MR ASSIGNMENT -5
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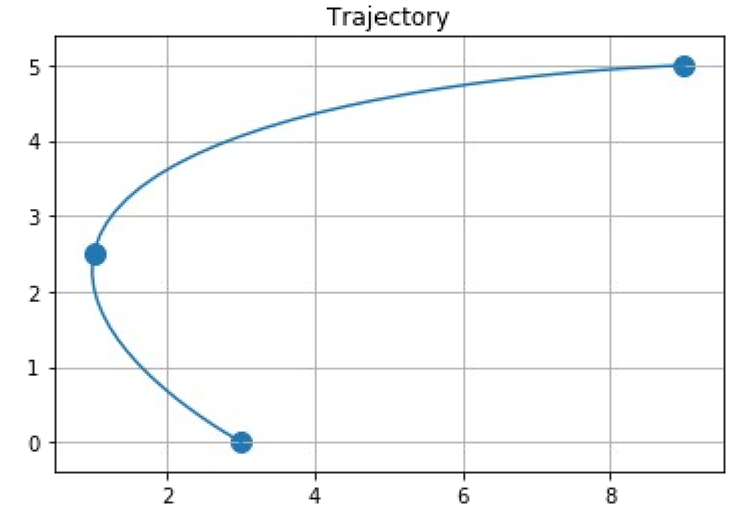
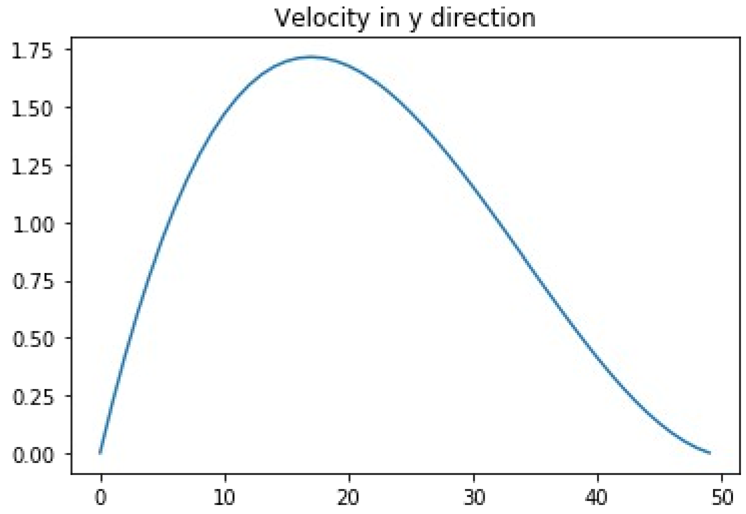
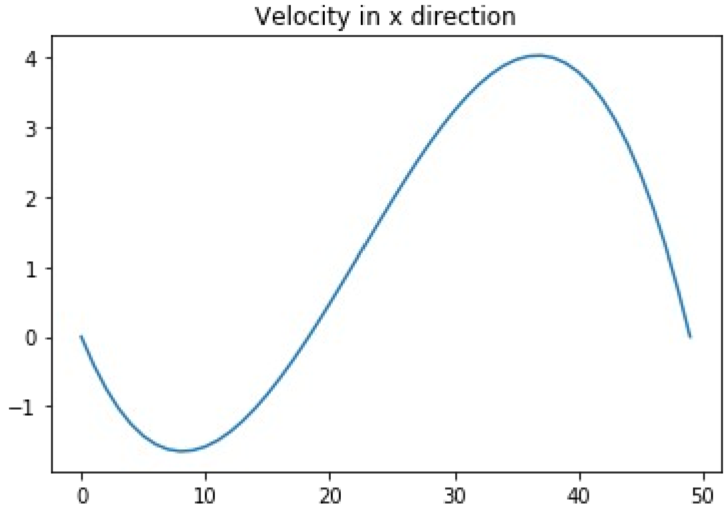
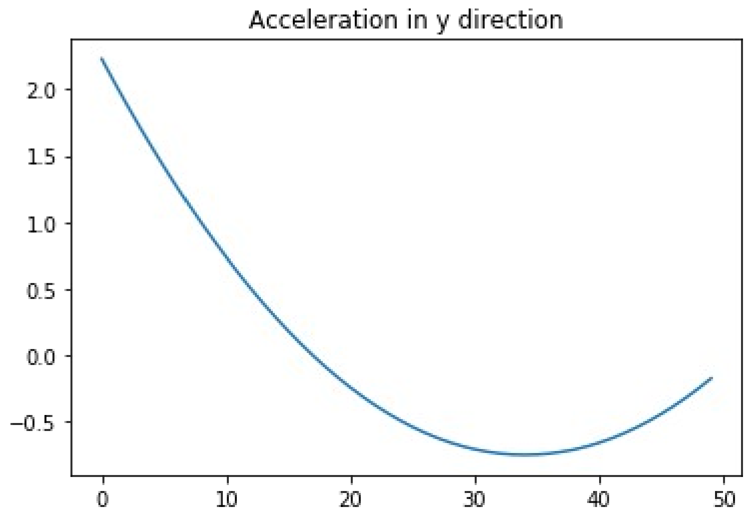
Trajectory planning using polynomials

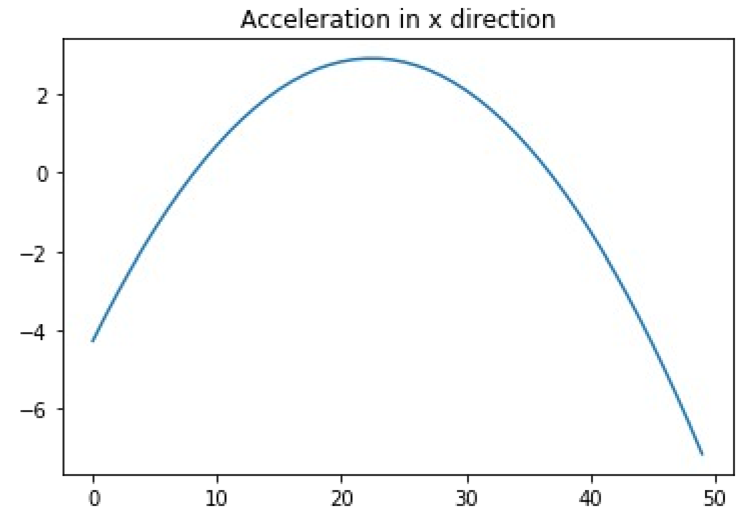
Plots for individual questions:
a. 4 degree polynomial



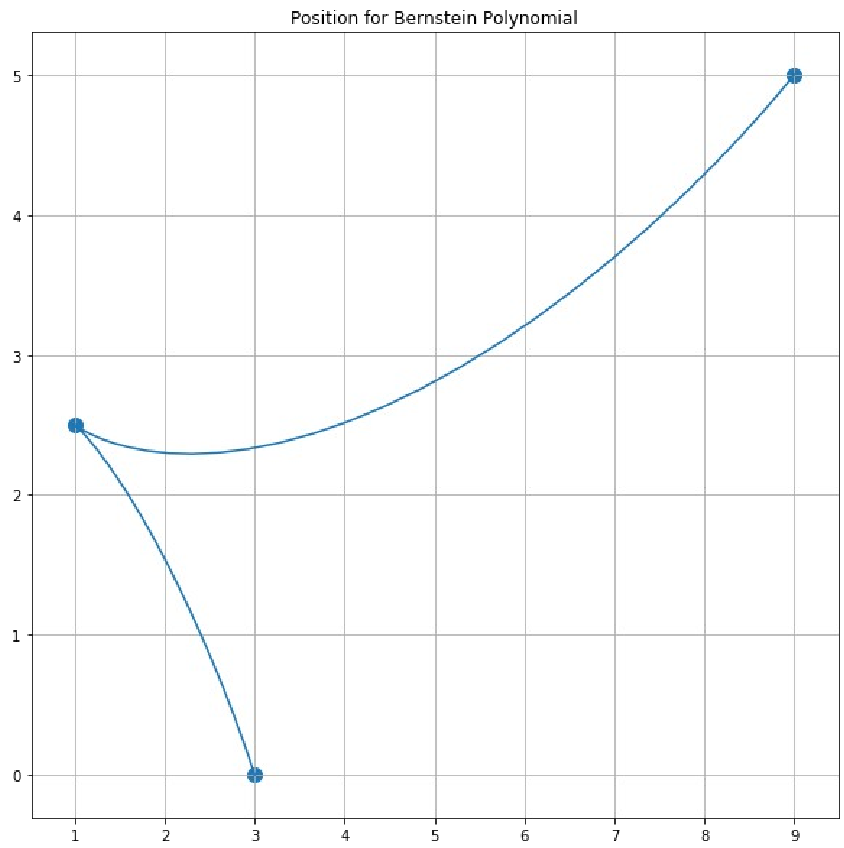


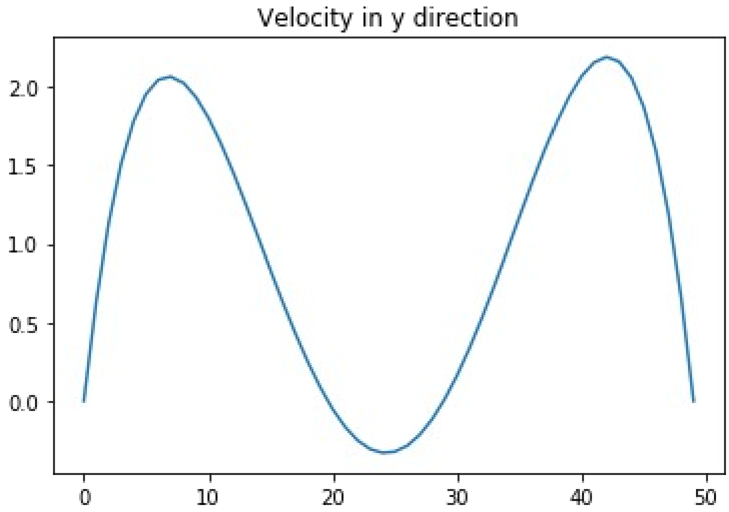


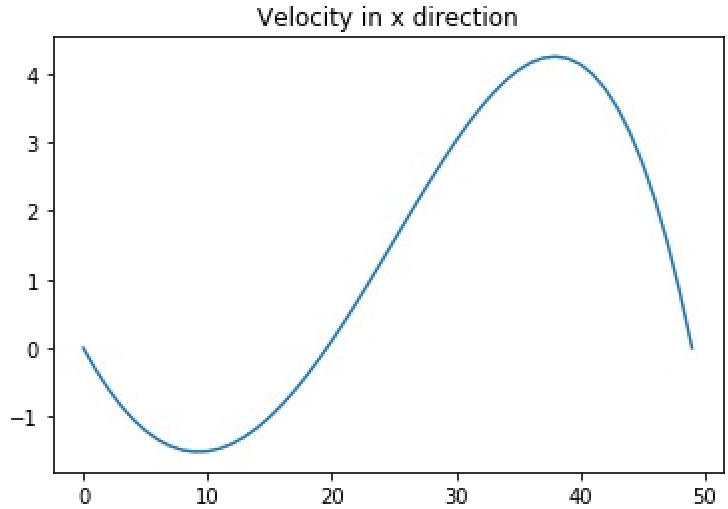


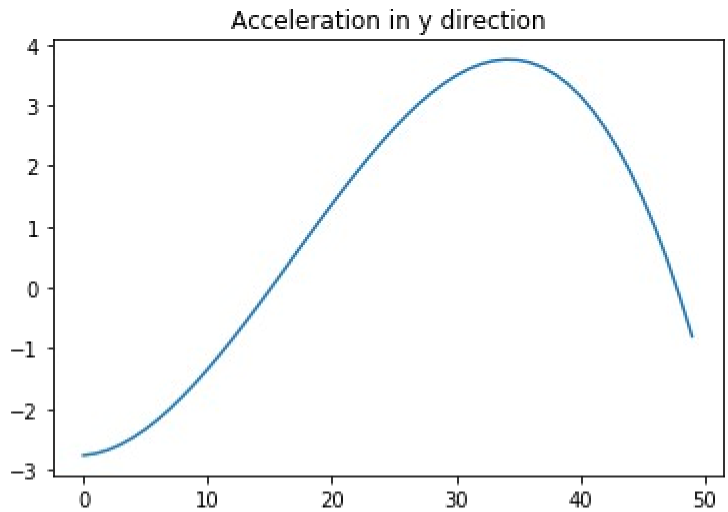


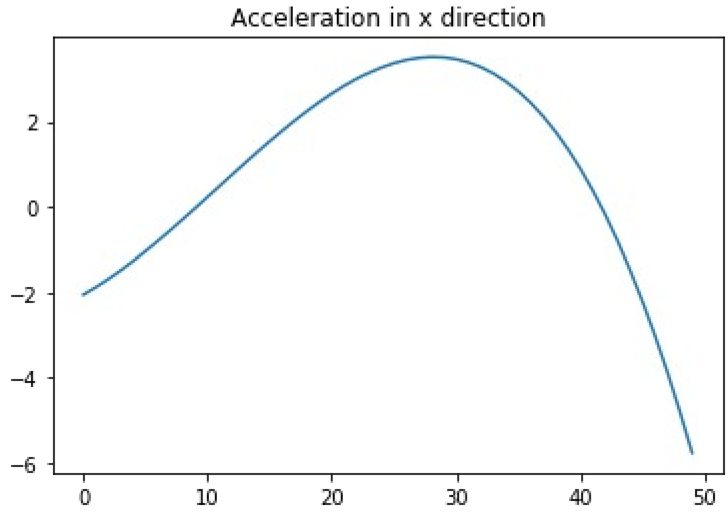
b. Bernstein Polynomial




Velocity in y direction
2.0
1.5
1.0
0.5
0.0
0
10
20
30
40
50

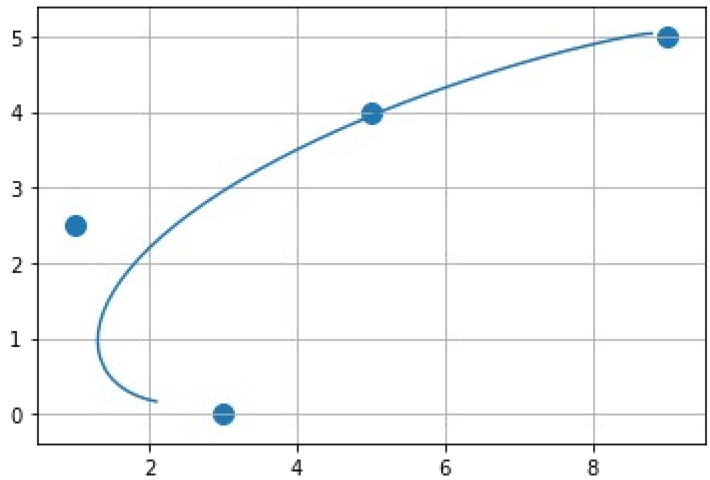

Velocity in x direction
4
3
2
1
0
−1
0
10
20
30
40
50


Acceleration in y direction
4
3
2
1
0
−1
−2
−3
0
10
20
30
40
50



Derivations are given on the next page.

Bonus:



We tried implementing the bonus question, but we were not able to find three waypoints that served as good constraints for the trajectrory given the circle from (5,4) with radius (3+threshold value) for the given quartic polynomial. The approach used here was correct, and two methods of implementing are derived in the handwritten notes that start in the next page.

Derivations

a. Polynomial of degree 4.

We have the parametric equations for $x(t)$ and $y(t)$

$$y(t) = w_5 t^4 + w_3 t^3 + w_2 t^2 + w_1 t^1 + w_0$$
$$x(t) = w_5' t^4 + w_3' t^3 + w_2' t^2 + w_1' t^1 + w_0'$$

we need to find the corresponding solutions w_5, w_4, w_2, w_2, w_1 given the constraints.

Constraints taken: At
$t=0,\ x=3, y=0$ — ①
$t=0,\ \dot{x}=0; \dot{y}=0$ — ②
$t=5\ \ x=9; y=5$ — ③
$t=5\ \ \dot{x}=0; \dot{y}=0$ — ④
$t=2\ \ x=1; y=2.5$ — ⑤
~~$t=2\ \ \dot{x}=0$ — ⑥~~

constraints marked by numbers for x

Now, we need to linearize for x and y and solve the equations $Ax = b$.

Hence, we get

$$\begin{array}{l} \text{Constraint ①} \\ \text{Constraint ②} \\ \text{Constraint ③} \\ \text{Constraint ④} \\ \text{Constraint ⑤} \end{array}
\overset{A}{\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 5^4 & 5^3 & 5^2 & 5 & 1 \\ 4.5^3 & 3.5^2 & 2.5 & 1 & 0 \\ 2^4 & 2^3 & 2^2 & 2 & 1 \end{bmatrix}}
\overset{x}{\begin{bmatrix} w_4 \\ w_3 \\ w_2 \\ w_1 \\ w_0 \end{bmatrix}}
= \overset{b}{\begin{bmatrix} 3 \\ 0 \\ 9 \\ 0 \\ 1 \end{bmatrix}}$$

Now, we solve for $x = A^{-1}b$, and we do the same for constraints on y. We take this and plot in increments of $t = 0.1$.

b. Bernstein Polynomial

We have the following $f(t) = \sum_{i=0}^{n} {}^nC_i \, \tau^i (1-\tau)^{n-i} P_i$

$$\tau = \frac{t - t_0}{t_f - t_0} = \frac{t}{5}$$

Hence, we have $\sum_{i=0}^{n} {}^nC_i \left(\frac{t}{5}\right)^i \left(1 - \frac{t}{5}\right)^{n-i} P_i$

So we get the coefficients of the following polynomial, and we need to solve for P_i.

corresponding to:

	For x	$\dot{x}$	$\ddot{x}$
$i=0$	$\left(1-\frac{t}{5}\right)^5$	$-\left(1-\frac{t}{5}\right)^4$	$\frac{4}{5}\left(1-\frac{t}{5}\right)^3$
$i=1$	$1-\left(1-\frac{t}{5}\right)^4$	$\left(1-\frac{t}{5}\right)^3(1-t)$	$\left(-\frac{3}{5}\right)\left(1-\frac{t}{5}\right)^2(1-t) - \left(1-\frac{t}{5}\right)^3$
$i=2$	$10\left(\frac{t}{5}\right)^2\left(1-\frac{t}{5}\right)^3$	$2t/5\left(1-\frac{t}{5}\right)^2(2-t)$	$-\frac{2}{5}\left(1-\frac{t}{5}\right)\left(\frac{4t}{5}-\frac{2t^2}{5}\right)+\left(1-\frac{t}{5}\right)^2\left(4/5 - 4t/5\right)$
$i=3$	$10\left(\frac{t}{5}\right)^3\left(1-\frac{t}{5}\right)^2$	$2\left(\frac{t}{5}\right)^2\left(1-\frac{t}{5}\right)(3-t)$	$-\frac{1}{5}\left(\frac{6t^2-2t^3}{25}\right)+\left(1-\frac{t}{5}\right)\left(\frac{12t}{25}\right)$
$i=4$	$\left(\frac{t}{5}\right)^4(5-t)$	$\left(\frac{t}{5}\right)^3(4-t)$	$(12t^2-4t^3)/125 - \frac{6t}{25}$
$i=5$	$\left(\frac{t}{5}\right)^5$	$\left(\frac{t}{5}\right)^4$	$\frac{4t^3}{5^4}$

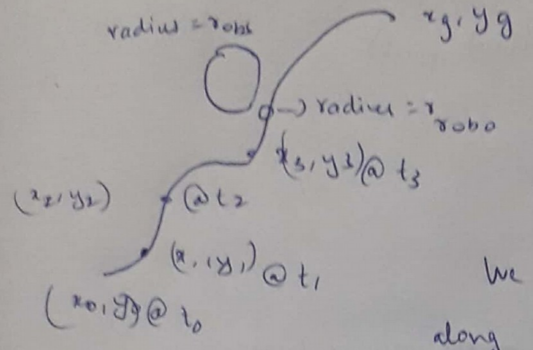
Now, we can apply the constraints

at
$t=0$ $x=0$ $y=0$
$t=0$ $\dot{x}=0$ $\dot{y}=0$
$t=2$ $x=2.5$ $y=2.5$
$t=2$ $\dot{x}=0$ $\dot{y}=0$
$t=5$ $x=9$ $y=5$
$t=5$ $\dot{x}=0$ $\dot{y}=0$

Now we can linearize and solve for $Ax=b$, same we do for y and plot in $t=0.1s$ time frames.

Bonus :

Approach 1 :



We have

$$(x_n - x_{ob})^2 + (y_n - y_{ob})^2 > R^2$$

$$-\left[(x_n - x_{ob})^2 + (y_y - y_{ob})^2 - R^2\right] < 0$$

We need to linearize the function along with the constraints for the obstacle.

So we get

$$-\left[(x_3 - x_{ob})^2 + (y_3 - y_{ob})^2 - R^2\right] < 0$$

Expanding

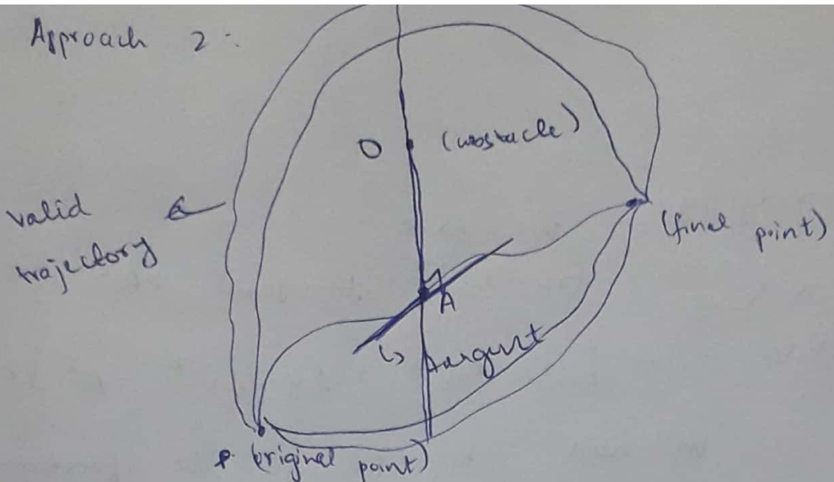
$$-\left[(x_0 + \dot{x}_0 + \dot{x}_1 + \dot{x}_2\,\delta t - x_{ob})^2 + (y_0 + (\dot{y}_0 + \dot{y}_1 + \dot{y}_2)\delta t - y_{ob})^2 - R^2\right] < 0$$

$$-\left[(x_0 - x_{ob} + (\dot{x}_0 + \dot{x}_1 + \dot{x}_2)\delta t)^2 + (y_0 - y_{ob} + (\dot{y}_0 + \dot{y}_1 + \dot{y}_2)\delta t)^2 - R^2\right] < 0$$

Now, we can apply Multivariate Taylor series for the approximation.

In our case, we tried to find 3 waypoints in order to ensure that the trajectory does not pass through a radius of $(\beta + \varepsilon)$ where ε is some threshold value from a center of (5,4)

Approach 2 :



From the obstacl and the ~~original~~ trajectory, we can find the point on the trajectory that is closest the line. Once we find this point A, we draw the tangent to the trajectory at point A and draw the perpendicular from O. Now that we have this ray OA, we can draw the trajectories for the original point to the final point that satisfy a certain radius and threshold constraint. This now generates a set of trajectories that are valid. To choose the most optimum, we find the trajectory for which the jerk is 0.