# Basketball Shot Trajectory Analysis With OpenCV and ARkit on IOS

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## 1 Goals

To compute the trajectory of a basketball using an iphone, Haar Cascades for object detection and anchor points from ARkit. Our goal is to make use of ARkits anchor points to measure the distance between the player and the basketball hoop in the xy-plane created between them.

Our goal is to obtain the following parameters from the basketballs shot trajectory:

1. Arc angle of release
2. Release velocity

Required measurements:

1. Distance from the base of the basketball hoop to the feet of the player
2. Vertical distance that the ball is released from the players hands
3. Time between when the basketball leaves the players hands and when the basketball enters within a certain distance of the basketball hoop (flight time)

Required Event Detections:

1. Detecting when the basketball leaves the players hands
2. Detecting when the basketball enters within a certain distance of the basketball hoop

In-order to measure flight time.
challenges: A goal box around the hoop
Calculations:
We first define a Cartesian coordinate system for calculations. We will use a 2 dimensional plane on which the pole of the basketball hoop lies. The origin

lies between the players feet and the x-axis connects the player to the base of the basketball hoop.

(In feet)

$$(0,0) := \text{location beneath players feet}$$

$$(x_{hoop}, 0) := \text{base of the basketball hoop}$$

$$(x_i, y_i) := \text{release location of basketball}$$

$$(x_f, y_f) := \text{final location of basketball}$$

$$t_f := \text{time in which the ball is detected in the goal zone}$$

$$t = 0, \text{ At moment of shots release}$$

$$v_x := \text{Horizontal velocity ignoring air resistance}$$

Horizontal calculations are straight forward.

$$v_x = \frac{x_i}{t_f}$$

$$x(t) = v_x t$$

Vertical calculations, assuming acceleration is constant and just composed of the force of gravity, $g$, and negative.

$$y''(t) = g$$

$$y'(t) = gt + c_1$$

$$y(t) = \frac{gt^2}{2} + c_1 t + c_2$$

Using initial conditions, $y(0) = y_i$ and $y(t_f) = y_f$

$$c_2 = y_i$$

$$c_1 = \frac{y_f - y_i}{t_f} - \frac{gt_f}{2}$$

$$y(t) = \frac{gt^2}{2} + \left(\frac{y_f - y_i}{t_f} - \frac{gt_f}{2}\right) t + y_i$$

Therefore we know the position at any time $t$ if $x_i, x_f, t_f$ are all known (initial height, final height, total time)

$$v_y(t) = y'(t) = gt + \frac{y_f - y_i}{t_f} - \frac{gt_f}{2}$$

where,

$$v_y(0) = \frac{y_f - y_i}{t_f} - \frac{gt_f}{2}$$

is the initial velocity in the $y$ axis.

We can also find the balls maximum height which occurs when the velocity is zero,

$$0 = gt + \frac{y_f - y_i}{t_f} - \frac{gt_f}{2}$$

$$gt = \frac{y_f - y_i}{t_f} - \frac{gt_f}{2}$$

$$t_{maxy} = \frac{y_f - y_i}{t_f g} - \frac{t_f}{2}$$

$$t_{maxy} = \frac{y_f - y_i}{t_f g} - \frac{t_f}{2}$$

Then the maximum height is $y(t_{maxy})$.

The magnitude of the initial release velocity vector is

$$\|\mathbf{v}\| = \sqrt{v_x^2 + v_y(0)^2} = \sqrt{v_x^2 + \left(\frac{y_f - y_i}{t_f} - \frac{gt_f}{2}\right)^2}$$

The Angle of release, $\theta_i$ is

$$\theta_i = \tan \text{inverse} \left(\frac{v_y(0)}{v_x}\right)$$