Problem Set 4A

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Functions Matrices and Iterates of Functions

Q1. Sketch a graph of the function $f(t) = \frac{x^2+1}{x^3-x}$ indicating any local extreme points and determine the asymptotic behaviour as $x \to \infty$.

Q2. Consider the function

$$z = f(x, y) = x^2 - 2xy + y^2 + 2x - 2y + 3$$

of two variables x and y. Using the standard search for local extreme values - this time with partial derivatives. Make sure to describe the nature of the extreme points $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$.

Q3. Given vectors $\mathbf{v}\ and \mathbf{w} \in \mathbb{R}^N$ then the scalar product is defined as

$$\mathbf{v}.\mathbf{w} = \sum_{i=1}^{N} v_i w_i$$

Briefly explain why it makes sense to define the scalar product of two functions $f(x)$ and $g(x)$ (thought of as vector elements of some vector space of functions) according to the

$$\mathbf{f}.\mathbf{g} = \int_a^b f(x)g(x)dx$$

Q4. Given below is a matrix of probabilities relating to a "walk" from node to node in a directed graph. Nodes are labelled 1 2 and 3.

$$\mathbb{P} = \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 0 & 3/4 \\ 1/4 & 3/4 & 0 \end{pmatrix}$$

where $\mathbb{P}_{ij}$ is the probability that one moves from node i to node j. Assuming a walk begins at node 1 what is the probability that a 10-step walk brings you to node 1. What about ending at node 2 ? And node 3 ? For extra marks use the method of diagonalisation.

Q5. Given the logistic map

$$f(x) = Ax(1 - x)$$

defined on $[0,1]$ and $A > 0$. Show that the condition that f^2 has two fixed points of period 2 is that $A > 3$.

Q6. Given the Fibonacci sequence

$$a_{N+1} = a_N + a_{N-1}$$

show that the sequence of ratios $R_i = \frac{a_i}{a_{i-1}}$ tends towards the limit value $L = \frac{1+\sqrt{5}}{2}$.

Q7. Given the following data as output of a procedure do you think the output is governed by a first order recurrence relation? Explain.

7.8391, 8.48737, 8.941159, 9.2588113

What about the data ? 7.7, 8.3, 8.7, 9.1 If not, can you approximate the values using a model that is governed by a homogeneous first order recurrence relation?