

# Problem Set 4A

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Functions Matrices and Iterates of Functions

Q1. Sketch a graph of the function  $f(t) = \frac{x^2+1}{x^3-x}$  indicating any local extreme points and determine the asymptotic behaviour as  $x \rightarrow \infty$ .

Q2. Consider the function

$$z = f(x, y) = x^2 - 2xy + y^2 + 2x - 2y + 3$$

of two variables  $x$  and  $y$ . Using the standard search for local extreme values - this time with partial derivatives  $\frac{\partial f}{\partial x} = 0$  and  $\frac{\partial f}{\partial y} = 0$ .

Q3. Given vectors  $\mathbf{v}$  and  $\mathbf{w} \in \mathbb{R}^N$  then the scalar product is defined as

$$\mathbf{v} \cdot \mathbf{w} = \sum_{i=1}^N v_i w_i$$

Briefly explain why it makes sense to define the scalar product of two functions  $f(x)$  and  $g(x)$  (thought of as vector elements of some vector space of functions) according to the

$$\mathbf{f} \cdot \mathbf{g} = \int_a^b f(x)g(x)dx$$

Q4. Given below is a matrix of probabilities relating to a "walk" from node to node in a directed graph. Nodes are labelled 1 2 and 3.

$$\mathbb{P} = \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 0 & 3/4 \\ 1/4 & 3/4 & 0 \end{pmatrix}$$

where  $\mathbb{P}_{ij}$  is the probability that one moves from node  $i$  to node  $j$ . Assuming a walk begins at node 1 what is the probability that a 10-step walk brings you to node 1. What about ending at node 2 ? And node 3 ?

Q5. Given the logistic map

$$f(x) = Ax(1 - x)$$

defined on  $[0, 1]$  and  $A > 0$ . Show that the condition that  $f^2$  has two fixed points of period 2 is that  $A > 3$ .

Q6. Given the Fibonacci sequence

$$a_{N+1} = a_N + a_{N-1}$$

show that the sequence of ratios  $R_i = \frac{a_i}{a_{i-1}}$  tends towards the limit value  $L = \frac{1+\sqrt{5}}{2}$ .

Q7. Given the following data as output of a procedure do you think the output is governed by a homogeneous first order recurrence relation? Explain.

7.8391, 8.48737, 8.941159, 9.2588113

What about the data ? 7.7, 8.3, 8.7, 9.1 If not, can you approximate the values using a model that is governed by a homogeneous first order homogeneous recurrence relation?