**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:- Given: x=60 , *µ = 45 +10 = 55 , σ = 8*

works begins 10 min after the car is dropped , the time left to complete work is 50 mins.

Probability that the service manager cannot meet his commitment is P(x>50) = 1 – pr(x<=50)

(where x is the time taken to complete work).

Convert 50 to z score

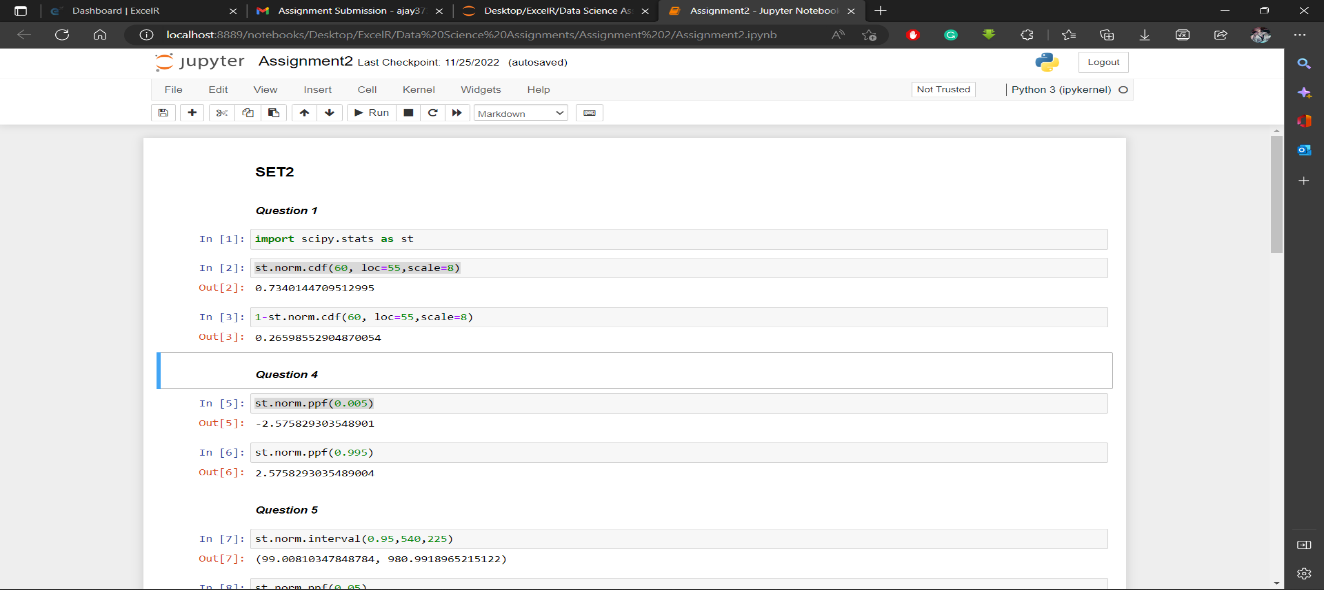
Standard normal variable Z=(X-mean)/ Standard Deviation

(X-45)/8

P(X<=50) = p(Z<=(50-45)/8) = pr(Z<=0.625) = 0.73237 = 73.237%(the number in Z table)

probability that the service manager cannot meet his commitment is : 100-73.237=26.763 i.e. 0.2676 ,

so the answer is B



1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans :- Given: *μ* = 38, *σ* =6, X = 44,38,30

So we calculate Z score by formula = (X-*μ* )/ *σ*

**A:-**  for 44 Z Score = (44-38)/6= 0.8413

For above 44 = 1-0.8413= 0.1587

Employees over 44 are = 0.1587\*400 = 63

Z-score for 38 = (38-38)/6 = 0.5

For between 38 and 44 = 0.8413-0.5=0.3413

Employees = 0.3413\*400=136

Statement A is false from above values

**B:** Now Z score for 30 = (30-38)/6= 0.09176

Employees = 0.09176\*400 = 36

Statement B is True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:-** 2X1 is simply a larger scale version of the random variable X1 if X1 is normally distribution then 2X1 is also normally distributed. X1 and X2 are normal distributed, the associated sums and random sample are exactly (and not just approximately) normal, with the appropriate parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:-** Given: here(pa<X<b)=0.99, mean = 100 & std.dev= 20 To Find: Identify symmetric values for the standard normal distribution such that the area enclosed is .99

From the above details, we have excluded area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

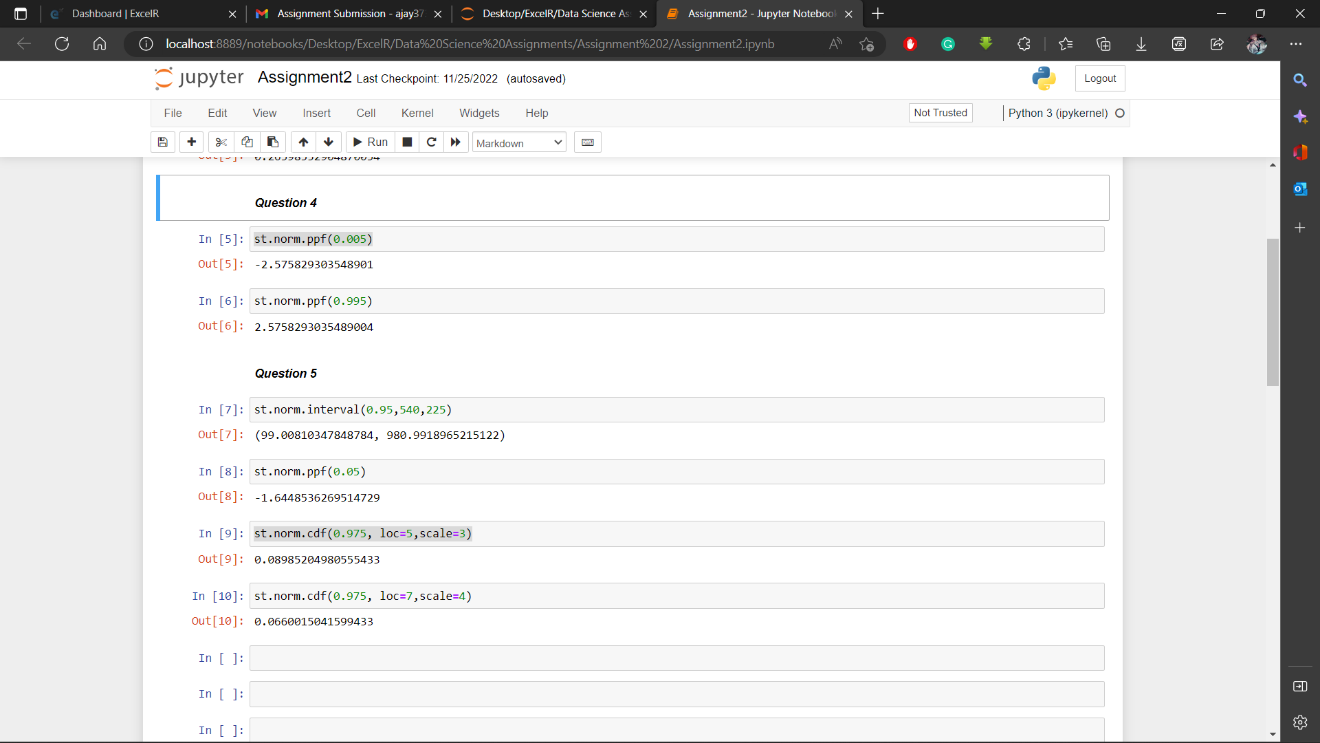
Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

a = -(20\*2.576) + 100= 48.5

b = (20\*2.576)+100= 151.5

Two values symmetric about mean for the given standard normal distribution are (48.5,151.5).



Hence the answer is option D*.*

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:-** from the empirical rule, approximately 95% of the data falls within two standard deviations of the mean.

a) μ ± 2σ = 540±2\*225=> (540-450, 540+450)=>(90,990) b) from the above normal distribution we can say that to find 5th percentile from the left side we can use the formula,

μ - 1.5σ => 540-(1.5\*225) =>202.5 million rupees.

c) this question concerns the original profit distribution. For division1= Z-score for a profit of zero: Z=(X-µ)/σ => (0-5)/3 =>

(or)

For division2= Z score for a profit of zero: Z=(X-µ)/σ =(0-7)/4 => -1.75= .0401

Division 2 has a higher probability of making a loss.

