of ele n/w →

Case(1) →

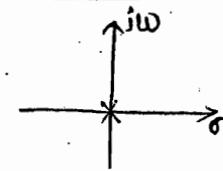


$$\frac{1}{s-1} \rightarrow e^t u(t)$$

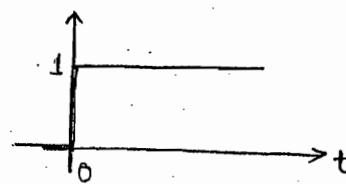


Unstable n/w

Case(2) →

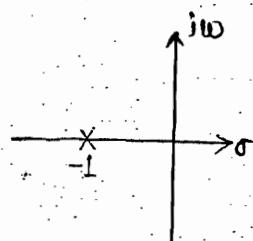


$$\frac{1}{s} \rightarrow u(t)$$



**ideal (OR) lossless
n/w**

Case(3) →

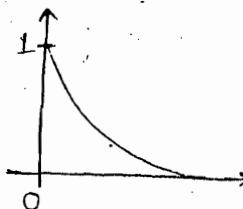


$$\frac{1}{s+1} \rightarrow e^{-t} u(t)$$

(OR)

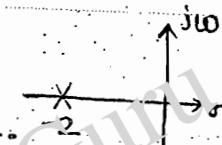
$$e^{-t/T} u(t)$$

($T = 1s$)



RC (OR) RL n/w

Case(4) →

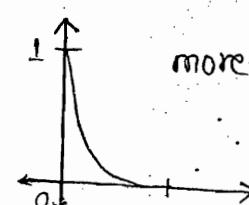


$$\frac{1}{s+2} \rightarrow e^{-2t} u(t)$$

(OR)

$$e^{-t/T} u(t)$$

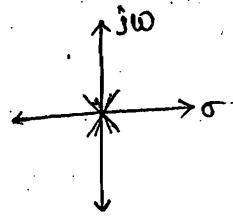
($T = 1/2$ sec)



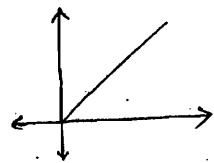
Stable ↑ TG ↓

more stable n/w

Case(5) →



$$\frac{1}{s^2} \rightarrow t \cdot u(t)$$



Unstable η/ω

* som...

* For a stable η/ω :-

(1) No pole should exist in RHP.

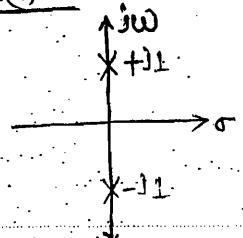
(2) No multiple pole should exist at the origin.

* When any pole lies along the -ve real axis the response is exponentially decaying & the rate of decay depends upon the exact loc'n of the pole along the -ve real axis.

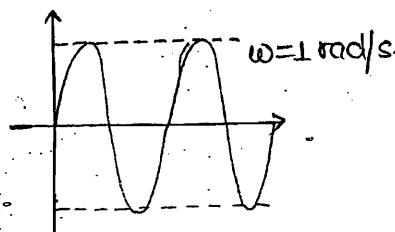
* When the pole is shifted away from origin along the -ve real axis the rate of decay of the exponential response increases, η/ω will

- become more stable & the TC of η/ω decreases.

Case(6) →



$$\frac{1}{(s-j1)(s+j1)} \rightarrow \frac{1}{s^2+1} \rightarrow \sin t \quad (\omega=1 \text{ rad/s})$$

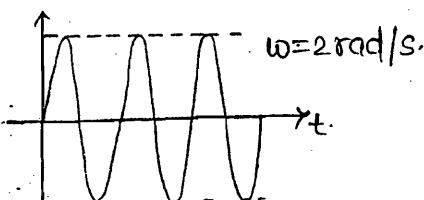
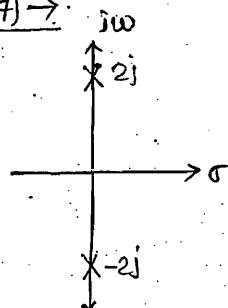


LC network

$$s=0+j1$$

$$\sigma=0$$

Case(7) →

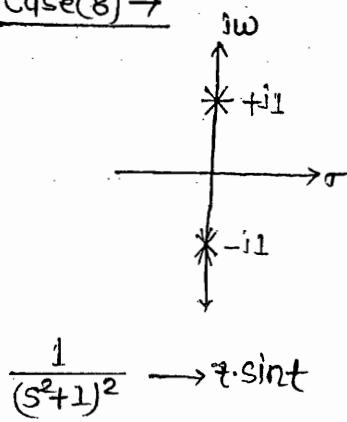


$$\frac{1}{(s-2j)(s+2j)} \rightarrow \frac{1}{s^2+4} \rightarrow \frac{1 \cdot 2}{(2)(s^2+4)}$$

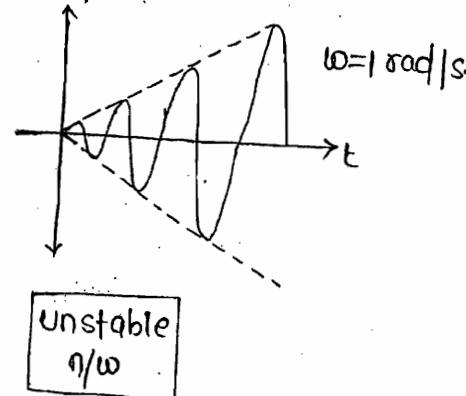
$$\Rightarrow \frac{1}{2} \sin 2t$$

$(\omega=2)$

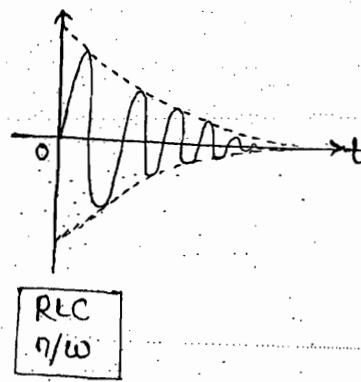
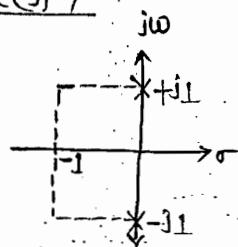
Case(8) →



$$\frac{1}{(s^2+1)^2} \rightarrow t \cdot \sin t$$



Case(9) →



Some points →

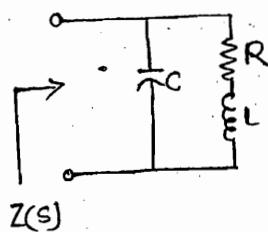
K. When the poles are located along the $j\omega$ axis the response is oscillatory in nature whose freq. is controlled by the exact locn of the poles along the $j\omega$ axis.

When the poles are shifted away from origin along the $j\omega$ axis the freq. of oscillatory response increases.

For a stable n/w multiple poles should not exist along the $j\omega$ axis.

When the poles are complex conjugate in nature & lie in LHP. then the response is oscillatory in nature whose amplitude decreases in an exponential manner.

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37



$$Z(0) = 1, \quad |Z(s)|_{s=0}$$

zero: $s = -1$

$$\text{poles: } s = \frac{-1 + j\sqrt{3}}{2}$$

$$Z(s) = \frac{\left(\frac{1}{sc}\right)(R+sl)}{\left(\frac{1}{sc}\right)+(R+sl)}$$

$$= \frac{(R+sl)}{1+sRc+s^2lc}$$

$$= \frac{l(s+r/l)}{lc(s^2+s\cdot R/l+\frac{1}{lc})}$$

$$Z(0) = \frac{R+0}{1+0+0} = 1$$

$$R = 1\Omega$$

Zeros \rightarrow form $s + \frac{R}{L} = 0$

$$s = -\frac{R}{L}$$

$$-1 = -\frac{R}{L}$$

$$L = 1H$$

Poles \rightarrow $s^2 + s \cdot \frac{R}{L} + \frac{1}{lc} = 0$

$$s_1, s_2 = \frac{-R \pm \sqrt{(R)^2 - 4 \times \frac{1}{lc}}}{2}$$

$$s_1, s_2 = \frac{-1 \pm \sqrt{(1-\frac{4}{c})}}{2}$$

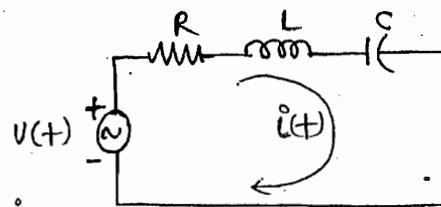
$$= -1 \pm 2j\sqrt{\frac{4}{c} \left(\frac{1}{c} - \frac{1}{4}\right)}$$

$$\frac{-1 \pm j\sqrt{3}}{2} = -1 \pm 2j\sqrt{\left(\frac{1}{c} - \frac{1}{4}\right)}$$

$$\frac{\sqrt{3}}{2} = \sqrt{\frac{1}{c} - \frac{1}{4}}$$

$$\frac{3}{4} = \frac{1}{c} - \frac{1}{4} \quad [c = 1F]$$

EFFECT OF VARIATION OF R ON THE RESPONSE IN RLC CIRCUIT



KVL \rightarrow

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C} \int i(t) dt = V(t)$$

$$R\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{1}{C}i(t) = \frac{dV(t)}{dt}$$

C/S EQUATION \rightarrow

$$Rs + Ls^2 + \frac{1}{C} = 0$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\Rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\Rightarrow 2\zeta\omega_n = \frac{R}{L}$$

$$\Rightarrow \omega_n = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \zeta = \frac{R}{2} \sqrt{\frac{C}{L}}$$

FINDING THE ROOTS OF C/S EQUATION \rightarrow

$$s_1, s_2 = \frac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2}$$

$$= -\zeta\omega_n \pm j\omega_n \sqrt{1 - \zeta^2}$$

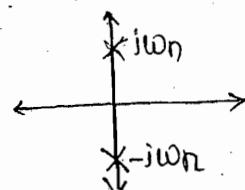
$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} \quad s_1, s_2 = -\zeta\omega_n \pm j\sqrt{1 - \zeta^2} \omega_n$$

Case (1) $\rightarrow \zeta = 0, R = 0$ (Undamped Response)

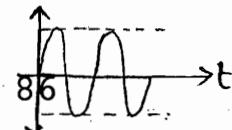
$$s_1, s_2 = \pm j\omega_n$$

$$\frac{1}{s^2 + \omega_n^2} \rightarrow \frac{1}{\omega_n} \sin \omega_n t$$

..... Roots are along $j\omega$ axis



..... Sinusoidal Response with Constant Amplitude.

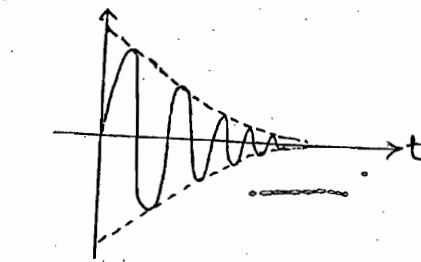
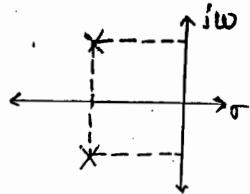


Case(2) $\rightarrow \xi < 1$ ($0 < \xi < 1$)

$$R < 2\sqrt{\frac{L}{C}}$$

$$s_1, s_2 = a \pm jb$$

..... Complex conjugate Root



..... sinusoidal Response
with decreasing amplitude.

Underdamped Response

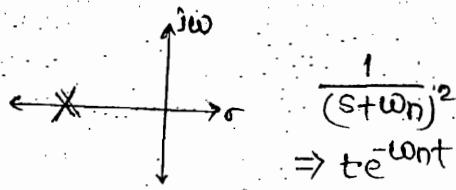
Case(3) $\rightarrow \xi = 1$

$$R = 2\sqrt{\frac{L}{C}}$$

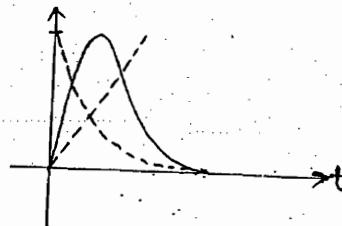
$$s_1, s_2 = -\omega_n$$

..... Double(equal) poles

along -ve real axis



$$\frac{1}{(s + \omega_n)^2} \\ \Rightarrow te^{-\omega_n t}$$



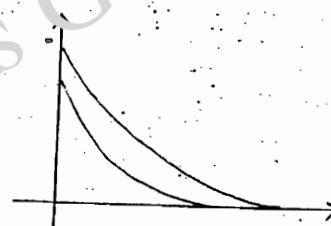
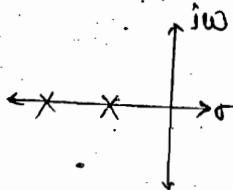
Critically damped

Case(4) $\rightarrow \xi > 1$

$$R > 2\sqrt{\frac{L}{C}}$$

$$s_1, s_2 = -a \pm b'$$

..... Unequal poles along -ve
real axis



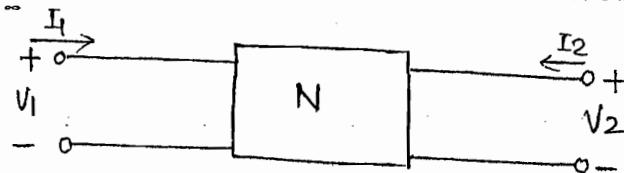
Over-damped

To avoid Oscillations

$$\xi \geq 1$$

Two Port n/w Parameters

..... Used to represent any 2 port n/w
mathematically.



Case(1) → Impedance (OR) Z (OR) OC parameter.

$$\boxed{V_1 = z_{11}I_1 + z_{12}I_2} \quad \dots \text{Series}$$

$$\boxed{V_2 = z_{21}I_1 + z_{22}I_2} \quad \dots \text{Series}$$

$y = mx + c$
 ↑
 dependant ↑
 independant

$I_2=0$	$I_1=0$
$z_{11} = \frac{V_1}{I_1} \Omega$	$z_{12} = \frac{V_1}{I_2} \Omega$
$z_{21} = \frac{V_2}{I_1} \Omega$	$z_{22} = \frac{V_2}{I_2} \Omega$

The n/w fns are calculated w/o any pre condn whereas the n/w parameters are calc. under some precondn either by OC (OR) SC the i/p OR O/p port.
 A single n/w fn is req. to specify any given ele. n/w whereas to represent the same n/w all the 4 parameters are req. simultaneously.

Case(2) → Admittance (OR) Y (OR) SC parameters.

$$\boxed{I_1 = y_{11}V_1 + y_{12}V_2} \quad \dots \text{parallel}$$

$$\boxed{I_2 = y_{21}V_1 + y_{22}V_2} \quad \dots \text{parallel}$$

$V_2=0$	$V_1=0$
$y_{11} = \frac{I_1}{V_1} S$	$y_{12} = \frac{I_1}{V_2} S$
$y_{21} = \frac{I_2}{V_1} S$	$y_{22} = \frac{I_2}{V_2} S$

Case(3) → Hybrid (OR) h-parameters

$V_1 = h_{11}I_1 + h_{12}V_2$ Series
$I_2 = h_{21}I_1 + h_{22}V_2$ Parallel

$V_2=0$	$I_1=0$
$h_{11} = \frac{V_1}{I_1} \Omega$	$h_{12} = \frac{V_1}{V_2}$
$h_{21} = \frac{I_2}{I_1}$	$h_{22} = \frac{I_2}{V_2} S$

* Because of mixed value of units it is known as hybrid parameter.

* This is only parameters used in the BJT because of this gives all the values of vol. gain, current gain, i/p & o/p imp. req. for BJT as amp.

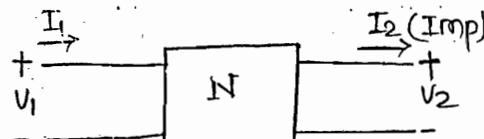
Case(4) → g-parameters.

$I_1 = g_{11}V_1 + g_{12}I_2$ parallel
$V_2 = g_{21}V_1 + g_{22}I_2$ series

$I_2=0 \rightarrow V_1=0$	
$g_{11} = \frac{I_1}{V_1} S$	$g_{12} = \frac{I_1}{I_2}$
$g_{21} = \frac{V_2}{I_1}$	$g_{22} = \frac{V_2}{I_2} \Omega$

* Here $V_1=0$, i.e. i/p is shorted. So it is not used in BJT for amp.

Case(5) → Transmission (OR) ABCD Parameters.

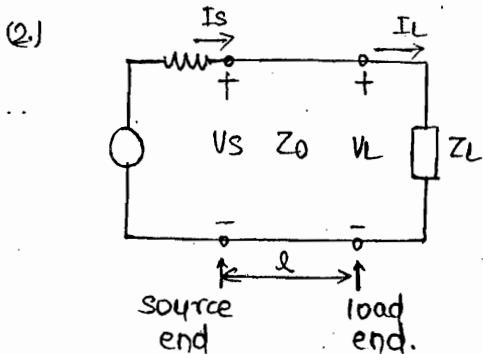
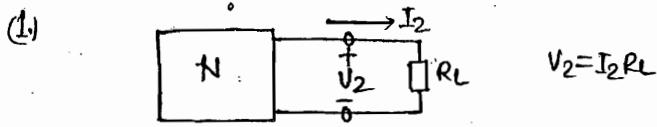


$V_1 = AV_2 + BI_2$	
$I_1 = CV_2 + DI_2$	

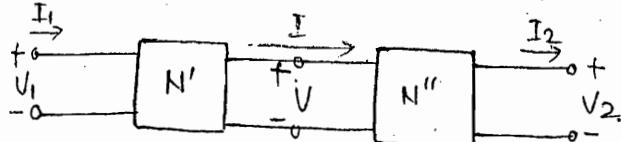
* If the value of vol. is doubled then B will be twice & C will halved.

$I_2=0$	$V_2=0$
$A = \frac{V_1}{I_2}$	$B = \frac{V_1}{I_2} \Omega$
$C = \frac{I_1 S}{V_2}$	$D = \frac{I_1}{I_2}$

* Reason behind taking sign of current opposite \rightarrow

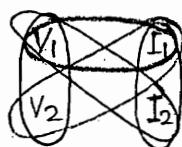


(3.1) Cascaded N/W \rightarrow



* Condition for any n/w to be reciprocal & symmetrical \rightarrow

Reciprocal	Symmetrical
* $Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
* $Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
* $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1, \Delta = 1$ $AD - BC = 1$	$A = D$
* $h_{12} = h_{21}$	$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1, \Delta h = 1$ $h_{11}h_{22} - h_{12}h_{21} = 1$
* $g_{12} = -g_{21}$	$\begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1, \Delta g = 1$ $g_{11}g_{22} - g_{21}g_{12} = 1$



* Conversion b/n parameters \rightarrow

* (1) Given: $[Z]$; To Find: $[Y]$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta Z} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} ; \quad Y_{21} = -\frac{Z_{21}}{\Delta Z}$$

$$Y_{12} = \frac{-Z_{12}}{\Delta Z} ; \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

$$\Delta Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} = Z_{11}Z_{22} - Z_{12} \cdot Z_{21}$$

* (2) Given: $h_{11}=2, h_{12}=-2, h_{21}=3, h_{22}=2$

To find: $[Z]$

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

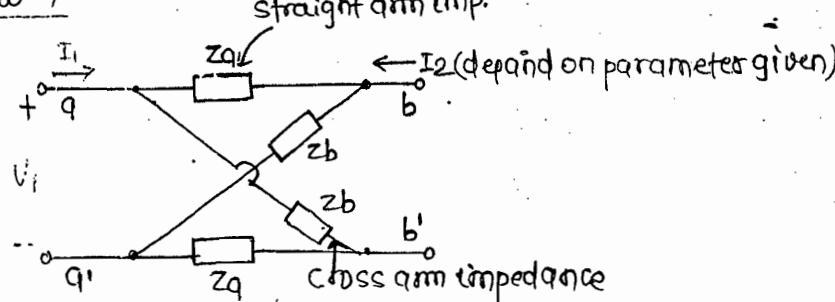
$$\begin{aligned} V_1 = 2I_1 - 2V_2 &\Rightarrow V_1 = 2I_1 - (I_2 - 3I_1) \\ I_2 = 3I_1 + 2V_2 &\Rightarrow 2V_2 = I_2 - 3I_1 \\ &\Rightarrow V_1 = 5I_1 + (-1)I_2 \\ &\Rightarrow V_2 = \left(\frac{3}{2}\right)I_1 + \left(\frac{1}{2}\right)I_2 \end{aligned}$$

$$[Z] = \begin{bmatrix} 5 & -1 \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}; \Omega$$

$Z_{11} \neq Z_{22} \Rightarrow$ Not symmetrical

$Z_{12} \neq Z_{21} \Rightarrow$ Not reciprocal.

* Lattice n/w \rightarrow



To find \rightarrow

(1) $[Z]$

(2) Zq, Zb in terms of $[Z]$

* A Lattice n/w said to be symmetrical if the 2 straight arm imp. are equal i.e.

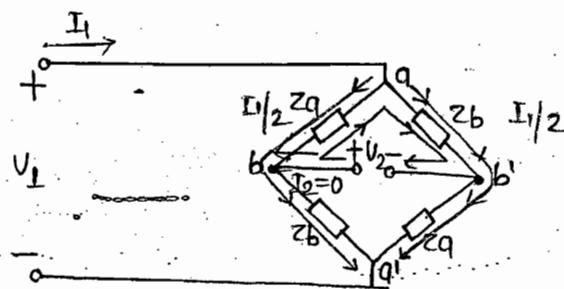
$$Z_{11} = Z_{22} \dots \text{Symmetrical N}$$

* A lattice n/w is said to be reciprocal if the 2 cross arm imp. are equal i.e.

$$Z_{12} = Z_{21} \dots \text{Reciprocal N}$$

$$\underline{I_2 = 0} \dots$$

$$Z_{11} = \frac{V_1}{I_1} \quad Z_{21} = \frac{V_2}{I_1}$$



By KVL →

$$V_1 = \frac{I_1}{2}(Z_q + Z_b) \dots (i)$$

$$0 = -V_2 - \frac{I_1}{2}Z_q + \frac{I_1}{2}(Z_b) \dots (ii)$$

$$V_2 = \frac{I_1}{2}(Z_b - Z_q) \dots (iii)$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{Z_b + Z_q}{2} = Z_{22}$$

$$\boxed{Z_{11} = Z_{22} = \frac{Z_b + Z_q}{2}}$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{Z_b - Z_q}{2} = Z_{12}$$

$$\boxed{Z_{21} = Z_{12} = \frac{Z_b - Z_q}{2}}$$

$$Z_b + Z_q = 2Z_{11}$$

$$Z_b - Z_q = 2Z_{12}$$

$$\text{Add: } Z_b = Z_{11} + Z_{12}$$

$$\text{Subtract: } Z_q = Z_{11} - Z_{12}$$

Lattice N/W

$$* [Z] = \frac{1}{2} \begin{bmatrix} Z_b + Z_q & Z_b - Z_q \\ Z_b - Z_q & Z_b + Z_q \end{bmatrix}$$

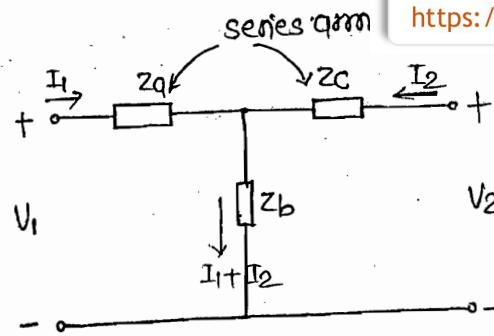
$$\underline{\text{T-N/W}}$$

$$* [Z] = \begin{bmatrix} Z_q + Z_b & Z_b \\ Z_b & Z_c + Z_b \end{bmatrix}$$

$$\underline{\text{π-N/W}}$$

$$* [Y] = \begin{bmatrix} Y_q + Y_b & -Y_b \\ -Y_b & Y_c - Y_b \end{bmatrix}$$

Example →



To find $\rightarrow [Z]$

T(OR) Y are star n/w

By KVL →

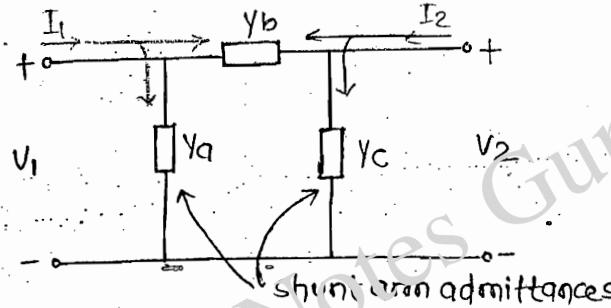
$$V_1 = \underbrace{(Z_q + Z_b)}_{Z_{11}} I_1 + \underbrace{Z_b I_2}_{Z_{12}}$$

$$V_2 = \underbrace{Z_b I_1}_{Z_{21}} + \underbrace{(Z_b + Z_c) I_2}_{Z_{22}}$$

* Any general T-n/w is always a reciprocal n/w but may not be a symmetrical n/w. Such n/w is symm. in nature if the 2 series arm impedances Z_q & Z_c are equal.

DATE - 03/12/14

Example → Π (OR) Δ n/w



To find
[Y]

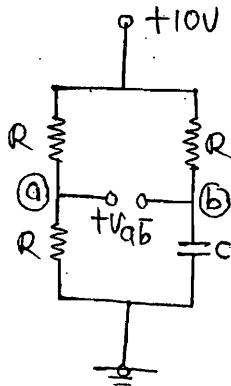
$$I_1 = V_1 Y_q + (V_1 - V_2) Y_b = V_1 \underbrace{(Y_q + Y_b)}_{Y_{11}} + \underbrace{(-Y_b)}_{Y_{12}} V_2$$

$$I_2 = V_2 Y_c + (V_2 - V_1) Y_b = \underbrace{(-Y_b)}_{Y_{21}} V_1 + \underbrace{(Y_c - Y_b)}_{Y_{22}} V_2$$

* Any general Π -n/w is always a reciprocal n/w but may not be symmetrical in nature. Such n/w is a symmetrical n/w if the 2 shunt arm admittances Y_q & Y_c are equal.

(4)
34

By rearranging the given n/w



..... All pass filter.

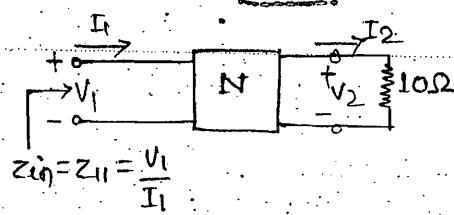
$$|V_{ab}| = |V_a - V_b|$$

$$V_{ab} = 5 - \frac{1/j\omega C}{R + 1/j\omega C} \times 10$$

$$= 5 \times \frac{-1 + j\omega RC}{1 + j\omega RC}$$

$$|V_{ab}| = 5 \text{ V.}$$

(5)
35



$$I_1 = A V_2 + B I_2$$

$$V_1 = C V_2 + D I_2$$

$$N: \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$V_1 = A V_2 + B I_2 = V_2 + 2I_2 = 10I_2 + 2I_2 = 12I_2$$

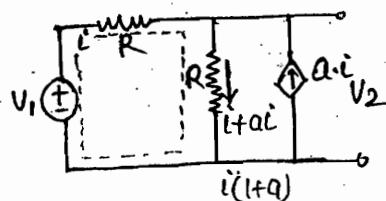
$$I_1 = C V_2 + D I_2 = V_2 + 3I_2 = 10I_2 + 3I_2 = 13I_2$$

$$V_2 = 10I_2$$

$$V_1 = 12I_2 ; I_1 = 13I_2$$

$$\boxed{\frac{V_1}{I_1} = \frac{12}{13} = Z_{11}}$$

(10)
35



$$V_1 = R + (1+q)R \quad \text{--- (i)}$$

$$V_2 = R(1+q)i \quad \text{--- (ii)}$$

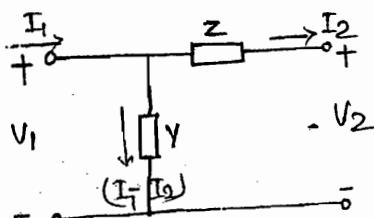
$$\frac{V_1}{V_2} = \frac{R + (1+q)R}{R(1+q)} = \frac{2+q}{1+q}$$

$$\frac{V_1}{V_2} = \frac{2+q}{1+q}$$

$$\boxed{\frac{V_2}{V_1} = \frac{1+q}{2+q}}$$

Ans.

(21)
37



for ABCD parameters

$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

$$(I_1 - I_2)Y = V_1 Y \quad \text{--- (i)}$$

$$V_1 = I_2 Z + V_2 \quad \text{--- (ii)}$$

$$V_1 = V_2 + Z I_2 = \frac{1}{A} V_2 + \frac{Z}{B} I_2$$

from eqn(i)

$$I_1 = V_1 Y + I_2 = Y(I_2 Z + V_2) + I_2 = V_2 + I_2(1 + YZ)$$

$$I_1 = \underbrace{YV_2}_{C} + \underbrace{(1+YZ)I_2}_{D}$$

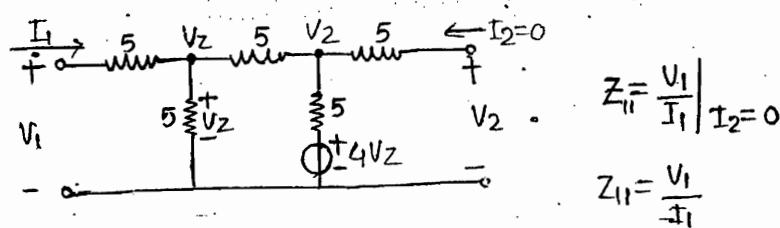
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ Y & 1+YZ \end{bmatrix}$$

Ans.

A ≠ D (not symmetrical)

$$AD - BC = 1(1+YZ) - YZ = 1 + YZ - YZ = 1 \quad (\text{Reciprocal})$$

(29)
39



$$Z_{11} = \left| \frac{V_1}{I_1} \right| \Big|_{I_2=0}$$

$$Z_{11} = \frac{V_1}{I_1}$$

In this eqn if we calculate either η/ω or η/ω parameter both will be same.

$$\frac{V_Z - V_1}{5} + \frac{V_Z - 0}{5} + \frac{V_Z - V_2}{5} = 0$$

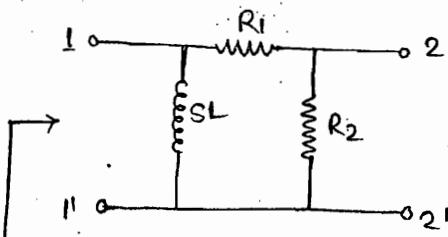
$$3V_Z = V_1 + V_2 \quad \dots \dots \text{(i)}$$

$$\frac{V_2 - V_Z}{5} + \frac{V_2 - 4V_Z}{5} = 0$$

$$2V_2 = 5V_Z \quad \dots \dots \text{(ii)}$$

$$\frac{V_1 - V_Z}{5} = I_1 \quad \dots \dots \text{(iii)}$$

(31)
39)



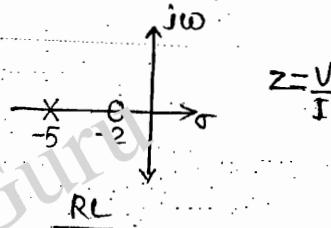
$$Z_{11} = \frac{(sL)(R_1 + R_2)}{(sL + R_1 + R_2)} = \frac{L(R_1 + R_2)}{L + s + q} \cdot s$$

$$\text{where; } q = \frac{R_1 + R_2}{L}$$

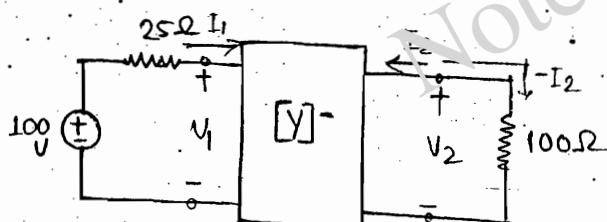
$$Z_{22} = \frac{(R_2)(R_1 + sL)}{(R_2 + R_1 + sL)}$$

$$= \frac{R_2 \cdot L}{L} \cdot \frac{s + R_1/L}{s + (R_1 + R_2)/L} = k_2 \frac{s + b}{s + q}$$

$$[Z_{11}, Z_{22}] = k_1 \frac{s}{s+q}, k_2 \frac{s+b}{s+q}$$



(4)



$$[Y] = \begin{bmatrix} 0.1s & -0.01s \\ 0.01s & 0.1s \end{bmatrix}$$

$$\text{To find: } \frac{V_2}{V_1} = g_{21}$$

$$I_1 = 0.1V_1 - 0.01V_2 \quad \dots \dots \text{(1)}$$

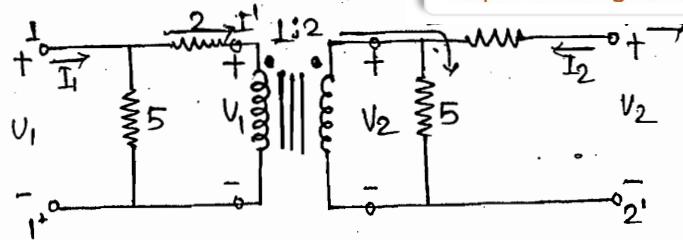
$$I_2 = 0.01V_1 + 0.01V_2 \quad \dots \dots \text{(ii)}$$

$$100 = 25I_1 + V_1 \quad \dots \dots \text{(iii)}$$

$$V_2 = -I_2 \cdot 100 \quad \dots \dots \text{(iv)}$$

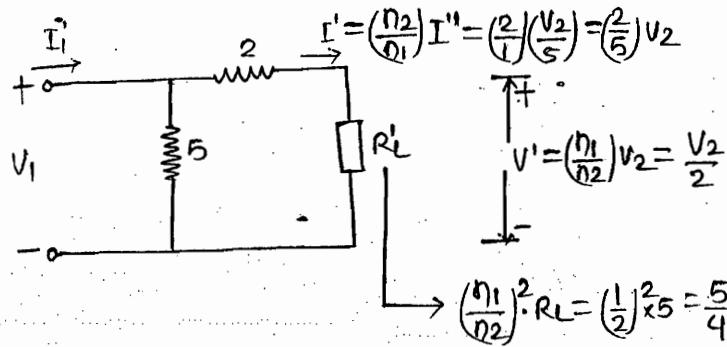
Eliminate I_2 from this 2 eqn

(36)
40



$$V_1 = AV_2 + BI_2$$

$$A = \frac{V_1}{V_2} \quad | \quad I_2 = 0$$



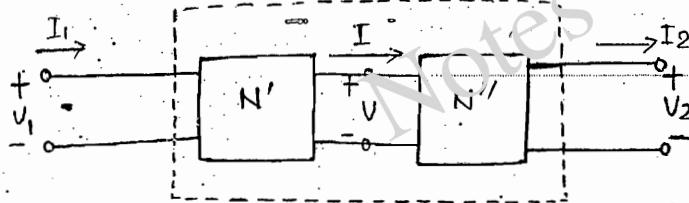
By KVL \rightarrow

$$\begin{aligned} V_1 &= 2I_1 + V' \\ &= 2 \times \frac{2}{5} V_2 + \frac{1}{2} V_2 \end{aligned}$$

$$\boxed{\frac{V_1}{V_2} = A = \frac{13}{10}}$$

* Inter connection b/w various 2-port n/w \rightarrow

Case(1) \rightarrow Cascaded N/W \rightarrow



$$N': \begin{cases} V_1 = AV + BI \\ I_1 = CV + DI \end{cases}$$

$$N'': \begin{cases} V = A''V_2 + B''I_2 \\ I = C''V_2 + D''I_2 \end{cases}$$

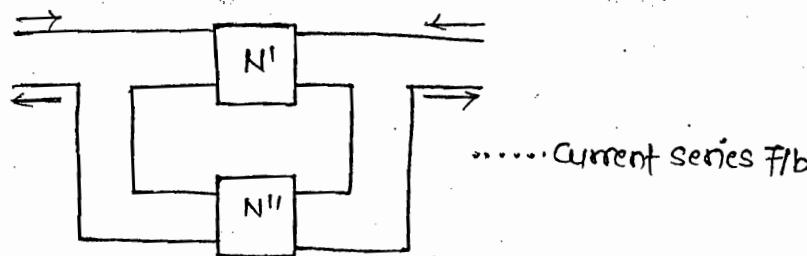
Overall N/W \rightarrow

$$\begin{cases} V_1 = AV_2 + BI_2 \\ I_1 = CV_2 + DI_2 \end{cases}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}$$

* When 2 n/w are connected in cascade then the ABCD parameters of the individual are multiplied to obtain overall ABCD parameters of the resulting n/w.

Cqse(2) → Series N/w →

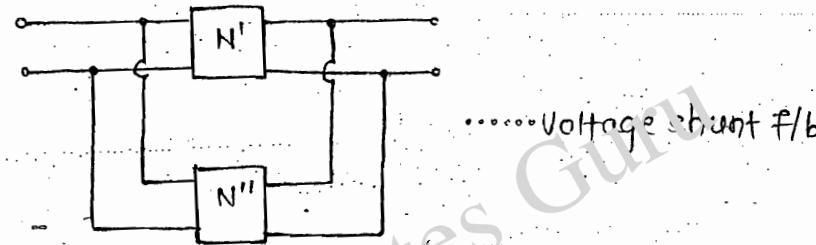


$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z'_{11} + Z''_{11} & Z'_{12} + Z''_{12} \\ Z'_{21} + Z''_{21} & Z'_{22} + Z''_{22} \end{bmatrix}$$

$$[Z] = [z'] + [z'']$$

When 2 n/w are connected in series at the i/p & o/p then the respective z-parameters of the 2 n/w are added to obtain overall z-parameters of the resulting n/w.

Cqse(3) → Parallel n/w →



$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y'_{11} + Y''_{11} & Y'_{12} + Y''_{12} \\ Y'_{21} + Y''_{21} & Y'_{22} + Y''_{22} \end{bmatrix}$$

$$[y] = [y'] + [y'']$$

When 2 n/w are connected in parallel (shunt) at i/p & o/p then the resulting respective y parameters of the 2 n/w are added to obtain the overall y parameters of resulting n/w.

Case(4) → Series parallel N/w →

$$[h] = [h'] + [h'']$$

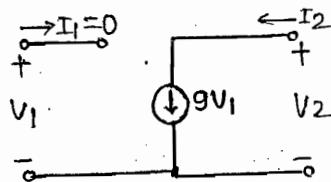
Case(5) → Parallel-Series N/w →

$$[g] = [g'] + [g'']$$

(14)
36

$$N' \Rightarrow y_{11} = y_{22} = -y_{12} = -y_{21} = y$$

$$N'' \Rightarrow I_1 = 0$$

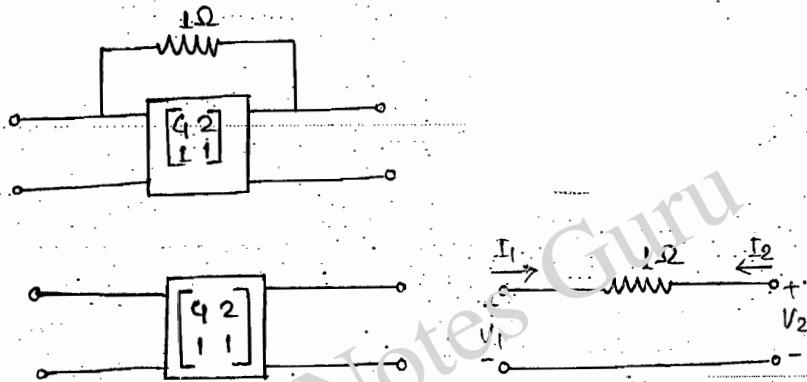


$V_2 = 0$	$V_1 = 0$
$y_{11} = \frac{I_1}{V_1} = 0$	$y_{21} = \frac{V_1}{V_2} = \frac{0}{V_2} = 0$
$y_{21} = \frac{I_2}{V_1} = \frac{gV_1}{V_1}$	$y_{22} = \frac{I_2}{V_2} = \frac{gV_1}{V_2} = 0$

$$\begin{aligned} y_{21}(\text{overall}) &= y_{21}(N') + y_{21}(N'') \\ &= -y + g \end{aligned}$$

$$y_{21} = -y + g$$

(1)
34



$$I_1 = -I_2 \quad \dots \quad (1)$$

$$V_1 = I_1 \times 1 + V_2$$

$$I_1 = \underbrace{1}_{Y_{11}} \times V_1 + \underbrace{(-1)}_{Y_{12}} V_2$$

$$I_2 = \underbrace{(-1)}_{Y_{21}} \times V_1 + \underbrace{(1)}_{Y_{22}} V_2$$

$$[y] = [y] + [y']$$

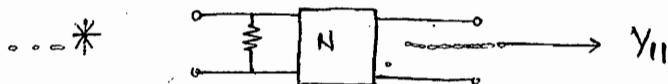
$$[y] = \begin{bmatrix} 4+1 & 2+(-1) \\ 1+(-1) & 1+(1) \end{bmatrix}; s$$

Circuit

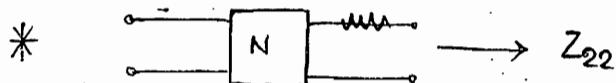
Parameters affected



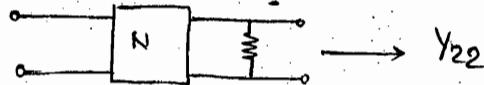
$$\rightarrow Z_{11}$$



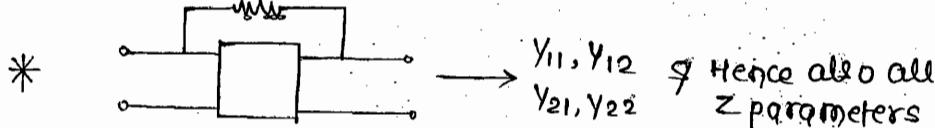
$$\rightarrow Y_{11}$$



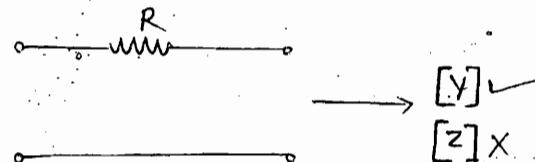
$$\rightarrow Z_{22}$$



$$\rightarrow Y_{22}$$



$\rightarrow Y_{11}, Y_{12}, Y_{21}, Y_{22}$ & Hence also all
Z parameters

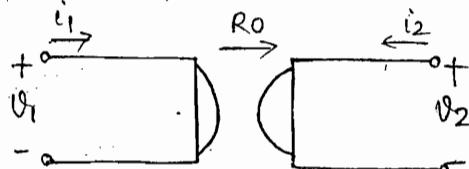


$\rightarrow [Y] \checkmark$
 $[Z] X$



$[Y] X$
 $\rightarrow [Z] \checkmark$

Example



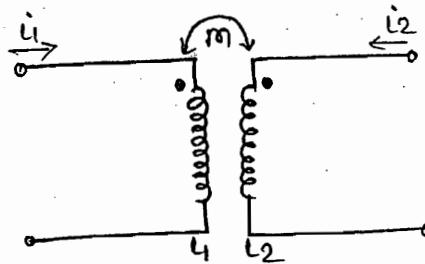
$$v_1 = +R_0 i_2 \Rightarrow v_1 = (0) i_1 + (+R_0) i_2$$

$$v_2 = -R_0 i_1 \Rightarrow v_2 = (-R_0) i_1 + (0) i_2$$

$$[Z] = \begin{bmatrix} 0 & +R_0 \\ -R_0 & 0 \end{bmatrix}; \Omega$$

..... symmetrical & not reciprocal.

Example 7



$$[Z] = \begin{bmatrix} j\omega L_1 + j\omega M & \\ +j\omega M & j\omega L_2 \end{bmatrix}$$

..... Reciprocal & not symmetrical

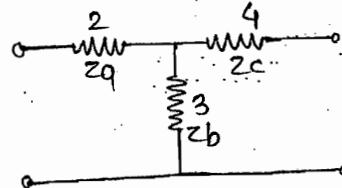
Example 8 A General Resistive T-N/W is shown below. 2 such identical N/W are connected :-

(1) cascade

(2) Series

Calculate Overall ABCD parameters & the Z parameters.

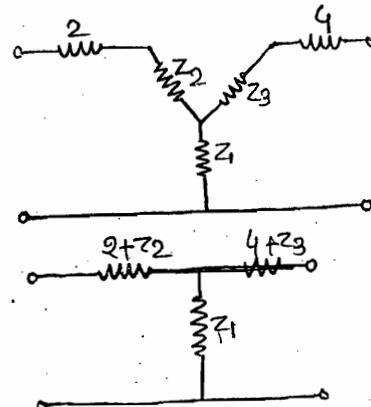
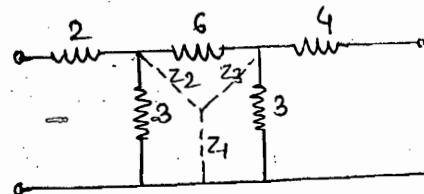
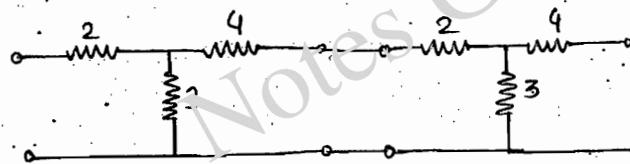
Soln →



$$\text{Soln } z_{11} = z_a + z_b = 5\Omega \quad z_{12} = z_{21} = z_b = 3\Omega$$

$$z_{22} = z_b + z_c = 7\Omega$$

Case(1) Cascade N/W

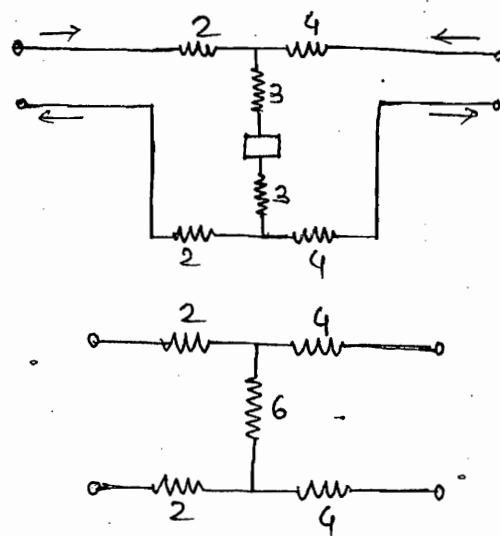


$$z_{11} = (2+z_2) + z_1$$

$$z_{22} = (4+z_3) + z_1$$

$$z_{12} = z_{21} = z_1$$

Case(2) Series N/W \rightarrow

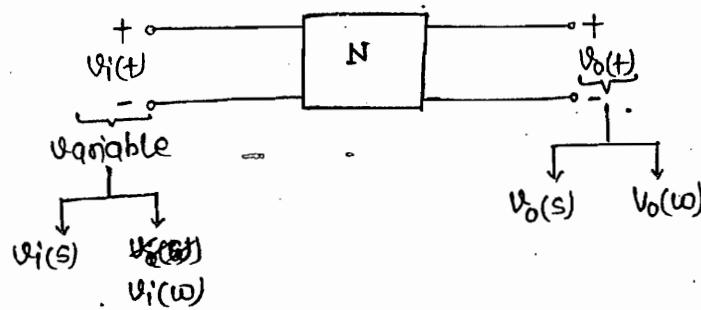


$$Z_{11} = 2 + 6 + 2 = 10 \Omega, Z_{22} = 4 + 4 + 6 = 14 \Omega$$

$$Z_{12} = Z_{21} = 6 \Omega$$

..... As a freq. selective N/W

* Principle →

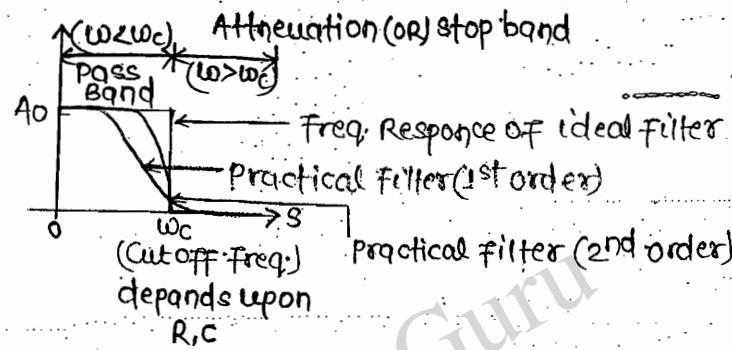


$$H(s) = \frac{V_o(s)}{V_i(s)} = A_v(s)$$

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = A_v(\omega)$$

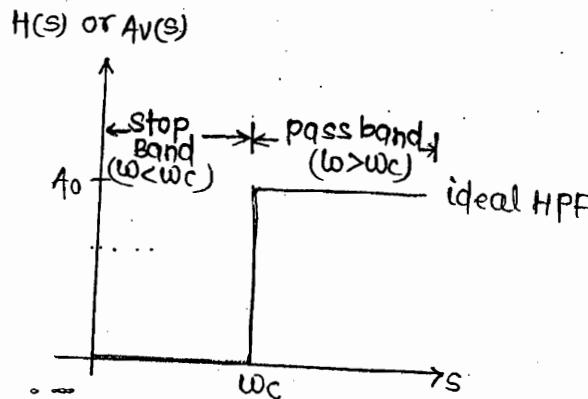
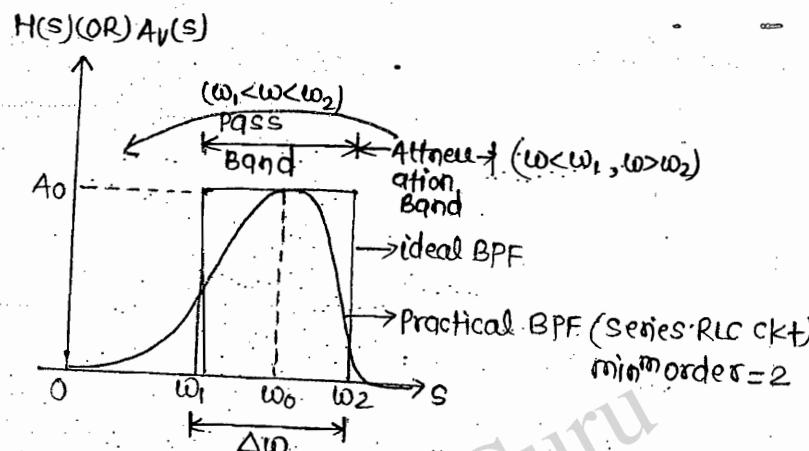
Case(1) → Low pass filter (LPF)

$$H(s) \text{ (OR) } A_v(s)$$



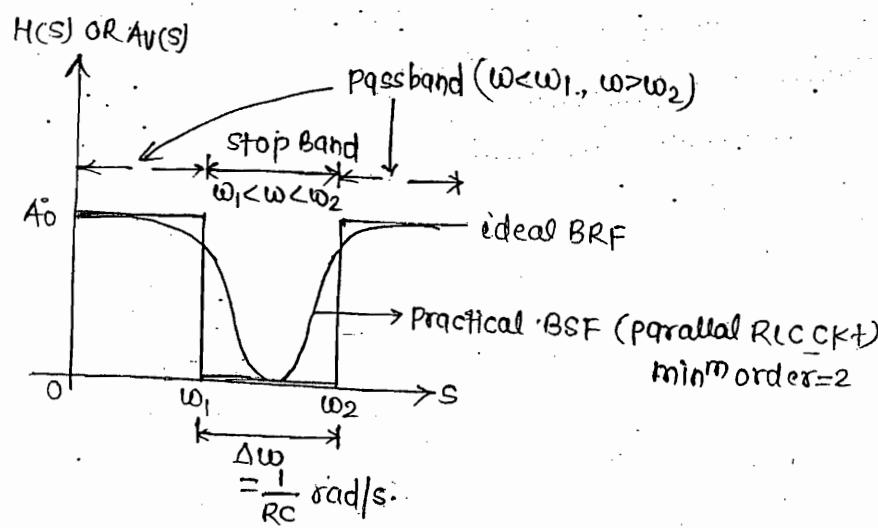
C/S of a practical filter

- * Cut off freq. of the filter can be adjusted by suitably selecting the numerical values of R & C components.
- * The gain during the pass band can be adjusted as per requirement by including an active device such as a variable gain opamp.
- The filter which include opamp as active device are called active filter.
- * An ideal filter has sharp cutoff c/s which can't be obtain practically. Since the response of any R-C n/w is limited by its finite time const. Therefore any practical filter has gradual cutoff c/s. This c/s can be approached to those of ideal filter by increasing the order of filter. This is done the no. of stages which are used in cascade in any type of filter.

Case(2) → High pass filter (HPF)Case(3) → Band pass Filter (BPF)

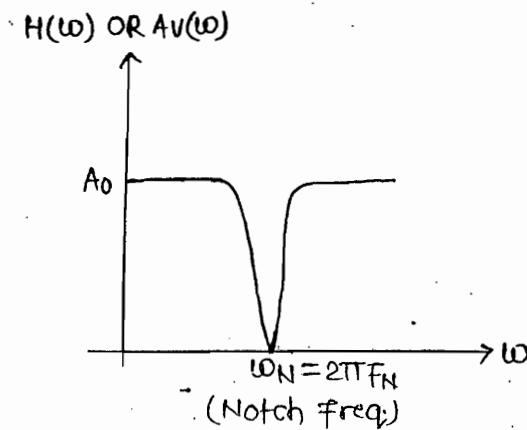
$$\Delta\omega = \omega_2 - \omega_1 = BW = \frac{R}{L} \text{ rad/sec}$$

$$\text{Also } Q_o = \frac{\omega_0}{\Delta\omega}$$

Case(4) → Band stop filter (or) BRF (Reject)

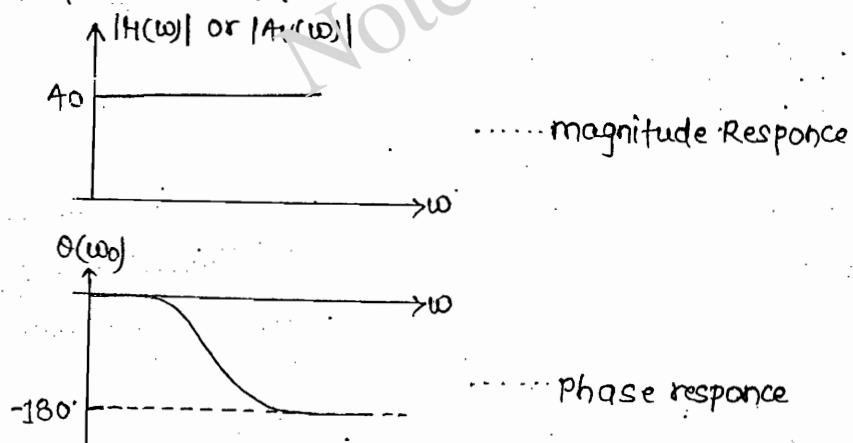
Case(5) → Notch filter

Because of appn of sinusoidal ($s \rightarrow \omega$) in x-axis



- * This filter is used for noise (or) harmonics minimization. (like 3rd harmonic distortion).
- * A Notch filter is special case of a BSF where a single freq; called the Notch freq; is not allowed to pass through sys.
- * This filter is always used as the i/p state of any audio sys where 50Hz main freq; is not allow to pass through the sys to avoid the distortion & the noise in the audio sys.
- * This filter is also used to minimise the 3rd harmonic distortion in any sys & therefore such filter is used as a i/p stage of that sys.

Case(6) → All pass filter (APF)

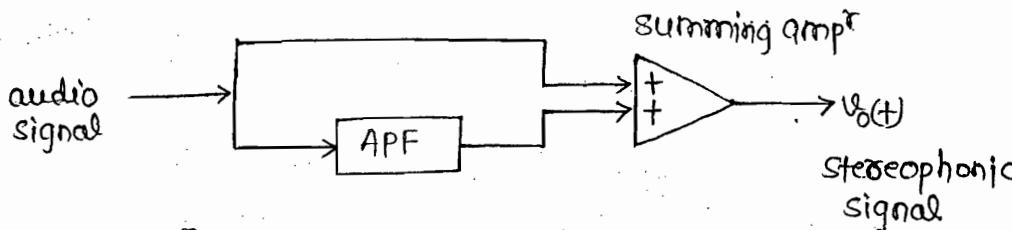


- * An APF passes all the freq; component with const. gain but various freq; component are subjected to diff. amt of phase shift having a min^m value of 0° & a max^m phase shift of -180° .

* Such filter has all the poles in LHP & all the zeros lie in RHP.

* Since in any n/w the no. of poles & no. of zeros are always equal then the no. of poles in LHP or the no. of zeros in RHP will control the order of the filter.

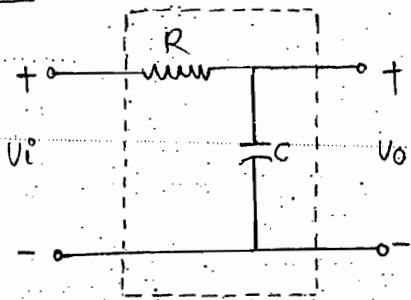
Application →



∴ An APF is used to produce stereophonic signal in any audio sys.

Example →

(1). 1st order LPF →



To find:-

$$(1) \text{TF } H(s) = \frac{V_o(s)}{V_i(s)}$$

(6) Justification

(2) Cut-off freq. (ω_c)

(3) Pole-Zero pattern

(4) Freq. response $H(s)$ vs s

(5) Magnitude res. $|H(\omega)|$ vs ω

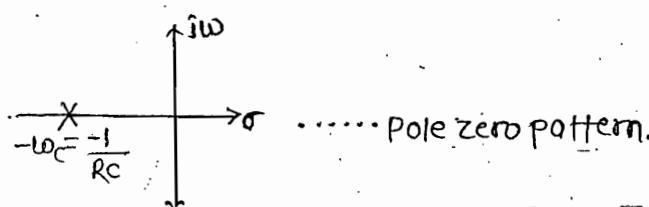
* Based on the time const (RC) this ckt is also known as integrator ckt.

$$\frac{V_o(s)}{V_i(s)} = \frac{1/SC}{R + 1/SC} = \frac{1}{1 + SRC} = \boxed{\frac{1}{1 + \frac{s}{\omega_c}} = H(s)}$$

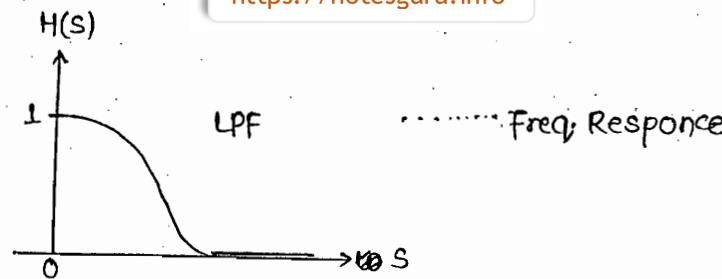
where $\omega_c = \frac{1}{RC} \dots \text{rad/s}$

$$f_c = \frac{1}{2\pi RC} \dots \text{Hz}$$

} cut off freq.



..... Pole zero pattern.



Now, for $s \rightarrow j\omega$

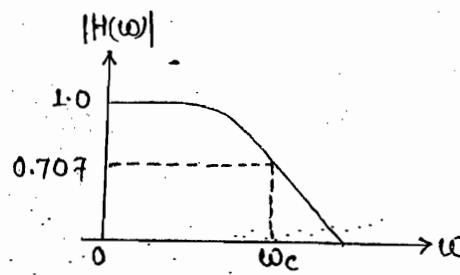
$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_c}}$$

(1) $\omega < \omega_c$; $|H(\omega)| \leq 1$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

(2) $\omega = \omega_c$; $|H(\omega)| = \frac{1}{\sqrt{2}} = 0.707$

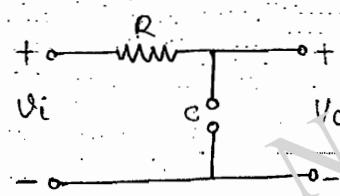
(3) $\omega > \omega_c$; $|H(\omega)| = \frac{\omega}{\omega_c}$



* Justification →

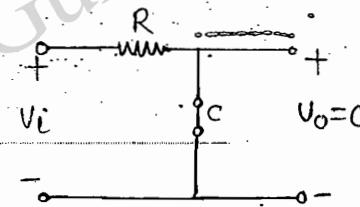
(1) $\omega = 0, s = 0$

$$Z_C = \frac{1}{sC} = \infty (\text{OC})$$

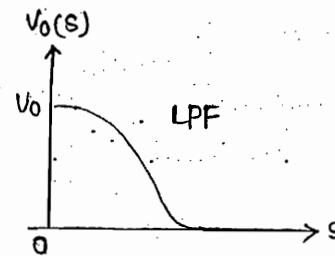


(2) $\omega = \infty, s = \infty$

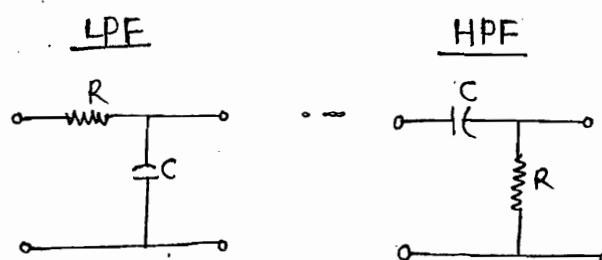
$$Z_C = \frac{1}{sC} = 0 \text{ (short)}$$



(3) $s = s_c$; U_o - finite value.



2) 1st Order HPF →



Comments:

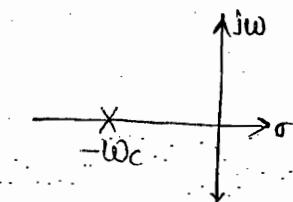
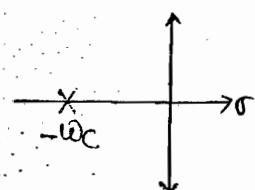
* Interchange the locn of R & C .

$$H(s) = \frac{1}{1 + \left(\frac{s}{\omega_c}\right)} \quad \frac{1}{1 + \left(\frac{\frac{1}{s}}{\omega_c}\right)} = \frac{s/\omega_c}{1 + \left(\frac{s}{\omega_c}\right)} \quad * \text{ Replace } \frac{s}{\omega_c} \text{ by } \frac{1}{(s/\omega_c)}$$

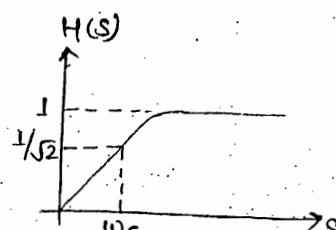
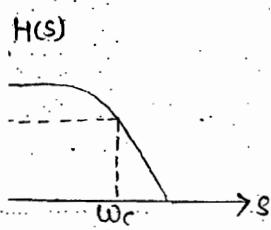
$$\omega_c = \frac{1}{RC}$$

$$\omega_c = \frac{1}{RC}$$

* Cut-off Freq.

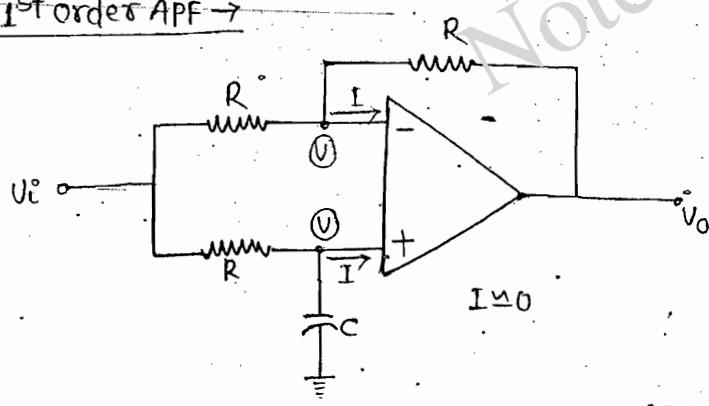


* Pole-zero plot



* Freq. Response

1st Order APF →



To find:-

$$(1) \text{ TF} = H(s) = \frac{V_o(s)}{V_i(s)}$$

(2) Pole-zero plot

(3) Magnitude response,
 $|H(\omega)|$ vs ω

Phase response

$$\theta(\omega) \text{ vs } \omega$$

The vol. at i/p terminal of OPAMP are nearly equal since the vol. gain in the differential mode is almost ∞ .

The current drawn by the i/p terminal of the OPAMP are nearly zero. since the i/p impedance of ideal OPAMP is ∞ .

* Always write KCL eqn at the i/p terminals of OPAMP.

* Never write KCL eqn at the o/p of OPAMP unless it is terminated by some Load resistor R_L .

KCL(1) inverting i/p :-

$$\frac{V - V_i}{R} + \frac{V - V_o}{R} + 0 = 0$$

$$2V = V_i + V_o \quad \text{--- (i)}$$

KCL(2) at NI i/p :-

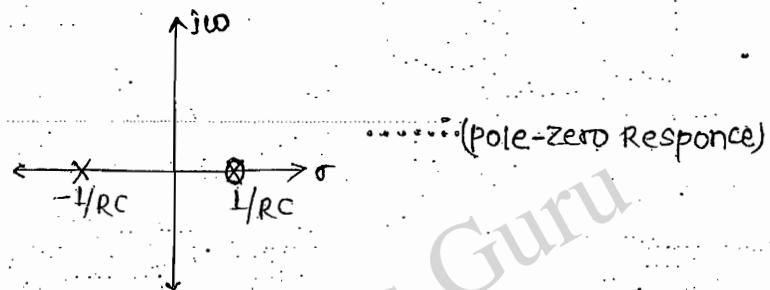
$$\frac{V - V_i}{R} + \frac{V - 0}{1/SC} + 0 = 0$$

$$V \left(\frac{1}{R} + SC \right) = \frac{V_i}{R}$$

$$V = \frac{V_i}{1 + RSC} \quad \text{--- (ii)}$$

from eqn (i) & (ii)

$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{1 - SRC}{1 + SRC}$$

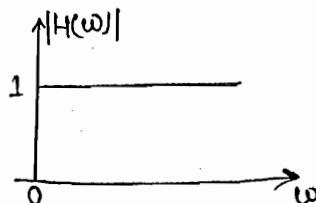


$s \rightarrow j\omega$

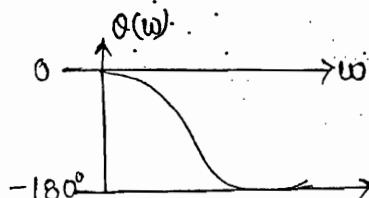
$$H(\omega) = \frac{1 - j\omega RC}{1 + j\omega RC}$$

$$|H(\omega)| = 1$$

$$\theta(\omega) = -2 \tan^{-1} \left(\frac{\omega RC}{1} \right)$$



..... magnitude
Response

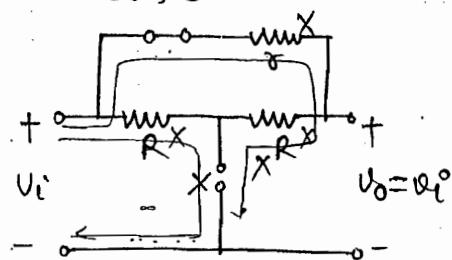


..... phase Response

(2)
52

$s \rightarrow 0$

$$C = OC; L = SC$$

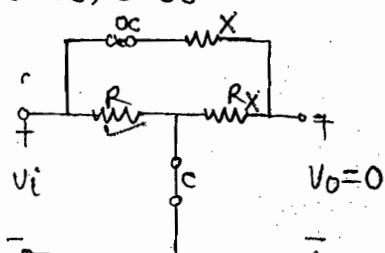


$X \rightarrow$ NO current flow

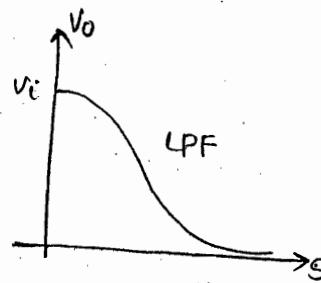
$$V_o = V_i$$

$s \rightarrow \infty$

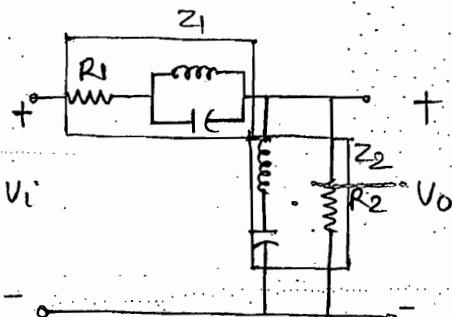
$$C = SC, L = OC$$



$$V_o = 0$$



(3)
52



$$V_o$$

(1) $s = 0, L \rightarrow \infty$

$C \rightarrow OC$

$$V_o = \frac{R_2}{R_1 + R_2} V_i \quad \text{finite}$$

BRF/BSF

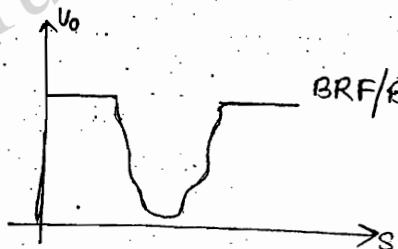
(2) $s = \infty, L \rightarrow OC$

$C \rightarrow SC$

$$V_o = \frac{R_2}{R_1 + R_2} V_i \quad \text{finite}$$

(3) $s = s$

$$if V_o = \frac{Z_2 V_i}{Z_1 + Z_2} V_i$$

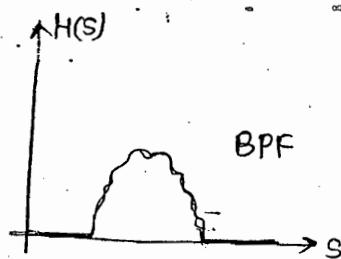


(4)
52

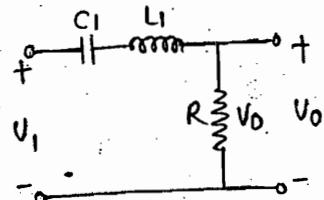
$$H(s) = \frac{10s}{s^2 + 10s + 100}$$

$$s = 0; \frac{0}{0+0+100} = 0$$

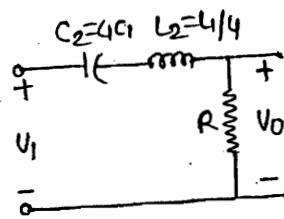
$$s = \infty; \frac{10/s}{(1+\frac{10}{s} + \frac{100}{s^2})} = 0$$



(53)



$$B_1 = \Delta\omega_1 = \frac{R}{L}$$



$$B_2 = \Delta\omega_2 = \frac{R}{L} = \frac{R}{4/4} = 4R = 4B_1$$

$$\frac{B_1}{B_2} = ?$$

$$\boxed{\frac{B_1}{B_2} = \frac{1}{4}}$$

ω_0 is same for above both n/w

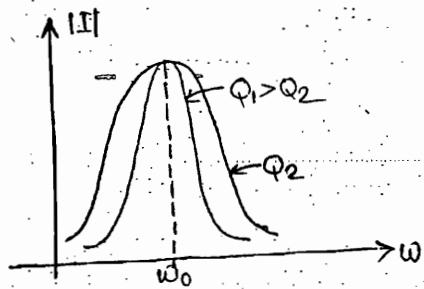
$$Q_0 = \frac{\omega_0}{B}$$

$$Q_0 \propto \frac{1}{B}$$

$B_1 < B_2 \} \text{ ckt } \& \text{ behaved as a better tuned ckt}$

$$Q_1 > Q_2 \}$$

$$B_1 > B_2 \\ Q_2 > Q_1$$

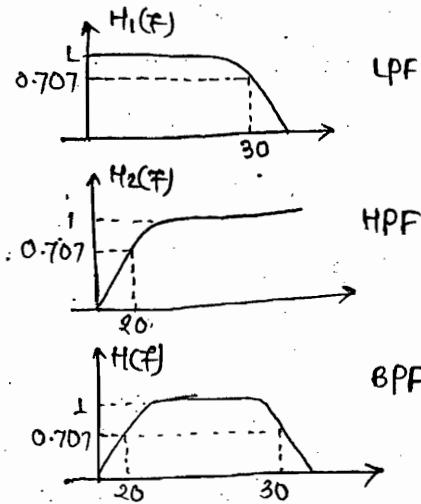


(10)
53

for cascaded filter;

$$H(s) = H_1(s) \cdot H_2(s)$$

$$H(f) = H_1(f) \cdot H_2(f)$$



$$BW = \Delta f = f_2 - f_1 = 10 \text{ Hz}$$

$$f_0 = \sqrt{f_1 \cdot f_2}$$

$$= \sqrt{20 \times 30} = 10\sqrt{6} \text{ Hz.}$$

$$Q_0 = \frac{f_0}{\Delta f} = \frac{10\sqrt{6}}{10}$$

$$Q_0 = \sqrt{6}$$

17

$$H(s) = H_1(s) \cdot H_2(s)$$

$$= \frac{1}{(1+SRC)^2}$$

$$= \frac{1/(RC)^2}{\underbrace{(s + \frac{1}{RC})^2}_{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

$\zeta = 1$; critically damped

.....Graph theory

* Used to solve any ele. N/W.

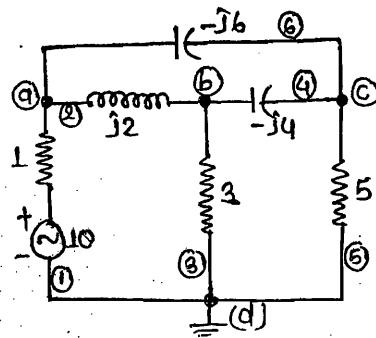
Example → Given: any ele. N/W

Find: (1) Graph of N/W

(2) Oriented graph

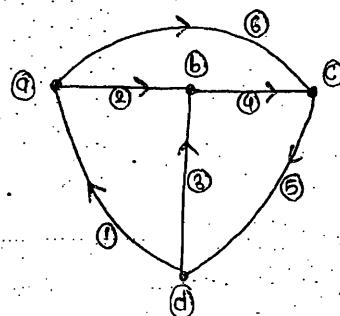
(3) Tree of graph

(4) Co-tree



Nodes(n) = 6

branches(b) = 6



Rank of graph [$r = (n-1)$] = 3

..... Oriented graph

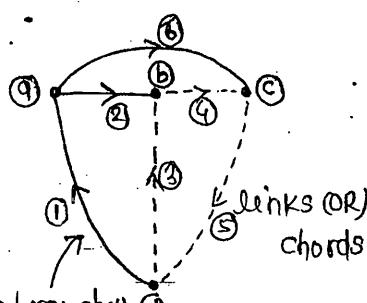
..... Fully connected Graph

(FC graph)

(all the nodes are directly connected by branch to each other)

no. of branches :- $nC_2 = \frac{n(n-1)}{2}$ [only applicable for FC graph]

Tree(1) →



Tree branches (OR)
Twigs.

Tree: [1, 2, 6] Tree branches
(OR)

Twigs

chords Co-tree [3, 4, 5] links (OR) chords

No. of tree branches = $n-1$

No. of links / chords = $b-(n-1)$

Total no. of trees = $n^{(n-2)}$ only for FC graph.

= $\det[A \cdot A^T]$ Applicable for both type (FC or NOT)
(General result)

A..... Reduced incidence matrix

* Branch Voltages in terms of branch currents :-

$$V_2 = j_2 \cdot i_2$$

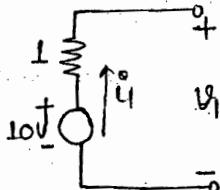
$$V_3 = 3 \cdot i_3$$

$$V_4 = -4j \cdot i_4$$

$$V_5 = 5 \cdot i_5$$

$$V_6 = -j_6 \cdot i_6$$

$$V_1 = 10 - i_1$$



$$V_2 - 10 + i_1 = 0$$

$$V_1 = 10 - i_1$$

ATE-05/12/14

The n/w topology is concern the manner in which various elements are connected b/n difference node irrespective of their values & types.

The tree of a graph →

i) The tree of a graph contains all tie nodes.

j) There is no closed path & hence any tree is ckt less.

l) The tree of a graph is not unique.

l) If the graph contains 'n' node its tree will contain $(n-1)$ branches.

The tree of a graph can be used to solve the given ele. n/w using :-

a) Tie-set matrix.

b) Cut-set matrix.

Any closed loop in a graph may not contain all the nodes of the graph.

In a fully connected graph each node is connected to every other node of the graph with a direct branch.

- * The degree of a node represents the no. of branches which are meeting at the specified node.
- * In a fully connected graph the degree of each node is same & represents the rank of the graph.
- * When the graph is not fully connected degree of each node is diff. but the highest degree of node represents the rank of graph.

* main points →

1) No. of branches in a FC Graph = $nC_2 = \frac{n(n-1)}{2}$

2) No. of trees of a graph = $n^{(n-2)}$ FC Graph

= $\det[AA^T]$ General Graph
(whether FC or not)

A Reduced incidence matrix

3) No. of links/chords = $b - (n-1)$

= no. of KVL eqns

= no. of fundamental loops [$l = b - (n-1)$]

= no. of fundamental tie sets

4) No. of tree branches/twigs = $n-1$

= no. of KCL eqns

= no. of node pair voltages

= Rank of graph

= Degree of each node in a FC Graph

= Max^m degree of node when graph is not FC

= No. of fundamental cut-sets

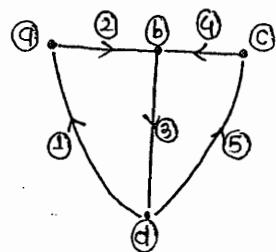
Example:- Given :-

Graph of a n/w

To find :-

(1) Incidence matrix

(2) Reduced incidence matrix [4]



* Not a FC Graph

$$\begin{cases} n=4 \\ b=5 \end{cases}$$

* Incidence matrix →

The incidence matrix translates all the geometrical feature of the n/w into an algebraic expn.

Every graph has an incidence matrix & vice-versa.

Each row of the matrix contains entries of +1, -1 (or) 0

→	= +1
←	= -1
•	= 0

Each column of the matrix contains only 1 entry of +1 & only 1 entry of -1 so that the algebraic sum of each column of matrix is zero.

The size of the matrix is given by [nxb]

$$n = \text{no. of nodes}$$

$$\Downarrow$$

$$n = \text{no. of rows}$$

$$b = \text{no. of branches}$$

$$\Downarrow$$

$$b = \text{no. of columns}$$

2 graphs having same incidence matrix are called ISOMORPHIC graphs.

The determinant of incidence matrix of any closed loop is always = 0.

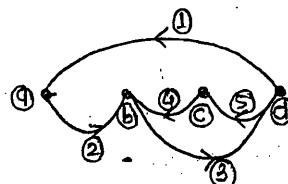
$$|\text{Incidence matrix}| = 0$$

* Incidence matrix →

nodes	Branches →				
①	②	③	④	⑤	
④	-1	+1	0	0	0
⑥	0	-1	+1	-1	0
⑦	0	0	0	+1	-1
⑧	+1	0	-1	0	+1

$[n \times b] = [4 \times 5]$

Obtaining Graph from matrix →



(using by column)

* Reduced incidence matrix → $[A]$

* Any one node is taken as a ref. node.

* Delete the row corresponding to the ref. node to obtain the reduced incidence matrix.

* The size of the matrix is $[(n-1) \times b]$.

⑥ → Ref. matrix

$$[A] = \begin{matrix} & \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} & \textcircled{5} \\ \textcircled{4} & -1 & +1 & 0 & 0 & 0 \\ \textcircled{6} & 0 & 0 & 0 & +1 & -1 \\ \textcircled{7} & +1 & 0 & -1 & 0 & +1 \end{matrix}$$

$[(n-1) \times b]$

$[3 \times 5]$

$[(n-1) \times b] = [3 \times 5]$

No. of trees = $\det[AAT]$

Example → Given:-

Tree of a graph

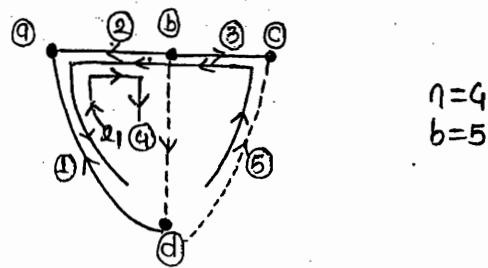
To find :-

(1) Fundamental tie set matrix

f - loop matrix

f - branch matrix $[B_f]$

(2) Cut-set matrix $[Q_f]$



* F-tie sets →

- (1) This represent optimum no. of tie sets with which the ckt. can be solved completely.
- (2) Each F-tie set represents a group of branches to make a closed loop
- (3) Each F-loop contains one & only one link or chord, remaining are the tree branches.
- (4) Dirn of each F-tie set is same as the dirn of the link or the chord.
- (5) No. of F-tie sets = no. of links/chords.

$$= b - (n - 1)$$

$$= 5 - (4 - 1) = 2$$
- (6) The fundamental tie-set matrix can be used to write the branch currents in terms of loop currents.

* F-tie sets matrix →

F-tie sets:-

$$l_1 := [\underline{1}, 2, \underline{4}]$$

→ link/chord

$$l_2 := [1, \underline{2}, 3, \underline{5}]$$

→ link/chord

F-tie set matrix →

$$[B_F] = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline l_1 & +1 & -1 & 0 & +1 & 0 \\ l_2 & -1 & +1 & -1 & 0 & +1 \\ \hline & B_t & & B_L = U & & \\ & (\text{tree}) & & (\text{link}) & & \end{array}$$

$$[B_F] = [B_t : U]$$

* Branch current →

$$\left. \begin{array}{l} i_1 = +I_1 - I_2 \\ i_2 = -I_1 + I_2 \\ i_3 = -I_2 \\ i_4 = +I_1 \\ i_5 = +I_2 \end{array} \right\} \text{(1)}$$

* F-cut set →

* The F-cut sets represent the optimum no. of cut sets with which we can solve the n/w completely.

* Each F-cut set represents a group of branches which must be cut so that the graph is divided into 2 parts.

* Each F-cut set contains one & only one tree branch remaining are the links (or) chords.

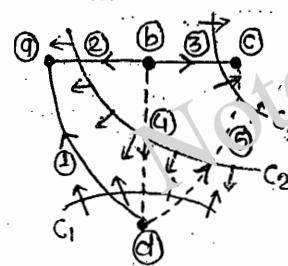
* The dirn of each F-cut set is same as the dirn of the tree branch.

* The no. of F-cut sets = no. of tree branches

$$= n-1$$

$$= 4-1 = 3$$

* The cut-set matrix can be used to represent the branch vol. in terms of nodal vol.



* F-cut set →

C₁: [3, 6] → tree branch.

C₂: [2, 4, 5] → tree branch.

C₃: [1, 4, 5]

↓ tree branch.

* F-cut set matrix →

$$[Q_F] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$Q_L = U$ Q_E

$$[Q_F] = [U \mid Q_E]$$

* Branch voltages →

$$\left. \begin{array}{l} v_1 = V_1 \\ v_2 = V_2 \\ v_3 = V_3 \\ v_4 = -v_1 + v_2 \\ v_5 = +v_1 - v_2 + v_3 \end{array} \right\} \text{II}$$

* The 3 set of eqn represent the equilibrium eqn since this eqn represents any given n/w completely & uniquely.

*
 $[B_T] = -[Q_T^T]$
 $[Q_T] = -[B_T^T]$

Hence T -tie set matrix corresponding to the tree branches is always equal to -ve of transpose of T -cut set matrix corresponding to the links or the chords & vice-versa.

This result is always valid irrespective of the type of tree selected for a particular graph of the corresponding ele. n/w.

While designing any ele. n/w, when the value of any element in any branch of n/w is valid again & again then the analysis of n/w becomes simplified using graph theory, since:-

1 Graph of n/w

1 Tree of graph.

T -tie set matrix

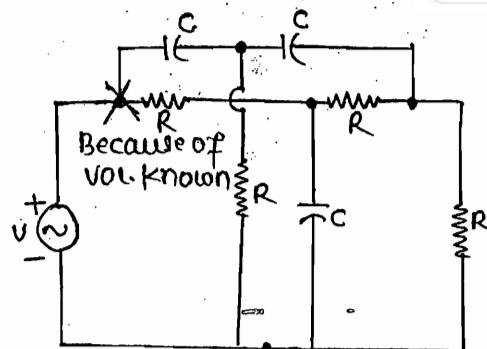
T -cut set matrix

It will remain same. Only one eqn is modified where the branch vol. al been represented in terms of branch current.

Under this condn the analysis of n/w using the KVL & KCL analysis becomes more complicated.

Even all the element value in any ele. n/w remain fixed then the analysis of n/w using KVL & KCL eqn becomes simplified.

Under this condn the method of Graph theory becomes more typical.

6
22

Twin-T n/w

 $n=4 \dots \text{independant nodes}$
 $b=7$

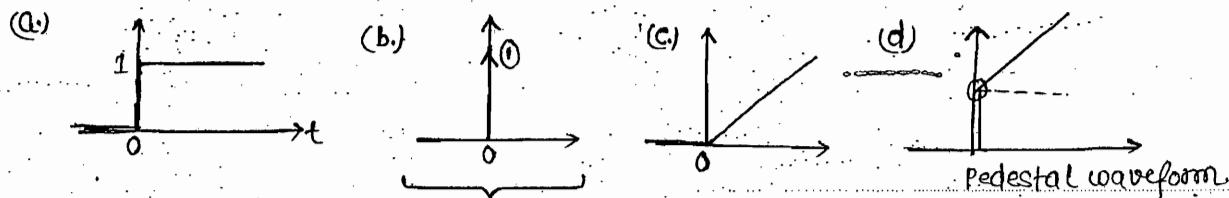
$\text{No. of KVL} = \text{No. of f-tie sets} = b - (n-1) = 7 - 3 = 4$

$\text{No. of KCL} = \text{No. of f-cut sets} = n-1$

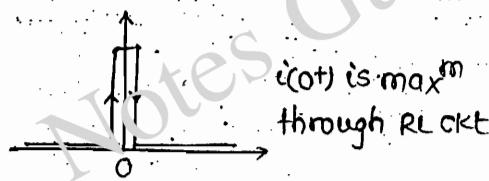
$= 4 - 1$
 $= 3$

So min^m no. of eqn will be ③

If we will make graph then 5th node will also included.

6
13

$i(0^+)$ is max^m for S(t)



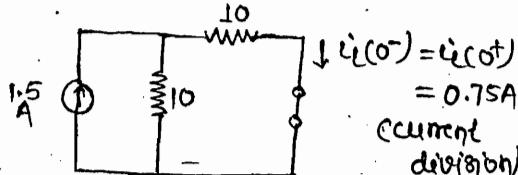
$V(t) = L(t) \cdot R + L \frac{di(t)}{dt}$

Because in impulse fⁿ value of deno. is 1.45
19

$\text{at } t=0^- \rightarrow$

S open

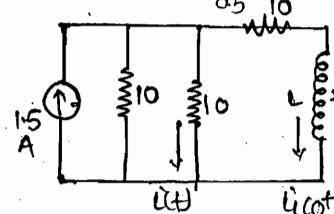
$L \Rightarrow \text{sc}$



$t > 0^+$

RL

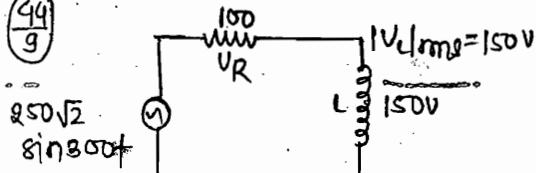
$i_L(t) = i_L(\infty) - [i_L(\infty) - i_L(0^+)] e^{-t/T}$



$T = \frac{L}{R_{\text{eq}}} \quad | 1.5A \Rightarrow 0C$
 $R_{\text{eq}} = (0.5 || 10) + 0.75 = 15$
 $T = \frac{15}{15} = 1 \text{ ms}$

$i_L(t) = V(t) - \frac{1}{10} [10 \cdot i_L(t) + L \frac{di_L(t)}{dt}]$

(44)
9



$$250\sqrt{2} \sin 30^\circ$$

$$U_i = \sqrt{V_R^2 + V_L^2}$$

$$250 \quad 150$$

$$V_R = 200V$$

$$V_R = 100\text{ rms}$$

$$200 = 100 \text{ rms}$$

$$I_{\text{rms}} = 2A$$

$$V_L = \omega L \cdot I_{\text{rms}}$$

$$L = \frac{V_L}{\omega I_{\text{rms}}}$$

$$= \frac{150}{900 \times 2}$$

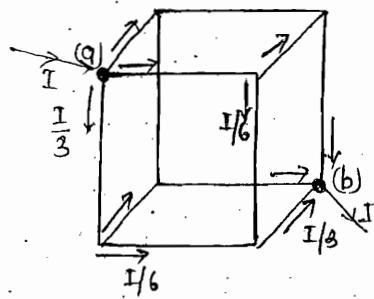
$$L = \frac{1}{4}H$$

(25)
6

$$Q = \int_0^t i(t) dt$$

$\underbrace{\hspace{2cm}}_{(q_{\text{req}})}$

(30)
7



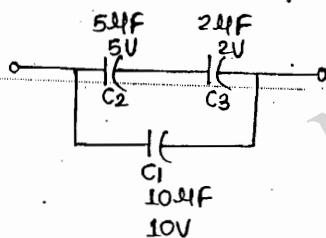
$$\text{Req.} = \frac{V_{ab}}{I}$$

(This is applicable only
for symmetrical γ/ω)

$$= \frac{1}{I} \left[\frac{I}{3} \times R + \frac{I}{6} R + \frac{I}{3} R \right]$$

$$= \frac{5}{6} \Omega$$

(36)
8



$$V_2 = \frac{Q_2}{C_2} = \frac{4}{5} = 0.8V$$

$$V_{qb} = 0.8 + 2 = 2.8V$$

$$V_3 = \frac{Q_3}{C_3} = \frac{4}{2} = 2V$$

$$Q_2 = C_2 V_2 = 25 \mu\text{C}$$

$$Q_3 = C_3 V_3 = 4 \mu\text{C}$$

(21)
15

$$V(t) = 10(t+0.01) e^{-100t}$$

$$V(s) = \frac{10}{(s+100)^2} = \frac{0.1}{(s+100)^2}$$

denom. $\Rightarrow (s+100)^2$

so $s = -100$

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n = 10,$$

$$Q = 1$$

$$Q_t = C_1 V_1 = 10 \times 2.8 = 28 \mu\text{C}$$

$$Q_t = 4 + 28 = 32 \mu\text{C}$$

(24)
6

$$\omega = 0, C \rightarrow \infty, i_2 = 0$$

(current starts from origin)

$$\omega = \infty, C \rightarrow 0; |i_2| = \frac{E_m}{R_2}$$

(Resistive)

$$|I| = \frac{V}{12\pi n}$$

$$V_R = |I|R = \frac{VR}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$



Network
Theorems

* Superposition theorems →

The response of any element in a linear bilateral RLC n/w containing more than 1 independent vol. (or) current source is the sum of responses produced by the sources each acting alone when:-

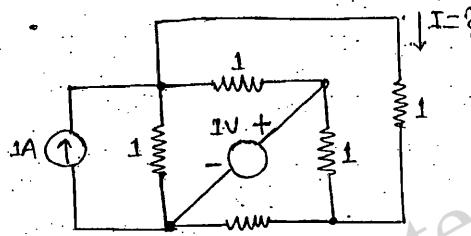
- (1) All other independent vol. sources are SC (or) replaced by their internal impedances.
- (2) All other independent current sources are OC (or) replaced by their internal impedances.
- (3) Dependent vol. (or) current sources remain as they are & therefore these sources are neither SC nor OC.

* The theorem is not applicable to the n/w containing:-

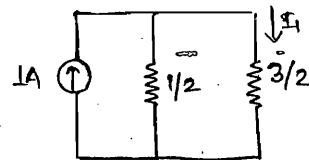
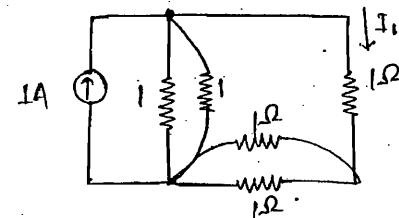
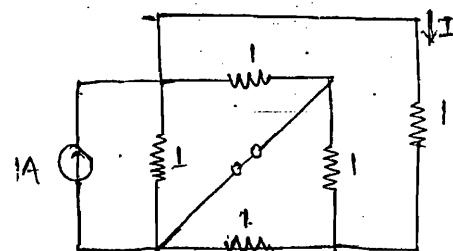
- (1) Non-linear elements
- (2) Unilateral elements (such as p-n junction diode)

DATE - 06/12/14

Que. →



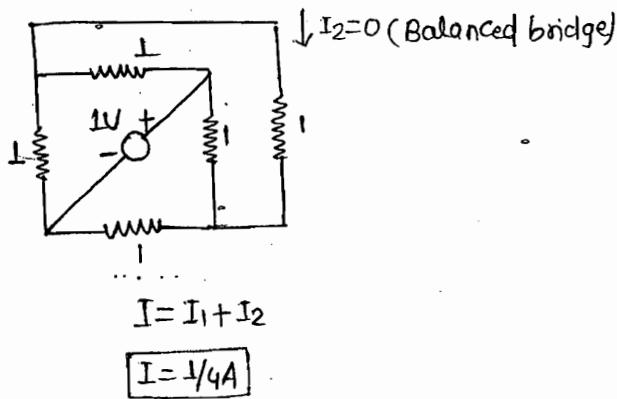
Sol. → I_1 (due to 1A source) -



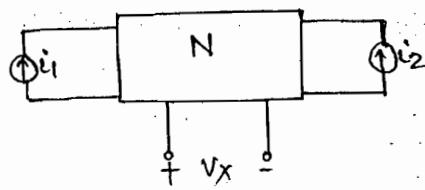
$$I_1 = 1 \times \frac{1/2}{1/2 + 3/2}$$

$$I_1 = \frac{1}{4} A$$

I_2 (due to $1V$) \rightarrow



Que:-



N :- Resistive & linear n/w
 If $i_1 = 8A$, $i_2 = 12A$; $v_x = 80V$
 $i_1 = -8A$, $i_2 = 4A$; $v_x = 0V$
 To find v_x if $i_1 = i_2 = 20A$

Sol'n \rightarrow

$$i_1 = 0, v_{x1} = i_2 R_1$$

$$i_2 = 0, v_{x2} = i_1 R_2$$

$$v_x = v_{x1} + v_{x2}$$

$$v_x = i_2 R_1 + i_1 R_2$$

$$80 = 8R_1 + 12R_2$$

$$0 = -8R_1 + 4R_2$$

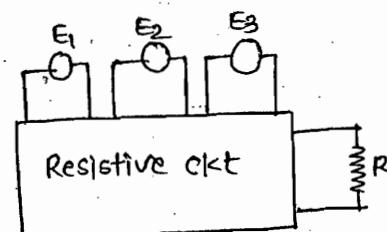
$$R_1 = 5/2, R_2 = 5$$

$$v_x = i_2 R_1 + i_1 R_2$$

$$= 20 \times \frac{5}{2} + 20 \times 5$$

$$v_x = 150V$$

Que:-



$$\begin{aligned} P_1 &= 18W \\ P_2 &= 60W \\ P_3 &= 98W \end{aligned}$$

$$\left. \begin{aligned} P_{\max} &\neq P_1 + P_2 + P_3 \\ P_{\min} &\neq P_3 - P_1 - P_2 \end{aligned} \right\} \text{Not correct.}$$

Solⁿ →

$$P = I^2 R = \frac{V^2}{R}$$

$$V = \sqrt{P} \sqrt{R}$$

$$V_{\max} = V_1 + V_2 + V_3$$

$$= (\sqrt{P_1} + \sqrt{P_2} + \sqrt{P_3}) \sqrt{R}$$

$$P_{\max} = \frac{V_{\max}^2}{R}$$

$$= \frac{(\sqrt{P_1} + \sqrt{P_2} + \sqrt{P_3})^2}{R} \times R$$

$$P_{\max} = (\sqrt{P_1} + \sqrt{P_2} + \sqrt{P_3})^2$$

$$V_{\min} = V_2 - V_1 - V_3$$

$$= (\sqrt{P_3} + \sqrt{P_2} - \sqrt{P_1}) \sqrt{R}$$

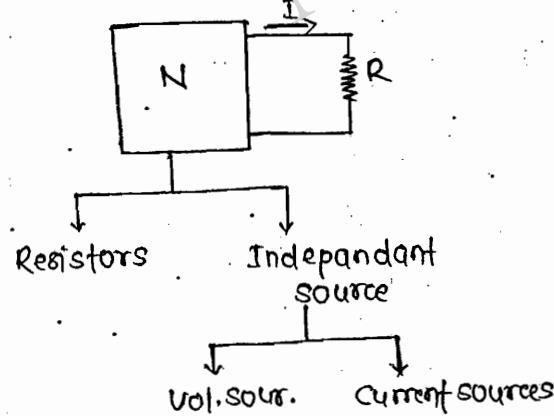
$$P_{\min} = \frac{V_{\min}^2}{R}$$

$$= \frac{(\sqrt{P_3} - \sqrt{P_2} - \sqrt{P_1})^2}{R} \times R$$

$$P_{\min} = (\sqrt{P_3} - \sqrt{P_2} - \sqrt{P_1})^2$$

$P_{\max} = 450W$
$P_{\min} = 50W$

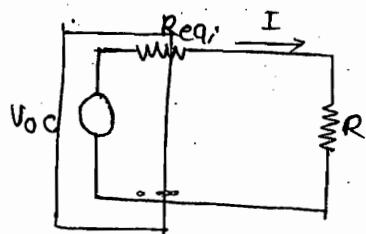
(22
27)



$$R=0, I=3A$$

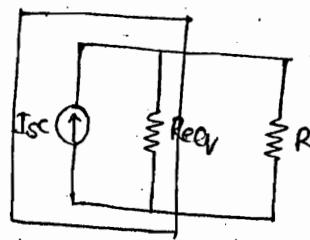
$$R=2, I=1.5A$$

$$I=? , R=?$$



Thevenin's
model

$$V_{oc} = I(Req + R)$$



Norton's eq:
model.

$$I = \left(\frac{Req}{Req + R} \right) Isc$$

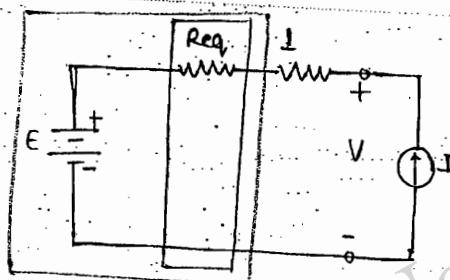
$$\left\{ \begin{array}{l} V_{oc} = 3(Req + 0) \\ V_{oc} = 1.5(Req + 2) \end{array} \right.$$

$$V_{oc} = 6, \quad Req = 2$$

$$G_{oc} = I(Req + R)$$

$$G = I(2 + 1)$$

$$I = 2A$$



Thevenin's model

$$(1) \quad E = E_1, \quad I = 0, \quad V = 5V$$

$$V = E_p = E_1 = 5V$$

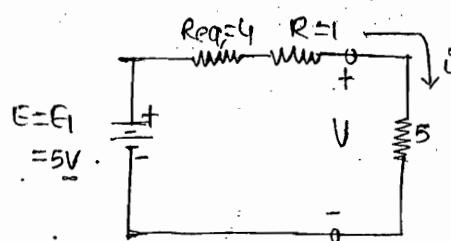
$$(2) \quad E = 0, \quad I = 1, \quad V = ?$$

$$V = I(1 + Req) + E$$

$$(3) \quad E = E_1, \quad I \Rightarrow \text{Replaced by } 5\Omega$$

$$V = ?$$

$$Req = 4 \Omega$$

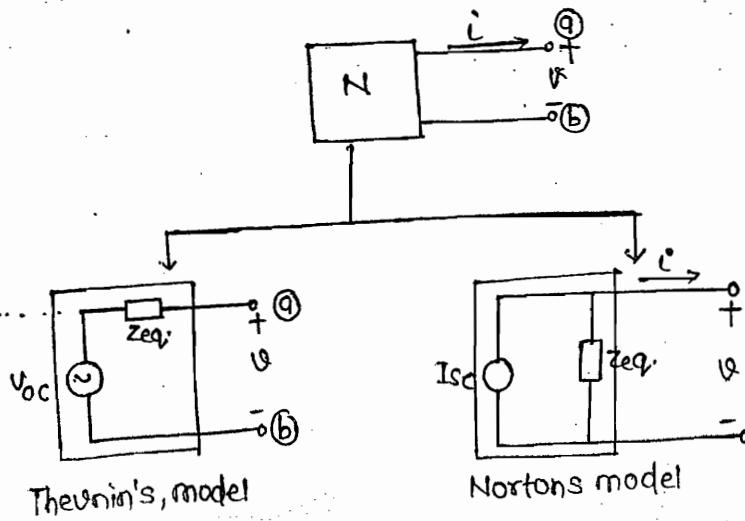


$$i = \frac{5}{4+1+5} = 1/2A$$

$$5i = V$$

$$V = 5/2A$$

* Thevenin's & Norton's theorem →



where: $V_{oc} = \text{OC vol. b/n 2-terminal (a & b)}$
when $i=0$

$I_{sc} = \text{SC current b/n 2-terminal (a, & b)}$
when $v=0$

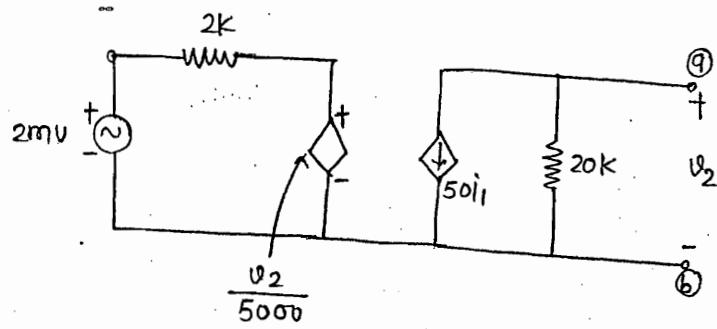
$$\frac{V_{oc}}{I_{sc}} = Z_{eq}$$

- * Thevenin's theorem → A linear active RLC n/w which contains one or more independant (or) dependant vol. or current sources can be replaced by a single vol. source V_{oc} & serie wth eq. impedance.
- * Z_{eq} represents the eq. impedance b/w the terminals a & b when:-
 (1) All independant vol. sources in sc (or) replaced by there internal impedances.
 (2) All independant current sources are oc (or) replaced by there internal impedances.
 (3) All dependant vol. & current sources remain as they are & therefore these sources are neither sc nor oc.
- * The theorem is always applicable irrespective of:-
 (1) The type of the vol. & current sources wether dc (or) ac sources.
 (2) Nature of the elements wether reactive (or) resistive.
- * The theorem is not applicable for the n/w containing:-
 (1) Non linear elements
 (2) Unilateral elements such as p-n junction diode.

* Norton's theorem →

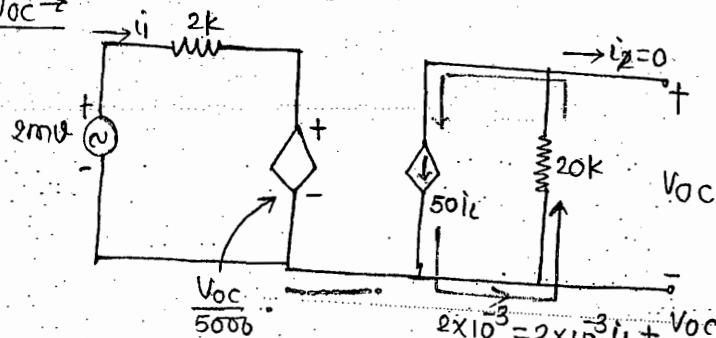
A linear, active RLC n/w which contains one or more independent (or) dependant vol. & current sources can be replaced by single current source I_{SC} in parallel with an eq. impedance Z_{eq} .

Ques. →



V_{oc} , Req. Thevenin's
 I_{sc} , Req. Norton's

Ques. → $V_{oc} \rightarrow$

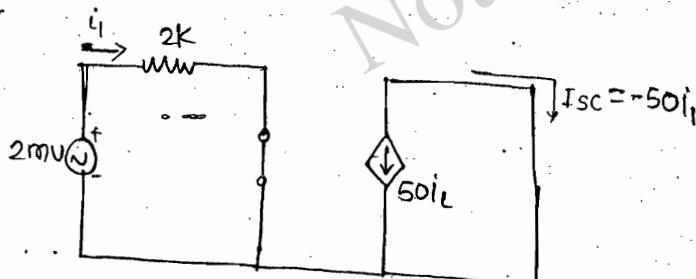


$$2 \times 10^3 = 2 \times 10^3 i_1 + \frac{V_{oc}}{5000} \quad \text{--- (i)}$$

$$0 = 20 \times 10^3 \times 50i_1 + V_{oc} \quad \text{--- (ii)}$$

$$V_{oc} = -\frac{10}{9} V$$

$I_{sc} \rightarrow$



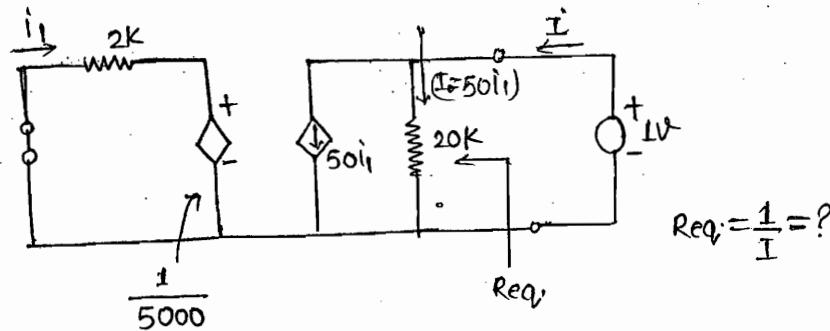
$$20 \times 10^3 = 20 \times 10^3 i_1$$

$$i_1 = 10^{-6}$$

$$\begin{aligned} I_{sc} &= -50i_1 \\ &= -50 \times 10^{-6} \end{aligned}$$

$$I_{sc} = -50 \mu A$$

* Req:-



$$\text{Req} = \frac{1}{I} = ?$$

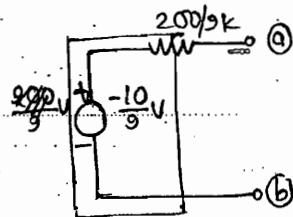
$$2 \times 10^3 i_1 + \frac{1}{5000} = 0 \quad \text{--- (i)}$$

$$i_1 = 20 \times 10^3 (I - 50i_1) \quad \text{--- (ii)}$$

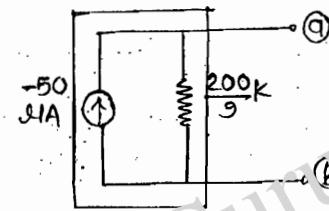
$$\frac{1}{I} = \text{Req} = \frac{200}{9} \text{ k}$$

Check →

$$\frac{V_{oc}}{I_{sc}} = \frac{-10/9}{-50 \times 10^{-6}} = \frac{200}{9} \text{ k} = \text{Req}$$

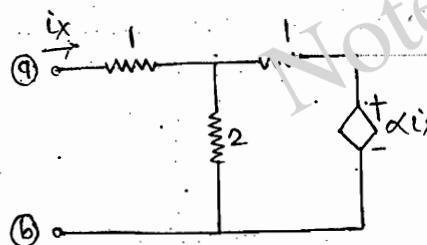


Thevenin's



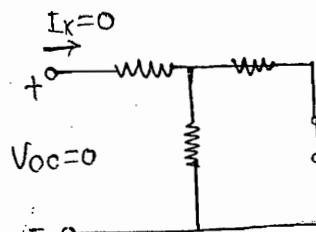
Norton's

(7/33)



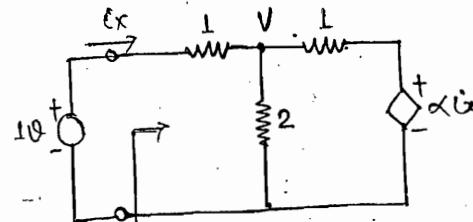
V_{oc} } Thevenin's
Req }

$V_{oc} \rightarrow$
 $(i_x=0); \quad i_x=0$
Voltage source \Rightarrow SC



$$V_{oc} = 0$$

Req →



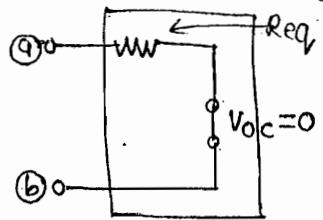
$$\text{Req} = \frac{1}{i_x} = ?$$

$$\frac{V-1}{1} + \frac{V-0}{2} + \frac{V-i_x}{1} = 0 \quad \text{--- (i)}$$

$$i_x = \frac{1-V}{1} \quad \text{--- (ii)}$$

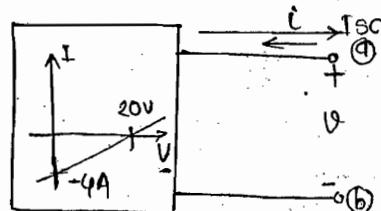
Eliminate v

Find $\frac{1}{i_x} = \text{Req.}$



Thevenin's
model

Ans. \rightarrow



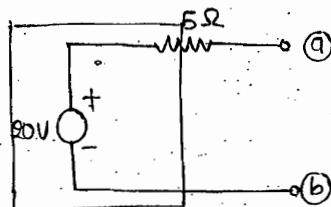
Draw Thevenin's & Norton's model

$$i=0; v=v_{oc}=20V$$

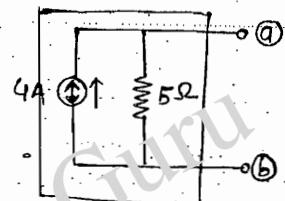
$$v=0; i=-4 \Rightarrow -I_{sc}$$

$$I_{sc}=4A$$

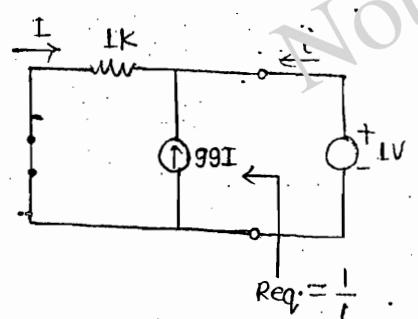
$$\frac{v_{oc}}{I_{sc}} = \text{Req.} = 5\Omega$$



Thevenin's



Norton's



$$I + 99I = -i$$

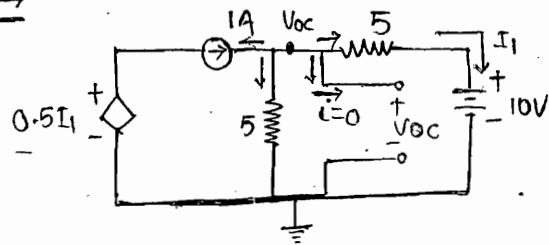
$$100I = -i \quad \text{---(i)}$$

$$I = -(1k)I \quad \text{---(ii)}$$

$$\text{Req.} = 0.01k = 10\Omega$$

(2)

$v_{oc} \rightarrow$

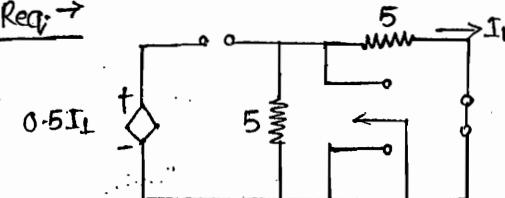


KCL \rightarrow

$$-1 + \frac{V_{OC}-0}{5} + 0 + \frac{V_{OC}-10}{5} = 0$$

$$V_{OC} = 7.5V$$

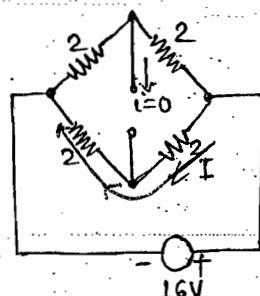
Req. \rightarrow



$$\text{Req.} = 5 || 5 \\ = 2.5\Omega$$

$$\text{Req.} = 2.5\Omega$$

(26)
29

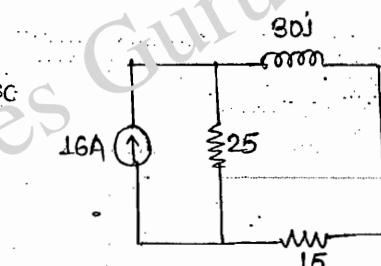
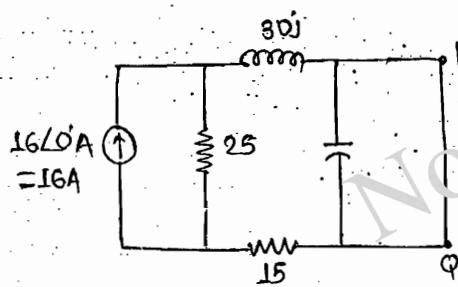


Balanced bridge

$$-16 + (2+2) I = 0$$

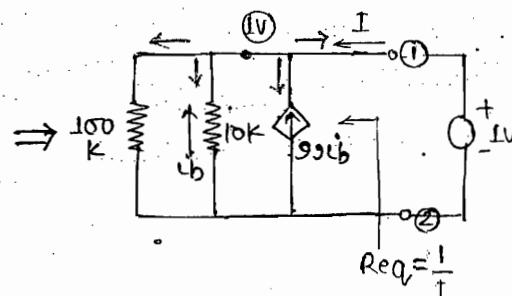
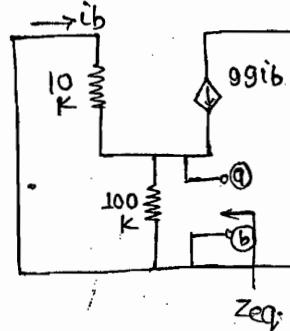
$$I = 4A$$

(45)
31



$$I_{SC} = \frac{25}{25 + 30j + 15} \times 16 \\ = 6.4 - 4.8j$$

(47)
31



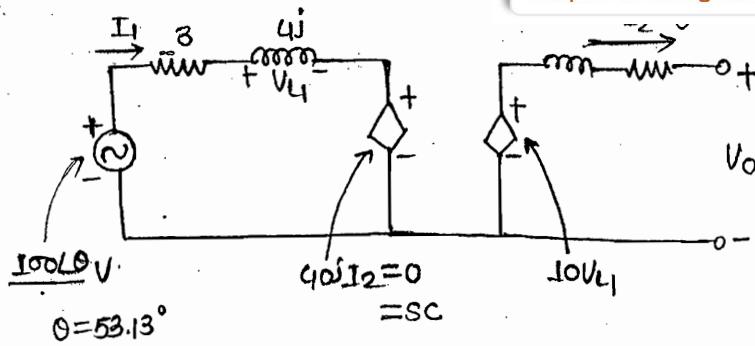
$$\text{Req.} = \frac{1}{I}$$

$$\frac{1-0}{100K} + \frac{1-0}{10K} + (-99ib) - I = 0 \quad \text{--- (i)}$$

$$-ib = \frac{1-0}{10K} \quad \text{--- (ii)}$$

$$\frac{1}{I} = \text{Req.} = 0.1K = 100\Omega$$

(49)
32



$$V_{OC} = 10V_{L1} = 10 \times 4jI_1$$

$$= \frac{40j \times 100\angle 90^\circ}{3+4j} = 540$$

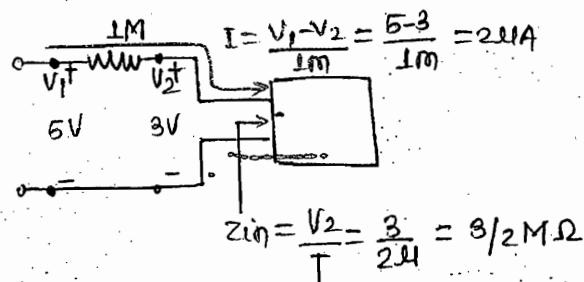
$$= 540$$

$$\left[\theta = \tan^{-1} \frac{4}{3} = 53.13^\circ \right]$$

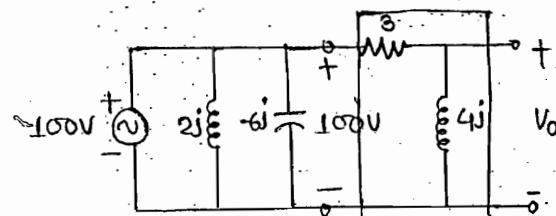
$$= 800\angle 90^\circ$$

$$= 800\angle 90^\circ \text{ V}$$

(26)
28



(7)
25

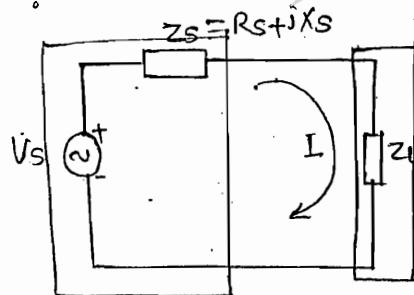


$$V_o = \frac{4j}{3+4j} \times 100$$

$$= \frac{j400}{3+4j} \times \frac{3-4j}{3-4j}$$

$$= \frac{j400(3-4j)}{9+16}$$

* max^m power transfer theorem →



★
 Z_L is variable
 $Z_L = R_L + jX_L$

Thevenin's model

To find → Condⁿ for max^m power Transfer from source to load.

$$I = \frac{Vs}{Zs + ZL} = \frac{Vs}{(Rs + jXs) + (Rl + jXl)} = \frac{Vs}{(Rs + Rl) + j(Xs + Xl)}$$

$$\begin{aligned} P_L &= I^2 Z_L \\ &= I^2 R_L \\ &= |I|^2 Z_L \\ &= |I|^2 R_L \end{aligned}$$

$$P_L = \frac{V_s^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2} \dots \dots \text{(i)}$$

for $P_L(\max)$

$$\frac{\partial}{\partial R_L} \left[R_L \right] = 0 ; \Rightarrow R_S = R_L$$

$$\frac{\partial}{\partial x_L} [P_L] = 0; \Rightarrow x_L = x_S$$

$$Z_L = R_L + jX_L$$

$$Z_L = R_S - jX_S \quad \leftarrow$$

$$\text{But } Z_s = R_s + jX_s;$$

$$Z_S^* = R_S - jX_S$$

$$Z_L = Z_S^*, \quad R_L = R_S, \quad X_L = -X_S$$

condⁿ for max^m power transfer

- * To maximise the power transferred from source to load the load impedance must be complex conjugate of the source impedance.
Therefore the source & the load must have equal resistive part.
 - * They must also have equal reactive parts but opposite in sign.
Therefore if the load is inductive the source must be capacitive & vice-versa.
 - * The maximum power transferred to the load is only 50% of the total power generated by the source.
 - * The results are valid only for variable load.
 - * To apply the maximum power transfer theorem any given ele. n/w has to be 1st transformed in the std. Thevenin's model.
 - * The theorem is always valid irrespective of:-
 - (i) Nature of the source & the load impedances
 - (ii) The nature of the vol. & the current sources.
 - * Hence the theorem is always valid for ac as well as dc ckt's.

DATE-07/12/19

$$P_L = \frac{V_S^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2} \quad \text{--- (i)}$$

Case(1) → General case

$$Z_S = R_S + jX_S$$

$$Z_L = R_L + jX_L$$

$$R_L = R_S$$

$$X_L = -X_S$$

$$Z_L = Z_S$$

* Case(2) →

$$Z_S = R_S + jX_S$$

$$Z_L = R_L$$

$$\frac{\partial}{\partial R_L} [P_L] = 0$$

$$R_L = |Z_S|$$

$$R_L = \sqrt{R_S^2 + X_S^2}$$

Case(3) →

$$Z_L = R_L$$

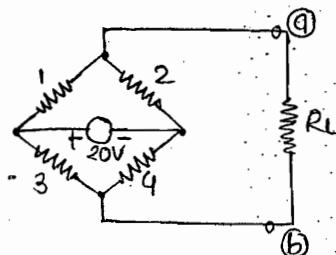
$$Z_S = R_S$$

dc ckt only

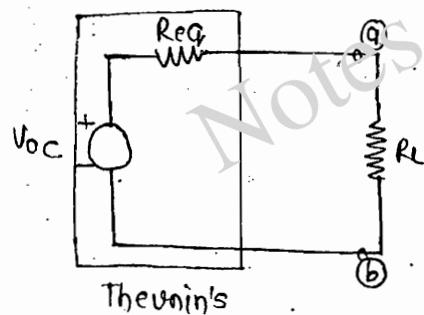
purely resistive

$$R_L = R_S$$

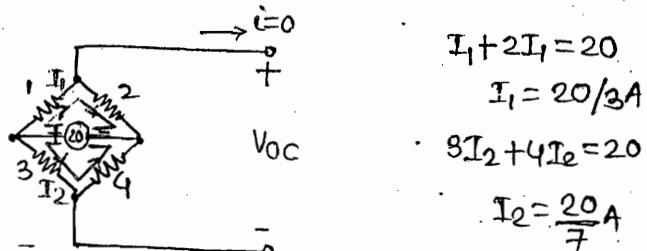
ue → In the ckt shown find the value of load resistance R_L so that max^m power is transferred to it. Hence cal. the max^m power transferred to the load?



SOLⁿ → for max^m Xfer convert the ckt in Thevinin's eq.



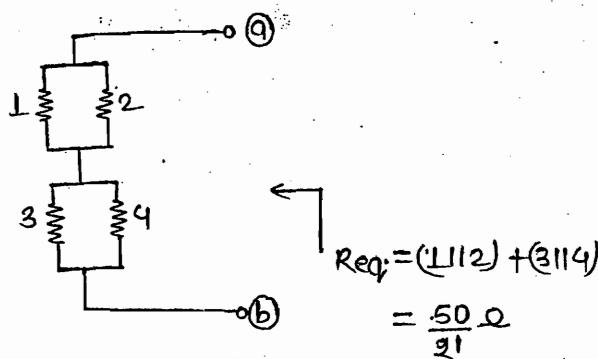
V_{oc} →



$$V_{oc} = 2I_1 - 4I_2$$

$$= 2 \times \frac{20}{3} - 4 \times \frac{20}{7} = \frac{40}{3} - \frac{80}{7} = \frac{280 - 240}{21} = \frac{40}{21} \text{ V}$$

Req →



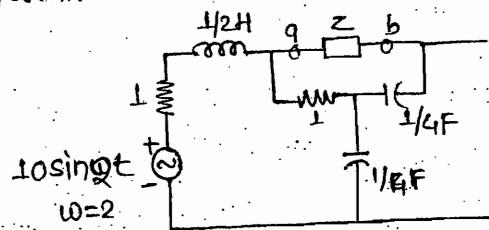
For P_{\max} →

$$R_L = \text{Req} = \frac{50}{21} \Omega$$

$$P_{\max} = I^2 R_L = \left(\frac{V_{OC}}{R_L + \text{Req}} \right)^2 R_L$$

$$= \frac{V_{OC}^2}{4R_L} = \frac{8}{21} W$$

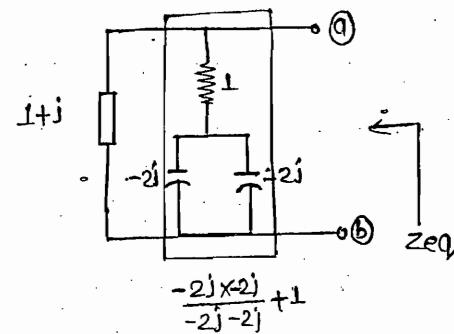
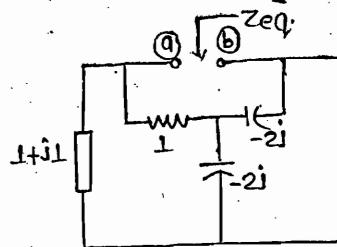
Que: In the circuit shown find the value of impe. z which must be connected b/n terminals a & b so that max power is transferred to it.



Soln →

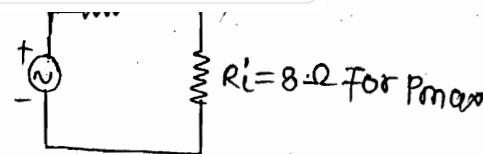
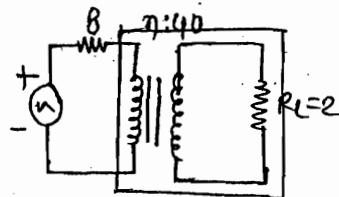
$$\frac{1}{2}H \Rightarrow j\omega L = j \times 2 \times \frac{1}{2} = j2$$

$$\frac{1}{4}F \Rightarrow \frac{1}{j\omega C} = \frac{1}{j \times 2 \times \frac{1}{4}} = -2j$$



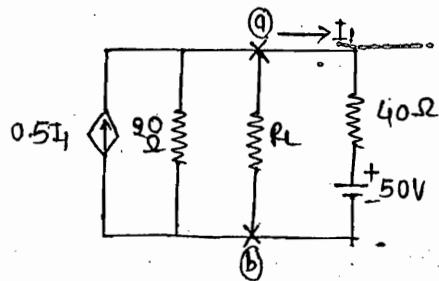
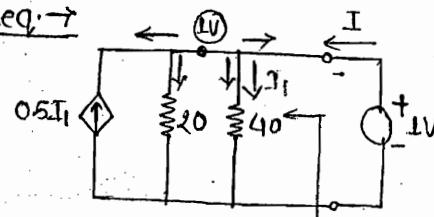
$$zeq = \frac{(1+j)(1-j)}{(1+j)(1-j)} = 1 \pm j0$$

$$\text{For } P_{\max}: z = zeq^* = (1+j0)^* = 1 \Omega$$

1
24

$$2 \left(\frac{\eta}{40} \right)^2 \times 8 = 8$$

$$\eta = 80$$

20
27.Req. \rightarrow 

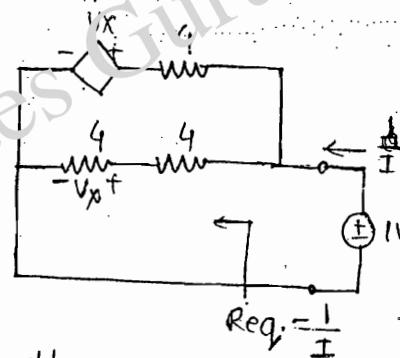
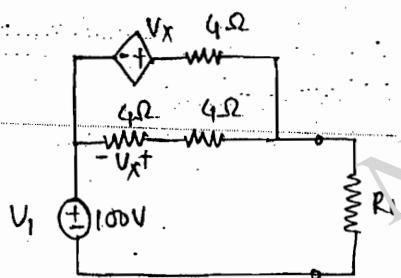
$$Req = \frac{1}{\frac{1}{I}} = ?$$

$$-0.5I_1 + \frac{1-0}{20} + \frac{1-0}{40} - I = 0 \quad \text{--- (i)}$$

$$\therefore I_1 = \frac{1}{40} \quad \text{--- (ii)}$$

$$\frac{1}{I} = Req = 16$$

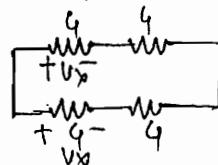
for P_{max} : $R_L = Req = 16 \Omega$

10
30.

$$Req = \frac{1}{I}$$

shortcut \rightarrow Because of same voltage across the

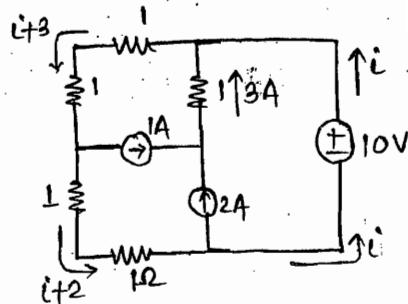
given two parallel branches (only applicable for same polarity vol. source)



$$(4+4) || (4+4) = 4 \Omega$$

for P_{max} : $R_L = Req = 4 \Omega$

(44)
31



$$\text{Power supplied} = 10 \times i$$

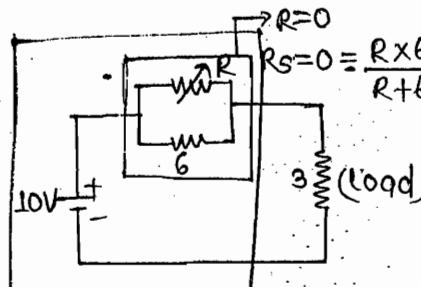
By KVL eqn:-

$$-10 + (1+1)(i+3) + (i+2)(1+1) = 0$$

$$i = 0$$

$$\text{Power supplied} = 0W$$

(46)
31



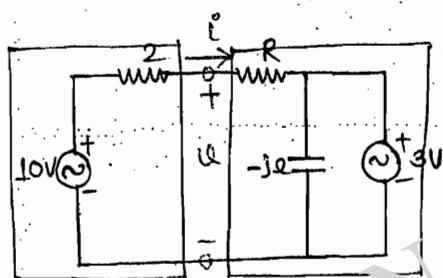
$$R = 0$$

$$R_S = 0 = \frac{R \times 6}{R+6}$$

$R_L \neq R_S$ for P_{max}

(Because of variable load max power will be delivered)

(48)
31



$$P = VI$$

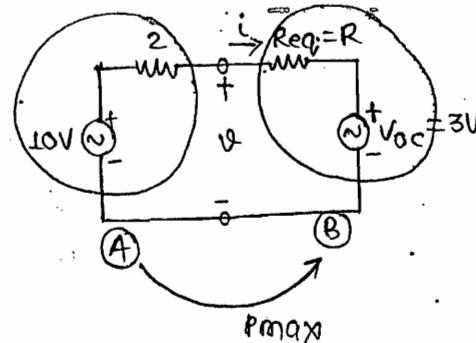
$$\frac{\partial P}{\partial R} = 0$$

Find R

Here the load is
block B (passive
elements included)

Thevenin's:

$$V_{OC} = 3V, R_{eq} = R$$



$$i = \frac{10-3}{2+R} = \frac{7}{2+R}$$

$$V = iR + 3 = \frac{7R}{2+R} + 3 = \frac{10R+6}{2+R}$$

$$P = VI$$

$$= \frac{7(10R+6)}{(2+R)^2}$$

$$\frac{\partial P}{\partial R} = 0$$

$$R = 0.8\Omega$$

* Reciprocity Theorem → In a linear, bilateral single source n/w the ratio of the excitation to response is constant when the positions of excitation & response are interchanged.

The basic of theorem is the symmetry of impedance & admittance matrices.

Impedance matrix :-

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

$$\begin{aligned} Z_{21} &= Z_{12} \\ Z_{31} &= Z_{13} \\ Z_{32} &= Z_{23} \end{aligned}$$

$$[V] = [z][I]$$

..... KVL

..... For reciprocal n/w

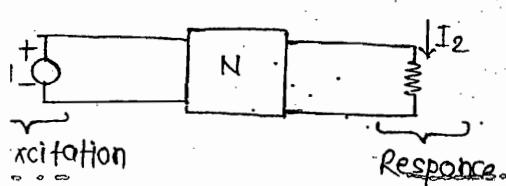
..... Reciprocity theorem is valid.

To verify the reciprocity theorem the basic configuration of the n/w remains same, only the external cond'n such as excitation & response are interchanged.

The theorem is not applicable for the n/w containing :-

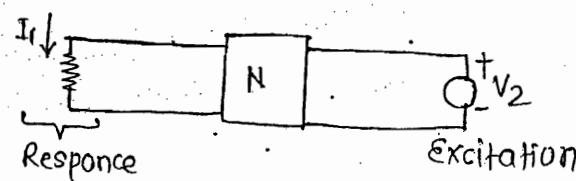
- 1) Non-linear elements.
- 2) Unilateral elements such as p-n junction diode.
- 3) Multiple independant (or) dependent vol. (or) current sources.

Case(1) →



$$\frac{\text{Excitation}}{\text{Response}} = \frac{V_1}{I_2} = Z_{12}$$

Case(2) →



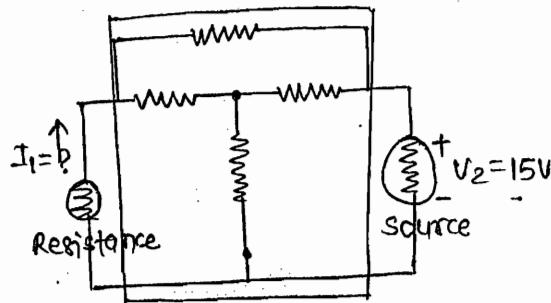
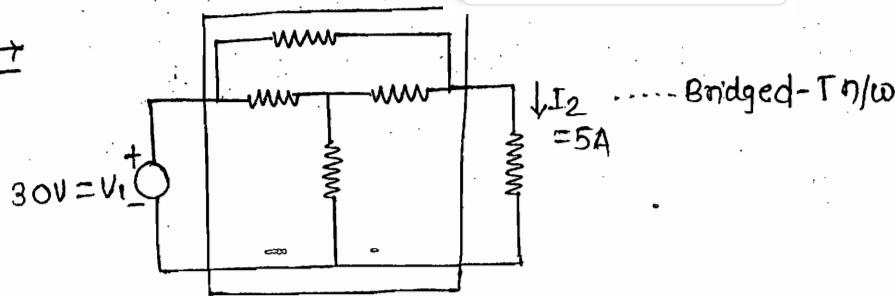
$$\frac{\text{Excitation}}{\text{Response}} = \frac{V_2}{I_1} = Z_{21}$$

$$\begin{aligned} \frac{V_1}{I_2} &= \frac{V_2}{I_1} \\ Z_{12} &= Z_{21} \end{aligned}$$

..... For reciprocal n/w

..... Reciprocity theorem is applicable

Que. →



Sol. →

$$\frac{V_1}{I_2} = \frac{V_2}{I_1}$$

$$\frac{30}{5} = \frac{15}{I_1} \quad ; \quad I_1 = -2.5A$$

Because of opposite current

Polarity of vol. source be same

* Tellegen's theorem → * In any n/w total instantaneous power consumed by various elements in different branches of n/w

is always = 0.

* In any n/w total instantaneous power delivered by various active element must always be equal to the total instantaneous power consumed by various passive elements & different branches of n/w.

$$\sum_{k=1}^b v_k i_k = 0$$

v_k branch voltages

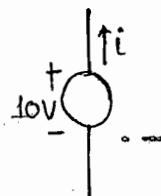
i_k Branch currents

b No. of branch.

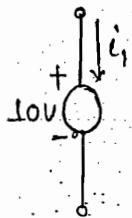
* The theorem is always valid irrespective of :-

- (1) The shape of n/w.
- (2) The type of elements contain in the n/w.
- (3) Value of each element in the n/w.

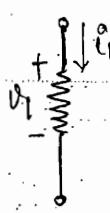
* The theorem is applicable so long as the KVL & KCL eqn are applicable to the given n/w.



$$\text{Power delivered} = 10xi$$

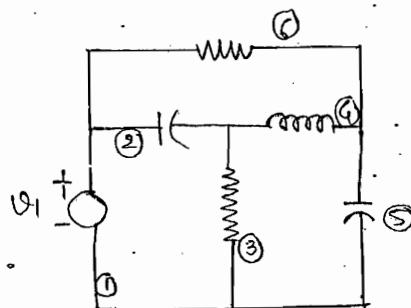


$$\star\star \quad \text{Power absorb/consumed} = 10xi$$



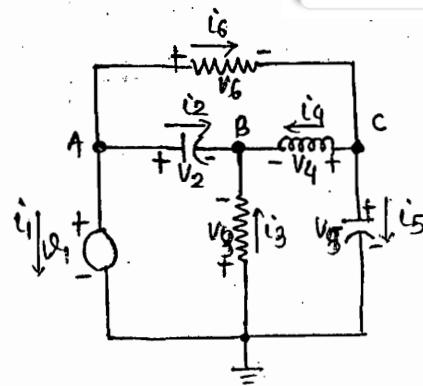
$$\star\star \quad \text{Power absorb/consumed} = v_1i$$

Ques:- In the ckt shown find the missing branch vol. & branch current. Find the power absorb by each element in various branches on n/w. Hence calc. the total power absorb by all the elements in diff branches of n/w.



$$\begin{array}{l|l} v_1 = 4V & i = 2A \\ v_2 = 2V & i_3 = 4A \\ v_4 = 3V & i_6 = 4A \end{array}$$

$\text{SOI}^n \rightarrow$



KEL

$$-v_1 + v_2 - v_3 = 0$$

$$-4 + 2 - v_3 = 0$$

$$V_3 = -2V$$

$$+v_3 - v_4 + v_5 = 0$$

$$-2 - 3 + \sqrt{5} = 0$$

$$U_5 = 5V$$

$$+v_6+v_4-v_2=0$$

$$v_6 + 3 - 2 = 0$$

$$V_6 = -1V$$

KCL-T

$$\text{at node } i_1 + i_2 + i_6 = 0$$

4

$$2 + i_2 + 4 = 0$$

$$l_2 = -6A$$

at node

$$\textcircled{B} \quad i_1 + i_3 + i_4 = 0$$

$$-6 + 4 + 14 = 0$$

$$l_4 = 2A$$

qtnode

⑥ $i_4 + i_5 = i_6$

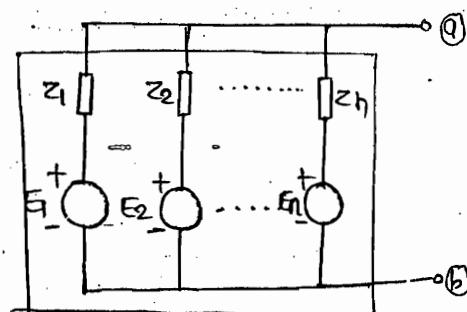
$$2+4=4$$

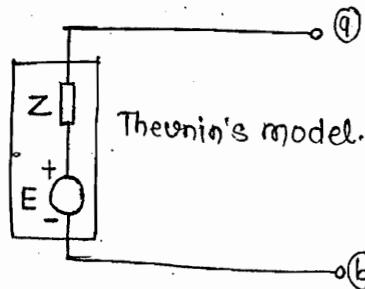
$$IS = 2A$$

	1	2	3	4	5	6
U_k	4	2	-2	3	5	-1
i_k	2	-6	4	2	2	4
$U_k i_k$	8	-12	-8	6	10	-4

$$\sum_{k=1}^6 v_k \cdot i_k = 0$$

* Millman's theorem →





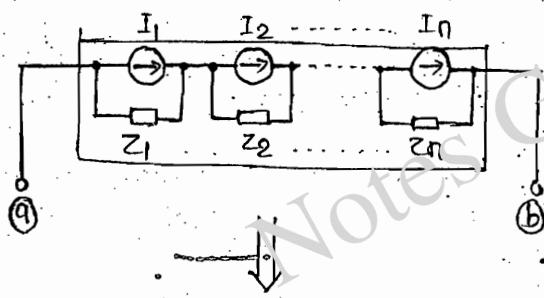
Thevenin's model.

$$\hat{E} = \frac{\sum E_i y_i}{\sum y_i}$$

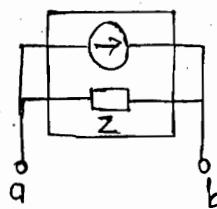
$$Z = \frac{1}{\sum y_i}$$

- * This theorem is an extension of the Thevenin's theorem & is useful whenever no. of vol. sources are large in the given n/w.
- * The theorem is applicable only when any given n/w can be rearranged in the std. format.

* Dual Millman's theorem →



Norton's model

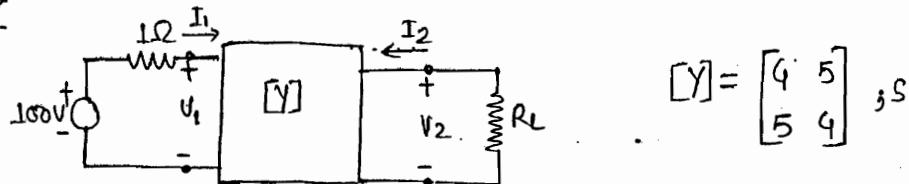


$$I = \frac{\sum I_i z_i}{\sum z_i}$$

$$Z = \sum z_i$$

- * The theorem is an extension of the Norton's theorem & is useful whenever there are large no. of current sources in the n/w
- * The theorem is applicable only when any given n/w can be rearranged in the std. format.

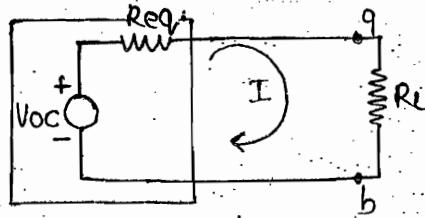
Ques. →



$$[Y] = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}; s$$

for the ckt shown cal. the value of R_L for max^m power transferred. Hence cal. the max^m power transferred to the load.

Soln. →



Thevenin's model

For P_{max} ; $R_L = \text{Req.}$

$$P_{max} = \frac{V_{oc}^2}{4R_L}$$

$$\left. \begin{array}{l} I_1 = 4U_1 + 5U_2 \dots \dots \dots (1) \\ I_2 = 5U_1 + 4U_2 \dots \dots \dots (2) \\ 100 = I_1 + U_1 \dots \dots \dots (3) \end{array} \right\} \text{From fig. 4 i.e. fig.}$$

$$100 = I_1 + U_1$$

V_{oc}

V_{oc} →

$$I_2 = 0$$

$$V_2 = V_{oc}$$

$$\left\{ \begin{array}{l} I_1 = 4U_1 + 5V_{oc} \\ 0 = 5U_1 + 4V_{oc} \\ 100 = I_1 + U_1 \\ \text{find } V_{oc} \end{array} \right.$$

I_{sc} →

$$V_2 = 0$$

$$I_{sc} = -I_2$$

$$\left\{ \begin{array}{l} I_1 = 4U_1 + 5 \times 0 \\ -I_{sc} = 5U_1 + 4V_2 \times 0 \\ 100 = I_1 + U_1 \\ \text{find } I_{sc} \end{array} \right.$$

Req. →

$$V_2 = 1V$$

$$\text{Req.} = \frac{1}{I_2}$$

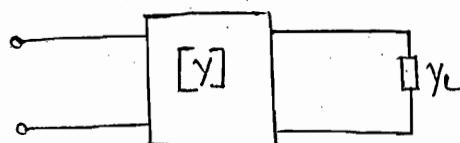
$$100V \Rightarrow SC$$

$$\left\{ \begin{array}{l} I_1 = 4U_1 + 5 \\ I_2 = 5U_1 + 4 \\ I_1 + U_1 = 0 \end{array} \right.$$

$$\text{find } \frac{1}{I_2} = \text{Req.}$$

$$\text{Req.} = \frac{V_{oc}}{I_{sc}}$$

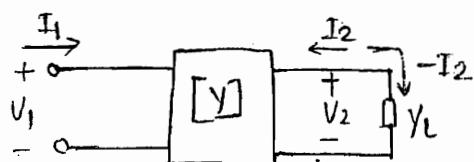
Que. 7



To find:-

$$G_{21} = \frac{V_2}{V_1}$$

Soln



$$I_1 = y_{11}V_1 + y_{12}V_2 \dots \text{(i)}$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \dots \text{(ii)}$$

$$-I_2 = V_2 y_L \dots \text{(iii)}$$

$$y_{21}V_1 + y_{22}V_2 = -V_2 y_L$$

$$V_2(y_{22} + y_L) = -y_{21}V_1$$

$$\frac{V_2}{V_1} = G_{21} = \frac{-y_{21}}{y_{22} + y_L}$$

$$\frac{V_2}{V_1} = \frac{-y_{21}}{y_{22} + y_L}$$

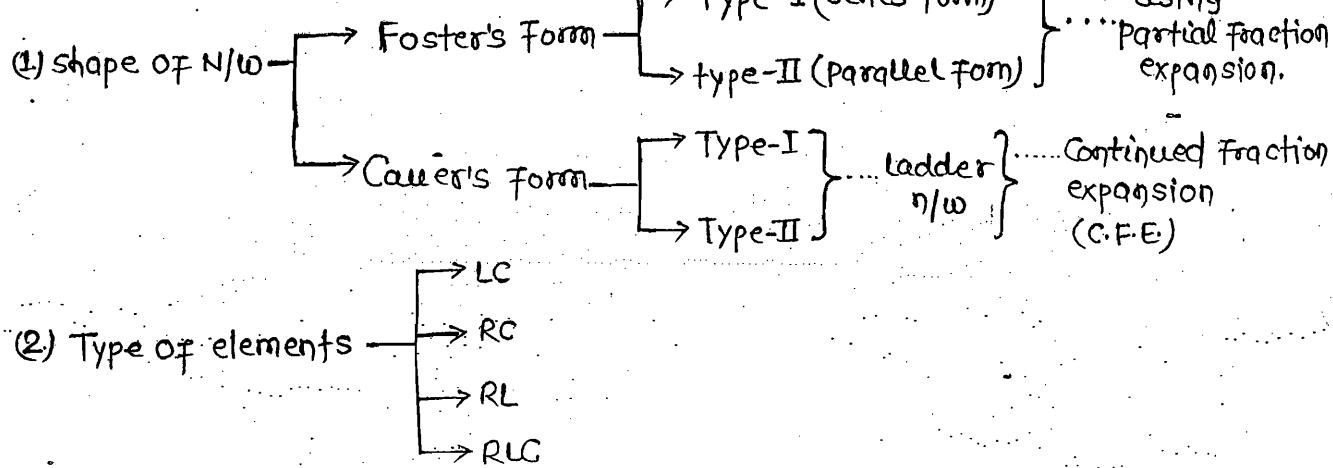
Network
Synthesis

Given:-Any N/w Fn in mathematical form } $F(s)$ → $Z_{11}, Z_{22}; Y_{11}, Y_{22}$ → $Z_{12}, Z_{21}; Y_{12}, Y_{21}; G_{12}, G_{21};$
 α_{12}, α_{21} To Find:-

(1) Shape of N/W.

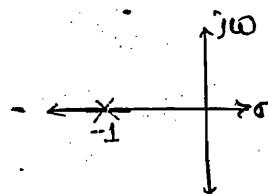
(2) Type of elements.

(3) Value of each element.

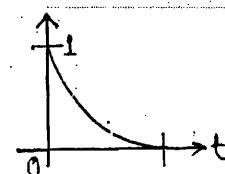
*Condition of any n/w to be:-

(1) Physically Realizable (causal).

(2) Stable.



$$\frac{1}{(s+1)} \Rightarrow e^{-t} u(t)$$

* For any ele. n/w to be stable n/w following cond'n must be satisfied:-(1) Network Fn $F(s)$ can't have pole in RHP.(2) N/w Fn $F(s)$ can't have multiple pole along the $j\omega$ axis including the origin.(3) The degree of numerator of n/w Fn $F(s)$ can't exceed the degree of denominator by more than unity.

For any stable n/w the response must be finite for finite excitation.

- * For a physically realisable n/w (or) for a causal n/w the o/p can't exist before the i/p is applied.
- * Any n/w. f_n(s) with poles in LHP has inverse laplace Xform which is 0 for t<0.
Therefore the stability implies causality.
- * Therefore any stable n/w is always a physically realisable n/w but the reverse may not be true. Hence any physically realisable n/w may (or) may not be stable n/w.

* Hurwitz polynomial $\rightarrow P(s)$

$$F(s) = \frac{\text{Num. poly}}{\text{denom. poly}} P(s)$$

- Controls poles of n/w
- must lie in LHP
- n/w must be stable
- P(s) must be Hurwitz
- The n/w must also be physically realisable.

* Features of P(s) \rightarrow

$$P(s) = 2s^6 + 2s^5 + 2s^4 + 6s^3 + 7s^2 + 9s + 5$$

Even parts	Odd parts
$P_e(s) = 2s^6 + 2s^4 + 7s^2 + 5$	$P_o(s) = 2s^5 + 6s^3 + 9s$

CFE of $\frac{P_e(s)}{P_o(s)}$

divisor) dividend (Quotient \rightarrow must be +ve

Reminder

If $P(s) = s^6 + 2s^4 + 6s^2 + 9$ is given
then; $p'(s) = 6s^5 + 8s^3 + 12s$

CFE of $\frac{P(s)}{P'(s)}$

If $P(s) = P_1(s) \cdot P_2(s)$

Even: $s^2 + 1$, $s^3 + 4s$ (odd)
 $s = \pm j$ $s = 0, \pm 2j$ (Both lying in $j\omega$)

* The Hurwitz poly. $P(s)$ represent the deno. poly. of a physically realisable & stable n/w $F(s)$.

* This poly. insures:-

(1) No pole lies in RHP.

(2) No multiple poles lie along the $j\omega$ axis including the origin.

* The poly. $P(s)$ is real for real value of s .

* All the coefficient of poly. must be real & +ve.

* No term must be missing in the Hurwitz poly. unless all the even term (or) all the odd terms are missing.

* The continued fraction expansion of the ratio of even to odd parts (or) odd to even parts must have all +ve coefficient term.

* The quotient term represent the numerical values of RLC compo. of N/W.

* If any poly. is either even (or) odd then it is Hurwitz if the CFE of $\frac{P(s)}{P'(s)}$ has all +ve quotient terms.

* If $P(s) = P_1(s) \cdot P_2(s)$ then the poly. $P(s)$ is Hurwitz if the 2 poly. $P_1(s)$ & $P_2(s)$ are separately Hurwitz.

* If any poly. is either even (or) odd then all its roots lie along the $j\omega$ axis only.

* If the deno. poly. of any n/w f_n is Hurwitz then all its roots lie in LHP, the n/w will be stable & therefore physically realisable.

Ques. → Find whether or not following poly. is Hurwitz (OR).

Find whether or not following poly. represents the deno. poly. of stable & physically realizable n/w.

$$P(s) = s^4 + s^3 + 3s^2 + 3s + 4$$

Soln →

$$P_e(s) = s^4 + 2s^2 + 4 \quad ; \quad P_o(s) = s^3 + 3s$$

$$\text{CFE of } \frac{P_e(s)}{P_o(s)}$$

check:- (i) +ve coefficients

(ii) Imag. coefficients (or) not

(iii) missing term.

$$\text{CFE} \rightarrow s^3 + 3s \overline{) s^4 + 3s^2 + 4 (s \leftarrow}$$

$$\underline{s^4 + 3s^2}$$

$$\underline{+ 2s^2 + 4) s^3 + 3s (2s \frac{2}{2} \leftarrow}$$

$$\underline{3s + 2s}$$

$$\underline{s) 2s^2 + 4 (2s \leftarrow}$$

$$\underline{2s^2 + }$$

$$\underline{4) s / (s \frac{4}{4} \leftarrow)}$$

$$\underline{\underline{0}}$$

+ve
quotient
term
↓
 $P(s)$ is
Hurwitz

$$\text{CFE} \rightarrow s + \frac{1}{\frac{s}{2} + \frac{1}{2s + \frac{s}{4}}}$$

Ques. → $P(s) = s^4 + 4$

Soln → Here s^2 is missing

$P(s)$ is not Hurwitz

Roots $\rightarrow s^4 + 4 = 0$

$$s^4 = -4$$

$s^2 = \pm i2$ Because of multiple pole along $j\omega$ axis.

Que. $\rightarrow P(s) = s^3 + 2s^2 + 3s + 6$

Soln $\rightarrow P_0(s) = s^3 + 3s \quad ; \quad P_e(s) = 2s^2 + 6$

CFE $\rightarrow \frac{P_0(s)}{P_e(s)}$

$$\begin{array}{r} 2s^2 + 6 \\ \hline s^3 + 3s \\ \hline 0 \end{array}$$

..... CFE has been terminated abruptly

let $s^3 + 3s$ is one factor of $P(s)$

$P_1(s)$

To find :- 2nd Factor $P_2(s)$

$$P(s) = \frac{P_1(s) \cdot P_2(s)}{s^3 + 3s}$$

$$P_2(s) = \frac{P(s)}{P_1(s)}$$

$$\begin{array}{r} s^3 + 9s \\ \hline s^3 + 2s^2 + 3s + 6 \\ \hline -s^3 - 3s \\ \hline 2s^2 + 6 \\ \hline -2s^2 - 6 \\ \hline 0 \end{array} \quad \left. \begin{array}{l} 1 + \frac{2}{s} \Rightarrow 1 + \frac{1}{s/2} \\ \} P_2(s) \end{array} \right.$$

$$P(s) = P_1(s) \cdot P_2(s)$$

$$= \underbrace{(s^3 + 3s)}_{s(s^2 + 3)} \underbrace{\left(1 + \frac{1}{s/2}\right)}_{\text{Hurwitz}}$$

$$\text{roots: } \underbrace{s=0}_{\text{Hurwitz}}, \pm j\sqrt{3}$$

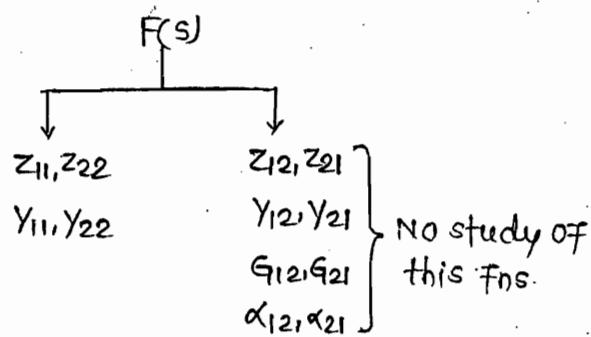
$P_1(s)$ is Hurwitz

$$P(s) = \underbrace{P_1(s)}_{\text{Hurwitz}} \cdot \underbrace{P_2(s)}_{\text{Hurwitz}}$$

$$\underbrace{\text{Hurwitz}}_{\text{Hurwitz}}$$

$\therefore P(s)$ is Hurwitz

* PRF (true Real Fn) \rightarrow



• The true real fn represent physically realizable & stable passive driving point impedance fun.

• Any Fn $F(s)$ is said to be PRF if the following cond'n is satisfied :-

$$\operatorname{Re}[F(s)] \Big|_{s=j\omega} \geq 0$$

..... for all ω

* features of PRF $\rightarrow F(s)$

1) If $F(s)$ is true real real fn then $1/F(s)$ is also PRF.

2) The sum of 2 true real fn is also a PRF

$$F(s) = F_1(s) + F_2(s)$$

3) All the zeros & poles lie in LHP.

4) All the poles & zeros lie along the -ve real axis (or) along $j\omega$ axis (or) represent complex conjugate in nature.

5) All the poles along $j\omega$ axis including the origin are simple in nature.

6) The degree of numerator poly. can't exceed the degree of deno. by more than unity.

7) The lowest power of s of numerator poly. can't exceed the lowest power of s of deno. poly. by more than unity.

* The necessary & sufficient cond'n for any Fn $F(s)$ to be a PRF are :-

(1) The poles & zeros must lie in LHP.

Therefore the numerator & deno. poly. must be Hurwitz.

(2) The residues of poles along the $j\omega$ axis must be real & +ve.

This is verified by making partial fraction expansion & checking the residues of only those pole which lie along $j\omega$ axis.

(3) $\operatorname{Re}[F(s)] \Big|_{s=j\omega} \geq 0$

..... for all ω .

* * * * * ③ Synthesis of ele. n/w containing specified element →

Case (i) → Driving point LC imittance fns.

* features →

- (1) This imittance fn is a ratio of even to odd (or) odd to even poly.
- (2) The poles & zeros are simple; lie along the jw axis & they interlace (alternate)
- (3) There must be either a pole (or) zero at the origin (or) at ∞ .
- (4) Highest power of s of numerator & deno. poly. must differ at the most by unity.
- (5) The lowest power of s of numerator & deno. poly. must also differ at most by unity.

*

Que → Find whether (or) not following fn represent physically realisable & stable driving point LC imittance fn.

$$(1) F_1(s) = \frac{s(s^2+4)}{(s^2+1)(s^2+3)}$$

Zeros:- $s=0,$

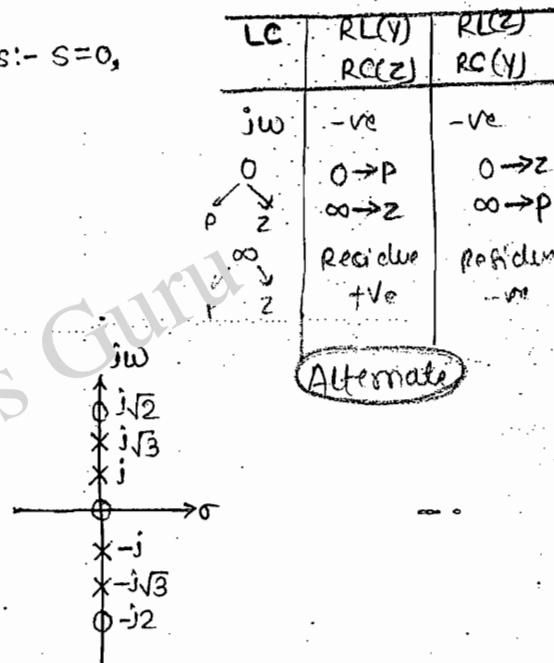
$$(2) F_2(s) = \frac{s^5 + 4s^3 + 5s}{3s^4 + 6s^2}$$

$$(3) F_3(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

Soln → * (1) Zeros:- $s=0, \pm j\sqrt{2}$

Poles:- $s=\pm j, \pm j\sqrt{3}$

Because of poles & zeros are not interlace/alternate; so this is not physically realisable.



* (2) zeros: In the deno. s^0 is missing in $F_2(s)$

$$F_2(s) = \frac{s(s^4 + 4s^2 + 5)}{s^2(3s^2 + 6)}$$

(i) Double pole exist

so it is also not realisable.

* (ii) Zeros: - $\pm j, \pm 3j$ poles: - $0, \pm j2$

By observation they are alternate.

All the condⁿ are satisfied so it can be physically realisable.

Cqse(2) → Driving point

RL admittance funⁿ (Y_{RL})

RC impedance f_n (Z_{RC})

* features:-

- (1) All the poles & zeros lie along the -ve real axis & they alternate.
- (2) The residues of poles must be real & +ve.
- (3) The singularity nearest to (or) at the origin must be a pole whereas the singularity nearest to (or) at ∞ must be a zero.

Ques → find whether following f_n represents physically realisable & stable

RC impedance f_n (or) RL admittance f_n

$$(1) F_1(s) = \frac{(s+1)(s+4)(s+8)}{s(s+2)(s+6)}$$

$$(2) F_2(s) = \frac{(s+1)(s+8)}{(s+2)(s+4)}$$

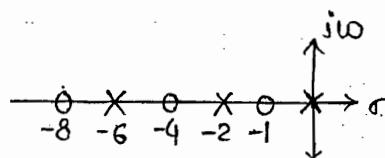
$$(3) F_3(s) = \frac{(s+1)(s+2)}{s(s+3)}$$

Soln → (1) Zeros: - $s = -1, -4, -8$

Poles: - $s = 0, -2, -6$

..... Yes (physically

Realisable)



for 3rd feature: - $F_1(s) \neq \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+6)}$ (Because of equal power in Num. & deno. it can't have Partial f_n)

(2) Zeros: - $s = -1, -8$ * Poles & zeros not alternate.

..... Poles: - $s = -2, -4$ * Nearest to origin is not pole.
..... No.

(3.) No, *they are not alternate (poles & zeros)

*Nearest to ∞ is not a zero.

* Case(3) \rightarrow Driving point

* RL impedance F^n (Z_{RL})

* RC admittance F^n (Y_{RC})

* Features:-

(1) The poles & zeros lie along the -ve real axis & they alternate.

(2) The singularity nearest to (OR) at origin must be a zero whereas the singularity nearest to (OR) at ∞ must be a pole.

(3) The residues of poles must be real & -ve.

Que. \rightarrow Find whether following f_n represent physically realisable & stable driving point RL imp. F^n (OR) RC admittance F^n :-

$$(1) F_1(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)} \quad \text{scale factor.}$$

$$(2) F_2(s) = \frac{4(s+1)(s+2)}{(s+3)(s+6)}$$

Sol n \rightarrow (1) Yes.

(2) No, Because of not alternating (poles & zeros)

* Case(4) \rightarrow Driving point immitance F^n

Z_{RLC}, Y_{RC}

* Features \rightarrow For RLC immitance F^n no set rules are followed for loc n of the poles & zeros & therefore the poles & zeros are present any where in LHP.

Ques. → Following Z_h represents the driving point impedance Z_h for physically realisable & stable impedance Z_h for RLC N/W. Realize it in the Cauer's type-I form

$$Z_{RLC} = \frac{s^2 + 2(s+2)}{s^2 + s + 1}$$

SOLN → Zeros $\therefore s = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j1$

Poles $\rightarrow s = \frac{-1 \pm \sqrt{1-4}}{2} = -0.5 \pm j0.866$

CFE of $\frac{s^2 + 2s + 2}{s^2 + s + 1}$

$$\frac{s^2 + s + 1}{s^2 + 2s + 2} (1 \longrightarrow z_1 \text{ (series)})$$

$$\frac{s^2 + s + 1}{s^2 + s} (s \longrightarrow y_2 \text{ (parallel)})$$

$$\frac{1}{s+1} (s \longrightarrow z_3 \text{ (series)})$$

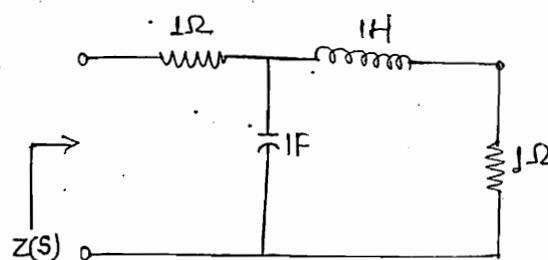
$$\frac{1}{s+1} (1 \longrightarrow y_4 \text{ (parallel)})$$

$z_1 = 1 = R_1$, $R_1 = 1\Omega$ (series)

$y_2 = s = SC_2$; $C_2 = 1F$ (parallel)

$z_3 = s = SL_3$; $L_3 = 1H$ (series)

$y_4 = 1 = \frac{1}{R_4}$; $R_4 = 1\Omega$ (parallel)



Cauer's type-I → ladder N/W

- * If in option 4 ckt elements given then $\frac{s^3}{s^2} = s \rightarrow s_4, z_1=4$ (series) $R_1=1\Omega$series (Because of 2 given)
- * If que. is in the form of $\frac{s^3}{s^2} = s \rightarrow s_4, z_1=4$ (series)
- * If $\frac{s^4}{s^2} \rightarrow s^2$ [No form for compare with s^2 , so not realizable]
- * Cauer's type-I, L-series, C-shunt & for Cauer's-II vice-versa.

Cauer's (I)	Cauer's (II)
CFE OF	CFE OF
$\frac{s^2+2s+2}{s^2+s+1}$	$\frac{2+2s+s^2}{1+s+s^2}$

* In Cauer's n/w & for RLC n/w total no. of elements in the ckt is equal to the algebraic sum of the no. of finite zeros & poles.

* If the impedance f_n is given then in the Cauer's form the 1st element represent an impedance element & represent a series element.

* In the Cauer's type-I n/w the inductor is always a series element & capacitor is always a shunt elements.

* In Cauer's type-II n/w C is always a series element & an inductor is always a shunt element.

* In Cauer's-I n/w CFE is found by arranging the num. & the deno. poly. in descending power of s.

* In Cauer's-II form the CFE is found by rearranging the num. & deno. poly. in ascending power of s.

* The Cauer's-I & II n/w are always independant of each other.

DATE - 09/12/14

Que. → The F^h shows physically realisable & stable driving point LC immitance
 F^h realize it in :-

(1) Foster's series form (2) Foster's parallel form.

$$F(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

Soln →

$$F_{LC}(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

$$F(s) = \frac{2s^4 + 20s^2 + 18}{s^3 + 4s} \quad s^3 + 4s \quad 2s^4 + 20s^2 + 18 \quad (2s)$$

$$= 2s + \underbrace{\frac{12s^2 + 18}{s(s^2 + 4)}}_{12s^2 + 18}$$

2B1

$$\frac{A}{s} + \frac{Bs+C}{s^2+4} ; \quad A = 9/2 \\ B = 15/2 \\ C = 0$$

$$F(s) = 2s + \frac{9/2}{s} + \frac{(15/2)s}{s^2+4}$$

QSE(1) → Foster's Series Form (type-I)

$$F(s) = Z(s)$$

$$Z(s) = 2s + \frac{(9/2)}{s} + \frac{(15/2)s}{s^2+4}$$

$$= Z_1 + Z_2 + Z_3$$

$$Z_1 = 2s = sL_1 ; \quad L_1 = 2H$$

$$Z_2 = \frac{9/2}{s} = \frac{1}{(2/9)s} = \frac{1}{sC_2} ; \quad C_2 = \frac{2}{9}F$$

$$Z_3 = \frac{(15/2)s}{(s^2+4)} = \frac{1}{Y_3} ; \quad Y_3 = \frac{s^2+4}{(15/2)s} = \frac{2}{15}s + \frac{8}{15s} = Y'_3 + Y''_3$$

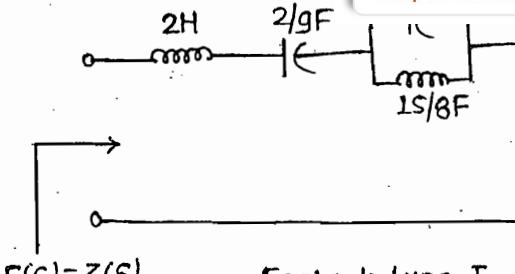
$$Y'_3 = \frac{2}{15}s = sC_3 ; \quad C_3 = \frac{2}{15}F$$

$$Y''_3 = \frac{8}{15s} = \frac{1}{(15/8)s} = \frac{1}{L_3s} ; \quad L_3 = \frac{15}{8}H$$

$$Z_1 = L_1 = 2H$$

$$Z_2 = C_2 = \frac{2}{9}F$$

$$Y'_3 = C_3 = \frac{2}{15}F ; \quad Y''_3 = \frac{15}{8}H = L_3$$



$F(s) = Z(s)$ Foster's type-I
 Series Form

Case(2) Foster's type-II

..... Parallel Form

$$F(s) = Y(s)$$

$$Y(s) = 2s + \frac{9/2}{s} + \frac{(15/2)s}{s^2+4}$$

$$= Y_1 + Y_2 + Y_3$$

$$Y_1 = 2s = sC_1 ; C_1 = 2F$$

$$Y_2 = \frac{1}{(\frac{9}{2})s} = \frac{1}{sL_2} ; L_2 = \frac{2}{9}H$$

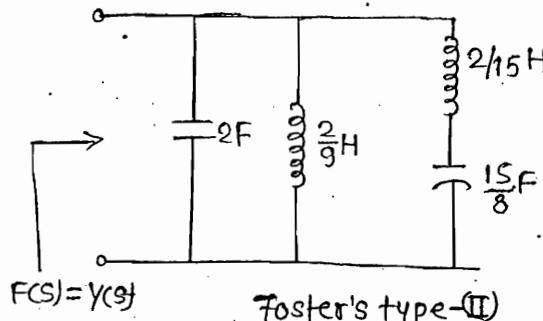
$$Y_3 = \frac{(15/2)s}{s^2+4} = \frac{1}{Z_3} ; Z_3 = \frac{s^2+4}{(15/2)s} = \frac{(2/15)s}{s} + \frac{1}{(15/8)s} = Z'_3 + Z''_3$$

$$Z'_3 = \frac{2}{15}s = L_3s \therefore L_3 = \frac{2}{15}H$$

$$Z''_3 = \frac{1}{(15/8)s} = \frac{1}{C_3s} ; C_3 = \frac{15}{8}F$$

$$Y_1 = C_1 = 2F, \quad Y_2 = L_2 = \frac{2}{9}H$$

$$Z'_3 = L_3 = \frac{2}{15}H, \quad Z''_3 = C_3 = \frac{15}{8}F$$



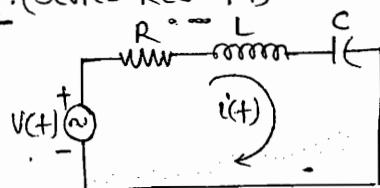
$F(s) = Y(s)$ Foster's type-II
 Parallel Form

Foster's type-I $\xleftrightarrow{\text{Dual Networks}}$ Foster's type-II

The Foster's type-I & II N/W are dual of each other. This 2 N/W are not the equivalent N/W & therefore type-I N/W can't be replaced by type-II N/W.

Duality \rightarrow

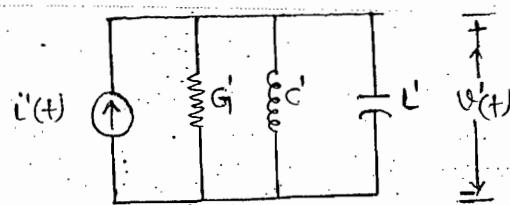
* N₁: (series RLC ckt)



KVL :-

$$R \cdot i(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int_0^t i(t) \cdot dt = v(t) \quad \text{--- (i)}$$

* N₂:- (parallel RLC ckt)
dual N/W



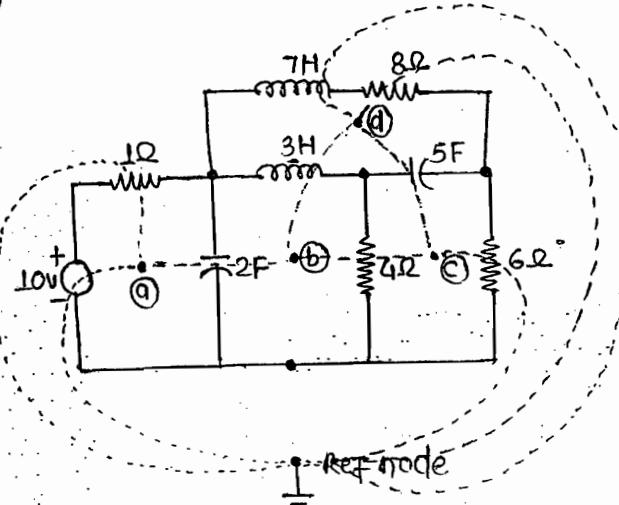
KCL \rightarrow

$$G'v'(t) + C' \frac{dv'(t)}{dt} + \frac{1}{L'} \int_0^t v'(t) dt = i'(t) \quad \text{--- (ii)}$$

N ₁	dual N/W N ₂
Series ckt	parallel ckt
KVL	KCL
V(t)	i(t)
i(t)	v(t)
Vol. source	Current source
Loop	Node
R	G
L	C
C	L
closing switch	Opening switch

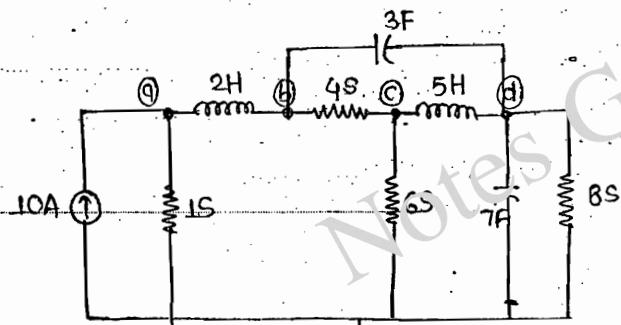
- * The original N/W & its dual N/W are not the eq. N/W & therefore the original N/W can't be replaced by its dual N/W.
 - * If we know the response of any series N/W in terms of current variable for specified excitation to the N/W then we can find the response of its dual N/W in terms of the vol. variable for similar current excitation.
 - This response can be written directly by inspection w/o actually solving it by using the transformation shown in table.

Que.→



calculate the duality
For given N/w

SOLⁿ →



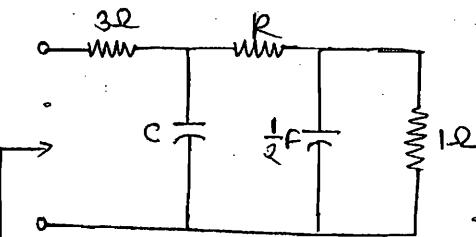
The diagram illustrates a mechanical system consisting of two vertical bars connected by a horizontal spring. The left bar has a velocity vector v_1 pointing upwards. The right bar has a velocity vector v_2 pointing downwards. A force vector F acts downwards on the left bar, and an equal and opposite force vector $-F$ acts upwards on the right bar. The spring connects the two bars at their midpoints. The center of mass of the system moves with velocity v_{cm} downwards. The total momentum of the system is zero. The equation $\Sigma p_i = \Sigma p_f$ is shown at the bottom.

Cauer's - (II) N/W

$$Z_{in} = 1 + \frac{sX(s+\frac{1}{s})}{s+(s+\frac{1}{s})} = \frac{s^3+2s^2+s+1}{2s^2+1}$$

$$\frac{s^3}{2s^2} = \frac{1}{2}s \Rightarrow sL = \frac{1+s+2s^2+s^3}{1+2s^2}$$

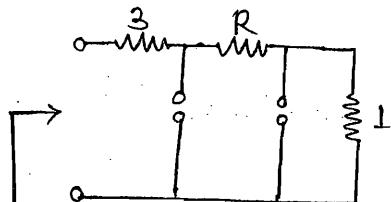
4
42



$$Z(s) = \frac{3(s^2 + 6s + 8)}{s^2 + 4s + 3}$$

1st method → By solving series parallel & calc. $Z(s)$ & then compare.

2nd method → $s=0$; $\text{OC} \dots \text{OC}$



$$Z(0) = 3 + R + L = \frac{3(0+0+8)}{0+0+3}$$

$$R = 4\Omega$$

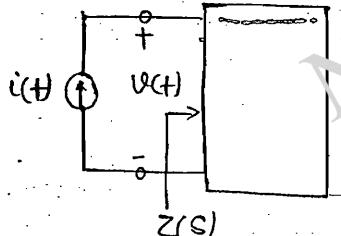
$$s=\infty, C \dots SC$$

C can't be found out
because of sc of C.

3rd method → Because of C as shunt element it is in causality (I).

CFE of $\frac{3s^2 + 18s + 24}{s^2 + 4s + 3}$ then find value of C

5
43



$$i(t) = u(t)$$

$$v(t) = 1 + e^{-t/C}$$

$$I(s) = 1/s$$

$$V(s) = \frac{1}{s} + \frac{1}{s+C}$$

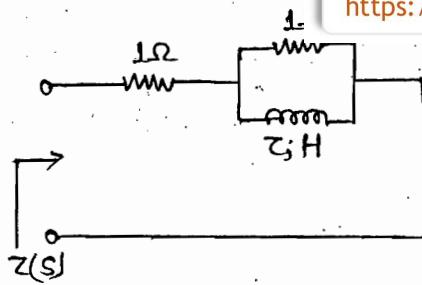
$$Z(s) = \frac{V(s)}{I(s)} = 1 + \frac{s}{s+C} = z_1 + z_2$$

$$z_1 = 1 \Rightarrow R_1 = 1\Omega$$

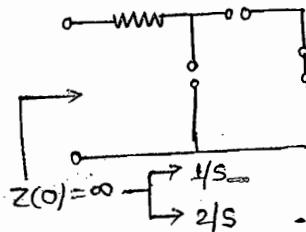
$$y_2 = \frac{1}{z_2} = 1 + \frac{1}{sC} = y_2' + y_2''$$

$$y_2' = 1 = \frac{1}{R_2} ; R_2 = 1\Omega$$

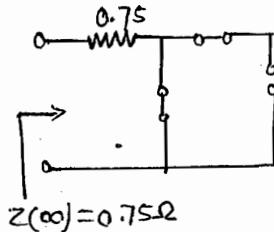
$$y_2'' = \frac{1}{sC} = \frac{1}{sL_2} ; L_2 = 2H$$



$$\frac{8}{43} \quad s=0 \quad L=SC, C \rightarrow OC$$

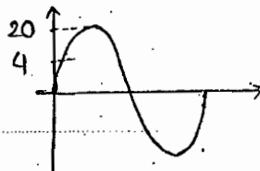


$$\frac{8}{43} \quad s=\infty \quad L \rightarrow OC; C \rightarrow SC$$



(b) $Z(0) = 1/s$; $Z(\infty) = 0.75\Omega$

(d) $Z(0) = 2/s$; $Z(\infty) = 0.75\Omega$



$$V_{\text{normalised}} = \frac{V}{V_{\max}} = \frac{4}{20}$$

so ans-(b) because of normalised is given

$$\frac{10}{43} \quad Z(s) = \frac{s\alpha}{s+\beta} = \frac{V}{I} = \frac{j\omega + \alpha}{j\omega + \beta}$$

V leads I

I/p sinusoidal

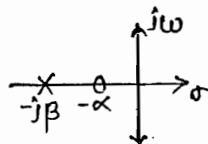
$$\phi = \tan^{-1} \frac{\omega}{\alpha} - \tan^{-1} \frac{\omega}{\beta} = +ve$$

$$\alpha < \beta$$

Another way;

RL ckt is given

$$Z_{RL} \quad \boxed{\alpha < \beta}$$



(13)
44

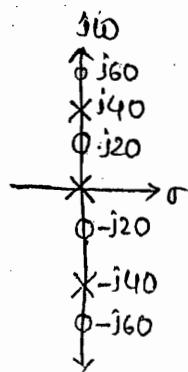
Reactive N/w \rightarrow LC ckt

Z_{LC}

Zeros: - $j\omega = 20, 60$

Poles: - $\omega = 0, 40, \infty$

$$Z|_{\omega=10} = -j70\Omega$$



$$Z(s) = \frac{k(s^2 + 20^2)(s^2 + 60^2)}{s^2(s^2 + 40^2)}$$

If we want to calc the value of k then put $s = j\omega \rightarrow \omega = 10$ & compare
 $Z(s) = -j70\Omega$.

$$Z(s) = \frac{s^4}{s^3} = sL \quad (1st \text{ term will be connected in series})$$

Ans.(a)

(5)
45

$$\frac{V(s)}{I(s)} = Z(s) = \frac{(s+1)}{(s+\sqrt{2})(s+\sqrt{1/2})}$$

$$i(t) = \sin t \quad v(t) = v \sin t \\ I = 1 \quad |V| = ?$$

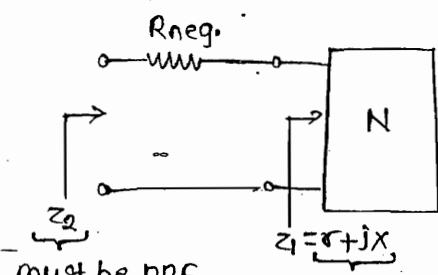
$$s = j\omega = j1$$

$$Z(j1) = \frac{(j1+1)}{(j1+\sqrt{2})(j1+\frac{1}{\sqrt{2}})} = \frac{V}{I=1}$$

$$|V| = \frac{\sqrt{2}}{\sqrt{1+2}\sqrt{1+1/2}}$$

$$|V| = \frac{2}{3}$$

(25)
45



must be PRF

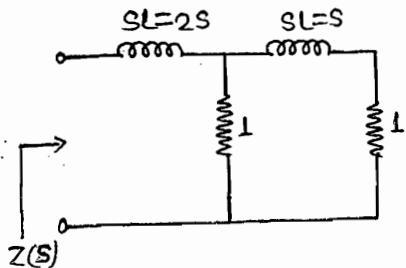
$Z_2 = s + jX$,
PRF
..... physically realizable

$$Z_2 = \tau_n + z_1 = \tau_n + (\tau \pm jx)$$

$$|\tau_n| \leq \tau$$

$$|\tau_n| \leq \operatorname{Re}[z_1]$$

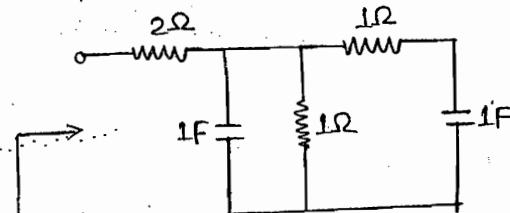
(29)
46



$$Z(s) = 2s + \frac{1 \times (s+1)}{1+(s+1)}$$

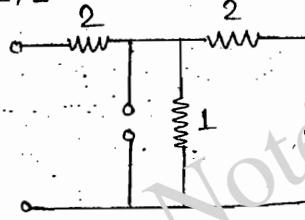
$$s = -2, \infty \dots \text{poles}$$

(92)
47



$$Z(s) = \frac{qs^2 + 7s + 3}{s^2 + 7s + b}$$

$$s \rightarrow 0; Z(0) = 2 + 1 = 3$$



$$Z(\infty) = \frac{0+0+3}{0+0+b}$$

$$3 = \frac{3}{b}$$

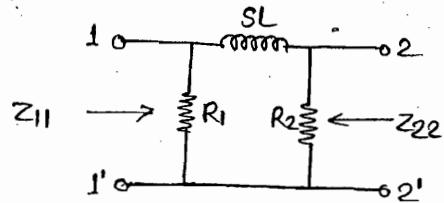
$$b = 1$$

$$s \rightarrow \infty;$$

C... (SC)

$$Z(\infty) = 2 = \frac{q + (7/s)^0 + (3/s^2)^0}{1 + (3/s)^0 + (b/s^2)^0}$$

$$q = 2$$

24
45

$$Z_{11} = k_1 \left(\frac{s+3}{s+8} \right)$$

$$Z_{11} = \frac{R_1 \times (R_2 + sL)}{R_1 + R_2 + sL} = \frac{R_1 k}{k} \cdot \frac{(s + \frac{R_2}{L})}{s + (\frac{R_1 + R_2}{L})}$$

$$k_1 \left(\frac{s+3}{s+8} \right) = R_1 \left(s + \frac{R_2}{L} \right) \\ \frac{s+3}{s+8} = \frac{s + \frac{R_1 + R_2}{L}}{s + \frac{R_2}{L}}$$

$$R_1 = k_1; \quad \frac{R_2}{L} = 3, \quad \frac{R_1 + R_2}{L} = 8$$

$$\frac{R_1}{L} = 5$$

$$Z_{22} = \frac{R_2 \times (R_1 + sL)}{(R_1 + R_2 + sL)} = \frac{R_2 k \times (s + \frac{R_1}{L})}{k \times (s + \frac{R_1 + R_2}{L})} \\ = R_2 \times \frac{(s+5)}{(s+8)} \\ = k_2 \times \frac{(s+5)}{(s+8)}$$

ie. → following Z^H represents physically realisable & stable driving point RLC impedance Z^H . Realise it in Foster's - (II) form.

$$Z(s) = \frac{(s+1)(s+4)}{(s+2)(s+3)} = Z_{RLC}(s)$$

 Z^H

$$Z(s) = \frac{(s+1)(s+4)}{(s+2)(s+3)} = Z_{RLC}(s)$$

..... Foster's type-II [Parallel Form]

$$\frac{1}{Z(s)} = Y(s) = \frac{(s+2)(s+3)}{(s+1)(s+4)} = \frac{2}{s+4} + \frac{1}{s+1} + \frac{2}{s+3} = \frac{1}{s+1} + \frac{A}{s+1} + \frac{B}{s+4}$$

Because of ratio of sq. is 2. $= 1 + \frac{(2/3)}{(s+1)} + \frac{(-2/3)}{(s+4)}$

It can't be physically realisable because of $-ve(B)$

$$\frac{Y(s)}{s} = \frac{(s+2)(s+3)}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+4)}$$

$$= \frac{3/2}{s} + \frac{(-2/3)s}{(s+1)} + \frac{1/6}{s+4}$$

$$Y(s) = \frac{3/2}{s} + \frac{(-2/3)s}{(s+1)} + \frac{s(1/6)}{(s+4)} \rightarrow \text{Need not to be devide because of +ve}$$

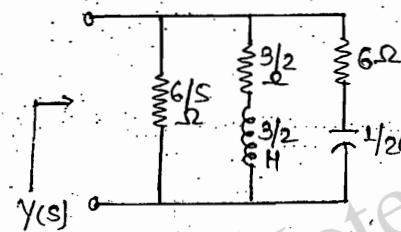
↓
same value of coefficient & -ve so we have to cal. the value so devide

$$\frac{s+1}{s+1} \frac{s}{s+1} (1)$$

$$Y(s) = \frac{3}{2} - \frac{2}{3} \left(1 - \frac{1}{s+1} \right) + \frac{1}{6} \cdot \frac{s}{s+4}$$

$$= \frac{5}{6} + \frac{2/3}{s+1} + \frac{1/6(s)}{(s+4)}$$

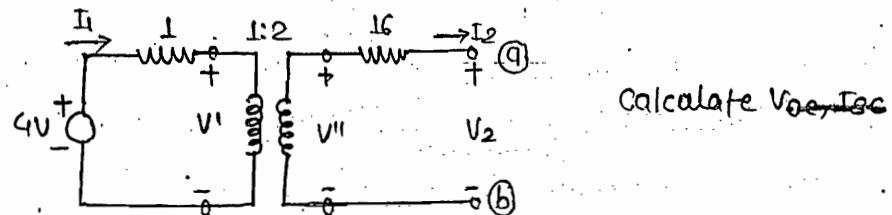
$$Y(s) = Y_1 + Y_2 + Y_3$$



Taiter's type (III)

Parallel form

Que. →



calculate V_{oe}, I_{ec} :

Soln →

$$Q = I_1 + V'$$

$$V_2 = -16I_2 + V''$$

$$\left. \begin{aligned} \frac{V'}{V''} &= \frac{1}{2} & ; V'' &= 2V' \\ \frac{I_1}{I_2} &= \frac{2}{1} & ; I_1 &= 2I_2 \end{aligned} \right\}$$

$V_{OC} \rightarrow$

$$V_2 = V_{OC}$$

$$I_2 = 0$$

$I_{SC} \rightarrow$

$$I_2 = I_{SC}$$

$$V_2 = 0$$

$Req. \rightarrow$

$$\frac{V_{OC}}{I_{SC}}$$

$Req. \rightarrow$

$$V_2 = 1$$

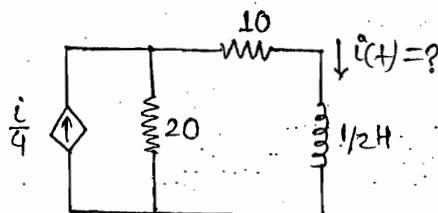
4V source \Rightarrow SC

$$Req. = \frac{1}{-I_2}$$

$$V_{OC} = 8V, Req. = 2\Omega$$

$$I_{SC} = 2/5A$$

Que. \rightarrow



$$i(0^+) = 10A$$

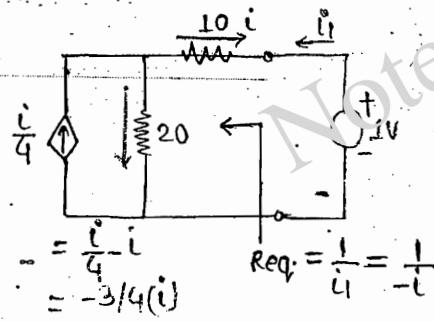
solt \rightarrow

$$i(t) = i(0^+) e^{-t/T}$$

$$10A$$

$$T = \frac{L}{Req.}$$

Req. \rightarrow



$$Req. = 25\Omega \quad ; \quad T = \frac{L}{Req.} = \frac{1}{50}$$

$$i(t) = i(0^+) \cdot e^{-t/T}$$

KVL \rightarrow

$$1 = -10i + 20\left(\frac{-3}{4}\right)i$$

$$1 = -25i$$

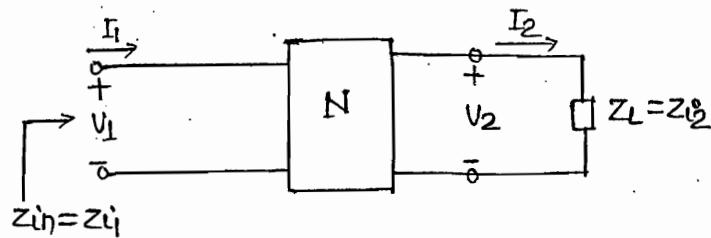
~~$$i = 1/25$$~~

$$\frac{1}{t} = -25$$

$$i(t) = 10 \times e^{-50t} \dots A$$

for $t > 0$

Image Impedances $\rightarrow Z_{i1}, Z_{i2}$



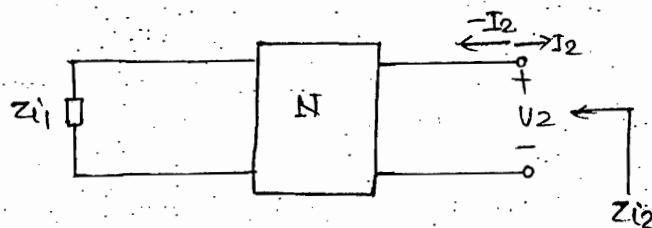
$$N: \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

$$V_2 = I_2 Z_L = I_2 Z_{i2}$$

$$\frac{V_1}{I_1} = Z_{in} = Z_{i1}$$



Z_{i1}, Z_{i2}
Image impedances

$$Z_{i1} = \sqrt{\frac{AB}{CD}}, \quad Z_{i2} = \sqrt{\frac{BD}{CA}}$$

If n/w is symmetrical

$$A=D$$

$$Z_{i1} = Z_{i2} = \sqrt{\frac{B}{C}}$$

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