Chapter 4 Reading Notes: The Solar System 4.1 – 4.4

4.1 Kepler's laws

$$F_G = \frac{G M_S M_E}{r^2} \qquad F_{G,x} = \frac{d^2x}{dt^2} M_E \qquad F_{G,y} = \frac{d^2y}{dt^2} \cdot M_E$$

for circular motion, we know: $F_G = \frac{M_E v^2}{r} = \frac{G M_S M_E}{r^2}$ $\qquad G M_S = v^2 r = 4\pi^2\ AU^3/yr^2$

a = radius of orbit

4.2: The inverse square law and the stability of planetary orbits

reduced mass $\mu = m_1 m_2 / (m_1 + m_2)$

displacement $\vec{r} = \vec{r}_2 - \vec{r}_1$ $\qquad$ if $m_1 \gg m_2$, $\mu \simeq m_2$

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{L^2} F(r) \qquad \frac{1}{r} = \left(\frac{\mu G M_S M_P}{L^2}\right)[1 - e\cos(\theta + \theta_0)]$$

$$r = \left(\frac{L^2}{\mu G M_S M_P}\right)\frac{1}{1 - e\cos\theta}$$

$r_{min} = a(1-e)$ $\qquad r_{max} = a(1+e)$

$a = L^2 / [\mu G M_S M_P a (1-e^2)]$

$L = \sqrt{\mu G M_S M_P a (1-e^2)}$

$$v_{max} = \sqrt{G M_S}\sqrt{\frac{1+e}{a(1-e)}\left(1 + \frac{M_P}{M_S}\right)}$$

$$v_{min} = \sqrt{G M_S}\sqrt{\frac{1-e}{a(1+e)}\left(1 + \frac{M_P}{M_S}\right)}$$

Kepler's 3rd law $\qquad T^2/a^3 = 4\pi^2/[G(M_S + M_P)] \simeq 4\pi^2/G M_S$

$$F_G = \frac{G M_S M_E}{r^\beta}$$

if $\beta = 2$, we have inverse square law

$\beta = 3$: inverse cube law.

the behavior is very sensitive to the inverse square law.

even $\beta = 2.01$ causes the orbit to look less circular.

4.3: Precession of the perihelion of Mercury.

Mercury's perihelion makes one complete rotation every $\simeq$ 230,000 years.

Force law predicted by general relativity:

$$F_G \simeq \frac{G M_S M_M}{r^2}\left(1 + \frac{\alpha}{r^2}\right) \qquad \alpha = 1.1 \times 10^{-8}\ AU^2$$

M_M = mass of mercury

$$-\frac{G M_S M_M}{r_1} + \frac{1}{2} M_M v_1^2 = -\frac{G M_S M_M}{r_2} + \frac{1}{2} M_M v_2^2$$

$r_1 v_1 = b v_2$

$$v_1 = \sqrt{2 G M_S \left[\frac{b^2}{a^2(1+e)^2 - b^2}\right]\left[\frac{1}{\sqrt{e^2 a^2 + b^2}} - \frac{1}{a + ea}\right]}$$

$$= \sqrt{\frac{G M_S (1-e)}{a(1+e)}} \qquad b = a\sqrt{1-e^2}$$

4.4: Three-body problem and the effect of Jupiter on Earth

$$F_{E,J} = \frac{G M_J M_E}{r_{EJ}^2} \qquad F_{EJ,x} = -\frac{G M_J M_E}{r_{EJ}^2}\cos\theta_{EJ} = -\frac{G M_J M_E (x_e - x_j)}{r_{EJ}^3}$$

$$\frac{dv_{x,e}}{dt} = -\frac{G M_S x_e}{r^3} - \frac{G M_J (x_e - x_j)}{r_{EJ}^3}$$

$G M_J = G M_S (M_J / M_S) = 4\pi^2 (M_J/M_S)$

Jupiter has negligible effect on Earth.

If Jupiter had much more mass, it would have an effect on Earth's orbit.