

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

IIT

ANALYSING HEAT TRANSFER

Assignment : 1

QUESTION : 1 Modes of heat transfer

- **1.1:** The primary mode of heat transfer responsible for heating water on a stove is **convection**. This is because as the water heats up, it becomes less dense and rises, while the cooler water descends, creating a convective current.
- **1.2:** The heat transfer mechanism involved when holding a metal rod over a flame is **conduction**. Heat travels along the rod from the high-temperature end near the flame to the lower temperature end that we are holding.

QUESTION-2: Heat Loss Analysis

- The hot water tank loses heat to the surrounding medium over time, resulting in a temperature distribution change until the water temperature in the tank becomes equal to the ambient temperature. This would be considered a **transient** heat transfer problem because the temperature of the water will change with time as it loses heat.
- The 200-L cylindrical hot water tank loses heat along the radial direction (from outside the tank to the inside) and in the axial direction (along the height of the cylinder). In most practical applications, radial heat transfer is more significant than axial heat transfer, which makes this heat transfer problem primarily one-dimensional. If the tank's geometry and accuracy require it, the axial heat

transfer may also need to be considered, resulting in a two-dimensional heat transfer problem.

QUESTION-3: Heat Conduction Equation

- Consider a cylindrical shell characterized by an inner radius r , outer radius $r+dr$, and length L . The rate of heat generation within this volume element is $g \cdot \text{volume} = g \cdot (2\pi r L dr)$
- According to Fourier's law, the rate of heat conduction into the volume element at radius r is given by $-K(2\pi r L) \frac{\partial T}{\partial r}$, where K denotes the thermal conductivity, and T represents the temperature. Similarly, the rate of heat conduction out of the volume element at radius $r+dr$ is $-K(2\pi(r+dr)L) \frac{\partial T}{\partial r}$ at $r+dr$.
- The net rate of heat conduction out of the volume element is expressed as:
 $g \cdot (2\pi r L dr) = -K(2\pi(r+dr)L) \frac{\partial T}{\partial r} \bigg|_{r+dr} + K(2\pi r L) \frac{\partial T}{\partial r} \bigg|_r$
- Expanding the left side, neglecting higher-order terms, and dividing through by $2\pi L dr$ while taking the limit as $dr \rightarrow 0$ we arrive at the differential equation:

$$\lim_{dr \rightarrow 0} \frac{-K(r+dr) \frac{\partial T}{\partial r} \bigg|_{r+dr} + K(r) \frac{\partial T}{\partial r} \bigg|_r}{dr} = g \cdot r$$
- Apply differentiation rules to simplify the left side and obtain the one-dimensional heat conduction equation for a long cylinder:
- The steady one dimension heat conduction equation for a long cylinder is given by:

$$\frac{d}{dr} \left(kr \frac{dT}{dr} \right) = g \cdot r$$

where (r) is the radial coordinate, (T) is the temperature, (k) is the thermal conductivity, and (g) is the rate of heat generation per unit volume.

QUESTION-4: Heat Transfer Characteristic

- (a) Heat transfer is **changing over time** because temperature varies ($\partial T / \partial t \neq 0$).
- (b) It's **one-dimensional**, focusing on heat conduction in the x direction.
- (c) No heat is generated in the medium; there's **no internal source**.

- (d) Thermal conductivity is **uniform** without variation across positions or temperatures.

QUESTION-5: Heat Flux Calculation

- We need to determine the heat flux on the surface of a large 3 cm thick stainless steel plate. Heat is generated uniformly at a rate of $5 \times 10^6 \text{ W/m}^3$ and the plate is losing heat from both sides. The provided solution involves the following
- $\text{Heat}_{\text{generated per unit area}} = \text{Heat}_{\text{generation rate}} * \text{Thickness}$
- Thickness is the thickness of the plate (0.03 m)
- $\text{Heat}_{\text{generated per unit area}} = (5 \times 10^6 \text{ W/m}^3) \times 0.03 \text{ m} = 150000 \text{ W/m}^2$
- $\text{Heat}_{\text{loss per side}} = (150000 \text{ W/m}^2) / 2 = 75000 \text{ W/m}^2$
- Therefore, the heat flux on the surface of the plate during steady operation is 75 kW/m^2

