# Problem A. Cards

| | |
|---|---|
| Input file: | *standard input* |
| Output file: | *standard output* |
| Time limit: | 1 second |
| Memory limit: | 256 mebibytes |

You are given a string $s$. Find out number of different strings you can get by permuting letter of $s$.

## Input

The only line of input contains one string of lowercase English letters $s$ ($1 \leqslant |s| \leqslant 15$).

## Output

Output one integer – answer to the problem.

## Examples

| standard input | standard output |
|---|---|
| solo | 12 |

# Problem B. Signed Derangements

| | |
|---|---|
| Input file: | *standard input* |
| Output file: | *standard output* |
| Time limit: | 1 second |
| Memory limit: | 256 mebibytes |

Signed permutation of size $n$ is an ordered set of $n$ numbers ranging from $-n$ to $n$ except 0, where absolute values of any two numbers are different. An example of a signed permutation is $\langle 4, -2, 3, -5, -1 \rangle$. Clearly, there are $2^n n!$ signed permutations of size $n$.

A signed permutation $\langle a_1, a_2, \ldots, a_n \rangle$ is called a signed derangmenent if $a_i \neq i$ for all $i$. For example, $\langle 4, -2, 3, -5, -1 \rangle$ is not a signed derangement, but $\langle 4, -2, -3, -5, -1 \rangle$ is.

Given $n$, find the number of signed derangements of size $n$.

## Input

Input file contains one integer number $n$ ($1 \leq n \leq 200$).

## Output

Output one integer number — the number of signed derangements of size $n$ modulo 998 244 353.

## Examples

| standard input | standard output |
|---|---|
| 2 | 5 |

# Problem C. Expectation

| | |
|---|---|
| Input file: | *standard input* |
| Output file: | *standard output* |
| Time limit: | 1 second |
| Memory limit: | 1024 mebibytes |

You have a box with balls of $n$ colors. For each color $i$ you know the number $a_i$ of balls having that color. You randomly pick a ball from the box using a uniform distribution, and then you put it back in the box.

What's the expected number of picks in order to choose a ball of each color at least once?

## Input

The first line contains a single integer $n$ ($1 \leqslant n \leqslant 20$).

The second line contains $n$ integers representing the elements of the array $1 \leqslant a_i \leqslant 100$.

## Output

Print a single number representing the expected number of picks. An answer is considered correct if the absolute difference between it and the official answer is less than $10^{-6}$.

## Examples

| standard input | standard output |
|---|---|
| 3<br>1 2 3 | 7.3000000000 |
| 5<br>1 1 1 1 1 | 11.4166666667 |

# Problem D. Expectation of Uniforms

| | |
|---|---|
| Input file: | *standard input* |
| Output file: | *standard output* |
| Time limit: | 1 second |
| Memory limit: | 256 mebibytes |

Consider $n$ independent continuous random variables $\xi_1, \ldots, \xi_n$ sampled from a uniform distribution over $[0, 1]$. Define $\xi_{(k)}$ as the $k$-th order statistic of sample $\xi$, so it holds that $\xi_{(1)} \leqslant \xi_{(2)} \leqslant \cdots \leqslant \xi_{(n)}$. Find out the expected value

$$\mathrm{E}\left[\prod \xi_{(i)}^{a_i}\right].$$

Print the answer modulo $10^9 + 7$. It can be proven that the answer is always a rational number. If the answer is the irreducible fraction $\frac{p}{q}$ and the denominator $q$ is not divisible by $10^9 + 7$, there is a unique $0 \leqslant x < 10^9 + 7$ such that $(p - xq)$ is divisible by $10^9 + 7$. You need to find this $x$.

## Input

The first line contains one integer $n$ ($n \leqslant 10^5$).

The second line contains $n$ integers $a_1, \ldots, a_n$ ($0 \leqslant a_i \leqslant 10^9$).

## Output

One integer — the answer to the problem.

It is guaranteed that the denominator of the answer in irreducible form is not divisible by $10^9 + 7$.

## Example

| standard input | standard output |
|---|---|
| 3<br>0 1 0 | 500000004 |
| 5<br>1 0 0 0 0 | 166666668 |
| 2<br>1 1 | 250000002 |

# Problem E. Expected LCP

| | |
|---|---|
| Input file: | *standard input* |
| Output file: | *standard output* |
| Time limit: | 2 seconds |
| Memory limit: | 256 mebibytes |

Consider a sequence $s_1$, $s_2$, ..., $s_n$ of $n$ infinite binary strings (that is, consisting only of zeros and ones), where each character of each string is generated uniformly at random independently from others. Denote

$$f(s_1, s_2, \ldots, s_n) = \max_{1 \le i < j \le n} LCP(s_i, s_j),$$

where $LCP$ is the maximum common prefix of two strings. Compute the expected value of $f(s_1, s_2, \ldots, s_n)$.

## Input

The only line of the input contains one integer $n$ ($2 \le n \le 10^4$).

## Output

Let the answer in the form of an irreducible fraction be $P/Q$. Then output $P \cdot Q^{-1} \bmod (10^9 + 7)$. It is guaranteed that $Q \bmod (10^9 + 7) \neq 0$.

## Examples

| standard input | standard output |
|---|---|
| 2 | 1 |
| 3 | 333333338 |

## Note

Note that the expected value is always finite, that is, $\mathtt{E} f(s_1, \ldots, s_n) < \infty$.

In the second sample the answer is $\frac{7}{3}$.

# Problem F. GCD Sum

| | |
|---|---|
| Input file: | *standard input* |
| Output file: | *standard output* |
| Time limit: | 2 seconds |
| Memory limit: | 256 mebibytes |

You are given a sequence of positive integers $a_1, \ldots, a_n$. Find the sum of greatest common divisors of all non-empty subsequences of the given sequence. Since the answer may be large, print it modulo $10^9 + 7$.

## Input

The first line contains an integer $n$ ($1 \leqslant n \leqslant 10^6$). The second line contains $n$ space-separated positive integers $a_1, a_2, \ldots, a_n$ ($1 \leqslant a_i \leqslant 10^6$).

## Output

One integer — the answer to the problem modulo $10^9 + 7$.

## Examples

| standard input | standard output |
|---|---|
| 3<br>2 3 4 | 14 |
| 10<br>2017 2007 2017 2007 2017 2007 2017<br>2007 2017 2007 | 125705 |

## Note

In the sample case, all subsequences are $(2), (3), (4), (2, 3), (2, 4), (3, 4), (2, 3, 4)$. GCD sum is $2 + 3 + 4 + 1 + 2 + 1 + 1 = 14$.

# Problem G. Count Graphs

| | |
|---|---|
| Input file: | *standard input* |
| Output file: | *standard output* |
| Time limit: | 1 second |
| Memory limit: | 256 mebibytes |

Count number of connected labeled graphs. More detailed statement is present below.

Consider all simple graphs without loops on $n$ vertices numbered $1, 2, \ldots, n$. Two graphs considered different is there's edge present in on of them and absent in other one. So, there are $2^{n(n-1)/2}$ labeled graph on $n$ vertices in total. Find out number of connected among them.

## Input

Input contains multiple testcases, one per line. Each testcase is one integer $n$ ($1 \leqslant n \leqslant 300$). Each test contains at most 300 testcases.

## Output

Output one integer — answer to the problem modulo 998 244 353.

## Examples

| standard input | standard output |
|---|---|
| 1 | 1 |
| 2 | 1 |
| 3 | 4 |
| 10 | 158488195 |

# Problem H. Matchings

| | |
|---|---|
| Input file: | *standard input* |
| Output file: | *standard output* |
| Time limit: | 1 second |
| Memory limit: | 256 mebibytes |

Let us consider a bipartite graph with $n + m$ vertexes. The left set contains $n$ vertexes. Every vertex has a integer written in it $a_1, a_2, \ldots, a_n$. The right set contains $m$ vertexes. Every vertex has a integer written in it $b_1, b_2, \ldots, b_m$. $i$-th vertex of left set and $j$-th vertex of right set are connected by edge only if $a_i \geq b_j$.

Count the number of different matchings containing $m$ edges in this graph.

## Input

First line contains two integers $n$ and $m$, $1 \leq n, m \leq 10^5$. Second line contains integers $a_1, a_2, \ldots, a_n$, $1 \leq a_i \leq 10^6$ separated by a space. Third line contains integers $b_1, b_2, \ldots, b_m$, $1 \leq b_i \leq 10^6$ separated by a space.

## Output

Output one integer - answer to the problem modulo $10^9 + 7$.

## Example

| standard input | standard output |
|---|---|
| 3 2<br>7 3 4<br>1 5 | 2 |

# Problem I. Product

| | |
|---|---|
| Input file: | *standard input* |
| Output file: | *standard output* |
| Time limit: | 1 second |
| Memory limit: | 512 mebibytes |

Let $p_1, p_2, p_3 \ldots$ be all prime numbers in ascending order. That is, $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, $p_4 = 7$, and so on. You are given $n$ numbers $a_1, \ldots, a_n$. Each $a_i$ is the exponent of the prime $p_i$ in the factorization of a large number $X$. Formally, $X = p_1^{a_1} * p_2^{a_2} * ... * p_n^{a_n}$.

Compute the number of sequences of positive integers such that each integer is greater than 1 and the product of all elements of the sequence is exactly $X$. Return that number modulo $1,000,000,007$.

## Input

The first line of input contains one integer $n$ ($1 \leqslant n \leqslant 50$). The second line contains $n$ integers $a_i$ ($1 \leqslant a_i \leqslant 50$).

## Output

Output one integer — answer to the problem.

## Examples

| standard input | standard output |
|---|---|
| 2<br>1 1 | 3 |
| 1<br>2 | 2 |
| 5<br>1 1 1 1 1 | 541 |
| 4<br>20 18 3 19 | 210662068 |
| 10<br>1 2 4 5 3 4 9 8 7 6 | 991940899 |

## Note

In first sample, $X = 2^1 * 3^1 = 6$. There are three valid sequences: (2,3), (3,2), and (6).

In the second sample we have $X = 2^2 = 4$. The two valid sequences are (2,2) and (4).

# Problem J. Sequential Game

| | |
|---|---|
| Input file: | *standard input* |
| Output file: | *standard output* |
| Time limit: | 1 second |
| Memory limit: | 1024 mebibytes |

Danil is playing a game at his phone. The game consists of $n$ sequential levels. Probability to win at $i$-th level is $\frac{p_i}{q_i}$. Each attempt to any level is independent from all previous attempts. If you win at $i$-th level, you proceed to the next. If you lose, you start the game from the beginnig. Danil starts at the first level. When he reaches the last level, the game ends. Each attempt to pass a level takes exactly one second (despite the outcome). All other actions take infinitesimal amount of time. Please, find the expected amount of time till the winning.

## Input

First line of input contains the only integer $1 \leqslant n \leqslant 10^5$. Next $n$ lines contains two space-separated integers $1 \leqslant p_i \leqslant q_i < 10^9 + 7$.

## Output

The result is always a rational number. However, it can be very large. Output the answer modulo $(10^9+7)$. If the answer is $\frac{a}{b}$ and $b$ is not divisible by $(10^9 + 7)$, there is a unique integer $0 \leqslant x < 10^9 + 7$ where $a \equiv bx \bmod (10^9 + 7)$. Output this integer, $x$. It is guaranteed that $b$ is not divisible by $(10^9 + 7)$ for all test cases.

## Examples

| standard input | standard output |
|---|---|
| 1<br>1 10 | 10 |
| 2<br>1 2<br>1 1 | 3 |