11/1/2025

COMPUTER VISION

Module 7:

3D point : $X = (x, y, z, 1)^T$

Two unknown cameras : $P = R_{3\times4}$, $P' = R_{3\times4}$

Image points : $x = (u, v, 1)^T \sim PX$, $x' = (u', v', 1)^T \sim P'X$

For any corresponding pair (x, x') we have epipolar constam

$$x'^T F x = 0$$

We seek Homographies H and H'

$\tilde{x} = Hx = (\tilde{u}, \tilde{v}, 1)^T$, $\tilde{x}' = H'x' = (\tilde{u}', \tilde{v}', 1)^T$

$\tilde{v} = \tilde{v}'$ (same row)

$d = \tilde{u} - \tilde{u}'$

We have focal length 'f' and base line 'B'

$$d = \frac{fB}{z} \Rightarrow z = \frac{fB}{d}$$

In uncaliberated case

$z = \frac{\alpha}{d}$, $\alpha = fB$ (unkown constant)

$\hat{z} = \frac{1}{d}$

$\underline{\underline{z = \alpha \hat{z}}}$

In the rectified pair the first camera can be chossen as

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$$P_0 = [I \mid 0]$$

$$P_0' = [I \mid b]$$

whose

$$b = (B, 0, 0)^{+} \text{ (unkown baseline)}$$

using pin hole model

$$\hat{x} = \frac{\tilde{u}}{d}, \quad \hat{y} = \frac{\tilde{v}}{d}, \quad \hat{z} = \frac{1}{d} \quad (d = \tilde{u} - \tilde{u}')$$

$$\hat{P} = \begin{vmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{vmatrix} = \frac{1}{d}\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ 1 \end{pmatrix}$$

True distance between A and B is L_{AB}^{true}

$$d_A = \tilde{u}_A - \tilde{u}_A', \quad d_B = \tilde{u}_B - \tilde{u}_B'$$

$$\hat{P}_A = \frac{1}{d_A}\begin{pmatrix} \tilde{u}_A \\ \tilde{v}_A \\ 1 \end{pmatrix}, \quad \hat{P}_B = \frac{1}{d_B}\begin{pmatrix} \tilde{u}_B \\ \tilde{v}_B \\ 1 \end{pmatrix}$$

$$\hat{L}_{AB} = \left| \hat{P}_A - \hat{P}_B \right|$$

$$\text{Scale Factor } s' = \frac{L_{AB}^{true}}{\hat{L}_{AB}}$$

→ Compute $\hat{P}_i, \hat{P}_j$

→ Projective distance $\hat{L}_{ij} = |\hat{P}_i - \hat{P}_j|$

$$L_{ij} = s\hat{L}_{ij} = \frac{L_{AB}^{tree}}{\hat{L}_{AB}} \hat{L}_{ij}$$

$$L_{ij} = s \| \hat{P}_i - \hat{P}_j \|$$