

Team notebook

September 3, 2015

Contents

1 Algorithms	1
1.1 sliding window	1
2 Data structures	2
2.1 hash table	2
2.2 heavy light decomposition	2
2.3 segment tree	3
2.4 sparse table	4
2.5 splay tree	4
2.6 trie	5
3 Graphs	6
3.1 directed mst	6
3.2 eulerian path	6
3.3 karp min mean cycle	7
3.4 konig's theorem	8
3.5 minimum path cover in DAG	8
3.6 tarjan scc	8
3.7 two sat (with kosaraju)	9
4 Math	10
4.1 euler totient	10
4.2 fft	10
5 Matrix	12
5.1 matrix	12
6 Misc	12
6.1 Template Java	12
6.2 io	13

7 Number theory	13
7.1 convolution	13
7.2 crt	14
7.3 discrete logarithm	14
7.4 ext euclidean	15
7.5 highest exponent factorial	15
7.6 miller rabin	15
7.7 mod inv	16
7.8 mod mul	16
7.9 mod pow	16
7.10 number theoretic transform	16
7.11 pollard rho factorize	17
8 Strings	17
8.1 minimal string rotation	17
8.2 suffix array	18
8.3 z algorithm	18

1 Algorithms

1.1 sliding window

```
/*
 * Given an array ARR and an integer K, the problem boils down to
 * computing for each index i: min(ARR[i], ARR[i-1], ..., ARR[i-K+1]).
 * if mx == true, returns the maximum.
 * http://people.cs.uct.ac.za/~ksmith/articles/sliding_window_minimum.html
 * */

vector<int> sliding_window_minmax(vector<int> & ARR, int K, bool mx) {
```

```

deque< pair<int, int> > window;
vector<int> ans;
for (int i = 0; i < ARR.size(); i++) {
    if (mx) {
        while (!window.empty() && window.back().first <= ARR[i])
            window.pop_back();
    } else {
        while (!window.empty() && window.back().first >= ARR[i])
            window.pop_back();
    }
    window.push_back(make_pair(ARR[i], i));

    while(window.front().second <= i - K)
        window.pop_front();

    ans.push_back(window.front().first);
}
return ans;
}

```

2 Data structures

2.1 hash table

```

/**
 * Micro hash table, can be used as a set.
 * Very efficient vs std::set
 * */

const int MN = 1001;
struct ht {
    int _s[(MN + 10) >> 5];
    int len;
    void set(int id) {
        len++;
        _s[id >> 5] |= (1LL << (id & 31));
    }
    bool is_set(int id) {
        return _s[id >> 5] & (1LL << (id & 31));
    }
};

```

2.2 heavy light decomposition

```

// Heavy-Light Decomposition
struct TreeDecomposition {
    vector<int> g[MAXN], c[MAXN];
    int s[MAXN]; // subtree size
    int p[MAXN]; // parent id
    int r[MAXN]; // chain root id
    int t[MAXN]; // index used in segtree/bit/...
    int d[MAXN]; // depht
    int ts;

    void dfs(int v, int f) {
        p[v] = f;
        s[v] = 1;
        if (f != -1) d[v] = d[f] + 1;
        else d[v] = 0;

        for (int i = 0; i < g[v].size(); ++i) {
            int w = g[v][i];
            if (w != f) {
                dfs(w, v);
                s[v] += s[w];
            }
        }
    }

    void hld(int v, int f, int k) {
        t[v] = ts++;
        c[k].push_back(v);
        r[v] = k;

        int x = 0, y = -1;
        for (int i = 0; i < g[v].size(); ++i) {
            int w = g[v][i];
            if (w != f) {
                if (s[w] > x) {
                    x = s[w];
                    y = w;
                }
            }
        }
        if (y != -1) {
            hld(y, v, k);
        }
    }
}

```

```

    for (int i = 0; i < g[v].size(); ++i) {
        int w = g[v][i];
        if (w != f && w != y) {
            hld(w, v, w);
        }
    }
}

void init(int n) {
    for (int i = 0; i < n; ++i) {
        g[i].clear();
    }
}

void add(int a, int b) {
    g[a].push_back(b);
    g[b].push_back(a);
}

void build() {
    ts = 0;
    dfs(0, -1);
    hld(0, 0, 0);
}
};

```

2.3 segment tree

```

/**
 * Taken from: http://codeforces.com/blog/entry/18051
 */

const int N = 1e5; // limit for array size
int n; // array size
int t[2 * N];

void build() { // build the tree
    for (int i = n - 1; i > 0; --i) t[i] = t[i<<1] + t[i<<1|1];
}

// Single modification, range query.
void modify(int p, int value) { // set value at position p

```

```

    for (t[p += n] = value; p > 1; p >>= 1) t[p>>1] = t[p] + t[p^1];
}

```

```

int query(int l, int r) { // sum on interval [l, r]
    int res = 0;
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) res += t[l++];
        if (r&1) res += t[--r];
    }
    return res;
}

```

// Range modification, single query.

```

void modify(int l, int r, int value) {
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) t[l++] += value;
        if (r&1) t[--r] += value;
    }
}

```

```

int query(int p) {
    int res = 0;
    for (p += n; p > 0; p >>= 1) res += t[p];
    return res;
}

```

```

/**
 * If at some point after modifications we need to inspect all the
 * elements in the array, we can push all the modifications to the
 * leaves using the following code. After that we can just traverse
 * elements starting with index n. This way we reduce the complexity
 * from  $O(n \log(n))$  to  $O(n)$  similarly to using build instead of n
 * modifications.
 */

```

```

void push() {
    for (int i = 1; i < n; ++i) {
        t[i<<1] += t[i];
        t[i<<1|1] += t[i];
        t[i] = 0;
    }
}

```

// Non commutative combiner functions.

```

void modify(int p, const S& value) {
    for (t[p += n] = value; p >= 1; ) t[p] = combine(t[p<<1], t[p<<1|1]);
}

S query(int l, int r) {
    S resl, resr;
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) resl = combine(resl, t[l++]);
        if (r&1) resr = combine(t[--r], resr);
    }
    return combine(resl, resr);
}

// To be continued ...

```

2.4 sparse table

```

// RMQ.
const int MN = 100000 + 10; // Max number of elements
const int ML = 18; // ceil(log2(MN));

struct st {
    int data[MN];
    int M[MN][ML];
    int n;

    void read(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            cin >> data[i];
    }

    void build() {
        for (int i = 0; i < n; ++i)
            M[i][0] = data[i];
        for (int j = 1, p = 2, q = 1; p <= n; ++j, p <= 1, q <= 1)
            for (int i = 0; i + p - 1 < n; ++i)
                M[i][j] = max(M[i][j - 1], M[i + q][j - 1]);
    }

    int query(int b, int e) {
        int k = log2(e - b + 1);
        return max(M[b][k], M[e + 1 - (1<<k)][k]);
    }
}

```

```

}
};

```

2.5 splay tree

```

using namespace std;
#include<bits/stdc++.h>
#define D(x) cout<<x<<endl;

typedef int T;

struct node{
    node *left, *right, *parent;
    T key;
    node (T k) : key(k), left(0), right(0), parent(0) {}
};

struct splay_tree{

    node *root;

    void right_rot(node *x) {
        node *p = x->parent;
        if (x->parent == p->parent) {
            if (x->parent->left == p) x->parent->left = x;
            if (x->parent->right == p) x->parent->right = x;
        }
        if (p->left == x->right) p->left->parent = p;
        x->right = p;
        p->parent = x;
    }

    void left_rot(node *x) {
        node *p = x->parent;
        if (x->parent == p->parent) {
            if (x->parent->left == p) x->parent->left = x;
            if (x->parent->right == p) x->parent->right = x;
        }
        if (p->right == x->left) p->right->parent = p;
        x->left = p;
        p->parent = x;
    }
}

```

```

void splay(node *x, node *fa = 0) {

    while( x->parent != fa and x->parent != 0) {
        node *p = x->parent;
        if (p->parent == fa)
            if (p->right == x)
                left_rot(x);
            else
                right_rot(x);
        else {
            node *gp = p->parent; //grand parent
            if (gp->left == p)
                if (p->left == x)
                    right_rot(x),right_rot(x);
                else
                    left_rot(x),right_rot(x);
            else
                if (p->left == x)
                    right_rot(x), left_rot(x);
                else
                    left_rot(x), left_rot(x);
        }
    }
    if (fa == 0) root = x;
}

void insert(T key) {
    node *cur = root;
    node *pcur = 0;
    while (cur) {
        pcur = cur;
        if (key > cur->key) cur = cur->right;
        else cur = cur->left;
    }
    cur = new node(key);
    cur->parent = pcur;
    if (!pcur) root = cur;
    else if (key > pcur->key ) pcur->right = cur;
    else pcur->left = cur;
    splay(cur);
}

node *find(T key) {
    node *cur = root;

```

```

        while (cur) {
            if (key > cur->key) cur = cur->right;
            else if(key < cur->key) cur = cur->left;
            else return cur;
        }
        return 0;
    }

    splay_tree(){ root = 0;};
};

```

2.6 trie

```

const int MN = 26; // size of alphabet
const int MS = 100010; // Number of states.

struct trie{
    struct node{
        int c;
        int a[MN];
    };

    node tree[MS];
    int nodes;

    void clear(){
        tree[nodes].c = 0;
        memset(tree[nodes].a, -1, sizeof tree[nodes].a);
        nodes++;
    }

    void init(){
        nodes = 0;
        clear();
    }

    int add(const string &s, bool query = 0){
        int cur_node = 0;
        for(int i = 0; i < s.size(); ++i){
            int id = gid(s[i]);
            if(tree[cur_node].a[id] == -1){
                if(query) return 0;
                tree[cur_node].a[id] = nodes;

```

```

        clear();
    }
    cur_node = tree[cur_node].a[id];
}
if(!query) tree[cur_node].c++;
return tree[cur_node].c;
}
};

```

3 Graphs

3.1 directed mst

```

const int inf = 1000000 + 10;

struct edge {
    int u, v, w;
    edge() {}
    edge(int a, int b, int c) : u(a), v(b), w(c) {}
};

/**
 * Computes the minimum spanning tree for a directed graph
 * - edges : Graph description in the form of list of edges.
 *   each edge is: From node u to node v with cost w
 * - root : Id of the node to start the DMST.
 * - n : Number of nodes in the graph.
 */

int dmst(vector<edge> &edges, int root, int n) {
    int ans = 0;
    int cur_nodes = n;
    while (true) {
        vector<int> lo(cur_nodes, inf), pi(cur_nodes, inf);
        for (int i = 0; i < edges.size(); ++i) {
            int u = edges[i].u, v = edges[i].v, w = edges[i].w;
            if (w < lo[v] and u != v) {
                lo[v] = w;
                pi[v] = u;
            }
        }
    }
}

```

```

lo[root] = 0;
for (int i = 0; i < lo.size(); ++i) {
    if (i == root) continue;
    if (lo[i] == inf) return -1;
}
int cur_id = 0;
vector<int> id(cur_nodes, -1), mark(cur_nodes, -1);
for (int i = 0; i < cur_nodes; ++i) {
    ans += lo[i];
    int u = i;
    while (u != root and id[u] < 0 and mark[u] != i) {
        mark[u] = i;
        u = pi[u];
    }
    if (u != root and id[u] < 0) { // Cycle
        for (int v = pi[u]; v != u; v = pi[v])
            id[v] = cur_id;
        id[u] = cur_id++;
    }
}

if (cur_id == 0)
    break;

for (int i = 0; i < cur_nodes; ++i)
    if (id[i] < 0) id[i] = cur_id++;

for (int i = 0; i < edges.size(); ++i) {
    int u = edges[i].u, v = edges[i].v, w = edges[i].w;
    edges[i].u = id[u];
    edges[i].v = id[v];
    if (id[u] != id[v])
        edges[i].w -= lo[v];
}
cur_nodes = cur_id;
root = id[root];
}

return ans;
}

```

3.2 eulerian path

```
// Taken from https://github.com/lbv/pc-code/blob/master/code/graph.cpp
// Eulerian Trail
```

```
struct Euler {
    ELV adj; IV t;
    Euler(ELV Adj) : adj(Adj) {}
    void build(int u) {
        while(! adj[u].empty()) {
            int v = adj[u].front().v;
            adj[u].erase(adj[u].begin());
            build(v);
        }
        t.push_back(u);
    }
};

bool eulerian_trail(IV &trail) {
    Euler e(adj);
    int odd = 0, s = 0;
    /*
        for (int v = 0; v < n; v++) {
            int diff = abs(in[v] - out[v]);
            if (diff > 1) return false;
            if (diff == 1) {
                if (++odd > 2) return false;
                if (out[v] > in[v]) start = v;
            }
        }
    */
    e.build(s);
    reverse(e.t.begin(), e.t.end());
    trail = e.t;
    return true;
}
```

3.3 karp min mean cycle

```
/**
 * Finds the min mean cycle, if you need the max mean cycle
 * just add all the edges with negative cost and print
 * ans * -1
 *
 * test: uva, 11090 - Going in Cycle!!
```

```
*/

const int MN = 1000;
struct edge{
    int v;
    long long w;
    edge(){} edge(int v, int w) : v(v), w(w) {}
};

long long d[MN][MN];
// This is a copy of g because increments the size
// pass as reference if this does not matter.
int karp(vector<vector<edge> > g) {
    int n = g.size();

    g.resize(n + 1); // this is important

    for (int i = 0; i < n; ++i)
        if (!g[i].empty())
            g[n].push_back(edge(i,0));
    ++n;

    for(int i = 0; i < n; ++i)
        fill(d[i], d[i] + (n+1), INT_MAX);

    d[n - 1][0] = 0;

    for (int k = 1; k <= n; ++k) for (int u = 0; u < n; ++u) {
        if (d[u][k - 1] == INT_MAX) continue;
        for (int i = g[u].size() - 1; i >= 0; --i)
            d[g[u][i].v][k] = min(d[g[u][i].v][k], d[u][k - 1] + g[u][i].w);
    }

    bool flag = true;

    for (int i = 0; i < n && flag; ++i)
        if (d[i][n] != INT_MAX)
            flag = false;

    if (flag) {
        return true; // return true if there is no a cycle.
    }

    double ans = 1e15;
```

```

for (int u = 0; u + 1 < n; ++u) {
    if (d[u][n] == INT_MAX) continue;
    double W = -1e15;

    for (int k = 0; k < n; ++k)
        if (d[u][k] != INT_MAX)
            W = max(W, (double)(d[u][n] - d[u][k]) / (n - k));

    ans = min(ans, W);
}

// printf("%.2lf\n", ans);
cout << fixed << setprecision(2) << ans << endl;

return false;
}

```

3.4 konig's theorem

In any bipartite graph, the number of edges in a maximum matching equals the number of vertices in a minimum vertex cover

3.5 minimum path cover in DAG

Given a directed acyclic graph $G = (V, E)$, we are to find the minimum number of vertex-disjoint paths to cover each vertex in V .

We can construct a bipartite graph $G' = (V_{out} \cup V_{in}, E')$ from G , where :

$$V_{out} = \{v \in V : v \text{ has positive out-degree}\}$$

$$V_{in} = \{v \in V : v \text{ has positive in-degree}\}$$

$$E' = \{(u, v) \in V_{out} \times V_{in} : (u, v) \in E\}$$

Then it can be shown, via König's theorem, that G' has a matching of size m if and only if there exists $n - m$ vertex-disjoint paths that cover each vertex in G , where n is the number of vertices in G and m is the maximum cardinality bipartite matching in G' .

Therefore, the problem can be solved by finding the maximum cardinality matching in G' instead.

NOTE: If the paths are not necessarily disjoint, find the transitive closure and solve the problem for disjoint paths.

3.6 tarjan scc

```

const int MN = 20002;

struct tarjan_scc {
    int scc[MN], low[MN], d[MN], stacked[MN];
    int ticks, current_scc;
    deque<int> s; // used as stack.

    tarjan_scc() {}

    void init () {
        memset(scc, -1, sizeof scc);
        memset(d, -1, sizeof d);
        memset(stacked, 0, sizeof stacked);
        s.clear();
        ticks = current_scc = 0;
    }

    void compute(vector<vector<int>> &g, int u) {
        d[u] = low[u] = ticks++;
        s.push_back(u);
        stacked[u] = true;
        for (int i = 0; i < g[u].size(); ++i) {
            int v = g[u][i];
            if (d[v] == -1)
                compute(g, v);
            if (stacked[v]) {
                low[u] = min(low[u], low[v]);
            }
        }

        if (d[u] == low[u]) { // root
            int v;
            do {
                v = s.back(); s.pop_back();
                stacked[v] = false;
                scc[v] = current_scc;
            } while (u != v);
            current_scc++;
        }
    }
};

```

3.7 two sat (with kosaraju)

```
/**
 * Given a set of clauses (a1 v a2)^(a2 v a3)....
 * this algorithm find a solution to it set of clauses.
 * test: http://lightoj.com/volume_showproblem.php?problem=1251
 */

#include<bits/stdc++.h>
using namespace std;
#define MAX 100000
#define endl '\n'

vector<int> G[MAX];
vector<int> GT[MAX];
vector<int> Ftime;
vector<vector<int> > SCC;
bool visited[MAX];
int n;

void dfs1(int n){
    visited[n] = 1;

    for (int i = 0; i < G[n].size(); ++i) {
        int curr = G[n][i];
        if (visited[curr]) continue;
        dfs1(curr);
    }

    Ftime.push_back(n);
}

void dfs2(int n, vector<int> &scc) {
    visited[n] = 1;
    scc.push_back(n);

    for (int i = 0; i < GT[n].size(); ++i) {
        int curr = GT[n][i];
        if (visited[curr]) continue;
        dfs2(curr, scc);
    }
}
```

```
void kosaraju() {
    memset(visited, 0, sizeof visited);

    for (int i = 0; i < 2 * n ; ++i) {
        if (!visited[i]) dfs1(i);
    }

    memset(visited, 0, sizeof visited);
    for (int i = Ftime.size() - 1; i >= 0; i--) {
        if (visited[Ftime[i]]) continue;
        vector<int> _scc;
        dfs2(Ftime[i], _scc);
        SCC.push_back(_scc);
    }
}

/**
 * After having the SCC, we must traverse each scc, if in one SCC are -b
 * y b, there is not a solution.
 * Otherwise we build a solution, making the first "node" that we find
 * truth and its complement false.
 */

bool two_sat(vector<int> &val) {
    kosaraju();
    for (int i = 0; i < SCC.size(); ++i) {
        vector<bool> tmpvisited(2 * n, false);
        for (int j = 0; j < SCC[i].size(); ++j) {
            if (tmpvisited[SCC[i][j] ^ 1]) return 0;
            if (val[SCC[i][j]] != -1) continue;
            else {
                val[SCC[i][j]] = 0;
                val[SCC[i][j] ^ 1] = 1;
            }
            tmpvisited[SCC[i][j]] = 1;
        }
    }
    return 1;
}

// Example of use

int main() {
```

```

int m, u, v, nc = 0, t; cin >> t;
// n = "nodes" number, m = clauses number

while (t--) {
    cin >> m >> n;
    Ftime.clear();
    SCC.clear();
    for (int i = 0; i < 2 * n; ++i) {
        G[i].clear();
        GT[i].clear();
    }

    // (a1 v a2) = (a1 -> a2) = (a2 -> a1)
    for (int i = 0; i < m; ++i) {
        cin >> u >> v;
        int t1 = abs(u) - 1;
        int t2 = abs(v) - 1;
        int p = t1 * 2 + ((u < 0)? 1 : 0);
        int q = t2 * 2 + ((v < 0)? 1 : 0);
        G[p ^ 1].push_back(q);
        G[q ^ 1].push_back(p);
        GT[p].push_back(q ^ 1);
        GT[q].push_back(p ^ 1);
    }

    vector<int> val(2 * n, -1);
    cout << "Case " << ++nc << ": ";
    if (two_sat(val)) {
        cout << "Yes" << endl;
        vector<int> sol;
        for (int i = 0; i < 2 * n; ++i)
            if (i % 2 == 0 and val[i] == 1)
                sol.push_back(i / 2 + 1);
        cout << sol.size() ;

        for (int i = 0; i < sol.size(); ++i) {
            cout << " " << sol[i];
        }
        cout << endl;
    } else {
        cout << "No" << endl;
    }
}
return 0;
}

```

4 Math

4.1 euler totient

```

for (int i = 0; i < MP; ++i) phi[i] = i;

for (int i = 0; primes[i] <= 5000000; ++i) {
    phi[primes[i]] = primes[i] - 1;
    for (int j = 2 * primes[i]; j <= 5000000; j += primes[i]) {
        phi[j] = phi[j] * (primes[i] - 1);
        phi[j] = phi[j] / primes[i];
    }
}

```

4.2 fft

```

/**
 * Fast Fourier Transform.
 * Useful to compute convolutions.
 * computes:
 *   C(f star g)[n] = sum_m(f[m] * g[n - m])
 * for all n.
 * test: icpc live archive, 6886 - Golf Bot
 * */

using namespace std;
#include<bits/stdc++.h>
#define D(x) cout << #x " = " << (x) << endl
#define endl '\n'

const int MN = 262144 << 1;
int d[MN + 10], d2[MN + 10];

const double PI = acos(-1.0);

struct cpx {
    double real, image;
    cpx(double _real, double _image) {
        real = _real;
        image = _image;
    }
}

```

```

    }
    cpx(){}
};

cpx operator + (const cpx &c1, const cpx &c2) {
    return cpx(c1.real + c2.real, c1.image + c2.image);
}

cpx operator - (const cpx &c1, const cpx &c2) {
    return cpx(c1.real - c2.real, c1.image - c2.image);
}

cpx operator * (const cpx &c1, const cpx &c2) {
    return cpx(c1.real*c2.real - c1.image*c2.image, c1.real*c2.image +
        c1.image*c2.real);
}

int rev(int id, int len) {
    int ret = 0;
    for (int i = 0; (1 << i) < len; i++) {
        ret <<= 1;
        if (id & (1 << i)) ret |= 1;
    }
    return ret;
}

cpx A[1 << 20];

void FFT(cpx *a, int len, int DFT) {
    for (int i = 0; i < len; i++)
        A[rev(i, len)] = a[i];
    for (int s = 1; (1 << s) <= len; s++) {
        int m = (1 << s);
        cpx wm = cpx(cos( DFT * 2 * PI / m), sin(DFT * 2 * PI / m));
        for(int k = 0; k < len; k += m) {
            cpx w = cpx(1, 0);
            for(int j = 0; j < (m >> 1); j++) {
                cpx t = w * A[k + j + (m >> 1)];
                cpx u = A[k + j];
                A[k + j] = u + t;
                A[k + j + (m >> 1)] = u - t;
                w = w * wm;
            }
        }
    }
}

```

```

    if (DFT == -1) for (int i = 0; i < len; i++) A[i].real /= len,
        A[i].image /= len;
    for (int i = 0; i < len; i++) a[i] = A[i];
    return;
}

cpx in[1 << 20];

void solve(int n) {
    memset(d, 0, sizeof d);
    int t;
    for (int i = 0; i < n; ++i) {
        cin >> t;
        d[t] = true;
    }
    int m;
    cin >> m;
    vector<int> q(m);
    for (int i = 0; i < m; ++i)
        cin >> q[i];

    for (int i = 0; i < MN; ++i) {
        if (d[i])
            in[i] = cpx(1, 0);
        else
            in[i] = cpx(0, 0);
    }

    FFT(in, MN, 1);
    for (int i = 0; i < MN; ++i) {
        in[i] = in[i] * in[i];
    }
    FFT(in, MN, -1);

    int ans = 0;
    for (int i = 0; i < q.size(); ++i) {
        if (in[q[i]].real > 0.5 || d[q[i]]) {
            ans++;
        }
    }
    cout << ans << endl;
}

int main() {
    ios_base::sync_with_stdio(false);cin.tie(NULL);
}

```

```

int n;
while (cin >> n)
    solve(n);
return 0;
}

```

5 Matrix

5.1 matrix

```

const int MN = 111;
const int mod = 10000;

struct matrix {
    int r, c;
    int m[MN][MN];

    matrix (int _r, int _c) : r (_r), c (_c) {
        memset(m, 0, sizeof m);
    }

    void print() {
        for (int i = 0; i < r; ++i) {
            for (int j = 0; j < c; ++j)
                cout << m[i][j] << " ";
            cout << endl;
        }
    }

    int x[MN][MN];
    matrix & operator *= (const matrix &o) {
        memset(x, 0, sizeof x);
        for (int i = 0; i < r; ++i)
            for (int k = 0; k < c; ++k)
                if (m[i][k] != 0)
                    for (int j = 0; j < c; ++j) {
                        x[i][j] = (x[i][j] + ((m[i][k] * o.m[k][j]) % mod) ) % mod;
                    }
        memcpy(m, x, sizeof(m));
        return *this;
    }
};

```

```

void matrix_pow(matrix b, long long e, matrix &res) {
    memset(res.m, 0, sizeof res.m);
    for (int i = 0; i < b.r; ++i)
        res.m[i][i] = 1;

    if (e == 0) return;
    while (true) {
        if (e & 1) res *= b;
        if ((e >>= 1) == 0) break;
        b *= b;
    }
}

```

6 Misc

6.1 Template Java

```

import java.io.*;
import java.util.StringTokenizer;

public class Template {

    public static void main(String []args) throws IOException {
        Scanner in = new Scanner(System.in);
        PrintWriter out = new PrintWriter(System.out);
        Task solver = new Task();
        solver.solve(in, out);
        out.close();
    }
}

class Task{
    public void solve(Scanner in, PrintWriter out){

    }
}

class Scanner{
    public BufferedReader reader;
    public StringTokenizer st;
}

```

```

public Scanner(InputStream stream){
    reader = new BufferedReader(new InputStreamReader(stream));
    st = null;
}

public String next(){
    while(st == null || !st.hasMoreTokens()){
        try{
            String line = reader.readLine();
            if(line == null) return null;
            st = new StringTokenizer(line);
        }catch (Exception e){
            throw (new RuntimeException());
        }
    }
    return st.nextToken();
}

public int nextInt(){
    return Integer.parseInt(next());
}

public long nextLong(){
    return Long.parseLong(next());
}

public double nextDouble(){
    return Double.parseDouble(next());
}
}

class OutputWriter{
    BufferedWriter writer;

    public OutputWriter(OutputStream stream){
        writer = new BufferedWriter(new OutputStreamWriter(stream));
    }

    public void print(int i) throws IOException {
        writer.write(i);
    }

    public void print(String s) throws IOException {
        writer.write(s);
    }
}

```

```

public void print(char []c) throws IOException {
    writer.write(c);
}

public void close() throws IOException {
    writer.close();
}
}

```

6.2 io

// taken from :
<https://github.com/lbv/pc-code/blob/master/solved/c-e/diablo/diablo.cpp>
 // this is very fast as well :
<https://github.com/lbv/pc-code/blob/master/code/input.cpp>

```

typedef unsigned int u32;
#define BUF 524288
struct Reader {
    char buf[BUF]; char b; int bi, bz;
    Reader() { bi=bz=0; read(); }
    void read() {
        if (bi==bz) { bi=0; bz = fread(buf, 1, BUF, stdin); }
        b = bz ? buf[bi++] : 0; }
    void skip() { while (b > 0 && b <= 32) read(); }
    u32 next_u32() {
        u32 v = 0; for (skip(); b > 32; read()) v = v*10 + b-48; return v; }
    int next_int() {
        int v = 0; bool s = false;
        skip(); if (b == '-') { s = true; read(); }
        for (; 48<=b&&b<=57; read()) v = v*10 + b-48; return s ? -v : v; }
    char next_char() { skip(); char c = b; read(); return c; }
};

```

7 Number theory

7.1 convolution

```

typedef long long int LL;
typedef pair<LL, LL> PLL;

```

```

inline bool is_pow2(LL x) {
    return (x & (x-1)) == 0;
}

inline int ceil_log2(LL x) {
    int ans = 0;
    --x;
    while (x != 0) {
        x >>= 1;
        ans++;
    }
    return ans;
}

/* Returns the convolution of the two given vectors in time proportional
   to n*log(n).
* The number of roots of unity to use nroots_unity must be set so that
   the product of the first
* nroots_unity primes of the vector nth_roots_unity is greater than the
   maximum value of the
* convolution. Never use sizes of vectors bigger than 2^24, if you need
   to change the values of
* the nth roots of unity to appropriate primes for those sizes.
*/
vector<LL> convolve(const vector<LL> &a, const vector<LL> &b, int
    nroots_unity = 2) {
    int N = 1 << ceil_log2(a.size() + b.size());
    vector<LL> ans(N,0), fA(N), fB(N), fC(N);
    LL modulo = 1;
    for (int times = 0; times < nroots_unity; times++) {
        fill(fA.begin(), fA.end(), 0);
        fill(fB.begin(), fB.end(), 0);
        for (int i = 0; i < a.size(); i++) fA[i] = a[i];
        for (int i = 0; i < b.size(); i++) fB[i] = b[i];
        LL prime = nth_roots_unity[times].first;
        LL inv_modulo = mod_inv(modulo % prime, prime);
        LL normalize = mod_inv(N, prime);
        ntfft(fA, 1, nth_roots_unity[times]);
        ntfft(fB, 1, nth_roots_unity[times]);
        for (int i = 0; i < N; i++) fC[i] = (fA[i] * fB[i]) % prime;
        ntfft(fC, -1, nth_roots_unity[times]);
        for (int i = 0; i < N; i++) {
            LL curr = (fC[i] * normalize) % prime;
            LL k = (curr - (ans[i] % prime) + prime) % prime;

```

```

            k = (k * inv_modulo) % prime;
            ans[i] += modulo * k;
        }
        modulo *= prime;
    }
    return ans;
}

```

7.2 crt

```

/**
 * Chinese remainder theorem.
 * Find z such that z % x[i] = a[i] for all i.
 */
long long crt(vector<long long> &a, vector<long long> &x) {
    long long z = 0;
    long long n = 1;
    for (int i = 0; i < x.size(); ++i)
        n *= x[i];

    for (int i = 0; i < a.size(); ++i) {
        long long tmp = (a[i] * (n / x[i])) % n;
        tmp = (tmp * mod_inv(n / x[i], x[i])) % n;
        z = (z + tmp) % n;
    }

    return (z + n) % n;
}

```

7.3 discrete logarithm

```

// Computes x which a ^ x = b mod n.
long long d_log(long long a, long long b, long long n) {
    long long m = ceil(sqrt(n));
    long long aj = 1;
    map<long long, long long> M;
    for (int i = 0; i < m; ++i) {
        if (!M.count(aj))
            M[aj] = i;
    }
}

```

```

    aj = (aj * a) % n;
}

long long coef = mod_pow(a, n - 2, n);
coef = mod_pow(coef, m, n);
// coef = a ^ (-m)
long long gamma = b;
for (int i = 0; i < m; ++i) {
    if (M.count(gamma)) {
        return i * m + M[gamma];
    } else {
        gamma = (gamma * coef) % n;
    }
}
return -1;
}

```

7.4 ext euclidean

```

void ext_euclid(long long a, long long b, long long &x, long long &y,
    long long &g) {
    x = 0, y = 1, g = b;
    long long m, n, q, r;
    for (long long u = 1, v = 0; a != 0; g = a, a = r) {
        q = g / a, r = g % a;
        m = x - u * q, n = y - v * q;
        x = u, y = v, u = m, v = n;
    }
}

```

7.5 highest exponent factorial

```

int highest_exponent(int p, const int &n){
    int ans = 0;
    int t = p;
    while(t <= n){
        ans += n/t;
        t*=p;
    }
    return ans;
}

```

7.6 miller rabin

```

const int rounds = 20;

// checks whether a is a witness that n is not prime, 1 < a < n
bool witness(long long a, long long n) {
    // check as in Miller Rabin Primality Test described
    long long u = n - 1;
    int t = 0;
    while (u % 2 == 0) {
        t++;
        u >>= 1;
    }
    long long next = mod_pow(a, u, n);
    if (next == 1) return false;
    long long last;
    for (int i = 0; i < t; ++i) {
        last = next;
        next = mod_mul(last, last, n);
        if (next == 1) {
            return last != n - 1;
        }
    }
    return next != 1;
}

// Checks if a number is prime with prob 1 - 1 / (2 ^ it)
// D(miller_rabin(99999999999999997LL) == 1);
// D(miller_rabin(999999999999971LL) == 1);
// D(miller_rabin(7907) == 1);
bool miller_rabin(long long n, int it = rounds) {
    if (n <= 1) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    for (int i = 0; i < it; ++i) {
        long long a = rand() % (n - 1) + 1;
        if (witness(a, n)) {
            return false;
        }
    }
    return true;
}

```

7.7 mod inv

```
long long mod_inv(long long n, long long m) {
    long long x, y, gcd;
    ext_euclid(n, m, x, y, gcd);
    if (gcd != 1)
        return 0;
    return (x + m) % m;
}
```

7.8 mod mul

```
// Computes (a * b) % mod
long long mod_mul(long long a, long long b, long long mod) {
    long long x = 0, y = a % mod;
    while (b > 0) {
        if (b & 1)
            x = (x + y) % mod;
        y = (y * 2) % mod;
        b /= 2;
    }
    return x % mod;
}
```

7.9 mod pow

```
// Computes (a ^ exp) % mod.
long long mod_pow(long long a, long long exp, long long mod) {
    long long ans = 1;
    while (exp > 0) {
        if (exp & 1)
            ans = mod_mul(ans, a, mod);
        a = mod_mul(a, a, mod);
        exp >>= 1;
    }
    return ans;
}
```

7.10 number theoretic transform

```
typedef long long int LL;
typedef pair<LL, LL> PLL;

/* The following vector of pairs contains pairs (prime, generator)
 * where the prime has an Nth root of unity for N being a power of two.
 * The generator is a number g s.t g^(p-1)=1 (mod p)
 * but is different from 1 for all smaller powers */
vector<PLL> nth_roots_unity {
    {1224736769,330732430},{1711276033,927759239},{167772161,167489322},
    {469762049,343261969},{754974721,643797295},{1107296257,883865065}};

PLL ext_euclid(LL a, LL b) {
    if (b == 0)
        return make_pair(1,0);
    pair<LL,LL> rc = ext_euclid(b, a % b);
    return make_pair(rc.second, rc.first - (a / b) * rc.second);
}

//returns -1 if there is no unique modular inverse
LL mod_inv(LL x, LL modulo) {
    PLL p = ext_euclid(x, modulo);
    if ( (p.first * x + p.second * modulo) != 1 )
        return -1;
    return (p.first+modulo) % modulo;
}

//Number theory fft. The size of a must be a power of 2
void ntfft(vector<LL> &a, int dir, const PLL &root_unity) {
    int n = a.size();
    LL prime = root_unity.first;
    LL basew = mod_pow(root_unity.second, (prime-1) / n, prime);
    if (dir < 0) basew = mod_inv(basew, prime);
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        LL w = 1;
        for (int i = 0; i < mh; i++) {
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                LL x = (a[j] - a[k] + prime) % prime;
                a[j] = (a[j] + a[k]) % prime;
                a[k] = (w * x) % prime;
            }
        }
    }
}
```



```

        w = (w * basew) % prime;
    }
    basew = (basew * basew) % prime;
}
int i = 0;
for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^ k); k >>= 1);
    if (j < i) swap(a[i], a[j]);
}
}

```

7.11 pollard rho factorize

```

long long pollard_rho(long long n) {
    long long x, y, i = 1, k = 2, d;
    x = y = rand() % n;
    while (1) {
        ++i;
        x = mod_mul(x, x, n);
        x += 2;
        if (x >= n) x -= n;
        if (x == y) return 1;
        d = __gcd(abs(x - y), n);
        if (d != 1) return d;
        if (i == k) {
            y = x;
            k *= 2;
        }
    }
    return 1;
}

// Returns a list with the prime divisors of n
vector<long long> factorize(long long n) {
    vector<long long> ans;
    if (n == 1)
        return ans;
    if (miller_rabin(n)) {
        ans.push_back(n);
    } else {
        long long d = 1;
        while (d == 1)

```

```

        d = pollard_rho(n);
        vector<long long> dd = factorize(d);
        ans = factorize(n / d);
        for (int i = 0; i < dd.size(); ++i)
            ans.push_back(dd[i]);
    }
    return ans;
}

```

8 Strings

8.1 minimal string rotation

```

// Lexicographically minimal string rotation
int lmsr() {
    string s;
    cin >> s;
    int n = s.size();
    s += s;
    vector<int> f(s.size(), -1);
    int k = 0;
    for (int j = 1; j < 2 * n; ++j) {
        int i = f[j - k - 1];
        while (i != -1 && s[j] != s[k + i + 1]) {
            if (s[j] < s[k + i + 1])
                k = j - i - 1;
            i = f[i];
        }
        if (i == -1 && s[j] != s[k + i + 1]) {
            if (s[j] < s[k + i + 1]) {
                k = j;
            }
            f[j - k] = -1;
        } else {
            f[j - k] = i + 1;
        }
    }
    return k;
}

```

8.2 suffix array

```
/**
 * O (n log^2 (n))
 * See http://web.stanford.edu/class/cs97si/suffix-array.pdf for reference
 */
using namespace std;
#include<bits/stdc++.h>
#define D(x) cout<<#x " = "<<(x)<<endl

struct entry{
    int a, b, p;
    entry(){
    entry(int x, int y, int z): a(x), b(y), p(z){}
    bool operator < (const entry &o) const {
        return (a == o.a) ? (b < o.b) : (a < o.a);
    }
};

struct SuffixArray{
    const int N;
    string s;
    vector<vector<int>> > P;
    vector<entry> M;

    SuffixArray(const string &s) : N(s.length()) , s(s), P(1, vector<int>
        (N, 0)), M(N) {
        for (int i = 0; i < N; ++i)
            P[0][i] = s[i];

        for (int skip = 1, level = 1; skip < N; skip *= 2, level++) {
            P.push_back(vector<int>(N, 0));
            for (int i = 0 ; i < N; ++i) {
                int next = ((i + skip) < N) ? P[level - 1][i + skip] : -10000;
                M[i] = entry(P[level - 1][i], next, i);
            }
            sort(M.begin(), M.end());
            for (int i = 0; i < N; ++i)
                P[level][M[i].p] = (i > 0 and M[i].a == M[i - 1].a and M[i].b ==
                    M[i - 1].b) ? P[level - 1][M[i - 1].p] : i;
        }

        vector<int> getSuffixArray(){
            return P.back();
        }
    }
};
```

```

    }

    // returns the length of the longest common prefix of s[i...L-1] and
    // s[j...L-1]
    int longestCommonPrefix(int i, int j) {
        int len = 0;
        if (i == j) return N - i;
        for (int k = P.size() - 1; k >= 0 && i < N && j < N; --k) {
            if (P[k][i] == P[k][j]) {
                i += 1 << k;
                j += 1 << k;
                len += 1 << k;
            }
        }
        return len;
    }
};
```

8.3 z algorithm

```
using namespace std;
#include<bits/stdc++.h>

vector<int> compute_z(const string &s){
    int n = s.size();
    vector<int> z(n,0);
    int l,r;
    r = l = 0;
    for(int i = 1; i < n; ++i){
        if(i > r) {
            l = r = i;
            while(r < n and s[r - 1] == s[r])r++;
            z[i] = r - l;r--;
        }else{
            int k = i-l;
            if(z[k] < r - i + 1) z[i] = z[k];
            else {
                l = i;
                while(r < n and s[r - l] == s[r])r++;
                z[i] = r - l;r--;
            }
        }
    }
}
```

```
    return z;
}

int main(){

    //string line;cin>>line;
    string line = "alfalfa";
    vector<int> z = compute_z(line);

    for(int i = 0; i < z.size(); ++i ){
        if(i)cout<<" ";
        cout<<z[i];
    }
    cout<<endl;

    // must print "0 0 0 4 0 0 1"

    return 0;
}
```
