Math Questions

Section A

1. If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \frac{1}{2}, |\vec{b}| = \frac{4}{\sqrt{3}}$ and $|\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}}$, then find $|\vec{a} \cdot \vec{b}|$.

2. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\vec{a} - \sqrt{2}\vec{b}$ to be a unit vector?

3. Find the distance between the planes $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) - 4 = 0$ and $\vec{r} \cdot (6\hat{i} - 9\hat{j} + 18\hat{k}) + 30 = 0$.

4. If A is a square matrix such that $|A| = 5$, write the value of $|AA^T|$.

5. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$, find $|AB|$.

6. If $A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$ and $KA = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$, find the values of k and a.

Section B

7. Differentiate $(\sin 2x)^x + \sin^{-1}(\sqrt{3}x)$ with respect to x.

   OR

   Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1}(x^2)$.

8. Find k, if
$$f(x) = \begin{cases} k \sin \frac{\pi}{2}(x+1), & x \leq 0, \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$$
   is continuous at $x = 0$.

9. Find equation of normal to the curve $ay^2 = x^3$ at the point whose x coordinate is am^2.

10. Find $\int \frac{1-\sin x}{\sin x(1+\sin x)}\, dx$

11. Find $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$

12. Evaluate $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\, dx$

    OR

    $\int_0^1 \cot^{-1}(1 - x + x^2)\, dx$

13. Solve the differential equation $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^3$