CONDUCTION THROUGH PLANE WALL

As in steady state, Q is constant along the path of heat flow. The Fourier's law equation can be integrated over the entire path from =0 to x=L (total thickness of the wall)

$$\dot{Q}_{cond,wall} = -k\,A\,\frac{dT}{dx} \qquad \text{(Fourier's law of conduction)} \longrightarrow 1$$

The variables in (1) are x and T

$$\int_{x=0}^{x=L} \dot{Q}_{cond,wall}\,dx = -\int_{T_1}^{T_2} kA\,dT \qquad \dot{Q}_{cond,wall} = k\,A\,\frac{T_1 - T_2}{L} \longrightarrow 2$$

Rearranging the above equation

$$\boxed{\dot{Q}_{cond,wall} = \frac{T_1 - T_2}{\left(\frac{L}{kA}\right)}} \longrightarrow 3$$

$$\boxed{\text{Thermal resistance, } R_{wall} = \left(\frac{L}{kA}\right)}$$

$$\dot{Q}_{cond,wall} = \frac{T_1 - T_2}{(R_{wall})}$$

CONDUCTION THROUGH PLANE WALL

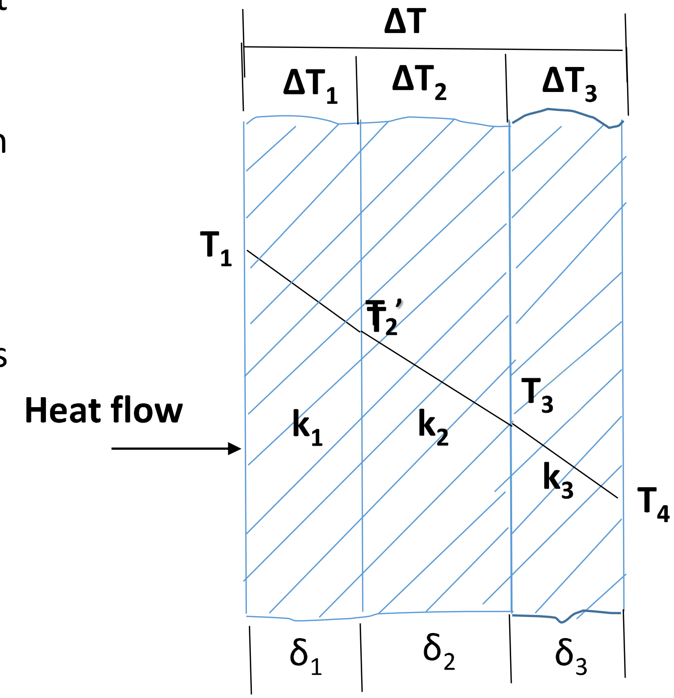
The reciprocal of resistance is called conductance, which for heat conduction is

Conductance = 1/ R = KA/L

Both the resistance and conductance depends upon the dimensions of a solid as well as on the thermal conductivity, a property of material.δ

CONDUCTION THROUGH COMPOSITE WALL

- Consider a flat wall constructed of a series of layers of three different materials.
- Let k_1, k_2 and k_3 be the thermal conductivities of the material of which layers are made.
- Let thickness of the layers be δ_1, δ_2 and δ_3 respectively.
- Let ΔT_1 be the temperature drop across the first layer, ΔT_2 that across the second layer and ΔT_3 that across the third layer.
- Let ΔT be the temperature drop across the entire composite wall.



CONDUCTION THROUGH COMPOSITE WALL

- Let T_1, T_2, T_3 and T_4 be the temperature at the faces of walls. T_1 is the temperature of the hot face and T_4 is the temperature of the cold face, assume that the layers are in excellent thermal contact.
- Let the area of the composite wall be A.

Over all temperature drop is related to the individual temperature drops over the layers by the equation

$$\Delta T = \Delta T_1 + \Delta T_2 + \Delta T_3$$

In the steady-state condition, the heat flow q is the same for all the layers and is constant. The equations of heat transfer through these layers

$$q = k_1 A \frac{T_1 - T_2}{\delta_1} \quad \text{for the first layer} \qquad \text{(i)}$$

$$q = k_2 A \frac{T_2 - T_3}{\delta_2} \quad \text{for the second layer} \qquad \text{(ii)}$$

$$q = k_3 A \frac{T_3 - T_4}{\delta_3} \quad \text{for the third layer.} \qquad \text{(iii)}$$

CONDUCTION THROUGH COMPOSITE WALL

The temperature differences across the layers, from above equations, are

$$T_1 - T_2 = q\left(\frac{\delta_1}{k_1 A}\right)$$

$$T_2 - T_3 = q\left(\frac{\delta_2}{k_2 A}\right)$$

$$T_3 - T_4 = q\left(\frac{\delta_3}{k_3 A}\right)$$

Adding the above equations, we get

$$T_1 - T_4 = q\left(\frac{\delta_1}{k_1 A} + \frac{\delta_2}{k_2 A} + \frac{\delta_3}{k_3 A}\right)$$

or

$$q = \frac{T_1 - T_4}{\dfrac{\delta_1}{k_1 A} + \dfrac{\delta_2}{k_2 A} + \dfrac{\delta_3}{k_3 A}}$$

R_1 , R_2, R_3 be the thermal resistance offered by layer 1,2 and 3

or

$$q = \frac{T_1 - T_4}{R_1 + R_2 + R_3}$$

CONDUCTION THROUGH COMPOSITE WALL

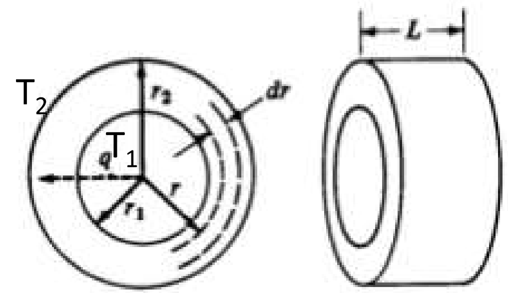
If R is the overall resistance, $R = R_1 + R_2 + R_3$

Therefore equation becomes $Q = \Delta T / R$

The temperatures at the interface of layers can be calculated using

$\Delta T / R = \Delta T_1 / R_1 = \Delta T_2 / R_2 = \Delta T_3 / R_3$

CONDUCTION THROUGH CYLINDER



- Consider a thick walled hollow cylinder of inside radius r_1, outside radius r_2, and length L. let k be the thermal conductivity of the material of cylinder

- let the temperature of inside and outside surface be T_1 and T_2 . Assume T_1 > T_2 , heat flows from inside of the cylinder to outside

- Consider a very thin cylinder concentric with the main cylinder of radius r , where r is in between r_1 and r_2 , thickness of wall be dr

  - Rate of heat flow at any radius r is given by

$$q_r = -kA_r \frac{dT}{dr}$$

$$q_r = -2\pi krL \frac{dT}{dr}$$

❖ Rearranging the above equation and integrating between the limits

Heat conduction through hollow cylinder

$$\dot{Q}_{cond,cyl} = -k\,A\,\frac{dT}{dr}$$ (Fourier's law of conduction)

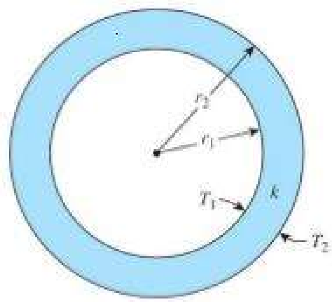
$$\int_{r_1}^{r_2} \frac{\dot{Q}_{cond,cyl}}{A}\,dr = -\int_{T_1}^{T_2} k\,dT$$

$A = 2\pi rL$

$$\dot{Q}_{cond,cyl} = 2\pi Lk\,\frac{T_1 - T_2}{\ln(r_2/r_1)}$$ L= length of cylinder

$$\boxed{\dot{Q}_{cond,cyl} = \frac{T_1 - T_2}{\left(\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi Lk}\right)}}$$ (Rearranging above equation)

$$\dot{Q}_{cond,cyl} = \frac{T_1 - T_2}{(R_{cyl})}$$ (Thermal Resistance Concept)

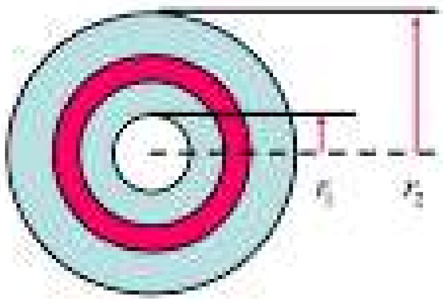


$$Q = -k\,\frac{2\pi L}{\ln\left(\frac{r_2}{r_1}\right)}(T_2 - T_1)$$

CONDUCTION THROUGH SPHERES

Geometric Considerations

Heat Conduction Through Hollow Spheres



$$\frac{Q}{A} = -k\frac{dT}{dr}$$

$$A = 4\pi r^2$$

$$\int \frac{Q}{4\pi r^2}\,dr = -k\int dT$$

Integrating both sides: $$\left(\frac{1}{r_1} - \frac{1}{r_2}\right)Q = -\frac{4\pi}{k}(T_2 - T_1)$$

$$\int_1^2 dt = -\int_1^2 \frac{\dot{Q}}{KA}\,dr$$

$$or,\ t_2 - t_1 = -\frac{\dot{Q}}{k}\int_1^2 \frac{dr}{4\pi r^2}$$

$$or,\ t_2 - t_1 = -\frac{\dot{Q}}{4\pi k}\left[-\frac{1}{r}\right]_1^2$$

$$or,\ t_1 - t_2 = -\frac{\dot{Q}}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]_1^2$$

$$or,\ \dot{Q} = \frac{4\pi k r_1 r_2 (t_1 - t_2)}{(r_2 - r_1)}$$

CONVECTION

- Convection is the transfer of heat from one point to another point within a fluid by mixing of hot and cold portions of the fluid.
- Heat transfer by convection occurs as a result of the movement of fluid on a macroscopic scale in the form of circulating current.
- The circulating currents may be set up either by heat transfer process itself or some external agency.
- There are two types of convection
    - Free or natural convection
    - Forced convection

FREE CONVECTION

- ✓ When the circulating currents arise from the heat transfer process itself
- ✓ i.e. from the density differences arising in turn due to temperature differences within the fluid mass, the mode of heat transfer is called free or natural convection.

Examples of natural convection:

Boiling water - The heat passes from the burner into the pot, heating the water at the bottom. Then it expands and rises because its density has become less than that of the remaining liquid. Cold water of higher density takes its place and moves down to replace it, causing a circular motion.

FORCED CONVECTION

- ❖ When the circulating currents are produced by an external agency, such as an agitator in a reaction vessel, pump, fan or blower, the mode of heat transfer is called forced convection.
- ❖ Here fluid motion is independent of density gradients.

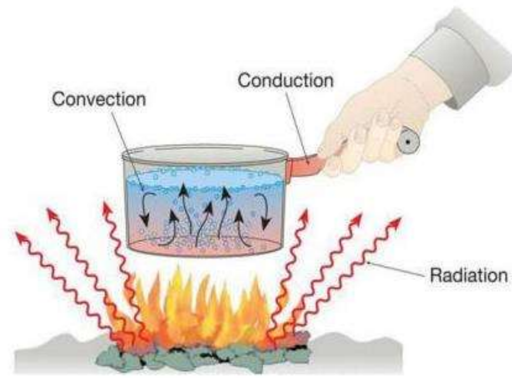
Examples

1. Heat flow to a liquid pumped through a heated pipe.
2. Heat or cool a home efficiently, such as using a fan.
3. Cooling of Internal combustion engines with fan in a radiator
4. Cooling in heat exchangers and in nuclear reactors

MECHANISM OF FREE CONVECTION

- In natural convection, the fluid motion occurs by natural means such as buoyancy.
- Buoyancy forces are developed due to density variations in the fluid caused by the temperature difference between the fluid and the adjacent surface.
- Since the fluid velocity associated with natural convection is relatively low, the heat transfer coefficient encountered in natural convection is also low.

RADIATION



- Radiation refers to the transport of energy through space by electromagnetic waves.
- Depends upon the electromagnetic waves as a means of transfer of energy from a source to a receiver.
- Mechanism of transmission is photon emission, this mode of energy transfer does not require any medium.
- The significance of this is that radiation will be the only mechanism for heat transfer whenever a vacuum is present.

Examples of heat transfer by radiation

- Transfer of heat from the sun to the earth
- Use of energy from the sun in solar heaters
- Heating of a cold room by a radiant electric heater

RADIATION

ABSORPTIVITY, REFLECTIVITY AND TRANSMISSIVITY

- Any substance receives and emits energy in the form of electromagnetic waves.
- When energy emitted by a heated body falls on a second body, it will be partly absorbed, partly reflected back and partly transmitted through the body.
- It is only the absorbed energy that appears as a heat in the body

  I.e. incident radiation = heat absorbed + heat transmitted + heat reflected

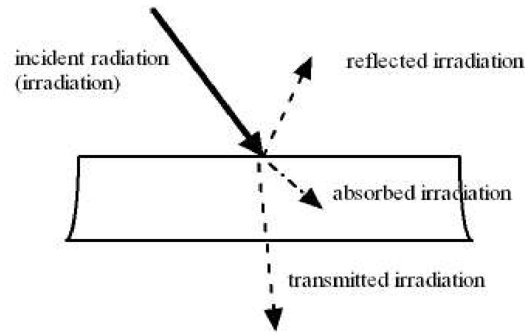
- The proportions of the incident energy that are absorbed, reflected and transmitted depends on the characteristics of a receiver.

RADIATION

Fraction of the incident radiation on a body that is absorbed **absorptivity** (**a**).

Fraction of the incident radiation on a body that is reflected **reflectivity (r).**

Fraction of the incident radiation on a body that is transmitted through the body is known as **transmissivity (τ).**



The energy balance about a receiver on which the total incident energy falling is unity (sum of all fractions is unity) $a + r + \tau = 1$

For an opaque material, $a + r = 1$ since $\tau = 0$; For perfectly transparent $\tau = 1$, $a = r = 0$

For non-reflecting surfaces $r = 0$, $r = 1$, perfectly reflector $a = 1$, perfectly absorbing surface or a black surface

RADIATION

BLACK BODY

- A body for which $a = 1$, $r = \tau = 0$, i.e. which absorbs all the incident radiant energy is called a black body.
- It neither reflects nor transmits but absorbs all the radiations incident on it, so it is treated as an ideal radiation receiver.
- The black body radiates maximum amount of energy at a given temperature.
- Lamp black is the nearest to a black body which absorbs 96 % of the visible light.
- Both the absorptivity and emissivity of perfectly black body are unit.

RADIATION

Monochromatic emissive power: It is the radiant energy emitted from a body per unit area per unit time per unit wavelength. It is denoted by E_λ. It has the unit of $W/(m^2\ \mu m)$.

Monochromatic emissivity: it is the ratio of monochromatic emissive power of a body to that of a black body at the same wavelength and temperature.

Gray body

A gray body is defined a body whose absorptivity of a surface does not vary with variation in temperature and wavelength of the incident radiation. I.e. A body having the same value of the monochromatic emissivity at all wavelengths is called a gray body and the emissivity is independent of wavelength.