MA2001D Mathematics III: Lecture 3

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Random Variable

A **random variable** X is function that associates a real number with each element in sample space, that is, $X : \Omega \to \mathbb{R}$.

Types of Random variable

- A **discrete random variable** is a random variable with a finite (or countably infinite) range.
- A **continuous random variable** is a random variable with an interval (either finite or infinite) of real numbers for its range.
  Examples of continuous random variables are age, height, weight etc.

Note

Random variables (r.v.) bare denoted by the capital letters X, Y, Z, etc., to distinguish them from their possible values given in lowercase x, y.

Examples

A random variable representing the number of automobiles sold at a particular dealership on one day would be discrete, while a random variable representing the weight of a person in kilograms (or pounds) would be continuous.

Example

Toss two fair coins, the sample space is

$$\Omega = \{HH, HT, TH, TT\}.$$

If we let X denotes the number of heads that appear, then X is a random variable taking on one of the values $0, 1$, and 2 with respective probabilities

$$P(X=0) = P(T,T) = \frac{1}{4}$$

$$P(X=1) = P((T,H),(H,T)) = \frac{2}{4}$$

$$P(X=2) = P(H,H) = \frac{1}{4}.$$

Probability distribution

The probability distribution is a mathematical function that gives the probabilities of occurrence of different outcomes for an experiment.

Discrete Random Variable

Probability Mass Function

If X is a discrete random variable and let R_X be the range space of r.v. X. A real valued function $f_X : R_X \to \mathbb{R}$ is said to be probability mass function (p.m.f.) if

$$f_X(x) = P(X = x) = P(\{s \in \Omega : X(s) = x\}), \quad x \in R_X.$$

Discrete Random Variable

Properties:

1. $f_X(x) \geq 0$.
2. $\sum_x f_X(x) = 1$.

Discrete Random Variable

For discrete random- variable, a knowledge of the probability mass function enables us to compute probabilities of arbitrary events. In fact, if A is an event then

$$P(x \in A) = \sum_{x \in A} f_X(x).$$

EXERCISE

An experiment consists of three independent tosses of a fair coin. Let

$$\begin{aligned} X &= \text{The number of heads} \\ Y &= \text{The number of head runs,} \\ Z &= \text{ The length of head runs,} \end{aligned}$$

a head run being defined as consecutive occurrence of at least two heads, its length then being the number of heads occurring together in three tosses of the coin.

Find the probability function of $(i)X,\ (ii)Y, (iii)Z, (iv)X+Y$ and $(v)XY$.

Cumulative Distribution Function (or distribution function)

Let X be a random variable. Then its cumulative distribution function (cdf) is defined by $F_X(x)$, where

$$F_X(x) = P(X \leq x) = P(\{s \in \Omega : X(s) \leq x\}), \quad x \in \mathbb{R}.$$

If X is discrete r.v. and $f(x)$ is its p.m.f., then

$$F_X(x) = \sum_{t \leq x} f(t).$$

PROPERTIES:

If F is a distribution function of the random variable X and if $a < b$. Then

- $P(a < X \leq b) = F(b) - F(a)$.

$$P(a < X \leq b) = P(X \leq b) - P(X \leq a) = F(b) - F(a).$$

- $P(a \leq X \leq b) = F(b) - F(a) + P(X = a)$.

- $P(a < X < b) = F(b) - F(a) - P(X = b)$.

$0 \leq F_X(x) \leq 1$

If $x \leq y$ then $F_X(x) \leq F_X(y)$.

$\lim_{x\to\infty} F_X(x) = 1$ and $\lim_{x\to-\infty} F_X(x) = 0$.

– End –