Team notebook

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1 Algorithms

1.1 sliding window

```
} else {
    while (!window.empty() && window.back().first >= ARR[i])
        window.pop_back();
}
window.push_back(make_pair(ARR[i], i));

while(window.front().second <= i - K)
    window.pop_front();

ans.push_back(window.front().first);
}
return ans;
}</pre>
```

2 Data structures

2.1 hash table

```
/**
 * Micro hash table, can be used as a set.
 * Very efficient vs std::set
 * */

const int MN = 1001;
struct ht {
  int _s[(MN + 10) >> 5];
  int len;
  void set(int id) {
    len++;
    _s[id >> 5] |= (1LL << (id & 31));
  }
  bool is_set(int id) {
    return _s[id >> 5] & (1LL << (id & 31));
  }
};</pre>
```

2.2 heavy light decomposition

```
// Heavy-Light Decomposition
struct TreeDecomposition {
```

```
vector<int> g[MAXN], c[MAXN];
int s[MAXN]; // subtree size
int p[MAXN]; // parent id
int r[MAXN]; // chain root id
int t[MAXN]; // index used in segtree/bit/...
int d[MAXN]; // depht
int ts;
void dfs(int v, int f) {
 p[v] = f;
 s[v] = 1;
 if (f != -1) d[v] = d[f] + 1;
 else d[v] = 0;
 for (int i = 0; i < g[v].size(); ++i) {</pre>
   int w = g[v][i];
   if (w != f) {
     dfs(w, v);
     s[v] += s[w];
 }
void hld(int v, int f, int k) {
 t[v] = ts++;
 c[k].push_back(v);
 r[v] = k;
 int x = 0, y = -1;
 for (int i = 0; i < g[v].size(); ++i) {</pre>
   int w = g[v][i];
   if (w != f) {
     if (s[w] > x) {
       x = s[w];
       y = w;
 if (y != -1) {
   hld(y, v, k);
 for (int i = 0; i < g[v].size(); ++i) {</pre>
   int w = g[v][i];
   if (w != f && w != y) {
```

```
hld(w, v, w);
     }
   }
 }
  void init(int n) {
   for (int i = 0; i < n; ++i) {</pre>
     g[i].clear();
   }
 }
  void add(int a, int b) {
    g[a].push_back(b);
    g[b].push_back(a);
  void build() {
   ts = 0;
    dfs(0, -1);
    hld(0, 0, 0);
 }
};
```

2.3 segment tree

```
/**
 * Taken from: http://codeforces.com/blog/entry/18051
 * */

const int N = 1e5; // limit for array size
int n; // array size
int t[2 * N];

void build() { // build the tree
  for (int i = n - 1; i > 0; --i) t[i] = t[i<<1] + t[i<<1|1];
}

// Single modification, range query.
void modify(int p, int value) { // set value at position p
  for (t[p += n] = value; p > 1; p >>= 1) t[p>>1] = t[p] + t[p^1];
}

int query(int l, int r) { // sum on interval [l, r)
```

```
int res = 0:
 for (1 += n, r += n; 1 < r; 1 >>= 1, r >>= 1) {
   if (l&1) res += t[l++];
   if (r&1) res += t[--r];
 return res;
// Range modification, single query.
void modify(int 1, int r, int value) {
 for (1 += n, r += n; 1 < r; 1 >>= 1, r >>= 1) {
   if (l&1) t[l++] += value;
   if (r&1) t[--r] += value;
}
int query(int p) {
 int res = 0;
 for (p += n; p > 0; p >>= 1) res += t[p];
 return res;
 * If at some point after modifications we need to inspect all the
 * elements in the array, we can push all the modifications to the
 * leaves using the following code. After that we can just traverse
 * elements starting with index n. This way we reduce the complexity
 * from O(n \log(n)) to O(n) similarly to using build instead of n
     modifications.
 * */
void push() {
 for (int i = 1; i < n; ++i) {</pre>
   t[i<<1] += t[i];
   t[i<<1|1] += t[i];
   t[i] = 0;
}
// Non commutative combiner functions.
void modify(int p, const S& value) {
 for (t[p += n] = value; p >>= 1;) t[p] = combine(t[p << 1], t[p << 1|1]);
```

```
S query(int 1, int r) {
    S resl, resr;
    for (1 += n, r += n; 1 < r; 1 >>= 1, r >>= 1) {
        if (1&1) resl = combine(resl, t[1++]);
        if (r&1) resr = combine(t[--r], resr);
    }
    return combine(resl, resr);
}
// To be continued ...
```

2.4 sparse table

```
// RMQ.
const int MN = 100000 + 10; // Max number of elements
const int ML = 18; // ceil(log2(MN));
struct st {
 int data[MN];
 int M[MN][ML];
 int n;
 void read(int _n) {
   n = _n;
   for (int i = 0; i < n; ++i)
     cin >> data[i];
 }
 void build() {
   for (int i = 0; i < n; ++i)
     M[i][0] = data[i];
   for (int j = 1, p = 2, q = 1; p \le n; ++j, p \le 1, q \le 1)
     for (int i = 0; i + p - 1 < n; ++i)
       M[i][j] = max(M[i][j-1], M[i+q][j-1]);
 int query(int b, int e) {
   int k = log2(e - b + 1);
   return max(M[b][k], M[e + 1 - (1<<k)][k]);</pre>
 }
};
```

2.5 splay tree

```
using namespace std;
#include<bits/stdc++.h>
#define D(x) cout<<x<<endl;</pre>
typedef int T;
struct node{
 node *left, *right, *parent;
 T key;
 node (T k) : key(k), left(0), right(0), parent(0) {}
};
struct splay_tree{
  node *root;
  void right_rot(node *x) {
   node *p = x->parent;
   if (x->parent = p->parent) {
     if (x->parent->left == p) x->parent->left = x;
     if (x->parent->right == p) x->parent->right = x;
   if (p->left = x->right) p->left->parent = p;
   x->right = p;
   p->parent = x;
  void left_rot(node *x) {
   node *p = x->parent;
   if (x->parent = p->parent) {
     if (x->parent->left == p) x->parent->left = x;
     if (x->parent->right == p) x->parent->right = x;
   if (p->right = x->left) p->right->parent = p;
   x \rightarrow left = p;
   p->parent = x;
  void splay(node *x, node *fa = 0) {
   while( x->parent != fa and x->parent != 0) {
     node *p = x->parent;
```

```
if (p->parent == fa)
     if (p->right == x)
       left_rot(x);
     else
       right_rot(x);
   else {
     node *gp = p->parent; //grand parent
     if (gp->left == p)
       if (p->left == x)
         right_rot(x), right_rot(x);
         left_rot(x),right_rot(x);
     else
       if (p->left == x)
         right_rot(x), left_rot(x);
         left_rot(x), left_rot(x);
   }
  }
  if (fa == 0) root = x;
}
void insert(T key) {
  node *cur = root;
  node *pcur = 0;
  while (cur) {
   pcur = cur;
   if (key > cur->key) cur = cur->right;
   else cur = cur->left;
  }
  cur = new node(key);
  cur->parent = pcur;
  if (!pcur) root = cur;
  else if (key > pcur->key ) pcur->right = cur;
  else pcur->left = cur;
  splay(cur);
}
node *find(T key) {
  node *cur = root;
  while (cur) {
   if (key > cur->key) cur = cur->right;
   else if(key < cur->key) cur = cur->left;
   else return cur;
  }
```

```
return 0;
}
splay_tree(){ root = 0;};
};
```

2.6 trie

```
const int MN = 26; // size of alphabet
const int MS = 100010; // Number of states.
struct trie{
 struct node{
   int c;
   int a[MN];
 node tree[MS];
 int nodes;
 void clear(){
   tree[nodes].c = 0;
   memset(tree[nodes].a, -1, sizeof tree[nodes].a);
   nodes++;
 void init(){
   nodes = 0;
   clear();
 int add(const string &s, bool query = 0){
   int cur_node = 0;
   for(int i = 0; i < s.size(); ++i){</pre>
     int id = gid(s[i]);
     if(tree[cur_node].a[id] == -1){
       if(query) return 0;
       tree[cur_node].a[id] = nodes;
       clear();
     cur_node = tree[cur_node].a[id];
   if(!query) tree[cur_node].c++;
```

```
return tree[cur_node].c;
};
```

3 Graphs

3.1 directed mst

```
const int inf = 1000000 + 10;
struct edge {
 int u, v, w;
 edge() {}
 edge(int a,int b,int c) : u(a), v(b), w(c) {}
};
/**
* Computes the minimum spanning tree for a directed graph
 * - edges : Graph description in the form of list of edges.
 * each edge is: From node u to node v with cost w
 * - root : Id of the node to start the DMST.
        : Number of nodes in the graph.
 * */
int dmst(vector<edge> &edges, int root, int n) {
 int ans = 0;
 int cur_nodes = n;
  while (true) {
   vector<int> lo(cur_nodes, inf), pi(cur_nodes, inf);
   for (int i = 0; i < edges.size(); ++i) {</pre>
     int u = edges[i].u, v = edges[i].v, w = edges[i].w;
     if (w < lo[v] and u != v) {
      lo[v] = w;
       pi[v] = u;
   }
   lo[root] = 0;
   for (int i = 0; i < lo.size(); ++i) {</pre>
     if (i == root) continue;
     if (lo[i] == inf) return -1;
```

```
int cur_id = 0;
  vector<int> id(cur_nodes, -1), mark(cur_nodes, -1);
  for (int i = 0; i < cur_nodes; ++i) {</pre>
   ans += lo[i];
   int u = i;
   while (u != root and id[u] < 0 and mark[u] != i) {</pre>
     mark[u] = i;
     u = pi[u];
   if (u != root and id[u] < 0) { // Cycle</pre>
      for (int v = pi[u]; v != u; v = pi[v])
        id[v] = cur_id;
      id[u] = cur_id++;
   }
 }
  if (cur_id == 0)
   break:
  for (int i = 0; i < cur_nodes; ++i)</pre>
   if (id[i] < 0) id[i] = cur_id++;</pre>
  for (int i = 0; i < edges.size(); ++i) {</pre>
   int u = edges[i].u, v = edges[i].v, w = edges[i].w;
   edges[i].u = id[u];
   edges[i].v = id[v];
   if (id[u] != id[v])
     edges[i].w -= lo[v];
  cur_nodes = cur_id;
  root = id[root];
return ans;
```

3.2 karp min mean cycle

```
/**
 * Finds the min mean cycle, if you need the max mean cycle
 * just add all the edges with negative cost and print
 * ans * -1
```

```
* test: uva, 11090 - Going in Cycle!!
const int MN = 1000;
struct edge{
 int v;
 long long w;
 edge(){} edge(int v, int w) : v(v), w(w) {}
};
long long d[MN][MN];
// This is a copy of g because increments the size
// pass as reference if this does not matter.
int karp(vector<vector<edge> > g) {
 int n = g.size();
 g.resize(n + 1); // this is important
 for (int i = 0; i < n; ++i)
   if (!g[i].empty())
     g[n].push_back(edge(i,0));
 ++n;
 for(int i = 0; i < n; ++i)
   fill(d[i],d[i]+(n+1),INT_MAX);
 d[n - 1][0] = 0;
 for (int k = 1; k \le n; ++k) for (int u = 0; u \le n; ++u) {
   if (d[u][k - 1] == INT_MAX) continue;
   for (int i = g[u].size() - 1; i >= 0; --i)
     d[g[u][i].v][k] = min(d[g[u][i].v][k], d[u][k-1] + g[u][i].w);
 }
 bool flag = true;
 for (int i = 0; i < n && flag; ++i)</pre>
   if (d[i][n] != INT_MAX)
     flag = false;
 if (flag) {
   return true; // return true if there is no a cycle.
```

```
double ans = 1e15;

for (int u = 0; u + 1 < n; ++u) {
    if (d[u][n] == INT_MAX) continue;
    double W = -1e15;

    for (int k = 0; k < n; ++k)
        if (d[u][k] != INT_MAX)
        W = max(W, (double)(d[u][n] - d[u][k]) / (n - k));

    ans = min(ans, W);
}

// printf("%.2lf\n", ans);
cout << fixed << setprecision(2) << ans << endl;
return false;
}</pre>
```

3.3 konig's theorem

In any bipartite graph, the number of edges in a maximum matching equals the number of vertices in a minimum vertex cover

3.4 minimum path cover in DAG

Given a directed acyclic graph G = (V, E), we are to find the minimum number of vertex-disjoint paths to cover each vertex in V.

We can construct a bipartite graph $G' = (Vout \cup Vin, E')$ from G, where :

```
Vout = \{v \in V : v \text{ has positive out } - degree\}
Vin = \{v \in V : v \text{ has positive } in - degree\}
E' = \{(u, v) \in Vout \times Vin : (u, v) \in E\}
```

Then it can be shown, via König's theorem, that G' has a matching of size m if and only if there exists n-m vertex-disjoint paths that cover each vertex in G, where n is the number of vertices in G and m is the maximum cardinality bipartite mathching in G'.

Therefore, the problem can be solved by finding the maximum cardinality matching in G' instead.

NOTE: If the paths are note necesarily disjoints, find the transitive closure and solve the problem for disjoint paths.

3.5 tarjan scc

```
const int MN = 20002;
struct tarjan_scc {
 int scc[MN], low[MN], d[MN], stacked[MN];
 int ticks, current_scc;
 deque<int> s; // used as stack.
 tarjan_scc() {}
 void init () {
   memset(scc, -1, sizeof scc);
   memset(d, -1, sizeof d);
   memset(stacked, 0, sizeof stacked);
   s.clear();
   ticks = current_scc = 0;
 }
 void compute(vector<vector<int> > &g, int u) {
   d[u] = low[u] = ticks++;
   s.push_back(u);
   stacked[u] = true;
   for (int i = 0; i < g[u].size(); ++i) {</pre>
     int v = g[u][i];
     if (d[v] == -1)
       compute(g, v);
     if (stacked[v]) {
       low[u] = min(low[u], low[v]);
    }
   }
   if (d[u] == low[u]) { // root
     int v;
     do {
       v = s.back();s.pop_back();
       stacked[v] = false;
       scc[v] = current_scc;
     } while (u != v):
     current_scc++;
   }
```

};

4 Matrix

4.1 matrix

```
const int MN = 111;
const int mod = 10000;
struct matrix {
 int r, c;
 int m[MN][MN];
 matrix (int _r, int _c) : r (_r), c (_c) {
   memset(m, 0, sizeof m);
 void print() {
   for (int i = 0; i < r; ++i) {
     for (int j = 0; j < c; ++j)
       cout << m[i][j] << " ";
     cout << endl:</pre>
   }
 }
  int x[MN][MN];
 matrix & operator *= (const matrix &o) {
   memset(x, 0, sizeof x);
   for (int i = 0; i < r; ++i)
     for (int k = 0; k < c; ++k)
       if (m[i][k] != 0)
         for (int j = 0; j < c; ++j) {
           x[i][j] = (x[i][j] + ((m[i][k] * o.m[k][j]) % mod)) % mod;
   memcpy(m, x, sizeof(m));
   return *this;
};
void matrix_pow(matrix b, long long e, matrix &res) {
 memset(res.m, 0, sizeof res.m);
```

```
for (int i = 0; i < b.r; ++i)
  res.m[i][i] = 1;

if (e == 0) return;
while (true) {
  if (e & 1) res *= b;
  if ((e >>= 1) == 0) break;
  b *= b;
}
```

5 Misc

5.1 Template Java

```
import java.io.*;
import java.util.StringTokenizer;
public class Template {
   public static void main(String []args) throws IOException {
       Scanner in = new Scanner(System.in);
       OutputWriter out = new OutputWriter(System.out);
       Task solver = new Task();
       solver.solve(in, out);
       out.close();
   }
}
class Task{
   public void solve(Scanner in, OutputWriter out){
class Scanner{
   public BufferedReader reader;
   public StringTokenizer st;
   public Scanner(InputStream stream){
       reader = new BufferedReader(new InputStreamReader(stream));
```

```
st = null:
   public String next(){
       while(st == null || !st.hasMoreTokens()){
          try{
              String line = reader.readLine();
              if(line == null) return null;
              st = new StringTokenizer(line);
          }catch (Exception e){
              throw (new RuntimeException());
          }
       }
       return st.nextToken();
   public int nextInt(){
       return Integer.parseInt(next());
   public long nextLong(){
       return Long.parseLong(next());
   public double nextDouble(){
       return Double.parseDouble(next());
}
class OutputWriter{
   BufferedWriter writer;
   public OutputWriter(OutputStream stream){
       writer = new BufferedWriter(new OutputStreamWriter(stream));
   public void print(int i) throws IOException {
       writer.write(i);
   public void print(String s) throws IOException {
       writer.write(s);
   public void print(char []c) throws IOException {
       writer.write(c);
```

```
public void close() throws IOException {
    writer.close();
}
```

5.2 io

```
// taken from :
    https://github.com/lbv/pc-code/blob/master/solved/c-e/diablo/diablo.cpp
// this is very fast as well :
    https://github.com/lbv/pc-code/blob/master/code/input.cpp
typedef unsigned int u32;
#define BUF 524288
struct Reader {
 char buf[BUF]; char b; int bi, bz;
 Reader() { bi=bz=0; read(); }
 void read() {
   if (bi==bz) { bi=0; bz = fread(buf, 1, BUF, stdin); }
   b = bz ? buf[bi++] : 0; }
 void skip() { while (b > 0 && b <= 32) read(); }</pre>
 u32 next u32() {
   u32 v = 0; for (skip(); b > 32; read()) v = v*10 + b-48; return v; }
  int next_int() {
   int v = 0; bool s = false;
   skip(); if (b == '-') { s = true; read(); }
   for (; 48<=b&&b<=57; read()) v = v*10 + b-48; return s ? -v : v; }</pre>
 char next_char() { skip(); char c = b; read(); return c; }
};
```

6 Number theory

6.1 convolution

```
typedef long long int LL;
typedef pair<LL, LL> PLL;
inline bool is_pow2(LL x) {
  return (x & (x-1)) == 0;
```

```
inline int ceil_log2(LL x) {
 int ans = 0;
 --x;
 while (x != 0) {
   x >>= 1;
   ans++:
 return ans;
/* Returns the convolution of the two given vectors in time proportional
    to n*log(n).
* The number of roots of unity to use nroots_unity must be set so that
     the product of the first
* nroots_unity primes of the vector nth_roots_unity is greater than the
     maximum value of the
* convolution. Never use sizes of vectors bigger than 2^24, if you need
     to change the values of
* the nth roots of unity to appropriate primes for those sizes.
vector<LL> convolve(const vector<LL> &a, const vector<LL> &b, int
    nroots_unity = 2) {
 int N = 1 \ll ceil_log2(a.size() + b.size());
 vector<LL> ans(N,0), fA(N), fB(N), fC(N);
 LL modulo = 1:
 for (int times = 0; times < nroots_unity; times++) {</pre>
   fill(fA.begin(), fA.end(), 0);
   fill(fB.begin(), fB.end(), 0);
   for (int i = 0; i < a.size(); i++) fA[i] = a[i];</pre>
   for (int i = 0; i < b.size(); i++) fB[i] = b[i];</pre>
   LL prime = nth_roots_unity[times].first;
   LL inv_modulo = mod_inv(modulo % prime, prime);
   LL normalize = mod_inv(N, prime);
   ntfft(fA, 1, nth_roots_unity[times]);
   ntfft(fB, 1, nth_roots_unity[times]);
   for (int i = 0; i < N; i++) fC[i] = (fA[i] * fB[i]) % prime;</pre>
   ntfft(fC, -1, nth_roots_unity[times]);
   for (int i = 0; i < N; i++) {</pre>
     LL curr = (fC[i] * normalize) % prime;
     LL k = (curr - (ans[i] % prime) + prime) % prime;
     k = (k * inv_modulo) % prime;
     ans[i] += modulo * k;
```

```
modulo *= prime;
}
return ans;
}
```

6.2 crt

```
/**
 * Chinese remainder theorem.
 * Find z such that z % x[i] = a[i] for all i.
 * */
long long crt(vector<long long> &a, vector<long long> &x) {
 long long z = 0;
 long long n = 1;
 for (int i = 0; i < x.size(); ++i)
    n *= x[i];

for (int i = 0; i < a.size(); ++i) {
    long long tmp = (a[i] * (n / x[i])) % n;
    tmp = (tmp * mod_inv(n / x[i], x[i])) % n;
    z = (z + tmp) % n;
}

return (z + n) % n;
}</pre>
```

6.3 discrete logarithm

```
// Computes x which a ^ x = b mod n.
long long d_log(long long a, long long b, long long n) {
  long long m = ceil(sqrt(n));
  long long aj = 1;
  map<long long, long long> M;
  for (int i = 0; i < m; ++i) {
    if (!M.count(aj))
        M[aj] = i;
        aj = (aj * a) % n;
}</pre>
```

```
long long coef = mod_pow(a, n - 2, n);
coef = mod_pow(coef, m, n);
// coef = a ^ (-m)
long long gamma = b;
for (int i = 0; i < m; ++i) {
   if (M.count(gamma)) {
      return i * m + M[gamma];
   } else {
      gamma = (gamma * coef) % n;
   }
}
return -1;
}</pre>
```

6.4 ext euclidean

```
void ext_euclid(long long a, long long b, long long &x, long long &y,
    long long &g) {
    x = 0, y = 1, g = b;
    long long m, n, q, r;
    for (long long u = 1, v = 0; a != 0; g = a, a = r) {
        q = g / a, r = g % a;
        m = x - u * q, n = y - v * q;
        x = u, y = v, u = m, v = n;
    }
}
```

6.5 highest exponent factorial

```
int highest_exponent(int p, const int &n){
  int ans = 0;
  int t = p;
  while(t <= n){
    ans += n/t;
    t*=p;
  }
  return ans;
}</pre>
```

6.6 miller rabin

```
const int rounds = 20;
// checks whether a is a witness that n is not prime, 1 < a < n
bool witness(long long a, long long n) {
 // check as in Miller Rabin Primality Test described
 long long u = n - 1;
 int t = 0;
 while (u % 2 == 0) {
   t++;
   u >>= 1:
 long long next = mod_pow(a, u, n);
 if (next == 1) return false;
 long long last;
 for (int i = 0; i < t; ++i) {</pre>
   last = next:
   next = mod_mul(last, last, n);
   if (next == 1) {
     return last != n - 1;
   }
 }
 return next != 1;
}
// Checks if a number is prime with prob 1 - 1 / (2 ^ it)
// D(miller_rabin(999999999999997LL) == 1);
// D(miller_rabin(999999999971LL) == 1);
// D(miller_rabin(7907) == 1);
bool miller_rabin(long long n, int it = rounds) {
 if (n <= 1) return false;</pre>
 if (n == 2) return true;
 if (n % 2 == 0) return false;
 for (int i = 0; i < it; ++i) {</pre>
   long long a = rand() \% (n - 1) + 1;
   if (witness(a, n)) {
     return false;
   }
 }
 return true;
```

6.7 mod inv

```
long long mod_inv(long long n, long long m) {
  long long x, y, gcd;
  ext_euclid(n, m, x, y, gcd);
  if (gcd != 1)
   return 0;
  return (x + m) % m;
}
```

6.8 mod mul

```
// Computes (a * b) % mod
long long mod_mul(long long a, long long b, long long mod) {
  long long x = 0, y = a % mod;
  while (b > 0) {
    if (b & 1)
        x = (x + y) % mod;
        y = (y * 2) % mod;
        b /= 2;
  }
  return x % mod;
}
```

6.9 mod pow

```
// Computes ( a ^ exp ) % mod.
long long mod_pow(long long a, long long exp, long long mod) {
  long long ans = 1;
  while (exp > 0) {
    if (exp & 1)
      ans = mod_mul(ans, a, mod);
    a = mod_mul(a, a, mod);
    exp >>= 1;
  }
  return ans;
}
```

6.10 number theoretic transform

```
typedef long long int LL;
typedef pair<LL, LL> PLL;
/* The following vector of pairs contains pairs (prime, generator)
* where the prime has an Nth root of unity for N being a power of two.
 * The generator is a number g s.t g^(p-1)=1 (mod p)
 * but is different from 1 for all smaller powers */
vector<PLL> nth roots unity {
 {1224736769,330732430},{1711276033,927759239},{167772161,167489322},
  {469762049,343261969},{754974721,643797295},{1107296257,883865065}};
PLL ext_euclid(LL a, LL b) {
 if (b == 0)
   return make_pair(1,0);
 pair<LL,LL> rc = ext_euclid(b, a % b);
 return make_pair(rc.second, rc.first - (a / b) * rc.second);
}
//returns -1 if there is no unique modular inverse
LL mod_inv(LL x, LL modulo) {
 PLL p = ext_euclid(x, modulo);
 if ( (p.first * x + p.second * modulo) != 1 )
   return -1:
 return (p.first+modulo) % modulo;
}
//Number theory fft. The size of a must be a power of 2
void ntfft(vector<LL> &a, int dir, const PLL &root_unity) {
 int n = a.size():
 LL prime = root_unity.first;
 LL basew = mod_pow(root_unity.second, (prime-1) / n, prime);
 if (dir < 0) basew = mod_inv(basew, prime);</pre>
 for (int m = n: m \ge 2: m \ge 1) {
   int mh = m >> 1;
   LL w = 1:
   for (int i = 0; i < mh; i++) {</pre>
     for (int j = i; j < n; j += m) {
       int k = j + mh;
       LL x = (a[j] - a[k] + prime) % prime;
       a[j] = (a[j] + a[k]) \% prime;
       a[k] = (w * x) \% prime;
```

```
w = (w * basew) % prime;
}
basew = (basew * basew) % prime;
}
int i = 0;
for (int j = 1; j < n - 1; j++) {
  for (int k = n >> 1; k > (i ^= k); k >>= 1);
  if (j < i) swap(a[i], a[j]);
}
</pre>
```

6.11 pollard rho factorize

```
long long pollard_rho(long long n) {
 long long x, y, i = 1, k = 2, d;
 x = y = rand() \% n;
 while (1) {
   ++i;
   x = mod_mul(x, x, n);
   x += 2;
   if (x \ge n) x = n;
   if (x == y) return 1;
   d = \_gcd(abs(x - y), n);
   if (d != 1) return d:
   if (i == k) {
     y = x;
     k *= 2;
 }
 return 1;
// Returns a list with the prime divisors of n
vector<long long> factorize(long long n) {
 vector<long long> ans;
 if (n == 1)
   return ans;
 if (miller_rabin(n)) {
   ans.push_back(n);
 } else {
   long long d = 1;
   while (d == 1)
```

```
d = pollard_rho(n);
vector<long long> dd = factorize(d);
ans = factorize(n / d);
for (int i = 0; i < dd.size(); ++i)
    ans.push_back(dd[i]);
}
return ans;
}</pre>
```

7 Strings

7.1 minimal string rotation

```
// Lexicographically minimal string rotation
int lmsr() {
 string s;
 cin >> s;
 int n = s.size();
 s += s;
 vector<int> f(s.size(), -1);
 int k = 0;
 for (int j = 1; j < 2 * n; ++j) {
   int i = f[j - k - 1];
   while (i != -1 && s[j] != s[k + i + 1]) {
    if (s[j] < s[k + i + 1])
      k = j - i - 1;
    i = f[i];
   if (i == -1 \&\& s[j] != s[k + i + 1]) {
    if (s[j] < s[k + i + 1]) {
       k = j;
    f[j-k] = -1;
   } else {
     f[j - k] = i + 1;
   }
 }
 return k;
```

7.2 suffix array

```
/**
 * 0 (n log^2 (n))
 * See http://web.stanford.edu/class/cs97si/suffix-array.pdf for reference
using namespace std;
#include<bits/stdc++.h>
#define D(x) cout<<#x " = "<<(x)<<endl
struct entry{
  int a, b, p;
  entry(){}
  entry(int x, int y, int z): a(x), b(y), p(z){}
  bool operator < (const entry &o) const {</pre>
   return (a == o.a)? (b < o.b): (a < o.a);
 }
};
struct SuffixArray{
  const int N;
  string s;
  vector<vector<int> > P;
  vector<entry> M;
  SuffixArray(const string &s) : N(s.length()) , s(s), P(1, vector<int>
      (N, O)), M(N) {
    for (int i = 0; i < N; ++i)</pre>
     P[0][i] = s[i];
    for (int skip = 1, level = 1; skip < N; skip *= 2, level++) {</pre>
     P.push_back(vector<int>(N, 0));
     for (int i = 0 ; i < N; ++i) {</pre>
       int next = ((i + skip) < N) ? P[level - 1][i + skip] : -10000;
       M[i] = entry(P[level - 1][i], next, i);
     sort(M.begin(), M.end());
     for (int i = 0; i < N; ++i)</pre>
       P[level][M[i].p] = (i > 0 \text{ and } M[i].a == M[i - 1].a \text{ and } M[i].b ==
            M[i - 1].b) ? P[level - 1][M[i - 1].p] : i;
   }
 }
  vector<int> getSuffixArray(){
    return P.back();
```

```
// returns the length of the longest common prefix of s[i...L-1] and
        s[j...L-1]
int longestCommonPrefix(int i, int j) {
    int len = 0;
    if (i == j) return N - i;
    for (int k = P.size() - 1; k >= 0 && i < N && j < N; --k) {
        if (P[k][i] == P[k][j]) {
            i += 1 << k;
            j += 1 << k;
            len += 1 << k;
        }
    }
    return len;
}
</pre>
```

7.3 z algorithm

```
using namespace std;
#include<bits/stdc++.h>
vector<int> compute_z(const string &s){
 int n = s.size();
 vector<int> z(n,0);
 int 1,r;
 r = 1 = 0;
 for(int i = 1; i < n; ++i){</pre>
   if(i > r) {
     1 = r = i;
     while(r < n and s[r - 1] == s[r])r++;
     z[i] = r - 1;r--;
   }else{
     int k = i-1;
     if(z[k] < r - i +1) z[i] = z[k];
     else {
      1 = i;
       while (r < n \text{ and } s[r - 1] == s[r])r++;
       z[i] = r - 1;r--;
     }
   }
```

```
return z;
}
int main(){

//string line;cin>>line;
string line = "alfalfa";
vector<int> z = compute_z(line);

for(int i = 0; i < z.size(); ++i ){
   if(i)cout<<" ";
   cout<<z[i];
}
cout<<endl;

// must print "0 0 0 4 0 0 1"

return 0;</pre>
```