The Traveling Salesman Problem (TSP)

Traveling salesman problem

- The traveling salesman problem consists of a salesman and a set of cities.
- The salesman has to visit each one of the cities starting from a certain one (e.g. the hometown) and returning to the same city.
- The **challenge of the problem is that the traveling salesman wants to minimize the total length of the trip.**

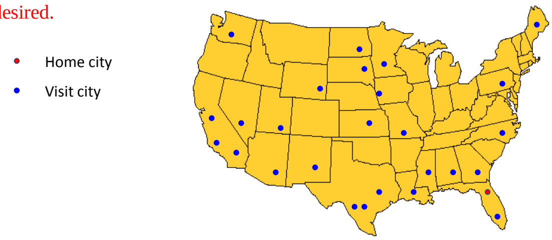
- The traveling salesman problem can be described as follows:
- TSP = {(G, f, t): G = (V, E) a complete graph,
  f is a function $V \times V \rightarrow Z$,
  $t \in Z$,
  G is a graph that contains a traveling salesman tour with cost that does not exceed t}.

A Hamiltonian cycle

- A Hamiltonian cycle is a cycle in a graph passing through all the vertices once.

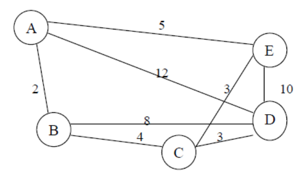
Applications Of TSP

•Once a month the salesperson starts from home and visits each city in his/her territory once. When done, the salesperson returns home.
•The salesperson wishes to minimize time spent in this activity.
•Assuming that the time spent at each city is fixed, total time is minimized by minimizing travel time. So, a tour of minimum length is desired.



Example

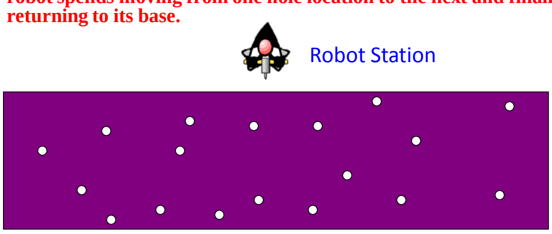
- Consider the following set of cities:



- The problem lies in finding a minimal path passing from all vertices once.
- For example the path Path1 {A, B, C, D, E, A} and the path Path2 {A, B, C, E, D, A} pass all the vertices but Path1 has a total length of 24 and Path2 has a total length of 31.

Applications Of TSP

- **Manufacturing**.
- A robot arm is used to drill n holes in a metal sheet.
- Time to drill the holes is fixed. **The only variable is the time the robot spends moving from one hole location to the next and finally returning to its base.**



n+1 vertex TSP.

A Salesman wishes to travel around a given set of cities, and return to the beginning, covering the smallest total distance

Easy to State

Difficult to Solve

A route returning to the beginning is known as a
Hamiltonian Circuit

A route not returning to the beginning is known as a
Hamiltonian Path

Applications of the TSP

Routing around Cities

Computer Wiring - connecting together computer components using minimum wire length

Genome Sequencing - arranging DNA fragments in sequence

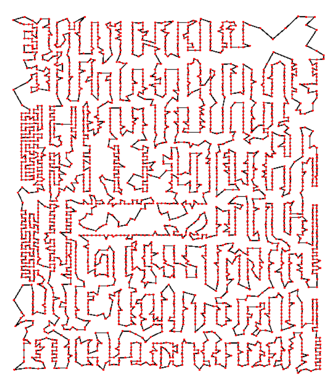
Job Sequencing - sequencing jobs in order to minimise total set-up time between jobs

History of TSP

1800's	**Irish Mathematician, Sir William Rowan Hamilton**
1930's	**Studied by Mathematicians Menger, Whitney, Flood etc.**
1954	**Dantzig, Fulkerson, Johnson, 49 cities (capitals of USA states) problem solved**
1971	**64 Cities**
1975	**100 Cities**
1977	**120 Cities**
1980	**318 Cities**
1987	**666 Cities**
1987	**2392 Cities (Electronic Wiring Example)**
1994	**7397 Cities**
1998	**13509 Cities (all towns in the USA with population > 500)**
2001	**15112 Cities (towns in Germany)**
2004	**24978 Cities (places in Sweden)**

But many smaller instances not yet solved (to proven optimality)
But there are still many *smaller* instances which have not been solved.

Printed Circuit Board 2392 cities 1987 Padberg and Rinaldi



USA Towns of 500 or more population 13509 cities 1998 Applegate, Bixby, Chvátal and Cook

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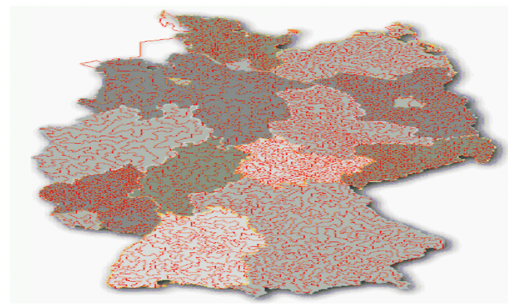
Towns in Germany 15112 Cities 2001Applegate, Bixby, Chvátal and Cook



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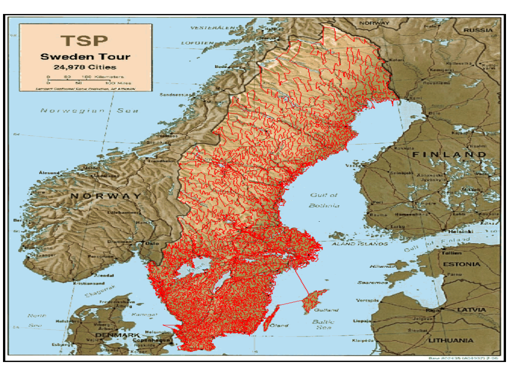
Sweden 24978 Cities 2004 Applegate, Bixby, Chvátal, Cook and Helsgaun



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