NE 401/501
Spring 2016

Computational Project

Due date: TBA

1. Create a computer code for solving k-eigenvalue problems for the multigroup neutron diffusion equations in 1D slab geometry that will enable you to determine a multiplication factor (largest eigenvalue) and fundamental mode (associated eigenfunction).

2. Use the finite-volume discretization method for discretizing the diffusion equation and power iteration method presented in class (see lecture notes on numerical methods for solving k-eigenvalue problems for the multigroup neutron diffusion equations at https://wolfware.ncsu.edu/).

3. The cross section for all materials needed in the project are given in Ref. [1]. This publication is available on the course website (https://wolfware.ncsu.edu/). In your calcualtions utilize data in columns with "transport" cross sections as total cross sections ($\Sigma_{t,g}$).

4. Use Fortran or MATLAB to write your code.

5. The stopping criteria are

$$||\bar{\boldsymbol{\phi}}^{(s)} - \bar{\boldsymbol{\phi}}^{(s-1)}||_\infty \leq \varepsilon_\phi ||\bar{\boldsymbol{\phi}}^{(s)}||_\infty \,, \quad \left|k^{(s)} - k^{(s-1)}\right| \leq \varepsilon_k k^{(s)} \,, \tag{1}$$

   where

$$\bar{\boldsymbol{\phi}} = \left(\bar{\phi}_1^1, \ldots, \bar{\phi}_i^1, \ldots, \bar{\phi}_N^1, \bar{\phi}_1^2, \ldots \bar{\phi}_i^g, \ldots, \bar{\phi}_N^G\right)^T \tag{2}$$

   is the vector of the cell-average group scalar flux. The vector norm is defined as

$$||\mathbf{y}||_\infty = \max_j |y_j| \,, \tag{3}$$

   The parameters of stopping criteria are $\varepsilon_k = 10^{-5}$, $\varepsilon_\phi = 10^{-5}$.

6. Normalize the obtained fundamental mode (ϕ^g) such that

$$\frac{1}{\mathcal{H}} \sum_{g=1}^{G} \int_0^{\mathcal{H}} \phi^g(x) dx = 1 \quad \text{and} \quad \text{hence} \quad \frac{1}{\mathcal{H}} \sum_{g=1}^{G} \sum_{i=1}^{N} \phi_i^g \Delta h_i = 1 \,, \tag{4}$$

   where $\mathcal{H}$ is the width of the spatial domain of a given problem.

7. Solve the numerical problems given below to calculate k_{eff} and fundamental mode.

8. You must show at 7 significant digits all your numerical results in the report ($k_{eff}^{(s)}$, $\bar{\phi}^g$, J^g). Thus, you need to use the following scientific format for all real numbers in tables etc:

$$\pm n_1.n_2n_3n_4n_5n_6n_7 \; e \pm m_1m_2 \,, \quad \text{where} \;\; n_1 \neq 0 \,.$$

   Use natural integer format for iteration numbers (s, M_s), and indices (i).

9. Don't use any markers in your graphs.

10. Prepare your report in a word processing software.

11. Use the lab report template to prepare the project report.

12. Submit a hard copy of your report.

13. Provide your source code as an appendix of the report.

- *Test A*

  Consider a fuel pellet made of MOX 8.7% with $\mathcal{H} = 1.08$ cm (see Figure 1). Boundary conditions are reflective. Find the numerical solution using a uniform spatial mesh with 10 intervals ($N = 10$). Solve this problem analytically and compare your numerical results with the analytic solution. They should match well. Show the following set of results:

  - number of power iterations,
  - numerical and analytical multiplication factors,
  - table presenting
    - the numerical solution ϕ_g at x=0 and x=0.54 versus g,
    - analytical ϕ_g versus g,
  - plot the numerical solution ϕ_g at x=0.54 as a function of g. *This will demonstrate the neutron spectrum.*

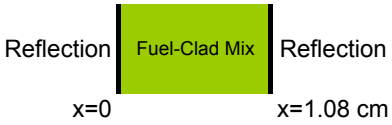

Figure 1: Fuel pellet.

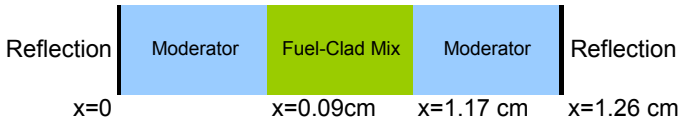

Figure 2: Pin-cell problem.

- *Test B*

  Consider a MOX 8.7% fuel pin cell with $\mathcal{H} = 1.26$ cm (see Figure 2). Boundary conditions are reflective. Use a uniform spatial mesh with 28 intervals ($N = 28$, Δh=0.045 cm). Present the following results:

  - number of power iterations,
  - multiplication factor,
  - table with ϕ^g at x=$\{0.045, 0.09, 0.63\}$ versus g,
  - single graph with plots of $\phi^g(x)$ versus g at x=$\{0.045, 0.09, 0.63\}$,
    (*Note that these positions correspond to (i) the center of the moderator region, (ii) moderator-fuel interface, and (iii) the center of the fuel region. Your data will show the spectrum at these locations. You can see variation in the spectrum versus position.*)
  - table of J^g at x=$\{0.045, 0.09, 0.63\}$, versus g,
  - single graph with plots of $J^g(x)$ versus g at x=$\{0.045, 0.09, 0.63\}$,
    (*Your data will show the net leakage rates in different groups and how they vary with locations.*)
  - graphs of
    - the cell-average values of the group scalar flux ($\bar{\phi}^g$) versus position x in every groups, i.e. $\frac{1}{2}(x_{i+1} + x_i) \rightarrow \bar{\phi}_i^g$,
    - the cell-average values of the total scalar flux ($\bar{\phi} = \sum_g \bar{\phi}^g$), i.e. $\frac{1}{2}(x_{i+1} + x_i) \rightarrow \bar{\phi}_i$,
    - the cell-edge values of the group current (J^g) versus position x in every group, i.e. $x_i \rightarrow J_i^g$.

- *Test C*

  This test has three regions shown in Figure 3 ($\mathcal{H} = 64.26$ cm). The left region is a UO_2 assembly. The middle region is a MOX assembly. The right region is a reflector with moderator. Boundary conditions are reflective. Both assemblies are made of 17 pin cells. All pin cells have the same dimensions presented in Figure 4. The designs of the assemblies are shown in Figures 5 and 6. Use uniform mesh with Δh=0.09 cm (N=714).

  Present the following set of results:

  - number of power iterations,
  - multiplication factor,
  - tables of ϕ^g at x={9.45, 21.42, 30.87, 42.84, 53.55} versus g,
  - single graph with plots of $\phi^g(x)$ at x={9.45, 21.42, 30.87, 42.84, 53.55} versus g,
    (*Note that these positions correspond to (i) the center of fuel pin No. 8 in the UO_2 assembly, (ii) interface between MOX and UO_2 assemblies, (iii) the center of the fuel pin No. 8 in MOX assembly, (iv) interface between MOX assembly and reflector region, (v) the center of the reflector region.*)
  - table of J^g at x={9.45, 21.42, 30.87, 42.84, 52.29} versus g,
  - single graph with plots of $J^g(x)$ at x={9.45, 21.42, 30.87, 42.84, 52.29} versus g,
  - graphs of
    * the cell-average values of the group scalar flux ($\bar{\phi}^g$) versus position,
    * the cell-average values of the total scalar flux ($\bar{\phi} = \sum_g \bar{\phi}^g$) versus position,
    * the cell-edge values of the group current (J^g).

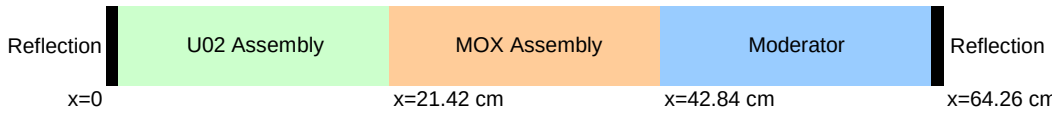

Figure 3: Test C.

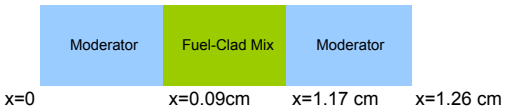

Figure 4: Pin-cell design.

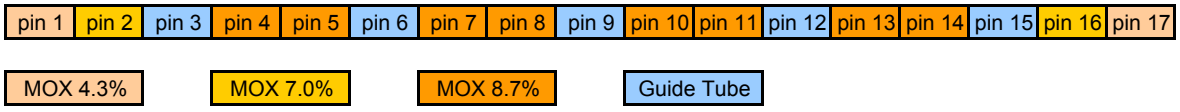

Figure 5: Design of MOX assembly.

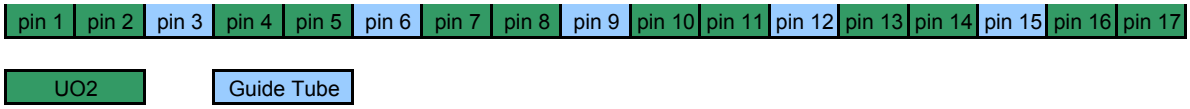

Figure 6: Design of UO_2 assembly.

References

[1] E. E. Lewis, M. A. Smith, N. Tsoulfanidis, G. Palmiotti, T. A. Taiwo & R. N. Blomquist. "Benchmark specification for Deterministic 2-D/3-D MOX fuel assembly transport calculations without spatial homogenization (C5G7)," Expert Group on 3-D Radiation Transport Benchmarks, NEA/NSC/DOC(2001)4