Knuth-Morris-Pratt Algorithm

The problem of String Matching

Given a string ‘S’, the problem of string matching deals with finding whether a pattern ‘p’ occurs in ‘S’ and if ‘p’ does occur then returning position in ‘S’ where ‘p’ occurs.

The Knuth-Morris-Pratt Algorithm

Knuth, Morris and Pratt proposed a linear time algorithm for the string matching problem.

A matching time of O(n) is achieved by avoiding comparisons with elements of ‘S’ that have previously been involved in comparison with some element of the pattern ‘p’ to be matched. i.e., backtracking on the string ‘S’ never occurs

Components of KMP algorithm

The prefix function, Π

The prefix function,Π for a pattern encapsulates knowledge about how the pattern matches against shifts of itself. This information can be used to avoid useless shifts of the pattern ‘p’. In other words, this enables avoiding backtracking on the string ‘S’.

The KMP Matcher

With string ‘S’, pattern ‘p’ and prefix function ‘Π’ as inputs, finds the occurrence of ‘p’ in ‘S’ and returns the number of shifts of ‘p’ after which occurrence is found.

The prefix function, Π

Following algorithm the prefix function, Π:

Compute-Prefix-Function (p)

```
m = length[p]          //'p' pattern to be matched
Π[1] = 0               // Π[1…m] be a new array
k = 0
    for q = 2 to m
        while k > 0 and p[k+1] != p[q]
            k = Π[k]
        If p[k+1] = p[q]
            k = k +1
        Π[q] = k
    return Π
```

Example: compute Π for the pattern ‘p’ below:

p a b a b a c a

Initially: m = length[p] = 7
Π[1] = 0
k = 0

Step 1: q = 2, k=0
Π[2] = 0

q	1	2	3	4	5	6	7
p	a	b	a	b	a	c	a
Π	0	0					

Step 2: q = 3, k = 0,
Π[3] = 1

q	1	2	3	4	5	6	7
p	a	b	a	b	a	c	a
Π	0	0	1				

Step 3: q = 4, k = 1
Π[4] = 2

q	1	2	3	4	5	6	7
p	a	b	a	b	a	c	A
Π	0	0	1	2			

Step 4: q = 5, k =2
Π[5] = 3

q	1	2	3	4	5	6	7
p	a	b	a	b	a	c	a
Π	0	0	1	2	3		

Step 5: q = 6, k = 3
Π[6] = 1

q	1	2	3	4	5	6	7
p	a	b	a	b	a	c	a
Π	0	0	1	2	3	1	

Step 6: q = 7, k = 1
Π[7] = 1

q	1	2	3	4	5	6	7
p	a	b	a	b	a	c	a
Π	0	0	1	2	3	1	1

After iterating 6 times, the prefix function computation is complete: →

q	1	2	3	4	5	6	7
p	a	b	A	b	a	c	a
Π	0	0	1	2	3	1	1

The KMP Matcher

The KMP Matcher, with pattern ‘p’, string ‘S’ and prefix function ‘Π’ as input, finds a match of p in S.
Following pseudocode computes the matching component of KMP algorithm:

```
KMP-Matcher(S,p)
1 n = length[S]
2 m = length[p]
3 Π = Compute-Prefix-Function(p)
4 q = 0                              //number of characters matched
5 for i =1 to n                      //scan S from left to right
6    while  q > 0 and p[q+1] != S[i]
7          q = Π[q]                  //next character does not match
8    if p[q+1] = S[i]
9          q = q + 1                 //next character matches
10   if q == m                       //is all of p matched?
11         print "Pattern occurs with shift" i – m
12         q ← Π[ q]                 // look for the next match
```

Note: KMP finds every occurrence of a ‘p’ in ‘S’. That is why KMP does not terminate in step 12, rather it searches remainder of ‘S’ for any more occurrences of ‘p’.

Illustration: given a String ‘S’ and pattern ‘p’ as follows:

S b a c b a b a b a b a c a c a

p a b a b a c a

Let us execute the KMP algorithm to find whether ‘p’ occurs in ‘S’.

For ‘p’ the prefix function, Π was computed previously and is as follows:

q	1	2	3	4	5	6	7
p	a	b	A	b	a	c	a
Π	0	0	1	2	3	1	1

Initially: n = size of S = 15;
m = size of p = 7

Step 1: i = 1, q = 0
comparing p[1] with S[1]

S b a c b a b a b a b a c a a b

p a b a b a c a

P[1] does not match with S[1]. ‘p’ will be shifted one position to the right.

Step 2: i = 2, q = 0
comparing p[1] with S[2]

S b a c b a b a b a b a c a a b

p a b a b a c a

P[1] matches S[2]. Since there is a match, p is not shifted.

Step 3: i = 3, q = 1
Comparing p[2] with S[3] p[2] does not match with S[3]

S b a c b a b a b a b a c a a b

p a b a b a c a

Backtracking on p, comparing p[1] and S[3]

Step 4: i = 4, q = 0
comparing p[1] with S[4] p[1] does not match with S[4]

S b a c b a b a b a b a c a a b

p a b a b a c a

Step 5: i = 5, q = 0
comparing p[1] with S[5] p[1] matches with S[5]

S b a c b a b a b a b a c a a b

p a b a b a c a

Step 6: i = 6, q = 1
Comparing p[2] with S[6] p[2] matches with S[6]

S b a c b a b a b a b a c a a b

p a b a b a c a

Step 7: i = 7, q = 2
Comparing p[3] with S[7] p[3] matches with S[7]

S b a c b a b a b a b a c a a b

p a b a b a c a

Step 8: i = 8, q = 3
Comparing p[4] with S[8] p[4] matches with S[8]

S b a c b a b a b a b a c a a b

p a b a b a c a

Step 9: i = 9, q = 4

Comparing p[5] with S[9] p[5] matches with S[9]

S b a c b a b a b a b a c a a b

p a b a b a c a

Step 10: i = 10, q = 5

Comparing p[6] with S[10] p[6] doesn't match with S[10]

S b a c b a b a b a b a c a a b

p a b a b a c a

Backtracking on p, comparing p[4] with S[10] because after mismatch q = Π[5] = 3

Step 11: i = 11, q = 4

Comparing p[5] with S[11] p[5] matches with S[11]

S b a c b a b a b a b a c a a b

p a b a b a c a

Step 12: i = 12, q = 5

Comparing p[6] with S[12] p[6] matches with S[12]

S b a c b a b a b a b a c a a b

p a b a b a c a

Step 13: i = 13, q = 6

Comparing p[7] with S[13] p[7] matches with S[13]

S b a c b a b a b a b a c a a b

p a b a b a c a

Pattern 'p' has been found to completely occur in string 'S'. The total number of shifts that took place for the match to be found are: i – m = 13 – 7 = 6 shifts.

Running - time analysis

- Compute-Prefix-Function (Π)

```
1  m ← length[p]          //'p' pattern to be matched
2  Π[1] ← 0
3  k ← 0
4      for q ← 2 to m
5              while k > 0 and p[k+1] != p[q]
6                  k ← Π[k]
7              If p[k+1] = p[q]
8                  k ← k +1
9              Π[q] ← k
10     return Π
```

In the above pseudocode for computing the prefix function, the for loop from step 4 to step 10 runs 'm' times. Step 1 to step 3 take constant time. Hence the running time of compute prefix function is Θ(m).

- KMP Matcher

```
1 n ← length[S]
2 m ← length[p]
3 Π ← Compute-Prefix-Function(p)
4 q ← 0
5 for i ← 1 to n
6      while  q > 0 and p[q+1] != S[i]
7              q ← Π[q]
8        if p[q+1] = S[i]
9              q ← q + 1
10       if q = m
11             print "Pattern occurs with shift" i – m
12             q ← Π[ q]
```

The for loop beginning in step 5 runs 'n' times, i.e., as long as the length of the string 'S'. Since step 1 to step 4 take constant time, the running time is dominated by this for loop. Thus running time of matching function is Θ(n).