EVRPTW-SMBS Optimization model

1 Quantities

1.1 Given values

- c = no. of customers.
- N = list of nodes = $\{0, 1, \ldots, c\}$.
- C = lis of customers = $\{1, 2, \ldots, c\}$
- demand = dict(C1015[demand]), ready_time = dict(C1015[ready_time]), due_date = dict(C1015[due_date]), service_time = dict(C1015[service_time])
- UBE = UBB = Upper bound on number of ECVs and BSVs respectively = c.
- ECVs = Set of ECV indices = $\{1, 2. \ldots, UBE\}$.
  BSVs = Set of BSV indices = $\{1, 2. \ldots, UBB\}$.
- Dist[j,k]$_{N\times N}$ = Distance between nodes j and k.
  max_dist = $\max\limits_{j,k}$ Dist[j,k].
- max_cap, max_cap$_b$ = Maximum capacity of ECVs and BSVs respectively.
- max_fuel, max_fuel$_b$ = Maximum fuel that can be carried by ECVs and BSVs.
- swap_time = Stored as a single value taken from the dataframe.

1.2 Parameters

1.2.1 Big M values

- M_{cap} = max(max_cap, max_cap_b).
- M_{fuel} = max(max_fuel, max_fuel_b) + max_dist.
- M_{time} = C1015['due_date'][0]+max_dist+swap_time+$\max\limits_{k}$ service_time[k].

1.2.2 Arc pruning

E_{arcs}, B_{arcs} = Set of node/customer pairs (j, k) that do not violate the following conditions:

1. $j = k$ to prevent self arcs.
2. `Dist[j,k]` $>$ `max_fuel` and `Dist[j,k]` $>$ `max_fuel`$_b$ for ECVs and BSVs respectively, to abide by fuel tank capacity.
3. `Dist[0,j] + Dist[j,k]` $>$ `due_date`$_k$ and `Dist[0,k]` $>$ `due_date`$_k$ to avoid arcs that cause arrival time outside the time window.
4. `Dist[k,0]` $>$ `max_fuel` and `Dist[k,0]` $>$ `max_fuel`$_b$ for ECVs and BSVs respectively, to avoid vehicles returning the depot when there is not enough fuel.

2 Variables

2.1 Real variables

- $ta_{UBE \times N}$: ta_{ik} is the arrival time of ECV i at node k.
- $td_{UBE \times N}$: td_{ij} is the departure time of ECV i from node j.
- $tba_{UBB \times N}$: tba_{bk} is the arrival time of BSV b at node k.
- $tbd_{UBB \times N}$: tbd_{bj} is the departure time of BSV b from node j.
- $ea_{UBE \times N}$: ea_{ik} is the fuel of ECV i when it arrives at node k.
- $ed_{UBE \times N}$: ed_{ij} is the fuel of ECV i when it departs node j.
- $eba_{UBB \times N}$: eba_{bk} is the fuel of BSV b when arrives at node k.
- $ebd_{UBB \times N}$: ebd_{bj} is the fuel of BSV b when it departs node j.

2.2 Integer Variables

- $ca_{UBE \times C}$: ca_{ik} is the capacity of ECV i when it arrives at customer node k.
- $cd_{UBE \times N}$: cd_{ij} is the capacity of ECV i when it departs node j.
- $cba_{UBB \times C}$: cba_{bk} is the capacity of BSV b when it arrives at customer node k.
- $cbd_{UBB \times N}$: cbd_{bj} is the capacity of BSV b when it departs node j.
- $u_{UBE \times C}$: u_{ij} is the order of customer j in the route of ECV i.
- $ub_{UBB \times C}$: ub_{bj} is the order of customer j in the route of BSV b.

2.3 Binary Variables

- $x_{UBE \times N \times N}$: $x_{ijk} = 1$ if ECV i travels from node j to k.
- $xb_{UBB \times N \times N}$: $xb_{bjk} = 1$ if BSV b travels from node j to k.
- y_{UBE} : $y_i = 1$ if ECV i is used.
- yb_{UBB} : $yb_b = 1$ if BSV b is used.
- $z_{UBE \times UBB \times C}$: $z_{ibk} = 1$ if there is a battery swap for ECV i by BSV b at customer k.

Notation:

$$i \in ECVs \rightarrow \text{ECV index i}$$

$$b \in BSVs \rightarrow \text{BSV index b}$$

$$j, k \in N, C \rightarrow \text{Nodes/Customers}$$

2.4 Additional quantities for final solution (Computed once solved)

- `total_swaps` $= \sum_{i \in ECVs} \sum_{b \in BSVs} \sum_{k \in C} z_{ibk}$.
- `total_vehicles` $= \sum_{i \in ECVs} y_i + \sum_{b \in BSVs} y_b$.
- `total_distance` $= \sum_{i \in ECVs} \sum_{j \in N} \sum_{k \in N} \texttt{dist}_{jk} \cdot x_{ijk} + \sum_{b \in BSVs} \sum_{j \in N} \sum_{k \in N} \texttt{dist}_{jk} \cdot xb_{bjk}$.

3 Model

3.1 Objective

Minimize

$$2 \sum_{i \in ECVs} \sum_{j \in N} \sum_{k \in N} \texttt{dist}_{jk} \cdot x_{ijk} + \sum_{b \in BSVs} \sum_{j \in N} \sum_{k \in N} \texttt{dist}_{jk} \cdot xb_{bjk}$$
$$+ \sum_{i \in ECVs} \sum_{b \in BSVs} \sum_{k \in C} z_{ibk} + 1.5 \sum_{i \in ECVs} y_i + \sum_{b \in BSVs} y_b$$

3.2 Constraints

3.2.1 Flow of vehicles

- One ECV per node:

$$\sum_{i \in ECVs} \sum_{\substack{j \in N \\ (j,k) \in E_{arcs}}} x_{ijk} = 1 \quad \forall k \in C$$

- At most one BSV per node:

$$\sum_{b \in BSVs} \sum_{\substack{j \in N \\ (j,k) \in B_{arcs}}} xb_{bjk} \leq 1 \quad \forall k \in C$$

- At most one arc per ECV/BSV:

$$\sum_{\substack{j \in N \\ (j,k) \in E_{arcs}}} x_{ijk} \leq 1 \quad \forall i \in ECVs, k \in C$$

$$\sum_{\substack{j \in N \\ (j,k) \in B_{arcs}}} xb_{bjk} \leq 1 \quad \forall b \in BSVs, k \in C$$

- Flow Conservation for ECVs and BSVs:

$$\sum_{\substack{j \in N \\ (j,k) \in E_{arcs}}} x_{ijk} = \sum_{\substack{l \in N \\ (k,l) \in E_{arcs}}} x_{ikl} \quad \forall i \in ECVs, k \in N$$

$$\sum_{\substack{l \in N \\ (j,k) \in B_{arcs}}} xb_{bjk} = \sum_{\substack{l \in N \\ (k,l) \in B_{arcs}}} xb_{bkl} \quad \forall b \in BSVs, k \in N$$

- BSV visits only if ECV visits a customer:

$$\sum_{b \in BSVs} \sum_{\substack{j \in N \\ (j,k) \in B_{arcs}}} xb_{bjk} \leq \sum_{i \in ECVs} \sum_{\substack{j \in N \\ (j,k) \in E_{arcs}}} x_{ijk} \quad \forall k \in C$$

3.2.2 Usage Vs Travel Vs Swap

- Use previous ECV/BSV before choosing to use next one:

$$y_i \leq y_{i-1} \forall i \in ECVs \backslash \{1\}$$

$$yb_b \leq yb_{b-1} \forall b \in BSVs \backslash \{1\}$$

- Relate ECV usage to ECV travel:

$$\sum_{\substack{k \in C \\ (0,k) \in E_{arcs}}} x_{i0k} = y_i \quad \forall i \in ECVs$$

- Relate BSV usage to BSV travel:

$$\sum_{\substack{k \in C \\ (0,k) \in B_{arcs}}} xb_{b0k} = yb_b \quad \forall b \in BSVs$$

- BSV used only if used in a swap:

$$\sum_{\substack{j \in N \\ (j,k) \in B_{arcs}}} xb_{bjk} \leq \sum_{i \in ECVs} z_{ibk} \quad \forall b \in BSVs, k \in C$$

- Only one BSV and ECV per swap:

$$\sum_{i \in ECVs} \sum_{b \in BSVs} z_{ibk} \leq 1 \quad \forall k \in C$$

- Swap only if ECV and BSV are used and travel to the node:

$$z_{ibk} \leq y_i \quad \forall i \in ECVs, b \in BSVs, k \in C$$

$$z_{ibk} \leq yb_b \quad \forall i \in ECVs, b \in BSVs, k \in C$$

$$z_{ibk} \leq \sum_{\substack{j \in N \\ (j,k) \in E_{arcs}}} x_{ijk} \quad \forall i \in ECVs, b \in BSVs, k \in C$$

$$z_{ibk} \leq \sum_{\substack{j \in N \\ (j,k) \in B_{arcs})}} xb_{bjk} \quad \forall i \in ECVs, b \in BSVs, k \in C$$

3.2.3 Subtour elimination

- Upper and lower bounds of u and ub:

$$u_{ij} \geq \sum_{\substack{k \in N \\ (j,k) \in E_{arcs}}} x_{ijk} \quad \forall i \in ECVs, j \in C$$

$$u_{ij} \leq n \cdot \sum_{\substack{k \in C \\ (j,k) \in E_{arcs}}} x_{ijk} \quad \forall i \in ECVs, j \in C$$

$$ub_{bj} \geq \sum_{\substack{k \in N \\ (j,k) \in B_{arcs}}} xb_{bjk} \quad \forall b \in BSVs, j \in C$$

$$ub_{bj} \leq n \cdot \sum_{\substack{k \in C \\ (j,k) \in B_{arcs}}} xb_{bjk} \quad \forall b \in BSVs, j \in C$$

- Prevent subtour:

$$u_{ij} - u_{ik} + n \cdot x_{ijk} \leq n - 1 \quad \forall i \in ECVs, j \in C, k \in C, j \neq k, (j,k) \in E_{arcs}$$

$$ub_{bj} - ub_{bk} + n \cdot xb_{bjk} \leq n - 1 \quad \forall b \in BSVs, j \in C, k \in C, j \neq k, (j,k) \in B_{arcs}$$

3.2.4 Capacity of Vehicles

- Maximum capacity bound on ECV trip:

$$\sum_{k \in C} \left(\texttt{demand}[k] \cdot \left(\sum_{\substack{j \in N \\ (j,k) \in E_{arcs}}} x_{ijk} \right) \right) \leq \texttt{max\_cap} \quad \forall i \in ECVs$$

- ECV capacity starts at maximum if used:

$$cd_{i0} = \texttt{max\_cap} \cdot y_i \quad \forall i \in ECVs$$

- Capacity update on ECVs:

$$cd_{ik} = ca_{ik} - \texttt{demand}[k] \cdot \sum_{\substack{j \in N \\ (j,k) \in E_{arcs}}} x_{ijk} \quad \forall i \in ECVs, k \in C$$

- Capacity consistency of ECVs:

$$cd_{ij} - ca_{ik} \leq M_{cap} \cdot (1 - x_{ijk}) \quad \forall i \in ECVs, j \in N, k \in C$$

$$cd_{ij} - ca_{ik} \geq -M_{cap} \cdot (1 - x_{ijk}) \quad \forall i \in ECVs, j \in N, k \in C$$

- BSV capacity starts at maximum:

$$cbd_{b0} = \texttt{max\_cap}_b \cdot yb_b \quad \forall b \in BSVs$$

- Capacity consistency of BSV:

$$cbd_{bj} - cba_{bk} \leq M_{cap} \cdot (1 - xb_{bjk}) \quad \forall b \in BSVs, j \in N, k \in C$$

$$cbd_{bj} - cba_{bk} \geq -M_{cap} \cdot (1 - xb_{bjk}) \quad \forall b \in BSVs, j \in N, k \in C$$

3.2.5 Fuel of Vehicles

- Maximum fuel at depot:

$$ed_{i0} = \texttt{max\_fuel} \quad \forall i \in ECVs$$

- ECV fuel update:

$$ed_{ik} \geq ea_{ik} \quad \forall i \in ECVs, k \in C$$

- Fuel consistency of ECVs:

$$ea_{ik} - ed_{ij} + \texttt{dist}_{jk} \leq M_{fuel} \cdot (1 - x_{ijk}) \quad \forall i \in ECVs, (j,k) \in E_{arcs}$$

$$ea_{ik} - ed_{ij} + \texttt{dist}_{jk} \geq -M_{fuel} \cdot (1 - x_{ijk}) \quad \forall i \in ECVs, (j,k) \in E_{arcs}$$

- Maximum fuel at depot for BSVs:

$$ebd_{b0} = \texttt{max\_fuel}_b \quad \forall b \in BSVs$$

- BSV fuel update:

$$ebd_{bk} = eba_{bk} \quad \forall b \in BSVs, k \in C$$

- Fuel amount consistency for BSVs:

$$eba_{bk} - ebd_{bj} + \texttt{dist}_{jk} \leq M_{fuel} \cdot (1 - xb_{bjk}) \quad \forall b \in BSVs, (j,k) \in B_{arcs}$$

$$eba_{bk} - ebd_{bj} + \texttt{dist}_{jk} \geq -M_{fuel} \cdot (1 - xb_{bjk}) \quad \forall b \in BSVs, (j,k) \in B_{arcs}$$

3.2.6 Time

- Departure and arrival at depot starts at 0:

$$td_{i0} = 0 \quad \forall i \in ECVs$$

- Departure and arrival connection at a node:

$$td_{ik} \geq ta_{ik} + \texttt{service\_time}[k] + \texttt{swap\_time} \cdot \sum_{b \in BSVs} z_{ibk} - M_{time} \cdot \left(1 - \sum_{(j,k) \in E_{arcs}} x_{ijk}\right) \quad \forall i \in ECVs, k \in C$$

$$td_{ik} \leq ta_{ik} + \texttt{service\_time}[k] + \texttt{swap\_time} \cdot \sum_{b \in BSVs} z_{ibk} + M_{time} \cdot \left(1 - \sum_{(j,k) \in E_{arcs}} x_{ijk}\right) \quad \forall i \in ECVs, k \in C$$

$$tbd_{bk} \geq tba_{bk} + \texttt{swap\_time} \cdot \sum_{i \in ECVs} z_{ibk} - M_{time} \cdot \left(1 - \sum_{(j,k) \in B_{arcs}} xb_{bjk}\right) \quad \forall b \in BSVs, k \in C$$

- Departure and arrival connection between nodes

$$td_{ij} + \texttt{dist}_{jk} - ta_{ik} \geq -M_{time} \cdot (1 - x_{ijk}) \quad \forall i \in ECVs, (j,k) \in E_{arcs}$$

$$tbd_{bj} + \texttt{dist}_{jk} - tba_{bk} \geq -M_{time} \cdot (1 - xb_{bjk}) \quad \forall b \in BSVs, (j,k) \in B_{arcs}$$

- Time windows:

$$ta_{ik} \geq \texttt{ready\_time}[k] - M_{time} \cdot \left(1 - \sum_{(j,k) \in E_{arcs}} x_{ijk}\right) \quad \forall i \in ECVs, k \in C$$

$$ta_{ik} \leq \texttt{due\_date}[k] + M_{time} \cdot \left(1 - \sum_{(j,k) \in E_{arcs}} x_{ijk}\right) \quad \forall i \in ECVs, k \in C$$

$$tba_{bk} \geq \texttt{ready\_time}[k] - M_{time} \cdot \left(1 - \sum_{i \in ECVs} z_{ibk}\right) \quad \forall b \in BSVs, k \in C$$

$$tba_{bk} \leq \texttt{due\_date}[k] + M_{time} \cdot \left(1 - \sum_{i \in ECVs} z_{ibk}\right) \quad \forall b \in BSVs, k \in C$$

3.2.7 Battery Swap

- ECV energy is maximum after swap:

$$ed_{ik} - ea_{ik} \leq M_{fuel} \cdot \sum_{b \in BSVs} z_{ibk} \quad \forall i \in ECVs, k \in C$$

$$ed_{ik} \geq \texttt{max\_fuel} \cdot \sum_{b \in BSVs} z_{ibk} \quad \forall i \in ECVs, k \in C$$

- BSV capacity decreases by one after swap:

$$cbd_{bk} = cba_{bk} - \sum_{i \in ECVs} z_{ibk} \quad \forall b \in BSVs, k \in C$$