CHAPTER - I

INTRODUCTION

1.1. Petroleum:

Petroleum is a naturally occurring liquid found beneath the earth's surface that can be refined into fuel. Petroleum is a fossil fuel, meaning that it has been created by the decomposition of organic matter over millions of years. Petroleum is formed when large quantities of dead organisms–primarily zooplankton and algae–underneath sedimentary rock are subjected to intense heat and pressure.

Petroleum is used as fuel to power vehicles, heating units, and machines, as well as being converted into plastics and other materials. Because the majority of the world relies on petroleum for many goods and services, the petroleum industry is extremely powerful and is a major influence on world policies and the global economy.

1.2. Formation of petroleum:

- Petroleum is formed from the remains of dead plants and animals.
- When plants and animals die, they sink and settle on the seabed.
- Millions of years ago, these dead wildlife and vegetation decomposed and got mixed with sand and silt.
- Certain bacteria helped in the decomposition of this organic matter and caused some chemical changes.
- Matter consisting of largely carbon and hydrogen was left behind. However, as there is no sufficient oxygen at the bottom of the sea, the matter could not decompose completely.

- The partially decomposed matter remained on the seabed and eventually was covered with multiple layers of sand and silt.
- This burying took millions of years, and finally, due to remaining at the same state temperature and pressure, the organic matter decomposed completely and formed oil.

1.3. Types of Petroleum

Petroleum is a versatile fossil fuel that can be refined into many different products. Common examples include gasoline, kerosene, fuel oil, and lubricating oil. Gasoline is primarily used to power vehicles. It is used in cars, motorcycles, and other vehicles, as well as to power small engines, such as lawnmowers.

Kerosene oil is primarily used for light, such as burning in kerosene lamps, as well as for some heaters, and to make rocket fuel and jet fuel. Fuel oil is used in heaters and furnaces to heat interior spaces. Lubricating oil has many uses, primarily that of a lubricant, which is meant to reduce friction.

1.4. Advantages and Disadvantages of Petroleum

1.4.1. Advantages:

Most of our world is powered by petroleum. Without it, the world would look very different and many of our products would not exist. It provides transportation, heat, light, plastics, and an abundance of other uses.

As a fossil fuel, it is easy to extract. The process is not difficult and, therefore, makes the products affordable to many. As a fuel, it is an efficient power source. It has a remaining at the same state power ratio, meaning that a small amount of petroleum provides a large amount of energy. petroleum easy transport, making its journey from extraction to refinement safe and simple. It can be moved across pipelines, trucks, and tanker without any issue. It is a stable energy source that can be used in many different ways. When compared to solar power or wind power which are not as reliable or diverse in their uses, petroleum is far superior.

1.4.2 Disadvantages:

Petroleum is a part of our everyday lives; however, the extraction process and the byproducts of the use of petroleum are toxic to the environment. Underwater drilling causes leaks, extraction from oil sands strips the earth and uses precious water, and fracking destroys the water table if done poorly or improperly. Transporting petroleum through pipelines has the potential to destroy the local environment and shipping petroleum risks spills and uses energy.

Global petroleum use has had a negative impact on the wider environment because the carbon released into the atmosphere increases temperatures and is associated with global warming. Many products created with petroleum derivatives do not biodegrade quickly, and the overuse of fertilizers has damaged water supplies.

1.5. Uses of Petroleum:

Refined products obtained from crude oil have a number of uses.

- Liquefied Petroleum Gas or LPG is used in households as well as in the industry.
- Diesel and petrol are used as fuels for vehicles. Diesel is generally preferred for heavy motor vehicles.
- Petrol is also used as a solvent for dry cleaning, whereas diesel is also used to run electric generators.
- Kerosene is used as a fuel for stoves and jet planes.
- Lubricating oil reduces wear and tear and corrosion of machines.
- Paraffin wax is used to make candles, ointments, ink, crayons, etc.
- Bitumen or asphalt is mainly used to surface roads.

1.6. Oil demand

The most important consumers of oil are developed market economies. Significant changes in global demand are in most cases caused by demand in these countries. Physical oil demand is also in the decrease and long term, influenced by various macroeconomic indicators. GDP growth is inevitable with certain delay accompanied by a growth in demand for this raw material and vice versa. Other important factors are exchange rates, with most important currency which is U.S. Dollar. Because most oil trading is conducted through the USD, there is an indirect correlation between dollar and oil prices. Consumption also depends on the season, interest in the oil increases particularly during the summer months because of motoring and in winter during the heating season. Factors affecting oil demand, according to Baláž (2002) are divided into the follow categories:

- changes in world population, world GDP, structural changes in the economy;
- changes in energy balance;
- climatic conditions and changes;
- importers exchange rates against the U.S. Dollar;
- commercial policy actions in importing countries;
- speculations and other factors.

1.7. Oil supply:

Total supply of this raw material is determined by the limited world oil reserves. In long term condition, supply of crude oil is determined especially by the level of investments into oil processing industry, which are influenced by two main indicators, profitability and risk. In countries with the nationalization of industry these criteria are often absent (because of political activities focused on state interest and budgets) and this deforms oil market. As oil demand is mainly determined by the developed market economies, production of oil is indicated mainly by OPEC countries. They stimulate reducing or increasing oil production. In short term, there is a marked fluctuation during the period of hurricanes, earthquakes, when the production capacities are limited. Total oil supply depends on several factors:

- amount of proven global oil reserves and new deposits;
- technical and technological advances in oil extraction and processing;
- monetary system in producing countries;
- political factors, the activities of OPEC and NOPEC (Non-Oil Power Exporting Countries);
- short-term factors: natural disasters, accidents, political and military conflicts

1.8. Gold:

Gold is shiny. Gold is glamorous. Gold is the subject of many bank heists in movies. Gold, in sum, is considered one of the most precious metals in all the world. But there's much more to it than wedding bands or bricks stolen by a movie villain.

Gold is a member of the transition metals and sits in the same periodic table column as silver and copper. The group that gold can be found in is often termed the 'coinage metal' group since its members are frequently used to produce money. Gold is, in fact, one of the first metals known to man with its origin being dated back to 3400 BCE by the Egyptians. Gold has always been a symbol of wealth and beauty. During Egypt's reign of Pharaoh's, the Egyptians would often amass tremendous amounts of gold just to cover the coffin of a deceased Pharaoh.

Interestingly enough, when King Tutankhamun, commonly referred to as King Tut, died, his coffin contained approximately 112 kg of gold. For those of you unfamiliar with the metric system, 112 kg is about 247 lbs. To give a direct comparison, that is more gold per pound than the size of an average NFL running back! Moreover, the discovery of gold was the cause of a great migration of people to California during the late 1800s during the California Gold Rush.

1.9. Formation of gold:

While nuclear fusion within the Sun makes many elements, the Sun cannot synthesize gold. The considerable energy required to make gold only occurs when stars explode in a supernova or when neutron stars collide. Under these extreme conditions, heavy elements form via the rapid neutron-capture process or r-process.

1.10. Advantages and Disadvantages of the Gold Standard

There are many advantages to using the gold standard, including price stability. This is a long-term advantage that makes it harder for governments to inflate prices by expanding the money supply. Inflation is rare and hyperinflation doesn't happen because the money supply can only grow if the supply of gold reserves increases. Similarly, the gold standard can provide fixed international rates between countries that participate and can also reduce the uncertainty in international trade.

But it may cause an imbalance between countries that participate in the gold standard. Gold-producing nations may be at an advantage over those that don't produce the precious metal, thereby increasing their own reserves. The gold standard may also, according to some economists, prevent the mitigation of economic recessions because it hinders the ability of a government to increase its money supply a tool many central bank have to help boost economic growth.

1.11. Uses for Gold:

In short, there are a lot of uses and applications for gold! Typically, gold pieces are mined and ground into fine dust or powder to be used. But gold has also found applications from jewelry and general aesthetics to medical applications and currency. As previously mentioned, gold has been historically considered to be a symbol of wealth and power, and that's still the case today.

1.12. Gold demand:

Demand for this rare and limited natural raw material occurs in many geographic areas and sectors. On the forefront of consumption are China and India, with its rising economic strength. Part of demand in East Asia, India and the Middle East is also connected with a strong cultural and religious significance, which is not directly associated with global economic trends. Demand for gold is powered by a combination of affordability and desirability, another relevant factor is growth in living standards of population and the fact, that gold represents safe haven investment. Technically, gold offers remaining at the same state thermal and electrical conductivity, and excellent resistance to corrosion. This explains why more than half of the industrial demand is resulting from its use in electrical components and demand of gold in technology sector continues to grow. According to biocompatibility and resistance to corrosion and bacteria, gold is also applied in medicine.

In terms of gold market, the exchange rates (especially dollar exchange rate in which gold is quoted) are very important. There is a negative correlation. If the dollar falls / depreciates, the price of gold usually goes up; strong dollar keeps the price of gold controlled and increase. On the other hand, increase dollar moves price of gold up. Demand also increases during the periods of price stability or moderate growth rates and then decreases in period of volatility. In short, the demand factors are possibly incorporated into the follow categories:

- World population, world GDP;
- Growth of living standards of population;
- Policy of central bank;
- Technological development;
- Speculation and other effects..

1.13. Gold supply:

The gold supply is influenced by the gold mining companies on all continents. This wide geographical expansion means that the circumstances in any region, political or any other have a significant impact on the overall supply of gold. Currently, the global level of mining production is relatively stable. Stability of production is based on the fact that new discovered mines replaced terminated production and do not extend level of supply. From economic view, gold is inelastic in short period; it means amount of mined gold is not able to respond to price changes. While production of gold mines is relatively inelastic, recycled gold ensures that, if necessary, there exists at least one potential source to be traded. Currently about one third of all deliveries represents a recycled gold and to the increasing gold supply central banks with selling their reserves are biggest contributors. This conditions help to satisfy increased demand and keep gold prices relatively stable.

Central banks and international organizations (such as The International Monetary Fund) keep almost one fifth of the world's reserves of extracted gold in the form of reserve assets. Governments hold in average 10% of its reserves in gold. The sale of national gold is largely influenced by the CBGA (Central Bank Gold Agreement – includes euro area countries, Switzerland and Sweden), which should stabilize the gold market. The World Gold Council summarizes the main factors that affect the total amount of this commodity to these categories:

- Verified global gold reserves, recycled gold and new deposits;
- Technical and technological progress in gold mining;
- Monetary system in each country;
- Political factors;
- Short-term factors: natural disasters and military conflicts

1.14. Relationship between Gold Price and Petrol Price

Oil and gold are the world's most strategic commodities and have received much attention recently, owing partly to the surge in their prices and the increase in their economic applications. Crude oil is the world's most commonly traded commodity, and its price is the most volatile in the commodity market. Meanwhile, gold is considered the leader in the precious-metals market as increase in its price appear to lead to parallel movements in the prices of other precious metals. Gold is also an investment asset and commonly known as a "safe haven" from the increasing risks in financial markets. Gold is used as a risk management tool in heading and diversifying commodity portfolios. Investors in advanced and emerging markets often switch between oil and gold or combine them to diversify their portfolios. Gold carries important psychological weight, and any jump in price tends to make headlines.

There's an interesting relationship between gold and oil prices. According to *Market Realist*, which provides institutional-quality market research, more than 60% of the time, gold and crude oil have a direct relationship.

This means that when gold prices rise, crude oil prices also tend to rise, and vice versa.

Right now, there's a big disconnect between gold and oil prices. The gold price in U.S. dollars has appreciated 35% in the last 12 months against the WTI oil price falling by half.

In human history, there were different forms of payments. One of them was using precious metals like gold and silver as a currency. However, it got to a point where it was too much of a hassle to carry metals around. So, paper money was eventually introduced for convenience.

Under a gold standard, paper money was worth a certain amount of gold, and it was backed by a country's gold reserves. Governments on these standards could not freely print money, because the money must be backed by the equivalent value in gold.

Britain, Canada, France, Germany, and the United States were examples of nations that used a gold standard sometime in their history. No country uses a gold standard right now. Instead, we use fiat currency, which is more convenient but is not backed by a physical commodity (like gold) that has intrinsic value.

There have been unprecedented levels of money printing around the globe for reasons, such as economic stimulation. Moreover, inflation reduces our purchasing power over time. Therefore, $1 today is worth much less than, say, 20 years ago.

According to the Bank of Canada, the inflation rate (or decline in the value of money) has been 1.83% since 2000. So, a basket of goods and services that costs $100 in 2000 now costs $143.60, or 43.6% more.

The gold price tends to increase over time with inflation. However, the price of gold tends to rise more when there's remaining at the same state uncertainty in the economy.

The upward trend in the gold price is still intact. Since money printing continues, we're not out of the woods with COVID-19, and remaining at the same state debt levels are weighing on the economies, the trend is likely to continue for some time.

1.15. Organization of the report

The project is organized with five Chapters and Bibliography section. Chapter I consists the Introduction about the Petrol and Gold Price, Relationship between the Gold and Petrol Price, uses and significance of the commodities. Chapter II describes the content of the schedule and the methodology used for the data analysis. Appropriate statistical techniques which are used to carry out the analysis are also briefly explained in this chapter. In Chapter III, the result of the descriptive measure of the data are presented with interpretation. In Chapter IV the Markov Model and Multivariate Markov Chain for the Petrol and Gold Price are constructed and predictions has been made. The steady state probabilities of the models are also found. Chapter V contains the summary and concluding remarks which are made from the statistical data analysis. Bibliography lists the books, materials referred to carry out this work.

CHAPTER-II

METERIALS AND METHODS

2.1. Introduction to Stochastic Process

Stochastic processes can be distinguished in different types depending upon the state space, index parameter and the dependence relations among the random variables through the specification of the joint distribution function. Among such processes Markov chain is a special type of random process. Markov chain was introduced by Andrei Markov (1856 – 1922). One of the important properties of Markov chain model is that the occurrence of any event in the future depends only in the present state. The set of prices taken by the Markov process is known as state space. A Markov process having discrete state space is termed as Markov chain. The fundamental difference between the Markov chain model and other statistical methods of projection like; regression model, time series analysis is that the Markov model does not require any mutual laws among the factors from complex predictor, it only requires the characteristic of evolution on the history of event (i.e. initial probability) to estimate the transition probability for different possible states at various time to come. By using Markov chain model it is easier to predict the possibility of state value in a certain period of time after knowing the initial probability distribution and transition probability matrix (TPM). Markov chain model has been extensively applied in predicting petrol and gold price.

2.2 Definition of Markov Chain

The sequence $\{X_n, n \geq 0\}$ is said to be a Markov chain if for all state prices $i_0, i_1, i_2, \dots, i_n \epsilon I$, then,

$$P\{\, X_{n+1} = j | X_0 = i_0, X_1 = i_1, \dots, X_n = i\} = P\{X_{n+1} = j | X_n = i\} \quad \dots (2.1)$$

where, $i_0, i_1, i_2, \dots, i_n$ are the states in the state space *I*. This type of probability is called Markov chain probability. This indicate that regardless of its history prior to time n, the probability that it will make a transition to another state *j* depends only on state *i*. Here it should be noted that whether the particle was in that state for only a short period or a long period of time does not matter.

2.3. Transition Probability and Transition Probability Matrix

The transition probability as defined by the Markov chain is called transition or jump probability from state i to state j. Then,

$$P\{ X_{n+1} = j|X_n = i\} = p_{ij} \quad \text{……… (2.2)}$$

This is also termed as one-step transition probability. If the transition probabilities defined above are independent of time (n), then such assumption is called homogenous or stationary Markov chain. Thus,

$$P\{X_{n+1} = j|X_n = i\} = P\{X_1 = j|X_0 = i\} = p_{ij} \quad \text{…….. (2.3)}$$

The transition probabilities p_{ij}'s can be written or arranged in a matrix form as,

$$P = [p_{ij}], i, j\epsilon I \quad \text{…….. (2.4)}$$

Here, the matrix *P* is called transition probability matrix (TPM) or stochastic matrix. The matrix *P* insists non-negative elements with row sum unity. Hence,

$$0 \le p_{ij} \le 1 \; and \; \sum_{j=1}^{n} p_{ij} = 1, i \in I \quad \text{... ... ... (2.5)}$$

The probability

$$p_{ij}\,(k) = P\{X_{n+k} = j|X_n = i\}, k > 0, n \ge 0, i, j \,\epsilon\, I \quad \text{……. (2.6)}$$

is the k-step transition probability from state *i* to state *j* in *k* steps.

The transition matrix *P* has the following property.

$$P(n) = P^{n-1} * P = P^n \quad \text{…….. (2.7)}$$

2.4. State Probability Matrix

The average transition process of Markov chain depends on the system's initial state and the transition probability matrix. The system initial state is a line matrix called initial probability vector defined as,

$$P(X_0 = i) = P(0) = [p_0(0)p_1(0) \dots \dots \dots p_n(0)] \qquad \dots\dots\dots (2.8)$$

$$0 \leq pi(0) \leq 1 \; and \; \sum_{i=0}^{n} p_i(0) = 1 \text{ for all states.}$$

Similarly, the probability vector at time n may be defined as,

$$P(X_n = i) = P(n) = [p_0(n)p_1(n) \dots \dots \dots p_n(n)] \qquad \dots\dots\dots (2.9)$$

$$0 \leq pi(n) \leq 1 \; and \; \sum_{i=0}^{n} p_i(n) = 1 \text{ for all states.}$$

By knowing the initial state of system and transition matrix after n^{th} step,

$$P(k+1) = P(k) * P \qquad \dots (2.10)$$

Which give, $P(1) = P(0) * P$

$$P(2) = P(1) * P = P(0) * P^2$$

$$P(k) = P(k-1) * P = P(k-2) * P^2 = \dots \dots \dots \dots = P(0) * P^k$$

Hence, $P(k+1) = P(0) * P^{k+1}, for \; k \geq 0$ ... (2.11)

This indicate that the transition probability matrix after *(k+1)* steps is the product of initial probability vector and $(k+1)^{\text{th}}$ power of the one-step transition probability matrix.

2.5. Irreducible Markov Chain

A Markov chain is said to be irreducible if it is not possible to partition the state space into two or more disjoint closing set. That means it consists only a single class.

2.6. Absolute Probability

The state probability distribution {(n), j ϵ I} shows the probability of finding the particle at state *j* at the n^{th} trial. If P*(0)* be the probability of finding such particle at state *i* at initial trial then,

$$P(X_n = j) = P_j(n) = \sum_i P(X_n = j, X_0 = i) = \sum_i P(X_n = j | X_0 = i)\ P(X_0 = i)$$

$$= \sum_i P_i\,(0).P_{ij}(n), n > 0 \qquad \ldots\ldots..(2.12)$$

Here, $\{P_i(0),\ i \in I\}$ is the initial probability distribution.

2.7. Stationary Distribution of a Markov Chain

This property of Markov chain states that regardless of the initial state of the system how does the stochastic process evolves, when the number of transition steps is sufficiently large, then the transition probability from state *i* to state *j* becomes settle down to some constant value. Thus,

$$\lim_{n\to\infty} P_{ij}\,(n) = \pi_j \qquad \ldots\ldots\ldots (2.13)$$

Such quantities are referred as steady state probabilities.

If the limits $\pi_j = \lim_{n\to\infty} P_j(n) = \lim_{n\to\infty} P_{ij}(n)$ exists and does not depend on the initial state, then,

$$P_j(n) = \sum_k P_k\,(n-1) P_{kj}\ becomes\ \pi_j = \sum_k \pi_j\, P_{kj}, as\ n \to \infty\ for\ j = 0, 1, 2, \ldots\ldots\ldots..$$

This is equivalent to $\pi = \pi * P$ $\ldots(2.14)$

The probability distribution $\{\pi_j,\ i \in I\}$ is called stationary or invariant for the given chain if, $\pi_i = \sum_{i\in I} \pi_i\, P_{ij} such\ that\ \pi_i \geq 0\ and\ \sum_i \pi_i = 1$

This property of Markov chain helps to determine the long–run behaviour of the chain.

2.8. Expected Return Time

For a finite irreducible Markov chain, the expected return time to state *j, j ϵ I* can obtain by taking the reciprocal of limiting probability $P_{ij}(n)$.

2.9. Multivariate Markov Chain

In multivariate Markov chain model represents the behaviour of multiple categorical sequences generated by similar sources or the same source. Here it is assumed that there are "s" categorical sequences and each has "m" possible states in M. Let $x_n^{(k)}$ be the state vector of the k^th sequence at time n. If the k^th sequence is in state j at time n then

$$x_n^{(k)} = ej = (0, \dots 0,1,0, \dots ,0)^T \qquad \dots\dots\dots (2.15)$$

It is assumed that there is the following relationship:

$$x_{n+1}^{(j)} = \sum_{k=1}^{s} \lambda_{jk} P^{(jk)} x_n^{(k)}, for\ j = 1, 2..., s \qquad \dots\dots\dots (2.16)$$

Where $\lambda_{jk} \geq 0, 1 \leq j, k \leq s\ and \sum_{k=1}^{s} \lambda_{jk}, for\ j = 1, 2 \dots s.$

The state probability distribution of the k^th sequence at the (n+1)^th step depends on the weighted average of $P^{(jk)} x_n^{(k)}$. Here $P^{(jk)}$ is a transition probability matrix from the states in the k^th sequence to the states in the j^th sequence, and $x_n^{(k)}$ is the state probability distribution of the k^th sequences at the n^th step. In matrix form it is written:

$$\boldsymbol{x_{n+1}} = \begin{bmatrix} x_{n+1}^{(1)} \\ x_{n+1}^{(2)} \\ \vdots \\ x_{n+1}^{(s)} \end{bmatrix} = \begin{bmatrix} \lambda_{11}P^{(11)} & \lambda_{12}P^{(12)} & \cdots & \lambda_{1s}P^{(1s)} \\ \lambda_{21}P^{(21)} & \lambda_{22}P^{(22)} & \cdots & \lambda_{2s}P^{(2s)} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{s1}P^{(s1)} & \lambda_{s2}P^{(s2)} & \cdots & \lambda_{ss}P^{(ss)} \end{bmatrix} \begin{bmatrix} x_n^{(1)} \\ x_n^{(2)} \\ \vdots \\ x_n^{(s)} \end{bmatrix} \equiv Qx_n \qquad \dots (2.17)$$

It is proposed that there are some methods for the estimations of $P^{(jk)}$ and λ_{jk}. For each data sequence, the transition probability matrix is estimated by the following method. Given the data sequence, the transition frequency from the states in the k^th sequence to the states in the j^th sequence is counted. Hence, the transition frequency matrix for the data sequence can be constructed. After the normalization, the estimates of the transition probability matrices for the multivariate Markov chain model have to be estimated. More precisely, the transition frequency $f_{i_j i_k}^{(jk)}$ from the state i_k in the sequence $\{x_n^{(k)}\}$ to the state i_j in the sequence $\{x_n^{(j)}\}$ is counted and therefore the transition frequency matrix for the sequences is constructed as follows:

$$F^{(jk)} = \begin{bmatrix} f_{11}^{(jk)} & f_{21}^{(jk)} & \cdots & f_{m1}^{(jk)} \\ f_{12}^{(jk)} & f_{22}^{(jk)} & \cdots & f_{m2}^{(jk)} \\ \vdots & \vdots & \vdots & \vdots \\ f_{1m}^{(jk)} & f_{2m}^{(jk)} & \cdots & f_{mm}^{(jk)} \end{bmatrix}$$

From $F^{(jk)}$, the estimates for $P^{(jk)}$ are obtained as follows:

$$\hat{P}^{(jk)} = \begin{bmatrix} \hat{P}_{11}^{(jk)} & \hat{P}_{21}^{(jk)} & \cdots & \hat{P}_{m1}^{(jk)} \\ \hat{P}_{12}^{(jk)} & \hat{P}_{22}^{(jk)} & \cdots & \hat{P}_{m2}^{(jk)} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{P}_{1m}^{(jk)} & \hat{P}_{2m}^{(jk)} & \cdots & \hat{P}_{mm}^{(jk)} \end{bmatrix}$$

Where,

$$\hat{P}^{(jk)} = \begin{cases} \frac{f_{i_j i_k}^{(jk)}}{\sum_{i_k=1}^{m} f_{i_j i_k}^{(jk)}} if \ \sum_{i_k=1}^{m} f_{i_j i_k}^{(jk)} \neq 0 \\ 0 \qquad\qquad otherwise \end{cases} \qquad \ldots(2.18)$$

2.10. Correlation Analysis:

The degree of the linear relationship between the two variables is measured by the correlation coefficient that is shown in (Eqn. 2.19) and denoted by "R". The correlation coefficient is the measure for the accord between the changes in the two variables and it varies from 1 to +1. If the two variables deviate in the same direction the correlation is said to be direct or positive. If they always deviate in the opposite direction the correlation is said to be inverse or negative. If the change in one variable corresponds to a proportional change in the other variable then the correlation is said to be perfect. The square of the correlation coefficient is the coefficient of determination.

$$\gamma_{xy} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{n\sigma_x\sigma_y} \qquad \dots (2.19)$$

Where $\bar{x}, \bar{y}$ are the arithmetic means and σ_x , σ_y the standard deviation of the variable x and y respectively.

CHAPTER - III

DATA DESCRIPTION AND DATA ANALYSIS

This chapter describes the describe the data and the results obtained by data analyzing the data using appropriate statistical techniques.

3.1 Data Description :

In this Project, the daily Petrol Price and Gold Price of Chennai Market has been considered. The study data taken from the web source www.chennailive.com. The daily Petrol Price and Gold Price of the period between 16 July 2017 and 09 April 2022.

Table 3.1 Descriptive Measurements of Average Petrol Rate of Chennai Market

Mean	82.67613
Median	78.4
Mode	101.4
Range	44.28
Sample Variance	110.5203
Kurtosis	-0.6530515
Skewness	0.8066373
Standard Deviation	10.51287
Minimum	66.57
Maximum	110.85
Sum	142864.3
Count	1728

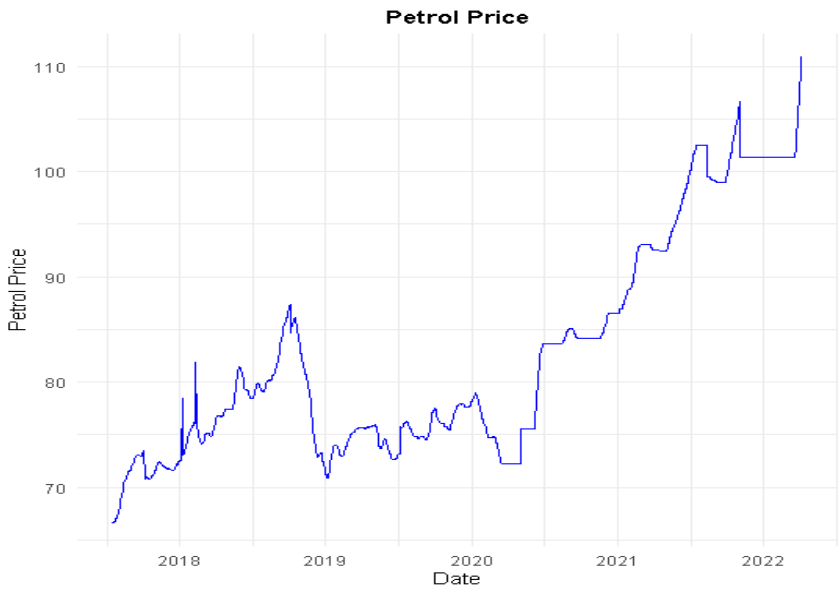

Figure 3.1: Petrol price of the Chennai market

From the table, the value of average petrol rate is calculated as 82.67613 with the standard deviation of 10.51287. The mean, median and mode of average Chennai market petrol rate are 82.67613, 78.4, and 101.4 respectively. The ordering mean > median > mode and the value of co-efficient of skewness (0.80804) indicates the distribution is positively skewed. The maximum and minimum average Chennai market petrol price are respectively 110.85 and 66.57.

Table 3.2 Descriptive Measurements of Average Petrol Rate of Chennai Market

Mean	3999.872
Median	3831
Mode	3197
Range	2906
Sample Variance	752824.3
Kurtosis	-1.677672
Skewness	0.09478182
Standard Deviation	867.6545
Minimum	2781
Maximum	5687
Sum	6911779
Count	1728

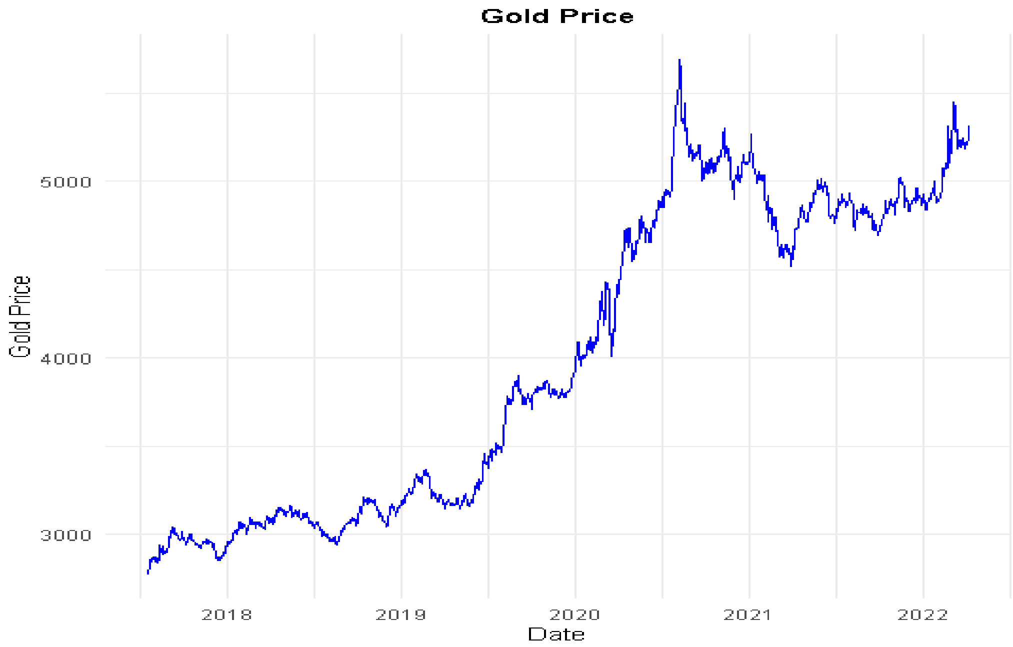

Figure 3.2: Line Chart of the Gold price of the Chennai market

From the table, the average gold rate is calculated as 3999.872 with the standard deviation of 867.6545. The mean, median and mode of average Chennai market gold rate are 3999.872, 3831, and 3197 respectively. The ordering mean > median > mode and the value of co-efficient of skewness (0.094947) indicates the distribution is positively skewed. The maximum and minimum average Chennai market petrol price is respectively 5687 and 2781.

3.2 Correlation Analysis

From the below table 3.3, it indicates that the value of the Karl Pearson Correlation Coefficient is 0.7219. the result signifies that the two variables such as petrol and gold price in Chennai market are moderately correlated with each other.

Table 3.3: Relationship between petrol and gold price in Chennai market

	Petrol price	**Gold price**
Petrol price	1	0.7219
Gold price	0.7219	1

The relationship between Petrol Price and Gold Price can be visualized using Scatterplot. The corresponding Scatterplot is shown below.

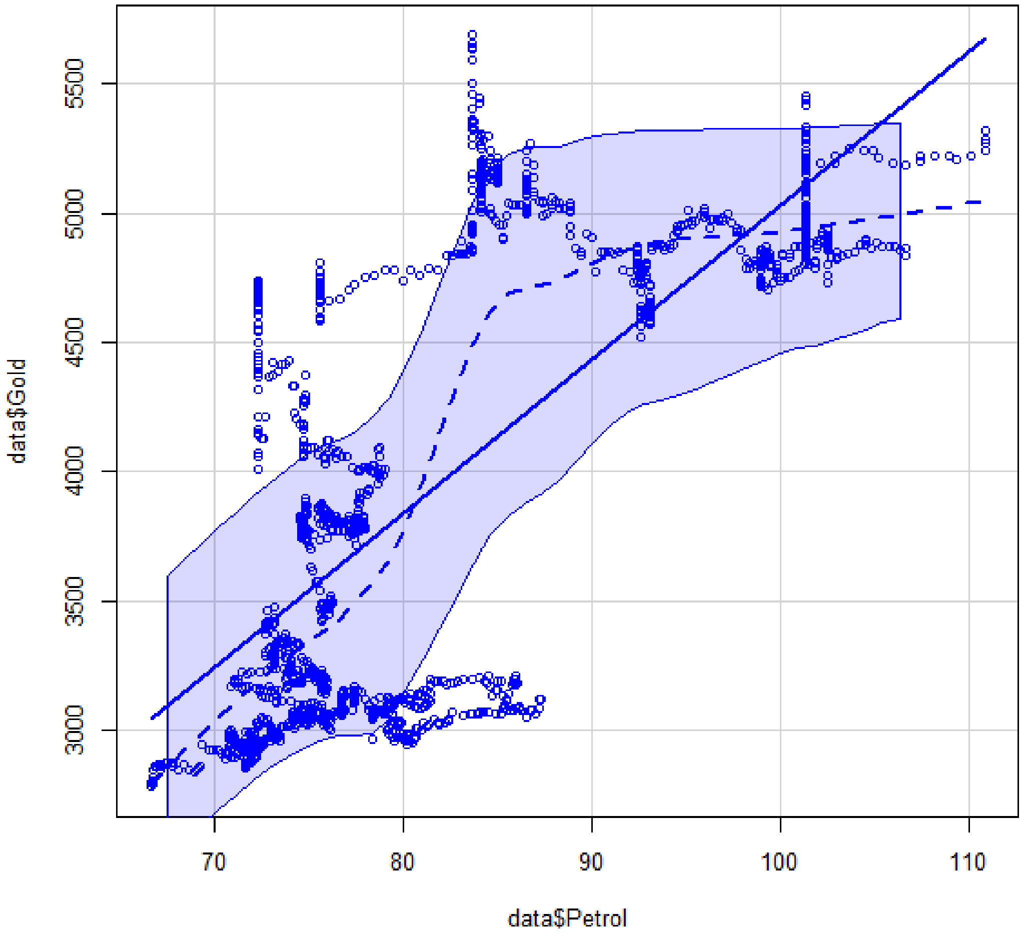

Fig 3.3 Scatter plot for petrol price and Gold price

To detect the Outliers, the box plot has drawn. The box plot for Petrol Price and Gold Price are as follows. From the plot, there is two outliers in Petrol Price and no outliers in Gold Price.

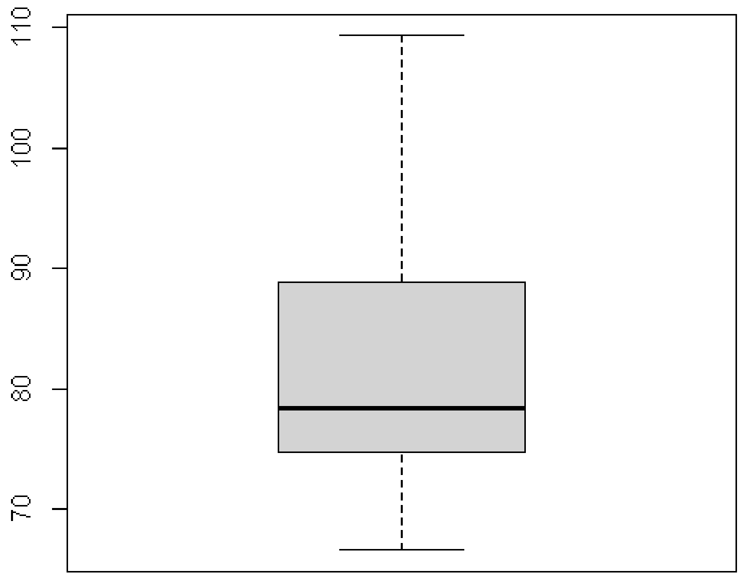

Fig 3.3 Box plot for petrol price

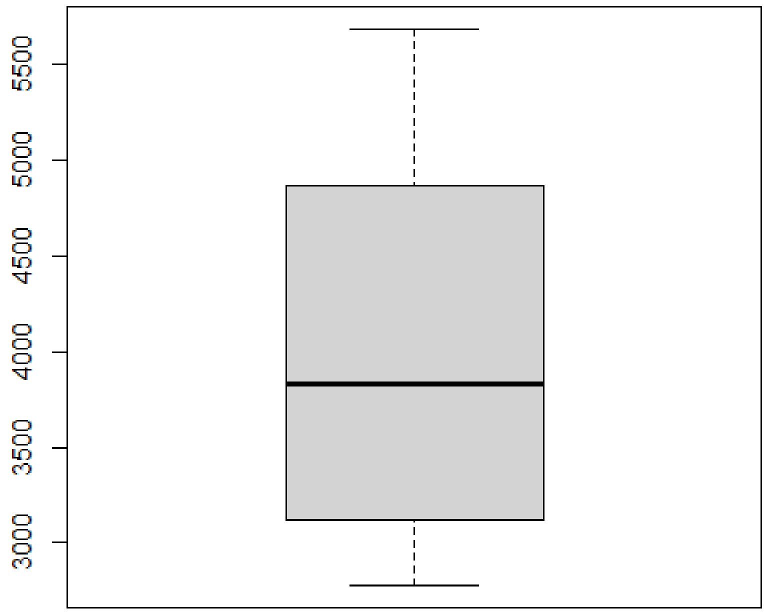

Fig 3.3 Box plot for gold price

CHAPTER - IV

RESULTS AND DISCUSSION

In this chapter deals with Markov Chain multivariate Markov model for petrol and gold price in Chennai Market. The values of the variables have converted into categorical data sequences. The categorical data sequences have to be correlated owing to the structure of the multivariate Markov chain model. In this application, value price is used for the data sequences. The change of values in Chennai market petrol and gold. According to the values of increase and decrease and remaining at the same state the data sequences which has been converted into categorical data sequences and denoted as P_1 and G_1.

P_1 ={I,I,R,I,I,I,I,I,I,I,I,I,I,I,I,D,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,I,R,I,I,I,I,I,R,I,I,I,I,I,I,
R,I,I,I,I,R,I,R,R,D,D,D,I,R,R,I,I,I,I,I,I,D,R,D,I,I,R,I,D,D,D,D,D,D,R,I,I,I,I,I,R,I,I,I,I,I,I,
I,I,I,I,I,I,I,I,R,I,R,D,D,R,D,D,D,D,D,D,D,I,D,D,D,D,D,D,D,R,D,D,R,I,R,I,D,D,D, D,D,
D,D,D,R,I,I,I,I,I,I,I,I,I,D,I,D,I,I,I,I,I,R,R,R,R,I,I,I,D,I,I,I,I, … }

G_1 ={D,I,I,I,I,I,R,I,D,D,I,D,I,R,D,I,D,D,I,D,D,I,I,I,I,D,D,R,D,R,D,I,I,D,R,D,I,D,D,I,R,R,I,I,
D,D,I,I,R,I,D,I,D,I,D,R,D,D,I,D,I,D,R,D,D,I,D,I,D,R,I,I,D,D, I,I,R,D,D,D,D,D,I, R,I,I,
D,I,D,I,R,D,D,D,D,I,I,R,D,I,D,D,D,I,R,D,D,D,D,D,D,R,D,I,I,I,I,R,R,D,D,I,D,I,I,R,D,
D,I,I,D,I,R,D,D,R,D,D,I,R,D,I,D,D,D,I,R,D,D,D,I, … }

P_1 data sequence shows the daily changes in the petrol price of Chennai market, G_1 data sequence shown the daily changes in the gold price in Chennai market.

4.1. Derivation of Three State Transition Probability Matrix

The close observation petrol and gold price over the study period shows that it involves one of the three different states of transition at the end of each day. The possible movements of Price in such states are either increase or decrease or remaining at the same state. For the purpose of development of transition probability matrix, these three different movements are considered as three different states in the Markov chain. The transition probability provides the information regarding the transition behavior of Markov chain. The elements of transition probability matrix indicate the probability of transitions from a particular state to another state. In other word, the transition probability refers to the probability of occurrence of a typical state from one of the existing states. This transition probability helps to make an idea about the likelihood of occurrence of future state and accordingly one can make the decisions.

The petrol price of 1728 days shows that it was decrease in 316 days, increase in 548 days and remaining at the same state on 862 days. The Gold price of 1728 of days shows that it was decrease in 684 days, increase in 743 days and remaining at the same state in 299 days.

Table 4.1: The intra transition frequency matrix of petrol price

	Decrease	**Increase**	**Remaining at the same state**	**Total**
Decrease	212	20	84	316
Increase	25	383	140	548
Remaining at the same state	79	144	639	862
Total	316	547	863	1726

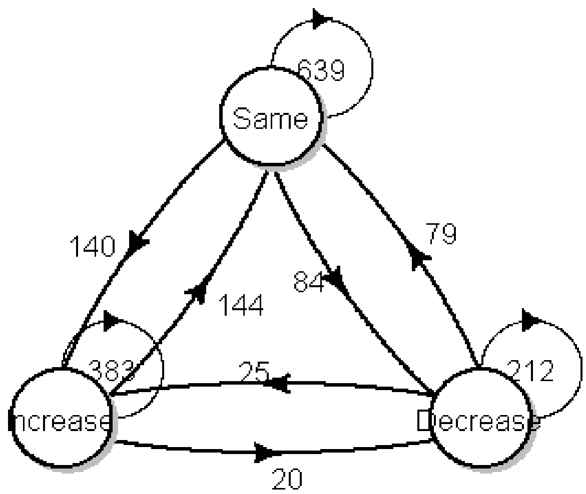

Fig 4.1 The transition digraph of the Petrol Price

Table 4.2: The intra transition frequency matrix of gold price

	Decrease	Increase	Remaining at the same state	Total
Decrease	264	317	103	684
Increase	243	324	176	743
Remaining at the same state	176	103	20	299
Total	683	744	299	1726

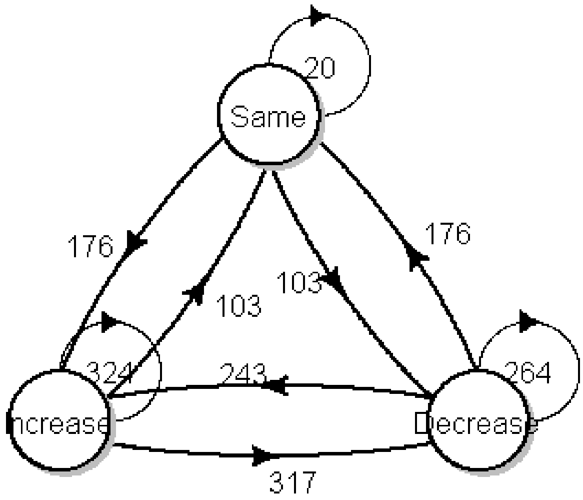

Fig 4.2 The transition digraph of the Gold Price

Table 4.3: The inter transition frequency matrix of petrol to gold price

		Gold			Total
		Decrease	**Increase**	**Remaining at the same state**	
Petrol	**Decrease**	130	125	61	316
	Increase	212	235	101	548
	Remaining at the same state	341	384	137	862
Total		683	744	299	1726

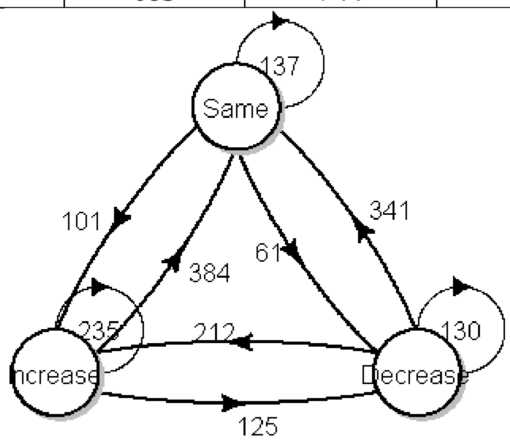

Fig 4.3 The transition digraph of the Petrol Price to Gold Price

Table 4.4: The inter transition frequency matrix of gold to petrol price

		Petrol			**Total**
		Decrease	**Increase**	**Remaining at the same state**	
Gold	**Decrease**	122	221	341	684
	Increase	134	226	383	743
	Remaining at the same state	60	100	139	299
Total		316	547	863	1726

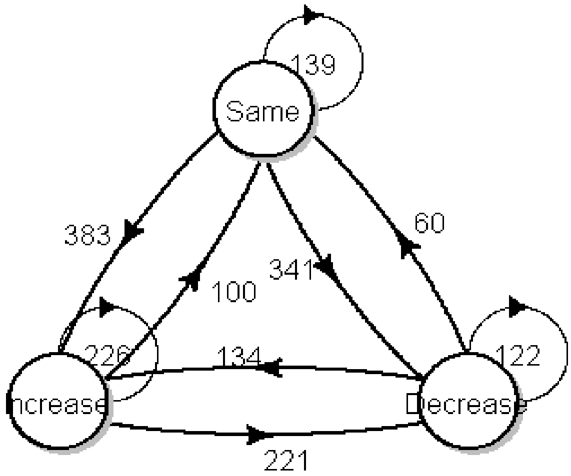

Fig 4.3 The transition digraph of the Gold Price to Petrol Price

The intra transition probability matrix of petrol price can be constructed as

$$P_{petrol} = \begin{bmatrix} 0.6708 & 0.0632 & 0.2658 \\ 0.0456 & 0.6989 & 0.2554 \\ 0.0916 & 0.1670 & 0.7412 \end{bmatrix}$$

The intra transition probability matrix of gold price can be constructed as

$$P_{gold} = \begin{bmatrix} 0.3859 & 0.4634 & 0.1505 \\ 0.3270 & 0.4360 & 0.2368 \\ 0.5886 & 0.3444 & 0.0668 \end{bmatrix}$$

The inter transition probability matrix of petrol to gold price can be constructed as

$$P_{petrol*gold} = \begin{bmatrix} 0.4113 & 0.3955 & 0.1930 \\ 0.3868 & 0.4288 & 0.1843 \\ 0.3955 & 0.4454 & 0.1589 \end{bmatrix}$$

The inter transition probability matrix of gold and petrol price can be constricted as

$$P_{gold*petrol} = \begin{bmatrix} 0.1783 & 0.3230 & 0.4985 \\ 0.1803 & 0.3041 & 0.5154 \\ 0.2006 & 0.3344 & 0.4648 \end{bmatrix}$$

4.2. Short-Term Forecasting Using State Probability in Markov Chain Models

4.2.1 Determination of Initial State Vector

The petrol and gold prices during the study period show three different states: Decrease (D), Increase (I), and Remaining at the same state (R). The probability of occurrence of these three states is captured by the initial state vector, denoted by $\eta(0)$. The initial state vector is defined as:

$$\eta(0) = (\eta_1, \eta_2, \eta_3)$$

Where:

- η_1: Probability of a Decrease (D) in price,
- η_2: Probability of an Increase (I) in price,
- η_3: Probability of Remaining at the same state (R).

4.2.1.1. Determination of initial state vector for Petrol price

The initial state vector of petrol price can be constructed as

$$\eta_{p1} = 316/1726 = 0.1830$$

$$\eta_{p2} = 548/1726 = 0.3174$$

$$\eta_{p3} = 862/1726 = 0.4994$$

Hence the initial state vector for petrol price is,

$$\eta_p(0) = (0.1830 \;\; 0.3174 \;\; 0.5017)$$

4.2.1.2 Determination of initial state vector for Gold price

The initial state vector of gold price can be constructed as

$$\eta_{g1} = 684/1726 = 0.3962$$

$$\eta_{g2} = 743/1726 = 0.4304$$

$$\eta_{g3} = 299/1726 = 0.1732$$

Hence the initial state vector for gold price is,

$$\eta_g(0) = (0.3962 \;\; 0.4304 \;\; 0.1732)$$

4.2.1.3. Determination of initial state vector for Petrol to Gold Price

The initial state vector of Petrol to gold price can be constructed as

$$\eta_{pg1} = 316/1726 = 0.1830$$

$$\eta_{pg2} = 548/1726 = 0.3174$$

$$\eta_{pg3} = 862/1726 = 0.4994$$

Hence the initial state vector for Petrol to gold price is,

$$\eta_{pg}(0) = (0.1830 \quad 0.3174 \quad 0.4994)$$

4.2.1.4. Determination of initial state vector for Gold to Petrol Price

The initial state vector of Petrol to gold price can be constructed as

$$\eta_{pg1} = 684/1726 = 0.3962$$

$$\eta_{pg2} = 743/1726 = 0.4304$$

$$\eta_{pg3} = 299/1726 = 0.1732$$

Hence the initial state vector for Petrol to gold price is,

$$\eta_{pg}(0) = (0.3962 \quad 0.4304 \quad 0.1732)$$

4.2.2 Short-Term Prediction

Short-term state probabilities can be obtained by iteratively multiplying the initial state vector by the transition probability matrix P. The relationship is defined as:

$$\eta(i+1) = \eta(i) * P$$

Where:

- $\eta(i)$ is the state vector at the i^{th} period.
- P is the transition probability matrix.

4.2.2.1. Short Term Prediction for Petrol Price

The state probabilities for the petrol price at the end of the 1729th day are as follows:

$$\eta_p(1) = \eta_p(0) * P_{petrol} = (0.1830 \;\; 0.3174 \;\; 0.5017) \begin{bmatrix} 0.6708 & 0.0632 & 0.2658 \\ 0.0456 & 0.6989 & 0.2554 \\ 0.0916 & 0.1670 & 0.7412 \end{bmatrix}$$

$$= (0.1830 \;\; 0.3169 \;\; 0.5000)$$

This result indicates the following probabilities for the end of the 1729th day:

- There is a **18.30%** chance that the petrol price will decrease.
- There is a **31.69%** chance that the petrol price will increase.
- There is a **50%** chance that the petrol price will remain the same.

Similarly, the state probabilities for the petrol price at the end of the 1730th day are as follows:

$$\eta_p(2) = \eta_p(1) * P_{petrol} = (0.1830 \;\; 0.3169 \;\; 0.5000) \begin{bmatrix} 0.6708 & 0.0632 & 0.2658 \\ 0.0456 & 0.6989 & 0.2554 \\ 0.0916 & 0.1670 & 0.7412 \end{bmatrix}$$

$$= (0.1831 \;\; 0.3166 \;\; 0.5002)$$

For the end of the 1730th day, the probabilities are as follows:

- There is a **18.31%** chance that the petrol price will decrease.
- There is a **31.16%** chance that the petrol price will increase.
- There is a **50.02%** chance that the petrol price will remain the same.

Finally, the state probabilities for the petrol price at the end of the 1731st day are as follows:

$$\eta_p(3) = \eta_p(2) * P_{petrol} = (0.1831 \;\; 0.3166 \;\; 0.5002) \begin{bmatrix} 0.6708 & 0.0632 & 0.2658 \\ 0.0456 & 0.6989 & 0.2554 \\ 0.0916 & 0.1670 & 0.7412 \end{bmatrix}$$

$$= (0.1831 \;\; 0.3164 \;\; 05004)$$

At the end of the 1731st day, the probabilities remain:

- There is a **18.31%** chance that the petrol price will decrease.
- There is a **31.64%** chance that the petrol price will increase.
- There is a **50.04%** chance that the petrol price will remain the same.

The subsequent probabilities follow the same pattern, indicating that the behaviour of the petrol price stabilizes over time.

4.2.2.2. Short Term Prediction for Gold Price

The state probabilities for the gold price at the end of the 1729th day are as follows:

$$\eta_g(1) = \eta_g(0) * P_{gold} = (0.3962 \quad 0.4304 \quad 0.1732) \begin{bmatrix} 0.3859 & 0.4634 & 0.1505 \\ 0.3270 & 0.4360 & 0.2368 \\ 0.5886 & 0.3444 & 0.0668 \end{bmatrix}$$

$$= (0.3957 \quad 0.4310 \quad 0.1732)$$

This result indicates the following probabilities for the end of the 1729th day:

- There is a **39.57%** chance that the gold price will decrease.
- There is a **43.10%** chance that the gold price will increase.
- There is a **17.32%** chance that the gold price will remain the same.

Similarly, the state probabilities for the gold price at the end of the 1730th day are as follows:

$$\eta_g(2) = \eta_g(1) * P_{gold} = (0.3957 \quad 0.4310 \quad 0.1732) \begin{bmatrix} 0.3859 & 0.4634 & 0.1505 \\ 0.3270 & 0.4360 & 0.2368 \\ 0.5886 & 0.3444 & 0.0668 \end{bmatrix}$$

$$= (0.3956 \quad 0.4310 \quad 0.1732)$$

For the end of the 1730th day, the probabilities are as follows:

- There is a **39.56%** chance that the gold price will decrease.
- There is a **43.10%** chance that the gold price will increase.
- There is a **17.32%** chance that the gold price will remain the same.

Finally, the state probabilities for the gold price at the end of the 1731st day are as follows:

$$\eta_g(3) = \eta_g(2) * P_{gold} = (0.3956 \quad 0.4310 \quad 0.1732) \begin{bmatrix} 0.3859 & 0.4634 & 0.1505 \\ 0.3270 & 0.4360 & 0.2368 \\ 0.5886 & 0.3444 & 0.0668 \end{bmatrix}$$

$$= (0.3956 \quad 0.4310 \quad 0.1732)$$

At the end of the 1731st day, the probabilities remain:

- There is a **39.56%** chance that the gold price will decrease.
- There is a **43.10%** chance that the gold price will increase.
- There is a **17.32%** chance that the gold price will remain the same.

The subsequent probabilities follow the same pattern, indicating that the behaviour of the gold price stabilizes over time.

4.2.2.3. Short Term Prediction for Petrol to Gold Price

The state probabilities for the petrol-to-gold price relationship at the end of the 1729th day are as follows:

$$\eta_{pg}(1) = \eta_{pg}(0) * P_{Petrol*gold} = (0.1830 \quad 0.3174 \quad 0.4994)\begin{bmatrix} 0.4113 & 0.3955 & 0.1930 \\ 0.3868 & 0.4288 & 0.1843 \\ 0.3955 & 0.4454 & 0.1589 \end{bmatrix}$$

$$= (0.3957 \quad 0.4310 \quad 0.1732)$$

This result indicates the following probabilities for the end of the 1729th day:

- There is a **39.57%** chance that the petrol price will influence the gold price to decrease.
- There is a **43.10%** chance that the petrol price will influence the gold price to increase.
- There is a **17.32%** chance that the petrol price will influence the gold price to remain the same.

Similarly, the state probabilities for the gold price at the end of the 1730th day are as follows:

$$\eta_{pg}(2) = \eta_{pg}(1) * P_{Petrol*gold} = (0.3957 \quad 0.4310 \quad 0.1732)\begin{bmatrix} 0.4113 & 0.3955 & 0.1930 \\ 0.3868 & 0.4288 & 0.1843 \\ 0.3955 & 0.4454 & 0.1589 \end{bmatrix}$$

$$= (0.3980 \quad 0.4185 \quad 0.1833)$$

For the end of the 1730th day, the probabilities are as follows:

- There is a **39.80%** chance that the petrol price will influence the gold price to decrease.
- There is a **41.85%** chance that the petrol price will influence the gold price to increase.

- There is an **18.33%** chance that the petrol price will influence the gold price to remain the same.

Finally, the state probabilities for the gold price at the end of the 1731st day are as follows:

$$\eta_{pg}(3) = \eta_{pg}(2) * P_{Petrol*gold} = (0.3980 \quad 0.4185 \quad 0.1833)\begin{bmatrix} 0.4113 & 0.3955 & 0.1930 \\ 0.3868 & 0.4288 & 0.1843 \\ 0.3955 & 0.4454 & 0.1589 \end{bmatrix}$$

$$= (0.3982 \quad 0.4186 \quad 0.1831)$$

At the end of the 1731st day, the probabilities are:

- There is a **39.82%** chance that the petrol price will influence the gold price to decrease.
- There is a **41.86%** chance that the petrol price will influence the gold price to increase.
- There is an **18.31%** chance that the petrol price will influence the gold price to remain the same.

The subsequent probabilities continue in the same manner, indicating that the behaviour of the petrol-to-gold price relationship stabilizes over time.

4.2.2.4. Short Term Prediction for Gold to Petrol Price

The state probabilities for the gold-to-petrol price relationship at the end of the 1729th day are as follows:

$$\eta_{gp}(1) = \eta_{gp}(0) * P_{gold*petrol} = (0.3962 \quad 0.4304 \quad 0.1732)\begin{bmatrix} 0.1783 & 0.3230 & 0.4985 \\ 0.1803 & 0.3041 & 0.5154 \\ 0.2006 & 0.3344 & 0.4648 \end{bmatrix}$$

$$= (\, 0.1830 \quad 0.3169 \quad 0.5000)$$

This result indicates the following probabilities for the end of the 1729th day:

- There is a **18.30%** chance that the gold price will influence the petrol price to decrease.
- There is a **31.69%** chance that the gold price will influence the petrol price to increase.

- There is a **50%** chance that the gold price will influence the petrol price to remain the same.

Similarly, the state probabilities for the petrol price at the end of the 1730th day are as follows:

$$\eta_{gp}(2) = \eta_{gp}(1) * P_{gold*petrol} = (\ 0.1830 \quad 0.3169 \quad 0.5000) \begin{bmatrix} 0.1783 & 0.3230 & 0.4985 \\ 0.1803 & 0.3041 & 0.5154 \\ 0.2006 & 0.3344 & 0.4648 \end{bmatrix}$$

$$= (0.1901 \quad 0.3227 \quad 0.4870)$$

For the end of the 1730th day, the probabilities are as follows:

- There is a **19.01%** chance that the gold price will influence the petrol price to decrease.
- There is a **32.27%** chance that the gold price will influence the petrol price to increase.
- There is an **48.70%** chance that the gold price will influence the petrol price to remain the same.

Finally, the state probabilities for the gold price at the end of the 1731st day are as follows:

$$\eta_{gp}(3) = \eta_{gp}(2) * P_{gold*petrol} = (0.1901 \quad 0.3227 \quad 0.4870) \begin{bmatrix} 0.1783 & 0.3230 & 0.4985 \\ 0.1803 & 0.3041 & 0.5154 \\ 0.2006 & 0.3344 & 0.4648 \end{bmatrix}$$

$$= (\ 0.1898 \quad 0.3225 \quad 0.4876)$$

At the end of the 1731st day, the probabilities are:

- There is a **18.98%** chance that the petrol price will influence the gold price to decrease.
- There is a **32.48%** chance that the petrol price will influence the gold price to increase.
- There is an **48.76%** chance that the petrol price will influence the gold price to remain the same.

The subsequent probabilities continue in the same manner, indicating that the behaviour of the gold-to-petrol price relationship stabilizes over time.

4.3. Long-Term Forecasting Using Steady-State Behavior in Markov Chain Models

4.3.1. Decision Making Under Long-Term Behaviour of Petrol Price and Gold Price

Long-term predictions are crucial for investors to identify future market states. These predictions guide investment decisions, as the market's steady-state behaviour offers insights into whether prices are likely to increase, decrease, or remain stable.

The long-term behaviour is determined by the n^{th} step transition probability matrix P^n. As increases, P^nconverges to a steady-state matrix, which provides the steady-state probabilities for each state.

4.3.1.1. Decision Making Under Long Run Behavior of Petrol Price

The long run behavior of Petrol Price is observed by determining the higher order transition probability matrix of Petrol Price by using R software as given below:

$$P_{petrol}^{2} = \begin{bmatrix} 0.4773 & 0.1311 & 0.3915 \\ 0.0859 & 0.5340 & 0.3800 \\ 0.1370 & 0.2463 & 0.6165 \end{bmatrix}$$

$$P_{petrol}^{3} = \begin{bmatrix} 0.3621 & 0.1872 & 0.4506 \\ 0.1168 & 0.4421 & 0.4410 \\ 0.1596 & 0.2838 & 0.5564 \end{bmatrix}$$

$$P_{petrol}^{4} = \begin{bmatrix} 0.2927 & 0.2290 & 0.4781 \\ 0.1389 & 0.3900 & 0.4709 \\ 0.1710 & 0.3014 & 0.5274 \end{bmatrix}$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$P_{petrol}^{38} = \begin{bmatrix} 0.1832 & 0.3161 & 0.5005 \\ 0.1832 & 0.3162 & 0.5005 \\ 0.1832 & 0.3162 & 0.5005 \end{bmatrix}$$

$$P_{petrol}^{39} = \begin{bmatrix} 0.1832 & 0.3162 & 0.5005 \\ 0.1832 & 0.3162 & 0.5005 \\ 0.1832 & 0.3162 & 0.5005 \end{bmatrix}$$

$$P_{petrol}^{40} = \begin{bmatrix} 0.1832 & 0.3162 & 0.5005 \\ 0.1832 & 0.3162 & 0.5005 \\ 0.1832 & 0.3162 & 0.5005 \end{bmatrix} = P_{petrol}^{41} = P_{petrol}^{43} = \ldots\ldots \text{ and so on.}$$

The higher order transition probability matrix of Petrol Price computed above shows that after the 39 days for 1728 days, the transition probability matrix tends to the steady state or the state of equilibrium. After then the transition probability matrix remains unchanged for the onward consecutive days. This steady state transition probability matrix of Petrol Price reveals the following information.

This steady-state transition probability matrix of Petrol Price reveals the following insights:

- **18.32%**: The probability that the Petrol Price will **decrease** in the near future, regardless of its initial state (whether it was decreasing, increasing, or remaining the same).
- **31.62%**: The probability that the Petrol Price will **increase** in the near future, regardless of its initial state.
- **50.05%**: The probability that the Petrol Price will **remain the same** in the near future, regardless of its initial state.

These probabilities suggest that over the long run, the Petrol Price is most likely to remain the same, with some chance of increasing and a smaller chance of decreasing.

4.3.1.2. Decision Making Under Long Run Behavior of Gold Price

The long run behavior of Gold Price is observed by determining the higher order transition probability matrix of Gold Price by using R software as given below:

$$P_{gold}^{2} = \begin{bmatrix} 0.3891 & 0.4328 & 0.1779 \\ 0.4082 & 0.4233 & 0.1683 \\ 0.3792 & 0.4460 & 0.1747 \end{bmatrix}$$

$$P_{gold}^{3} = \begin{bmatrix} 0.3965 & 0.4304 & 0.1730 \\ 0.3951 & 0.4318 & 0.1730 \\ 0.3950 & 0.4304 & 0.1744 \end{bmatrix}$$

$\vdots$

$\vdots$

$\vdots$

$$P_{gold}^{8} = \begin{bmatrix} 0.3955 & 0.4310 & 0.1732 \\ 0.3956 & 0.4310 & 0.1732 \\ 0.3955 & 0.4310 & 0.1732 \end{bmatrix}$$

$$P_{gold}^{9} = \begin{bmatrix} 0.3956 & 0.4310 & 0.1732 \\ 0.3956 & 0.4310 & 0.1732 \\ 0.3956 & 0.4310 & 0.1732 \end{bmatrix}$$

$$P_{gold}^{10} = \begin{bmatrix} 0.3956 & 0.4310 & 0.1732 \\ 0.3956 & 0.4310 & 0.1732 \\ 0.3956 & 0.4310 & 0.1732 \end{bmatrix} = P_{gold}^{11} = P_{gold}^{12} = \ldots\ldots \text{ and so on.}$$

The higher order transition probability matrix of Gold Price computed above shows that after the 9th days for 1728 days, the transition probability matrix tends to the steady state or the state of equilibrium. After then the transition probability matrix remains unchanged for the onward consecutive days. This steady state transition probability matrix of Gold Price reveals the following information.

This steady-state transition probability matrix of Gold Price reveals the following insights:

- **39.56%**: The probability that the Gold Price will **decrease** in the near future, regardless of its initial state (whether it was decreasing, increasing, or remaining the same).
- **43.10%**: The probability that the Gold Price will **increase** in the near future, regardless of its initial state.
- **17.32%**: The probability that the Gold Price will **remain the same** in the near future, regardless of its initial state.

These probabilities suggest that over the long run, the Gold Price is more likely to increase rather than decrease or remain the same.

4.3.1.3. Decision Making Under Long Run Behavior of Petrol to Gold price

The long run behavior of Petrol to Gold Price is observed by determining the higher order transition probability matrix of Petrol to Gold Price by using R software as given below:

$$P^2{}_{petrol*gold} = \begin{bmatrix} 0.3986 & 0.4183 & 0.1830 \\ 0.3979 & 0.4190 & 0.1830 \\ 0.3979 & 0.4183 & 0.1837 \end{bmatrix}$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$P^5{}_{petrol*gold} = \begin{bmatrix} 0.3982 & 0.4186 & 0.1831 \\ 0.3981 & 0.4186 & 0.1831 \\ 0.3982 & 0.4186 & 0.1831 \end{bmatrix}$$

$$P^6{}_{petrol*gold} = \begin{bmatrix} 0.3982 & 0.4186 & 0.1831 \\ 0.3982 & 0.4186 & 0.1831 \\ 0.3982 & 0.4186 & 0.1831 \end{bmatrix}$$

$P^7{}_{petrol*gold} = \begin{bmatrix} 0.3982 & 0.4186 & 0.1831 \\ 0.3982 & 0.4186 & 0.1831 \\ 0.3982 & 0.4186 & 0.1831 \end{bmatrix} = P^8{}_{petrol*gold} = P^9{}_{petrol*gold} = \dots$ and so on.

The higher order transition probability matrix of Petrol Price Gold Price computed above shows that after the 6th days for 1728 days, the transition probability matrix tends to the steady state or the state of equilibrium. After then the transition probability matrix remains unchanged for the onward consecutive days. This steady state transition probability matrix of Petrol to Gold Price reveals the following information.

This steady-state transition probability matrix of Petrol to Gold price reveals the following insights:

- **39.82%**: The probability that the Petrol to Gold price will **decrease** in the near future, regardless of its initial state (whether it was decreasing, increasing, or remaining the same).
- **41.86%**: The probability that the Petrol to Gold price will **increase** in the near future, regardless of its initial state.
- **18.31%**: The probability that the Petrol to Gold price will **remain the same** in the near future, regardless of its initial state.

These probabilities suggest that, over the long run, there is a slightly higher chance that the Petrol to Gold price will increase rather than decrease or remain the same.

4.3.1.4. Decision Making Under Long Run Behavior of Gold to Petrol price:

The long run behavior of Gold to Petrol Price is observed by determining the higher order transition probability matrix of Gold Price to Petrol Price by using R software as given below:

$$P^2{}_{gold*petrol} = \begin{bmatrix} 0.1901 & 0.3226 & 0.4872 \\ 0.1904 & 0.3231 & 0.4863 \\ 0.1893 & 0.3220 & 0.4885 \end{bmatrix}$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$P^5{}_{gold*petrol} = \begin{bmatrix} 0.1898 & 0.3224 & 0.4875 \\ 0.1898 & 0.3224 & 0.4875 \\ 0.1898 & 0.3225 & 0.4874 \end{bmatrix}$$

$$P^6{}_{gold*petrol} = \begin{bmatrix} 0.1898 & 0.3225 & 0.4875 \\ 0.1898 & 0.3225 & 0.4875 \\ 0.1898 & 0.3225 & 0.4875 \end{bmatrix}$$

$P^7{}_{gold*petrol} = \begin{bmatrix} 0.1898 & 0.3225 & 0.4875 \\ 0.1898 & 0.3225 & 0.4875 \\ 0.1898 & 0.3225 & 0.4875 \end{bmatrix} = P^8{}_{gold*petrol} = P^9{}_{gold*petrol} = \dots$ and so on.

The higher order transition probability matrix of Gold to Petrol Price computed above shows that after the 6th days for 1728 days, the transition probability matrix tends to the steady state or the state of equilibrium. After then the transition probability matrix remains unchanged for the onward consecutive days. This steady state transition probability matrix of Gold to Petrol Price reveals the following information.

This steady-state transition probability matrix of Gold to Petrol price reveals the following insights:

- **18.98%**: The probability that the Gold to Petrol price will **decrease** in the near future, regardless of its initial state (whether it was decreasing, increasing, or remaining the same).

- **32.25%**: The probability that the Gold to Petrol price will **increase** in the near future, regardless of its initial state.
- **48.75%**: The probability that the Gold to Petrol price will **remain the same** in the near future, regardless of its initial state.

These probabilities suggest that, over the long run, there is a higher likelihood that the Gold to Petrol price will remain the same rather than increase or decrease.

4.3.2. Long Term Prediction

In Markov chain analysis, long-term predictions are possible under steady-state conditions, where the state vector stabilizes over time. The formula for long-term prediction is given by:

$$\pi = \eta(0) * P^n$$

Where:

- $\eta(0)$ represents the initial state vector.
- P^n represents the transition matrix raised to the power n, where n is sufficiently large to reach steady-state.

4.3.2.1. Long Term Prediction for Petrol price

If the Petrol Price starts in a given state with the initial state vector $\eta_p(0) =$ $(0.1830\ \ 0.3174\ \ 0.5017)$, then the long-term probabilities of the Petrol Price decreasing, increasing, or remaining the same can be determined by multiplying the initial state vector by the higher-order transition probability matrix obtained at the state of equilibrium. This calculation is shown as follows:

$$\eta_p(0) * P_{petrol}^{39} = (0.1830 \ \ 0.3174 \ \ 0.5017) \begin{bmatrix} 0.1832 & 0.3162 & 0.5005 \\ 0.1832 & 0.3162 & 0.5005 \\ 0.1832 & 0.3162 & 0.5005 \end{bmatrix}$$

$$= (0.1832 \ \ 0.3162 \ \ 0.5005)$$

The result indicates the long-run probabilities for Petrol Price behaviour at equilibrium:

- **18.32%**: The probability that the Petrol Price will **decrease** in the long run.
- **31.62%**: The probability that the Petrol Price will **increase** in the long run.
- **50.05%**: The probability that the Petrol Price will **remain the same** in the long run.

Thus, the invariant (steady-state) distribution of the Markov chain is π=(0.1832 0.3162 0.5005) Since the elements of this matrix are non-negative and sum up to 1, it can be concluded that the Markov chain is **ergodic**. An ergodic Markov chain is one in which every state is reachable from every other state, and the chain will converge to a unique steady-state distribution regardless of the initial state.

4.3.2.2. Long Term Prediction for Gold price

If the Gold Price starts in a given state with the initial state vector $\eta_p(0) = (0.3962 \ \ 0.4304 \ \ 0.1732)$, then the long-term probabilities of the Gold Price decreasing, increasing, or remaining the same can be determined by multiplying the initial state vector by the higher-order transition probability matrix obtained at the state of equilibrium. This calculation is shown as follows:

$$\eta_g(0) * P_{gold}^{9} = (0.3962 \ \ 0.4304 \ \ 0.1732) \begin{bmatrix} 0.3956 & 0.4310 & 0.1732 \\ 0.3956 & 0.4310 & 0.1732 \\ 0.3956 & 0.4310 & 0.1732 \end{bmatrix}$$

$$= (0.3956 \ \ 0.4310 \ \ 0.1732)$$

The result indicates the long-run probabilities for Gold Price behaviour at equilibrium:

- **39.56%**: The probability that the Gold Price will **decrease** in the long run.
- **43.10%**: The probability that the Gold Price will **increase** in the long run.
- **17.32%**: The probability that the Gold Price will **remain the same** in the long run.

Thus, the invariant (steady-state) distribution of the Markov chain is π=$(0.3956 \quad 0.4310 \quad 0.1732)$ Since the elements of this matrix are non-negative and sum up to 1, it can be concluded that the Markov chain is **ergodic**. An ergodic Markov chain is one in which every state is reachable from every other state, and the chain will converge to a unique steady-state distribution regardless of the initial state.

4.3.2.3. Long Term Prediction for Petrol to Gold price

If the Petrol to Gold Price starts in a given state with the initial state vector $\eta_p(0)$ = $(0.1830 \quad 0.3174 \quad 0.4994)$, then the long-term probabilities of the Petrol to Gold Price decreasing, increasing, or remaining the same can be determined by multiplying the initial state vector by the higher-order transition probability matrix obtained at the state of equilibrium. This calculation is shown as follows:

$$\eta_{pg}(0) * P^{6}_{petrol*gold} = (0.1830 \quad 0.3174 \quad 0.4994)\begin{bmatrix} 0.3982 & 0.4186 & 0.1831 \\ 0.3982 & 0.4186 & 0.1831 \\ 0.3982 & 0.4186 & 0.1831 \end{bmatrix}$$

$$= (0.3982 \quad 0.4186 \quad 0.1831)$$

The result indicates the long-run probabilities for Petrol to Gold Price behaviour at equilibrium:

- **39.82%**: The probability that the petrol price will influence the gold price **decrease** in the long run.
- **41.86%**: The probability the petrol price will influence the gold price **increase** in the long run.

- **18.31%**: The probability that the petrol price will influence the gold price **remain the same** in the long run.

Thus, the invariant (steady-state) distribution of the Markov chain is π=$(0.3982 \quad 0.4186 \quad 0.1831)$ Since the elements of this matrix are non-negative and sum up to 1, it can be concluded that the Markov chain is **ergodic**. An ergodic Markov chain is one in which every state is reachable from every other state, and the chain will converge to a unique steady-state distribution regardless of the initial state.

4.3.2.4. Long Term Prediction for Gold to Petrol price

If the Gold to Petrol Price starts in a given state with the initial state vector $\eta_p(0)$ = $(0.1830 \quad 0.3174 \quad 0.4994)$, then the long-term probabilities of the Gold to Petrol Price decreasing, increasing, or remaining the same can be determined by multiplying the initial state vector by the higher-order transition probability matrix obtained at the state of equilibrium. This calculation is shown as follows:

$$\eta_{gp}(0) * P^6_{gold*petrol} = (0.3962 \quad 0.4304 \quad 0.1732)\begin{bmatrix} 0.1898 & 0.3225 & 0.4875 \\ 0.1898 & 0.3225 & 0.4875 \\ 0.1898 & 0.3225 & 0.4875 \end{bmatrix}$$

$$= (0.1898 \quad 0.3225 \quad 0.4875)$$

The result indicates the long-run probabilities for Gold to Petrol Price behaviour at equilibrium:

- **18.98%**: The probability that the gold price will influence the petrol price **decrease** in the long run.
- **32.25%**: The probability the gold price will influence the petrol price **increase** in the long run.
- **48.75%**: The probability that the gold price will influence the petrol price **remain the same** in the long run.

Thus, the invariant (steady-state) distribution of the Markov chain is π=$(0.1898 \quad 0.3225 \quad 0.4875)$ Since the elements of this matrix are non-negative and sum up to 1, it can be concluded that the Markov chain is **ergodic**. An ergodic Markov chain is one in which every state is reachable from every other state, and the chain will converge to a unique steady-state distribution regardless of the initial state.

CHAPTER -V

SUMMARY AND CONCLUSION

In this project work, the Markov chain model and the Multivariate Markov Chain model is constructed to predict the behavior of Chennai market. Petrol and gold price of the from July 16, 2017 to March 9,2022 are considered for the study. The predicted results are expressed in terms of probability of certain state in the future. The model does not provide the forecasting results in an absolute state. The initial state vector and the transition probability matrices are used to estimate the probability being in different states in the upcoming days. The steady state probabilities are obtained from the n^{th} step transition probability matrices.

The result of steady state probability matrix of petrol prices shows that the chance of petrol price will decrease in the near future is **18.31%**. The probability that the Petrol prices will increase in near future is **31.62%** .and the petrol price will be remaining the same state with probability **50.05%**.

The result of steady state probability matrix of gold prices shows that the chance of gold prices will decrease in the near future is **39.56%.** The probability that the gold prices will increase in near future is **43.10%** and the gold prices will be remaining the same state with probability **17.32%**.

The result of steady state probability matrix of petrol to gold prices shows that the chance of the petrol prices is decrease, increase or remaining the same state will gold in the state decrease in the near future is **39.82%**. The probability that the petrol prices is decrease, increase or remaining at the same state will gold in the state increase in near future is **41.86%**. and the petrol price is decrease, increase or remaining the same state with probability **18.31%.**

The result of steady state probability matrix of gold to petrol prices shows that the chance of the petrol prices is decrease, increase or remaining the same state will gold in the state decrease in the near future is **18.98%**. The probability that the petrol prices is decrease, increase or remaining the same state will gold in the state increase in near future is **32.25%**and the gold price is decrease, increase or remaining same state with probability **48.75%**.

Thus, the Markov Chain and Multivariate Markov Chain models are constructed and the behavior of Gold and Petrol Price are predicted. The invariant solutions are also determined. These solutions satisfy the invariance property. So, the Corresponding matrices can be known as Ergodic Markov Chain. Using the model constructed in the study, we can predict the future behavior of Gold and Petrol Prices.

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