LCR circuit

- **Define capacitance.**

  The capacitance of a capacitor is the measure of amount of charge it can store per unit voltage.
  Denoted C.
  SI unit: farad or F.
  $$C = \frac{Q}{V}.$$

- **Define one farad.**

  When potential difference (voltage) across the capacitor is 1 V and if it stores 1 C of charge then the capacitance is said to be 1 F.

- **Define inductance.**

  It is the measure of opposition to the change in current in a coil when an EMF is induced in that coil due to the change in current.
  Denoted L.
  SI unit: henry or H.
  $$E_{ind} = -L\frac{\mathrm{d}I}{\mathrm{d}t},$$
  E_{ind} is EMF induced due to changing current in the coil (Faraday's law).

- **Define one henry.**

  If the current in the coil changes by $1\,\mathrm{As}^{-1}$ inducing an EMF of 1 V, then the inductance of the coil is said to be 1 H.

- **What is frequency in alternating current or voltage?**

  In AC, the current and voltage oscillates with time. By frequency we mean how many cycles of oscillations are completed in one second. Say if 1 kHz is the frequency set in the AC source, it means 1000 cycles of oscillations are completed in one second. Frequency is denoted by f. f is related to angular frequency ω as $f = \omega/2\pi$.
  SI unit: hertz or Hz.

- **What is peak current?**

  It is the maximum value of current (amplitude of alternating current).
  Denoted I_0.
  If I_0=2 mA, it means the current oscillates between 2 mA and −2 mA with time. See figure (1).

- **What is capacitive reactance and inductive reactance?**

  The resistance to current offered by a capacitor is capacitive reactance (X_C).
  The resistance to current offered by an inductor is inductive reactance (X_L).
  Both are measured in ohms or Ω.

- **How X_C and X_L are related to frequency of AC source?**

  $$X_L = \omega L,$$
  $$X_C = \frac{1}{\omega C}.$$
  Therefore, X_C and X_L are frequency dependent.

- **What is series LCR circuit?**

  Inductor, capacitor and resistor are connected end to end (in series). Here the current through L, C & R are same but the voltages across them differ.

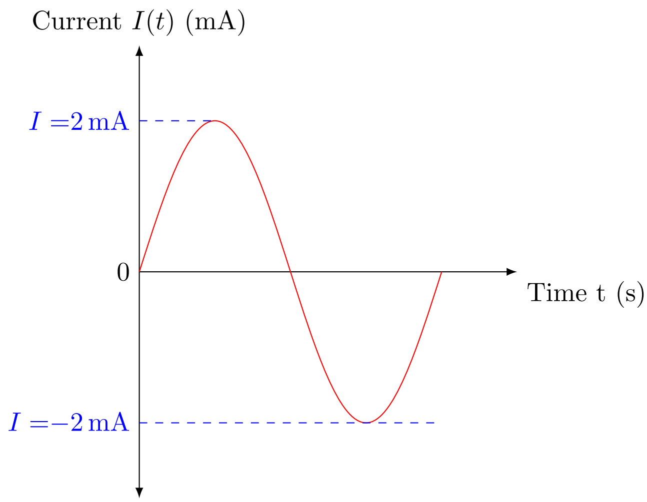

Figure 1: Figure depicts one cycle of oscillation of alternating current for $I_0 = 2\,\mathrm{mA}$.

- **What is parallel LCR circuit?**

  Inductor, capacitor and resistor are connected between two common points (in parallel). Here the voltage across $L, C \,\&\, R$ are same but the currents through them are different.

- **Which current does the ammeter measure?**

  It measures RMS current. Peak current and RMS current are related as follows: $I_{rms} = I_0/\sqrt{2}$. They only differ by a factor of $1/\sqrt{2}$.

- **What is electrical resonance in LCR circuit?**

  It is a condition when the capacitive reactance and inductive reactance become numerically equal.
  The frequency at which $X_L = X_C$ is called the **resonant frequency** (f_r).
  For series LCR, at resonance, peak current is maximum and impedance is minimum.
  For parallel LCR, at resonance, peak current is minimum and impedance is maximum.

- **What is impedance?**

  Impedance is the effective resistance offered by resistor, capacitor and inductor.
  Denoted Z.
  It is the ratio of peak voltage to peak current.

  $$Z = \frac{V_0}{I_0}.$$

  For series LCR circuit,

  $$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

  Notice if $R \neq 0$, impedance will be minimum when $X_L = X_C$. Then $Z_{min} = R$.

- **Why/How does peak current vary with frequency of AC source?**

  - For series LCR, see figure (2).

    $$I_0 = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}}.$$

    Here X_L and X_C are frequency dependent.

    * When $f < f_r$, $X_L < X_C$. As f is increased difference between X_L and X_C reduces, hence I_0 increases.
    * When $f = f_r$, $X_L = X_C$, resonance occurs, I_0 is maximum.
    * When $f > f_r$, $X_L > X_C$. As f is increased $(X_L - X_C)^2$ increases, I_0 decreases.
  - For parallel LCR,

$$I_0 = V_0 \left[\frac{1}{R} + j \left(\omega C - \frac{1}{\omega L} \right) \right],$$

  where $j = \sqrt{-1}$.

  I_0 is minimum when $\omega C = \dfrac{1}{\omega L}$ at resonance. Hence the opposite behavior is shown, see figure (3).

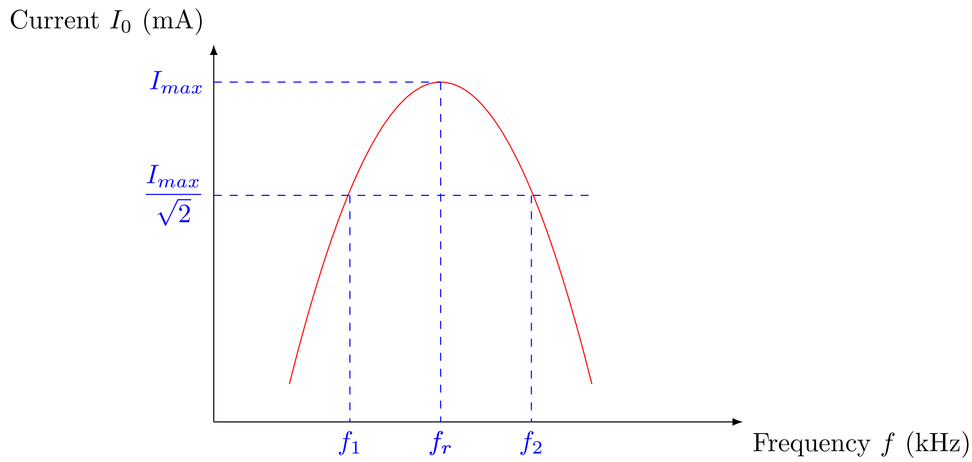

Figure 2: Frequency response curve for series LCR circuit.

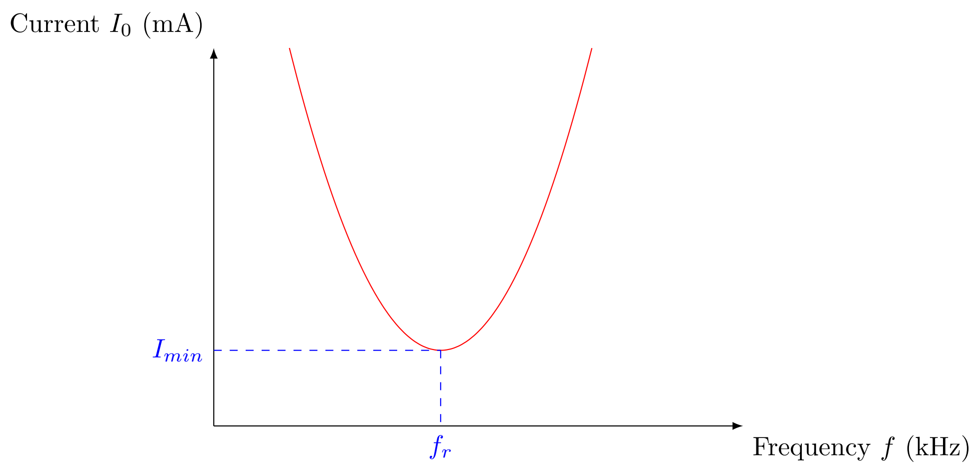

Figure 3: Frequency response curve for parallel LCR circuit

- **Define bandwidth.**

  The difference between two frequencies of the applied voltage at which the current in LCR circuit drops to $1/\sqrt{2}$ times its resonant value is called bandwidth. See figure (2).

Bandwidth $\Delta f = f_2 - f_1$.

- **What are half power frequencies?**

  The frequencies for which $I_0 = \frac{1}{\sqrt{2}}(I_0)_{max}$ are called half power frequencies.

- **Define Quality factor.**

  Quality factor is denoted by Q.

  $$Q = \frac{\text{Resonant Frequency}}{\text{Bandwidth}} = \frac{f_r}{\Delta f}.$$

  Q has no units. It measures the sharpness of the circuit.

- **Does changing the value of R in the circuit change the resonant frequency?**

  $$f_r = \frac{1}{2\pi\sqrt{LC}}.$$

  f_r is independent of the value of R. Changing resistance does not alter the resonant frequency.