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# COMPUTER VISION:

## ASSIGNMENT 5

### 1(a) Derivation of the motion tracking equation:

Let $I(x, y, t)$ denote the image intensity at pixel $(x, y)$ at time $t$.

**Brightness constancy assumption:**

The intensity of a moving point does not change between two very close frames, so

$$I(x, y, t) = I(x+u, y+v, t+\Delta t)$$

where $u$ and $v$ are small motions in the $x$ and $y$ directions over time $\Delta t$.

**Taylor expansion:**

Expand the right-hand side with a first-order Taylor series around $(x, y, t)$:

$$I(x+u, y+v, t+\Delta t) \approx I(x, y, t) + I_x u + I_y v + I_t \Delta t$$

where

$$I_x = \frac{\partial I}{\partial x}, \quad I_y = \frac{\partial I}{\partial y}, \quad I_t = \frac{\partial I}{\partial t}.$$

Substituting this into the brightness constancy equation:

$$I(x, y, t) = I(x, y, t) + I_x u + I_y v + I_t \Delta t.$$

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Substract $I(x,y,t)$ from both sides:

$$0 = I_x u + I_y v + I_t \Delta t$$

For small consecutive frames we can take $\Delta t = 1$, so the optical flow equation is:

$$I_x u + I_y v + I_t = 0$$

This eqn relates the motion vector $(u,v)$ at each pixe to the image gradients.

→ Computing a motion estimate from a small patch.

Now we take a small window around same point in two consecutive frames $t$ and $t+1$.

$$I_x \approx \frac{I(x+1,y,t) - I(x-1,y,t)}{2}, \quad I_y \approx \frac{I(x,y+1,t) - I(x,y-1,t}{2}$$

$$I_t \approx I(x,y,t+1) - I(x,y,t)$$

Suppose for four pixels in the patch we have computed the following gradient values;

| pixels | $I_x$ | $I_y$ | $I_t$ |
|---|---|---|---|
| 1 | 2.0 | 0.0 | -2 |
| 2 | 1.5 | -1.5 | -2 |
| 3 | 1.0 | -1.5 | -1 |
| 4 | 1.5 | -1.5 | -1 |

For each pixels the optical flow equation:

$I_x u + I_y v + I_t = 0$

gives

1: $2.0u + 0.0v - 2 = 0$

2: $1.5u - 1.5v - 2 = 0$

3: $1.0u - 1.5v - 1 = 0$

4: $1.5u - 1.5v - 1 = 0$

Rewriting in the form $I_x u + I_y v = -I_t$;

1: $2.0u + 0.0v = 2$

2: $1.5u \cancel{*} - 1.5v = 2$

3: $1.0u - 1.5v = 1$

4: $1.5u - 1.5v = 1$

Stacking these equations in matrix form,

$$\underbrace{\begin{bmatrix} 2.0 & 0.0 \\ 1.5 & -1.5 \\ 1.0 & -1.5 \\ 1.5 & -1.5 \end{bmatrix}}_{A} \begin{bmatrix} u \\ v \end{bmatrix} \underbrace{\begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}}_{b}$$

In a small window we have more equations than unknowns, so we solve $(u,v)$ in the least-squares sense:

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$$\begin{bmatrix} u \\ v \end{bmatrix} = (A^T A)^{-1} A^T b.$$

⇒ For the numbers above this gives approximate

$u \approx 1 \qquad v \approx 0$

So, from two consecutive frames and the chosen path, the estimated motion is about 1 pixel the right and 0 pixels vertically.