Problem Set 1

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1 Functions - Derivatives,Integrals and Series

Q1. Sketch a graph of the function $f(x) = 2x^2 - 3x + 5$ indicating the minimum value of f and the value of t for which that minimum occurs.

Q2. Consider the function $f(t) = Ae^{-kt}$ where A and $k > 0$ are parameters. Show that the function satisfies the differential equation $\frac{df}{dt} = -kf(t)$. Suppose we know that a quantity f satisfies this differential equation and that we have the following data - $(1, .541)$ and $(2, .073)$. Calculate the half-life of the function - that is the time taken for the function to halve its output value.

Q3. Read the article about fads in the toy industry. Can you think of two other examples of behaviour that is "S-shaped". Find a mathematical function that has the S-shape.

Q4. Find the average value of the function $f(t) = t^3 - t$ on the interval $[-1, 1]$.

Q5. Suppose wooden fencing costs 15 E per meter and barbed-wire fencing costs 5 E per meter. Suppose further that you wish to make a rectangular sheep pen of area 100 sq.meters on your land that has one border that runs along a straight cliff face where barbed-wire fence will be erected. Wooden fencing is to be used for sides not running along the cliff. Determine the arrangement that minimises the cost of the fencing.

Q6. Consider the sum $\Sigma_N = 1 + \alpha + 2\alpha^2 + 3\alpha^3.... + N\alpha^N$.

Compute a simple expression that describes Σ_N.

Now, assuming $|\alpha| < 1$ determine $\lim \Sigma_N$ as $N \longrightarrow \infty$.