

# Team notebook

August 3, 2017

## Contents

<b>1 Algorithms</b>	<b>2</b>
1.1 Mo's algorithm . . . . .	2
1.2 sliding window . . . . .	3
<b>2 DP Optimizations</b>	<b>3</b>
2.1 convex hull trick . . . . .	3
<b>3 Data structures</b>	<b>4</b>
3.1 STL Treap . . . . .	4
3.2 STL order statistics tree II . . . . .	4
3.3 STL order statistics tree . . . . .	5
3.4 binary index tree . . . . .	5
3.5 hash table . . . . .	6
3.6 heavy light decomposition . . . . .	6
3.7 persistent array . . . . .	7
3.8 persistent seg tree . . . . .	7
3.9 persistent trie . . . . .	9
3.10 segment tree . . . . .	9
3.11 sparse table . . . . .	10
3.12 splay tree . . . . .	10
3.13 trie . . . . .	11
<b>4 Geometry</b>	<b>12</b>
4.1 center 2 points + radius . . . . .	12
4.2 closest pair problem . . . . .	12
4.3 squares . . . . .	13
4.4 triangles . . . . .	14

<b>5 Graphs</b>	<b>14</b>
5.1 bridges . . . . .	14
5.2 directed mst . . . . .	15
5.3 eulerian path . . . . .	16
5.4 karp min mean cycle . . . . .	16
5.5 konig's theorem . . . . .	17
5.6 minimum path cover in DAG . . . . .	17
5.7 planar graph (euler) . . . . .	17
5.8 query with lca . . . . .	18
5.9 tarjan scc . . . . .	18
5.10 two sat (with kosaraju) . . . . .	19
<b>6 Math</b>	<b>21</b>
6.1 Lucas theorem . . . . .	21
6.2 cumulative sum of divisors . . . . .	21
6.3 fft . . . . .	21
6.4 fibonacci properties . . . . .	22
6.5 sigma function . . . . .	23
<b>7 Matrix</b>	<b>23</b>
7.1 matrix . . . . .	23
<b>8 Misc</b>	<b>24</b>
8.1 Template Java . . . . .	24
8.2 dates . . . . .	25
8.3 fraction . . . . .	25
8.4 io . . . . .	25
<b>9 Number theory</b>	<b>26</b>
9.1 convolution . . . . .	26
9.2 crt . . . . .	26
9.3 discrete logarithm . . . . .	27

9.4	ext euclidean	27
9.5	highest exponent factorial	27
9.6	miller rabin	27
9.7	mod inv	28
9.8	mod mul	28
9.9	mod pow	28
9.10	number theoretic transform	28
9.11	pollard rho factorize	29
9.12	primes	29
9.13	totient sieve	30
9.14	totient	30
<b>10</b>	<b>Strings</b>	<b>30</b>
10.1	Incremental Aho Corasick	30
10.2	minimal string rotation	32
10.3	suffix array	32
10.4	suffix automaton	33
10.5	z algorithm	34

# 1 Algorithms

## 1.1 Mo's algorithm

---

```

const int MN = 5 * 100000 + 100;
const int SN = 708;

struct query {
    int a, b, id;
    query() {}
    query(int x, int y, int i) : a(x), b(y), id(i) {}

    bool operator < (const query &o) const {
        return b < o.b;
    }
};

vector<query> s[SN];
int ans[MN];

struct DS {

```

```

    void clear() {}
    void insert(int x) {}
    void erase(int x) {}
    long long query() {}
};

DS data;

int main() {
    int n, q;
    while (cin >> n >> q) {
        for (int i = 0; i < SN; ++i)
            s[i].clear();

        vector<int> a(n);
        for (auto &i : a) cin >> i;

        for (int i = 0; i < q; ++i) {
            int b, e;
            cin >> b >> e;
            b--; e--;
            s[b / SN].emplace_back(b, e, i);
        }

        for (int i = 0; i < SN; ++i) {
            if (s[i].size())
                sort(s[i].begin(), s[i].end());
        }

        for (int b = 0; b < SN; ++b) {
            if (s[b].size() == 0) continue;
            int i = s[b][0].a;
            int j = s[b][0].a - 1;

            data.clear();
            for (int k = 0; k < (int)s[b].size(); ++k) {
                int L = s[b][k].a;
                int R = s[b][k].b;

                while (j < R) {
                    j++;
                    data.insert(a[j]);
                }
            }
        }
    }
}

```

```

while (j > R) {
    data.erase(a[j]);
    j--;
}

while (i < L) {
    data.erase(a[i]);
    i++;
}

while (i > L) {
    i--;
    data.insert(a[i]);
}
ans[s[b][k].id] = data.query();
}
}

for (int i = 0; i < q; ++i) {
    cout << ans[i] << endl;
}

return 0;
};

```

---

## 1.2 sliding window

---

```

/*
 * Given an array ARR and an integer K, the problem boils down to
 * computing for each index i: min(ARR[i], ARR[i-1], ..., ARR[i-K+1]).
 * if mx == true, returns the maximum.
 * http://people.cs.uct.ac.za/~ksmith/articles/sliding_window_minimum.html
 */

vector<int> sliding_window_minmax(vector<int> & ARR, int K, bool mx) {
    deque< pair<int, int> > window;
    vector<int> ans;
    for (int i = 0; i < ARR.size(); i++) {
        if (mx) {
            while (!window.empty() && window.back().first <= ARR[i])
                window.pop_back();

```

```

        } else {
            while (!window.empty() && window.back().first >= ARR[i])
                window.pop_back();
        }
        window.push_back(make_pair(ARR[i], i));

        while(window.front().second <= i - K)
            window.pop_front();

        ans.push_back(window.front().first);
    }
    return ans;
}

```

---

## 2 DP Optimizations

### 2.1 convex hull trick

---

```

struct line {
    long long m, b;
    line (long long a, long long c) : m(a), b(c) {}
    long long eval(long long x) {
        return m * x + b;
    }
};

long double inter(line a, line b) {
    long double den = a.m - b.m;
    long double num = b.b - a.b;
    return num / den;
}

struct convex_hull_trick {
    vector<line> ch;
    int idx; // id of last "best" in query
    convex_hull_trick() {
        idx = 0;
    }

    void add(line cur) {
        // new line's slope is less than all the previous
        while (ch.size() > 1 &&

```

```

        (inter(cur, ch[ch.size() - 2]) >= inter(cur, ch[ch.size() - 1])) {
            // f(x) is better in interval [inter(ch.back(), cur), inf)
            ch.pop_back();
        }

        ch.push_back(cur);
    }

    long long query(long long x) {
        // current x is greater than all the previous x,
        // if that is not the case we can make binary search.
        idx = min<int>(idx, ch.size() - 1);
        while (idx + 1 < (int)ch.size() && ch[idx + 1].eval(x) <=
            ch[idx].eval(x))
            idx++;

        return ch[idx].eval(x);
    }
};

```

## 3 Data structures

### 3.1 STL Treap

```

#include <ext/rope> //header with rope
using namespace std;
using namespace __gnu_cxx; //namespace with rope and some additional stuff
int main()
{
    ios_base::sync_with_stdio(false);
    rope<int> v; //use as usual STL container
    int n, m;
    cin >> n >> m;
    for(int i = 1; i <= n; ++i)
        v.push_back(i); //initialization
    int l, r;
    for(int i = 0; i < m; ++i)
    {
        cin >> l >> r;
        --l, --r;
        rope<int> cur = v.substr(l, r - l + 1);
        v.erase(l, r - l + 1);
    }
}

```

```

        v.insert(v.mutable_begin(), cur);
    }
    for(rope<int>::iterator it = v.mutable_begin(); it !=
        v.mutable_end(); ++it)
        cout << *it << " ";
    return 0;
}

```

### 3.2 STL order statistics tree II

```

#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace std;
using namespace __gnu_pbds;

typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> order_set;

order_set X;

int get(int y) {
    int l=0, r=1e9+1;
    while(l<r) {
        int m=l+((r-l)>>1);
        if(m-X.order_of_key(m+1)<y)
            l=m+1;
        else
            r=m;
    }
    return l;
}

main(){
    ios::sync_with_stdio(0);
    cin.tie(0);
    int n,m;
    cin>>n>>m;

    for(int i=0;i<m;i++) {
        char a;
        int b;
    }
}

```

```

cin>>a>>b;
if(a=='L')
    cout<<get(b)<<endl;
else
    X.insert(get(b));
}
}

```

```

/**
Input
20 7

```

```

L 5
D 5
L 4
L 5
D 5
L 4
L 5

```

```

Output

```

```

5
4
6
4
7
***/

```

### 3.3 STL order statistics tree

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <bits/stdc++.h>

```

```

using namespace __gnu_pbds;
using namespace std;

```

```

typedef
tree<
    pair<int,int>,
    null_type,
    less<pair<int,int>>,
    rb_tree_tag,
    tree_order_statistics_node_update>

```

```

ordered_set;

main()
{
    ios::sync_with_stdio(0);
    cin.tie(0);
    int n;
    int sz=0;
    cin>>n;
    vector<int> ans(n,0);

    ordered_set t;
    int x,y;
    for(int i=0;i<n;i++)
    {
        cin>>x>>y;
        ans[t.order_of_key({x,++sz})]++;
        t.insert({x,sz});
    }

    for(int i=0;i<n;i++)
        cout<<ans[i]<<'\\n';
}

```

```

/**
Input
5
1 1
5 1
7 1
3 3
5 5

```

```

Output

```

```

1
2
1
1
0
***/

```

### 3.4 binary index tree

```

struct binary_index_tree {
    int n;
    int t[2 * N];

    void add(int where, long long what){
        for (where++; where <= n; where += where & -where){
            t[where] += what;
        }
    }

    void add(int from, int to, long long what) {
        add(from, what);
        add(to + 1, -what);
    }

    long long query(int where){
        long long sum = t[0];
        for (where++; where > 0; where -= where & -where){
            sum += t[where];
        }
        return sum;
    }
};

```

### 3.5 hash table

```

/**
 * Micro hash table, can be used as a set.
 * Very efficient vs std::set
 * */

const int MN = 1001;
struct ht {
    int _s[(MN + 10) >> 5];
    int len;
    void set(int id) {
        len++;
        _s[id >> 5] |= (1LL << (id & 31));
    }
    bool is_set(int id) {
        return _s[id >> 5] & (1LL << (id & 31));
    }
};

```

### 3.6 heavy light decomposition

```

// Heavy-Light Decomposition
struct TreeDecomposition {
    vector<int> g[MAXN], c[MAXN];
    int s[MAXN]; // subtree size
    int p[MAXN]; // parent id
    int r[MAXN]; // chain root id
    int t[MAXN]; // index used in segtree/bit/...
    int d[MAXN]; // depht
    int ts;

    void dfs(int v, int f) {
        p[v] = f;
        s[v] = 1;
        if (f != -1) d[v] = d[f] + 1;
        else d[v] = 0;

        for (int i = 0; i < g[v].size(); ++i) {
            int w = g[v][i];
            if (w != f) {
                dfs(w, v);
                s[v] += s[w];
            }
        }
    }

    void hld(int v, int f, int k) {
        t[v] = ts++;
        c[k].push_back(v);
        r[v] = k;

        int x = 0, y = -1;
        for (int i = 0; i < g[v].size(); ++i) {
            int w = g[v][i];
            if (w != f) {
                if (s[w] > x) {
                    x = s[w];
                    y = w;
                }
            }
        }
        if (y != -1) {
            hld(y, v, k);
        }
    }
};

```

```

    for (int i = 0; i < g[v].size(); ++i) {
        int w = g[v][i];
        if (w != f && w != y) {
            hld(w, v, w);
        }
    }
}

void init(int n) {
    for (int i = 0; i < n; ++i) {
        g[i].clear();
    }
}

void add(int a, int b) {
    g[a].push_back(b);
    g[b].push_back(a);
}

void build() {
    ts = 0;
    dfs(0, -1);
    hld(0, 0, 0);
}
};

```

### 3.7 persistent array

```

struct node {
    node *l, *r;
    int val;

    node (int x) : l(NULL), r(NULL), val(x) {}
    node () : l(NULL), r(NULL), val(-1) {}
};

typedef node* pnode;

pnode update(pnode cur, int l, int r, int at, int what) {
    pnode ans = new node();

    if (cur != NULL) {

```

```

        *ans = *cur;
    }
    if (l == r) {
        ans->val = what;
        return ans;
    }
    int m = (l + r) >> 1;
    if (at <= m) ans->l = update(ans->l, l, m, at, what);
    else ans->r = update(ans->r, m + 1, r, at, what);
    return ans;
}

int get(pnode cur, int l, int r, int at) {
    if (cur == NULL) return 0;
    if (l == r) return cur->val;
    int m = (l + r) >> 1;
    if (at <= m) return get(cur->l, l, m, at);
    else return get(cur->r, m + 1, r, at);
}

```

### 3.8 persistent seg tree

```

/**
 * Important:
 * When using lazy propagation remembert to create new
 * versions for each push_down operation!!!
 * */

struct node {
    node *l, *r;
    long long acc;
    int flip;

    node (int x) : l(NULL), r(NULL), acc(x), flip(0) {}
    node () : l(NULL), r(NULL), acc(0), flip(0) {}
};

typedef node* pnode;

pnode create(int l, int r) {
    if (l == r) return new node();
    pnode cur = new node();
    int m = (l + r) >> 1;

```

```

    cur-> l = create(l, m);
    cur-> r = create(m + 1, r);
    return cur;
}

pnode copy_node(pnode cur) {
    pnode ans = new node();
    *ans = *cur;
    return ans;
}

void push_down(pnode cur, int l, int r) {
    assert(cur);
    if (cur-> flip) {
        int len = r - l + 1;
        cur-> acc = len - cur-> acc;
        if (cur-> l) {
            cur-> l = copy_node(cur-> l);
            cur-> l -> flip ^= 1;
        }
        if (cur-> r) {
            cur-> r = copy_node(cur-> r);
            cur-> r -> flip ^= 1;
        }
        cur-> flip = 0;
    }
}

int get_val(pnode cur) {
    assert(cur);
    assert((cur-> flip) == 0);
    if (cur) return cur-> acc;
    return 0;
}

pnode update(pnode cur, int l, int r, int at, int what) {
    pnode ans = copy_node(cur);
    if (l == r) {
        assert(l == at);
        ans-> acc = what;
        ans-> flip = 0;
        return ans;
    }
    int m = (l + r) >> 1;
    push_down(ans, l, r);

```

```

    if (at <= m) ans-> l = update(ans-> l, l, m, at, what);
    else ans-> r = update(ans-> r, m + 1, r, at, what);

    push_down(ans-> l, l, m);
    push_down(ans-> r, m + 1, r);
    ans-> acc = get_val(ans-> l) + get_val(ans-> r);
    return ans;
}

pnode flip(pnode cur, int l, int r, int a, int b) {
    pnode ans = new node();

    if (cur != NULL) {
        *ans = *cur;
    }
    if (l > b || r < a)
        return ans;

    if (l >= a && r <= b) {
        ans-> flip ^= 1;
        push_down(ans, l, r);
        return ans;
    }

    int m = (l + r) >> 1;
    ans-> l = flip(ans-> l, l, m, a, b);
    ans-> r = flip(ans-> r, m + 1, r, a, b);
    push_down(ans-> l, l, m);
    push_down(ans-> r, m + 1, r);
    ans-> acc = get_val(ans-> l) + get_val(ans-> r);
    return ans;
}

long long get_all(pnode cur, int l, int r) {
    assert(cur);
    push_down(cur, l, r);
    return cur-> acc;
}

void traverse(pnode cur, int l, int r) {
    if (!cur) return;
    cout << l << " - " << r << " : " << (cur-> acc) << " " << (cur-> flip)
        << endl;
    traverse(cur-> l, l, (l + r) >> 1);
    traverse(cur-> r, 1 + ((l + r) >> 1), r);
}

```



```
}
```

### 3.9 persistent trie

```
/**
 * Persistent version of trie:
 * could be used as a persistent BST of integers.
 * add: O(log2(n))
 * query: O(log2(n)), works equal than the non-persistent version.
 */

const int MD = 31;
const int MAX_CHILD = 2;

struct node {
    node *child[MAX_CHILD];
};

typedef node* pnode;

pnode copy_node(pnode cur) {
    pnode ans = new node();
    if (cur)
        *ans = *cur;
    return ans;
}

pnode add(pnode cur, int val, int id = MD) {
    pnode ans = copy_node(cur);
    if (id == -1) return ans;
    int t = (val >> id) & 1;
    ans->child[t] = add(ans->child[t], val, id - 1);
    return ans;
}
```

### 3.10 segment tree

```
/**
 * Taken from: http://codeforces.com/blog/entry/18051
 */
```

```
const int N = 1e5; // limit for array size
int n; // array size
int t[2 * N];

void build() { // build the tree
    for (int i = n - 1; i > 0; --i) t[i] = t[i<<1] + t[i<<1|1];
}

// Single modification, range query.
void modify(int p, int value) { // set value at position p
    for (t[p += n] = value; p > 1; p >>= 1) t[p>>1] = t[p] + t[p^1];
}

int query(int l, int r) { // sum on interval [l, r)
    int res = 0;
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) res += t[l++];
        if (r&1) res += t[--r];
    }
    return res;
}

// Range modification, single query.

void modify(int l, int r, int value) {
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) t[l++] += value;
        if (r&1) t[--r] += value;
    }
}

int query(int p) {
    int res = 0;
    for (p += n; p > 0; p >>= 1) res += t[p];
    return res;
}

/**
 * If at some point after modifications we need to inspect all the
 * elements in the array, we can push all the modifications to the
 * leaves using the following code. After that we can just traverse
 * elements starting with index n. This way we reduce the complexity
 * from O(n log(n)) to O(n) similarly to using build instead of n
 * modifications.
 */
```

```

void push() {
    for (int i = 1; i < n; ++i) {
        t[i<<1] += t[i];
        t[i<<1|1] += t[i];
        t[i] = 0;
    }
}

// Non commutative combiner functions.

void modify(int p, const S& value) {
    for (t[p += n] = value; p >>= 1; ) t[p] = combine(t[p<<1], t[p<<1|1]);
}

S query(int l, int r) {
    S resl, resr;
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) resl = combine(resl, t[l++]);
        if (r&1) resr = combine(t[--r], resr);
    }
    return combine(resl, resr);
}

// To be continued ...

```

---

### 3.11 sparse table

```

// RMQ.
const int MN = 100000 + 10; // Max number of elements
const int ML = 18; // ceil(log2(MN));

struct st {
    int data[MN];
    int M[MN][ML];
    int n;

    void read(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            cin >> data[i];
    }
}

```

```

void build() {
    for (int i = 0; i < n; ++i)
        M[i][0] = data[i];
    for (int j = 1, p = 2, q = 1; p <= n; ++j, p <= 1, q <= 1)
        for (int i = 0; i + p - 1 < n; ++i)
            M[i][j] = max(M[i][j - 1], M[i + q][j - 1]);
}

int query(int b, int e) {
    int k = log2(e - b + 1);
    return max(M[b][k], M[e + 1 - (1<<k)][k]);
}
};

```

---

### 3.12 splay tree

```

using namespace std;
#include<bits/stdc++.h>
#define D(x) cout<<x<<endl;

typedef int T;

struct node{
    node *left, *right, *parent;
    T key;
    node (T k) : key(k), left(0), right(0), parent(0) {}
};

struct splay_tree{

    node *root;

    void right_rot(node *x) {
        node *p = x->parent;
        if (x->parent = p->parent) {
            if (x->parent->left == p) x->parent->left = x;
            if (x->parent->right == p) x->parent->right = x;
        }
        if (p->left = x->right) p->left->parent = p;
        x->right = p;
        p->parent = x;
    }
}

```

```

void left_rot(node *x) {
    node *p = x->parent;
    if (x->parent = p->parent) {
        if (x->parent->left == p) x->parent->left = x;
        if (x->parent->right == p) x->parent->right = x;
    }
    if (p->right = x->left) p->right->parent = p;
    x->left = p;
    p->parent = x;
}

```

```

void splay(node *x, node *fa = 0) {

    while( x->parent != fa and x->parent != 0) {
        node *p = x->parent;
        if (p->parent == fa)
            if (p->right == x)
                left_rot(x);
            else
                right_rot(x);
        else {
            node *gp = p->parent; //grand parent
            if (gp->left == p)
                if (p->left == x)
                    right_rot(x),right_rot(x);
                else
                    left_rot(x),right_rot(x);
            else
                if (p->left == x)
                    right_rot(x), left_rot(x);
                else
                    left_rot(x), left_rot(x);
        }
    }
    if (fa == 0) root = x;
}

```

```

void insert(T key) {
    node *cur = root;
    node *pcur = 0;
    while (cur) {
        pcur = cur;
        if (key > cur->key) cur = cur->right;
        else cur = cur->left;
    }
}

```

```

    cur = new node(key);
    cur->parent = pcur;
    if (!pcur) root = cur;
    else if (key > pcur->key ) pcur->right = cur;
    else pcur->left = cur;
    splay(cur);
}

```

```

node *find(T key) {
    node *cur = root;
    while (cur) {
        if (key > cur->key) cur = cur->right;
        else if(key < cur->key) cur = cur->left;
        else return cur;
    }
    return 0;
}

```

```

splay_tree(){ root = 0;};
};

```

---

### 3.13 trie

---

```

const int MN = 26; // size of alphabet
const int MS = 100010; // Number of states.

```

```

struct trie{
    struct node{
        int c;
        int a[MN];
    };

    node tree[MS];
    int nodes;

    void clear(){
        tree[nodes].c = 0;
        memset(tree[nodes].a, -1, sizeof tree[nodes].a);
        nodes++;
    }

    void init(){
        nodes = 0;
    }
}

```

```

clear();
}

int add(const string &s, bool query = 0){
    int cur_node = 0;
    for(int i = 0; i < s.size(); ++i){
        int id = gid(s[i]);
        if(tree[cur_node].a[id] == -1){
            if(query) return 0;
            tree[cur_node].a[id] = nodes;
            clear();
        }
        cur_node = tree[cur_node].a[id];
    }
    if(!query) tree[cur_node].c++;
    return tree[cur_node].c;
}

};

```

---

## 4 Geometry

### 4.1 center 2 points + radius

```

vector<point> find_center(point a, point b, long double r) {
    point d = (a - b) * 0.5;
    if (d.dot(d) > r * r) {
        return vector<point> ();
    }
    point e = b + d;
    long double fac = sqrt(r * r - d.dot(d));
    vector<point> ans;
    point x = point(-d.y, d.x);
    long double l = sqrt(x.dot(x));
    x = x * (fac / l);
    ans.push_back(e + x);
    x = point(d.y, -d.x);
    x = x * (fac / l);
    ans.push_back(e + x);
    return ans;
}

```

---

### 4.2 closest pair problem

```

struct point {
    double x, y;
    int id;
    point() {}
    point (double a, double b) : x(a), y(b) {}
};

double dist(const point &o, const point &p) {
    double a = p.x - o.x, b = p.y - o.y;
    return sqrt(a * a + b * b);
}

double cp(vector<point> &p, vector<point> &x, vector<point> &y) {
    if (p.size() < 4) {
        double best = 1e100;
        for (int i = 0; i < p.size(); ++i)
            for (int j = i + 1; j < p.size(); ++j)
                best = min(best, dist(p[i], p[j]));
        return best;
    }

    int ls = (p.size() + 1) >> 1;
    double l = (p[ls - 1].x + p[ls].x) * 0.5;
    vector<point> xl(ls), xr(p.size() - ls);
    unordered_set<int> left;
    for (int i = 0; i < ls; ++i) {
        xl[i] = x[i];
        left.insert(x[i].id);
    }
    for (int i = ls; i < p.size(); ++i) {
        xr[i - ls] = x[i];
    }

    vector<point> yl, yr;
    vector<point> pl, pr;
    yl.reserve(ls); yr.reserve(p.size() - ls);
    pl.reserve(ls); pr.reserve(p.size() - ls);
    for (int i = 0; i < p.size(); ++i) {
        if (left.count(y[i].id))
            yl.push_back(y[i]);
        else
            yr.push_back(y[i]);
    }
}

```

```

    if (left.count(p[i].id))
        pl.push_back(p[i]);
    else
        pr.push_back(p[i]);
}

double dl = cp(pl, xl, yl);
double dr = cp(pr, xr, yr);
double d = min(dl, dr);
vector<point> yp; yp.reserve(p.size());
for (int i = 0; i < p.size(); ++i) {
    if (fabs(y[i].x - l) < d)
        yp.push_back(y[i]);
}
for (int i = 0; i < yp.size(); ++i) {
    for (int j = i + 1; j < yp.size() && j < i + 7; ++j) {
        d = min(d, dist(yp[i], yp[j]));
    }
}
return d;
}

double closest_pair(vector<point> &p) {
    vector<point> x(p.begin(), p.end());
    sort(x.begin(), x.end(), [](const point &a, const point &b) {
        return a.x < b.x;
    });
    vector<point> y(p.begin(), p.end());
    sort(y.begin(), y.end(), [](const point &a, const point &b) {
        return a.y < b.y;
    });
    return cp(p, x, y);
}

```

### 4.3 squares

```

typedef long double ld;

const ld eps = 1e-12;
int cmp(ld x, ld y = 0, ld tol = eps) {
    return (x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1;
}

```

```

struct point{
    ld x, y;
    point(ld a, ld b) : x(a), y(b) {}
    point() {}
};

struct square{
    ld x1, x2, y1, y2,
        a, b, c;
    point edges[4];
    square(ld _a, ld _b, ld _c) {
        a = _a, b = _b, c = _c;
        x1 = a - c * 0.5;
        x2 = a + c * 0.5;
        y1 = b - c * 0.5;
        y2 = b + c * 0.5;
        edges[0] = point(x1, y1);
        edges[1] = point(x2, y1);
        edges[2] = point(x2, y2);
        edges[3] = point(x1, y2);
    }
};

ld min_dist(point &a, point &b) {
    ld x = a.x - b.x,
        y = a.y - b.y;
    return sqrt(x * x + y * y);
}

bool point_in_box(square s1, point p) {
    if (cmp(s1.x1, p.x) != 1 && cmp(s1.x2, p.x) != -1 &&
        cmp(s1.y1, p.y) != 1 && cmp(s1.y2, p.y) != -1)
        return true;
    return false;
}

bool inside(square &s1, square &s2) {
    for (int i = 0; i < 4; ++i)
        if (point_in_box(s2, s1.edges[i]))
            return true;

    return false;
}

```

```

bool inside_vert(square &s1, square &s2) {
    if ((cmp(s1.y1, s2.y1) != -1 && cmp(s1.y1, s2.y2) != 1) ||
        (cmp(s1.y2, s2.y1) != -1 && cmp(s1.y2, s2.y2) != 1))
        return true;
    return false;
}

bool inside_hori(square &s1, square &s2) {
    if ((cmp(s1.x1, s2.x1) != -1 && cmp(s1.x1, s2.x2) != 1) ||
        (cmp(s1.x2, s2.x1) != -1 && cmp(s1.x2, s2.x2) != 1))
        return true;
    return false;
}

ld min_dist(square &s1, square &s2) {
    if (inside(s1, s2) || inside(s2, s1))
        return 0;

    ld ans = 1e100;
    for (int i = 0; i < 4; ++i)
        for (int j = 0; j < 4; ++j)
            ans = min(ans, min_dist(s1.edges[i], s2.edges[j]));

    if (inside_hori(s1, s2) || inside_hori(s2, s1)) {
        if (cmp(s1.y1, s2.y2) != -1)
            ans = min(ans, s1.y1 - s2.y2);
        else
            if (cmp(s2.y1, s1.y2) != -1)
                ans = min(ans, s2.y1 - s1.y2);
    }

    if (inside_vert(s1, s2) || inside_vert(s2, s1)) {
        if (cmp(s1.x1, s2.x2) != -1)
            ans = min(ans, s1.x1 - s2.x2);
        else
            if (cmp(s2.x1, s1.x2) != -1)
                ans = min(ans, s2.x1 - s1.x2);
    }

    return ans;
}

```

## 4.4 triangles

Let  $a, b, c$  be length of the three sides of a triangle.

$$p = (a + b + c) * 0.5$$

The inradius is defined by:

$$iR = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$$

The radius of its circumcircle is given by the formula:

$$cR = \frac{abc}{\sqrt{(a+b+c)(a+b-c)(a+c-b)(b+c-a)}}$$

## 5 Graphs

### 5.1 bridges

```

struct edge{
    int to, id;
    edge(int a, int b) : to(a), id(b) {}
};

struct graph {
    vector<vector<edge>> > g;
    vector<int> vi, low, d, pi, is_b;

    int ticks, edges;

    graph(int n, int m) {
        g.assign(n, vector<edge>());
        is_b.assign(m, 0);
        vi.resize(n);
        low.resize(n);
        d.resize(n);
        pi.resize(n);
        edges = 0;
    }

    void add_edge(int u, int v) {
        g[u].push_back(edge(v, edges));
        g[v].push_back(edge(u, edges));
    }
}

```

```

    edges++;
}

void dfs(int u) {
    vi[u] = true;
    d[u] = low[u] = ticks++;
    for (int i = 0; i < g[u].size(); ++i) {
        int v = g[u][i].to;
        if (v == pi[u]) continue;
        if (!vi[v]) {
            pi[v] = u;
            dfs(v);
            if (d[u] < low[v])
                is_b[g[u][i].id] = true;

            low[u] = min(low[u], low[v]);
        } else {
            low[u] = min(low[u], d[v]);
        }
    }
}

// Multiple edges from a to b are not allowed.
// (they could be detected as a bridge).
// If you need to handle this, just count
// how many edges there are from a to b.
void comp_bridges() {
    fill(pi.begin(), pi.end(), -1);
    fill(vi.begin(), vi.end(), 0);
    fill(low.begin(), low.end(), 0);
    fill(d.begin(), d.end(), 0);
    ticks = 0;
    for (int i = 0; i < g.size(); ++i)
        if (!vi[i]) dfs(i);
}
};

```

## 5.2 directed mst

```
const int inf = 1000000 + 10;
```

```
struct edge {
```

```

    int u, v, w;
    edge() {}
    edge(int a, int b, int c) : u(a), v(b), w(c) {}
};

/**
 * Computes the minimum spanning tree for a directed graph
 * - edges : Graph description in the form of list of edges.
 *   each edge is: From node u to node v with cost w
 * - root : Id of the node to start the DMST.
 * - n     : Number of nodes in the graph.
 */

int dmst(vector<edge> &edges, int root, int n) {
    int ans = 0;
    int cur_nodes = n;
    while (true) {
        vector<int> lo(cur_nodes, inf), pi(cur_nodes, inf);
        for (int i = 0; i < edges.size(); ++i) {
            int u = edges[i].u, v = edges[i].v, w = edges[i].w;
            if (w < lo[v] and u != v) {
                lo[v] = w;
                pi[v] = u;
            }
        }

        lo[root] = 0;
        for (int i = 0; i < lo.size(); ++i) {
            if (i == root) continue;
            if (lo[i] == inf) return -1;
        }
        int cur_id = 0;
        vector<int> id(cur_nodes, -1), mark(cur_nodes, -1);
        for (int i = 0; i < cur_nodes; ++i) {
            ans += lo[i];
            int u = i;
            while (u != root and id[u] < 0 and mark[u] != i) {
                mark[u] = i;
                u = pi[u];
            }
            if (u != root and id[u] < 0) { // Cycle
                for (int v = pi[u]; v != u; v = pi[v])
                    id[v] = cur_id;
                id[u] = cur_id++;
            }
        }
    }
}

```

```

}

if (cur_id == 0)
    break;

for (int i = 0; i < cur_nodes; ++i)
    if (id[i] < 0) id[i] = cur_id++;

for (int i = 0; i < edges.size(); ++i) {
    int u = edges[i].u, v = edges[i].v, w = edges[i].w;
    edges[i].u = id[u];
    edges[i].v = id[v];
    if (id[u] != id[v])
        edges[i].w -= lo[v];
}
cur_nodes = cur_id;
root = id[root];
}

return ans;
}

```

## 5.3 eulerian path

// Taken from <https://github.com/lbv/pc-code/blob/master/code/graph.cpp>  
// Eulerian Trail

```

struct Euler {
    ELV adj; IV t;
    Euler(ELV Adj) : adj(Adj) {}
    void build(int u) {
        while(! adj[u].empty()) {
            int v = adj[u].front().v;
            adj[u].erase(adj[u].begin());
            build(v);
        }
        t.push_back(u);
    }
};

bool eulerian_trail(IV &trail) {
    Euler e(adj);
    int odd = 0, s = 0;
    /*

```

```

        for (int v = 0; v < n; v++) {
            int diff = abs(in[v] - out[v]);
            if (diff > 1) return false;
            if (diff == 1) {
                if (++odd > 2) return false;
                if (out[v] > in[v]) start = v;
            }
        }
        /*
    e.build(s);
    reverse(e.t.begin(), e.t.end());
    trail = e.t;
    return true;
}

```

## 5.4 karp min mean cycle

```

/**
 * Finds the min mean cycle, if you need the max mean cycle
 * just add all the edges with negative cost and print
 * ans * -1
 *
 * test: uva, 11090 - Going in Cycle!!
 * */

```

```

const int MN = 1000;
struct edge{
    int v;
    long long w;
    edge(){} edge(int v, int w) : v(v), w(w) {}
};

```

```

long long d[MN][MN];
// This is a copy of g because increments the size
// pass as reference if this does not matter.
int karp(vector<vector<edge> > g) {
    int n = g.size();

    g.resize(n + 1); // this is important

    for (int i = 0; i < n; ++i)
        if (!g[i].empty())
            g[n].push_back(edge(i,0));

```



```

++n;

for(int i = 0; i < n; ++i)
    fill(d[i], d[i] + (n + 1), INT_MAX);

d[n - 1][0] = 0;

for (int k = 1; k <= n; ++k) for (int u = 0; u < n; ++u) {
    if (d[u][k - 1] == INT_MAX) continue;
    for (int i = g[u].size() - 1; i >= 0; --i)
        d[g[u][i].v][k] = min(d[g[u][i].v][k], d[u][k - 1] + g[u][i].w);
}

bool flag = true;

for (int i = 0; i < n && flag; ++i)
    if (d[i][n] != INT_MAX)
        flag = false;

if (flag) {
    return true; // return true if there is no a cycle.
}

double ans = 1e15;

for (int u = 0; u + 1 < n; ++u) {
    if (d[u][n] == INT_MAX) continue;
    double W = -1e15;

    for (int k = 0; k < n; ++k)
        if (d[u][k] != INT_MAX)
            W = max(W, (double)(d[u][n] - d[u][k]) / (n - k));

    ans = min(ans, W);
}

// printf("%.2lf\n", ans);
cout << fixed << setprecision(2) << ans << endl;

return false;
}

```

## 5.5 konig's theorem

In any bipartite graph, the number of edges in a maximum matching equals the number of vertices in a minimum vertex cover

## 5.6 minimum path cover in DAG

Given a directed acyclic graph  $G = (V, E)$ , we are to find the minimum number of vertex-disjoint paths to cover each vertex in  $V$ .

We can construct a bipartite graph  $G' = (V_{out} \cup V_{in}, E')$  from  $G$ , where :

$$V_{out} = \{v \in V : v \text{ has positive out-degree}\}$$

$$V_{in} = \{v \in V : v \text{ has positive in-degree}\}$$

$$E' = \{(u, v) \in V_{out} \times V_{in} : (u, v) \in E\}$$

Then it can be shown, via König's theorem, that  $G'$  has a matching of size  $m$  if and only if there exists  $n - m$  vertex-disjoint paths that cover each vertex in  $G$ , where  $n$  is the number of vertices in  $G$  and  $m$  is the maximum cardinality bipartite matching in  $G'$ .

Therefore, the problem can be solved by finding the maximum cardinality matching in  $G'$  instead.

**NOTE:** If the paths are not necessarily disjoint, find the transitive closure and solve the problem for disjoint paths.

## 5.7 planar graph (euler)

Euler's formula states that if a finite, connected, planar graph is drawn in the plane without any edge intersections, and  $v$  is the number of vertices,  $e$  is the number of edges and  $f$  is the number of faces (regions bounded by edges, including the outer, infinitely large region), then:

$$f + v = e + 2$$

It can be extended to non connected planar graphs with  $c$  connected components:

$$f + v = e + c + 1$$

## 5.8 query with lca

```
struct lowest_ca {
    int T[MN], L[MN], W[MN];
    int P[MN][ML], MI[MN][ML], MA[MN][ML];

    void dfs(vector<vector<edge>> &g, int root, int pi = -1) {
        if (pi == -1) {
            L[root] = W[root] = 0;
            T[root] = -1;
        }
        for (int i = 0; i < (int)g[root].size(); ++i) {
            int to = g[root][i].v;
            if (to != pi) {
                T[to] = root;
                W[to] = g[root][i].w;
                L[to] = L[root] + 1;
                dfs(g, to, root);
            }
        }
    }

    void init(vector<vector<edge>> &g, int root) {
        // g is undirected
        dfs(g, root);
        int N = g.size(), i, j;

        for (i = 0; i < N; i++) {
            for (j = 0; 1 << j < N; j++) {
                P[i][j] = -1;
                MI[i][j] = inf;
            }
        }

        for (i = 0; i < N; i++) {
            P[i][0] = T[i];
            MI[i][0] = W[i];
        }

        for (j = 1; 1 << j < N; j++)
            for (i = 0; i < N; i++)
                if (P[i][j - 1] != -1) {
                    P[i][j] = P[P[i][j - 1]][j - 1];
                    MI[i][j] = min(MI[i][j - 1], MI[P[i][j - 1]][j - 1]);
                }
    }
};
```

```
};

int query(int p, int q) {
    int tmp, log, i;

    int mmin = inf;
    if (L[p] < L[q])
        tmp = p, p = q, q = tmp;

    for (log = 1; 1 << log <= L[p]; log++);
    log--;

    for (i = log; i >= 0; i--)
        if (L[p] - (1 << i) >= L[q]) {
            mmin = min(mmin, MI[p][i]);
            p = P[p][i];
        }

    if (p == q)
        // return p;
        return mmin;

    for (i = log; i >= 0; i--)
        if (P[p][i] != -1 && P[p][i] != P[q][i]) {
            mmin = min(mmin, min(MI[p][i], MI[q][i]));
            p = P[p][i], q = P[q][i];
        }

    // return T[p];
    return min(mmin, min(MI[p][0], MI[q][0]));
};
```

## 5.9 tarjan scc

```
const int MN = 20002;

struct tarjan_scc {
    int scc[MN], low[MN], d[MN], stacked[MN];
    int ticks, current_scc;
    deque<int> s; // used as stack.

    tarjan_scc() {}
};
```

```

void init () {
    memset(scc, -1, sizeof scc);
    memset(d, -1, sizeof d);
    memset(stacked, 0, sizeof stacked);
    s.clear();
    ticks = current_scc = 0;
}

void compute(vector<vector<int> > &g, int u) {
    d[u] = low[u] = ticks++;
    s.push_back(u);
    stacked[u] = true;
    for (int i = 0; i < g[u].size(); ++i) {
        int v = g[u][i];
        if (d[v] == -1)
            compute(g, v);
        if (stacked[v]) {
            low[u] = min(low[u], low[v]);
        }
    }

    if (d[u] == low[u]) { // root
        int v;
        do {
            v = s.back(); s.pop_back();
            stacked[v] = false;
            scc[v] = current_scc;
        } while (u != v);
        current_scc++;
    }
}
};

```

## 5.10 two sat (with kosaraju)

```

/**
 * Given a set of clauses (a1 v a2)^(a2 v a3)....
 * this algorithm find a solution to it set of clauses.
 * test: http://lightoj.com/volume\_showproblem.php?problem=1251
 */

#include<bits/stdc++.h>

```

```

using namespace std;
#define MAX 100000
#define endl '\n'

vector<int> G[MAX];
vector<int> GT[MAX];
vector<int> Ftime;
vector<vector<int> > SCC;
bool visited[MAX];
int n;

void dfs1(int n){
    visited[n] = 1;

    for (int i = 0; i < G[n].size(); ++i) {
        int curr = G[n][i];
        if (visited[curr]) continue;
        dfs1(curr);
    }

    Ftime.push_back(n);
}

void dfs2(int n, vector<int> &scc) {
    visited[n] = 1;
    scc.push_back(n);

    for (int i = 0; i < GT[n].size(); ++i) {
        int curr = GT[n][i];
        if (visited[curr]) continue;
        dfs2(curr, scc);
    }
}

void kosaraju() {
    memset(visited, 0, sizeof visited);

    for (int i = 0; i < 2 * n; ++i) {
        if (!visited[i]) dfs1(i);
    }

    memset(visited, 0, sizeof visited);
    for (int i = Ftime.size() - 1; i >= 0; i--) {

```

```

    if (visited[Ftime[i]]) continue;
    vector<int> _scc;
    dfs2(Ftime[i], _scc);
    SCC.push_back(_scc);
}

/**
 * After having the SCC, we must traverse each scc, if in one SCC are -b
 * y b, there is not a solution.
 * Otherwise we build a solution, making the first "node" that we find
 * truth and its complement false.
 */

bool two_sat(vector<int> &val) {
    kosaraju();
    for (int i = 0; i < SCC.size(); ++i) {
        vector<bool> tmpvisited(2 * n, false);
        for (int j = 0; j < SCC[i].size(); ++j) {
            if (tmpvisited[SCC[i][j] ^ 1]) return 0;
            if (val[SCC[i][j]] != -1) continue;
            else {
                val[SCC[i][j]] = 0;
                val[SCC[i][j] ^ 1] = 1;
            }
            tmpvisited[SCC[i][j]] = 1;
        }
    }
    return 1;
}

// Example of use

int main() {

    int m, u, v, nc = 0, t; cin >> t;
    // n = "nodes" number, m = clauses number

    while (t--) {
        cin >> m >> n;
        Ftime.clear();
        SCC.clear();
        for (int i = 0; i < 2 * n; ++i) {
            G[i].clear();

```

```

            GT[i].clear();
        }

        // (a1 v a2) = (a1 -> a2) = (a2 -> a1)
        for (int i = 0; i < m; ++i) {
            cin >> u >> v;
            int t1 = abs(u) - 1;
            int t2 = abs(v) - 1;
            int p = t1 * 2 + ((u < 0)? 1 : 0);
            int q = t2 * 2 + ((v < 0)? 1 : 0);
            G[p ^ 1].push_back(q);
            G[q ^ 1].push_back(p);
            GT[p].push_back(q ^ 1);
            GT[q].push_back(p ^ 1);
        }

        vector<int> val(2 * n, -1);
        cout << "Case " << ++nc << ": ";
        if (two_sat(val)) {
            cout << "Yes" << endl;
            vector<int> sol;
            for (int i = 0; i < 2 * n; ++i)
                if (i % 2 == 0 and val[i] == 1)
                    sol.push_back(i / 2 + 1);
            cout << sol.size() ;

            for (int i = 0; i < sol.size(); ++i) {
                cout << " " << sol[i];
            }
            cout << endl;
        } else {
            cout << "No" << endl;
        }
    }
    return 0;
}

```

## 6 Math

### 6.1 Lucas theorem

For non-negative integers  $m$  and  $n$  and a prime  $p$ , the following congruence relation holds: :

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p},$$

where :

$$m = m_k p^k + m_{k-1} p^{k-1} + \cdots + m_1 p + m_0,$$

and :

$$n = n_k p^k + n_{k-1} p^{k-1} + \cdots + n_1 p + n_0$$

are the base  $p$  expansions of  $m$  and  $n$  respectively. This uses the convention that  $\binom{m}{n} = 0$  if  $m \leq n$ .

### 6.2 cumulative sum of divisors

---

/\*\*

The function SOD(n) (sum of divisors) is defined as the summation of all the actual divisors of an integer number n. For example,

$$\text{SOD}(24) = 2+3+4+6+8+12 = 35.$$

The function CSOD(n) (cumulative SOD) of an integer n, is defined as below:

$$\text{csod}(n) = \sum_{i=1}^n \text{sod}(i)$$

It can be computed in  $O(\sqrt{n})$ :

\*/

```
long long csod(long long n) {
    long long ans = 0;
    for (long long i = 2; i * i <= n; ++i) {
        long long j = n / i;
        ans += (i + j) * (j - i + 1) / 2;
        ans += i * (j - i);
    }
    return ans;
}
```

---

### 6.3 fft

---

/\*\*

\* Fast Fourier Transform.  
\* Useful to compute convolutions.  
\* computes:  
\* C(f star g)[n] = sum\_m(f[m] \* g[n - m])  
\* for all n.  
\* test: icpc live archive, 6886 - Golf Bot  
\* \*/

```
using namespace std;
#include<bits/stdc++.h>
#define D(x) cout << #x " = " << (x) << endl
#define endl '\n'
```

```
const int MN = 262144 << 1;
int d[MN + 10], d2[MN + 10];
```

```
const double PI = acos(-1.0);
```

```
struct cpx {
    double real, image;
    cpx(double _real, double _image) {
        real = _real;
        image = _image;
    }
    cpx(){}
};
```

```
cpx operator + (const cpx &c1, const cpx &c2) {
    return cpx(c1.real + c2.real, c1.image + c2.image);
}
```

```
cpx operator - (const cpx &c1, const cpx &c2) {
    return cpx(c1.real - c2.real, c1.image - c2.image);
}
```

```
cpx operator * (const cpx &c1, const cpx &c2) {
    return cpx(c1.real*c2.real - c1.image*c2.image, c1.real*c2.image +
        c1.image*c2.real);
}
```

```

int rev(int id, int len) {
    int ret = 0;
    for (int i = 0; (1 << i) < len; i++) {
        ret <<= 1;
        if (id & (1 << i)) ret |= 1;
    }
    return ret;
}

cpx A[1 << 20];

void FFT(cpx *a, int len, int DFT) {
    for (int i = 0; i < len; i++)
        A[rev(i, len)] = a[i];
    for (int s = 1; (1 << s) <= len; s++) {
        int m = (1 << s);
        cpx wm = cpx(cos( DFT * 2 * PI / m), sin(DFT * 2 * PI / m));
        for(int k = 0; k < len; k += m) {
            cpx w = cpx(1, 0);
            for(int j = 0; j < (m >> 1); j++) {
                cpx t = w * A[k + j + (m >> 1)];
                cpx u = A[k + j];
                A[k + j] = u + t;
                A[k + j + (m >> 1)] = u - t;
                w = w * wm;
            }
        }
    }
    if (DFT == -1) for (int i = 0; i < len; i++) A[i].real /= len,
        A[i].image /= len;
    for (int i = 0; i < len; i++) a[i] = A[i];
    return;
}

cpx in[1 << 20];

void solve(int n) {
    memset(d, 0, sizeof d);
    int t;
    for (int i = 0; i < n; ++i) {
        cin >> t;
        d[t] = true;
    }
    int m;
    cin >> m;

```

```

vector<int> q(m);
for (int i = 0; i < m; ++i)
    cin >> q[i];

for (int i = 0; i < MN; ++i) {
    if (d[i])
        in[i] = cpx(1, 0);
    else
        in[i] = cpx(0, 0);
}

FFT(in, MN, 1);
for (int i = 0; i < MN; ++i) {
    in[i] = in[i] * in[i];
}
FFT(in, MN, -1);

int ans = 0;
for (int i = 0; i < q.size(); ++i) {
    if (in[q[i]].real > 0.5 || d[q[i]]) {
        ans++;
    }
}
cout << ans << endl;
}

int main() {
    ios_base::sync_with_stdio(false); cin.tie(NULL);
    int n;
    while (cin >> n)
        solve(n);
    return 0;
}

```

## 6.4 fibonacci properties

Let A, B and n be integer numbers.

$$k = A - B \quad (1)$$

$$F_A F_B = F_{k+1} F_A^2 + F_k F_A F_{A-1} \quad (2)$$

$$\sum_{i=0}^n F_i^2 = F_{n+1}F_n \quad (3)$$

$ev(n)$  = returns 1 if  $n$  is even.

$$\sum_{i=0}^n F_i F_{i+1} = F_{n+1}^2 - ev(n) \quad (4)$$

$$\sum_{i=0}^n F_i F_{i-1} = \sum_{i=0}^{n-1} F_i F_{i+1} \quad (5)$$

## 6.5 sigma function

the sigma function is defined as:

$$\sigma_x(n) = \sum_{d|n} d^x$$

when  $x = 0$  is called the divisor function, that counts the number of positive divisors of  $n$ .

Now, we are interested in find

$$\sum_{d|n} \sigma_0(d)$$

if  $n$  is written as prime factorization:

$$n = \prod_{i=1}^k P_i^{e_k}$$

we can demonstrate that:

$$\sum_{d|n} \sigma_0(d) = \prod_{i=1}^k g(e_k + 1)$$

where  $g(x)$  is the sum of the first  $x$  positive numbers:

$$g(x) = (x * (x + 1)) / 2$$

## 7 Matrix

### 7.1 matrix

---

```
const int MN = 111;
const int mod = 10000;

struct matrix {
    int r, c;
    int m[MN][MN];

    matrix(int _r, int _c) : r(_r), c(_c) {
        memset(m, 0, sizeof m);
    }

    void print() {
        for (int i = 0; i < r; ++i) {
            for (int j = 0; j < c; ++j)
                cout << m[i][j] << " ";
            cout << endl;
        }
    }

    int x[MN][MN];

    matrix & operator *= (const matrix &o) {
        memset(x, 0, sizeof x);
        for (int i = 0; i < r; ++i)
            for (int k = 0; k < c; ++k)
                if (m[i][k] != 0)
                    for (int j = 0; j < c; ++j) {
                        x[i][j] = (x[i][j] + (m[i][k] * o.m[k][j]) % mod) % mod;
                    }
        memcpy(m, x, sizeof(m));
        return *this;
    }
};

void matrix_pow(matrix b, long long e, matrix &res) {
    memset(res.m, 0, sizeof res.m);
    for (int i = 0; i < b.r; ++i)
        res.m[i][i] = 1;

    if (e == 0) return;
    while (true) {
```

```

    if (e & 1) res *= b;
    if ((e >>= 1) == 0) break;
    b *= b;
}
}

```

---

## 8 Misc

### 8.1 Template Java

```

import java.io.*;
import java.util.StringTokenizer;

public class Template {

    public static void main(String []args) throws IOException {
        Scanner in = new Scanner(System.in);
        PrintWriter out = new PrintWriter(System.out);
        Task solver = new Task();
        solver.solve(in, out);
        out.close();
    }

}

class Task{
    public void solve(Scanner in, PrintWriter out){

    }

}

class Scanner{
    public BufferedReader reader;
    public StringTokenizer st;

    public Scanner(InputStream stream){
        reader = new BufferedReader(new InputStreamReader(stream));
        st = null;
    }

    public String next(){
        while(st == null || !st.hasMoreTokens()){

```

```

            try{
                String line = reader.readLine();
                if(line == null) return null;
                st = new StringTokenizer(line);
            }catch (Exception e){
                throw (new RuntimeException());
            }
        }
        return st.nextToken();
    }

    public int nextInt(){
        return Integer.parseInt(next());
    }

    public long nextLong(){
        return Long.parseLong(next());
    }

    public double nextDouble(){
        return Double.parseDouble(next());
    }
}

class PrintWriter{
    BufferedWriter writer;

    public PrintWriter(OutputStream stream){
        writer = new BufferedWriter(new OutputStreamWriter(stream));
    }

    public void print(int i) throws IOException {
        writer.write(i);
    }

    public void print(String s) throws IOException {
        writer.write(s);
    }

    public void print(char []c) throws IOException {
        writer.write(c);
    }

    public void close() throws IOException {
        writer.close();
    }
}

```

---



## 8.2 dates

---

```
//
// Time - Leap years
//

// A[i] has the accumulated number of days from months previous to i
const int A[13] = { 0, 0, 31, 59, 90, 120, 151, 181, 212, 243, 273, 304,
334 };
// same as A, but for a leap year
const int B[13] = { 0, 0, 31, 60, 91, 121, 152, 182, 213, 244, 274, 305,
335 };
// returns number of leap years up to, and including, y
int leap_years(int y) { return y / 4 - y / 100 + y / 400; }
bool is_leap(int y) { return y % 400 == 0 || (y % 4 == 0 && y % 100 !=
0); }
// number of days in blocks of years
const int p400 = 400*365 + leap_years(400);
const int p100 = 100*365 + leap_years(100);
const int p4 = 4*365 + 1;
const int p1 = 365;
int date_to_days(int d, int m, int y)
{
    return (y - 1) * 365 + leap_years(y - 1) + (is_leap(y) ? B[m] : A[m]) +
d;
}
void days_to_date(int days, int &d, int &m, int &y)
{
    bool top100; // are we in the top 100 years of a 400 block?
    bool top4; // are we in the top 4 years of a 100 block?
    bool top1; // are we in the top year of a 4 block?

    y = 1;
    top100 = top4 = top1 = false;

    y += ((days-1) / p400) * 400;
    d = (days-1) % p400 + 1;

    if (d > p100*3) top100 = true, d -= 3*p100, y += 300;
    else y += ((d-1) / p100) * 100, d = (d-1) % p100 + 1;

    if (d > p4*24) top4 = true, d -= 24*p4, y += 24*4;
    else y += ((d-1) / p4) * 4, d = (d-1) % p4 + 1;

    if (d > p1*3) top1 = true, d -= p1*3, y += 3;
```

```
else y += (d-1) / p1, d = (d-1) % p1 + 1;

const int *ac = top1 && (!top4 || top100) ? B : A;
for (m = 1; m < 12; ++m) if (d <= ac[m + 1]) break;
d -= ac[m];
}
```

---

## 8.3 fraction

---

```
struct frac{
    long long x, y;
    frac(long long a, long long b) {
        long long g = __gcd(a, b);
        x = a / g;
        y = b / g;
    }
    bool operator < (const frac &o) const {
        return (x * o.y < y * o.x);
    }
};
```

---

## 8.4 io

---

```
// taken from :
// https://github.com/lbv/pc-code/blob/master/solved/c-e/diablo/diablo.cpp
// this is very fast as well :
// https://github.com/lbv/pc-code/blob/master/code/input.cpp
```

```
typedef unsigned int u32;
#define BUF 524288
struct Reader {
    char buf[BUF]; char b; int bi, bz;
    Reader() { bi=bz=0; read(); }
    void read() {
        if (bi==bz) { bi=0; bz = fread(buf, 1, BUF, stdin); }
        b = bz ? buf[bi++] : 0; }
    void skip() { while (b > 0 && b <= 32) read(); }
    u32 next_u32() {
        u32 v = 0; for (skip(); b > 32; read()) v = v*10 + b-48; return v; }
    int next_int() {
        int v = 0; bool s = false;
```

```

    skip(); if (b == '-') { s = true; read(); }
    for (; 48<=b&&b<=57; read()) v = v*10 + b-48; return s ? -v : v; }
char next_char() { skip(); char c = b; read(); return c; }
};

```

---

## 9 Number theory

### 9.1 convolution

```

typedef long long int LL;
typedef pair<LL, LL> PLL;

inline bool is_pow2(LL x) {
    return (x & (x-1)) == 0;
}

inline int ceil_log2(LL x) {
    int ans = 0;
    --x;
    while (x != 0) {
        x >>= 1;
        ans++;
    }
    return ans;
}

/* Returns the convolution of the two given vectors in time proportional
   to n*log(n).
   * The number of roots of unity to use nroots_unity must be set so that
   the product of the first
   * nroots_unity primes of the vector nth_roots_unity is greater than the
   maximum value of the
   * convolution. Never use sizes of vectors bigger than 2^24, if you need
   to change the values of
   * the nth roots of unity to appropriate primes for those sizes.
   */
vector<LL> convolve(const vector<LL> &a, const vector<LL> &b, int
    nroots_unity = 2) {
    int N = 1 << ceil_log2(a.size() + b.size());
    vector<LL> ans(N,0), fA(N), fB(N), fC(N);
    LL modulo = 1;
    for (int times = 0; times < nroots_unity; times++) {

```

```

        fill(fA.begin(), fA.end(), 0);
        fill(fB.begin(), fB.end(), 0);
        for (int i = 0; i < a.size(); i++) fA[i] = a[i];
        for (int i = 0; i < b.size(); i++) fB[i] = b[i];
        LL prime = nth_roots_unity[times].first;
        LL inv_modulo = mod_inv(modulo % prime, prime);
        LL normalize = mod_inv(N, prime);
        ntfft(fA, 1, nth_roots_unity[times]);
        ntfft(fB, 1, nth_roots_unity[times]);
        for (int i = 0; i < N; i++) fC[i] = (fA[i] * fB[i]) % prime;
        ntfft(fC, -1, nth_roots_unity[times]);
        for (int i = 0; i < N; i++) {
            LL curr = (fC[i] * normalize) % prime;
            LL k = (curr - (ans[i] % prime) + prime) % prime;
            k = (k * inv_modulo) % prime;
            ans[i] += modulo * k;
        }
        modulo *= prime;
    }
    return ans;
}

```

---

### 9.2 crt

```

/**
 * Chinese remainder theorem.
 * Find z such that z % x[i] = a[i] for all i.
 */
long long crt(vector<long long> &a, vector<long long> &x) {
    long long z = 0;
    long long n = 1;
    for (int i = 0; i < x.size(); ++i)
        n *= x[i];

    for (int i = 0; i < a.size(); ++i) {
        long long tmp = (a[i] * (n / x[i])) % n;
        tmp = (tmp * mod_inv(n / x[i], x[i])) % n;
        z = (z + tmp) % n;
    }

    return (z + n) % n;
}

```

---

### 9.3 discrete logarithm

---

```
// Computes x which a ^ x = b mod n.

long long d_log(long long a, long long b, long long n) {
    long long m = ceil(sqrt(n));
    long long aj = 1;
    map<long long, long long> M;
    for (int i = 0; i < m; ++i) {
        if (!M.count(aj))
            M[aj] = i;
        aj = (aj * a) % n;
    }

    long long coef = mod_pow(a, n - 2, n);
    coef = mod_pow(coef, m, n);
    // coef = a ^ (-m)
    long long gamma = b;
    for (int i = 0; i < m; ++i) {
        if (M.count(gamma)) {
            return i * m + M[gamma];
        } else {
            gamma = (gamma * coef) % n;
        }
    }
    return -1;
}
```

---

### 9.4 ext euclidean

---

```
void ext_euclid(long long a, long long b, long long &x, long long &y,
               long long &g) {
    x = 0, y = 1, g = b;
    long long m, n, q, r;
    for (long long u = 1, v = 0; a != 0; g = a, a = r) {
        q = g / a, r = g % a;
        m = x - u * q, n = y - v * q;
        x = u, y = v, u = m, v = n;
    }
}
```

---

### 9.5 highest exponent factorial

---

```
int highest_exponent(int p, const int &n){
    int ans = 0;
    int t = p;
    while(t <= n){
        ans += n/t;
        t*=p;
    }
    return ans;
}
```

---

### 9.6 miller rabin

---

```
const int rounds = 20;

// checks whether a is a witness that n is not prime, 1 < a < n
bool witness(long long a, long long n) {
    // check as in Miller Rabin Primality Test described
    long long u = n - 1;
    int t = 0;
    while (u % 2 == 0) {
        t++;
        u >>= 1;
    }
    long long next = mod_pow(a, u, n);
    if (next == 1) return false;
    long long last;
    for (int i = 0; i < t; ++i) {
        last = next;
        next = mod_mul(last, last, n);
        if (next == 1) {
            return last != n - 1;
        }
    }
    return next != 1;
}

// Checks if a number is prime with prob 1 - 1 / (2 ^ it)
// D(miller_rabin(9999999999999997LL) == 1);
// D(miller_rabin(9999999999971LL) == 1);
// D(miller_rabin(7907) == 1);
```

---

```

bool miller_rabin(long long n, int it = rounds) {
    if (n <= 1) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    for (int i = 0; i < it; ++i) {
        long long a = rand() % (n - 1) + 1;
        if (witness(a, n)) {
            return false;
        }
    }
    return true;
}

```

---

## 9.7 mod inv

---

```

long long mod_inv(long long n, long long m) {
    long long x, y, gcd;
    ext_euclid(n, m, x, y, gcd);
    if (gcd != 1)
        return 0;
    return (x + m) % m;
}

```

---

## 9.8 mod mul

---

```

// Computes (a * b) % mod
long long mod_mul(long long a, long long b, long long mod) {
    long long x = 0, y = a % mod;
    while (b > 0) {
        if (b & 1)
            x = (x + y) % mod;
        y = (y * 2) % mod;
        b /= 2;
    }
    return x % mod;
}

```

---

## 9.9 mod pow

---

```

// Computes (a ^ exp) % mod.
long long mod_pow(long long a, long long exp, long long mod) {
    long long ans = 1;
    while (exp > 0) {
        if (exp & 1)
            ans = mod_mul(ans, a, mod);
        a = mod_mul(a, a, mod);
        exp >>= 1;
    }
    return ans;
}

```

---

## 9.10 number theoretic transform

---

```

typedef long long int LL;
typedef pair<LL, LL> PLL;

/* The following vector of pairs contains pairs (prime, generator)
 * where the prime has an Nth root of unity for N being a power of two.
 * The generator is a number g s.t g^(p-1)=1 (mod p)
 * but is different from 1 for all smaller powers */
vector<PLL> nth_roots_unity {
    {1224736769,330732430},{1711276033,927759239},{167772161,167489322},
    {469762049,343261969},{754974721,643797295},{1107296257,883865065}};

PLL ext_euclid(LL a, LL b) {
    if (b == 0)
        return make_pair(1,0);
    pair<LL,LL> rc = ext_euclid(b, a % b);
    return make_pair(rc.second, rc.first - (a / b) * rc.second);
}

//returns -1 if there is no unique modular inverse
LL mod_inv(LL x, LL modulo) {
    PLL p = ext_euclid(x, modulo);
    if ( (p.first * x + p.second * modulo) != 1 )
        return -1;
    return (p.first+modulo) % modulo;
}

```

---

//Number theory fft. The size of a must be a power of 2

```

void ntfft(vector<LL> &a, int dir, const PLL &root_unity) {
    int n = a.size();
    LL prime = root_unity.first;
    LL basew = mod_pow(root_unity.second, (prime-1) / n, prime);
    if (dir < 0) basew = mod_inv(basew, prime);
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        LL w = 1;
        for (int i = 0; i < mh; i++) {
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                LL x = (a[j] - a[k] + prime) % prime;
                a[j] = (a[j] + a[k]) % prime;
                a[k] = (w * x) % prime;
            }
            w = (w * basew) % prime;
        }
        basew = (basew * basew) % prime;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^= k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
}

```

---

## 9.11 pollard rho factorize

```

long long pollard_rho(long long n) {
    long long x, y, i = 1, k = 2, d;
    x = y = rand() % n;
    while (1) {
        ++i;
        x = mod_mul(x, x, n);
        x += 2;
        if (x >= n) x -= n;
        if (x == y) return 1;
        d = __gcd(abs(x - y), n);
        if (d != 1) return d;
        if (i == k) {
            y = x;
            k *= 2;
        }
    }
}

```

```

}
return 1;
}

// Returns a list with the prime divisors of n
vector<long long> factorize(long long n) {
    vector<long long> ans;
    if (n == 1)
        return ans;
    if (miller_rabin(n)) {
        ans.push_back(n);
    } else {
        long long d = 1;
        while (d == 1)
            d = pollard_rho(n);
        vector<long long> dd = factorize(d);
        ans = factorize(n / d);
        for (int i = 0; i < dd.size(); ++i)
            ans.push_back(dd[i]);
    }
    return ans;
}

```

---

## 9.12 primes

```

namespace primes {
    const int MP = 100001;
    bool sieve[MP];
    long long primes[MP];
    int num_p;
    void fill_sieve() {
        num_p = 0;
        sieve[0] = sieve[1] = true;
        for (long long i = 2; i < MP; ++i) {
            if (!sieve[i]) {
                primes[num_p++] = i;
                for (long long j = i * i; j < MP; j += i)
                    sieve[j] = true;
            }
        }
    }
}

```

```
// Finds prime numbers between a and b, using basic primes up to sqrt(b)
vector<long long> seg_sieve(long long a, long long b) {
    long long ant = a;
    a = max(a, 3LL);
    vector<bool> pmap(b - a + 1);
    long long sqrt_b = sqrt(b);
    for (int i = 0; i < num_p; ++i) {
        long long p = primes[i];
        if (p > sqrt_b) break;
        long long j = (a + p - 1) / p;
        for (long long v = (j == 1) ? p + p : j * p; v <= b; v += p) {
            pmap[v - a] = true;
        }
    }
    vector<long long> ans;
    if (ant == 2) ans.push_back(2);
    int start = a % 2 ? 0 : 1;
    for (int i = start, I = b - a + 1; i < I; i += 2)
        if (pmap[i] == false)
            ans.push_back(a + i);
    return ans;
}

vector<pair<int, int>> factor(int n) {
    vector<pair<int, int>> ans;
    if (n == 0) return ans;
    for (int i = 0; primes[i] * primes[i] <= n; ++i) {
        if ((n % primes[i]) == 0) {
            int expo = 0;
            while ((n % primes[i]) == 0) {
                expo++;
                n /= primes[i];
            }
            ans.emplace_back(primes[i], expo);
        }
    }

    if (n > 1) {
        ans.emplace_back(n, 1);
    }
    return ans;
}
}
```

## 9.13 totient sieve

---

```
for (int i = 1; i < MN; i++)
    phi[i] = i;

for (int i = 1; i < MN; i++)
    if (!sieve[i]) // is prime
        for (int j = i; j < MN; j += i)
            phi[j] -= phi[j] / i;
```

---

## 9.14 totient

---

```
long long totient(long long n) {
    if (n == 1) return 0;
    long long ans = n;
    for (int i = 0; primes[i] * primes[i] <= n; ++i) {
        if ((n % primes[i]) == 0) {
            while ((n % primes[i]) == 0) n /= primes[i];
            ans -= ans / primes[i];
        }
    }
    if (n > 1) {
        ans -= ans / n;
    }
    return ans;
}
```

---

# 10 Strings

## 10.1 Incremental Aho Corasick

---

```
class IncrementalAhoCorasick {
    static const int Alphabets = 26;
    static const int AlphabetBase = 'a';
    struct Node {
        Node *fail;
        Node *next[Alphabets];
        int sum;
        Node() : fail(NULL), next={}, sum(0) {}
    };
};
```

```

struct String {
    string str;
    int sign;
};

public:
    //totalLen = sum of (len + 1)
    void init(int totalLen) {
        nodes.resize(totalLen);
        nNodes = 0;
        strings.clear();
        roots.clear();
        sizes.clear();
        que.resize(totalLen);
    }

    void insert(const string &str, int sign) {
        strings.push_back(String{ str, sign });
        roots.push_back(nodes.data() + nNodes);
        sizes.push_back(1);
        nNodes += (int)str.size() + 1;
        auto check = [&]() { return sizes.size() > 1 && sizes.end()[-1] ==
            sizes.end()[-2]; };
        if(!check())
            makePMA(strings.end() - 1, strings.end(), roots.back(), que);
        while(check()) {
            int m = sizes.back();
            roots.pop_back();
            sizes.pop_back();
            sizes.back() += m;
            if(!check())
                makePMA(strings.end() - m * 2, strings.end(), roots.back(), que);
        }
    }

    int match(const string &str) const {
        int res = 0;
        for(const Node *t : roots)
            res += matchPMA(t, str);
        return res;
    }

private:

```

```

static void makePMA(vector<String>::const_iterator begin,
    vector<String>::const_iterator end, Node *nodes, vector<Node*>
    &que) {
    int nNodes = 0;
    Node *root = new(&nodes[nNodes ++]) Node();
    for(auto it = begin; it != end; ++ it) {
        Node *t = root;
        for(char c : it->str) {
            Node *&n = t->next[c - AlphabetBase];
            if(n == nullptr)
                n = new(&nodes[nNodes ++]) Node();
            t = n;
        }
        t->sum += it->sign;
    }
    int qt = 0;
    for(Node *&n : root->next) {
        if(n != nullptr) {
            n->fail = root;
            que[qt ++] = n;
        } else {
            n = root;
        }
    }
    for(int qh = 0; qh != qt; ++ qh) {
        Node *t = que[qh];
        int a = 0;
        for(Node *n : t->next) {
            if(n != nullptr) {
                que[qt ++] = n;
                Node *r = t->fail;
                while(r->next[a] == nullptr)
                    r = r->fail;
                n->fail = r->next[a];
                n->sum += r->next[a]->sum;
            }
            ++ a;
        }
    }
}

static int matchPMA(const Node *t, const string &str) {
    int res = 0;
    for(char c : str) {
        int a = c - AlphabetBase;

```

```

        while(t->next[a] == nullptr)
            t = t->fail;
        t = t->next[a];
        res += t->sum;
    }
    return res;
}

vector<Node> nodes;
int nNodes;
vector<String> strings;
vector<Node*> roots;
vector<int> sizes;
vector<Node*> que;
};

int main() {
    int m;
    while(~scanf("%d", &m)) {
        IncrementalAhoCorasic iac;
        iac.init(600000);
        rep(i, m) {
            int ty;
            char s[300001];
            scanf("%d%s", &ty, s);
            if(ty == 1) {
                iac.insert(s, +1);
            } else if(ty == 2) {
                iac.insert(s, -1);
            } else if(ty == 3) {
                int ans = iac.match(s);
                printf("%d\n", ans);
                fflush(stdout);
            } else {
                abort();
            }
        }
    }
    return 0;
}

```

## 10.2 minimal string rotation

---

```
// Lexicographically minimal string rotation
```

```

int lmsr() {
    string s;
    cin >> s;
    int n = s.size();
    s += s;
    vector<int> f(s.size(), -1);
    int k = 0;
    for (int j = 1; j < 2 * n; ++j) {
        int i = f[j - k - 1];
        while (i != -1 && s[j] != s[k + i + 1]) {
            if (s[j] < s[k + i + 1])
                k = j - i - 1;
            i = f[i];
        }
        if (i == -1 && s[j] != s[k + i + 1]) {
            if (s[j] < s[k + i + 1]) {
                k = j;
            }
            f[j - k] = -1;
        } else {
            f[j - k] = i + 1;
        }
    }
    return k;
}

```

---

## 10.3 suffix array

---

```

/**
 * O(n log^2(n))
 * See http://web.stanford.edu/class/cs97si/suffix-array.pdf for reference
 */

struct entry{
    int a, b, p;
    entry(){}
    entry(int x, int y, int z): a(x), b(y), p(z){}
    bool operator < (const entry &o) const {
        return (a == o.a) ? (b == o.b) ? (p < o.p) : (b < o.b) : (a < o.a);
    }
};

```



```

struct SuffixArray{
    const int N;
    string s;
    vector<vector<int> > P;
    vector<entry> M;

    SuffixArray(const string &s) : N(s.length()) , s(s), P(1, vector<int>
        (N, 0)), M(N) {
        for (int i = 0; i < N; ++i)
            P[0][i] = (int) s[i];

        for (int skip = 1, level = 1; skip < N; skip *= 2, level++) {
            P.push_back(vector<int>(N, 0));
            for (int i = 0 ; i < N; ++i) {
                int next = ((i + skip) < N) ? P[level - 1][i + skip] : -10000;
                M[i] = entry(P[level - 1][i], next, i);
            }
            sort(M.begin(), M.end());
            for (int i = 0; i < N; ++i)
                P[level][M[i].p] = (i > 0 and M[i].a == M[i - 1].a and M[i].b ==
                    M[i - 1].b) ? P[level][M[i - 1].p] : i;
        }
    }

    vector<int> getSuffixArray(){
        vector<int> &rank = P.back();
        vector<pair<int, int> > inv(rank.size());
        for (int i = 0; i < rank.size(); ++i)
            inv[i] = make_pair(rank[i], i);
        sort(inv.begin(), inv.end());
        vector<int> sa(rank.size());
        for (int i = 0; i < rank.size(); ++i)
            sa[i] = inv[i].second;
        return sa;
    }

    // returns the length of the longest common prefix of s[i...L-1] and
    // s[j...L-1]
    int lcp(int i, int j) {
        int len = 0;
        if (i == j) return N - i;
        for (int k = P.size() - 1; k >= 0 && i < N && j < N; --k) {
            if (P[k][i] == P[k][j]) {
                i += 1 << k;

```

```

                j += 1 << k;
                len += 1 << k;
            }
        }
        return len;
    }
};

```

## 10.4 suffix automaton

```

/*
 * Suffix automaton:
 * This implementation was extended to maintain (online) the
 * number of different substrings. This is equivalent to compute
 * the number of paths from the initial state to all the other
 * states.
 *
 * The overall complexity is O(n)
 * can be tested here:
 *   https://www.urionlinejudge.com.br/judge/en/problems/view/1530
 */

struct state {
    int len, link;
    long long num_paths;
    map<int, int> next;
};

const int MN = 200011;
state sa[MN << 1];
int sz, last;
long long tot_paths;

void sa_init() {
    sz = 1;
    last = 0;
    sa[0].len = 0;
    sa[0].link = -1;
    sa[0].next.clear();
    sa[0].num_paths = 1;
    tot_paths = 0;
}

```

```

void sa_extend(int c) {
    int cur = sz++;
    sa[cur].len = sa[last].len + 1;
    sa[cur].next.clear();
    sa[cur].num_paths = 0;
    int p;
    for (p = last; p != -1 && !sa[p].next.count(c); p = sa[p].link) {
        sa[p].next[c] = cur;
        sa[cur].num_paths += sa[p].num_paths;
        tot_paths += sa[p].num_paths;
    }

    if (p == -1) {
        sa[cur].link = 0;
    } else {
        int q = sa[p].next[c];
        if (sa[p].len + 1 == sa[q].len) {
            sa[cur].link = q;
        } else {
            int clone = sz++;
            sa[clone].len = sa[p].len + 1;
            sa[clone].next = sa[q].next;
            sa[clone].num_paths = 0;
            sa[clone].link = sa[q].link;
            for (; p != -1 && sa[p].next[c] == q; p = sa[p].link) {
                sa[p].next[c] = clone;
                sa[q].num_paths -= sa[p].num_paths;
                sa[clone].num_paths += sa[p].num_paths;
            }
            sa[q].link = sa[cur].link = clone;
        }
    }
    last = cur;
}

```

---

## 10.5 z algorithm

```

using namespace std;
#include<bits/stdc++.h>

vector<int> compute_z(const string &s){
    int n = s.size();
    vector<int> z(n,0);

```

```

    int l,r;
    r = l = 0;
    for(int i = 1; i < n; ++i){
        if(i > r) {
            l = r = i;
            while(r < n and s[r - l] == s[r])r++;
            z[i] = r - l;r--;
        }else{
            int k = i-l;
            if(z[k] < r - i +1) z[i] = z[k];
            else {
                l = i;
                while(r < n and s[r - l] == s[r])r++;
                z[i] = r - l;r--;
            }
        }
    }
    return z;
}

int main(){

    //string line;cin>>line;
    string line = "alfalfa";
    vector<int> z = compute_z(line);

    for(int i = 0; i < z.size(); ++i ){
        if(i)cout<<" ";
        cout<<z[i];
    }
    cout<<endl;

    // must print "0 0 0 4 0 0 1"

    return 0;
}

```

---