Continuous random variables

Definition 1.17

A random variable $X : S \to \mathbb{R}$ is said to continuous if
$P(X = x) = 0, \forall x \in \mathbb{R}.$

$$P(\{s \in S : X(s) = x\}) = 0.$$

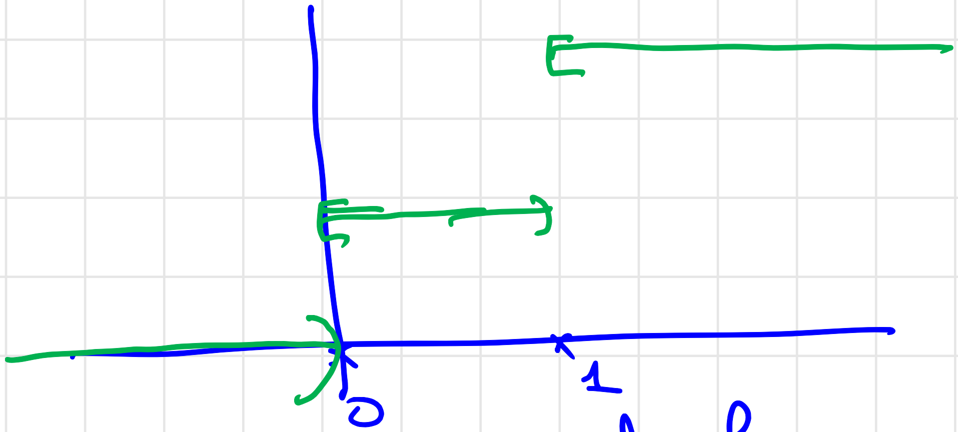
??

$$P(S) = 1$$

$$f(x) = P(X = x).$$

$$F(x) = P(X \le x)$$

p.d.f



c.d.f

Remark

If $F(x) = P(X \leq x)$ is the distribution function of a continuous random variable X, then F is a continuous function and

$$F(b) - F(a) = P(a < X \leq b) = P(a < X < b) = P(a \leq X < b).$$

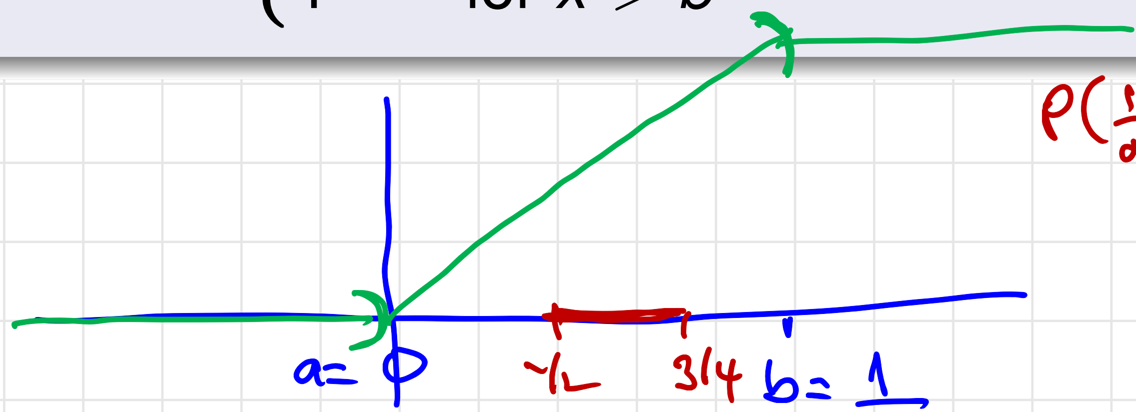
Example 1.10 (Continuous uniform distribution)

$$F(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } a \leq x \leq b \\ 1 & \text{for } x > b \end{cases}$$

$P(\frac{1}{2} \leq X \leq \frac{3}{4})$

$= F(3/4) - F(1/2)$

$= 3/4 - \frac{1}{2} = \frac{1}{4}$



$P(\frac{1}{2} \leq X \leq \frac{3}{2}) =$

$1 - \frac{1}{2} = \frac{1}{2}.$

Probability density function

Definition 1.18

A function $f : \mathbb{R} \to \mathbb{R}$ is said to be the probability density function of a random variable with distribution function F, if

$$F(x) = \int_{-\infty}^{x} f(t)\, dt.$$

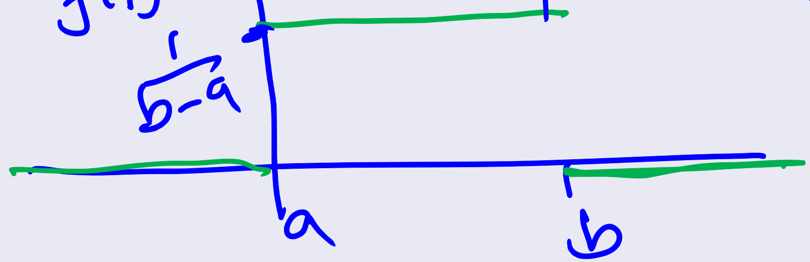
f

Remark

in this case,

1. $\int_{-\infty}^{\infty} f(t)\, dt = 1$ $\qquad \lim_{x \to \infty} F(x)$
2. $P(a < X < b) = \int_a^b f(t)\, dt$
3. If f is continuous at x, F is differentiable at x and $F'(x) = f(x)$. (Fundamental Theorem of Calculus)

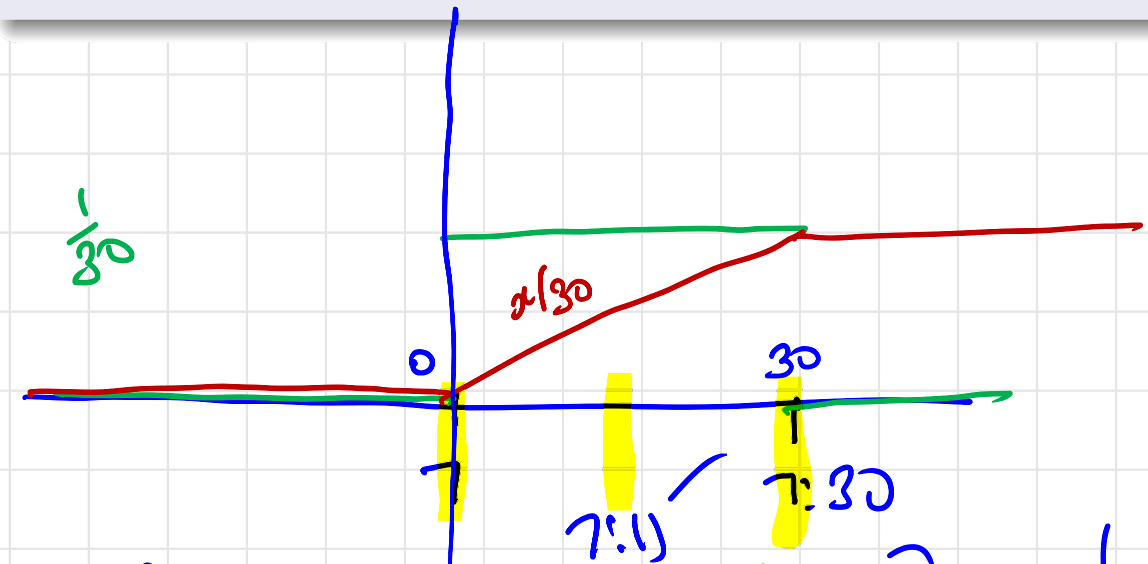
$f(t) = \frac{1}{b-a}$ uniform density



Uniform density

Exercise 1.20

Buses arrive at a specified stop at 15-minute intervals starting at 7 A.M. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7 : 30, find the probability that he waits (a) less than 5 minutes for a bus; (b) at least 12 minutes for a bus.



$f(x)$, p.d.f

$F(x)$ - c.d.f

a) P(passenger waits less than 5 minutes) = P(passenger arrives between 7:10 and 7:15 OR between 7:25 and 7:30)

$= F(7{:}15 - 7{:}10) + F(7{:}30 - 7{:}25)$

$= \int_{10}^{15} + \int_{25}^{30}$

b)