FIR digital filter design and MATLAB simulation

Zhenhe Sun, Wei Yu, Chen He
School of Automation
Harbin University of Science and Technology,
Harbin China
Sun zhenhe@hrbust.edu.cn

***Abstract*—In this paper, window function method was expatiated on how to design digital filters .Band-pass filter was designed by using MATALB which possesses superior characteristics of devising filters in a fast and effective way when using window functions originated in it, and the filter was applied to a mixed sine wave signal. Simulation model was established by using MATLAB and the waveforms from oscilloscopes were observed in order to verify the performance.**

Keywords-finite impulse response; window function method; MATLAB

I. INTRODUCTION

Digital filter is a type of digital system that filters discrete-time signal and aims at dealing with frequency domain filtering by processing sample data. It can be divided into two categories in accordance with the time domain features of unit impulse response function: Infinite impulse response (IIR) and finite impulse response (FIR).Compared with IIR filter, FIR filter has access to strict linear phase characteristic while satisfying amplitude frequency response[1]. Therefore, it is widely used in high-fidelity signal processing, such as digital audio, image processing, data transmission, biomedicine and other fields. SIMULINK is a software package in MATLAB which is specially used for dynamic system modeling, simulation and analysis. We can directly present simulation results in a virtual-realized way by using Signal Processing Box built-in MATLAB that enables MATLAB to use graphing technology[2].

II. THE BASIC STRUCTURE OF THE FILTER

Assuming the unit impulse response h(n) is a sequence of N points of the filter, $0 \le n \le N-1$,thus the system function of the filter is illustrated as follows:

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}. \qquad (1)$$

A. Direct-form structrue

The function above can be expressed as a difference equation:

$$y(n) = \sum_{m=0}^{N-1} h(m) x(n-m). \qquad (2)$$

Obviously, equation（2） is a convolution sum formula of linear shift-invariant system as well as a horizontal structure of the delay chain of $x(n)$, thus FIR horizontal structure is as follows:

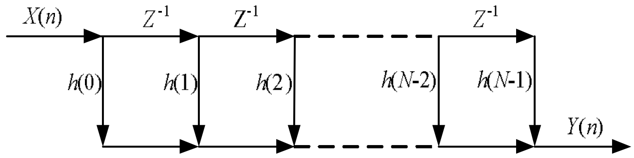

Figure 1. Direct-form structure

Noting that N represents tap numbers of the FIR filter; $x(n)$ is for the input sample of nth time; $h(m)$ acts as the m-level tap coefficients of the FIR filter[3].

B. Cascated structure

The system function $H(z)$ can be decomposed into second-order real coefficients factor form. As (3) shows:

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} = \prod_{i=1}^{\frac{N}{2}} (\beta_{0i} + \beta_{1i} z^{-1} + \beta_{2i} z^{-2}). \qquad (3)$$

Its structure form as follows:

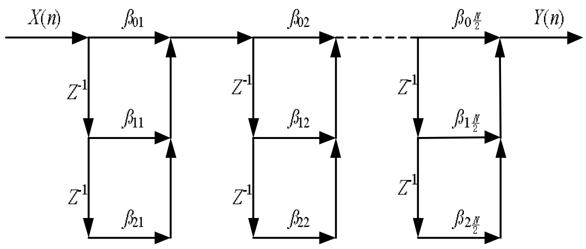

Figure 2. Cascaded structure

Among this structure every joint controls a pair of conjugate poles for the convenience of adjusting transmission zeros. However, this kind of structure is not often among the options as it needs more coefficients and multiplications than the direct one[4].

C. linear structure

We are acquainted that if unit impulse response function $h(n)$ of FIR filter is real and satisfies the following conditions:

$$\begin{cases} h(n) = h(N-1-n) \\ h(n) = -h(N-1-n). \end{cases} \tag{4}$$

It means the center of the symmetry has the coordinate of N-1）/2, thus the filter qualifies the conditions possesses linear phase and the linear structure of FIR filter can be deduced according to the restrictions[5]. The symmetrical characteristic of the FIR filter can simplify network structure (here only for discussion when $h(n)$ is an even-symmetry case and N is an even number. It's the same theory when $h(n)$ is an odd-symmetry presentation) when $h(n)$ is even symmetry and N is an even number:

If a given unit impulse response of linear FIR filter is $h(n)$, $0 \le n \le N-1$, and it satisfies either of the symmetrical conditions as (4) illustrated.

When h(n) is for the dual symmetry ,N is an even number:

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n} = \sum_{n=0}^{(N/2)-1} h(n)z^{-n} + \sum_{n=N/2}^{N-1} h(n)z^{-n}. \tag{5}$$

Among the second $\sum$, we proceed with $n = N-1-m$, and change m with n to get the expression:

$$H(z) = \sum_{n=0}^{(N/2)-1} h(n)z^{-n} + \sum_{n=0}^{(N/2)-1} h(N-1-n)z^{-(N-1-n)}. \tag{6}$$

Equation (6) is embedded with linear parity-symmetry conditions: $h(n) = \pm h(N-1-n)$, thus a new formula is established as:

$$H(z) = \sum_{n=0}^{(N/2)-1} h(n)[z^{-n} \pm z^{-(N-1-n)}]. \tag{7}$$

Its linear network structure is presented as following picture:

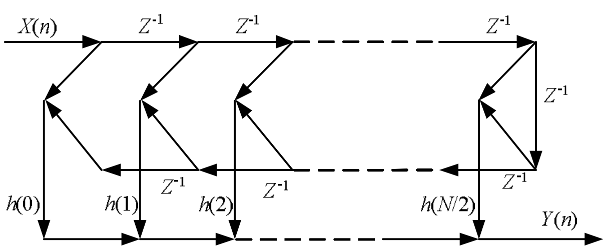

Figure 3. h(n) for the dual symmetry, N is an even number

When $h(n)$ is for the dual symmetry, N is an odd number, its linear network structure is presented as follows[6]:

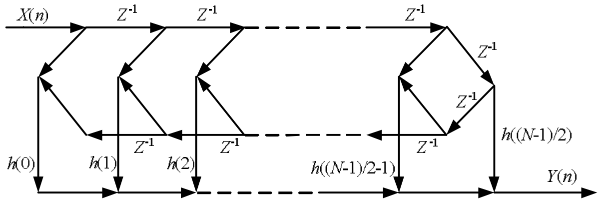

Figure 4. $h(n)$ for the dual symmetry, N is an odd number

A conclusion can be drawn according to the analysis above: linear phase FIR filter can save half of multiplications compared with a direct-form one. However, we adopt the direct form due to its convenience to be designed and the same linear phase restrictions will be satisfied when it is consistent with linear restrictions[7].

III. WINDOW FUNCTION

A. The princple of the window function

An ideal frequency response $H_d(e^{j\omega})$ is generally given the first step when we try to use FIR window design method to devise a digital filter, after that using the frequency response:

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-jwn}. \tag{8}$$

which was defined by h (n) sought after the first step to approximate $H_d(e^{j\omega})$ .Using Fourier inverse transmission of $H_d(e^{j\omega})$ to derive $h_d(n)$ as for in the time domain:

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega. \tag{9}$$

It appears that the system function of the filter can be obtained after Z transmission as soon as $h_d(n)$ is derived from $H_d(e^{j\omega})$ .But, in fact, as $H_d(e^{j\omega})$ is generally piecewise constant, the points are not continuous at the frequency boundary, so that the corresponding $h_d(n)$ is a time-limitless wide sequence, and non-causal, physically impossible to realize[8]. The length of the $h(n)$ of the FIR filter we are designing must be finite, therefore a length-limited $h(n)$ is needed to approximate $h_d(n)$, one effective way is to cut off $h_d(n)$,which means using a length-finite window sequence to intercept $h_d(n)$, namely: $h(n)=\omega(n)h_d(n)$, and turning non-casual sequence into a casual one.

B. Various window functions

Frequently-used window functions can be listed as Triangular windows, Hann window, Hamming window, Kaiser windows and Blackman windows and so forth. All kinds of functional parameters list is as follows:

TABLE I. PARAMETERS TABLE OF VARIOUS WINDOW FUNCTIONS

Window function	Over band width (P/N)	Minimum stopband attenuation (dB)	Sidelobe peak amplitude (dB)
Rectangle	4	21	13
Triangle	8	25	25
Hann	8	44	31
Hamming	8	53	41
Kaiser (β=5.6)	7.442	60	51
Blackman	12	74	71

According to engineering experience, the Kaiser windows function design experience is as follows:

Normalized transition zone:

$$\Delta f = \frac{\omega_s - \omega_p}{2\pi}. \tag{10}$$

ω_p represents the passband cutoff frequency, ω_s represents the stopband cutoff frequency.

The order of the filter

$$M = \frac{A_s - 7.95}{14.36\Delta f}. \tag{11}$$

A_s is the minimum stopband attenuation.

When A_s is separately defined as :

$A_s < 21,\ \ 21 < A_s < 50,\ \ A_s \ge 50.$ We can obtain values of β correspondently as follows:

$$\begin{cases} \beta = 0, \\ \beta = 0.584(A_s - 21)^{0.4} + 0.07886(A_s - 21). \\ \beta = 0.1102(A_s - 8.7) \end{cases} \tag{12}$$

The performance of various window functions has been listed in the table and the appropriate window function can be selected according to the parameters in order to accomplish a given design[9].

Specific design flow of window function is charted as follows:

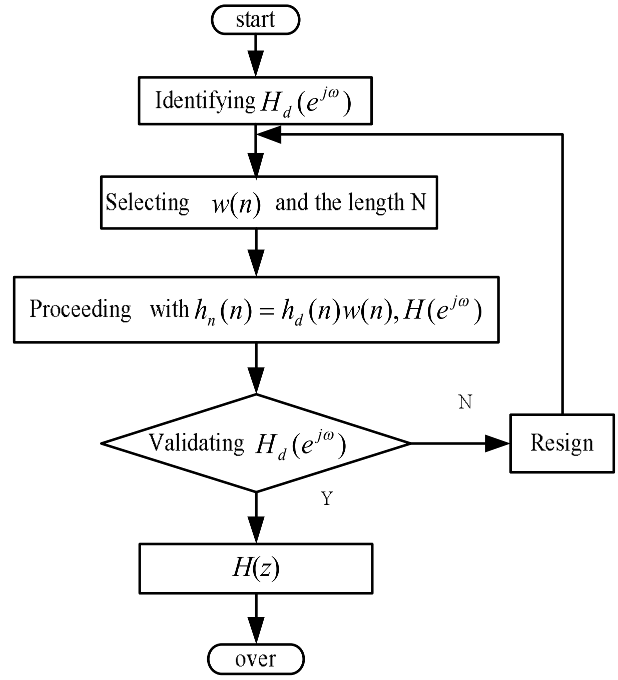

Figure 5. Design flow of window function

C. The necessary steps for the window function

1) Given the desired frequency response function;

2) Calculating $h_d(n) = IDTFT[H_d(e^{j\omega})]$*;*

3) Judging by the chart tabel I,The shape of the window and the size of N should be certain due to the requirements of the transition bandwidth and the minimum stopband attenuation ;

4) Obtaining the unit sampling response of the FIR design h(n),n=0,1,...N-1

5) Calculating $H(e^{j\omega}) = DTFT[h(n)]$*, and verifying if it meets the requirements otherwise you need to redesign it*[10].

In short, window function method is both the most simple and effective way as well as the most common path in devising FIR filter considering its simpleness to realize, closed form formula to reference, enhanced work efficiency characteristics. Undoubtedly window function method is my article's priority.

IV. MATLAB SIMULATION

In this article we will direct at a sine wave single which contained a mixed signals of 5Hz、15Hz and 30Hz to design a FIR band-pass filter. Required parameters: sampling frequency fs=100Hz, lower cut-off frequency of passband fc1=10 Hz, upper cut-off frequency of passband fc2=20Hz, transition band 6Hz, the fluctuations of passband and stopband 0.01, adopting Kaiser Windows to fulfill the design.

Fig. 6 is an amplitude-frequency response characteristic diagram designed with Kaiser windows. The order of the filter n=38，beta=3.4.

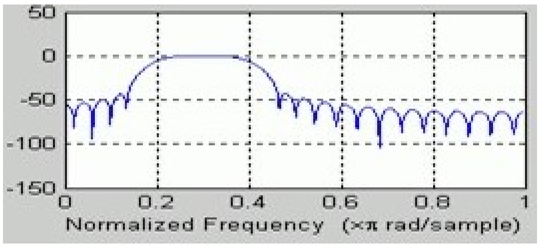

Figure 6. Figure1.amplitude-frequency characteristic diagram

Running SIMULINK in MATLAB and establishing simulation model, inserting the filter originated from the design and starting simulation, simulation diagram is as follows:

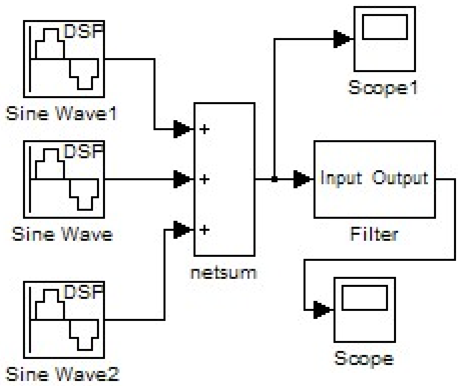

Figure 7. SIMULINK simulation diagram

Observing the wave forms of the two oscilloscopes and analyzing if it satisfies the demands.

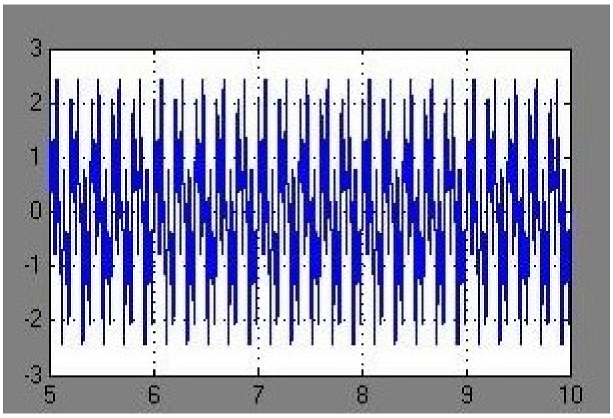

Figure 8. Waveform before filtering

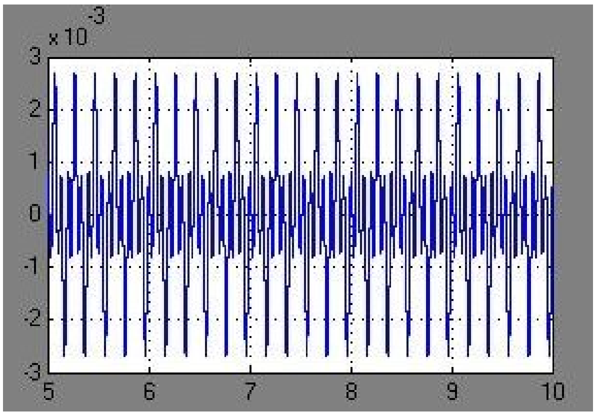

Figure 9. Waveform after filtering

Judging by the simulation figures Fig. 8 and Fig. 9 we can conclude the design of the FIR filter meets the requirements.

V. CONCLUSIONS

Window function method is to obtain a length-finite impulse response sequence by using a window function with certain width to intercept an infinite impulse response sequence. The application of window function design method can quickly and efficiently design FIR digital filters formed by the software and combine SIMULINK to establish simulation model for analyzing the simulation results. Superior properties can be achieved by using MATLAB to deal with the link of design and simulation such as: good practicability, altering parameters at any time by contrasting characteristics of the filter, in order to achieve the optimal design of the filter.

REFERENCES

[1] Cetin A E, Gerek O N, Yardimci Y, “Equiripple FIR filter design by the FFT algorithm,” Signal Processing Magazine, 1997,14(2), pp. 60-64.

[2] Yagyu M, Yoshida T, Nishihara A, “Design of FIR Digital Filters with Minimum Weight Representation,” IEEE Transactions on Circuits and Systems, 1995, 40(3), pp. 227-230.

[3] Bougas P, Kalivas P, “Pipelined array-based FIR filter folding,” IEEE Transactions on Circuits and Systems, 2005, 52(1), pp. 108-118.

[4] Gang G Y, Qing Z, Yuanjin L, “Discrete-time Approximation of Filters,” Journal of Control Theory and Applications, 2004, pp 1-10.

[5] Jongsun P, Muhammad K, Roy K， “High-performance FIR filter design based on sharing multiplication,” IEEE Transactions on VLSI, 2003, 11(2), pp. 244-253.

[6] Cheng C, Parhi K K, “Low-cast Parallel FIR Filter Structures with 2-stage Parallelism,” IEEE Transactions on Circuits and Systems, 2007, 54(2), pp. 280-290.

[7] Hermanowicz E, Johansson H, “A Complex Variable Fractional-Delay FIR filter Structure,” IEEE Transactions on Circuits and Systems II, 2007, 54(9), pp 785-789.

[8] Chi-Tsong C, “Digital Signal Processing-Spectral Computation and Filter Design,” London: Oxford University Press, 2001, pp. 198-205.

[9] Shengli Yan, “FIR filter principle and design method,” Changchun Engineering college journal, 2003, 4(1), pp 63-65.

[10] Kaiyu Xu, Zhongming Zhu, Yaling Lu, “Digital signal processing,” Electronics and industry press, 2005, pp. 137-138.