Computational Physics - Chapter 3: Oscillatory Motion and Chaos

Simple harmonic Motion:

Example: Particle of mass m Connected to a rigid Support by a massless String.

- The Parallel forces add to zero
- force perpendicular: $F_g = -mg\sin\theta$
- Based on Newton's 2nd law: $F = Md^2s/dt^2$. displacement: $s = l\theta$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$

Central equation of Simple harmonic motion: $\theta = \theta_0 \sin(\Omega t + \phi)$

$\Omega = \sqrt{g/l}$

$$\frac{d\omega}{dt} = -\frac{g}{l}\theta \qquad \frac{d\theta}{dt} = \omega$$

$\omega_{i+1} =$

$\omega_{i+1} = \omega_i - (g/l)\theta_i \Delta t$

$\theta_{i+1} = \theta_i + \omega_i \Delta t \qquad t_{i+1} = t_i + \Delta t$

The total energy of the pendulum: $E = \underbrace{\frac{1}{2} ml^2\omega^2}_{\text{kinetic}} + \underbrace{mgl(1-\cos\theta)}_{\text{gravitational}}$

for Small θ: $E = \frac{1}{2} ml^2 (\omega^2 + \frac{g}{l}\theta^2)$

Subbing the it1: $E_{i+1} = E_i + \frac{1}{2} mgl(\omega_i^2 + \frac{g}{l}\theta_i^2)(\Delta t)^2$

For Problems that involve oscillatory motion, using the Euler-Cromer method shows a graph where energy is Conserved over each Period of motion.

3.2: Adding dissipation, nonlinearity, And driving Force.

- the frictional force takes the form $-q(d\theta/dt)$

the equation of motion will be: $\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt}$

Over-damped: $\theta(t) = \theta_0 e^{-(q/2 \pm \sqrt{q^2/4 - \Omega^2})t}$

Critically damped: $\theta(t) = (\theta_0 + Ct)e^{-qt/2}$

driving force: example: electric Charge on mass → equation of motion:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\left(\frac{d\theta}{dt}\right) + F_D \sin(\Omega_D t)$$

3.3 Chaos in the Driven nonlinear Pendulum

- no Small-angle approximation - Include friction: $-q(d\theta/dt)$
- Sinusoidal driving Force (F_D): $F_D \sin(\Omega_D t)$

Eq of Motion: $\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta - q\frac{d\theta}{dt} + F_D \sin(\Omega_D t)$ $= \frac{d\omega}{dt}$, $\frac{d\theta}{dt} = \omega$

- we See that the value of driving force never repeats, thus being chaotic behavior

λ = Lyapunov exponent more driving Force = more Chaos. - Shown in phase plot

3.4 Routes to Chaos: Period Doubling

bifurcation diagram: Shows how the transition to chaos Comes about.

F_n = value of driving force for transition to period-2^n

$\delta_n = \frac{F_n - F_{n-1}}{F_{n+1} - F_n}$: describes the Shrinkage of the size of the periodic windows.

3.5: The logistic map: Why the Period Doubles.