Unit II

Permutations & Combinations

$n!$ → Product of first n natural numbers

$0! = 1$

$1! = 1$

$2! = 2 \times 1 = 2$

$3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 5 \times 4! = 5 \times 24 = 120$

$$\frac{6!}{4!} = \frac{6 \times 5 \times 4!}{4!} = 30$$

(F.P.C) Product rule | **Fundamental Principle of Counting**

If one task can be done in m ways & another task can be done in n ways, then both of them can be done together in $m \times n$ ways.

Combinations	Permutations
↓	↓
selection / choosing (order does not matter)	arrangement (order matters)
${}^nC_r$ → no. of ways of selecting r objects out of available n objects	${}^nP_r$ → no. of ways of arranging r objects out of available n objects
$\binom{n}{r}$ or ${}^nC_r$ or $C(n,r)$ $= \frac{n!}{r!(n-r)!}$	${}^nP_r = \frac{n!}{(n-r)!}$

$^nP_r = r! \, {}^nC_r$

Questions based on FPC

Q1. How many 3 digit no's can be formed using the digits 1,2,3,4,5,6 if
a) repetition of digits is allowed
b) repetition of digits is not allowed
c) repetition is not allowed & no should be divisible by 5.

A1. a) $\underset{H}{6} \times \underset{T}{6} \times \underset{O}{6} = 216$

b) $\underset{H}{6} \times \underset{T}{5} \times \underset{O}{4} = 120$

c) $\underset{H}{5} \times \underset{T}{4} \times \underset{O\ (only\ 5)}{1} = 20$

Q2. How many 3 digit no's can be formed using the digits 0-9 if a) repetition of digits is allowed
b) " " " " not "

A2 a) $\underset{H\ (except\ 0)}{9} \times \underset{T}{10} \times \underset{O}{10} = 900$

b) $\underset{H}{9} \times \underset{T}{9} \times \underset{O}{8} = 648$

Note and → × or → +

Q3. How many different no. plates are there that involve 1, 2 or 3 letters followed by 4 digits?

A3 Case I 1 letter

$$26 \times 10 \times 10 \times 10 \times 10 = 26 \times 10^4$$

Case II 2 letters

$$26 \times 26 \times 10^4 = 26^2 \times 10^4$$

Case III 3 letters

$$26 \times 26 \times 26 \times 10^4 = 26^3 \times 10^4$$

Answer → $(26 \times 10^4) + (26^2 \times 10^4) + (26^3 \times 10^4)$

Q4. How many 3 digit even numbers can be formed if no repetition is allowed?

A4 Case I 0 at one's place

$$\underset{H}{9} \times \underset{T}{8} \times \underset{O}{1} = 72$$

Case II 2, 4, 6 or 8 at one's place

$$\underset{H}{8} \times \underset{T}{8} \times \underset{O}{4} = 256$$

Answer → $72 + 256 = 328$

Q. Each user on a computer system has a password which is 6 to 7 characters long where each character is an upper case letter or a digit. Each password must contain atleast one digit. How many possible passwords are there?

Ans There are 26 letters & 10 digits
Total characters = 26 + 10 = 36

No. of strings of 6 characters = 36^6
No. of " " " " = 26^6
with no digits

No. of strings of 6 characters = $36^6 - 26^6$
with atleast one digit

Similarly no. of passwords of 7 characters with atleast one digit = $36^7 - 26^7$

Answer → $(36^6 - 26^6) + (36^7 - 26^7)$

Questions based on Combinations

Q1. How many ways are there to select 11 cricketers from available 14 cricketers?

A1 $^{14}C_{11} = \frac{14!}{11!\,3!} = \frac{14 \times 13 \times 12 \times 11!}{11! \times 3 \times 2 \times 1}$

$= 364$

Q2. How many ways are there to select 5 cards out of 52 cards?

A2 $^{52}C_5 = \frac{52!}{5!\,47!}$

Q3. How many different committees of 3 students can be formed from a group of 5 students?

A3 $^5C_3 = \frac{5!}{3!\,2!} = \frac{5 \times 4^2 \times 2!}{3! \times 2 \times 1} = 10$

Q4. Suppose there are 9 faculty members of Maths & 11 of CS. How many ways are there to select a committee so that it consists of 3 faculty members from Maths & 4 from CS department?

A4 No. of ways of selecting 3 Maths faculty members out of 9 members $= {}^9C_3$

$= \frac{9!}{3!\,6!} = \frac{{}^3\cancel{9} \times 8 \times 7 \times \cancel{6!}}{3 \times 2 \times 1 \times \cancel{6!}}$

$= 84$

No. of ways of selecting 4 CS faculty members out of 11 members $= {}^{11}C_4$

$= \frac{11!}{4!\,7!} = \frac{11 \times 10 \times \cancel{9}^3 \times 8 \times \cancel{7!}}{4 \times 3 \times 2 \times 1 \times \cancel{7!}}$

$= 330$

Total no of ways $= 84 \times 330$

$= 27720$

Q5 There are 6 men & 7 women in a group. How many different committees of 5 members can be formed if the committee should have 2 men & 3 women?

A5 ~~No~~

6M → 2M & 7W → 3W

$${}^6C_2 \times {}^7C_3$$

$$= \frac{6!}{2!\,4!} \times \frac{7!}{3!\,4!}$$

$$= \frac{6 \times 5 \times 4!}{2! \times 4!} \times \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!}$$

$$= \frac{30}{2} \times 35 = 15 \times 35 = 525$$

Q6 If ${}^nC_{12} = {}^nC_8$ then find the value of n.

A6 We know that ${}^nC_r = {}^nC_{n-r}$

$$\Rightarrow {}^nC_8 = {}^nC_{n-8}$$

$${}^nC_{12} = {}^nC_8$$

$$\Rightarrow {}^nC_{12} = {}^nC_{n-8}$$

$$\Rightarrow n - 8 = 12$$

$$n = 12 + 8$$

$$n = 20$$

Questions based on permutations

Q1. How many words can be formed using all letters of the word CHAPTER?

A1 No. of words that can be formed using all letters of the word CHAPTER $= {}^{7}P_{7} = \frac{7!}{0!}$

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

Q2. How many words can be formed using all letters of the word
a) APPLE b) MISSISSIPPI c) MATHEMATICS

A2 a) APPLE 5 letters with 2 P's

No. of words $= \frac{5!}{2!} = \frac{120}{2} = 60$

b) MISSISSIPPI

11 letters with 4S, 4I, 2P

No. of words $= \frac{11!}{4!\,4!\,2!}$

c) MATHEMATICS

11 letters with 2M, 2A, 2T

No. of words $= \frac{11!}{2!\,2!\,2!}$

Q3 In how many ways 6 girls can be seated in a ~~to~~ row having 6 chairs?

A3 No of seating arrangements = ${}^6P_6$

$= 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

Q4. In how many ways letters of the word FATHER be arranged? How many of these words begin with T & end with R?

A4 FATHER has 6 letters.

Total no of words = ${}^6P_6 = 6! = 720$

No of words beginning with T & ending with R (First & last place is fixed)

$= {}^4P_4 = 4! = 4 \times 3 \times 2 \times 1 = 24$

Q5 How many words can be formed using all the letters of the word 'DAUGHTER' such that vowels are always together?

A5 DAUGHTER → 8 letters.

3 vowels (A, U, E) 5 consonants (D, G, H, T, R)

Consider A, U, E as one letter

Now there are 6 letters D, G, H, T, R, (AUE) which can be arranged in

${}^6P_6 = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways

A, U, E can be arranged among themselves in ${}^3P_3 = 3! = 3 \times 2 \times 1 = 6$ ways

Total no of words having vowels together $= 720 \times 6 = 4320$

Q6. A bookshelf has 4 books of Maths, 3 books of Statistics & 2 books of Computer Science ~~Statistics~~. In how many ways they can arranged such that books of same subject are placed together

Ans6 $(M_1 M_2 M_3 M_4) \rightarrow 1$ book

$(S_1 S_2 S_3) \rightarrow 1$ book $(C_1 C_2) \rightarrow 1$ book

Now there are 3 books which can be arranged in ${}^3P_3 = 3! = 6$ ways

Maths books can be arranged in ${}^4P_4 = 4! = 24$ ways

Stats " " " " " ${}^3P_3 = 3! = 6$ ways

CS " " " " " $2! = 2$ ways

Total no of arrangements $= 6 \times 24 \times 6 \times 2$
$= 1728$

Circular permutations → No. of circular arrangements of n objects is given by $(n-1)!$

Example No. of ways of arranging 5 persons around a circular table $= 4! = 24$