Kunci Jawaban Ulangan Bentuk Pangkat dan Akar (seri 002)

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July 26, 2019

1. Simplify
$$\frac{5(6^{11}+6^{10}+...+6^2+6+1)}{6^6-1}-6^6$$
Solution: Let $s = 1+6+6^2+...+6^{10}+6^{11}$, then $6s = 6+6^2+...+6^{12}$, and we have
$$(1-6)s = 1-6^{12} \implies s = \frac{1-6^{12}}{-5}$$
so we have
$$\begin{aligned}\frac{5(6^{11}+6^{10}+...+6^2+6+1)}{6^6-1}-6^6 &= \frac{6^{12}-1}{6^6-1}-6^6\\ &= (6^6+1)-6^6 = 1\end{aligned}$$

2. Simplify
$$\frac{a+b}{a^{2/3}-a^{1/3}b^{1/3}+b^{2/3}}-\frac{a-b}{a^{2/3}+a^{1/3}b^{1/3}+b^{2/3}}-\frac{a^{2/3}-b^{2/3}}{a^{1/3}-b^{1/3}}$$
Solution:

Notice that all powers are multiple of 1/3. Now let $x = a^{1/3}, y = b^{1/3}$, then the form becomes:
$$\frac{x^3+y^3}{x^2-xy+y^2}-\frac{x^3-y^3}{x^2+xy+y^2}-\frac{x^2-y^2}{x-y}$$

$$= \frac{(x+y)(x^2-xy+y^2)}{x^2-xy+y^2} - \frac{(x-y)(x^2+xy+y^2)}{x^2+xy+y^2} - \frac{x^2-y^2}{x-y}$$

$$= (x+y)-(x-y)-(x+y) = y-x = b^{1/3}-a^{1/3}$$

3. pass

4. Simplify

$$\sqrt{2\left[\sqrt{3}\left(\sqrt{7+4\sqrt{3}}\right)-\sqrt{2}\left(\sqrt{2+\sqrt{3}}\right)\right]\sqrt{3+\sqrt{5-\sqrt{13+\sqrt{48}}}}}$$

Solution:

Notice that

$$(2+\sqrt{3})^2 = 4+4\sqrt{3}+3 = 7+4\sqrt{3}$$
$$(1+\sqrt{3})^2 = 1+2\sqrt{3}+3 = 4+2\sqrt{3}$$
$$(1+2\sqrt{3})^2 = 1+4\sqrt{3}+12 = 13+4\sqrt{3}$$

So the expression becomes:

$$\sqrt{2\left[\sqrt{3}\left(2+\sqrt{3}\right)-\left(1+\sqrt{3}\right)\right]\sqrt{3+\sqrt{5-(1+2\sqrt{3})}}}$$

$$= \sqrt{2\left[2+\sqrt{3}\right]\sqrt{3+\sqrt{4-2\sqrt{3}}}}$$

$$= \sqrt{2\left[2+\sqrt{3}\right]\sqrt{3+\sqrt{(1-\sqrt{3})^2}}} = \sqrt{2\left[2+\sqrt{3}\right]\sqrt{4-\sqrt{3}}}$$

$$= \sqrt{(1+\sqrt{3})^2\sqrt{4-\sqrt{3}}} = (1+\sqrt{3})(4-\sqrt{3})^{1/4}$$

5. Given $x = \frac{1}{2}\left(\sqrt{a/b} - \sqrt{b/a}\right)$, with $a, b > 0$. Evaluate

$$\frac{2a\sqrt{1+x^2}}{x+\sqrt{1+x^2}}$$

Solution:

$$\frac{A \to 2a\sqrt{1+x^2}}{x+\sqrt{1+x^2}}$$

$$x^2 = \frac{((a/b) - 2 + (b/a))}{4} = \frac{a^2+b^2}{4ab} - \frac{1}{2}$$

$$1 + x^2 = (1/2) + \frac{a^2+b^2}{4ab} = \frac{2ab + a^2 + b^2}{4ab} = \frac{(a+b)^2}{4ab}$$

so we have

$$\sqrt{1+x^2} = \frac{(a+b)}{2\sqrt{ab}} = \frac{(a+b)\sqrt{ab}}{2ab}$$

$$A = \frac{(a+b)\sqrt{ab}}{b}$$

The expression becomes:

$$\frac{\frac{(a+b)\sqrt{ab}}{b}}{x+\sqrt{1+x^2}}$$

$$= \frac{(a+b)\sqrt{ab}}{b} \cdot (\sqrt{1+x^2} - x)$$

$$= \frac{(a+b)\sqrt{ab}}{b} \cdot \left[\frac{(a+b)\sqrt{ab}}{2ab} - \frac{1}{2}\left(\sqrt{a/b} - \sqrt{b/a}\right)\right]$$

$$= \frac{(a+b)^2 ab}{2ab^2} - \left(\frac{(a+b)a}{2b} - \frac{(a+b)b}{2b}\right)$$

$$= \frac{(a+b)^2 - (a^2 - b^2)}{2b}$$

$$= \frac{2ab + 2b^2}{2b} = a + b$$

6. If $x = \sqrt{\frac{n-1}{n+1}}$, show that

$$\left(\frac{x}{x-1}\right)^2 + \left(\frac{x}{x+1}\right)^2 = n(n-1)$$

Solution:

$$x - 1 = \sqrt{\frac{n-1}{n+1}} - \sqrt{\frac{n+1}{n+1}} = \frac{\sqrt{n-1} - \sqrt{n+1}}{\sqrt{n+1}}$$

$$x + 1 = \sqrt{\frac{n-1}{n+1}} + \sqrt{\frac{n+1}{n+1}} = \frac{\sqrt{n-1} + \sqrt{n+1}}{\sqrt{n+1}}$$

$$\frac{x}{x-1} = \frac{\sqrt{n-1}}{\sqrt{n-1} - \sqrt{n+1}} = \frac{\sqrt{n-1}(\sqrt{n-1} + \sqrt{n+1})}{-2} = \frac{((n-1) + \sqrt{n^2-1})}{-2}$$

$$\frac{x}{x+1} = \frac{\sqrt{n-1}}{\sqrt{n-1} + \sqrt{n+1}} = \frac{\sqrt{n-1}(\sqrt{n-1} - \sqrt{n+1})}{-2} = \frac{((n-1) - \sqrt{n^2-1})}{-2}$$

So

$$\left(\frac{x}{x-1}\right)^2 + \left(\frac{x}{x+1}\right)^2$$

$$= \frac{(n-1)^2 + 2(n-1)\sqrt{n^2-1} + (n^2-1)}{4} + \frac{(n-1)^2 - 2(n-1)\sqrt{n^2-1} + (n^2-1)}{4}$$

$$= \frac{(n-1)^2 + (n^2-1)}{2} = \frac{(n-1)\left[(n-1) + (n+1)\right]}{2} = n(n-1)$$