Kunci Jawaban Ulangan Bentuk Pangkat dan Akar (Seri 001)

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1. Simplify

$$\left((a^{1/4}+b^{1/2})\frac{a+a^{3/4}b^{1/2}+a^{1/4}b^{3/2}+b^2}{a^{1/2}+2a^{1/4}b^{1/2}+b} + \frac{3b^{1/2}\left(a^{1/2}-b\right)}{a^{-1/4}(a^{1/4}-b^{1/2})} \right)^{-1/3} : (a^{1/4}+b^{1/2})^{-1}$$

Solution:

$$\frac{A \to (a^{1/4}+b^{1/2})}{B \to \left((a^{1/4}+b^{1/2})\frac{a+a^{3/4}b^{1/2}+a^{1/4}b^{3/2}+b^2}{a^{1/2}+2a^{1/4}b^{1/2}+b} + \frac{3b^{1/2}\left(a^{1/2}-b\right)}{a^{-1/4}(a^{1/4}-b^{1/2})} \right)^{1/3}}$$

$$B = \left((a^{1/4}+b^{1/2})\frac{a+a^{3/4}b^{1/2}+a^{1/4}b^{3/2}+b^2}{a^{1/2}+2a^{1/4}b^{1/2}+b} + \frac{3b^{1/2}\left(a^{1/2}-b\right)}{a^{-1/4}(a^{1/4}-b^{1/2})} \right)^{1/3}$$

$$= \left((a^{1/4}+b^{1/2})\frac{a+a^{3/4}b^{1/2}+a^{1/4}b^{3/2}+b^2}{(a^{1/4}+b^{1/2})^2} + \frac{3b^{1/2}\left(a^{1/4}-b^{1/2}\right)\left(a^{1/4}+b^{1/2}\right)}{a^{-1/4}(a^{1/4}-b^{1/2})} \right)^{1/3}$$

$$= \left(\frac{a+a^{3/4}b^{1/2}+a^{1/4}b^{3/2}+b^2}{(a^{1/4}+b^{1/2})} + \frac{3b^{1/2}\left(a^{1/4}+b^{1/2}\right)}{a^{-1/4}} \right)^{1/3}$$

$$= \left(\frac{a+a^{3/4}b^{1/2}+a^{1/4}b^{3/2}+b^2+3b^{1/2}\left(a^{1/2}+a^{1/4}b^{1/2}\right)\left(a^{1/4}+b^{1/2}\right)}{(a^{1/4}+b^{1/2})} \right)^{1/3}$$

$$= \left(\frac{a+a^{3/4}b^{1/2}+a^{1/4}b^{3/2}+b^2+(3b^{1/2}a^{1/2}+3a^{1/4}b)a^{1/4}+(3b^{1/2}a^{1/2}+3a^{1/4}b)b^{1/2}}{(a^{1/4}+b^{1/2})} \right)^{1/3}$$

$$= \left(\frac{a+a^{3/4}b^{1/2}+a^{1/4}b^{3/2}+b^2+(3b^{1/2}a^{3/4}+3a^{1/2}b)+(3ba^{1/2}+3a^{1/4}b^{3/2})}{(a^{1/4}+b^{1/2})} \right)^{1/3}$$

$$= \left(\frac{a+4a^{3/4}b^{1/2}+4a^{1/4}b^{3/2}+6a^{1/2}b+b^2}{(a^{1/4}+b^{1/2})} \right)^{1/3}$$

$$= \left(\frac{(a^{1/4} + b^{1/2})^4}{(a^{1/4} + b^{1/2})} \right)^{1/3} = (a^{1/4} + b^{1/2})$$

So $A/B = 1$.

2. Simplify

$$\left(\frac{(1 + a^{-1/2})^{1/6}}{(1 + a^{1/2})^{-1/3}} - \frac{(a^{1/2} - 1)^{1/3}}{(1 - a^{-1/2})^{-1/6}} \right)^{-2} \cdot \frac{(1/3)a^{1/12}}{\sqrt{a} + \sqrt{a - 1}}$$

Solution:

$$A = \left(\frac{(1 + a^{-1/2})^{1/6}}{(1 + a^{1/2})^{-1/3}} - \frac{(a^{1/2} - 1)^{1/3}}{(1 - a^{-1/2})^{-1/6}} \right)^{-2}, \quad B = \frac{(1/3)a^{1/12}}{\sqrt{a} + \sqrt{a - 1}}$$

$$A = \left(\frac{(1 + a^{-1/2})^{1/6}}{(1 + a^{1/2})^{-1/3}} - \frac{(a^{1/2} - 1)^{1/3}}{(1 - a^{-1/2})^{-1/6}} \right)^{-2}$$

$$= \left(\frac{(1 + a^{-1/2})^{1/6}(1 - a^{-1/2})^{-1/6} - (a^{1/2} - 1)^{1/3}(1 + a^{1/2})^{-1/3}}{(1 + a^{1/2})^{-1/3}(1 - a^{-1/2})^{-1/6}} \right)^{-2}$$

$$= \left(\frac{\left[\frac{(1+a^{-1/2})}{(1-a^{-1/2})}\right]^{1/6} - \left[\frac{(a^{1/2}-1)}{(1+a^{1/2})}\right]^{1/3}}{(1 + a^{1/2})^{-1/3}(1 - a^{-1/2})^{-1/6}} \right)^{-2}$$

$$= \left((1 + a^{1/2})^{1/3}(1 + a^{-1/2})^{1/6} - (1 - a^{-1/2})^{1/6}(a^{1/2} - 1)^{1/3}\right)^{-2}$$

$$= \frac{1}{\left((1 + a^{1/2})^{1/3}(1 + a^{-1/2})^{1/6} - (1 - a^{-1/2})^{1/6}(a^{1/2} - 1)^{1/3}\right)^2}$$

$$= \frac{1}{(1 + a^{1/2})^{2/3}(1 + a^{-1/2})^{1/3} - 2(a - 1)^{1/3}(1 - a^{-1})^{1/6} + (1 - a^{-1/2})^{1/3}(a^{1/2} - 1)^{2/3}}$$

$$= \frac{1}{a^{-1/6}(1 + a^{1/2}) - 2(a - 1)^{1/3}(1 - a^{-1})^{1/6} + (a^{1/2} - 1)a^{-1/6}}$$

$$= \frac{1}{-2(a-1)^{1/3}(1-a^{-1})^{1/6} + 2a^{1/2}a^{-1/6}} = \frac{a^{1/6}}{2(a^{1/2}-(a-1)^{1/2})}$$

so

$$A \cdot B = \frac{a^{1/6}}{2(a^{1/2}-(a-1)^{1/2})} \frac{(1/3)a^{1/12}}{(\sqrt{a}+\sqrt{a-1})}$$

$$= \frac{a^{1/4}}{6(a-(a-1))} = \frac{a^{1/4}}{6}$$

3. If

$$x + \sqrt{x^2-1} + \frac{1}{x-\sqrt{x^2-1}} = 20$$

find the value of

$$x^2 + \sqrt{x^4-1} + \frac{1}{x^2+\sqrt{x^4-1}}$$

Solution:

Notice that

$$\frac{(x+\sqrt{x^2-1})(x-\sqrt{x^2-1})}{(x-\sqrt{x^2-1})} + \frac{1}{x-\sqrt{x^2-1}} = 20$$

$$\frac{2}{x-\sqrt{x^2-1}} = 20$$

$$x - \sqrt{x^2-1} = 1/10$$

this implies that

$$(2x)/10 = (1/100) + 1$$

$$x = (101/200)$$

Now

$$x^2 + \sqrt{x^4-1} + \frac{1}{x^2+\sqrt{x^4-1}} = \frac{1}{x^2-\sqrt{x^4-1}} + \frac{1}{x^2+\sqrt{x^4-1}} = \frac{2x^2}{1}$$

So the answer is

$$x^2 + \sqrt{x^4-1} + \frac{1}{x^2+\sqrt{x^4-1}} = 2(101/20)^2$$

4. Simplify the following:

$$\frac{(38+17\sqrt{5})^{1/3}-(17\sqrt{5}-38)^{1/3}}{((1+3x)+(3+x)\sqrt{x})^{1/3}-((3+x)\sqrt{x}-(1+3x))^{1/3}}$$

Solution:

$$\begin{aligned}
&\frac{(38+17\sqrt{5})^{1/3}-(17\sqrt{5}-38)^{1/3}}{((1+3x)+(3+x)\sqrt{x})^{1/3}-((3+x)\sqrt{x}-(1+3x))^{1/3}}\\
&=\frac{(38+17\sqrt{5})^{1/3}-(17\sqrt{5}-38)^{1/3}}{((\sqrt{x}+1)^3)^{1/3}-((\sqrt{x}-1)^3)^{1/3}}\\
&=\frac{(38+17\sqrt{5})^{1/3}-(17\sqrt{5}-38)^{1/3}}{2}
\end{aligned}$$

now notice that

$$(2+\sqrt{5})^3=(9+4\sqrt{5})(2+\sqrt{5})=18+17\sqrt{5}+20=38+17\sqrt{5}$$

$$(\sqrt{5}-2)^3=(9-4\sqrt{5})(\sqrt{5}-2)=-38+17\sqrt{5}$$

so the original expression is equivalent with

$$\frac{(2+\sqrt{5})-(\sqrt{5}-2)}{2}=2$$

5. Find the value of

$$\frac{1}{(\sqrt{1}+\sqrt{2})(1^{1/4}+2^{1/4})}+\frac{1}{(\sqrt{2}+\sqrt{3})(2^{1/4}+3^{1/4})}+...+\frac{1}{(\sqrt{255}+\sqrt{256})(255^{1/4}+256^{1/4})}$$

Solution:

We have to rationalize the denominator.

$$\frac{1}{(\sqrt{1}+\sqrt{2})(1^{1/4}+2^{1/4})}=\frac{\sqrt{1}-\sqrt{2}}{(1-2)(1^{1/4}+2^{1/4})}=\frac{(\sqrt{1}-\sqrt{2})(1^{1/4}-2^{1/4})}{(1-2)(\sqrt{1}-\sqrt{2})}=2^{1/4}-1^{1/4}$$

The others also will have the same pattern, so the summation is equivalent with:

$$(2^{1/4} - 1^{1/4}) + (3^{1/4} - 2^{1/4}) + (4^{1/4} - 3^{1/4}) + ... + (256^{1/4} - 255^{1/4})$$
$$= 256^{1/4} - 1$$

6. Find value of x that satisfy

$$(3^{1/5})^x + (3^{1/10})^{x-10} = 84$$

Solution:

$$(3^{1/5})^x + (3^{1/10})^{x-10} = 84$$

Let $a = 3^{1/5}$, then

$$a^x + (\sqrt{a})^{x-10} = 84$$
$$a^x + (\sqrt{a})^x a^{-5} = 84$$

Let $y = \sqrt{a}^x$, then

$$y^2 + \frac{y}{3} - 84 = 0$$

$$y_{1,2} = \frac{-(1/3) \pm \sqrt{(1/9) - 4(-84)}}{2} = \frac{-1 \pm \sqrt{1 + 9(336)}}{6} = \frac{-1 \pm 55}{6}$$

the negative value is impossible, so we just take the positive value $y = 9$.

$$a^{x/2} = 3^{x/10} = 9 = 3^2$$

$$(x/10) = 2 \implies x = 20$$

7. Raymond has a number of square tiles, each measuring 1cm by 1cm. He tries to put these small square tiles together to form a larger square of side length n cm, but finds that he has 92 tiles left over. If he had increased the side length to $(n + 2)$cm, he would have been 100 tiles short of completing the larger square. How many tiles does Raymond have?

Solution:

For side length n, the square needs n^2 tiles. 92 tiles left over (*sisa*) means that Raymond has n^2+92 tiles, let the number of tiles Raymond has be r.

$$r = n^2 + 92$$

If the side length is $n+2$ then the square needs $(n+2)^2$ tiles. 100 tiles short means that Raymond needs 100 tiles more for the larger square, so

$$r = (n+2)^2 - 100$$

so we get

$$n^2 - (n+2)^2 + 192 = 0$$

$$(2n+2)(2) = 192$$

$$n = 47$$

So $r = 92 + 47^2$