# Problem A. Graph Game

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 mebibytes

Alice and Bob play a game with the following rules:

- $\bullet$  Before the game, they draw N vertices without edges.
- On her turn, the player can draw a bidirectional edge between two different vertices that has not yet been drawn. Only one edge is allowed between any pair of vertices.
- The player after whose turn the graph becomes connected loses the game.

Please find out which player wins if both play optimally and Bob moves first.

#### Input

The first line of input contains the number of test cases T ( $1 \le T \le 100$ ). Next T lines contain one number  $N \ge 2$  each which is the amount of vertices in the graph. The total amount of vertices in all test cases does not exceed  $2 \cdot 10^5$ .

## Output

For each test case, print the name of the winner ("Alice" or "Bob") on a separate line.

| standard input | standard output |
|----------------|-----------------|
| 2              | Alice           |
| 2              | Bob             |
| 3              |                 |
| 1              | Bob             |
| 10             |                 |

# Problem B. Flatland Fencing

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 mebibytes

The King of Flatland will organize a knights' tournament! The winner will get half the kingdom and the favor of the princess of legendary beauty and wisdom. The final test of the applicants' courage and strength will be a fencing tournament. The tournament is held by the following rules: the participants fight one on one, the winner (or rather, the survivor) transfers to the next round.

Before the battle both participants stand at the specified points on the Ox axis with integer coordinates. Then they make moves in turn. The first participant moves first, naturally. During a move, the first participant can transfer from the point x to any integer point of the interval [x + a; x + b]. The second participant can transfer during a move to any integer point of the interval [x - b; x - a]. That is, the options for the players' moves are symmetric (note that the numbers a and b are not required to be positive, and if  $a \le 0 \le b$ , then staying in one place is a correct move). At any time the participants can be located arbitrarily relative to each other, that is, it is allowed to "jump" over the enemy in any direction. A participant wins if he uses his move to transfer to the point where his opponent is.

Of course, the princess has already chosen a husband and now she wants to make her sweetheart win the tournament. He has already reached the tournament finals and he is facing the last battle. The princess asks the tournament manager to arrange the tournament finalists in such a way that her sweetheart wins the tournament, considering that both players play optimally. However, the initial location of the participants has already been announced, and we can only pull some strings and determine which participant will be first and which one will be second. But how do we know which participant can secure the victory? Alas, the princess is not learned in the military affairs... Therefore, she asks you to determine how the battle will end considering that both opponents play optimally. Also, if the first player wins, your task is to determine his winning move.

## Input

The first line contains four space-separated integers  $-x_1$ ,  $x_2$ , a and b ( $x_1 \neq x_2$ ,  $a \leq b$ ,  $-10^9 \leq x_1, x_2, a, b \leq 10^9$ ) — coordinates of the points where the first and the second participant start, and the numbers that determine the players' moves, correspondingly.

## Output

On the first line print the outcome of the battle as "FIRST" (without the quotes), if both players play optimally and the first player wins. Print "SECOND" (without the quotes) if the second player wins and print "DRAW" (without the quotes), if nobody is able to secure the victory.

If the first player wins, print on the next line the single integer x — the coordinate of the point where the first player should transfer to win. The indicated move should be valid, that is, it should meet the following condition:  $x_1 + a \le x \le x_1 + b$ . If there are several winning moves, print any of them. If the first participant can't secure the victory, then you do not have to print anything.

| standard input | standard output |
|----------------|-----------------|
| 0 2 0 4        | FIRST           |
|                | 2               |
| 0 2 1 1        | SECOND          |
| 0 2 0 1        | DRAW            |

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## Note

In the first sample the first player can win in one move.

In the second sample the first participant must go to point 1, where the second participant immediately goes and wins.

In the third sample changing the position isn't profitable to either participant, so nobody wins.

## Problem C. Game

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 mebibytes

Petya and Vasya got bored playing Tic Tac Toe, so they decided to invent their own game. Unfortunately, in the moment of true inspiration, they had nothing to use as equipment except for pen and paper. After long debates, they finally agreed that the game board will be composed from N blocks of lines. The first block consists from  $a_1$  lines with  $b_1$  cells in each of them. The second one contains  $a_2$  lines with  $b_2$  cells each, and so on. The N-th one contains  $a_N$  lines with  $b_N$  cells each. Before the game starts, Petya randomly fills the board cells with digits 0 and 1. By the special game requirement, every line ends with 1. According to the rules of the game, players are allowed to choose a line and remove any suffix that satisfies the following:

- After removal, line should end with a 1.
- Length of the suffix should be present in the special valid moves set.

Petya makes the first move. Player which can not make a move loses.

In the filling phase, Petya is cheating because he knows that the game can be easily analyzed. Thus, whenever Petya and Vasya are playing the game, Petya wins. Although Vasya is not smart enough to directly reveal Petya, he can start suspecting Petya if the initial state of the game will be repeated twice. Naturally, Petya wants to know how many different winning (of course, for Petya) initial states of the game exists. Find this number modulo  $10^9 + 7$ .

#### Input

The first line of input contains one integer N  $(1 \le N \le 100)$  — the number of line blocks. Each of the next N lines contains a pair of integers  $a_i$  and  $b_i$   $(1 \le a_i \le 10^{18}, \ 1 \le b_i \le 100)$  — the parameters described above. The next line contains an integer K  $(0 \le K \le 6)$  — the number of moves in the valid moves set. The last line contains K distinct integers — lengths of suffixes in the valid moves set. Each of them is between 1 and 6, inclusive.

## Output

The only line of output should contain the number of games in which Petya wins. Print this number modulo  $10^9 + 7$ .

| standard input | standard output |
|----------------|-----------------|
| 2              | 64              |
| 1 4            |                 |
| 1 5            |                 |
| 3              |                 |
| 4 5 6          |                 |

## Problem D. Graph Game

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 mebibytes

Nick and Peter like to play the following game when attending their complexity theory lectures. They draw an undirected bipartite graph G on a sheet of paper, and put a token to one of its vertices. After that they make moves in turn. Nick moves first.

A move consists of moving the token along the graph edge. After it the vertex where the token was before the move, together with all edges incident to it, are removed from the graph. The player who has no valid moves loses the game.

You are given the graph that Nick and Peter have drawn. For each vertex of the graph find out who wins if the token is initially placed in that vertex. Assume that both Nick and Peter play optimally.

#### Input

The first line of the input file contains three integer numbers  $n_1$ ,  $n_2$ , and m — the number of vertices in each part, and the number of edges, respectively ( $1 \le n_1, n_2 \le 500, 0 \le m \le 50\,000$ ). The following m lines describe edges — each line contains the numbers of vertices connected by the corresponding edge. Vertices in each part are numbered independently, starting from 1.

## Output

Output two lines. The first line must contain  $n_1$  characters, the *i*-th character must be 'N' in case Nick wins if the token is initially placed in the *i*-th vertex of the first part, and 'P' if Peter does. The second line must contain  $n_2$  characters and describe the second part in the same way.

| standard input | standard output |
|----------------|-----------------|
| 3 3 5          | NPP             |
| 1 1            | NPP             |
| 1 2            |                 |
| 1 3            |                 |
| 2 1            |                 |
| 3 1            |                 |

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# Problem E. Function

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 mebibytes

You are given directed acyclic graph. Calculate Grundy's function for each vertex.

#### Input

The first line contains two integers n and m — number of vertices and edges respectively  $(1 \le n, m \le 100\,000)$ . Next m lines contains description of edges u, v  $(1 \le u, v \le n)$ .

Note that graph can contains multiple edges.

## Output

Output n numbers — Grundy values for vertices.

| standard input | standard output |
|----------------|-----------------|
| 3 3            | 2               |
| 1 2            | 1               |
| 2 3            | 0               |
| 1 3            |                 |
| 2 1            | 0               |
| 2 1            | 1               |

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## Problem F. Kvass

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 mebibytes

Alice and Bob play the game "Break the chocolate".

Initially, they have n rectangular pieces of chocolate. The i-th piece has size  $w_i \times h_i$  and is divided into  $1 \times 1$  squares by horizontal and vertical lines.

At her move, Alice may break some piece along some horizontal dividing line, creating two new pieces.

At his move, Bob may break some piece along some vertical dividing line, creating two new pieces.

The obtained pieces can not be rotated.

The player who can't make a move loses the game.

Who will win if Alice is the first player, they must alternate their moves and both are playing optimally?

## Input

The first line contains the number of test cases T ( $1 \le T \le 1000$ ). After that, T testcases follow.

The first line of each test case contains an integer n  $(1 \le n \le 10^3)$ .

The next n lines contain the description of pieces (one per each line): integers  $w_i$  and  $h_i$  ( $1 \le w_i$ ,  $h_i \le 10^9$ ).

The sum of n over all test cases does not exceed 1000.

## Output

Print the name of the winner for each test case on a separate lines: "Alice" or "Bob" (without quotes).

| standard input | standard output |
|----------------|-----------------|
| 1              | Bob             |
| 1              |                 |
| 2 2            |                 |

# Problem G. One Heap

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 mebibytes

Two players play a game with one pile that contains n stones.

On corresponding step player can remove from the pile  $a_1, a_2, \ldots, a_k$  stones.

Find winner!

## Input

The first line contains one integer k.

The second line contains  $a_1, a_2, \ldots, a_k$ .

The third line contains m — number of n-s for which you need to find winner.

Next line contains m integers  $n_1, n_2, \ldots, n_m$ .

Constraints:  $1 \le k \le 20, m \le 10^4, 1 \le N_i, a_i \le 10^6$ .

#### Output

Output m lines, each containing winning player — First or Second.

| standard input  | standard output |
|-----------------|-----------------|
| 3               | First           |
| 1 2 3           | First           |
| 8               | First           |
| 1 2 3 4 5 6 7 8 | Second          |
|                 | First           |
|                 | First           |
|                 | First           |
|                 | Second          |

## Problem H. La Battala

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 mebibytes

Saruman and Gandalf are playing a game with chains of rings.

In the beginning of the game there are k chains of rings. For each ring its power is known. Power of the ring is positive integer between 1 and 100.

Players make moves one by one, Gandalf goes first.

On each move a player can chose a chain and a ring from this chain. After this player removes all rings from the chain that have power less than or equal that power of chosen ring. After this chain can split into multiple chains.

Find if Gandalf can win, and possible first move.

#### Input

The first line contains one integer k — number of chains.

Next k lines contains description of chains. First integer in each description is  $m_i$  — number of rings in a chain. Next  $m_i$  integers are powers of rings in their order in a chain.

## Output

Output "S" is Saruman will win.

Otherwise output "G" in the first line.

In the second line output two integers — number of chain and number of ring in a chain Gandalf should chose to win. If there are multiple answers, find one with smallest number of chain. If there is still multiple answers, find one with smallest number of ring.

Chains are numerated from 1 to k, rings in a chain numerated from 1.

| standard input | standard output |
|----------------|-----------------|
| 2              | G               |
| 3 1 2 1        | 1 1             |
| 1 1            |                 |
| 2              | S               |
| 3 2 1 2        |                 |
| 1 1            |                 |

# Problem I. Juego Injusto

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 mebibytes

David and Vasya playing a game!

Game position is positive integer n.

On each move, player have to divide n by some of its prime divisors p. After this player can, but not obliged to, multiply n by prime number 1 < q < p that is less than p.

David goes first. Determine who wins!

## Input

The first line contains one integer  $1 \le n \le 10^{12}$ .

#### Output

Output Vasya is Vasya have winning strategy, otherwise output David.

| standard input | standard output |
|----------------|-----------------|
| 16             | Vasya           |
| 1              | Vasya           |
| 2              | David           |